Joints and Doublers

Bill Gran

GRAN Corporation

Joints and Doublers

by

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President, CEO & Super Genius

***GRAN** CORPORATION*

First Edition

***GRAN** CORPORATION*

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Joints and Doublers

GRAN Corporation Whitney, Texas

Legal Department

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Table of Contents

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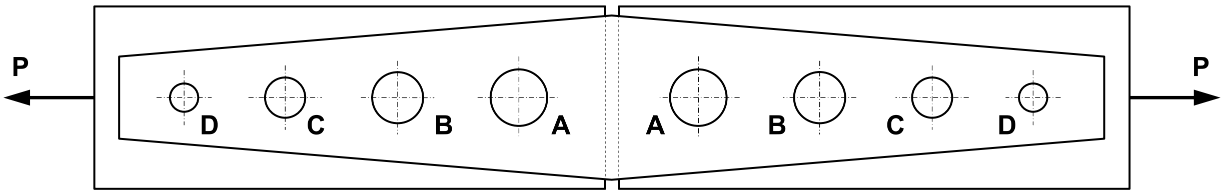
1.0 Bolt Load Distribution

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* page D1.3

P = 100,000 lb



Shear Strength

ANC-5 *Strength of Metal Aircraft Elements* page 32

Table 2.6111(a) Shear and Tensile Strengths … of Steel Bolts and Pins

Bolt	AN Bolt	Diameter	Single Shear	Double Shear
A	AN-10	5 / 8	23,000	46,000 lb
B	AN-9	9 / 16	18,700	37,400 lb
C	AN-7	7 / 16	11,250	22,500 lb
D	AN-5	5 / 16	5,750	11,500 lb
			Σ	117,400 lb

Load Distribution

$$P_a = P \frac{P_{sa}}{\sum P_s} = 100{,}000\ lb \left(\frac{46{,}000\ lb}{117{,}400\ lb} \right) = 39{,}182\ lb$$

$$P_b = P \frac{P_{sb}}{\sum P_s} = 100{,}000\ lb \left(\frac{37{,}400\ lb}{117{,}400\ lb} \right) = 31{,}857\ lb$$

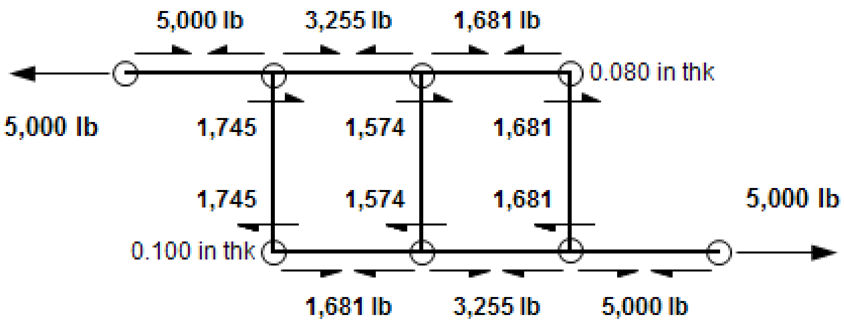
$$P_c = P \frac{P_{sc}}{\sum P_s} = 100{,}000\ lb \left(\frac{22{,}500\ lb}{117{,}400\ lb} \right) = 19{,}165\ lb$$

$$P_d = P \frac{P_{sd}}{\sum P_s} = 100{,}000\ lb \left(\frac{11{,}500\ lb}{117{,}400\ lb} \right) = 9{,}796\ lb$$

Check

P = 39,182 lb + 31,857 lb + 19,165 lb + 9,796 lb = 100,000 lb

2.0 Fastener Flexibility



Pitch = 1.00 inch   0.250 inch diameter steel fasteners

b = 1.00 inch t_1 = 0.080 inch t_2 = 0.100 inch

References

Heimo Huth

Influence of Fastener Flexibility on the Prediction of Load Transfer and Fatigue Life for Multiple-Row Joints

Correct equation is...

$$C = [(t_1 + t_2) / 2d]^a \; (b / n) \; [1 / (t_1 E_1) + 1 / (n t_2 E_2) + 1 / (2 t_1 E_3) + 1 / (2 n t_2 E_3)]$$

"Zum Einflub der Nietnachgiebigkeit mehrreihiger Nietverbindungen auf die Lastübertragungs - und Lebensdauervorhersage"

LBF Report No. FB-172, Dissertation, Technische Universität München, Munich, Germany, 1984.

Thanks to analyst64 on the www.eng-tips.com website.

Manford B. Tate and Samuel J. Rosenfeld

NACA TN-1051   *Preliminary Investigation of the Loads Carried by Individual Bolts in Bolted Joints*

Samuel J. Rosenfeld

ASTM STP 927   *Fatigue in Mechanically Fastened Composite and Metallic Joints*

NACA TN-1458   *Analytical and Experimental Investigation of Bolted Joints*

T. Swift   *Development of the Fail-Safe Design Features of the DC-10*

William F. McCombs, James C. McQueen, Jeffrey L. Perry

AFFDL-TR-67-184   *Analytical Design Methods for Aircraft Structural Joints*

http://www.dtic.mil/

Data for Calculating Joint Spring Constants

William F. McCombs

Engineering Column Analysis - The Analysis of Compression Members Appendix E, page E1

$$k_{o\ joint} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{k_1\ k_2}{k_1 + k_2} \qquad where \quad k_n = A_n\ k_{n\ tabulated}$$

Figure E-1

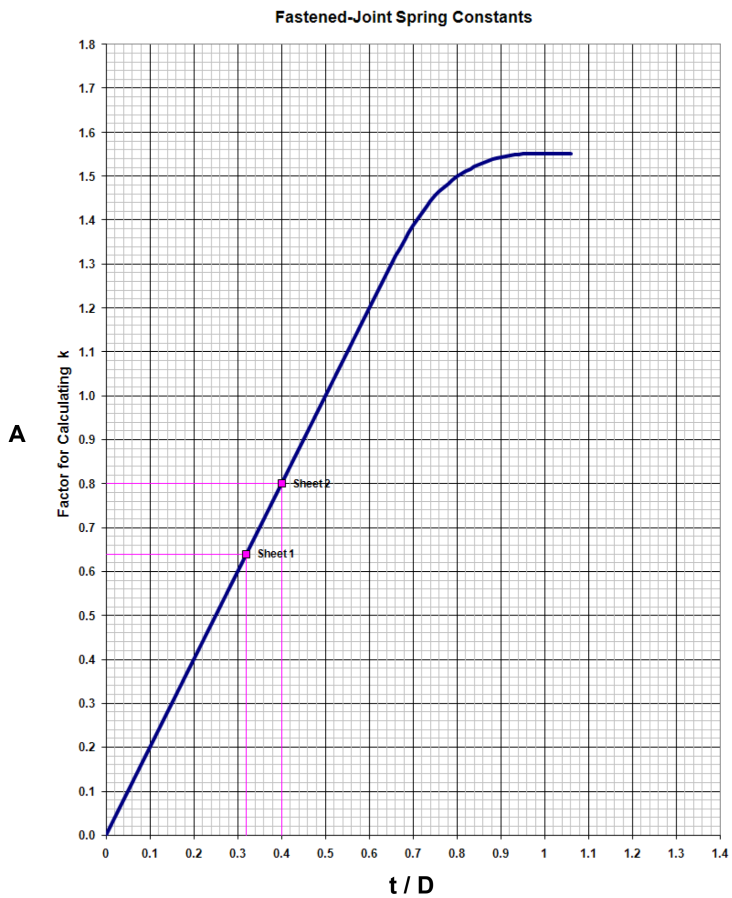


Table E-2

Fsnr Dia	k_n x 10^{-6}		
D	Alum	Steel	Other
0.125	0.212	4.71	0.715
0.156	0.264	5.88	0.892
0.188	0.317	7.07	1.073
0.250	0.423	9.43	1.431
0.313	0.528	11.80	1.790
0.375	0.633	14.20	2.154
0.438	0.732	16.40	2.488
0.540	0.845	18.90	2.867
0.563	0.952	21.20	3.216
0.625	1.060	23.50	3.565

Plate Constants

Samuel J. Rosenfeld

NACA TN-1458 *Analytical and Experimental Investigation of Bolted Joints* page 6

$$K = \frac{p}{b\ t\ E}$$

p *Fastener Pitch*

b *Plate Width*

t *Plate Thickness*

E *Young's Modulus*

$$K_a = K_b = \frac{p}{b\ t\ E} = \frac{1.00\ inch}{1.00\ inch\ (\ 0.080\ inch\ )\ 10.5\ msi} = 1.19\ \mathrm{E}-06\ \ \mathrm{inch}/lb$$

$$K_c = K_d = \frac{p}{b\ t\ E} = \frac{1.00\ inch}{1.00\ inch\ (\ 0.100\ inch\ )\ 10.5\ msi} = 0.92\ \mathrm{E}-06\ \ \mathrm{inch}/lb$$

Fastened-Joint Spring Constants

William F. McCombs

Engineering Column Analysis - The Analysis of Compression Members Appendix E, page E1

t_1 / d = 0.080 inch / 0.25 inch = 0.32 A = 0.64 $k_1' = k_3'$ = 0.64 (423,000) = 270,720 lb / inch

t_2 / d = 0.100 inch / 0.25 inch = 0.40 A = 0.80 k_2' = 0.80 (423,000) = 338,400 lb / inch

Three Equations, Three Unknowns

$$\left(\frac{R_1}{k_1{}'} \right) + R_1 K_a = (P - R_1)\ K_b + \left(\frac{R_2}{k_2{}'} \right)$$

$$\left(\frac{R_2}{k_2{}'} \right) + (R_1 + R_2)\ K_c = (P - R_1 - R_2)\ K_d + \left(\frac{R_3}{k_3{}'} \right)$$

$$R_1 + R_2 + R_3 = \mathrm{P}$$

$$6.075\ R_1 - 2.955\ R_2 = 5{,}952.4$$

$$1.905\ R_1 + 4.860\ R_2 - 3.694\ R_3 = 4{,}761.9$$

$$R_1 + R_2 + R_3 = 5{,}000$$

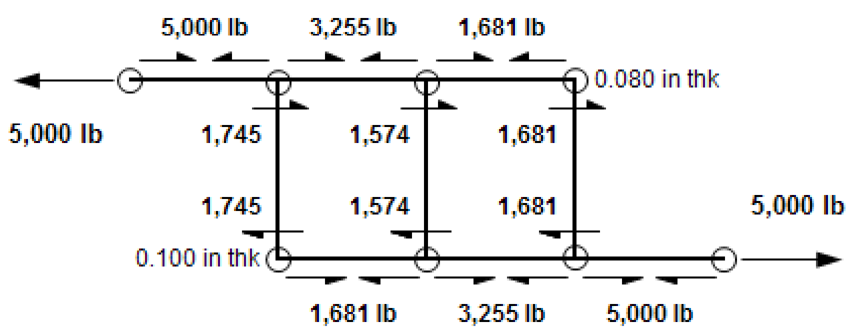
Solve

$$\begin{bmatrix} 6.075 & -2.955 & 0 \\ 1.905 & 4.860 & -3.694 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} 5{,}952 \\ 4{,}762 \\ 5{,}000 \end{Bmatrix}$$

$$\begin{bmatrix} 6.075 & -2.955 & 0 \\ 1.905 & 4.860 & -3.694 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0.125 & 0.043 & 0.159 \\ -0.082 & 0.089 & 0.328 \\ -0.043 & -0.132 & 0.513 \end{bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} 5{,}952 \\ 4{,}762 \\ 5{,}000 \end{Bmatrix} \begin{bmatrix} 0.125 & 0.043 & 0.159 \\ -0.082 & 0.089 & 0.328 \\ -0.043 & -0.132 & 0.513 \end{bmatrix} = \begin{Bmatrix} 1{,}745 \\ 1{,}574 \\ 1{,}681 \end{Bmatrix}$$

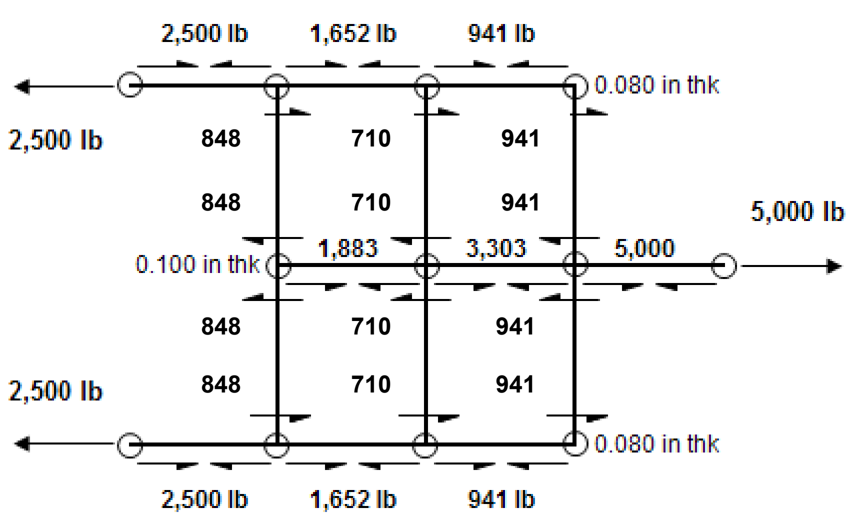
Single Shear



Take this analysis with a block of salt. Compare to 5,000 lb / 3 = 1,667 lb per fastener.

Double Shear

In a similar fashion:



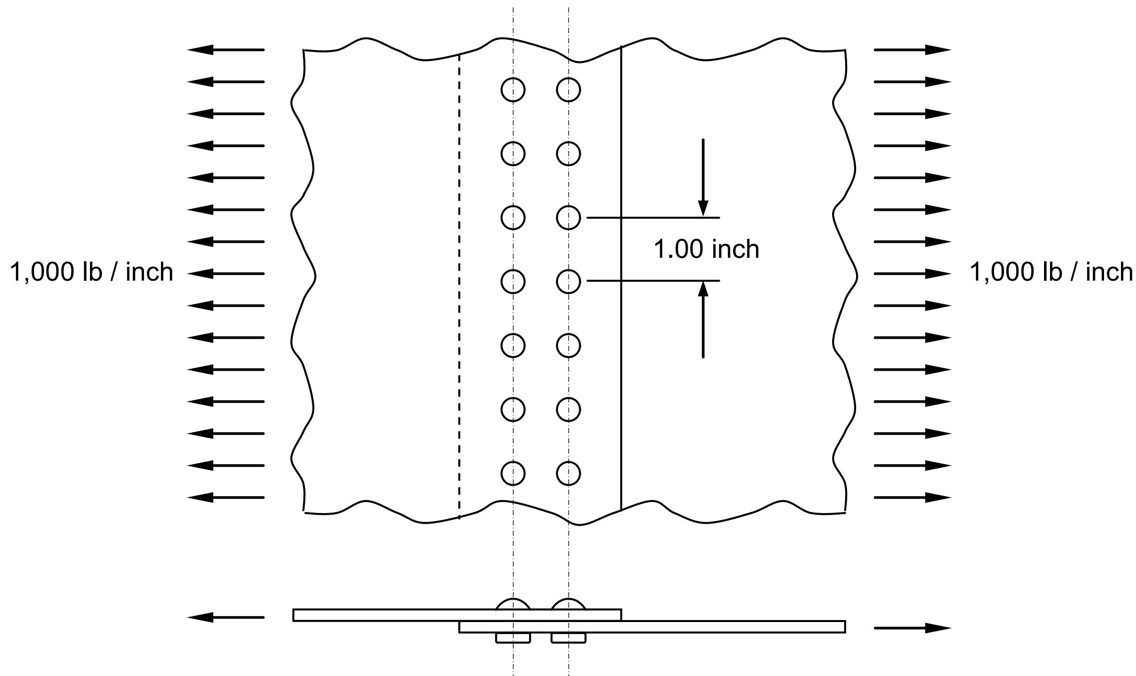
Compare to 5,000 lb / 6 = 833 lb

3.0 Lap Joint

William F. McCombs *Analysis and Design of Flight Vehicle Structures* page D1.24

See page D3.14 Good Practice - Holes in Line Poor Practice - Holes Staggered

Ultimate Load = 869.6 lb / inch (1.15 Fitting Factor) = 1,000 lb / inch



Tension in Sheet

Elmer F. Bruhn *Analysis and Design of Flight Vehicle Structures* Table D1.6, page D1.21

MMPDS-01 *Metallic Materials Properties Development and Standardization* page 8-12

Table 8.1.2(a) Standard Rivet-Hole Drill Sizes and Nominal Hole Diameters

Rivet Size, inch	1/16	3/32	1/8	5/32	3/16	1/4	5/16	3/8
Drill No.	51	41	30	21	11	F	P	W
Nominal Hole Diameter, inch	0.067	0.096	0.1285	0.159	0.191	0.257	0.323	0.386

AD Rivets, Driven AL 2117-T3 5/32 inch diameter Nominal Hole Diameter = 0.159 inch

Ultimate Tension F_{tu} = 60 ksi Clad 2024-T3 0.040 inch thick sheet

Net Area A_{net} = 0.040 inch (1.00 inch - 0.159 inch) = 0.0336 in^2

Tension Allowable $P_{allow} = F_{tu} \, A_{net}$ = 60,000 psi (0.0336 in^2) = 2,018 lb

Sheet Tension Margin of Safety

$$MS = \frac{P_{allow}}{P} - 1 = \frac{2{,}018\ lb}{1{,}000\ lb} - 1 = 1.02$$

Shear

MMPDS-01 *Metallic Materials Properties Development and Standardization* page 8-13

Table 8.1.2(b) Single Shear Strength of Solid Rivets [a]

Undriven			Driven		Rivet Designation	Rivet Size							
Rivet Material	F_{su} (ksi) Min	F_{su} (ksi) Max	Rivet Material	F_{su} (ksi)		1/16	3/32	1/8	**5/32**	3/16	1/4	5/16	3/8
						Driven Single Shear Strength (lbs) [c]							
5056-H32	24	n/a	5056-H321[d]	28[e]	B[f]	99	203	363	556	802	1,450	2,290	3,275
2117-T4	**26**	**n/a**	**2117-T3**	**30[e]**	**AD**	106	217	389	**596**	860	1,555	2,455	3,510
2017-T4	35	42	2017-T3	38[e]	D	134	275	493	755	1,085	1,970	3,115	4,445
2024-T4	37	n/a	2024-T31	41[g]	DD	145	297	532	814	1,175	2,125	3,360	4,795
7050-T73	41	46	7050-T731[d]	43[e]	E[h]	152	311	558	854	1,230	2,230	3,520	5,030
Monel	49	59	Monel	52[e]	M	183	376	674	1,030	1,490	2,695	4,260	6,085
Ti-45Cb	50	59	Ti-45Cb	53[e]	T	187	384	687	1,050	1,515	2,745	4,340	6,200
A-286	85	95	A-286 90[e]	90[e]	-	317	651	1,165	1,785	2,575	4,665	7,375	10,500

MMPDS-01 *Metallic Materials Properties Development and Standardization* page 8-15

Table 8.1.2.1(b) Shear Strength Correction Factors for Solid Protruding Head Rivets [a]

Thick (inch)	Fastener Diameter (inch)							
	1/16	3/32	1/8	**5/32**	3/16	1/4	5/16	3/8
	Single-Shear Rivet Strength Factors							
0.016	0.964	...	...	...	...	...	...	...
0.018	0.981	0.912	...	...	...	...	...	...
0.020	0.995	0.933	...	...	...	...	...	...
0.025	1.000	0.970	0.920	...	...	...	...	...
0.032	...	1.000	0.964	0.925	...	...	...	...
0.036	...	...	0.981	0.946	0.912	...	...	...
0.040	...	...	0.995	**0.964**	0.933	...	...	...
0.045	...	...	1.000	0.981	0.953	...	...	...
0.050	...	...	...	0.995	0.970	0.920	...	...
0.063	...	...	...	1.000	1.000	0.961	0.922	...
0.071	...	...	...	...	...	0.979	0.944	0.909
0.080	...	...	...	...	...	0.995	0.964	0.933
0.090	...	...	...	...	...	1.000	0.981	0.953
0.100	...	...	...	...	...	...	0.995	0.972
0.125	...	...	...	...	...	...	1.000	1.000

[a] Sheet thickness is that of the thinnest sheet in single-shear joints and the middle sheet in double-shear joints.

MMPDS-01 Section 8.1.2.1 Protruding-Head Solid Rivet Joints page 8-11

In computing protruding-head rivet design shear strengths, the shear strength values obtained from Table 8.1.2(b) should be multiplied by the correction factors given in Table 8.1.2.1(b). This compensates for the reduction in rivet shear strength resulting from high bearing stresses on the rivet at t/D ratios less than 0.33 for single-shear joints and 0.67 for double-shear joints.

Single Shear Strength

$P_{allow} = P_s\ C$ = 596 lb per rivet (0.964) 2 rivets = 1,149 lb

Single Shear Margin of Safety

$$MS = \frac{P_{allow}}{P} - 1 = \frac{1{,}149\ lb}{1{,}000\ lb} - 1 = 0.15$$

Bearing

Unit Bearing Strength

MMPDS-01 *Metallic Materials Properties Development and Standardization* page 8-14

Table 8.1.2.1(a) Unit Bearing Strength of Sheet on Rivets F_{br} = 100 ksi

Sheet Thickness (inch)	Fastener Diameter (inch)							
	1/16	3/32	1/8	5/32	3/16	1/4	5/16	3/8
	Unit Bearing Strength for Indicated Rivet Diameter (lbs)							
0.012	80	...	...	...	...	...	...	...
0.016	107	...	...	...	...	...	...	...
0.018	121	173	...	...	...	...	...	...
0.020	134	192	...	...	...	...	...	...
0.025	168	240	321	...	...	...	...	...
0.032	214	307	411	509	...	...	...	...
0.036	241	346	462	572	688	...	...	...
0.040	268	384	514	**636**	764	...	...	...
0.045	302	432	578	716	860	...	...	...
0.050	335	480	642	795	955	1,285	...	...
0.063	422	605	810	1,002	1,203	1,619	2,035	...
0.071	476	682	912	1,129	1,356	1,825	2,293	2,741
0.080	536	768	1,028	1,272	1,528	2,056	2,584	3,088
0.090	603	864	1,156	1,431	1,719	2,313	2,907	3,474
0.100	670	960	1,285	1,590	1,910	2,570	3,230	3,860
0.125	838	1,200	1,606	1,988	2,388	3,212	4,038	4,825
0.160	1,072	1,536	2,056	2,544	3,056	4,112	5,168	6,176
0.190	1,273	1,824	2,442	3,021	3,629	4,883	6,137	7,334
0.250	1,670	2,400	3,210	3,975	4,775	6,425	8,075	9,650

Bearing Factor

William F. McCombs *Analysis and Design of Flight Vehicle Structures* Table D1.8, page D1.22

ANC-5 ***Strength of Metal Aircraft Elements***

Table 3.6111(b) Aluminum Alloy and Plate – Bearing Factors, page 84

Clad 2024-T3 0.040 inch thick A Values Edge Margin = 5 / 16 inch e / D = 2

K = 1.14 Ratio of actual bearing strength of sheet to F_{br} = 100 ksi

Bearing Strength

From Table 8.1.2.1(a) Unit Bearing Strength P_{br} = 636 lb

$P_{allow} = P_{br}\ K$ = 636 lb per rivet (1.14) 2 rivets = 1,450 lb

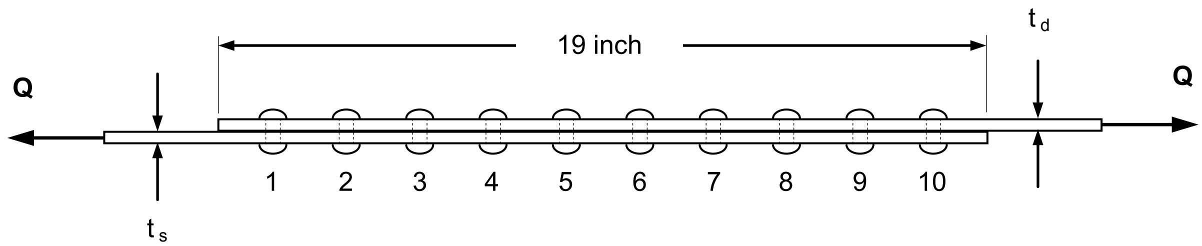
Bearing Margin of Safety

$$MS = \frac{P_{allow}}{P} - 1 = \frac{1{,}450\ lb}{1{,}000\ lb} - 1 = 0.45$$

4.0 Splice

William F. McCombs, James C. McQueen, Jeffrey L. Perry

AFFDL-TR-67-184 *Analytical Design Methods for Aircraft Structural Joints* pages 28 - 30



Given

Q = 8,000 lb	t_s = 0.20 inch	t_d = 0.20 inch	Pitch, p = 1.90 inch
Width = 1.84 inch	Steel, E = 29 msi	D = 0.375 inch	k_F = 800,000 lb / inch

Data

Net Effective Area A_s = (Width - 0.80 D) t_s Note: cos 40° = 0.766

A_s = [1.84 inch - 0.80 (0.375 inch)] 0.20 inch = 0.308 in^2

A_d = [1.84 inch - 0.80 (0.375 inch)] 0.20 inch = 0.308 in^2

$$k = \frac{k_F}{p} = = \frac{800{,}000\ lb/inch}{1.90\ inch} = 421{,}053\ \ lb/inch\ per\ inch$$

$$N = \frac{k\ Q}{A_s\ E} = = \frac{421{,}052.6\ \frac{lb}{in}\ per\ inch\ \ (8{,}000\ lb)}{0.308\ in^2\ \ (29\ E06\ psi)} = 377.118$$

$$M = k\ \left(\frac{1}{A_s\ E_s} + \frac{1}{A_d\ E_d}\right)$$

$$M = 421{,}052.6\ \frac{lb}{in}\ per\ inch\ \left[\frac{1}{0.308\ in^2\ \ (29\ E06\ psi)} + \frac{1}{0.308\ in^2\ \ (29\ E06\ psi)}\right] = 0.09428$$

$$C_1 = \frac{Q - \frac{N}{M}\left(1 - e^{-\sqrt{M}\,L}\right)}{e^{\sqrt{M}\,L} - e^{-\sqrt{M}\,L}}$$

$$C_1 = \frac{8{,}000\ lb - \frac{377.118}{0.09428}\left(1 - e^{-19\sqrt{0.09428}}\right)}{e^{19\sqrt{0.09428}} - e^{-19\sqrt{0.09428}}} = \frac{8{,}000 - 3{,}988.3}{341.709 - 0.003} = 11.740$$

$$P_n = C_1\left(e^{\sqrt{M}\,x} - e^{-\sqrt{M}\,x}\right) + \frac{N}{M}\left(1 - e^{-\sqrt{M}\,x}\right)$$

Fastener Loads

$$P_n = C_1 \left(e^{\sqrt{M}\,x} - e^{-\sqrt{M}\,x} \right) + \frac{N}{M} \left(1 - e^{-\sqrt{M}\,x} \right)$$

$$P_1 = 11.740 \left(e^{1.9\sqrt{0.09428}} - e^{-1.9\sqrt{0.09428}} \right) + \frac{377.118}{0.09428} \left(1 - e^{-1.9\sqrt{0.09428}} \right)$$

$$P_1 = P_{F1} = 1{,}782.5 \ lb$$

$$P_2 = 11.740 \left(e^{3.8\sqrt{0.09428}} - e^{-3.8\sqrt{0.09428}} \right) + \frac{377.118}{0.09428} \left(1 - e^{-3.8\sqrt{0.09428}} \right)$$

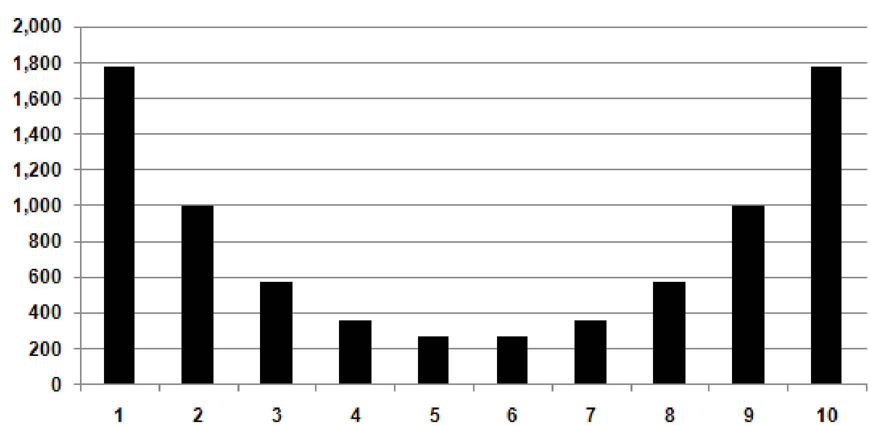
$$P_2 = 2{,}788.6 \ lb \qquad P_{F2} = 2{,}7886.6 \ lb - 1{,}782.5 \ lb = 1{,}006.1 \ lb$$

$$P_3 = 11.740 \left(e^{5.7\sqrt{0.09428}} - e^{-5.7\sqrt{0.09428}} \right) + \frac{377.118}{0.09428} \left(1 - e^{-5.7\sqrt{0.09428}} \right)$$

Table

Fastener	x	P_n	P_F	%
n	inch	lb	lb	
1	1.9	1,782.5	1,782	22.3%
2	3.8	2,788.6	1,006	12.6%
3	5.7	3,370.6	582	7.3%
4	7.6	3,732.2	362	4.5%
5	9.5	4,000.0	268	3.3%
6	11.4	4,267.8	268	3.3%
7	13.3	4,629.4	362	4.5%
8	15.2	5,211.4	582	7.3%
9	17.1	6,217.5	1,006	12.6%
10	19.0	8,000.0	1,782	22.3%

Load Distribution Chart



Load in Center Segment

$$P = 11.740 \left(e^{9.5\sqrt{0.09428}} - e^{-9.5\sqrt{0.09428}} \right) + \frac{377.118}{0.09428} \left(1 - e^{-9.5\sqrt{0.09428}} \right)$$

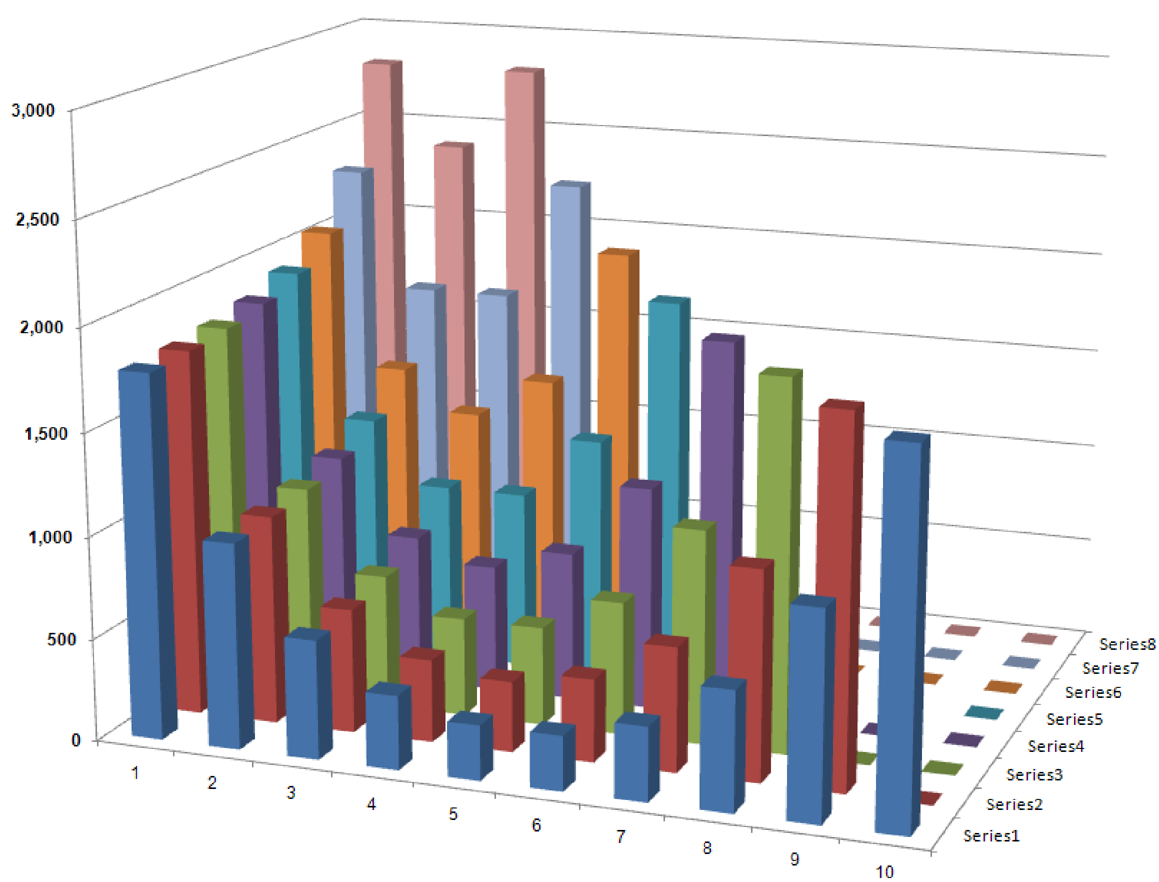
$$P = 4{,}000 \ lb$$

Compare Splices with Three to Ten Fasteners

Table

Series	1	2	3	4	5	6	7	8
L	19	17.1	15.2	13.3	11.4	9.5	7.6	5.7
C_1	14.675	26.361	47.441	85.663	155.63	285.96	536.78	1,051.4
n	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**
Fasteners	P_F	P_F	P_F	P_F	P_F	P_F	P_F	P_F
	lb	lb	lb	lb	lb	lb	lb	lb
1	1,782	1,794	1,815	1,853	1,922	2,050	2,298	2,806
2	1,006	1,022	1,050	1,101	1,194	1,368	1,702	2,388
3	582	607	652	734	884	1,164	1,702	2,806
4	362	405	483	625	884	1,368	2,298	
5	268	345	483	734	1,194	2,050		
6	268	405	652	1,101	1,922			
7	362	607	1,050	1,853				
8	582	1,022	1,815					
9	1,006	1,794						
10	1,782							

Column Chart



Numerical Method

William F. McCombs, James C. McQueen, Jeffrey L. Perry

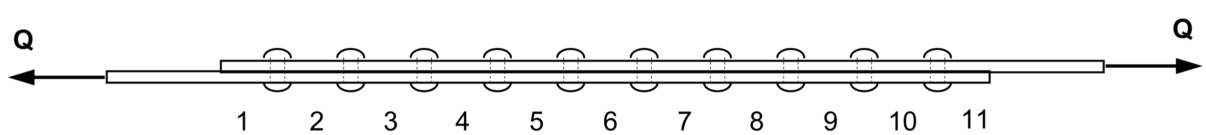
AFFDL-TR-67-184 *Analytical Design Methods for Aircraft Structural Joints* pages 39 to 43

$$k_d = \frac{A_d \; E_d}{p} = \frac{0.308 \; in^2 \; (\, 29 \text{ E06 psi} \,)}{1.90 \; inch} = 470{,}105 \;\; lb/inch \; per \; inch$$

$$k_s = \frac{A_s \; E_s}{p} = \frac{0.308 \; in^2 \; (\, 29 \text{ E06 psi} \,)}{1.90 \; inch} = 470{,}105 \;\; lb/inch \; per \; inch$$

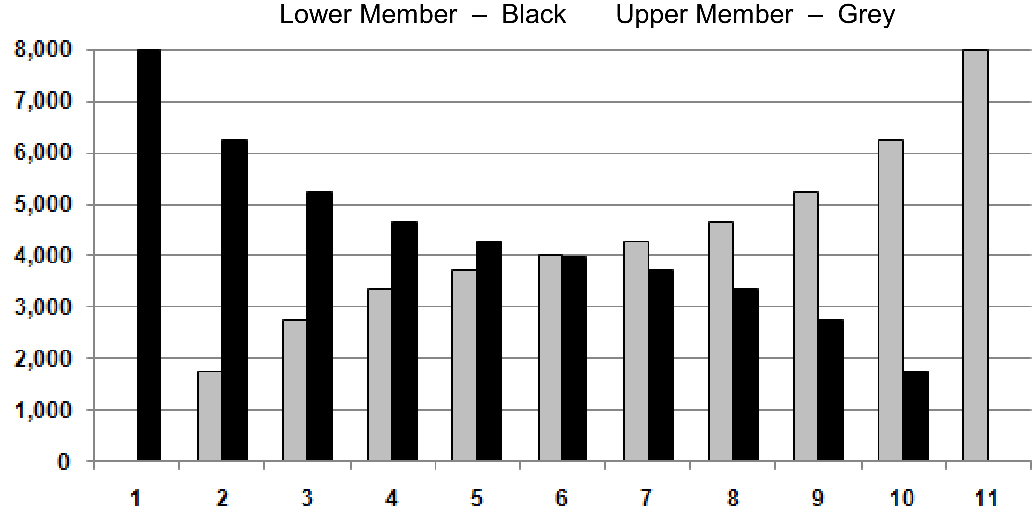
Table

1	2	4	5	6	7	8	9	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Member Load P_D	Spring Constant k_D	Member Strain $\Delta\delta_D$	Applied Load $q_L + (11)$	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_L + (11)$	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	2,207	2,207	0.80	**1,766**	1,766	4.70	376	8,000	6,234	4.70	1,326	951
2	1,256	1,256	0.80	1,005	2,771	4.70	589	8,000	5,229	4.70	1,112	523
3	733	733	0.80	587	3,357	4.70	714	8,000	4,643	4.70	988	273
4	460	460	0.80	368	3,725	4.70	792	8,000	4,275	4.70	909	117
5	343	343	0.80	275	4,000	4.70	851	8,000	4,000	4.70	851	0
6	343	343	0.80	275	4,275	4.70	909	8,000	3,725	4.70	792	-117
7	460	460	0.80	368	4,643	4.70	988	8,000	3,357	4.70	714	-273
8	733	733	0.80	587	5,229	4.70	1,112	8,000	2,771	4.70	589	-523
9	1,256	1,256	0.80	1,005	6,234	4.70	1,326	8,000	1,766	4.70	376	-951
10	2,207	2,207	0.80	1,766	**8,000**							



Load Distribution Chart

Lower Member – Black Upper Member – Grey



Oversize Hole

William F. McCombs, James C. McQueen, Jeffrey L. Perry

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Assume the first fastener hole is “sloppy”. Without a 0.003 inch oversize hole …

$$P_{F1} = \frac{Q}{n/4}\left(\frac{A_d\,E_d}{A_d\,E_d + A_s\,E_s}\right)$$

$$P_{F1} = \frac{Q}{10/4}\left[\frac{0.308\ in^2\ (29\text{ E06 psi})}{0.308\ in^2\ (29\text{ E06 psi}) + 0.308\ in^2\ (29\text{ E06 psi})}\right] = 1{,}600\ lb$$

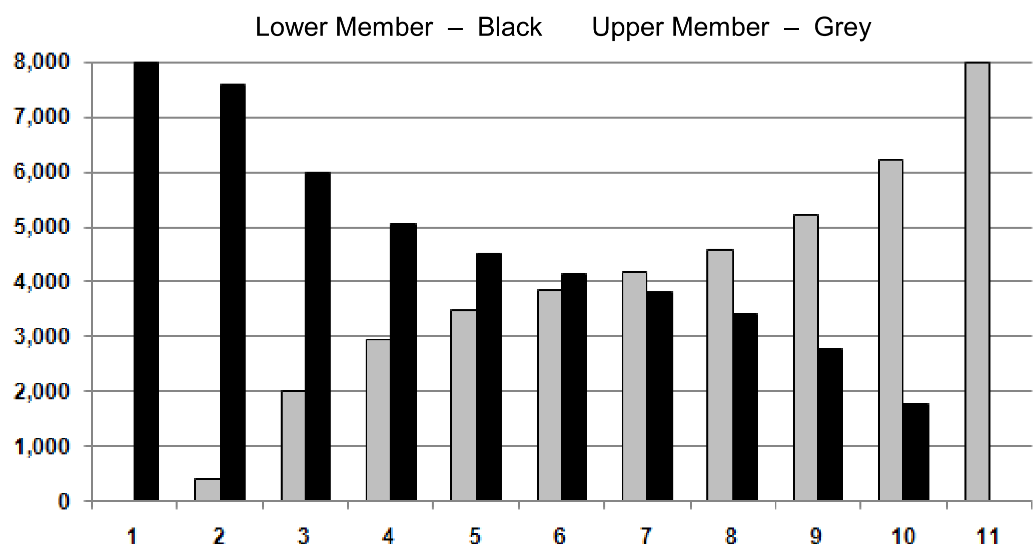
Assume that the first fastener takes half of the 1,600 lb for the first iteration in column six.

Use a trial and error method changing the load in the first fastener (Column 6) until the load in the far end of the upper member (Column 7) is equal to Q, in this case 8,000 pounds.

Table

1	2	3	4	5	6	7	8	9	12	13	14	15	16
Fastener Number	**Strain Delta** $\Delta(\delta_s - \delta_d)$	**Fastener "Slop"** k_F	**Fastener Strain** $\Delta\delta_F$	**Spring Constant** k_F	**Fastener Load** P_F	**Member Load** P_D	**Spring Constant** k_D	**Member Strain** $\Delta\delta_D$	**Applied Load** q_L + (11)	**Load in Base** P_S	**Spring Constant** k_S	**Base Strain** δ_S	**Strain Delta** $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	µ inch	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	3,520	3,000	520	0.80	**415.3**	415	4.70	88	8,000	7,585	4.70	1,613	1,525
2	1,995	0	1,995	0.80	1,596	2,011	4.70	428	8,000	5,989	4.70	1,274	846
3	1,149	0	1,149	0.80	919	2,930	4.70	623	8,000	5,070	4.70	1,078	455
4	694	0	694	0.80	555	3,485	4.70	741	8,000	4,515	4.70	960	219
5	475	0	475	0.80	380	3,865	4.70	822	8,000	4,135	4.70	880	57
6	418	0	418	0.80	334	4,199	4.70	893	8,000	3,801	4.70	808	-85
7	502	0	502	0.80	402	4,601	4.70	979	8,000	3,399	4.70	723	-256
8	758	0	758	0.80	606	5,208	4.70	1,108	8,000	2,792	4.70	594	-514
9	1,272	0	1,272	0.80	1,017	6,225	4.70	1,324	8,000	1,775	4.70	378	-947
10	2,218	0	2,218	0.80	1,775	**8,000**							

Load Distribution Chart



5.0 Doublers

Doubler

William F. McCombs, James C. McQueen, Jeffrey L. Perry

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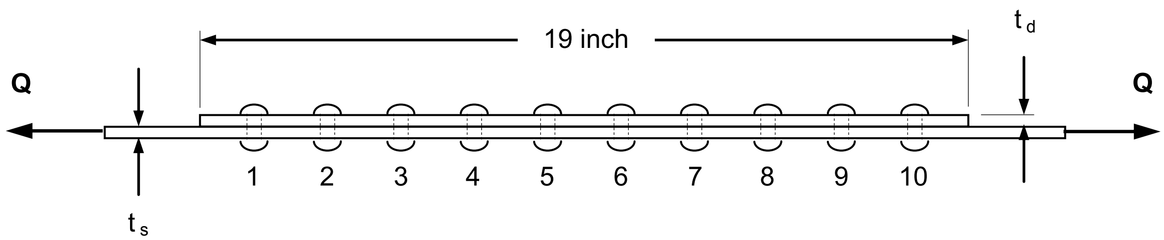


Given

Q = 8,000 lb	t_s = 0.200 inch	t_d = 0.200 inch	Pitch, p = 1.90 inch
Width = 1.84 inch	Steel, E = 29 msi	D = 0.375 inch	k_F = 800,000 lb / inch

Data

Net Effective Area A_s = (Width - 0.80 D) t_s Note: cos 40° = 0.766

A_s = [1.84 inch - 0.80 (0.375 inch)] 0.20 inch = 0.308 in^2

A_d = [1.84 inch - 0.80 (0.375 inch)] 0.20 inch = 0.308 in^2

$$k \;=\; \frac{k_F}{p} \;=\;=\; \frac{800{,}000\ lb/inch}{1.90\ inch} \;=\; 421{,}052.6\ \frac{lb}{in}\ per\ inch$$

$$N \;=\; \frac{k\ \ Q}{A_s\ \ E} \;=\;=\; \frac{421{,}052.6\ \ lb/inch\ \ per\ inch\ \ (\ 8{,}000\ lb\ )}{0.308\ in^2\ \ (\ 29\ E06\ psi\ )} \;=\; 377.118$$

$$M \;=\; k\ \left(\frac{1}{A_s\ \ E_s} + \frac{1}{A_d\ \ E_d}\right)$$

$$M \;=\; 421{,}052.6\ \ lb/inch\ per\ inch\ \left[\ \frac{1}{0.308\ in^2\ \ (\ 29\ E06\ psi\ )} + \frac{1}{0.308\ in^2\ \ (\ 29\ E06\ psi\ )}\ \right] \;=\; 0.09428$$

$$C_1 \;=\; \frac{Q \;-\; \dfrac{N}{M}\left(1 \;-\; e^{\,-\sqrt{M}\,L}\right)}{e^{\,\sqrt{M}\,L} \;-\; e^{\,-\sqrt{M}\,L}}$$

$$C_1 \;=\; \frac{8{,}000\ lb - \dfrac{377.118}{0.09428}\left(1 - e^{\,-19\ \sqrt{0.09428}}\ \right)}{e^{\,19\ \sqrt{0.09428}} \;-\; e^{\,-19\ \sqrt{0.09428}}} \;=\; \frac{8{,}000 -\ 3{,}988.3}{341.709 -\ 0.003} \;=\; 11.740$$

$$P_n \;\cong\; \frac{N}{M}\ \left(1 \;-\; e^{\,-\sqrt{M}\,x}\ \right)$$

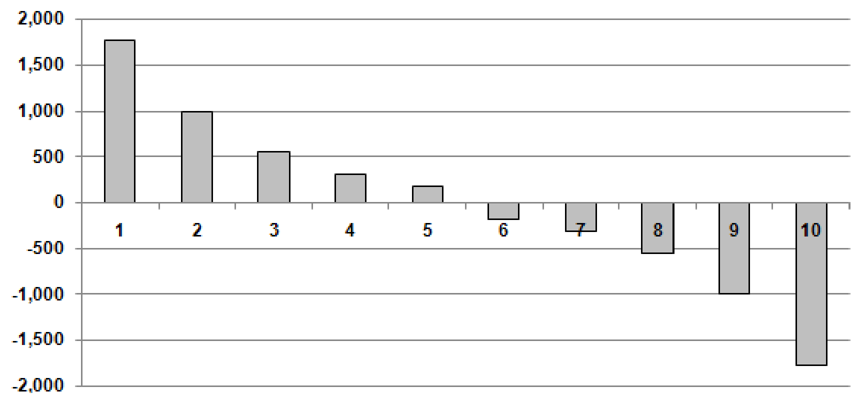
End Fastener Load

$$P_1 \cong \frac{N}{M}\left(1 - e^{-\sqrt{M}\,x}\right) \cong \frac{377.118}{0.09428}\left(1 - e^{-1.9\sqrt{0.09428}}\right) = P_{F1} = 1{,}768\ lb$$

Table

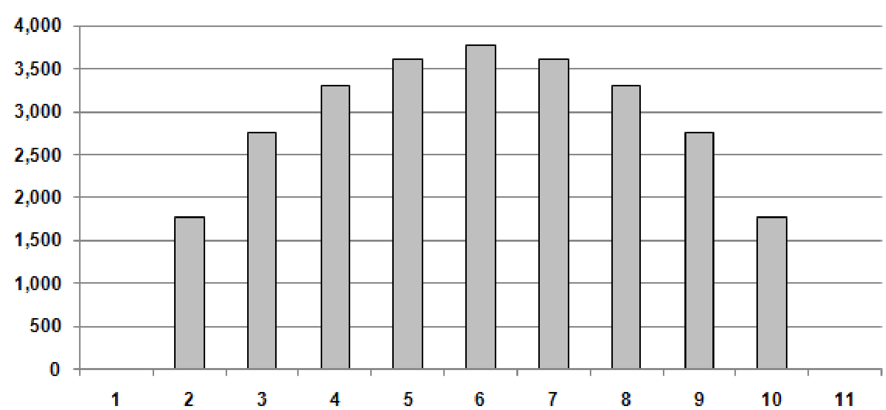
Fastener	x	P_n	P_F	%
n	inch	lb	lb	
1	1.9	1,768.0	1,768	22.1%
2	3.8	2,754.5	987	12.3%
3	5.7	3,305.0	550	6.9%
4	7.6	3,612.2	307	3.8%
5	9.5	3,783.6	171	2.1%
6	11.4	3,783.6	-171	2.1%
7	13.3	3,612.2	-307	3.8%
8	15.2	3,305.0	-550	6.9%
9	17.1	2,754.5	-987	12.3%
10	19.0	1,768.0	-1,768	22.1%

Fastener Loads



Center of Doubler

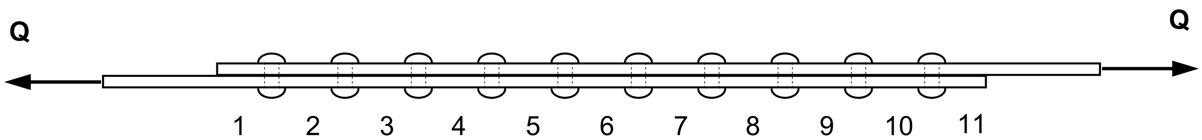
$$P_5 \cong \frac{N}{M}\left(1 - e^{-\sqrt{M}\,x}\right) \cong \frac{377.118}{0.09428}\left(1 - e^{-9.5\sqrt{0.09428}}\right) = 3{,}783.6\ lb$$

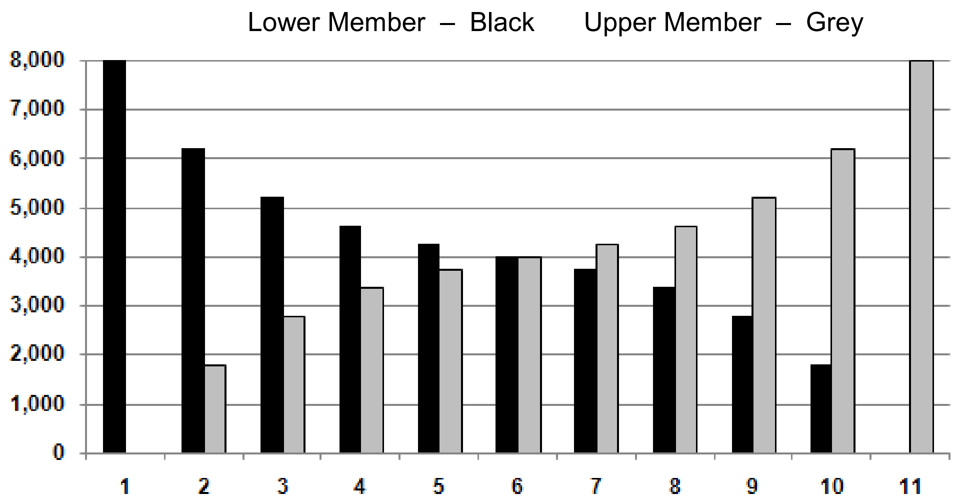


Splice and Doubler Comparison

Splice Load Distribution

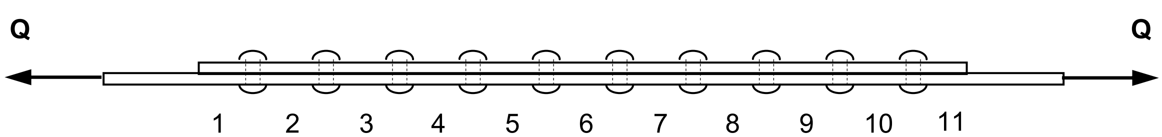
A splice's function is to transfer a given load. It is kept as short as possible in accomplishing this. Page 42

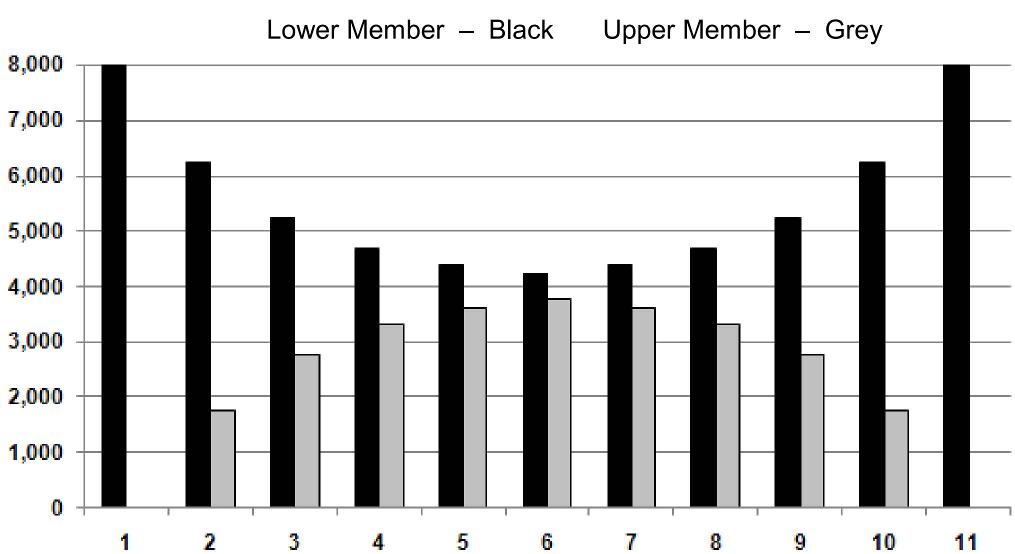




Doubler Load Distribution

A doubler's function is to pick up load (and relieve another member). In order to do this efficiently it must have some considerable length, although this is kept to a minimum. Therefore doublers are, by nature, relatively long members compared to splices. Page 42





Numerical Method I

William F. McCombs, James C. McQueen, Jeffrey L. Perry

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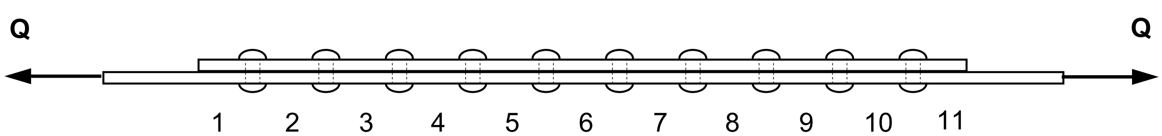
$$k_d = \frac{A_d \; E_d}{p} = \frac{0.308 \; in^2 \; (29 \text{ E06 psi})}{1.90 \; inch} = 470{,}105 \;\; lb/inch \; per \; inch$$

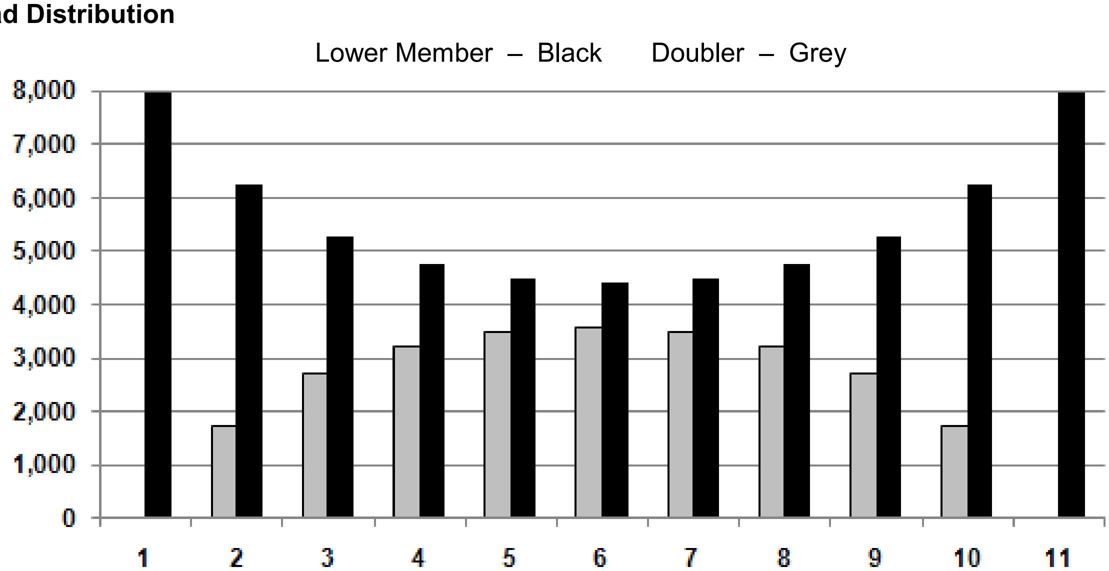
$$k_s = \frac{A_s \; E_s}{p} = \frac{0.308 \; in^2 \; (29 \text{ E06 psi})}{1.90 \; inch} = 470{,}105 \;\; lb/inch \; per \; inch$$

Table

1	2	4	5	6	7	8	9	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_S - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Applied Load $q_L + (11)$	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_L + (11)$	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	2,169	2,169	0.80	**1,735**	1,735	4.70	369	8,000	6,266	4.70	1,333	964
2	1,205	1,205	0.80	964	2,698	4.70	574	8,000	5,302	4.70	1,128	554
3	651	651	0.80	521	3,219	4.70	685	8,000	4,781	4.70	1,017	332
4	319	319	0.80	255	3,475	4.70	739	8,000	4,525	4.70	963	224
5	95.5	95.5	0.80	76	3,551	4.70	755	8,000	4,449	4.70	946	191
6	-95.5	-95.5	0.80	-76	3,475	4.70	739	8,000	4,525	4.70	963	224
7	-319	-319	0.80	-255	3,219	4.70	685	8,000	4,781	4.70	1,017	332
8	-651	-651	0.80	-521	2,699	4.70	574	8,000	5,301	4.70	1,128	554
9	-1,205	-1,205	0.80	-964	1,735	4.70	369	8,000	6,265	4.70	1,333	964
10	-2,169	-2,169	0.80	-1,735	**0**	4.70		8,000		4.70		



Load Distribution



Numerical Method II

William F. McCombs, James C. McQueen, Jeffrey L. Perry

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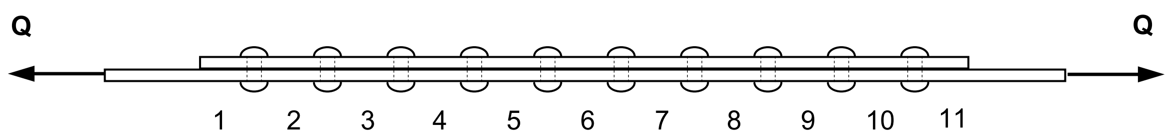
Using the approximate stiffness equation:

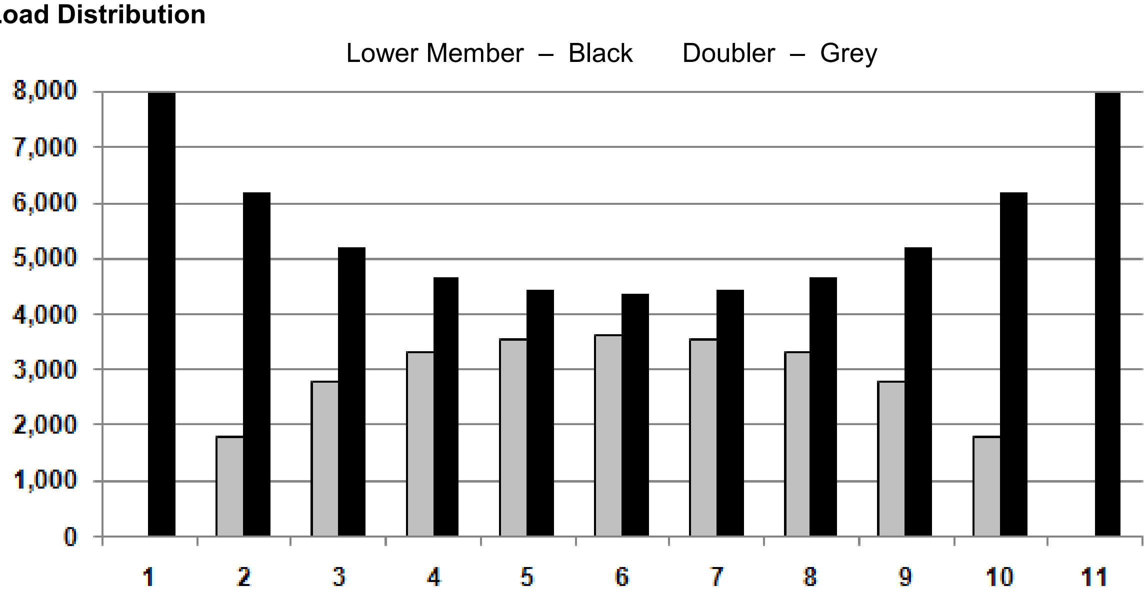
$$k = \frac{k_F}{p} = \frac{800{,}000\ lb/inch}{1.90\ inch} = 421{,}053\ \ lb/inch\ per\ inch$$

Table

1	2	4	5	6	7	8	9	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Applied Load $q_L + (11)$	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_L + (11)$	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	2,260	2,260	0.8	**1,809**	1,809	4.21	430	8,000	6,191	4.21	1,470	1,041
2	1,219	1,219	0.8	975	2,784	4.21	661	8,000	5,216	4.21	1,239	578
3	642	642	0.8	513	3,297	4.21	783	8,000	4,703	4.21	1,117	334
4	308	308	0.8	246	3,544	4.21	842	8,000	4,456	4.21	1,058	217
5	91.1	91.1	0.8	73	3,616	4.21	859	8,000	4,384	4.21	1,041	182
6	-91.1	-91.1	0.8	-73	3,544	4.21	842	8,000	4,456	4.21	1,058	217
7	-308	-308	0.8	-246	3,297	4.21	783	8,000	4,703	4.21	1,117	334
8	-642	-642	0.8	-513	2,784	4.21	661	8,000	5,216	4.21	1,239	578
9	-1,219	-1,219	0.8	-976	1,808	4.21	429	8,000	6,192	4.21	1,471	1,041
10	-2,260	-2,260	0.8	-1,808	**0**	4.21		8,000		4.21		



Load Distribution



Oversize Hole

William F. McCombs, James C. McQueen, Jeffrey L. Perry

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Assume the first fastener hole is "sloppy". Without a 0.003 inch oversize hole …

$$P_{F1} = \frac{Q}{n\,/\,4}\left(\frac{A_d\;E_d}{A_d\;E_d\;+\;A_s\;E_s}\right)$$

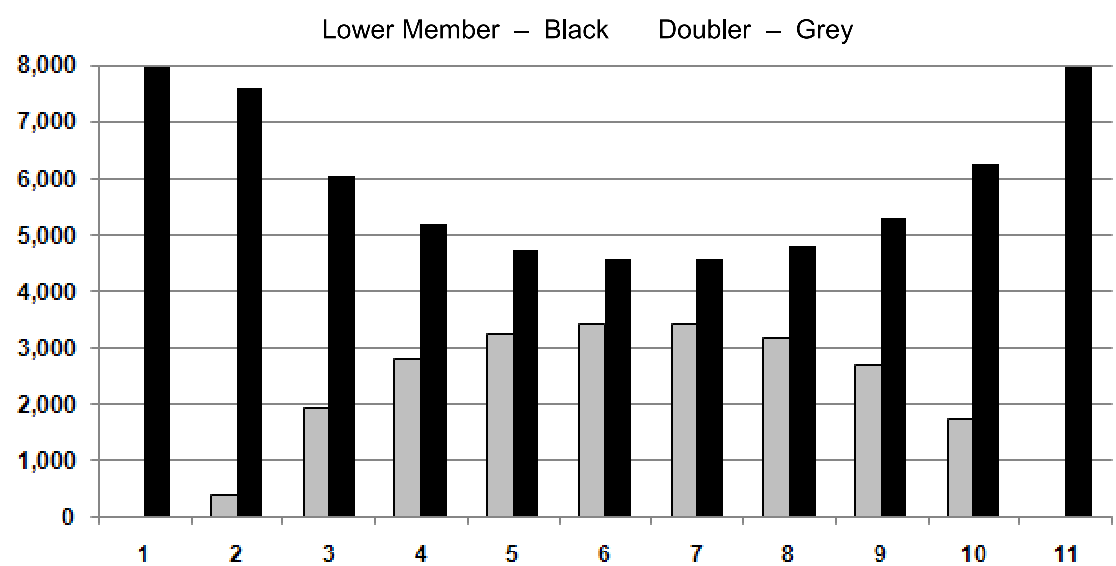
$$P_{F1} = \frac{Q}{10\,/\,4}\left[\frac{0.308\;in^2\;\;(\,29\text{ E06 psi}\,)}{0.308\;in^2\;\;(\,29\text{ E06 psi}\,)\;+\;\;0.308\;in^2\;\;(\,29\text{ E06 psi}\,)}\right] = 1{,}600\;lb$$

Assume that the first fastener takes half of the 1,600 lb for the first iteration. Using a trial and error method change the load in the first fastener (until the load in the far end of the doubler (Column 7) is zero.

Table

1	2	3	4	5	6	7	8	9	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_S - \delta_d)$	Fastener "Slop" k_F	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	µ inch	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	3,482	3,000	482	0.8	**384**	384	4.70	82	8,000	7,616	4.70	1,620	1,538
2	1,944	0	1,944	0.8	1,555	1,939	4.70	412	8,000	6,061	4.70	1,289	877
3	1,067	0	1,067	0.8	853	2,792	4.70	594	8,000	5,208	4.70	1,108	514
4	553	0	553	0.8	442	3,234	4.70	688	8,000	4,766	4.70	1,014	326
5	227	0	227	0.8	182	3,416	4.70	727	8,000	4,584	4.70	975	248
6	-21	0	-21	0.8	-17	3,399	4.70	723	8,000	4,601	4.70	979	256
7	-277	0	-277	0.8	-221	3,178	4.70	676	8,000	4,822	4.70	1,026	350
8	-626	0	-626	0.8	-501	2,677	4.70	569	8,000	5,323	4.70	1,132	563
9	-1,189	0	-1,189	0.8	-951	1,725	4.70	367	8,000	6,275	4.70	1,335	968
10	-2,157	0	-2,157	0.8	-1,726	**0**	4.70		8,000		4.70		

Load Distribution



Loads in the Plastic Range

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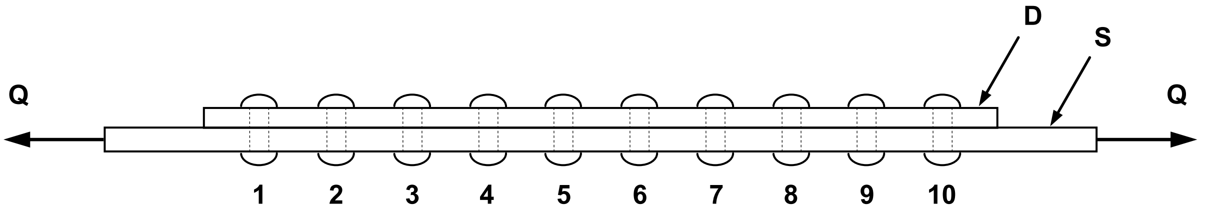
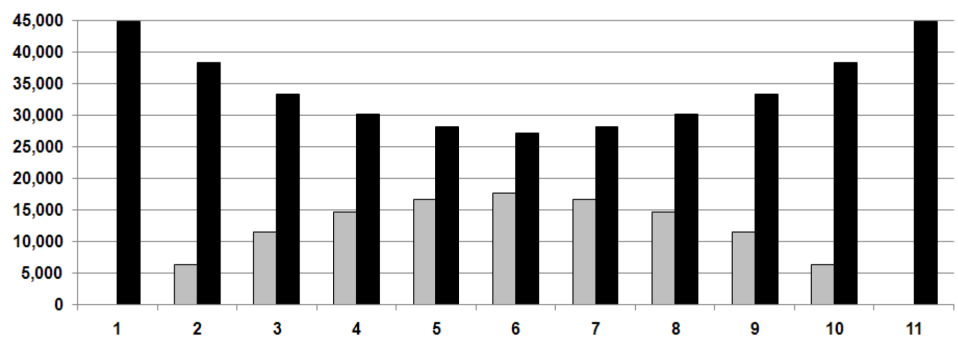
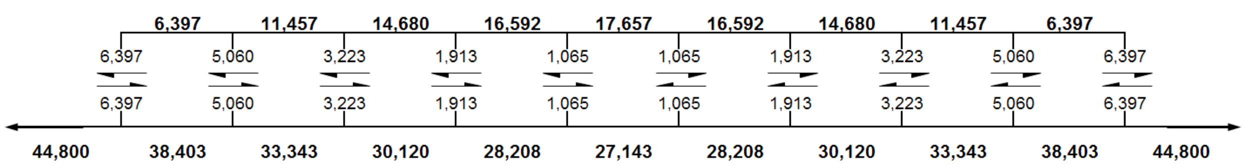


Q = 44,800 lb Pitch = 1.00 inch K_S = 2,360,000 lb / inch K_D = 1,920,000 lb / inch

Load Distribution



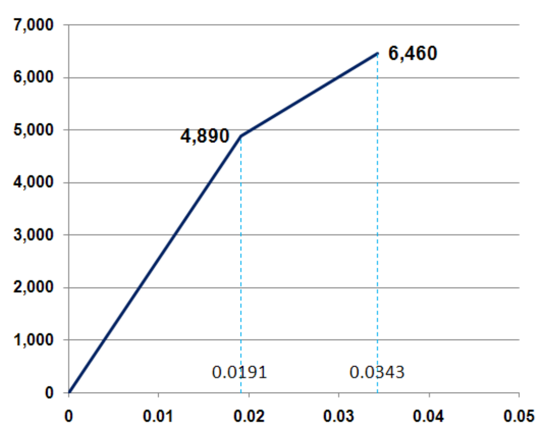
Free Body Diagram



Fastener Load versus Joint Deflection

K_{FA} = 4,889.6 lb / 0.0191 inch = 256,000 lb / inch

K_{FB} = (6459.8 lb - 4,889.6 lb) / (0.0343 inch - 0.0191 inch) = 103,300 lb / inch



First Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	**Strain Delta** $\Delta(\delta_s - \delta_d)$	**Fastener Strain** $\Delta\delta_F$	**Spring Constant** k_F	**Fastener Load** P_F	**Doubler Load** P_D	**Spring Constant** k_D	**Doubler Strain** $\Delta\delta_D$	**Intermed Loads** $q_a L_n$	**Intermed Loads** Sum (10)	**Applied Load** q_L + (11)	**Load in Base** P_S	**Spring Constant** k_S	**Base Strain** δ_S	**Strain Delta** $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	30,121	30,121	0.256	**7,711**	7,711	1.92	4,016	0	0	44,800	37,089	2.36	15,716	11,700
2	18,421	18,421	0.256	**4,716**	12,427	1.92	6,472	0	0	44,800	32,373	2.36	13,718	7,245
3	11,176	11,176	0.256	**2,861**	15,288	1.92	7,962	0	0	44,800	29,512	2.36	12,505	4,543
4	6,633	6,633	0.256	**1,698**	16,986	1.92	8,847	0	0	44,800	27,814	2.36	11,786	2,939
5	3,694	3,694	0.256	**946**	17,931	1.92	9,339	0	0	44,800	26,869	2.36	11,385	2,046
6	-3,694	-3,694	0.256	-946	17,931	1.92	9,339	0	0	44,800	26,869	2.36	11,385	2,046
7	-6,633	-6,633	0.256	-1,698	16,986	1.92	8,847	0	0	44,800	27,814	2.36	11,786	2,939
8	-11,176	-11,176	0.256	-2,861	15,288	1.92	7,962	0	0	44,800	29,512	2.36	12,505	4,543
9	-18,421	-18,421	0.256	-4,716	12,427	1.92	6,472	0	0	44,800	32,373	2.36	13,718	7,245
10	-30,121	-30,121	0.256	-7,711	7,711	1.92	4,016	0	0	44,800	37,089	2.36	15,716	11,700

Second Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	**Strain Delta** $\Delta(\delta_s - \delta_d)$	**Fastener Strain** $\Delta\delta_F$	**Spring Constant** k_F	**Fastener Load** P_F	**Doubler Load** P_D	**Spring Constant** k_D	**Doubler Strain** $\Delta\delta_D$	**Intermed Loads** $q_a L_n$	**Intermed Loads** Sum (10)	**Applied Load** q_L + (11)	**Load in Base** P_S	**Spring Constant** k_S	**Base Strain** δ_S	**Strain Delta** $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	14,291	14,291	0.103	**1,476**	1,476	1.920	769	0	0	16,392	14,915	2.36	6,320	5,551
2	8,740	8,740	0.256	**2,237**	3,714	1.920	1,934	0	0	16,392	12,678	2.36	5,372	3,438
3	5,302	5,302	0.256	**1,357**	5,071	1.920	2,641	0	0	16,392	11,320	2.36	4,797	2,155
4	3,147	3,147	0.256	**806**	5,877	1.920	3,061	0	0	16,392	10,515	2.36	4,455	1,395
5	1,752	1,752	0.256	**449**	6,325	1.920	3,295	0	0	16,392	10,066	2.36	4,265	971
6	-1,752	-1,752	0.256	-449	6,325	1.920	3,295	0	0	16,392	10,066	2.36	4,265	971
7	-3,147	-3,147	0.256	-806	5,877	1.920	3,061	0	0	16,392	10,515	2.36	4,455	1,395
8	-5,302	-5,302	0.256	-1,357	5,071	1.920	2,641	0	0	16,392	11,320	2.36	4,797	2,155
9	-8,740	-8,740	0.256	-2,237	3,714	1.920	1,934	0	0	16,392	12,678	2.36	5,372	3,438
10	-14,291	-14,291	0.103	-1,476	1,476	1.920	769	0	0	16,392	14,915	2.36	6,320	5,551

Third Iteration

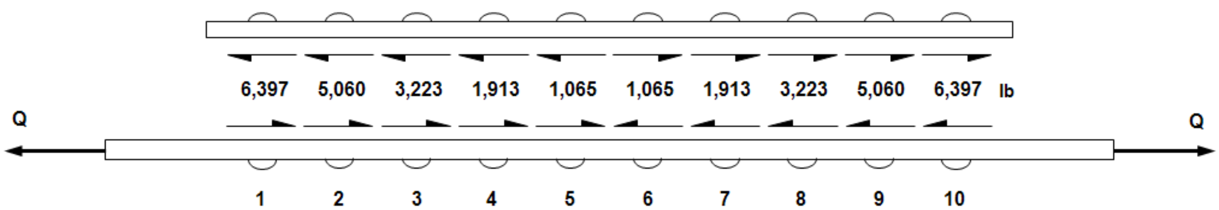
1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	**Strain Delta** $\Delta(\delta_s - \delta_d)$	**Fastener Strain** $\Delta\delta_F$	**Spring Constant** k_F	**Fastener Load** P_F	**Doubler Load** P_D	**Spring Constant** k_D	**Doubler Strain** $\Delta\delta_D$	**Intermed Loads** $q_a L_n$	**Intermed Loads** Sum (10)	**Applied Load** q_L + (11)	**Load in Base** P_S	**Spring Constant** k_S	**Base Strain** δ_S	**Strain Delta** $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	2,460	2,460	0.103	**254**	254	1.920	132	0	0	2,478	2,224	2.36	942	810
2	1,650	1,650	0.103	**170**	425	1.920	221	0	0	2,478	2,053	2.36	870	649
3	1,001	1,001	0.256	**256**	681	1.920	355	0	0	2,478	1,797	2.36	761	407
4	594	594	0.256	**152**	833	1.920	434	0	0	2,478	1,645	2.36	697	263
5	331	331	0.256	**85**	918	1.920	478	0	0	2,478	1,560	2.36	661	183
6	-331	-331	0.256	-85	918	1.920	478	0	0	2,478	1,560	2.36	661	183
7	-594	-594	0.256	-152	833	1.920	434	0	0	2,478	1,645	2.36	697	263
8	-1,001	-1,001	0.256	-256	681	1.920	355	0	0	2,478	1,797	2.36	761	407
9	-1,650	-1,650	0.103	-170	425	1.920	221	0	0	2,478	2,053	2.36	870	649
10	-2,460	-2,460	0.103	-254	254	1.920	132	0	0	2,478	2,224	2.36	942	810

Internal Loads in the Plastic Range

Table III.3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Load	First Unit Solution	Limiting Load Level	Possible Limiting Ratios	First Loading Increm.	Sum of Loads	Second Unit Solution	Limiting Load Level	Possible Limiting Ratios	Second Loading Increm.	Sum of Loads	Third Unit Solution	Limiting Load Level	Possible Limiting Ratios	Third Loading Increm.	Sum of Loads	Fourth Unit Solution
			r_1	Δ_1	$\Sigma\Delta_1$			r_2	Δ_2	$\Sigma\Delta_2$			r_3	Δ_3	$\Sigma\Delta$	
			(3) / (2)	$(4)_{min}$* (2)	(5)	(2) - (6)		$\frac{(8)-(6)}{(7)}$	$(9)_{min}$* (7)	(6) + (10)	(2) - (11)		$\frac{(13)-(11)}{(12)}$	$(14)_{min}$* (12)	(11) + (13)	(2) - (16)
		k_1 to k_5	**256,000**	lb/in per inch			k_1	**103,300**	k_2 to k_5	**256,000**		k_1 & k_2	**103,300**	k_3 to k_5	**256,000**	
Q_L	44,800	44,800	1.000	28,408	28,408	16,392	44,800	1.000	13,914	42,322	2,478	44,800	1.000	2,478	44,800	0
P_{F1}	7,711	4,889.6	0.634	4,890	4,890	1,476	6,459.8	1.064	1,253	6,143	254	6,459.8	1.248	254	**6,397**	1,314
P_{F2}	4,716	4,889.6	1.037	2,990	2,990	2,237	4,889.6	0.849	1,899	4,890	170	6,459.8	9.213	170	**5,060**	-344
P_{F3}	2,861	4,889.6	1.709	1,814	1,814	1,357	4,889.6	2.266	1,152	2,966	256	4,889.6	7.505	256	**3,223**	-362
P_{F4}	1,698	4,889.6	2.880	1,077	1,077	806	4,889.6	4.733	684	1,761	152	4,889.6	20.575	152	**1,913**	-215
P_{F5}	946	4,889.6	5.171	600	600	449	4,889.6	9.562	381	980	85	4,889.6	46.159	85	**1,065**	-120
P_{D5}	16,740			10,615	10,615	5,369			4,558	15,173	918			918	16,090	650
P_{S5}	28,060			17,794	17,794	11,022			9,356	27,150	1,560			1,560	28,710	-650

Fastener Loads



Base and Doubler Loads

Load	P_F	P_D	P_S
		0	44,800
P_{F1}	6,397	6,397	38,403
P_{F2}	5,060	11,457	33,343
P_{F3}	3,223	14,680	30,120
P_{F4}	1,913	16,592	28,208
P_{F5}	1,065	17,657	27,143
P_{F6}	-1,065	17,657	27,143
P_{F7}	-1,913	16,592	28,208
P_{F8}	-3,223	14,680	30,120
P_{F9}	-5,060	11,457	33,343
P_{F10}	-6,397	6,397	38,403
		0	44,800

Parabolic WAG Method

If I were to invent my own doubler analysis method for preliminary analysis, I would assume that the load at the center of the base member is equal to the “EA ratio” times the load.

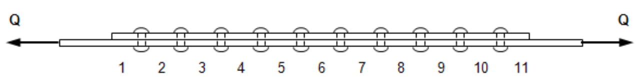
$$Load\ at\ Center\ of\ Base\ Member\ =\ Q\ \left(\frac{E_s\,A_s}{E_d\,A_d\ +\ E_s\,A_s}\right)$$

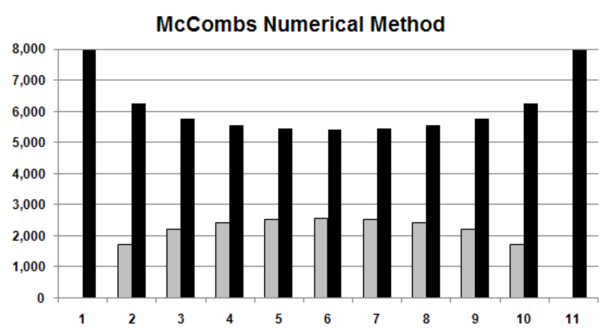
You then have three points of the load distribution curve. Solve for the equation of a parabola. Voilà. Splice analysis could be accomplished in similar fashion.

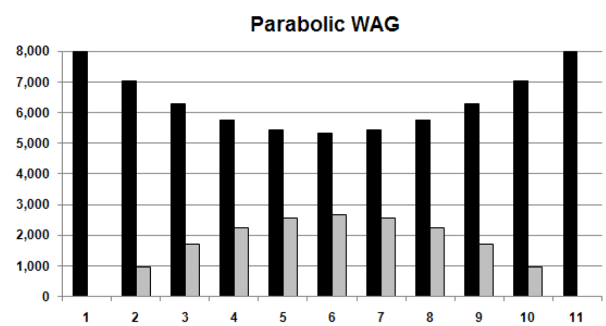
Stick with William McCombs, Manford Tate and Samuel Rosenfeld.

Example 1

t_d = 0.100 inch $\qquad$ t_s = 0.200 inch

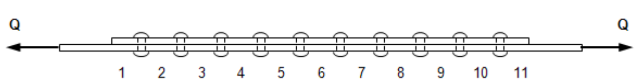


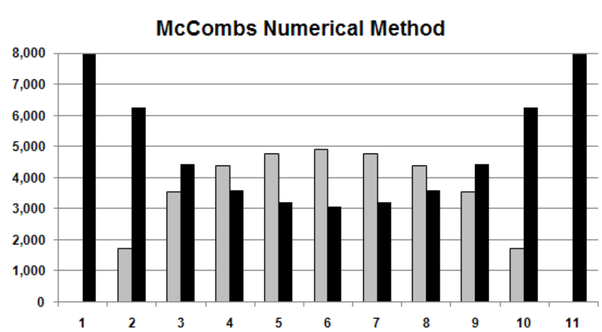


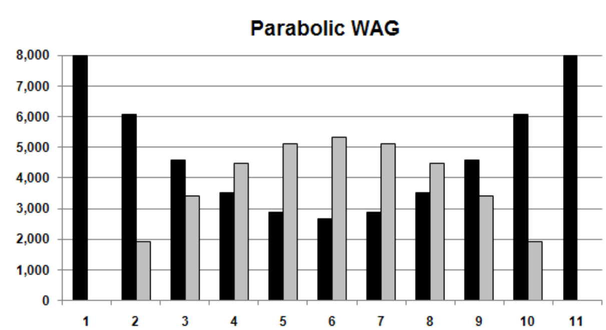


Example 2

t_d = 0.200 inch $\qquad$ t_s = 0.100 inch



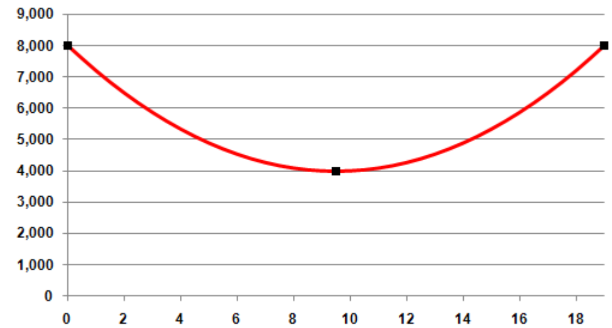




Three Equations, Three Unknowns

DATA

x_1	0	inch	y_1	8,000	lb
x_2	9.50	inch	y_2	4,000	lb
x_3	19.00	inch	y_3	8,000	lb



3 Equations, 3 Unknowns

$y_1 = a\,x_1^2 + b\,x_1 + c$	0	8,000
$y_2 = a\,x_2^2 + b\,x_2 + c$	9.5	4,000
$y_3 = a\,x_3^2 + b\,x_3 + c$	19.0	8,000

In Matrix Form

$$\begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix}$$

Inverse

$$\begin{vmatrix} 0 & 0 & 1 \\ 90.25 & 9.5 & 1 \\ 361 & 19 & 1 \end{vmatrix}^{-1} = \begin{vmatrix} 0.006 & -0.011 & 0.006 \\ -0.158 & 0.211 & -0.053 \\ 1.000 & 0.000 & 0.000 \end{vmatrix}$$

Therefore:

$$\begin{vmatrix} 0 & 0 & 1 \\ 90.3 & 9.5 & 1 \\ 361.0 & 19.0 & 1 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 8{,}000 \\ 4{,}000 \\ 8{,}000 \end{vmatrix}$$

Check

$$\begin{vmatrix} 0 & 0 & 1 \\ 90.25 & 9.5 & 1 \\ 361 & 19 & 1 \end{vmatrix} \begin{vmatrix} 0.006 & -0.011 & 0.006 \\ -0.158 & 0.211 & -0.053 \\ 1.000 & 0.000 & 0.000 \end{vmatrix}$$

Solution

$$\begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 0.006 & -0.011 & 0.006 \\ -0.158 & 0.211 & -0.053 \\ 1.000 & 0.000 & 0.000 \end{vmatrix} \begin{vmatrix} 8{,}000 \\ 4{,}000 \\ 8{,}000 \end{vmatrix} = \begin{vmatrix} 44.3 \\ -842.1 \\ 8{,}000 \end{vmatrix}$$

Identity Matrix $= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

a = 44.3

b = -842.1

c = 8,000

$$\boxed{y = a\,x^2 + b\,x + c}$$

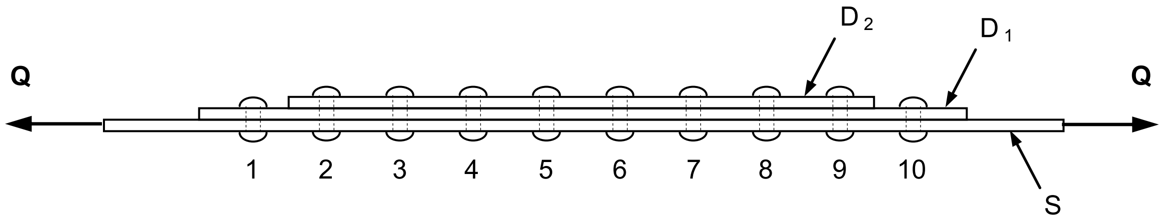
$y = 44.32\ x^2 + (\ -842.11\ )\ x + 8{,}000$

Multiple Doublers

William F. McCombs, James C. McQueen, Jeffrey L. Perry

AFFDL-TR-67-184 *Analytical Design Methods for Aircraft Structural Joints* pages 57 thru 63

Q = 32,000 lb Pitch = 1.00 inch

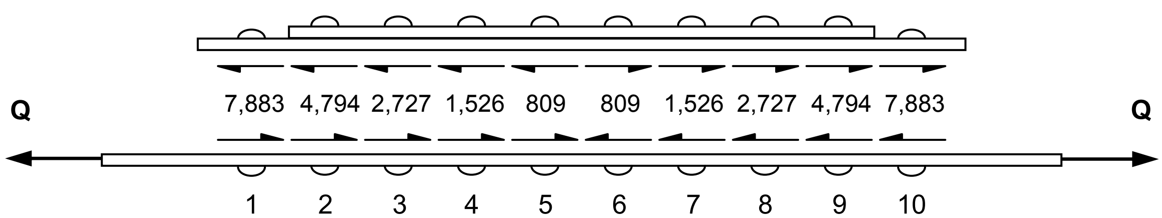


K_{Fn} = 0.47 x10^6 lb/in per inch K_{Sn} = K_{D1n} = 2.47 x10^6 K_{D2n} = 1.23 x10^6

See page 59:

- *a* *Combine the stacked doublers D_1 and D_2 into one member, D , (by adding the k values) as in Figure III.13c. This assumes the fasteners between them to be rigid.*

- *b* *Determine the corresponding fastener loads between this assumed member, D_1 , and the base structure, S, in the conventional tabular manner. Note the strains, Column 9 of the table.*



- *c.* *Then consider only the two doublers, as they actually exist, to be a structure subjected to the loads of (b) above, applied to the member D_1 , as in Figure III.13d.*

- *d.* *Determine the internal loads for this configuration and loading and also note the strains in the member D_1 Column 15 of the table. Member D_1 is the "base structure" in this analysis.*

- *e* *Calculate an effective k_D value for the combined members D_1 and D_2 using the member strains from (b) and (d) above is follows:*

  For any segment the effective k_D of the combined members is taken as

$$(k_d)_{eff} = (k_d)_{assumed}\left(\frac{\delta D}{\delta D_1}\right)$$

Repeat steps (b) through (e) until the doubler strains equal the base strains.

A rougher estimate can, of course, be obtained simply by carrying out steps (a) and (b) only one time. This assumes the doublers to be one integral member ard therefore results in the fastener loads and the doubler load being larger than they actually are.

First Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,772	16,772	0.47	7,883	7,883	2.47	3,191	0	0	32,000	24,117	2.47	9,764	6,572
2	10,200	10,200	0.47	4,794	12,677	3.70	3,426	0	0	32,000	19,323	2.47	7,823	4,397
3	5,803	5,803	0.47	2,727	15,404	3.70	4,163	0	0	32,000	16,596	2.47	6,719	2,556
4	3,247	3,247	0.47	1,526	16,930	3.70	4,576	0	0	32,000	15,070	2.47	6,101	1,525
5	1,722	1,722	0.47	809	17,740	3.70	4,795	0	0	32,000	14,260	2.47	5,773	979
6	-1,722	-1,722	0.47	-809	17,740	3.70	4,795	0	0	32,000	14,260	2.47	5,773	979
7	-3,247	-3,247	0.47	-1,526	16,930	3.70	4,576	0	0	32,000	15,070	2.47	6,101	1,525
8	-5,803	-5,803	0.47	-2,727	15,404	3.70	4,163	0	0	32,000	16,596	2.47	6,719	2,556
9	-10,200	-10,200	0.47	-4,794	12,677	3.70	3,426	0	0	32,000	19,323	2.47	7,823	4,397
10	-16,772	-16,772	0.47	-7,883	7,883	2.47	3,191	0	0	32,000	24,117	2.47	9,764	6,572

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	0	0	0	0	0	0	8,517	7,883	7,883	7,883	7,883	2.47	3,191	-5,325
2	5,325	5,325	0.47	2,503	2,503	1.23	2,035	4,794	12,677	12,677	10,174	2.47	4,119	2,084
3	3,241	3,241	0.47	1,523	4,026	1.23	3,273	2,727	15,404	15,404	11,378	2.47	4,606	1,333
4	1,908	1,908	0.47	897	4,923	1.23	4,002	1,526	16,930	16,930	12,007	2.47	4,861	859
5	1,049	1,049	0.47	493	5,416	1.23	4,403	809	17,740	17,740	12,324	2.47	4,989	586
6	-1,049	-1,049	0.47	-493	5,416	1.23	4,403	-809	17,740	17,740	12,324	2.47	4,989	586
7	-1,908	-1,908	0.47	-897	4,923	1.23	4,002	-1,526	16,930	16,930	12,007	2.47	4,861	859
8	-3,241	-3,241	0.47	-1,523	4,026	1.23	3,273	-2,727	15,404	15,404	11,378	2.47	4,606	1,333
9	-5,325	-5,325	0.47	-2,503	2,503	1.23	2,035	-4,794	12,677	12,677	10,174	2.47	4,119	2,084
10	0	0	0	0	0	0	8,578	-7,883	7,883	7,883	7,883	2.47	3,191	-5,386

Strain and Effective Stiffness

Compare the doubler strains from Column 9 in the first table to base strains in Column 15 of the second table. Use the effective stiffness you calculate in the first iteration for the second calculation and so on.

$$(k_d)_{eff} = (k_d)_{assumed} \left(\frac{\delta D}{\delta D_1} \right)$$

Doubler Strain	Base Strain	Ratio	K	K_{eff}
inch / inch	inch / inch			lb/in per inch
0.00319	0.00319	1.00	2.47	2.470
0.00343	0.00412	0.83	3.70	3.078
0.00416	0.00461	0.90	3.70	3.344
0.00458	0.00486	0.94	3.70	3.483
0.00479	0.00499	0.96	3.70	3.555
0.00479	0.00499	0.96	3.70	3.555
0.00458	0.00486	0.94	3.70	3.483
0.00416	0.00461	0.90	3.70	3.344
0.00343	0.00412	0.83	3.70	3.078
0.00319	0.00319	1.00	2.47	2.470

Second Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,425	16,425	0.47	7,720	7,720	2.470	3,125	0	0	32,000	24,280	2.47	9,830	6,705
2	9,721	9,721	0.47	4,569	12,289	3.078	3,993	0	0	32,000	19,711	2.47	7,980	3,987
3	5,733	5,733	0.47	2,695	14,983	3.344	4,481	0	0	32,000	17,017	2.47	6,889	2,409
4	3,325	3,325	0.47	1,563	16,546	3.483	4,751	0	0	32,000	15,454	2.47	6,257	1,506
5	1,819	1,819	0.47	855	17,401	3.555	4,894	0	0	32,000	14,599	2.47	5,911	1,017
6	-1,819	-1,819	0.47	-855	17,401	3.555	4,894	0	0	32,000	14,599	2.47	5,911	1,017
7	-3,325	-3,325	0.47	-1,563	16,546	3.483	4,751	0	0	32,000	15,454	2.47	6,257	1,506
8	-5,733	-5,733	0.47	-2,695	14,983	3.344	4,481	0	0	32,000	17,017	2.47	6,889	2,409
9	-9,721	-9,721	0.47	-4,569	12,289	3.078	3,993	0	0	32,000	19,711	2.47	7,980	3,987
10	-16,425	-16,425	0.47	-7,720	7,720	2.470	3,125	0	0	32,000	24,280	2.47	9,830	6,705

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	4,153	4,153	0	0	0	0	4,153	7,720	7,720	7,720	7,720	2.47	3,125	-1,028
2	5,181	5,181	0.47	2,435	2,435	1.23	1,980	4,569	12,289	12,289	9,853	2.47	3,989	2,009
3	3,172	3,172	0.47	1,491	3,926	1.23	3,192	2,695	14,983	14,983	11,057	2.47	4,477	1,285
4	1,887	1,887	0.47	887	4,813	1.23	3,913	1,563	16,546	16,546	11,733	2.47	4,750	837
5	1,050	1,050	0.47	493	5,306	1.23	4,314	855	17,401	17,401	12,094	2.47	4,896	582
6	-1,050	-1,050	0.47	-493	5,306	1.23	4,314	-855	17,401	17,401	12,094	2.47	4,896	582
7	-1,887	-1,887	0.47	-887	4,813	1.23	3,913	-1,563	16,546	16,546	11,733	2.47	4,750	837
8	-3,172	-3,172	0.47	-1,491	3,926	1.23	3,192	-2,695	14,983	14,983	11,057	2.47	4,477	1,285
9	-5,181	-5,181	0.47	-2,435	2,435	1.23	1,980	-4,569	12,289	12,289	9,853	2.47	3,989	2,009
10	-4,159	-4,159	0	0	0	0	4,147	-7,720	7,720	7,720	7,720	2.47	3,125	-1,022

Strain and Effective Stiffness

$$(k_d)_{eff} = (k_d)_{assumed} \left(\frac{\delta D}{\delta D_1} \right)$$

Doubler Strain	Base Strain	Ratio	K	K eff
inch / inch	inch / inch			lb/in per inch
0.00313	0.00313	1.000	2.470	2.47
0.00399	0.00399	1.001	3.078	3.080
0.00448	0.00448	1.001	3.344	3.347
0.00475	0.00475	1.000	3.483	3.483
0.00489	0.00490	0.999	3.555	3.554
0.00489	0.00490	0.999	3.555	3.554
0.00475	0.00475	1.000	3.483	3.483
0.00448	0.00448	1.001	3.344	3.347
0.00399	0.00399	1.001	3.078	3.080
0.00313	0.00313	1.000	2.470	2.47

Third Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,427	16,427	0.47	**7,721**	7,721	2.470	3,126	0	0	32,000	24,279	2.47	9,830	6,704
2	9,723	9,723	0.47	**4,570**	12,290	3.080	3,990	0	0	32,000	19,710	2.47	7,980	3,990
3	5,733	5,733	0.47	**2,695**	14,985	3.347	4,477	0	0	32,000	17,015	2.47	6,889	2,412
4	3,321	3,321	0.47	**1,561**	16,546	3.483	4,750	0	0	32,000	15,454	2.47	6,257	1,506
5	1,815	1,815	0.47	**853**	17,399	3.554	4,896	0	0	32,000	14,601	2.47	5,911	1,015
6	-1,815	-1,815	0.47	**-853**	17,399	3.554	4,896	0	0	32,000	14,601	2.47	5,911	1,015
7	-3,321	-3,321	0.47	**-1,561**	16,546	3.483	4,750	0	0	32,000	15,454	2.47	6,257	1,506
8	-5,733	-5,733	0.47	**-2,695**	14,985	3.347	4,477	0	0	32,000	17,015	2.47	6,889	2,412
9	-9,723	-9,723	0.47	**-4,570**	12,290	3.080	3,990	0	0	32,000	19,710	2.47	7,980	3,990
10	-16,427	-16,427	0.47	**-7,721**	7,721	2.470	3,126	0	0	32,000	24,279	2.47	9,830	6,704

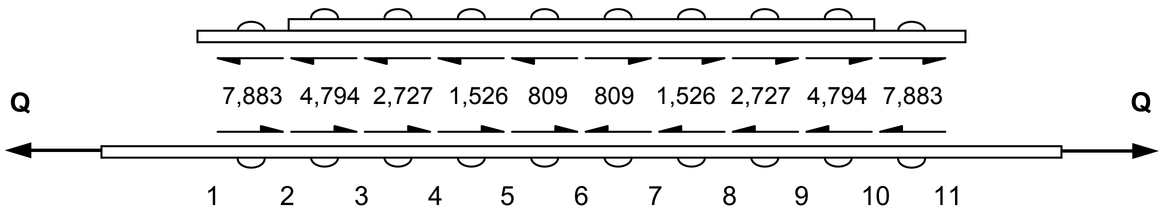
1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	4,154	4,154	0	**0**	0	0	4,154	7,721	7,721	7,721	7,721	2.47	3,126	-1,028
2	5,182	5,182	0.47	**2,435**	2,435	1.23	1,980	4,570	12,290	12,290	9,855	2.47	3,990	2,010
3	3,172	3,172	0.47	**1,491**	3,926	1.23	3,192	2,695	14,985	14,985	11,059	2.47	4,477	1,285
4	1,887	1,887	0.47	**887**	4,813	1.23	3,913	1,561	16,546	16,546	11,733	2.47	4,750	837
5	1,049	1,049	0.47	**493**	5,306	1.23	4,314	853	17,399	17,399	12,093	2.47	4,896	582
6	-1,049	-1,049	0.47	**-493**	5,306	1.23	4,314	-853	17,399	17,399	12,093	2.47	4,896	582
7	-1,887	-1,887	0.47	**-887**	4,813	1.23	3,913	-1,561	16,546	16,546	11,733	2.47	4,750	837
8	-3,172	-3,172	0.47	**-1,491**	3,926	1.23	3,192	-2,695	14,985	14,985	11,059	2.47	4,477	1,285
9	-5,182	-5,182	0.47	**-2,435**	2,435	1.23	1,980	-4,570	12,290	12,290	9,855	2.47	3,990	2,010
10	25,173	25,173	0	**0**	0	0	33,481	-7,721	7,721	7,721	7,721	2.47	3,126	-30,355

Strain and Effective Stiffness

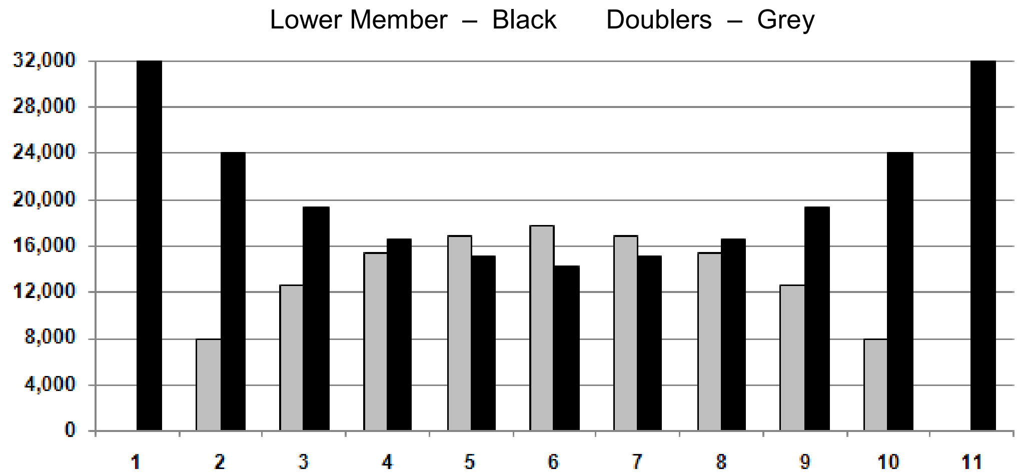
$$(k_d)_{eff} = (k_d)_{assumed} \left(\frac{\delta D}{\delta D_1} \right)$$

Doubler Strain	Base Strain	Ratio	K	K_{eff}
inch / inch	inch / inch			lb/in per inch
0.00313	0.00313	1.00	2.470	2.470
0.00399	0.00399	1.00	3.080	3.080
0.00448	0.00448	1.00	3.347	3.347
0.00475	0.00475	1.00	3.483	3.483
0.00490	0.00490	1.00	3.554	3.554
0.00490	0.00490	1.00	3.554	3.554
0.00475	0.00475	1.00	3.483	3.483
0.00448	0.00448	1.00	3.347	3.347
0.00399	0.00399	1.00	3.080	3.080
0.00313	0.00313	1.00	2.470	2.470

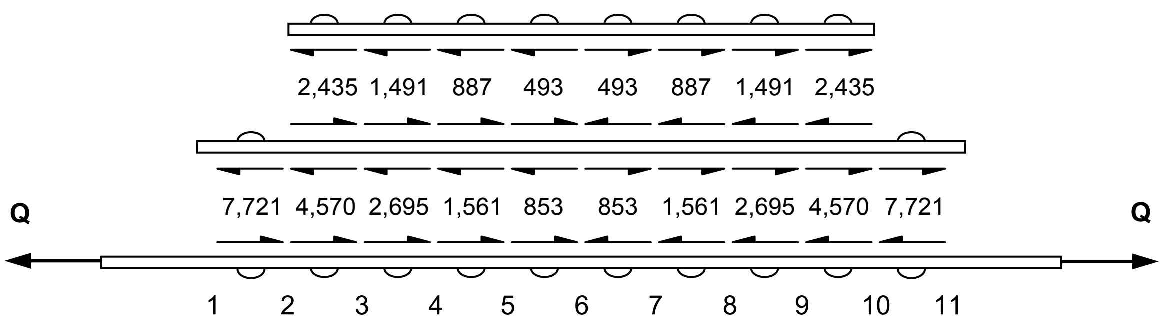
Case I - Free Body Diagram



Load Distribution



Case II - Free Body Diagram



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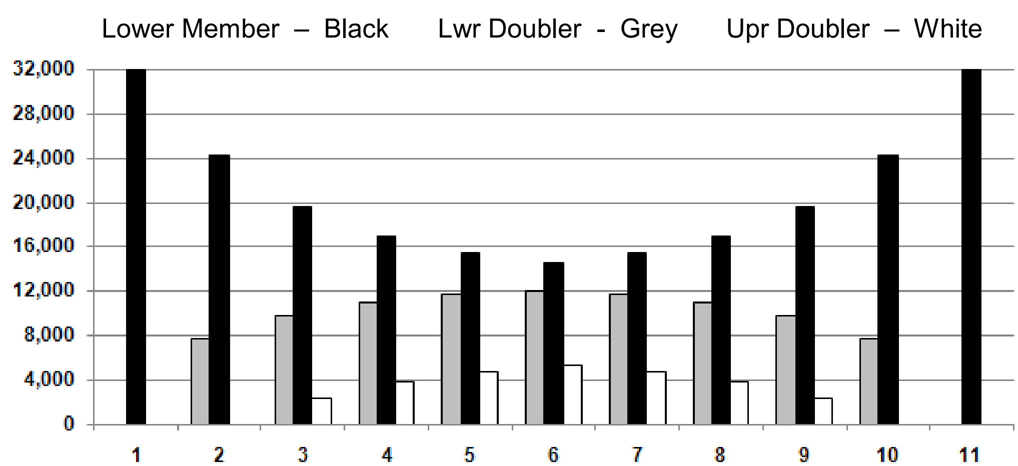
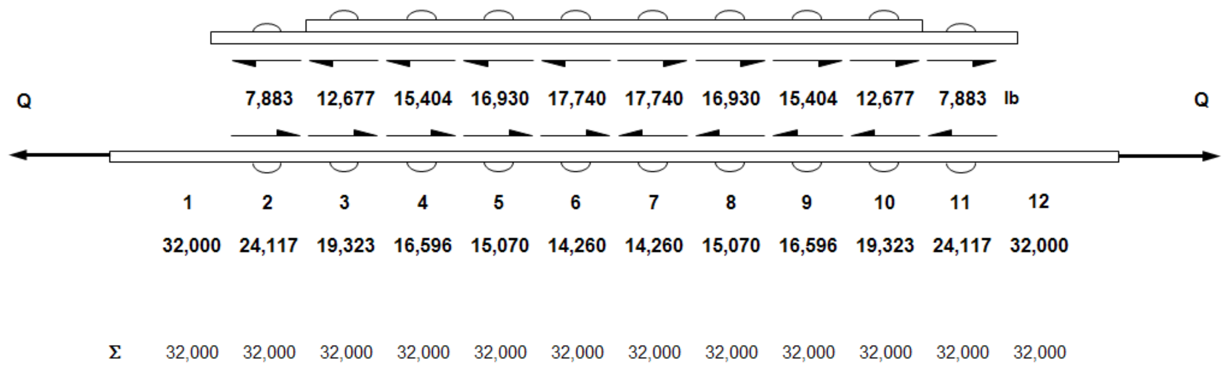
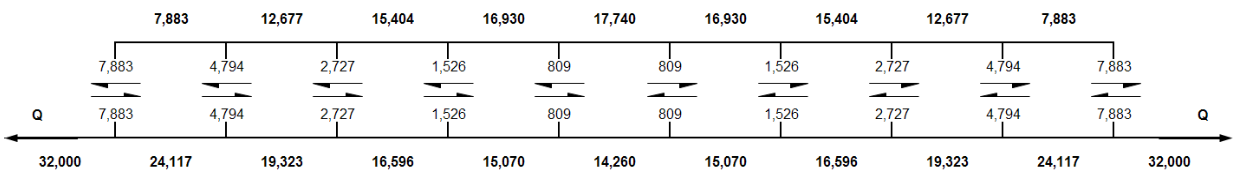
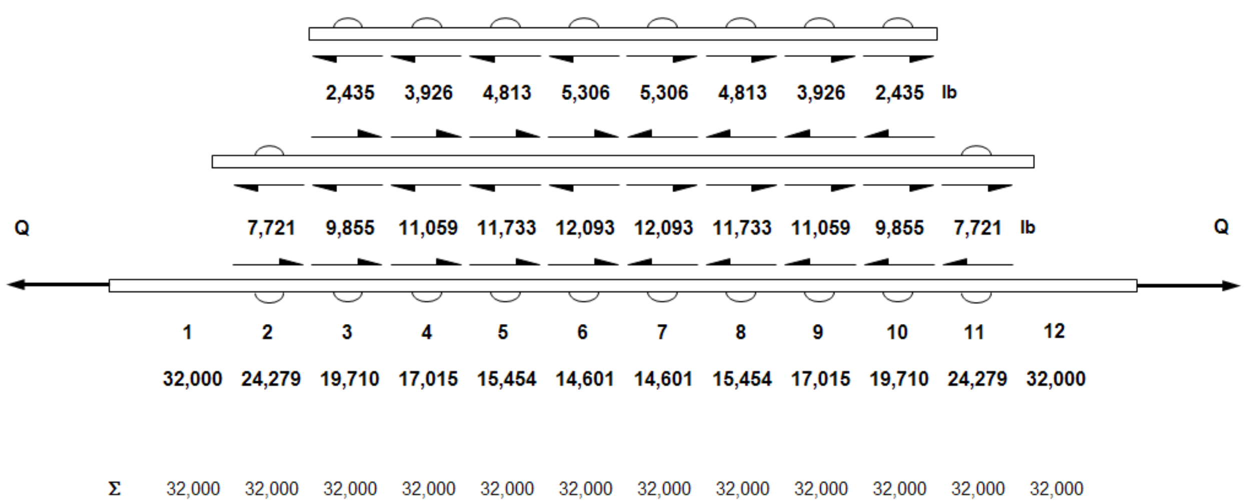


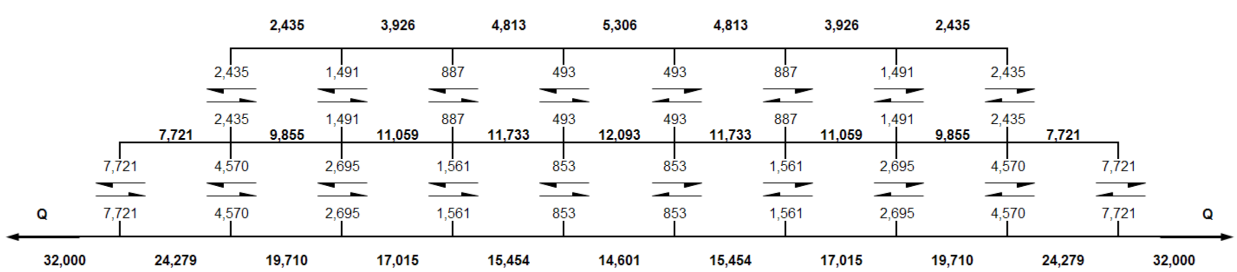
Case I – Load Distribution





Case II – Load Distribution





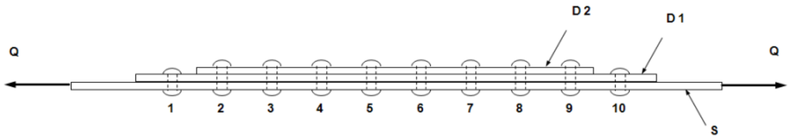
Spreadsheet

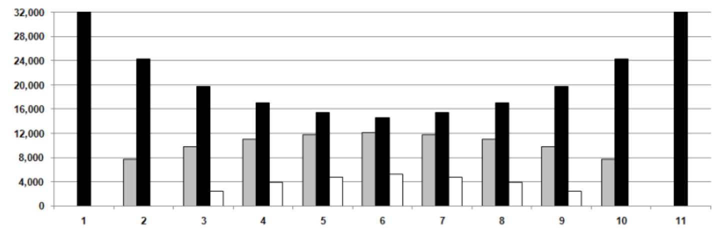
Numerical Method for Multiple Doublers

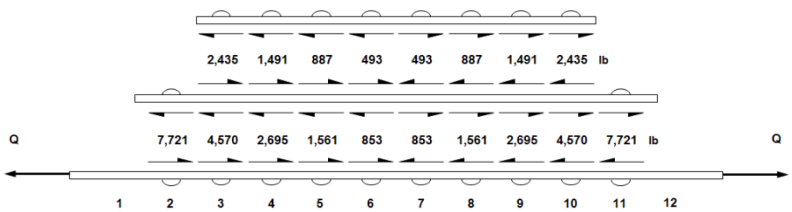
William F. McCombs AFFDL-TR AFFDL-TR-67-184 *Analytical Design Methods for Aircraft Structural Joints* pages 57 thru 63

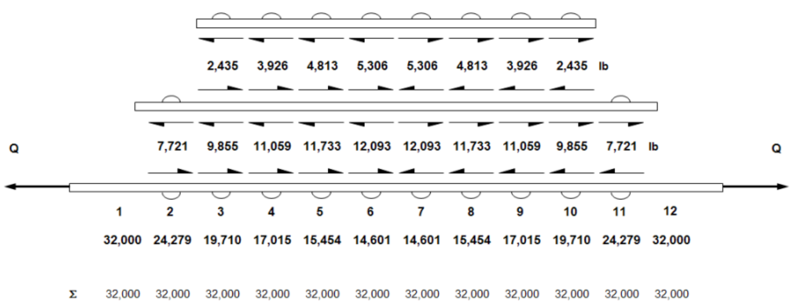
GRAN Corporation Made in America

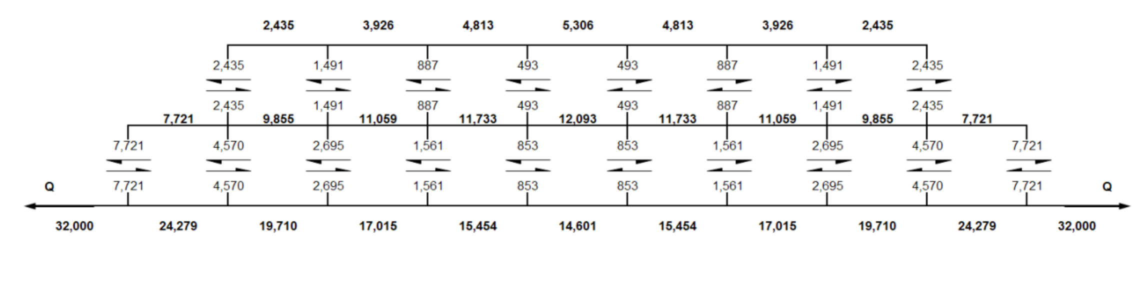
n	10	fasteners
D	0.375	inch
Width	1.84	inch
Pitch	1.00	inch
k_F	800,000	lb / inch
t_d	0.200	inch
t_s	0.200	inch
E_d	2.90E+07	psi
E_s	2.90E+07	psi
A_s	0.3080	in^2
A_d	0.3080	in^2
$A_d\ E_d$	8.93E+06	
$A_s\ E_s$	8.93E+06	
C_1	184.878	
k	800,000	lb / inch per inch
k_s	893,200	lb / inch per inch
L	10.0	inch
M	0.20000	
$\sqrt{M}$	0.44721	
N	3,200.0	
k_d	893,200	lb / inch per inch
Q	32,000	lb
K_{D1n}	2.47E+06	lb/in per inch
K_{D2n}	1.23E+06	lb/in per inch
K_{Dn}	3.70E+06	lb/in per inch
K_{S1n}	2.47E+06	lb/in per inch
K_{Fn}	4.70E+05	lb/in per inch











William F. McCombs, James C. McQueen, Jeffrey L. Perry

AFFDL-TR-67-184 *Analytical Design Methods for Aircraft Structural Joints*

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,772	16,772	0.47	7,883	7,883	2.47	3,191	0	0	32,000	24,117	2.47	9,764	6,572
2	10,200	10,200	0.47	4,794	12,677	3.70	3,426	0	0	32,000	19,323	2.47	7,823	4,397
3	5,803	5,803	0.47	2,727	15,404	3.70	4,163	0	0	32,000	16,596	2.47	6,719	2,556
4	3,247	3,247	0.47	1,526	16,930	3.70	4,576	0	0	32,000	15,070	2.47	6,101	1,525
5	1,722	1,722	0.47	809	17,740	3.70	4,795	0	0	32,000	14,260	2.47	5,773	979
6	743	743	0.47	349	18,089	3.70	4,889	0	0	32,000	13,911	2.47	5,632	743
7	0	0	0.47	0	18,089	3.70	4,889	0	0	32,000	13,911	2.47	5,632	743
8	-743	-743	0.47	-349	17,740	3.70	4,795	0	0	32,000	14,260	2.47	5,773	979
9	-1,722	-1,722	0.47	-809	16,930	3.70	4,576	0	0	32,000	15,070	2.47	6,101	1,525
10	-3,247	-3,247	0.47	-1,526	15,404	3.70	4,163	0	0	32,000	16,596	2.47	6,719	2,556
11	-5,803	-5,803	0.47	-2,727	12,677	3.70	3,426	0	0	32,000	19,323	2.47	7,823	4,397
12	-10,200	-10,200	0.47	-4,794	7,883	2.47	3,191	0	0	32,000	24,117	2.47	9,764	6,572
	-16,772	-16,772	0.47	-7,883										
	0				0				Goal Seek Set = 0		0	Changing Cell C45		

Goal Seek

Sum Doubler	Lwr Doubler	Load in Base	Upper Doubler	Σ
	0	*32,000*		
7,883	7,883	24,117	0	32,000
12,677	10,174	19,323	2,503	32,000
15,404	11,378	16,596	4,026	32,000
16,930	12,007	15,070	4,923	32,000
17,740	12,324	14,260	5,416	32,000
18,089	12,456	13,911	5,633	32,000
18,089	12,456	13,911	5,633	32,000
17,740	12,324	14,260	5,416	32,000
16,930	12,007	15,070	4,923	32,000
15,404	11,378	16,596	4,026	32,000
12,677	10,174	19,323	2,503	32,000
7,883	7,883	24,117	0	32,000
	0	*32,000*		

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	0	0	0.47	0	0	0	8,517	7,883	7,883	7,883	7,883	2.47	3,191	-5,325
2	5,325	5,325	0.47	2,503	2,503	1.23	2,035	4,794	12,677	12,677	10,174	2.47	4,119	2,084
3	3,241	3,241	0.47	1,523	4,026	1.23	3,273	2,727	15,404	15,404	11,378	2.47	4,606	1,333
4	1,908	1,908	0.47	897	4,923	1.23	4,002	1,526	16,930	16,930	12,007	2.47	4,861	859
5	1,049	1,049	0.47	493	5,416	1.23	4,403	809	17,740	17,740	12,324	2.47	4,989	586
6	463	463	0.47	217	5,633	1.23	4,580	349	18,089	18,089	12,456	2.47	5,043	463
7	0	0	0.47	0	5,633	1.23	4,580	0	18,089	18,089	12,456	2.47	5,043	463
8	-463	-463	0.47	-217	5,416	1.23	4,403	-349	17,740	17,740	12,324	2.47	4,989	586
9	-1,049	-1,049	0.47	-493	4,923	1.23	4,002	-809	16,930	16,930	12,007	2.47	4,861	859
10	-1,908	-1,908	0.47	-897	4,026	1.23	3,273	-1,526	15,404	15,404	11,378	2.47	4,606	1,333
11	-3,241	-3,241	0.47	-1,523	2,503	1.23	2,035	-2,727	12,677	12,677	10,174	2.47	4,119	2,084
12	-5,325	-5,325	0.47	-2,503	0	0	8,578	-4,794	7,883	7,883	7,883	2.47	3,191	-5,386
								-7,883						
	0				0				Goal Seek Set = 0		0	Changing Cell C70		

Goal Seek

Table I Column 9 Doubler Strain inch / inch	Table II Column 15 Base Strain inch / inch	Ratio	k	k_{eff} lb/in per inch
0.00319	0.00319	1.00	2.47	2.47
0.00343	0.00412	0.83	3.70	3.08
0.00416	0.00461	0.90	3.70	3.34
0.00458	0.00486	0.94	3.70	3.48
0.00479	0.00499	0.96	3.70	3.56
0.00489	0.00504	0.97	3.70	3.59
0.00489	0.00504	0.97	3.70	3.59
0.00479	0.00499	0.96	3.70	3.56
0.00458	0.00486	0.94	3.70	3.48
0.00416	0.00461	0.90	3.70	3.34
0.00343	0.00412	0.83	3.70	3.08
0.00319	0.00319	1.00	2.47	2.47

Second Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,425	16,425	0.47	7,720	7,720	2.470	3,125	0	0	32,000	24,280	2.47	9,830	6,705
2	9,721	9,721	0.47	4,569	12,289	3.078	3,993	0	0	32,000	19,711	2.47	7,980	3,987
3	5,733	5,733	0.47	2,695	14,983	3.344	4,481	0	0	32,000	17,017	2.47	6,889	2,409
4	3,325	3,325	0.47	1,563	16,546	3.483	4,751	0	0	32,000	15,454	2.47	6,257	1,506
5	1,819	1,819	0.47	855	17,401	3.555	4,894	0	0	32,000	14,599	2.47	5,911	1,017
6	802	802	0.47	377	17,778	3.587	4,956	0	0	32,000	14,222	2.47	5,758	802
7	0	0	0.47	0	17,778	3.587	4,956	0	0	32,000	14,222	2.47	5,758	802
8	-802	-802	0.47	-377	17,401	3.555	4,894	0	0	32,000	14,599	2.47	5,911	1,017
9	-1,819	-1,819	0.47	-855	16,546	3.483	4,751	0	0	32,000	15,454	2.47	6,257	1,506
10	-3,325	-3,325	0.47	-1,563	14,983	3.344	4,481	0	0	32,000	17,017	2.47	6,889	2,409
11	-5,733	-5,733	0.47	-2,695	12,289	3.078	3,993	0	0	32,000	19,711	2.47	7,980	3,987
12	-9,721	-9,721	0.47	-4,569	7,720	2.470	3,125	0	0	32,000	24,280	2.47	9,830	6,705
	-16,425	-16,425	0.47	-7,720										
	0				0				Goal Seek Set = 0		0	Changing Cell C98		

Goal Seek

Sum Doubler	Lwr Doubler	Load in Base	Upper Doubler	Σ
	0	*32,000*		
7,720	7,720	24,280	0	32,000
12,289	9,853	19,711	2,435	32,000
14,983	11,057	17,017	3,926	32,000
16,546	11,733	15,454	4,813	32,000
17,401	12,094	14,599	5,306	32,000
17,778	12,252	14,222	5,526	32,000
17,778	12,252	14,222	5,526	32,000
17,401	12,094	14,599	5,306	32,000
16,546	11,733	15,454	4,813	32,000
14,983	11,057	17,017	3,926	32,000
12,289	9,853	19,711	2,435	32,000
7,720	7,720	24,280	0	32,000
	0	*32,000*		

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	4,153	4,153	0	0	0	0	4,153	7,720	7,720	7,720	7,720	2.47	3,125	-1,028
2	5,181	5,181	0.47	2,435	2,435	1.23	1,980	4,569	12,289	12,289	9,853	2.47	3,989	2,009
3	3,172	3,172	0.47	1,491	3,926	1.23	3,192	2,695	14,983	14,983	11,057	2.47	4,477	1,285
4	1,887	1,887	0.47	887	4,813	1.23	3,913	1,563	16,546	16,546	11,733	2.47	4,750	837
5	1,050	1,050	0.47	493	5,306	1.23	4,314	855	17,401	17,401	12,094	2.47	4,896	582
6	467	467	0.47	220	5,526	1.23	4,493	377	17,778	17,778	12,252	2.47	4,960	467
7	0	0	0.47	0	5,526	1.23	4,493	0	17,778	17,778	12,252	2.47	4,960	467
8	-467	-467	0.47	-220	5,306	1.23	4,314	-377	17,401	17,401	12,094	2.47	4,896	582
9	-1,050	-1,050	0.47	-493	4,813	1.23	3,913	-855	16,546	16,546	11,733	2.47	4,750	837
10	-1,887	-1,887	0.47	-887	3,926	1.23	3,192	-1,563	14,983	14,983	11,057	2.47	4,477	1,285
11	-3,172	-3,172	0.47	-1,491	2,435	1.23	1,980	-2,695	12,289	12,289	9,853	2.47	3,989	2,009
12	-5,181	-5,181	0.47	-2,435	0	0	4,147	-4,569	7,720	7,720	7,720	2.47	3,125	-1,022
	-4,159	-4,159	0	0				-7,720						
	0				0				Goal Seek Set = 0		0	Changing Cell C123		

Goal Seek

Table I Column 9 Doubler Strain inch / inch	Table II Column 15 Base Strain inch / inch	Ratio	k	k_{eff} lb/in per inch
0.00313	0.00313	1.00	2.47	2.47
0.00399	0.00399	1.00	3.08	3.08
0.00448	0.00448	1.00	3.34	3.35
0.00475	0.00475	1.00	3.48	3.48
0.00489	0.00490	1.00	3.56	3.55
0.00496	0.00496	1.00	3.59	3.58
0.00496	0.00496	1.00	3.59	3.58
0.00489	0.00490	1.00	3.56	3.55
0.00475	0.00475	1.00	3.48	3.48
0.00448	0.00448	1.00	3.34	3.35
0.00399	0.00399	1.00	3.08	3.08
0.00313	0.00313	1.00	2.47	2.47

Third Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,427	16,427	0.47	**7,721**	7,721	2.47	3,126	0	0	32,000	24,279	2.47	9,830	6,704
2	9,723	9,723	0.47	**4,570**	12,290	3.080	3,990	0	0	32,000	19,710	2.47	7,980	3,990
3	5,733	5,733	0.47	**2,695**	14,985	3.347	4,477	0	0	32,000	17,015	2.47	6,889	2,412
4	3,321	3,321	0.47	**1,561**	16,546	3.483	4,750	0	0	32,000	15,454	2.47	6,257	1,506
5	1,815	1,815	0.47	**853**	17,399	3.554	4,896	0	0	32,000	14,601	2.47	5,911	1,015
6	800	800	0.47	376	17,775	3.584	4,959	0	0	32,000	14,225	2.47	5,759	800
7	0	0	0.47	0	17,775	3.584	4,959	0	0	32,000	14,225	2.47	5,759	800
8	-800	-800	0.47	-376	17,399	3.554	4,896	0	0	32,000	14,601	2.47	5,911	1,015
9	-1,815	-1,815	0.47	**-853**	16,546	3.483	4,750	0	0	32,000	15,454	2.47	6,257	1,506
10	-3,321	-3,321	0.47	**-1,561**	14,985	3.347	4,477	0	0	32,000	17,015	2.47	6,889	2,412
11	-5,733	-5,733	0.47	**-2,695**	12,290	3.080	3,990	0	0	32,000	19,710	2.47	7,980	3,990
12	-9,723	-9,723	0.47	**-4,570**	7,721	2.47	3,126	0	0	32,000	24,279	2.47	9,830	6,704
	-16,427	-16,427	0.47	**-7,721**										

Goal Seek

Sum Doubler	Lwr Doubler	Load in Base	Upper Doubler	Σ
	0	*32,000*		
7,721	7,721	24,279	0	32,000
12,290	9,855	19,710	2,435	32,000
14,985	11,059	17,015	3,926	32,000
16,546	11,733	15,454	4,813	32,000
17,399	12,093	14,601	5,306	32,000
17,775	12,249	14,225	5,525	32,000
17,775	12,249	14,225	5,525	32,000
17,399	12,093	14,601	5,306	32,000
16,546	11,733	15,454	4,813	32,000
14,985	11,059	17,015	3,926	32,000
12,290	9,855	19,710	2,435	32,000
7,721	7,721	24,279	0	32,000
	0	*32,000*		

0 0 Goal Seek Set = 0 **0** Changing Cell C151

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	4,154	4,154	0	0	0	0	4,154	7,721	7,721	7,721	7,721	2.47	3,126	-1,028
2	5,182	5,182	0.47	**2,435**	2,435	1.23	1,980	4,570	12,290	12,290	9,855	2.47	3,990	2,010
3	3,172	3,172	0.47	**1,491**	3,926	1.23	3,192	2,695	14,985	14,985	11,059	2.47	4,477	1,285
4	1,887	1,887	0.47	**887**	4,813	1.23	3,913	1,561	16,546	16,546	11,733	2.47	4,750	837
5	1,049	1,049	0.47	**493**	5,306	1.23	4,314	853	17,399	17,399	12,093	2.47	4,896	582
6	467	467	0.47	220	5,525	1.23	4,492	376	17,775	17,775	12,249	2.47	4,959	467
7	0	0	0.47	0	5,525	1.23	4,492	0	17,775	17,775	12,249	2.47	4,959	467
8	-467	-467	0.47	-220	5,306	1.23	4,314	-376	17,399	17,399	12,093	2.47	4,896	582
9	-1,049	-1,049	0.47	**-493**	4,813	1.23	3,913	-853	16,546	16,546	11,733	2.47	4,750	837
10	-1,887	-1,887	0.47	**-887**	3,926	1.23	3,192	-1,561	14,985	14,985	11,059	2.47	4,477	1,285
11	-3,172	-3,172	0.47	**-1,491**	2,435	1.23	1,980	-2,695	12,290	12,290	9,855	2.47	3,990	2,010
12	-5,182	-5,182	0.47	**-2,435**	0	0	4,119	-4,570	7,721	7,721	7,721	2.47	3,126	-993
	-4,189	-4,189	0	0				-7,721						

Goal Seek

Table I Column 9 Doubler Strain inch / inch	Table II Column 15 Base Strain inch / inch	Ratio	k	k_{eff} lb/in per inch
0.00313	0.00313	1.00	2.47	2.47
0.00399	0.00399	1.00	3.08	3.08
0.00448	0.00448	1.00	3.35	3.35
0.00475	0.00475	1.00	3.48	3.48
0.00490	0.00490	1.00	3.55	3.55
0.00496	0.00496	1.00	3.58	3.58
0.00496	0.00496	1.00	3.58	3.58
0.00490	0.00490	1.00	3.55	3.55
0.00475	0.00475	1.00	3.48	3.48
0.00448	0.00448	1.00	3.35	3.35
0.00399	0.00399	1.00	3.08	3.08
0.00313	0.00313	1.00	2.47	2.47

0 0 Goal Seek Set = 0 **0** Changing Cell C176

Fourth Iteration

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Sum (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	16,427	16,427	0.47	**7,721**	7,721	2.47	3,126	0	0	32,000	24,279	2.47	9,830	6,704
2	9,723	9,723	0.47	**4,570**	12,290	3.08	3,990	0	0	32,000	19,710	2.47	7,980	3,990
3	5,733	5,733	0.47	**2,695**	14,985	3.35	4,477	0	0	32,000	17,015	2.47	6,889	2,412
4	3,322	3,322	0.47	**1,561**	16,546	3.48	4,750	0	0	32,000	15,454	2.47	6,257	1,506
5	1,815	1,815	0.47	**853**	17,399	3.55	4,896	0	0	32,000	14,601	2.47	5,911	1,015
6	800	800	0.47	376	17,775	3.58	4,959	0	0	32,000	14,225	2.47	5,759	800
7	0	0	0.47	0	17,775	3.58	4,959	0	0	32,000	14,225	2.47	5,759	800
8	-800	-800	0.47	-376	17,399	3.55	4,896	0	0	32,000	14,601	2.47	5,911	1,015
9	-1,815	-1,815	0.47	**-853**	16,546	3.48	4,750	0	0	32,000	15,454	2.47	6,257	1,506
10	-3,322	-3,322	0.47	**-1,561**	14,985	3.35	4,477	0	0	32,000	17,015	2.47	6,889	2,412
11	-5,733	-5,733	0.47	**-2,695**	12,290	3.08	3,990	0	0	32,000	19,710	2.47	7,980	3,990
12	-9,723	-9,723	0.47	**-4,570**	7,721	2.47	3,126	0	0	32,000	24,279	2.47	9,830	6,704
	-16,427	-16,427	0.47	**-7,721**										

Goal Seek

Sum Doubler	Lwr Doubler	Load in Base	Upper Doubler	Σ
	0	*32,000*		
7,721	7,721	24,279	0	32,000
12,290	9,855	19,710	2,435	32,000
14,985	11,059	17,015	3,926	32,000
16,546	11,733	15,454	4,813	32,000
17,399	12,093	14,601	5,306	32,000
17,775	12,250	14,225	5,525	32,000
17,775	12,250	14,225	5,525	32,000
17,399	12,093	14,601	5,306	32,000
16,546	11,733	15,454	4,813	32,000
14,985	11,059	17,015	3,926	32,000
12,290	9,855	19,710	2,435	32,000
7,721	7,721	24,279	0	32,000
	0	*32,000*		

0 0 Goal Seek Set = 0 **0** Changing Cell C204

1	2	4	5	6	7	8	9	10	11	12	13	14	15	16
Fastener Number	Strain Delta $\Delta(\delta_s - \delta_d)$	Fastener Strain $\Delta\delta_F$	Spring Constant k_F	Fastener Load P_F	Doubler Load P_D	Spring Constant k_D	Doubler Strain $\Delta\delta_D$	Intermed Loads $q_a L_n$	Intermed Loads Σ (10)	Applied Load q_L + (11)	Load in Base P_S	Spring Constant k_S	Base Strain δ_S	Strain Delta $\delta_S - \delta_D$
n	$(2)_{n-1} - (16)_{n-1}$	(2) + (3)	x 10^6 lb/in	(4) * (5)	Σ (6)	x 10^6 lb/in	(7) / (8)	$q_a L_n$	Σ (10)	q_L + (11)	(12) - (7)	x 10^6 lb/in	(13) / (14)	(15) - (9)
1	4,154	4,154	0	0	0	0	4,154	7,721	7,721	7,721	**7,721**	2.47	3,126	-1,028
2	5,182	5,182	0.47	**2,435**	2,435	1.23	1,980	4,570	12,290	12,290	**9,855**	2.47	3,990	2,010
3	3,172	3,172	0.47	**1,491**	3,926	1.23	3,192	2,695	14,985	14,985	**11,059**	2.47	4,477	1,285
4	1,887	1,887	0.47	**887**	4,813	1.23	3,913	1,561	16,546	16,546	**11,733**	2.47	4,750	837
5	1,049	1,049	0.47	**493**	5,306	1.23	4,314	853	17,399	17,399	**12,093**	2.47	4,896	582
6	467	467	0.47	220	5,525	1.23	4,492	376	17,775	17,775	12,250	2.47	4,959	467
7	0	0	0.47	0	5,525	1.23	4,492	0	17,775	17,775	12,250	2.47	4,959	467
8	-467	-467	0.47	-220	5,306	1.23	4,314	-376	17,399	17,399	**12,093**	2.47	4,896	582
9	-1,049	-1,049	0.47	**-493**	4,813	1.23	3,913	-853	16,546	16,546	**11,733**	2.47	4,750	837
10	-1,887	-1,887	0.47	**-887**	3,926	1.23	3,192	-1,561	14,985	14,985	**11,059**	2.47	4,477	1,285
11	-3,172	-3,172	0.47	**-1,491**	2,435	1.23	1,980	-2,695	12,290	12,290	**9,855**	2.47	3,990	2,010
12	-5,182	-5,182	0.47	**-2,435**	0	0	4,222	-4,570	7,721	7,721	**7,721**	2.47	3,126	-1,097
	-4,085	-4,085	0	0				-7,721						

Goal Seek

Table I Column 9 Doubler Strain inch / inch	Table II Column 15 Base Strain inch / inch	Ratio	k	k_{eff} lb/in per inch
0.00313	0.00313	1.00	2.47	2.47
0.00399	0.00399	1.00	3.08	3.08
0.00448	0.00448	1.00	3.35	3.35
0.00475	0.00475	1.00	3.48	3.48
0.00490	0.00490	1.00	3.55	3.55
0.00496	0.00496	1.00	3.58	3.58
0.00496	0.00496	1.00	3.58	3.58
0.00490	0.00490	1.00	3.55	3.55
0.00475	0.00475	1.00	3.48	3.48
0.00448	0.00448	1.00	3.35	3.35
0.00399	0.00399	1.00	3.08	3.08
0.00313	0.00313	1.00	2.47	2.47

0 0 Goal Seek Set = 0 **0** Changing Cell C229

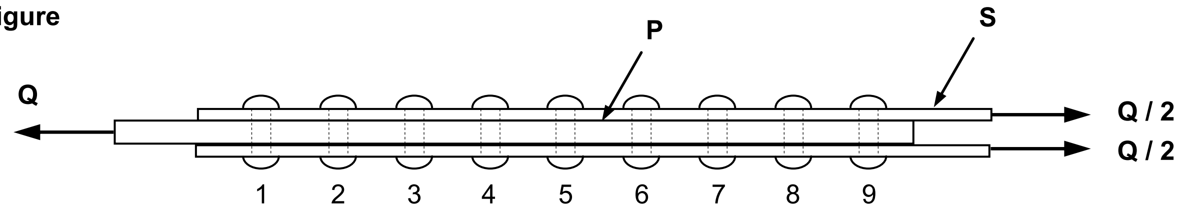
6.0 Butt Joints

Recurrence Formula

Samuel J. Rosenfeld

NACA TN-1458 *Analytical and Experimental Investigation of Bolted Joints* Appendix B, page 21

Figure



24S-T Plates E = 10,500 ksi Pitch, p = 1.25 inch t_p = 0.375 inch t_s = 0.1875 inch

Alloy Steel Bolts E_{bb} = 29,000 ksi D = 0.250 inch Width, b = 3.50 inch

Plate Constant

$$K_s = \frac{p}{b\ t_s\ E} = \frac{1.25\ inch}{3.50\ inch\ \ (\ 0.1875\ inch\ )\ \ 10{,}500\ ksi} = 0.0001814$$

Bolt Constant

General Expression

NACA TN-1051 *Preliminary Investigation of the Loads Carried by Individual Bolts in Bolted Joints*

$$C = \frac{2\ t_s + t_p}{3\ G_b\ A_b} + \frac{8\ t_s^{\ 3} + 16\ t_s^{\ 2}\ t_p + 8\ t_s\ t_p^{\ 2} + t_p^{\ 3}}{192\ E_{bb}\ I_b} + \frac{2\ t_s + t_p}{t_s\ t_p\ E_{b\,br}} + \frac{1}{t_s\ E_{s\,br}} + \frac{2}{t_s\ E_{p\,br}}$$

Equal Plate Areas

For 24S-T plates with alloy steel bolts and equal plate areas:

$$t_s = \frac{t_p}{2}$$

$$C = \frac{8}{t_p\ E_{bb}} \left\{ 0.13 \left(\frac{t_p}{D}\right)^2 \left[2.12 + \left(\frac{t_p}{D}\right)^2 \right] + 1.87 \right\}$$ NACA TN-1051 Equation 4, page 7

$$C = \frac{8}{0.375\ (\ 29{,}000\ ksi\ )} \left\{ 0.13 \left(\frac{0.375\ inch}{0.250\ inch}\right)^2 \left[2.12 + \left(\frac{0.375\ inch}{0.250\ inch}\right)^2 \right] + 1.87 \right\} = 0.002316$$

Loads in Successive Bolts – General Relationship NACA TN-1051 Equation 1, page 6

$$R_{i+1} = \frac{C_i}{C_{i+1}}\ R_i + \frac{2\ K_p + K_s}{C_{i+1}}\ R_i - \frac{2\ K_p}{C_{i+1}}\ Q + \frac{2\ K_p + K_s}{C_{i+1}} \sum_{1}^{i-1} R$$

For a butt strap thickness half of the the main plate thickness NACA TN-1458 Equation 7, page 7

$$R_{i+1} = R_{i-1} - \left(2 + \frac{2\ K_s}{C} \right) R_i + R_{i+1}$$

Matrix Form

$$\begin{bmatrix} -1.1567 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2.1567 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2.1567 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2.1567 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2.1567 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.1567 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2.1567 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.1567 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1.1567 \end{bmatrix} \begin{Bmatrix} R_0 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \end{Bmatrix} = \begin{Bmatrix} -0.0783\ Q \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.0783\ Q \end{Bmatrix}$$

Invert Matrix and Multiply

$$\begin{bmatrix} -2.080 & -1.406 & -0.952 & -0.647 & -0.443 & -0.309 & -0.224 & -0.173 & -0.150 \\ -1.406 & -1.626 & -1.101 & -0.748 & -0.513 & -0.358 & -0.259 & -0.200 & -0.173 \\ -0.952 & -1.101 & -1.422 & -0.967 & -0.663 & -0.462 & -0.334 & -0.259 & -0.224 \\ -0.647 & -0.748 & -0.967 & -1.337 & -0.916 & -0.639 & -0.462 & -0.358 & -0.309 \\ -0.443 & -0.513 & -0.663 & -0.916 & -1.313 & -0.916 & -0.663 & -0.513 & -0.443 \\ -0.309 & -0.358 & -0.462 & -0.639 & -0.916 & -1.337 & -0.967 & -0.748 & -0.647 \\ -0.224 & -0.259 & -0.334 & -0.462 & -0.663 & -0.967 & -1.422 & -1.101 & -0.952 \\ -0.173 & -0.200 & -0.259 & -0.358 & -0.513 & -0.748 & -1.101 & -1.626 & -1.406 \\ -0.150 & -0.173 & -0.224 & -0.309 & -0.443 & -0.647 & -0.952 & -1.406 & -2.080 \end{bmatrix} \begin{Bmatrix} -0.0783\ Q \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.0783\ Q \end{Bmatrix} = \begin{Bmatrix} 0.1746\ Q \\ 0.1237\ Q \\ 0.0921\ Q \\ 0.0749\ Q \\ 0.0695\ Q \\ 0.0749\ Q \\ 0.0921\ Q \\ 0.1237\ Q \\ 0.1746\ Q \end{Bmatrix}$$

$R_0 = 0.1746\ Q$

$R_1 = 0.1237\ Q$

$R_2 = 0.0921\ Q$

$R_3 = 0.0749\ Q$

$R_4 = 0.0695\ Q$

$R_5 = 0.0749\ Q$

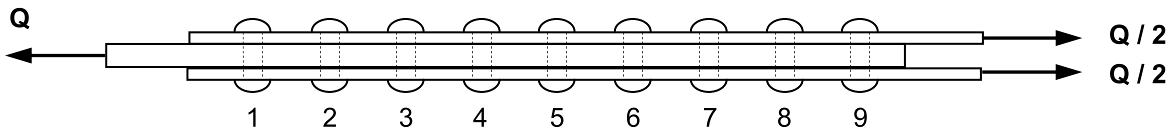
$R_6 = 0.0921\ Q$

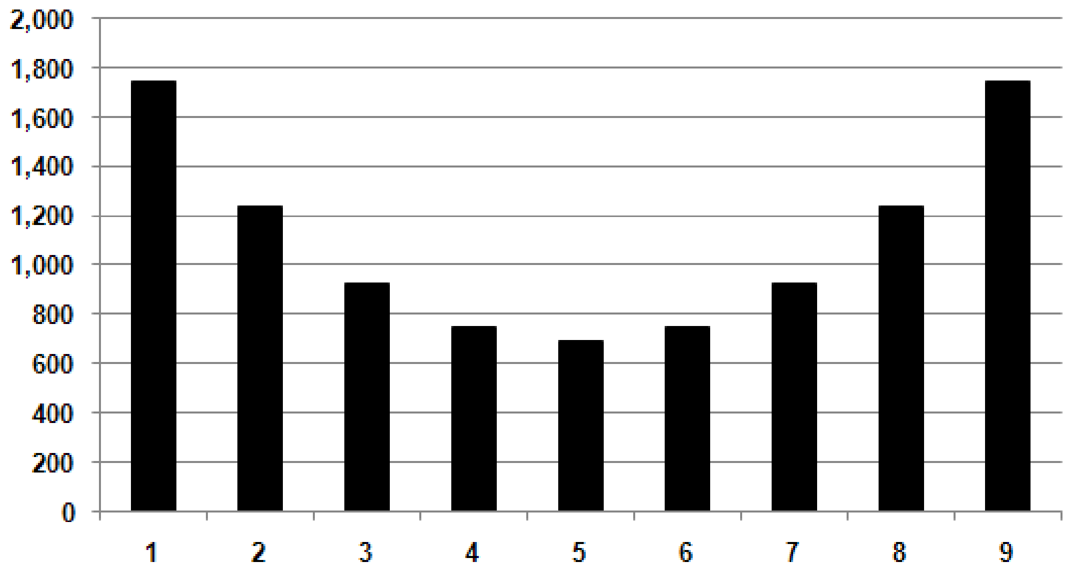
$R_7 = 0.1237\ Q$

$R_8 = 0.1746\ Q$

Fastener Loads

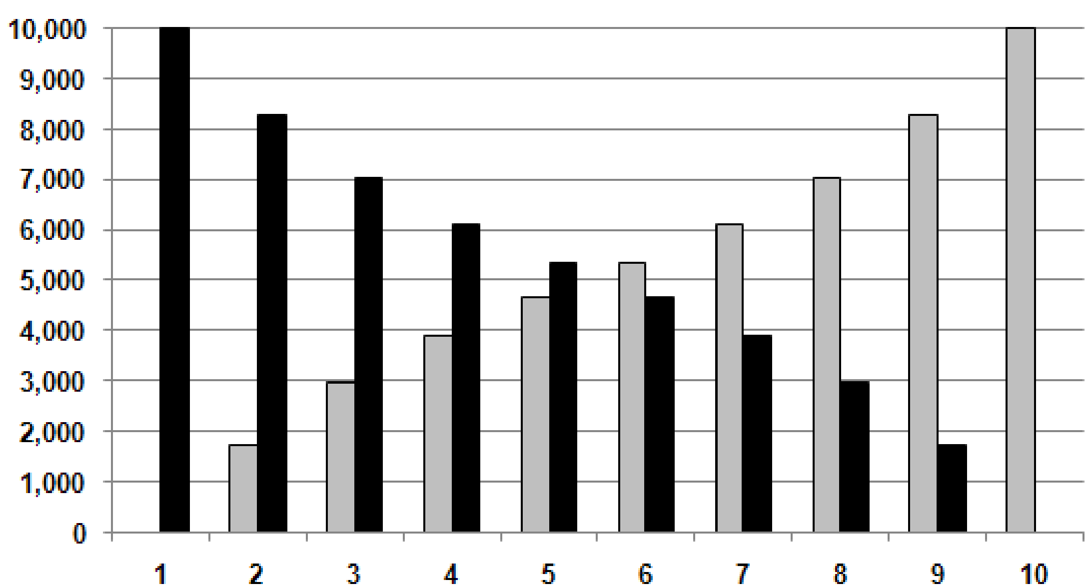
For Q = 10,000 lb





Load Distribution

For Q = 10,000 lb

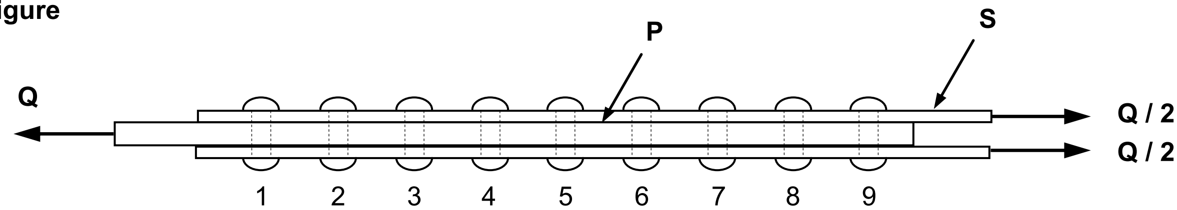


Finite Difference Equation

Samuel J. Rosenfeld

NACA TN-1458 *Analytical and Experimental Investigation of Bolted Joints* Appendix B, page 22

Figure



24S-T Plates E = 10,500 ksi Pitch, p = 1.25 inch t_p = 0.375 inch t_s = 0.1875 inch
Alloy Steel Bolts E_{bb} = 29,000 ksi D = 0.250 inch Width, b = 3.50 inch

Constants

$$2\ K_p \ = \ K_s \ = \ \frac{p}{b\ t_s\ E} = \frac{1.25\ inch}{3.50\ inch\ \ (0.1875\ inch)\ \ 10{,}500\ ksi} \ = \ 0.0001814$$

$$C \ = \ \frac{8}{t_p\ E_{bb}} \left\{ 0.13 \ \left(\frac{t_p}{D}\right)^2 \left[2.12 \ + \ \left(\frac{t_p}{D}\right)^2 \right] \ + \ 1.87 \right\}$$

$$C \ = \ \frac{8}{0.375\ \ (29{,}000\ ksi)} \left\{ 0.13 \ \left(\frac{0.375\ inch}{0.250\ inch}\right)^2 \left[2.12 \ + \ \left(\frac{0.375\ inch}{0.250\ inch}\right)^2 \right] \ + \ 1.87 \right\} \ = \ 0.002316$$

$$\varphi \ = \ \frac{2\ K_p \ + \ K_s}{C} \ = \ \frac{0.0001814 \ + \ 0.0001814}{0.002316} \ = \ 0.15666$$

$$\lambda \ = \ cosh^{-1} \ \left(1 \ + \ \frac{\varphi}{2} \right) \ = \ cosh^{-1} \ \left(1 \ + \ \frac{0.15666}{2} \right) \ = \ 0.39326$$

$$\alpha \ = -\left[\frac{\varphi \ + \ (\ 2\ K_p \ / \ C\ )\ (\ e^{-n\lambda} \ - \ 1\ )}{(\ e^{n\lambda} - \ e^{-n\lambda}\ )\ (\ e^{-\lambda} \ - \ 1\ )} \right] \ Q$$

$$\alpha \ = -\left\{ \frac{0.15666 \ + \ (\ 0.0001814 \ / \ 0.002316\ )\ \left[e^{-9\ (0.39326)} \ - \ 1 \right]}{\left[e^{9\ (0.39326)} \ - \ e^{-9\ (0.39326)} \right]\ (\ e^{-0.39326} \ - \ 1\ )} \right\} \ Q \ = \ 0.007203\ Q$$

$$\beta \ = \left[\frac{\varphi \ + \ (\ 2\ K_p \ / \ C\ )\ (\ e^{n\lambda} \ - \ 1\ )}{(\ e^{n\lambda} - \ e^{-n\lambda}\ )\ (\ e^{\lambda} \ - \ 1\ )} \right] \ Q$$

$$\beta \ = \left\{ \frac{0.15666 \ + \ (\ 0.0001814 \ / \ 0.002316\ )\ \left[e^{9\ (0.39326)} \ - \ 1 \right]}{\left[e^{9\ (0.39326)} \ - \ e^{-9\ (0.39326)} \right]\ (\ e^{0.39326} \ - \ 1\ )} \right\} \ Q \ = \ 0.167435\ Q$$

Bolt Expressions

$$R_0 = \alpha + \beta = 0.007203\ Q + 0.167435\ Q = 0.1746\ Q$$

$$R_1 = \alpha\ e^{\lambda} + \beta\ e^{-\lambda} = 0.007203\ Q\ e^{0.39326} + 0.167435\ Q\ e^{-0.39326} = 0.1237\ Q$$

$$R_2 = \alpha\ e^{2\lambda} + \beta\ e^{-2\lambda} = 0.007203\ Q\ e^{2(0.39326)} + 0.167435\ Q\ e^{-2(0.39326)} = 0.0921\ Q$$

$$R_3 = \alpha\ e^{3\lambda} + \beta\ e^{-3\lambda} = 0.007203\ Q\ e^{3(0.39326)} + 0.167435\ Q\ e^{-3(0.39326)} = 0.0749\ Q$$

$$R_4 = \alpha\ e^{4\lambda} + \beta\ e^{-4\lambda} = 0.007203\ Q\ e^{4(0.39326)} + 0.167435\ Q\ e^{-4(0.39326)} = 0.0695\ Q$$

$$R_5 = \alpha\ e^{3\lambda} + \beta\ e^{-3\lambda} = 0.007203\ Q\ e^{3(0.39326)} + 0.167435\ Q\ e^{-3(0.39326)} = 0.0749\ Q$$

$$R_6 = \alpha\ e^{2\lambda} + \beta\ e^{-2\lambda} = 0.007203\ Q\ e^{2(0.39326)} + 0.167435\ Q\ e^{-2(0.39326)} = 0.0921\ Q$$

$$R_7 = \alpha\ e^{\lambda} + \beta\ e^{-\lambda} = 0.007203\ Q\ e^{0.39326} + 0.167435\ Q\ e^{-0.39326} = 0.1237\ Q$$

$$R_8 = \alpha + \beta = 0.007203\ Q + 0.167435\ Q = 0.1746\ Q$$

$R_0 = 0.1746\ Q$

$R_1 = 0.1237\ Q$

$R_2 = 0.0921\ Q$

$R_3 = 0.0749\ Q$

$R_4 = 0.0695\ Q$

$R_5 = 0.0749\ Q$

$R_6 = 0.0921\ Q$

$R_7 = 0.1237\ Q$

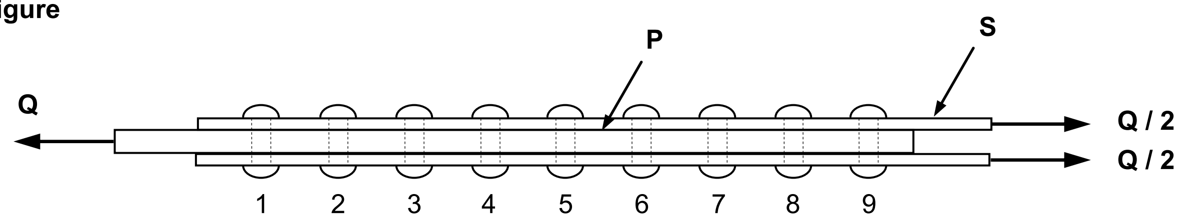
$R_8 = 0.1746\ Q$

Shear Lag Analogy

Samuel J. Rosenfeld

NACA TN-1458 *Analytical and Experimental Investigation of Bolted Joints* Appendix B, page 24

Figure



24S-T Plates E = 10,500 ksi Pitch, p = 1.25 inch t_p = 0.375 inch t_s = 0.1875 inch
Alloy Steel Bolts E_{bb} = 29,000 ksi D = 0.250 inch Width, b = 3.50 inch

Bolt Constant

General Expression

NACA TN-1458 Equation 2, page 5 NACA TN-1051 Equation A14, page 30

$$C = \frac{2\ t_s + t_p}{3\ G_b\ A_b} + \frac{8\ t_s^{\,3} + 16\ t_s^{\,2}\ t_p + 8\ t_s\ t_p^{\,2} + t_p^{\,3}}{192\ E_{bb}\ I_b} + \frac{2\ t_s + t_p}{t_s\ t_p\ E_{b\,br}} + \frac{1}{t_s\ E_{s\,br}} + \frac{2}{t_s\ E_{p\,br}}$$

Equal Plate Areas

NACA TN-1458 Equation 8, page 7 NACA TN-1051 Equation 4, page 7

When the joints are made of 24S-T plates with $t_s = t_p / 2$, fastened by alloy steel bolts, the expression for the bolt constant (equation 2) reduces to

$$C = \frac{8}{t_p\ E_{bb}} \left\{ 0.13 \left(\frac{t_p}{D}\right)^2 \left[2.12 + \left(\frac{t_p}{D}\right)^2 \right] + 1.87 \right\}$$

$$C = \frac{8}{0.375\ (\,29{,}000\ ksi\,)} \left\{ 0.13 \left(\frac{0.375\ inch}{0.250\ inch}\right)^2 \left[2.12 + \left(\frac{0.375\ inch}{0.250\ inch}\right)^2 \right] + 1.87 \right\} = 0.002316$$

Modified Shear Lag Parameter

$$A_s = t_s\ b$$

$$k^2 = \frac{2}{p\ E\ C\ A_s} = \frac{2}{1.25\ inch\ (\,10{,}500\ ksi\,)\ \ 0.002316\ (\,0.1875\ inch\,)\ \ 3.50\ inch} = 0.1003$$

$$k = \sqrt{0.1003} = 0.3166$$

Bolt Expressions

$$R_i = k\ p \left[\frac{\cosh k\ (\ L/2\ -\ x\ )}{2\ \sinh\left(\frac{k\ L}{2}\right)} \right]\ Q$$

$$R_i = \left\{ \left[\frac{k\ p}{2\ \sinh\left(\frac{k\ L}{2}\right)} \right] \cosh k\ (\ L/2\ -\ x\ ) \right\}\ Q$$

$$R_i = \left\{ \left[\frac{0.3166\ (\ 1.25\,inch\ )}{2\ \sinh\left[\frac{0.3166\ (\ 11.25\,inch\ )}{2}\right]} \right] \cosh 0.3166\ (\ 11.25\,inch\ /\ 2\ -\ x\ ) \right\}\ Q$$

$$R_0 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 0.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.1742\ Q$$

$$R_1 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 1.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.1230\ Q$$

$$R_2 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 2.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.0913\ Q$$

$$R_3 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 3.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.0741\ Q$$

$$R_4 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 4.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.0686\ Q$$

$$R_5 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 3.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.0741\ Q$$

$$R_6 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 2.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.0913\ Q$$

$$R_7 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 1.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.1230\ Q$$

$$R_8 = 0.0686\ \cosh\ \{\ 0.3166\ [\,5.625\,inch\ -\ 0.5\ (\ 1.25\,inch\ )\ ]\ \}\ Q = 0.1742\ Q$$

$R_0 = 0.1742\ Q$

$R_1 = 0.1230\ Q$

$R_2 = 0.0913\ Q$

$R_3 = 0.0741\ Q$

$R_4 = 0.0686\ Q$

$R_5 = 0.0741\ Q$

$R_6 = 0.0913\ Q$

$R_7 = 0.1230\ Q$

$R_8 = 0.1742\ Q$

Comparison

NACA TN-1458 *Analytical and Experimental Investigation of Bolted Joints* Appendix B, page 30

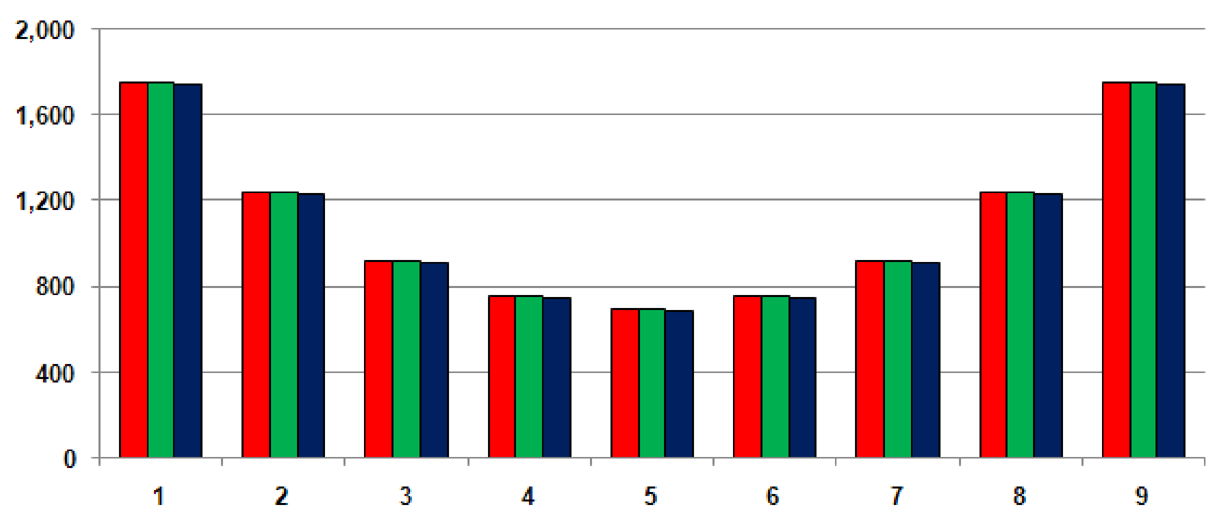
	Recurrence Formula		Finite Difference		Shear Lag Analogy	
Bolt	**Fraction**	**R / R_{avg}**	**Fraction**	**R / R_{avg}**	**Fraction**	**R / R_{avg}**
1	0.1746	1.5717	0.1746	1.5717	0.1742	1.5674
2	0.1237	1.1130	0.1237	1.1130	0.1230	1.1066
3	0.0921	0.8286	0.0921	0.8286	0.0913	0.8214
4	0.0749	0.6741	0.0749	0.6741	0.0741	0.6666
5	0.0695	0.6251	0.0695	0.6251	0.0686	0.6176
6	0.0749	0.6741	0.0749	0.6741	0.0741	0.6666
7	0.0921	0.8286	0.0921	0.8286	0.0913	0.8214
8	0.1237	1.1130	0.1237	1.1130	0.1230	1.1066
9	0.1746	1.5717	0.1746	1.5717	0.1742	1.5674
	1.0000		1.0000		0.9935	

$R_{avg} = Q / 9$ fasteners

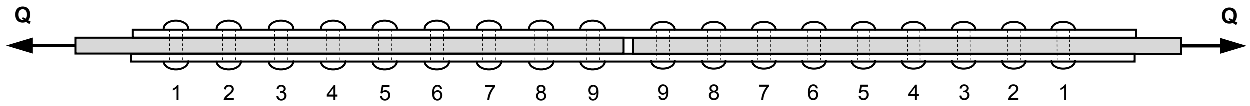
Fastener Loads

For $Q = 10,000$ lb

Recurrence Formula - Red **Finite Difference - Green** **Shear Lag Analogy - Blue**

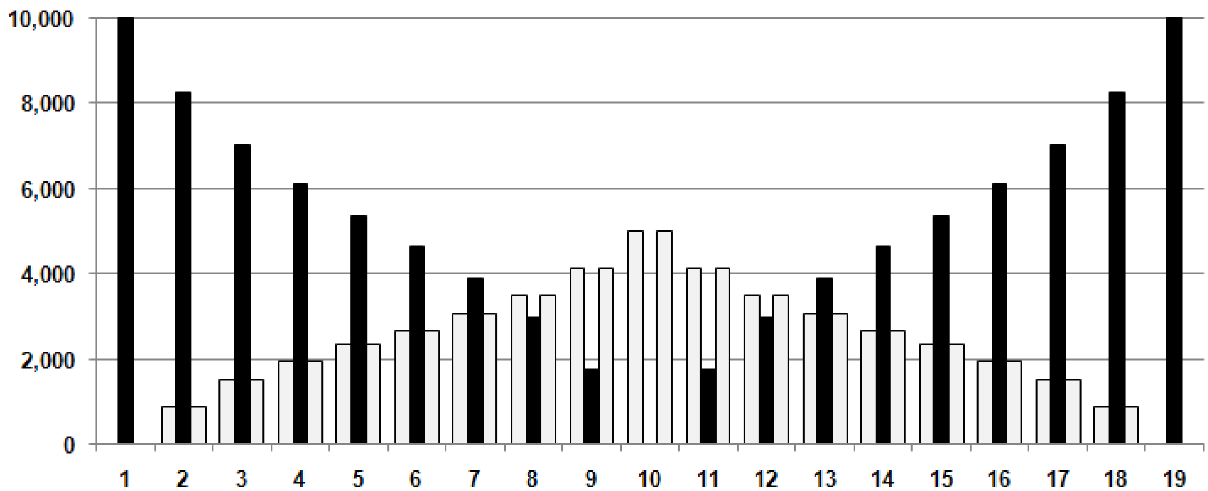


Butt Joint

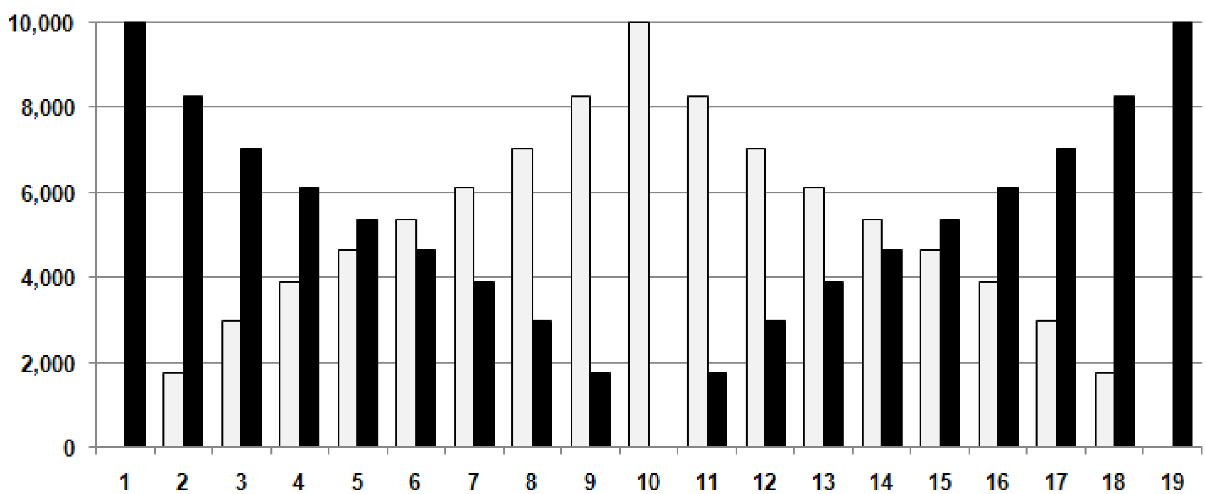


Load Distribution

Three Members



Combining Upper and Lower Straps



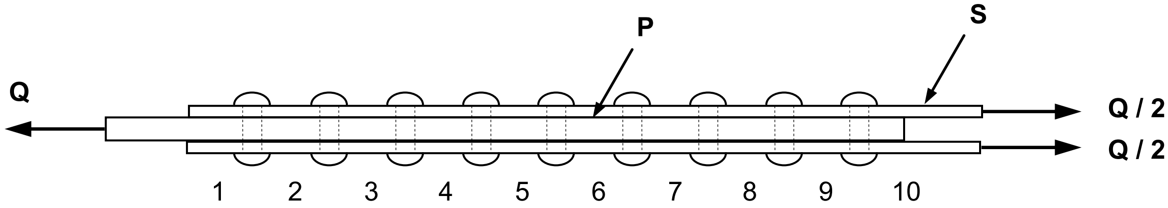
Note:

AFFDL-TR-67-184 *Analytical Design Methods for Aircraft Structural Joints* page 22

If desired, hyperbolic functions can be used to replace some of the exponential forms since

$$e^{z} - e^{-z} = 2 \sinh z \qquad \text{and} \qquad e^{z} + e^{-z} = 2 \cosh z$$

Parabolic WAG Method



24S-T Plates $E = 10{,}500$ ksi Pitch, $p = 1.25$ inch $t_p = 0.375$ inch $t_s = 0.1875$ inch

Alloy Steel Bolts $E_{bb} = 29{,}000$ ksi $D = 0.250$ inch Width, $b = 3.50$ inch

Average Bolt Load

$P_{avg} = 10{,}000 \text{ lb} \ / \ 9 \text{ bolts} = 1{,}111.1 \text{ lb}$

Assume Bolt Loads

Assume a parabolic bolt load distribution where:

End Bolts $P_1 = P_9 = P_{avg} \ (1 + n\,\%) = 1{,}111.1 \text{ lb} \ (1 + n\,\%)$

Center Bolt $P_5 = P_{avg} \ (n\,\%) = 1{,}111.1 \text{ lb} \ (n\,\%)$

$$\left(\frac{E_s\,A_s}{E_p\,A_p + E_s\,A_s} \right) = \left[\frac{10{,}500\ ksi\ (3.50\ inch)\ 0.1875\ inch\ (2)}{10{,}500\ ksi\ (3.50\ inch)\ 0.375\ inch + 10{,}500\ ksi\ (3.50\ inch)\ 0.1875\ inch\ (2)} \right]$$

$$\left(\frac{E_s\,A_s}{E_p\,A_p + E_s\,A_s} \right) = 0.50$$

Goal Seek

Solve for *n* by *Trial and Error*. You might use the “EA Ratio” of 50% for the first guess.

		x (inch)		P_F (lb)	P_S (lb)	P_P (lb)	
P_{avg}	1,111.1 lb	0			0	10,000	
		1.25	R_0	**1,721**	1,721	8,279	
$E_s\ A_s$	6.50E+06	2.5	R_1	1,282	3,003	6,997	
		3.75	R_2	953	3,955	6,045	
$E_p\ A_p$	1.30E+07	5	R_3	733	4,688	5,312	
		6.25	R_4	**623**	5,312	4,688	
	Changing	7.5	R_5	733	6,045	3,955	
n	**54.9%**	8.75	R_6	953	6,997	3,003	
		10	R_7	1,282	8,279	1,721	
		11.25	R_8	**1,721**	10,000	**0**	Goal Seek

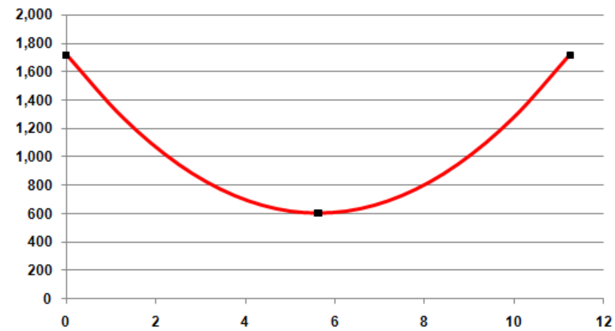
End Bolts $P_1 = P_9 = P_{avg} \ (1 + n\,\%) = 1{,}111.1 \text{ lb} \ (1 + \mathit{0.549}) = 1{,}721 \text{ lb}$

Center Bolt $P_5 = P_{avg} \ (n\,\%) = 1{,}111.1 \text{ lb} \ (\mathit{0.549}) = 610 \text{ lb}$

Parabolic Equation

DATA

x_1	0	inch	y_1	1,721	lb
x_2	5.625	inch	y_2	610	lb
x_3	11.250	inch	y_3	1,721	lb



3 Equations, 3 Unknowns

$y_1 = a\,x_1^2 + b\,x_1 + c$	0.000	1,721
$y_2 = a\,x_2^2 + b\,x_2 + c$	5.625	610
$y_3 = a\,x_3^2 + b\,x_3 + c$	11.250	1,721

In Matrix Form

$$\begin{vmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix}$$

Inverse

$$\begin{vmatrix} 0 & 0 & 1 \\ 31.64063 & 5.625 & 1 \\ 126.5625 & 11.25 & 1 \end{vmatrix}^{-1} = \begin{vmatrix} 0.016 & -0.032 & 0.016 \\ -0.267 & 0.356 & -0.089 \\ 1.000 & 0.000 & 0.000 \end{vmatrix}$$

Therefore:

$$\begin{vmatrix} 0 & 0 & 1 \\ 31.6 & 5.6 & 1 \\ 126.6 & 11.3 & 1 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 1{,}721 \\ 610 \\ 1{,}721 \end{vmatrix}$$

Check

$$\begin{vmatrix} 0 & 0 & 1 \\ 31.64063 & 5.625 & 1 \\ 126.5625 & 11.25 & 1 \end{vmatrix} \begin{vmatrix} 0.016 & -0.032 & 0.016 \\ -0.267 & 0.356 & -0.089 \\ 1.000 & 0.000 & 0.000 \end{vmatrix}$$

Solution

$$\begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 0.016 & -0.032 & 0.016 \\ -0.267 & 0.356 & -0.089 \\ 1.000 & 0.000 & 0.000 \end{vmatrix} \begin{vmatrix} 1{,}721 \\ 610 \\ 1{,}721 \end{vmatrix} = \begin{vmatrix} 35.1 \\ -395.1 \\ 1{,}721 \end{vmatrix}$$

$$\textit{Identity Matrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

a = 35.1

b = -395.1

c = 1,721

$$\boxed{y = a\,x^2 + b\,x + c}$$

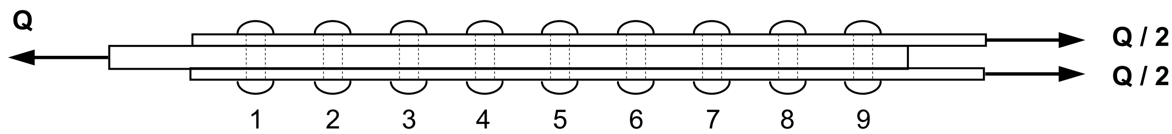
$$y = 35.12\ x^2 + (-395.06)\ x + 1{,}721$$

Solution

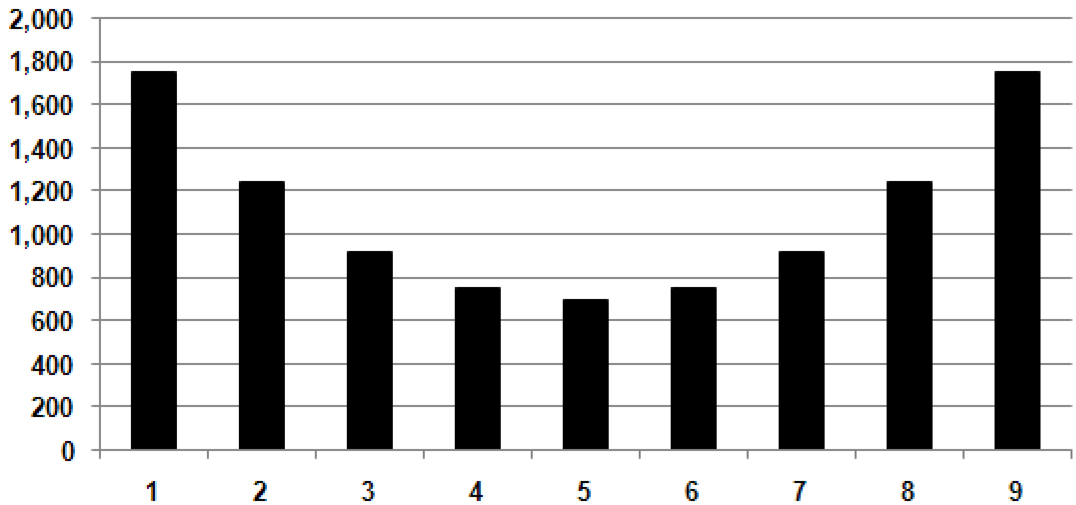
		Recurrence Formula				Parabolic WAG Method		
x		P_F	P_S	P_P		P_F	P_S	P_P
(inch)		(lb)	(lb)	(lb)		(lb)	(lb)	(lb)
0		0	0	10,000		0	0	10,000
1.25	R_0	1,746	1,746	8,254	R_0	1,721	1,721	8,279
2.50	R_1	1,237	2,983	7,017	R_1	1,282	3,003	6,997
3.75	R_2	921	3,904	6,096	R_2	953	3,955	6,045
5.00	R_3	749	4,653	5,347	R_3	733	4,688	5,312
6.25	R_4	695	5,347	4,653	R_4	623	5,312	4,688
7.50	R_5	749	6,096	3,904	R_5	733	6,045	3,955
8.75	R_6	921	7,017	2,983	R_6	953	6,997	3,003
10.00	R_7	1,237	8,254	1,746	R_7	1,282	8,279	1,721
11.25	R_8	1,746	10,000	0	R_8	1,721	10,000	**0**
		0				0		

Fastener Loads

Recurrence Formula

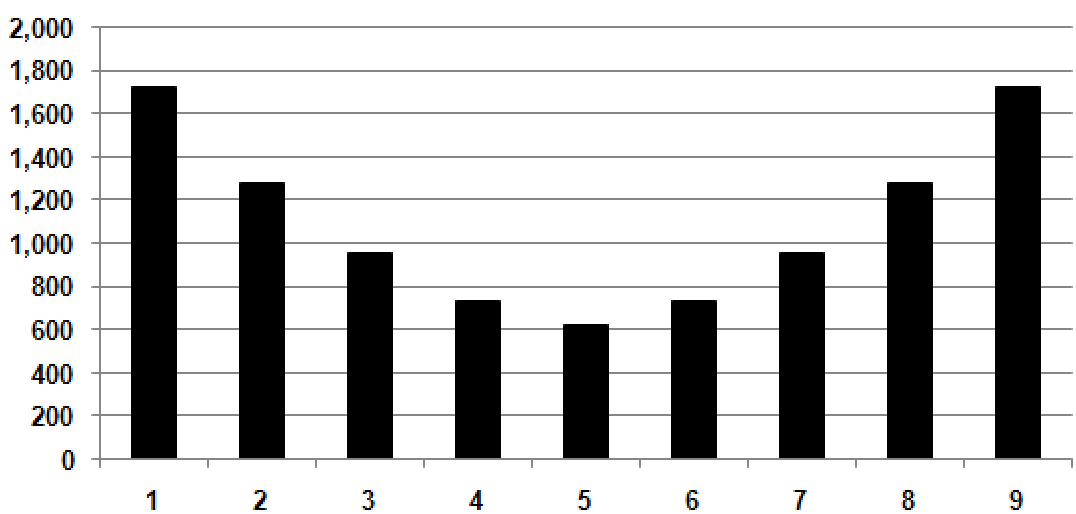


For Q = 10,000 lb



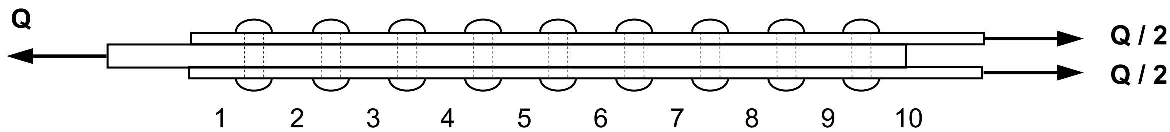
Parabolic WAG Method

For Q = 10,000 lb

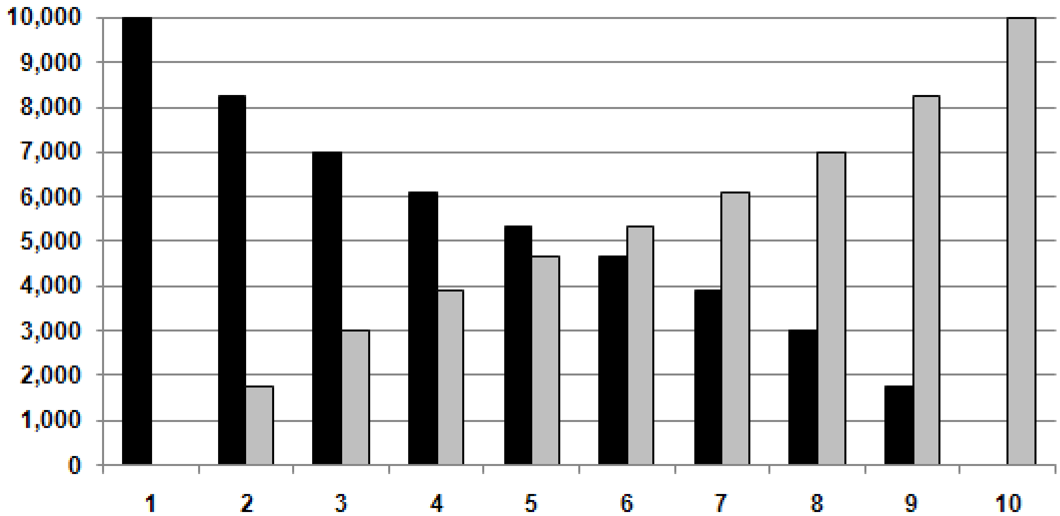


Load Distribution

Recurrence Formula

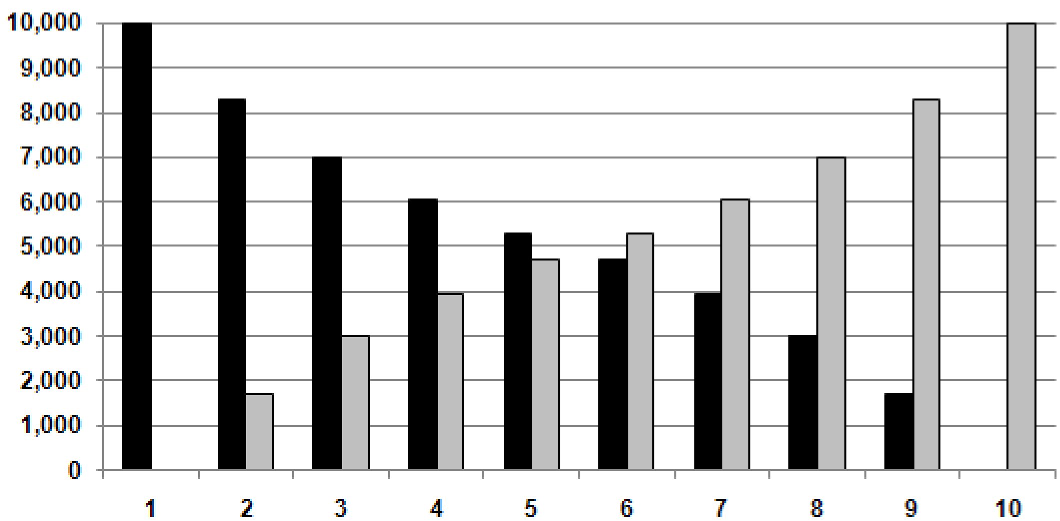


For Q = 10,000 lb



Parabolic WAG Method

For Q = 10,000 lb

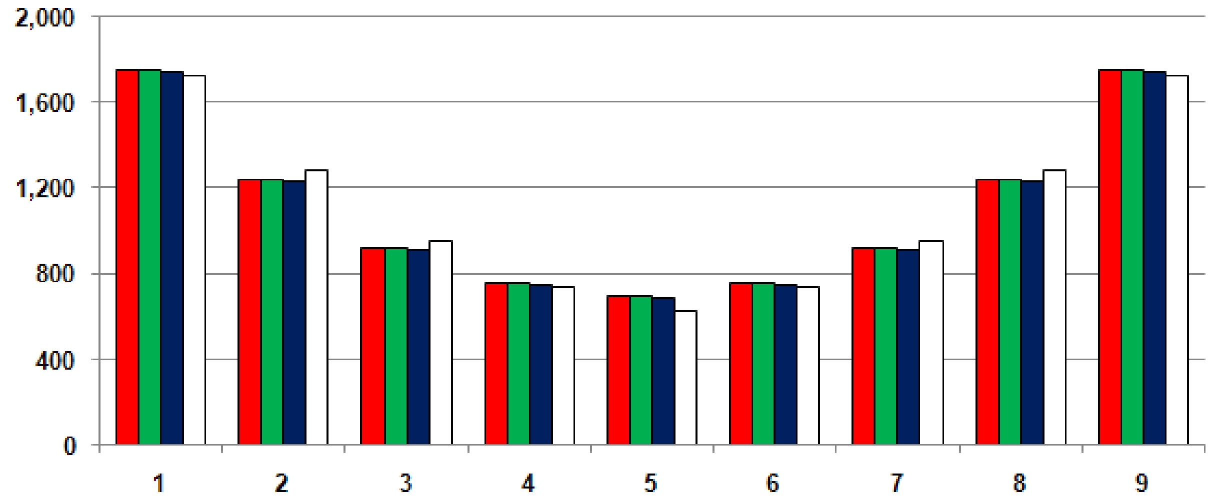


Comparison

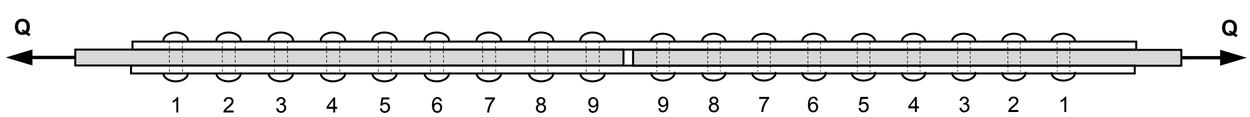
For Q = 10,000 lb

Parabolic WAG Method – White

Recurrence Formula – Red **Finite Difference** – Green **Shear Lag Analogy** – Blue

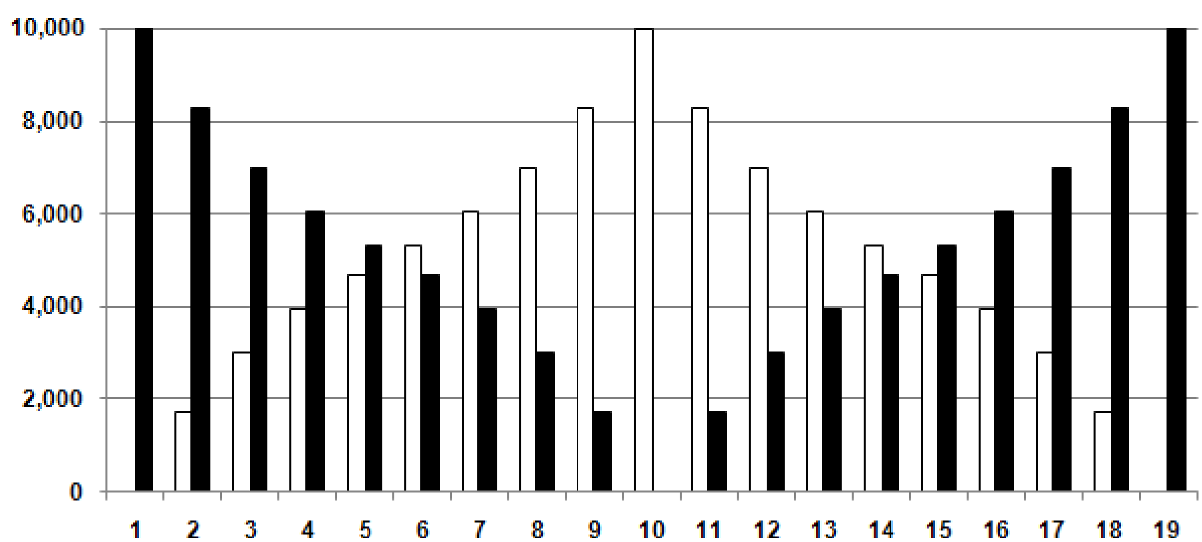


Butt Joint



Load Distribution

Combining Upper and Lower Straps

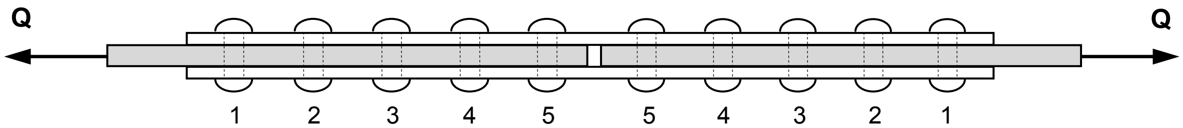


Butt Joint

Manford B. Tate and Samuel J. Rosenfeld

NACA TN-1051 *Preliminary Investigation of the Loads Carried by Individual Bolts in Bolted Joints*

Appendix B pages 34 to 36



24S-T Plates $E = 10{,}500$ ksi Pitch, $p = 1.00$ inch $t_p = 0.3125$ inch $t_s = 0.1875$ inch

Alloy Steel Bolts $E_{bb} = 29{,}000$ ksi $D = 0.250$ inch Width, $b = 2.00$ inch

Plate Constants

$$K_s = \frac{p}{b\ t_s\ E} = \frac{1.00\ inch}{2.00\ inch\ (0.1875\ inch)\ 10{,}500\ ksi} = 0.000254$$

$$2\ K_p = \frac{2\ p}{b\ t_s\ E} = \frac{2\ (1.00\ inch)}{2.00\ inch\ (0.3125\ inch)\ 10{,}500\ ksi} = 0.000305$$

Bolt Constant

General Expression

NACA TN-1051 *Preliminary Investigation of the Loads Carried by Individual Bolts in Bolted Joints*

$$C = \frac{2\ t_s + t_p}{3\ G_b\ A_b} + \frac{8\ t_s^{\ 3} + 16\ t_s^{\ 2}\ t_p + 8\ t_s\ t_p^{\ 2} + t_p^{\ 3}}{192\ E_{bb}\ I_b} + \frac{2\ t_s + t_p}{t_s\ t_p\ E_{b\,br}} + \frac{1}{t_s\ E_{s\,br}} + \frac{2}{t_s\ E_{p\,br}}$$

Unequal Plate Areas

Using average thickness in the equation for 24S-T plates with alloy steel bolts and equal plate areas:

$$t_{avg} = \frac{2\,t_s + t_p}{2} = \frac{2\ (0.1875\ inch) + 0.3125\ inch}{2} = 0.3438\ inch$$

NACA TN-1051 Equation 4, page 7

$$C_{avg} = \frac{8}{t_{avg}\ E_{bb}} \left\{ 0.13\ \left(\frac{t_{avg}}{D}\right)^2 \left[2.12 + \left(\frac{t_{avg}}{D}\right)^2 \right] + 1.87 \right\}$$

$$C_{avg} = \frac{8}{0.3438\ (29{,}000\ ksi)} \left\{ 0.13\ \left(\frac{0.3438\ inch}{0.250\ inch}\right)^2 \left[2.12 + \left(\frac{0.34385\ inch}{0.250\ inch}\right)^2 \right] + 1.87 \right\} = 0.002292$$

Loads in Successive Bolts – General Relationship

NACA TN-1051 Equation 1, page 6

$$R_{i+1} = \frac{C_i}{C_{i+1}} R_i + \frac{2 K_p + K_s}{C_{i+1}} R_i - \frac{2 K_p}{C_{i+1}} Q + \frac{2 K_p + K_s}{C_{i+1}} \sum_1^{i-1} R$$

Bolts are identical …

$$C_i = C_{i+1}$$

$$R_{i+1} = R_i + \frac{2 K_p + K_s}{C} R_i - \frac{2 K_p}{C} Q + \frac{2 K_p + K_s}{C} \sum_1^{i-1} R$$

$$\frac{2 K_p + K_s}{C_{avg}} = \frac{0.000305 + 0.000254}{0.002292} = 0.2438$$

$$\frac{2 K_p}{C_{avg}} = \frac{0.000305}{0.002292} = 0.1330$$

Loads in Successive Bolts

$$R_1 = R_1$$

$$R_2 = R_1 + 0.2438 R_1 - 0.1330 Q = 1.2438 R_1 - 0.1330 Q$$

$$R_3 = R_2 + 0.2438 R_2 - 0.1330 Q + 0.2438 R_1$$

$$R_3 = 1.2438 (1.2438 R_1 - 0.1330 Q) - 0.1330 Q + 0.2438 R_1$$

$$R_3 = 1.7908 R_1 - 0.2984 Q$$

$$R_4 = R_3 + 0.2438 R_3 - 0.1130 Q + 0.2438 (R_1 + R_2)$$

$$R_4 = 1.2438 (1.7908 R_1 - 0.2984 Q) - 0.1330 Q + 0.2438 (2.2438 R_1 - 0.1330 Q)$$

$$R_4 = 2.7744 R_1 - 0.5365 Q$$

$$R_5 = R_4 + 0.2438 R_4 - 0.1130 Q + 0.2438 (R_1 + R_2 + R_3)$$

$$R_5 = 1.2438 (1.7908 R_1 - 0.2984 Q) - 0.1330 Q + 0.2438 (4.0346 R_1 - 0.4314 Q)$$

$$R_5 = 4.4346 R_1 - 0.9055 Q$$

$$Q = \sum R = 11.244 R_1 - 1.873 Q$$

Fastener Loads

$$Q = \sum R = 11.244\, R_1 - 1.873\ Q$$

$$R_1 = \frac{2.873}{11.244}\ Q = 0.2556\, Q$$

$$R_2 = 1.2438\ (0.2556\, Q) - 0.1330\ Q = 0.1849\, Q$$

$$R_3 = 1.7908\ (0.2556\, Q) - 0.2984\ Q = 0.1593\, Q$$

$$R_4 = 2.7744\ (0.2556\, Q) - 0.5365\ Q = 0.1725\, Q$$

$$R_5 = 4.4346\ (0.2556\, Q) - 0.9055\ Q = 0.2278\, Q$$

$$R_{avg} = 0.20\ Q$$

$$R_1 / R_{avg} = 1.278$$

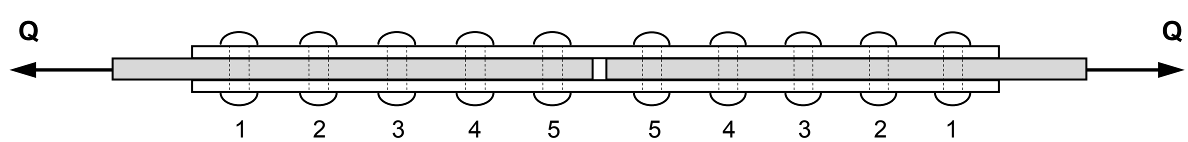
$$R_2 / R_{avg} = 0.924$$

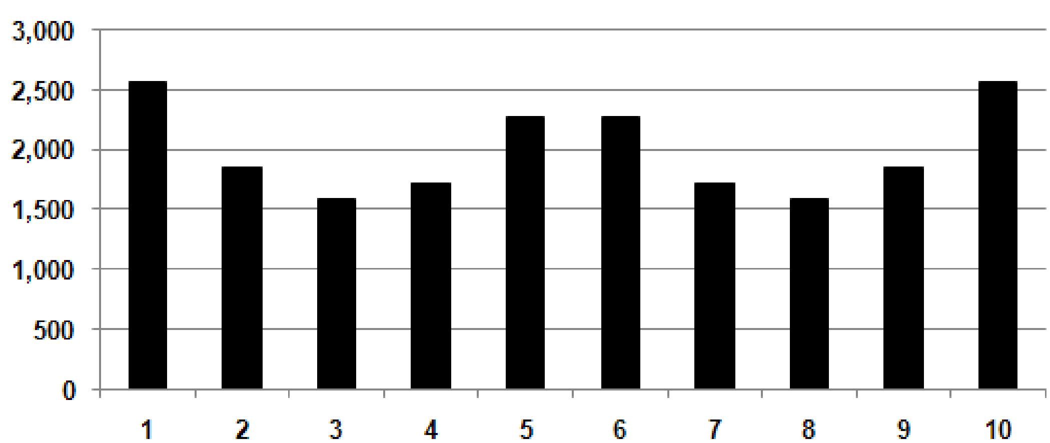
$$R_3 / R_{avg} = 0.796$$

$$R_4 / R_{avg} = 0.863$$

$$R_5 / R_{avg} = 1.139$$

For Q = 10,000 lb





7.0 Fatigue

NAVWEPS 00-25-559 *Tips on Fatigue* Clarence R. Smith

Department of the Navy Prepared for the Bureau of Naval Weapons Selected quotes below.

Lap Joints

While the lap joint is the simplest of all joints, its main problem is that, when the two sheets of material are joined, they tend to align themselves with each other. This causes the sheet to be bent at the first fastener, which is already suffering from too much load. This offset in alignment is commonly called eccentricity.

Butt Joints

Single Shear

The single shear butt joint is really two lap joints facing each other, so it has the same problem as the lap joint. One of its advantages is that the doubler can be made thicker than the material being spliced. This reduces the effects of bending, but it creates an additional problem: the rivet nearest the doubler's edge now carries most of the load just as in the case of the clevis joint …

Double Shear

Double shear butt joints are superior to those of the single shear type. This is because the symmetry of the double shear type eliminates the bending effects found in the single shear. However, the double shear type also has the problem of load distribution between fasteners. Scarfing, or providing auxiliary doublers as for single shear joints, will improve fatigue life.

Doublers

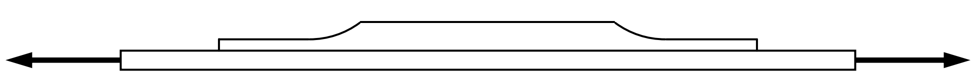
Some degree of deformation can be achieved by thinning the doubler material between the first two rows of fasteners so the second row can carry some of the load. Since doubler material must stretch in order to do this, the thickness at the first fastener should be less than half of the material being spliced.

The auxiliary doubler should be long enough to engage an extra row of rivets outside the main splice area. Here again there is a compromise between the practical and theoretical optimum thickness of auxiliary doublers.

Auxiliary thin doublers when properly used will increase the lifetime more than ten times.

Intuition should tell us that the doubler material between the first two rows of fasteners should be thinned down so it will stretch without overloading the first row of fasteners.

Stepped Doubler



References

NAVWEPS 00-25-559	*Tips on Fatigue*	Clarence R. Smith
Department of the Navy	Prepared for the Bureau of Naval Weapons	

ASTM International – Special Technical Publications

STP 9 *References on Fatigue*
STP 91 *Manual on Fatigue Testing*
STP 203 *Fatigue on Aircraft Structures*
STP 237 *Symposium on Basic Mechanism of Fatigue*
STP 274 *Symposium on Fatigue of Aircraft Structures*
STP 284 *Symposium on Acoustical Fatigue*
STP 338 *Symposium on Fatigue Tests of Aircraft Structures: Low-cycle, Full-scale, and Helicopters*

Note: ASTM International was originally called the American Society for Testing and Materials.

http://www.astm.org/COMMIT/filtrexx40.cgi?-P+COMMIT+E08+commitpubs_stp.frm

American Society of Mechanical Engineers – ASME

Conference (International) on Fatigue of Metals – Proceedings

ASM International (American Society for Metals)

Metals Handbook

Society for Experimental Mechanics – SEM

Handbook of Experimental Stress .... Edited by M. Hetenyi

Note: "The Society for Experimental Mechanics, originally called The Society for Experimental Stress Analysis, was founded in 1943 as a nonprofit scientific and educational organization …" (Wikipedia)

Miscellaneous

Fatigue of Metals .... P.G. Forrest
Fatigue in Aircraft Structure .... A.M. Freudenthal
The Fatigue of Metals and Structures .... Gordon Grover & Jackson
Metallic Fatigue with Particular Reference to Significance of Certain Standard Aircraft Fabrication and Finishing Process .... W.J. Harris
Designing by Photoelasticity .... R.B. Heywood
Designing Against Fatigue of Metals .... R.B. Heywood
Stress Concentration Design Factors .... R.E. Peterson
Full-Scale Fatigue Testing of Aircraft Structures .... R.J. Plantema & Schijve (Editors)
Metal Fatigue .... George Sines and J.L. Waisman
Fatigue Testing and Analysis of Results .... W. Weibull