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Flexibility of, and load distribution in, multi-bolt lap joints subject to in-plane axial loads

Associated software: ESDUpac A9812

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FLEXIBILITY OF, AND LOAD DISTRIBUTION IN, MULTI-BOLT LAP JOINTS SUBJECT TO IN-PLANE AXIAL LOADS

CONTENTS

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1. INTRODUCTION, THEORY AND APPLICATION

1.1 INTRODUCTION

This Data Item, together with ESDUpac A9812, provides a computer program for the analysis of multi-bolt, single row, lap joints. The Data Item is provided in two main sections. Section 1 describes the theoretical solution and its application to practical bolted joints. Section 2 describes the use of the Fortran program, its input, output and all messages that might be encountered. Example calculations are also provided. The program is available in the form of a Fortran source code and corresponding executable file. Both files can be found in ESDUscope (the CD version of the Series), on the internet version of the Series, and on disk in the Series Software Volume.

The analysis applies to multi-bolt, single row, lap joints loaded in either axial tension or compression. It requires the flexibilities of each of the bolts under shear loading. A theory for the calculation of these flexibilities is also described and may be used when they are not available. However, without test data to support the values of the flexibilities used, it is recommended that the calculation of the flexibilities of individual bolts via the program is only used for comparative purposes.

The analysis allows for joints in which the bolts are of different sizes and are not at a regular pitch and in which the thickness and/or width of the joined plates vary between bolt pitches. Although the analysis is performed in terms of a single lap joint, it may be applied to symmetric double lap joints. It may also be applied to joints having multiple rows of bolts provided that the rows are identical. Notes on the appropriate factors for the plate stiffness and bolt head restraints are included together with details of their derivation. Figures are provided that illustrate the influence of the major parameters on the bolt loads and joint flexibility.

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

A	cross-sectional area of plate $(t \times w)$	m^2	in^2
A_b	bolt cross-sectional area	m^2	in^2
A_{ps}	effective area of plate over which shear acts (see Equation (1.13))	m^2	in^2
D_b	bolt diameter	m	in
D_{ref}	reference bolt diameter used when estimating k (see Equation (1.21))	m	in
E	modulus of elasticity	N/m^2	lbf/in^2
F_b	flexibility of single bolt/hole* joining two plates in single shear, $F_b = f_{bA} + f_{bC} = 1/S_b = \delta_{bT}/P_b$	m/N	in/lbf

* *For footnote see end of Notation Section*

F_j	flexibility of joint between reference points defined by L_A and L_C	m/N	in/lbf
f_b	flexibility of single bolt/hole* in single plate, $f_b = \delta_b/P_b$	m/N	in/lbf
G	shear modulus	N/m^2	lbf/in^2
I_b	second moment of area of bolt cross-section	m^4	in^4
k	effective stiffness, per unit thickness, of plate supporting bolt (see Sections 1.3.3.1 and 1.4.1.1 and the examples of Section 2.5)	N/m^2	lbf/in^2
L	length measured along joint	m	in
L_A	length from arbitrary reference point to first bolt, see Sketch 1.1	m	in
L_C	length from last bolt to arbitrary reference point, see Sketch 1.1	m	in
l_{inf}	length of plate loaded in tension over which stress distribution is influenced by presence of a bolt (see Section 1.4.1.4)	m	in
M_{bx}	internal moment in bolt at distance x (see Section 1.3.3)	N m	lbf in
N	number of bolts along joint		
P_b	shear load on bolt	N	lbf
P_j	total shear load on joint, tension positive, compression negative	N	lbf
Q_{bx}	shear in bolt at distance x (see Section 1.3.3)	N	lbf
q_{px}	reaction in plate supporting bolt per unit length at distance x (see Equation (1.13))	N/m	lbf/in
S_b	stiffness of bolt/hole* at joint of two plates in single shear, $S_b = 1/F_b$	N/m	lbf/in
S_j	joint stiffness between arbitrary reference points defined by L_A and L_C, $S_j = 1/F_j$	N/m	lbf/in
t	thickness of plate	m	in
w	width of plate	m	in
w_{eff}	effective value of w for plate loaded in tension (see Section 1.4.1.4)	m	in
β	rotation of bolt axis due to shear, $\beta = Q_{bx}/\mu\, G_b A_b$	rad	rad
x	longitudinal bolt co-ordinate, see Sketch 1.3	m	in
y	bolt deflection, see Sketch 1.3	m	in

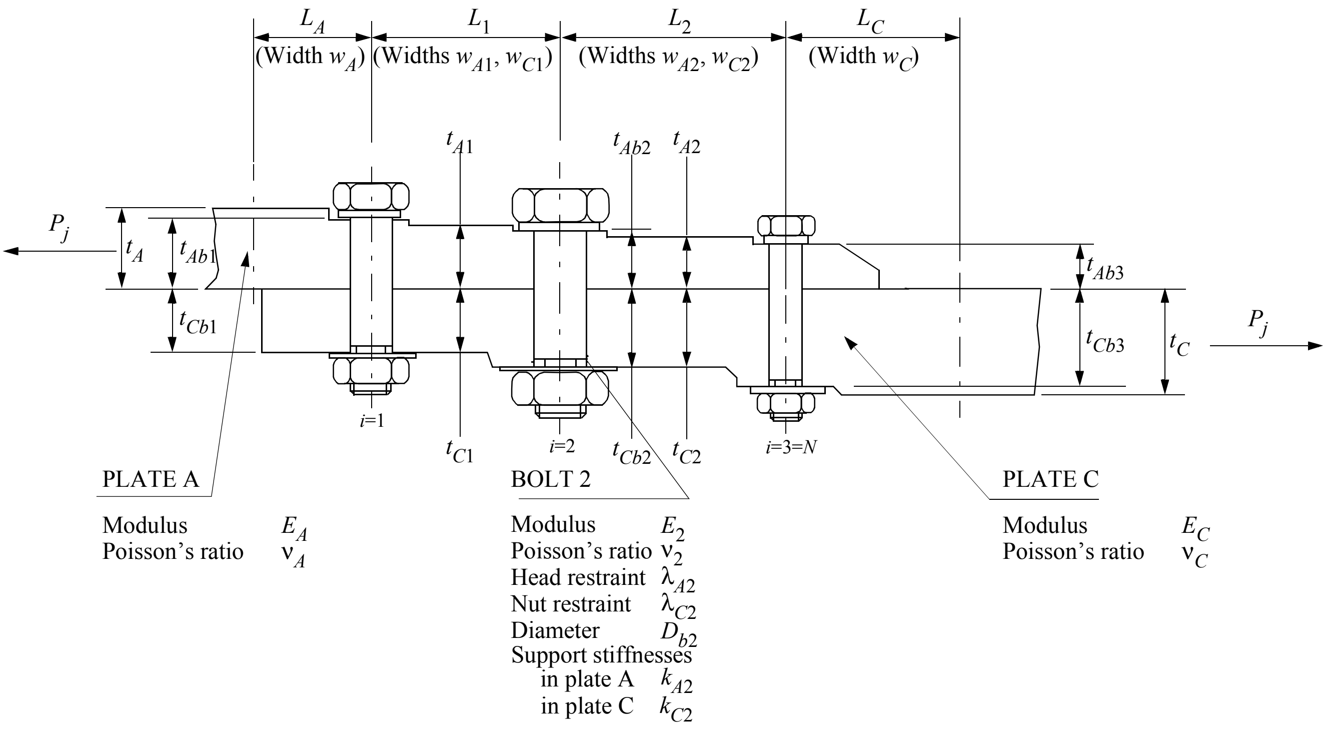
* *For footnote see end of Notation Section*

δ	in-plane longitudinal extension	m	in
δ_b	total deformation of bolt/hole* in a single plate as shown in Sketch 1.3	m	in
δ_{bT}	total deformation of bolt/hole* at a joint of two plates in single shear, $\delta_{bT} = \delta_{bA} + \delta_{bC}$	m	in
δ_j	total in-plane extension of joint between arbitrary reference points defined by L_A and L_C	m	in
η	function, see Equation (1.16)		
λ	rotational stiffness provided by bolt head or nut	N m/rad	lbf in/rad
μ	shape factor for circular beam, $\mu = \dfrac{6(1+\nu_b)}{(7+6\nu_b)}$		
ν	Poisson's ratio of material		
ψ_b	rotation of bolt axis due to bending	rad	rad

* The term 'bolt/hole', used throughout this Item, refers to the bolt plus the material surrounding it which loads and/or supports it.

1.2.1 Suffixes

A	relates to plate A of a single lap joint, see Sketch 1.1
b	relates to bolt material, or load or plate dimension at a bolt
C	relates to plate C of a single lap joint, see Sketch 1.1
eff	denotes effective value
i	identifies ith bolt or pitch along joint (pitch i lies between bolts i and $i+1$, see Sketch 1.1)
m	bolt number (see Sketch 1.2 and Equations (1.2), (1.3) and (1.5) to (1.7))
max	denotes maximum value
p	denotes plate (see Section 1.3.3)
α	1, 2, 3 or 4, see Equation (1.16)



Sketch 1.1 Multi-bolt single lap joint showing notation (Equilibrium moment omitted for clarity)

1.3 ANALYSIS OF MULTI-BOLT SINGLE LAP JOINTS

The analysis is based upon Derivation 4 of Section 1.6 in which the elastic behaviour of multi-bolt joints of the type illustrated in Sketch 1.1 is analysed to determine the load transferred at each bolt and the overall joint flexibility using the following assumptions.

(a) All components of the joint behave elastically.

(b) The flexibilities of the assemblies of individual bolts in the plate are available. (If the required flexibilities are not available, Section 1.3.3 provides a theoretical method for estimating them.)

(c) The transfer of load by friction is ignored and load transfer takes place only by shear through the bolts.

(d) The increase in flexibility of the plates under tensile loading attributable to the presence of the bolt holes can be accounted for by use of reduced plate areas. Under compressive loading the bolt holes are ignored, see Section 1.4.1.4.

(e) The overall moment arising from the offset loading in a single lap joint is not considered.

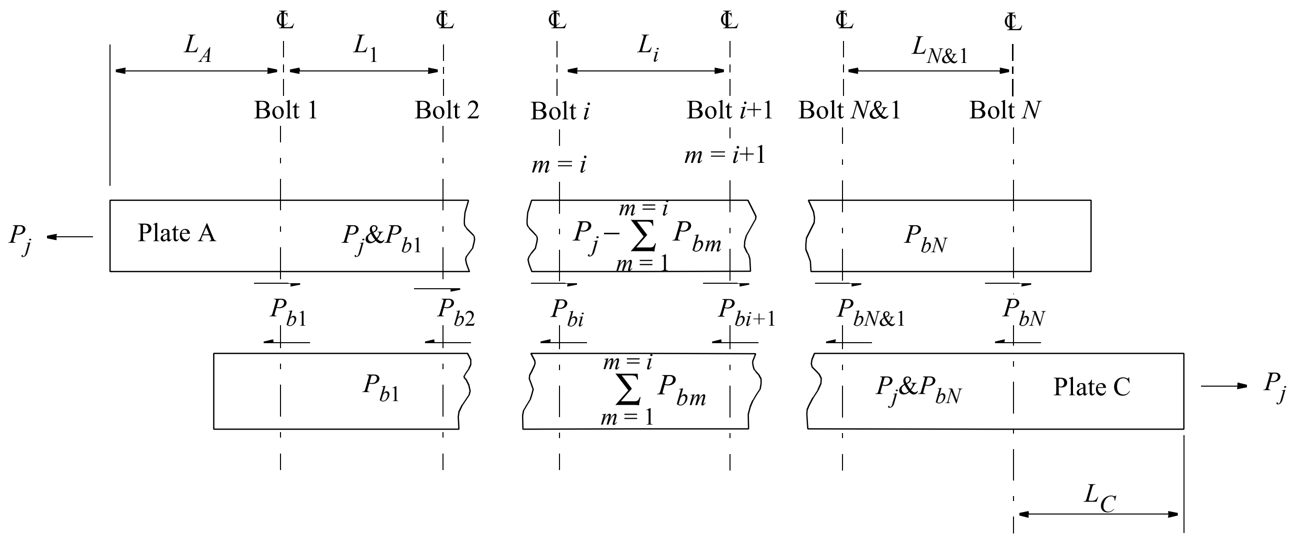
(f) The loading can be tensile or compressive. Under compressive loading no account is taken of the possibility of buckling.

(g) The bolts are assumed to be perfect fits in both plates and therefore to have perfectly matched pitches in the plates.

Mismatching pitches and/or lack of bolt fit in the plates can significantly influence the load distribution among the bolts.

1.3.1 Analysis of Load Distribution in a Joint

Joints of the type illustrated in Sketch 1.1 may be idealised as illustrated in Sketch 1.2. Note that the dimensions L_A and L_C define the ends of the joint and can be arbitrarily chosen. Over each pitch the end load is constant at the values shown within each plate.



Sketch 1.2 Idealised single lap joint

Each pitch between the bolts, as shown in Sketch 1.2, is considered in turn and the effects summed, from $i = 1$ to $N - 1$. For compatibility the extensions over pitch i are equal so that

$$\delta_{iA} = \delta_{iC} \; . \tag{1.1}$$

The pitch extensions are made up of two components: (a) plate extensions and (b) bolt/hole deformations.

Note that throughout the following equations 'effective' areas are used. These are used to take account of the increased flexibility of the plates in tension owing to the presence of the bolt holes, see Section 1.4.1.4.

(a) Plate extensions only

For plate A,

$$\delta_{iAp} = \frac{L_{iA}}{A_{iAeff} E_{iA}} \left[P_j - \sum_{m=1}^{m=i} P_{bm} \right] \tag{1.2}$$

and for plate C,

$$\delta_{iCp} = \frac{L_{iC}}{A_{iCeff}E_{iC}}\left[\sum_{m=1}^{m=i} P_{bm}\right] . \qquad (1.3)$$

(b) Bolt/hole extensions

Since the bolt/hole flexibility, F_b, relates to both plates A and C, the effective extension of pitch i owing to bolt/hole flexibility is the difference between the displacements of the bolts at each end of the pitch, i, in both plates A and C. Thus,

$$\delta_{bTi} = P_{b(i+1)}F_{b(i+1)} - P_{bi}F_{bi} . \qquad (1.4)$$

Combining Equations (1.1) to (1.4) to give the compatibility equation for each pitch gives $N - 1$ equations as follows.

$$\left(\frac{L_{iA}}{A_{iAeff}E_{iA}}\right)\left[P_j - \sum_{m=1}^{m=i} P_{bm}\right] - \left(\frac{L_{iC}}{A_{iCeff}E_{iC}}\right)\left[\sum_{m=1}^{m=i} P_{bm}\right] + P_{b(i+1)}F_{b(i+1)} - P_{bi}F_{bi} = 0 . \qquad (1.5)$$

For overall load equilibrium the sum of the bolt loads must equal the applied load, or

$$P_j = \sum_{m=1}^{m=N} P_{bm} . \qquad (1.6)$$

Equations (1.5) and (1.6) now represent N simultaneous equations which may be solved to yield the values of the bolt loads.

1.3.2 Joint Displacement, Flexibility and Stiffness

The joint extension is found by adding (a) the plate A (pitch) extensions, (b) the extension due to the flexibility of the end bolt/hole, N, and (c) the free plate extensions in plates A and C at each end over the lengths L_A and L_C.

$$\delta_j = \left[\sum_{i=1}^{i=N-1} \frac{L_{iA}}{A_{iAeff}E_{iA}}\left[P_j - \sum_{m=1}^{m=i} P_{bm}\right]\right] + P_{bN}F_{bN} + \frac{P_jL_A}{A_{Aeff}E_A} + \frac{P_jL_C}{A_{Ceff}E_C} . \qquad (1.7)$$

Joint flexibility is then

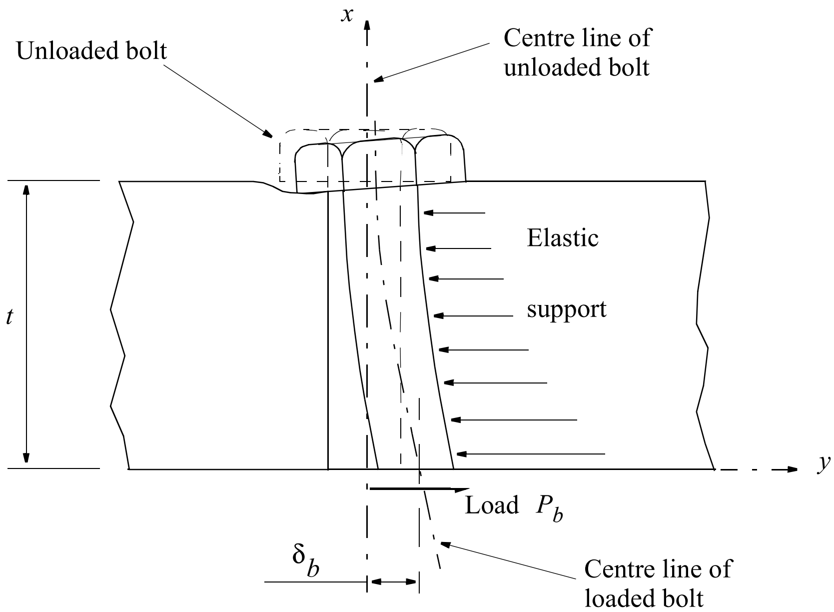
$$F_j = \frac{\delta_j}{P_j} \qquad (1.8)$$

and joint stiffness is

$$S_j = \frac{1}{F_j} \quad . \tag{1.9}$$

1.3.3 Calculation of Bolt Flexibility

The calculation of bolt flexibility is based on the theoretical analysis of Derivation 7 of Section 1.6. The theory, summarised below, considers elastic behaviour of an individual bolt in a single plate subjected to a shear load as shown in Sketch 1.3 and so leads to the calculation of f_b. The bolt is assumed to act as a beam on an elastic foundation. Bending and shear of the bolt are considered together with the through-the-thickness shear stiffness of the plate and the effect of bolt head rotational restraint. The plate width and length are both assumed to be sufficiently large for plate boundary effects not to influence the solution significantly.



Sketch 1.3 Theoretical bolt and plate configuration showing reference axes

1.3.3.1 Basic relationships

At any point, x, along the bolt the following relationships apply.

Rotation:

$$\frac{dy}{dx} = \beta + \psi_b . \tag{1.10}$$

Beam equilibrium:

$$\frac{dM_{bx}}{dx} + Q_{bx} = 0 \tag{1.11}$$

and

$$\frac{\mathrm{d}Q_{bx}}{\mathrm{d}x} + q_{px} = 0 . \tag{1.12}$$

The plate reaction, q_{px}, may be simply represented by the product of the foundation stiffness and the bolt deflection, ky. However, this only accounts for the direct stiffness of the foundation acting as if it were a set of independent springs acting on the bolt. The plate material shear stiffness couples adjacent regions of the foundation. This can be accounted for by taking the slope dy/dx, together with the material through-the-thickness shear stiffness, G_p, and assuming action over an effective area, A_{ps}, to give an effective shear force in the plate. The rate of change of this shear force gives an additional plate reaction so that, in total

$$q_{px} = -ky + G_p A_{ps} \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} . \tag{1.13}$$

Note that in Equation (1.13) k and A_{ps} are specific to the bolt under consideration. Hence, changing bolt diameter changes both k and A_{ps}.

Linear small displacement assumption:

$$M_{bx} = E_b I_b \frac{\mathrm{d}\psi_b}{\mathrm{d}x} . \tag{1.14}$$

1.3.3.2 Governing equation

From Equations (1.10) to (1.14) the following relationship is obtained.

$$\left[1 + \frac{G_p A_{ps}}{\mu G_b A_b}\right] \frac{\mathrm{d}^4 y}{\mathrm{d}x^4} - \left[\frac{k}{\mu G_b A_b} + \frac{G_p A_{ps}}{E_b I_b}\right] \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{k}{E_b I_b} y = 0 . \tag{1.15}$$

The general solution of Equation (1.15) is given by,

$$y = A_\alpha \, \mathrm{e}^{\eta_\alpha x} \tag{1.16}$$

where α is 1, 2, 3 or 4 and η is a function of the coefficients in Equation (1.15).

1.3.3.3 Boundary conditions

The four boundary conditions are as follows.

At $x = 0$,

$$Q_{bx} = P_b - G_p A_{ps} \frac{\mathrm{d}y}{\mathrm{d}x} \tag{1.17}$$

and a point of contraflexure is assumed in the bolt so that

$$M_{bx} = 0 . \tag{1.18}$$

This latter condition is strictly only valid in single lap joints in which the plates are of the same thickness and stiffness; however, this is still a reasonable assumption in most practical structural joints.

At $x = t$,

$$Q_{bx} = -G_p A_{ps} \frac{\mathrm{d}y}{\mathrm{d}x} \tag{1.19}$$

and

$$M_{bx} = -\lambda\psi_b . \tag{1.20}$$

Equation (1.19) relates the shear load at the bolt head to the bolt slope, dy/dx, and the plate shear stiffness. Since, from Equation (1.10), the slope dy/dx includes a component due to bolt bending, ψ_b, Equation (1.19) is only strictly valid when ψ_b is negligible, that is when the bolt head restraint, λ, is large. This will be the case in most practical structural joints, see Section 1.4.1.2.

By using Equations (1.10) to (1.14) and (1.17) to (1.20) equations may be developed to solve for the coefficients A_α of Equation (1.16). The coefficients are all that is required to calculate the required deflection at $x = 0$ via Equation (1.16) since all other terms are functions of e^x and are therefore unity at $x = 0$.

1.4 APPLICATION OF THE THEORY FOR CALCULATING THE FLEXIBILITY OF A BOLT IN A SINGLE PLATE

1.4.1 Parameters Required to Calculate Joint Flexibilities

The theory described in Section 1.3.3 for calculating the flexibility of a bolt in a single plate requires knowledge of the following.

(a) The bolt diameter, the plate thickness and the bolt and plate material properties.

(b) The effective areas of the plates (reduced from their nominal areas by the presence of the bolt holes).

(c) The rotational restraint, λ, provided by the nut or bolt head.

(d) The effective area of plate, A_{ps}, over which the bolt shear is taken to act.

(e) The effective stiffness, k, of the plate as an elastic foundation supporting the bolt.

The tests reported in Derivation 2 on single bolt joints show that the measured flexibility of nominally identical joints usually varies by a factor in excess of 1.3 and can vary by a factor of up to 2.

The values listed in (a) above are readily available. The effective areas of (b) may be estimated from the dimensions and properties of the plates using the method of Section 1.4.1.4 and that method is incorporated in the program.

Figure 1 shows that, for two different joint geometries, as λ is increased, variations in bolt flexibility as a result of errors in estimating the value of λ become insignificant, see Section 1.4.1.2.

The value of A_{ps} is only approximately known, see Section 1.4.1.3. However, its influence on the joint flexibility is small. This is illustrated by Figure 2 where, for two different joint geometries, the change in f_b for a range of values of A_{ps} is shown to be relatively small.

The remaining factor, k, is a major influence on the flexibility of a joint and so it is essential that a good estimate of k is available if an accurate value of joint flexibility is to be obtained.

1.4.1.1 Effective stiffness, *k*

While the best estimate of k may be obtained by experiment, the practical difficulties inherent in measuring a particular parameter when testing a representative joint should not be underestimated. Appendix A describes a simple test on a specially manufactured double lap joint having relatively rigid outer plates. This design eliminates all overall joint eccentricity effects, minimises bolt bending effects and allows measurement of the extension that arises principally from the deformation of the material around the inner plate bolt hole. The relatively small plate extensions are calculated and subtracted from the measured extension and the resulting value is used to estimate k.

It should be noted that the value of k used in Equation (1.13) depends on bolt diameter. For example, doubling the bolt diameter may be expected to double approximately the value of k.

Derivation 2 gives details of some 300 tests and covers the types of joint listed in Table 1.1. The larger groups of nominally identical tests have been analysed in order to determine an approximate value for k. For this it was assumed that λ was effectively infinite and that $A_{ps} = 0.1 D_b^2$ (see Section 1.4.1.3); k was then varied until, by trial, the average value of F_j for each set of tests was obtained. This indicated that k is approximately related to the modulus of the plates and the bolt diameter. The values of k obtained were then related to an arbitrarily chosen reference bolt diameter, D_{ref}, of 6.35 mm (0.25 in). The mean value of k was then found to be given by

$$k = 0.32\, E_p \frac{D_b}{D_{ref}} \quad . \tag{1.21}$$

This analysis indicated that the factor of 0.32 in Equation (1.21) has a standard deviation of about 0.1 and consequently a coefficient of variation in the region of 0.3. The analysis assumes that all variability between the test results is attributable to k, which is clearly not the case. However, in the absence of specific information on k for the joint under design, application of Equation (1.21) should lead to reasonable estimates of F_j.

TABLE 1.1

Joint type	*Maximum and minimum bolt diameters* mm (in)	*Bolt head type*	*Bolt material*	*Maximum and minimum plate thicknesses* mm (in)	*Plate material*
Double lap	6.35 (0.25) (only)	Countersunk	Titanium alloy	3.1 (0.124) (only) outer plates 6.5 (0.256) (only) inner plates	Aluminium alloy
Single lap	4.0 (0.156) 12.7 (0.50)	Countersunk	Titanium alloy	2.0 (0.080) 12.7 (0.50)	Titanium alloy
Single lap	4.0 (0.156) 4.8 (0.188)	Protruding head	Titanium alloy	1.0 (0.040) 4.8 (0.188)	Titanium alloy
Single lap	4.8 (0.188) 9.5 (0.375)	Protruding head	Steel	0.8 (0.031) 4.8 (0.188)	Aluminium alloy

1.4.1.2 Rotational restraint, λ

In the majority of structural joints the value of λ is sufficiently high to be taken as infinite with minimal loss of accuracy, see Section 1.4.2. The effect of decreasing the rotational restraint, λ, is to increase joint flexibility.

1.4.1.3 Effective shear area, A_{ps}

In Derivation 6 it has been found by experiment that a suitable value for the effective shear area of the plate, A_{ps}, is $0.1D_b^2$. This is the default value used by the program of ESDUpac A9812. A facility for using other values for A_{ps} is provided in the program.

Figure 2 shows the effect on bolt/hole flexibility of varying A_{ps} from 1.0 to 10.0 mm^2 for the 6.35 mm bolt of Figure 1 in 10 mm and 25 mm thick plates. Also shown on the figure is the suggested value of $A_{ps} = 0.1D_b^2$.

1.4.1.4 Effective areas of the plates to account for the presence of bolt holes

Throughout the description of the theory in Section 1.3 the plate areas used were termed effective values. The effective values are designed to account for the presence of the bolt holes in the flexibility calculation. Under a tensile load the presence of the bolt hole increases the flexibility of the plates. For plates loaded in compression the bolt is capable of passing the load directly through its diameter and, to account for this, the plate flexibility is calculated as if the plate contains no bolt holes.

For joints in tension the increased flexibility of the plates arising from the presence of the bolt holes has been assessed using finite element analyses and the experimental work of Derivation 5. The finite element analyses took no account of the presence of the bolt in the hole. However, the experimental work of Derivation 5 measured the flexibility of plates containing a bolt and good agreement between that work and the finite element analyses was observed. This has led to the determination of an effective width for the plates that may be used to calculate the area for the flexibility calculation. The analysis of the ends of the plates, lengths L_A and L_C, led to the conclusion that L_A and L_C should exceed a limiting value, l_{inf}, since below this value the stress distribution across the ends of the plates is not sensibly uniform. The value of l_{inf} is given by

$$\frac{l_{inf}}{w} = 0.403 + 2.24\frac{D_b}{w} - 1.57\left(\frac{D_b}{w}\right)^2 . \tag{1.22}$$

For plates where L_A, or L_C, exceeded l_{inf} the effective width was found to be given by

$$\frac{w_{eff}}{w} = L_A\left[L_A - l_{inf} + \frac{l_{inf}}{1.0 - 0.189\frac{D_b}{w} - 0.671\left(\frac{D_b}{w}\right)^2}\right]^{-1} . \tag{1.23}$$

The analysis was restricted to plates for which $0.1 < D_b/w < 0.6$.

If the joint design is such that L_A and/or L_C are less than l_{inf}, Equation (1.23) is not strictly valid but in practice may be expected to give an acceptable result.

For multi-bolt joints the plate between bolts has also been analysed using finite elements. This has led to the following equation for w_{eff}/w for $L/D_b = 2.0$,

$$\frac{w_{eff}}{w} = -0.61\left(\frac{D_b}{w}\right)^2 - 0.537\frac{D_b}{w} + 1.0 . \tag{1.24}$$

Figure 3 illustrates the use of Equations (1.22), (1.23) and (1.24) for pitches of uniform width between bolts of equal diameter. For pitches for which L_i exceeds $2l_{inf}$, Equation (1.23) with $L_i/2$ substituted for L_A is used, and the boundary where $2l_{inf} = L_i$ is determined from Equation (1.22). Equation (1.24) gives the values of w_{eff}/w for pitches for which $L_i/D_b = 2$ and these values are taken to be connected to the values of Equation (1.23) at the $L_i = 2l_{inf}$ values by straight lines as shown in the figure. Figure 3 has been found to give reasonable agreement with the available finite element data. Where adjacent bolts differ in diameter leading to different values of w_{eff} for the plate between them, the program described in Section 2 uses the average value.

It should be noted that for joints loaded in tension variations in F_j resulting from differences between using w_{eff} and w are relatively small, usually less than 10 per cent of F_j. Thus, variations in F_j arising from errors in w_{eff} are only a very small fraction of F_j.

1.4.2 Influence of the Major Factors Governing the Flexibility of a Single Bolt

Figure 1 shows the variation of f_b with t for single 6.35 mm and 12.7 mm (0.25 in and 0.5 in) diameter steel bolts in a single aluminium alloy plate. For both bolts the value of A_{ps} has been taken to be $0.1D_b^2$. The figure shows that at large values of t (long bolts), the curves for each bolt approach an asymptote.

The presence of an asymptote indicates that load transfer occurs close to the joint interface and that when t is large the head restraint is too far away from this region to influence significantly the value of f_b. The curves for each bolt also merge as t approaches zero, the flexibility becoming dominated by t and k. In the central region λ influences the value of f_b and increasing λ leads to a decrease in f_b. However, for the 6.35 mm bolt the greatest possible decrease is in the region of 6.5 mm where increasing λ from 0 to

1×10^{18} kN mm/rad leads to a decrease in f_b of only 40 per cent. Thus, in most joints, increasing λ from one practical value to another is unlikely to yield a large change in f_b. This conclusion is derived from an analysis that assumes the bolt to be a perfect fit in the plate, see Section 1.3.

1.4.3 Application of Flexibility of a Bolt in a Single Plate to a Bolt in a Single Lap Joint

The flexibility, f_b, obtained from the equations of Section 1.3.3 relates to a bolt under shear load in a single plate. The flexibility of the bolt when connecting two unequal plates, as in a single lap joint, can be obtained by determining the flexibility of the bolt in the individual plates, f_{bA} and f_{bC}, and adding them so that

$$F_b = f_{bA} + f_{bC} \qquad (1.25)$$

and therefore the stiffness is

$$S_b = \frac{1}{f_{bA} + f_{bC}}. \qquad (1.26)$$

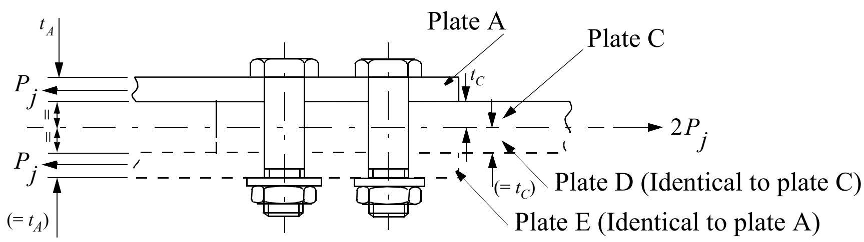
Because of the boundary condition assumptions at the joint interface, see Section 1.3.3.3, the above equations are strictly only valid for plates of the same material and equal thickness. However, for practical purposes, the sum of the flexibilities of a bolt joining two dissimilar plates will not usually be significantly different from that obtained using Equation (1.25). This equation is used in the program of Section 2 to calculate F_b for each bolt in a single lap joint.

1.4.4 Application of Flexibility of a Bolt in a Single Plate to a Bolt in a Symmetric Double Lap Joint

The flexibility of a symmetric double lap joint may be calculated by representing it as two identical single lap joints, see Sketch 1.4. The extension of a symmetric double lap joint under load $2P_j$ is the same as that of one of its component single lap joints under load P_j. Thus, the flexibility F_j of a double lap joint (that is, plates A, C, D and E) based upon the total joint load $2P_j$ is obtained as follows.

$$(F_j)\text{Double lap} = \frac{(\delta_j)\text{Single lap (plates A and C) under load } P_j}{2P_j}. \qquad (1.27)$$

Note that, owing to symmetry, the effective value of λ at the joint centre-line between plates C and D is infinity, see Sketch 1.4.



Sketch 1.4 Showing representation of a symmetric double lap joint as two identical single lap joints

1.5 APPLICATION OF THE THEORY FOR CALCULATING JOINT FLEXIBILITY

The program of Section 2 has been used to construct Figures 4 to 9 which show examples of the effect of typical geometrical parameters on the maximum and minimum bolt loads, joint flexibilities and bolt load distributions for single lap joints loaded in tension that are either symmetrical or asymmetrical about the shear face. The results have been obtained assuming that the individual plate cross-sectional areas and the corresponding plate moduli, E_A and E_C, are constant along the length of the joint. Figures 6 and 9 are non-dimensionalised using the value of F_b $(= f_{bA} + f_{bC})$ which is a function of plate thickness. For the particular joints used to generate these figures, F_b is constant at 0.001 862 × 10^{-3} in/lbf for $t_C/t_A = 3$ and at 0.002 234 × 10^{-3} in/lbf at $t_C/t_A = 1$.

In Figures 4 and 5 the effect on the individual bolt load levels of varying the number of equally spaced bolts in a particular single lap joint of fixed length is shown. Figure 6 shows the variation of the maximum bolt load and the joint flexibility with the number of bolts for the same fixed-length joint as in Figures 4 and 5. In Figures 7 to 9 the data given in Figures 4 to 6 are repeated for the case where the pitch is uniform and constant and N is varied; consequently the joint length varies.

1.6 DERIVATION

This section lists selected sources that have assisted in the preparation of this Item.

1.	TATE, M.E. ROSENFELD, S.J.	Preliminary investigation of the loads carried by individual bolts in bolted joints. NACA TN 1051, 1947.
2.	HARRIS, H.G. OJALVO, I.U. HOOSON, R.E.	Stress and deflection analysis of mechanically fastened joints. Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, USA. Technical Report AFFDL-TR-70-49, May 1970.
3.	MOORE, K. McCORD, S.J.	Load distribution in bolted and riveted joints. McDonnell Aircraft Corp. Memo. No. 237-A-47, April 1971.
4.	GREEN, E.H.	Report on the stiffness of bolted and riveted joints. Report on work carried out by British Aircraft Corporation under Ministry of Defence Contract, October 1971. Private communication, September 1985.
5.	COURSE, D. PRESSNELL, M.S.	Load distribution in bolted joints. Project Report, The Hatfield Polytechnic, Department of Mechanical and Aeronautical Engineering, May 1972.
6.	PARKINSON, M.S.	Further investigation of the stiffness of simple bolted joints. Report on work carried out by British Aircraft Corporation under Ministry of Defence Contract, April 1976. Private communication, September 1985.
7.	HUGHES, G.W.	Investigation of the stiffness of structural joints. Report on work carried out by British Aircraft Corporation under Ministry of Defence Contract, November 1976. Private communication, September 1985.
8.	EGGWERTZ, S. MAGNUSSEN, A.	Review of Swedish work on fatigue of aircraft structures during period May 1977 to April 1979, F.F.A. TN H.E. 2159, I.C.A.F., Brussels 1979.

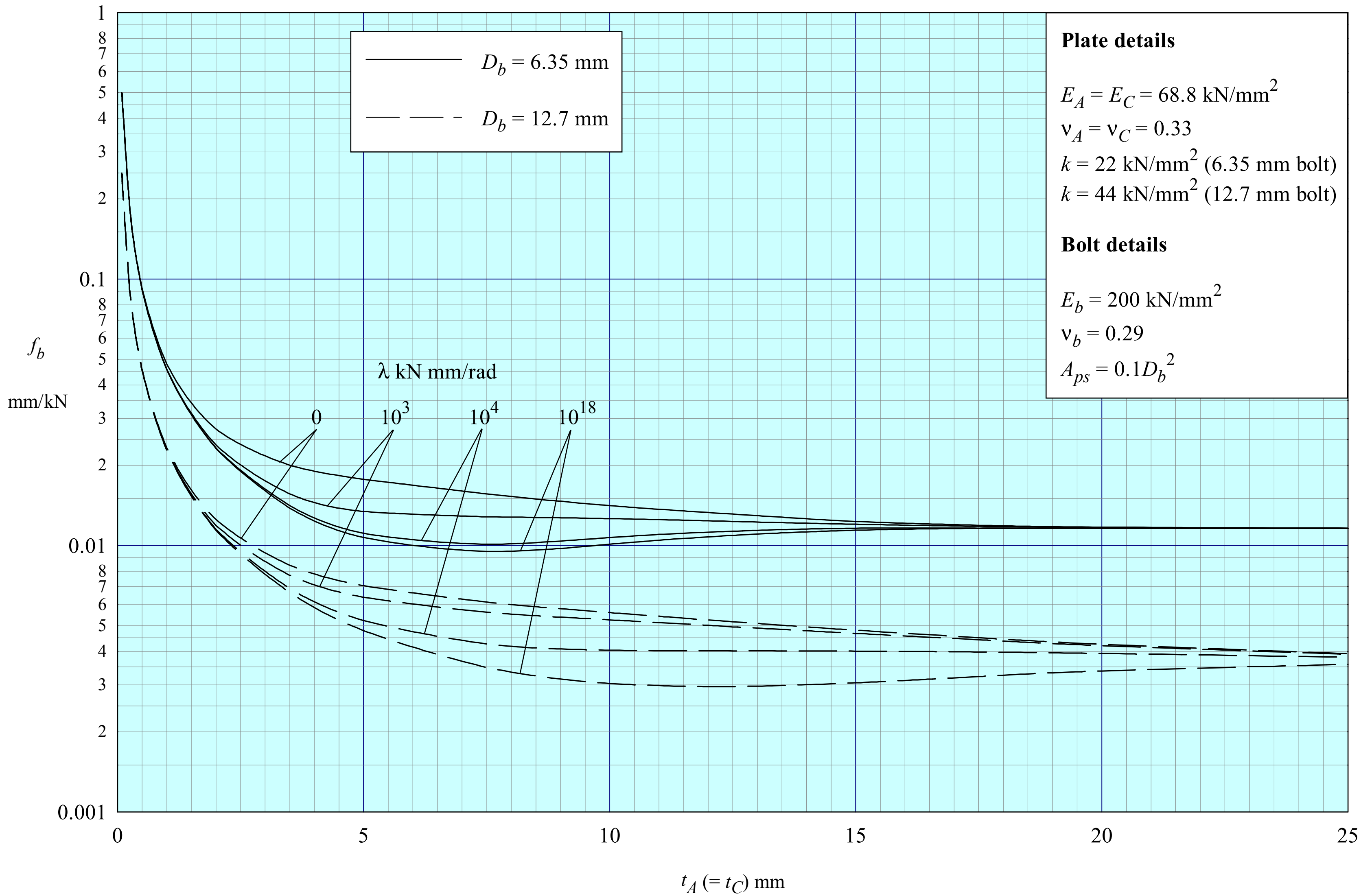


FIGURE 1 VARIATION OF f_b IN ONE PLATE WITH t AND λ FOR 6.35 AND 12.7 mm DIAMETER BOLTS

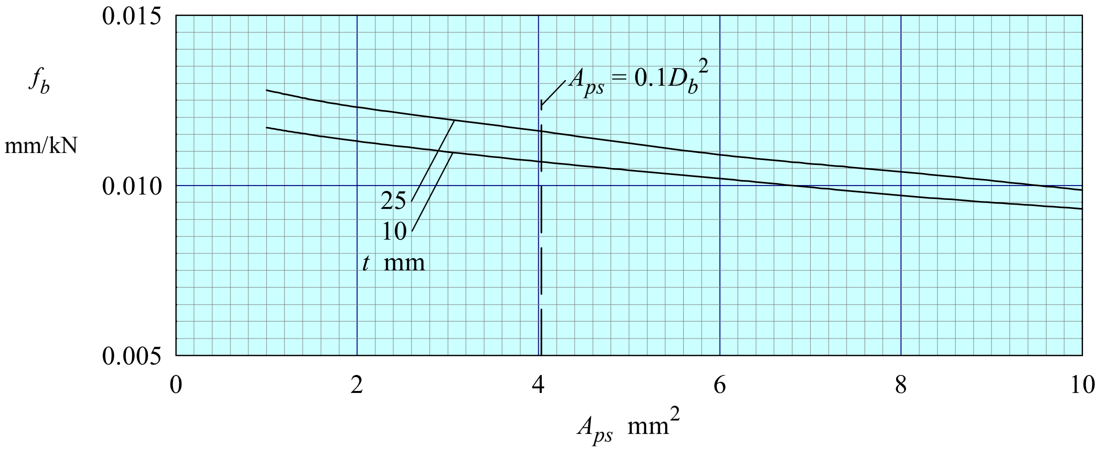


FIGURE 2 VARIATION OF f_b WITH A_{ps} FOR THE 6.35 mm DIAMETER BOLTS OF FIGURE 1 AT PLATE THICKNESSES OF 10 AND 25 mm AND WITH $\lambda = 1 \times 10^4$ kN mm/rad

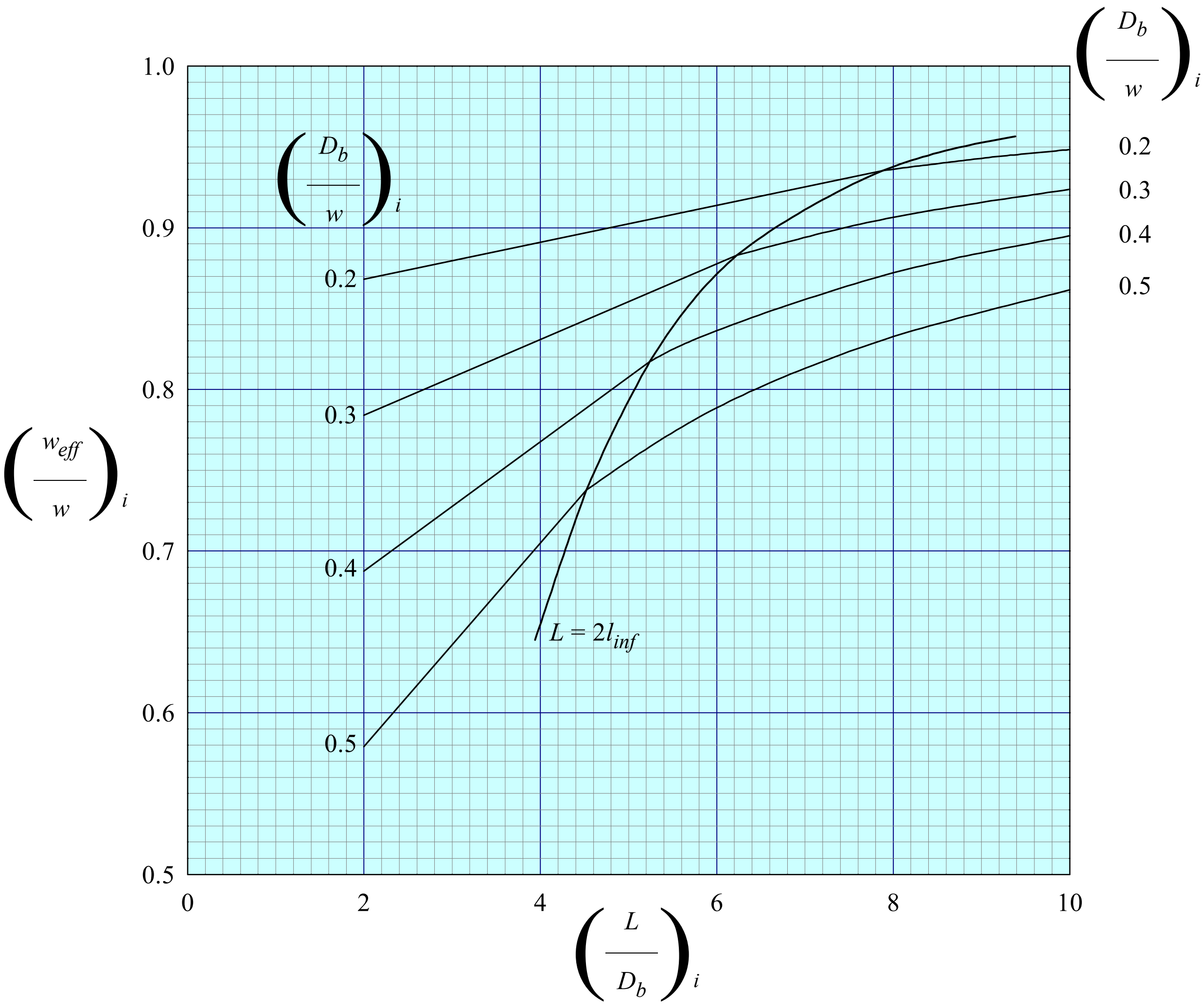


FIGURE 3 EFFECTIVE WIDTH OF PLATE BETWEEN IDENTICAL BOLTS (SEE SECTION 1.4.1.4)

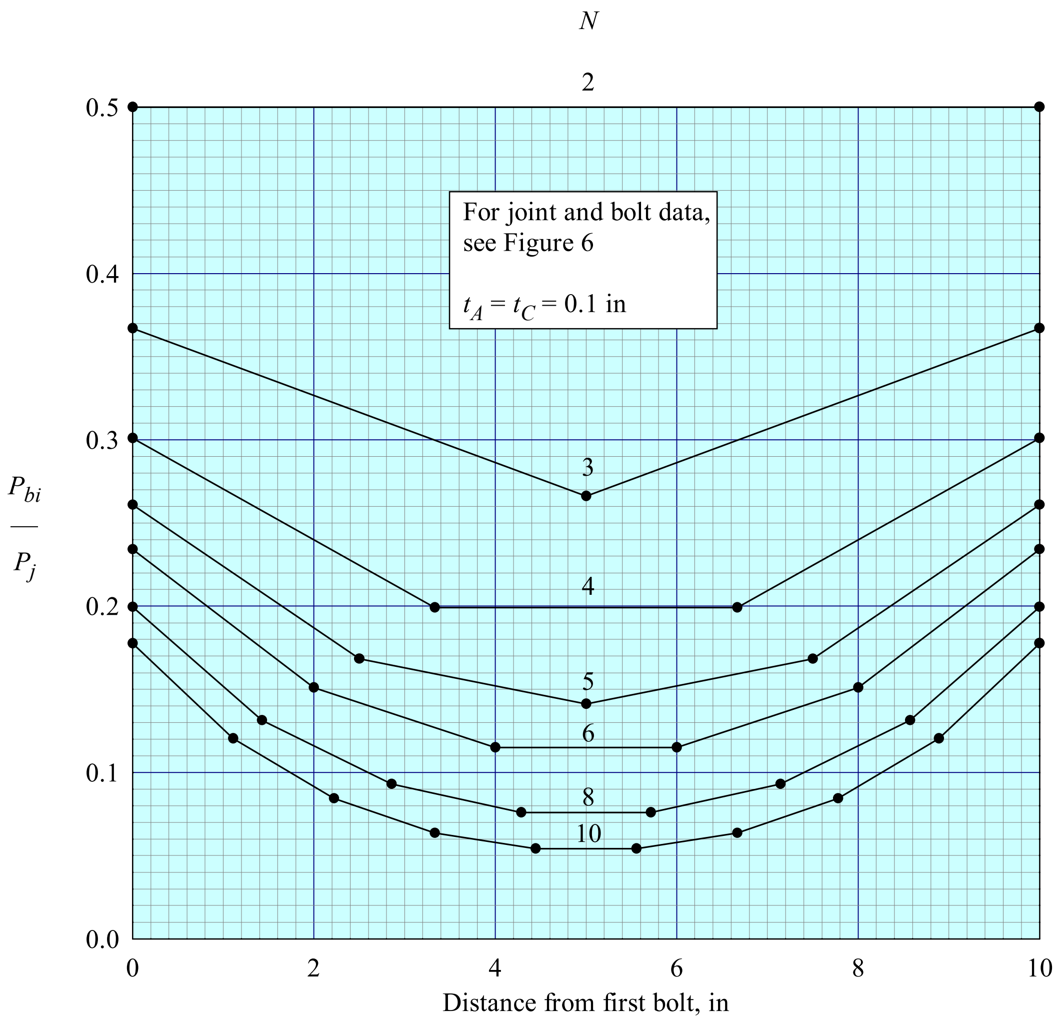


FIGURE 4 EFFECT OF VARYING *N* ON INDIVIDUAL BOLT LOADS IN A SINGLE LAP SYMMETRIC JOINT OF CONSTANT LENGTH WITH EQUALLY SPACED BOLTS, LOADED IN TENSION

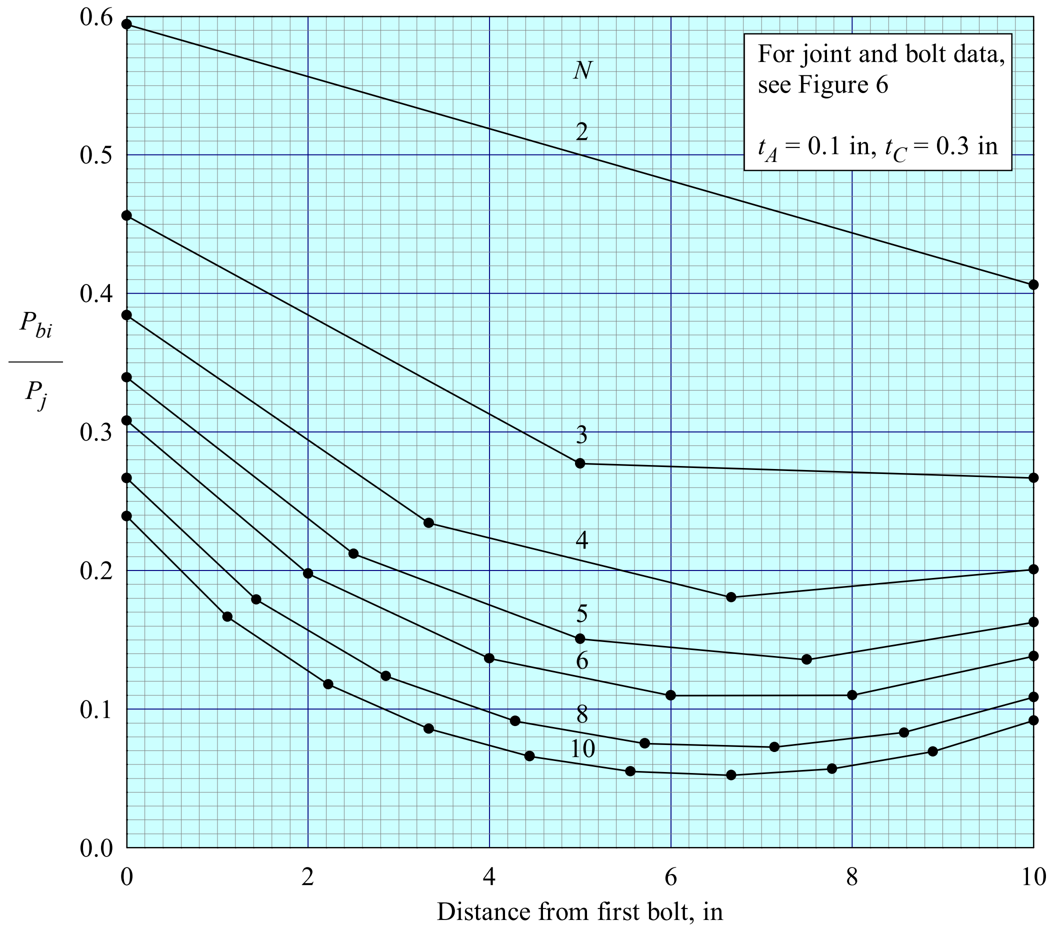


FIGURE 5 EFFECT OF VARYING *N* ON INDIVIDUAL BOLT LOADS IN A SINGLE LAP ASYMMETRIC JOINT OF CONSTANT LENGTH WITH EQUALLY SPACED BOLTS, LOADED IN TENSION

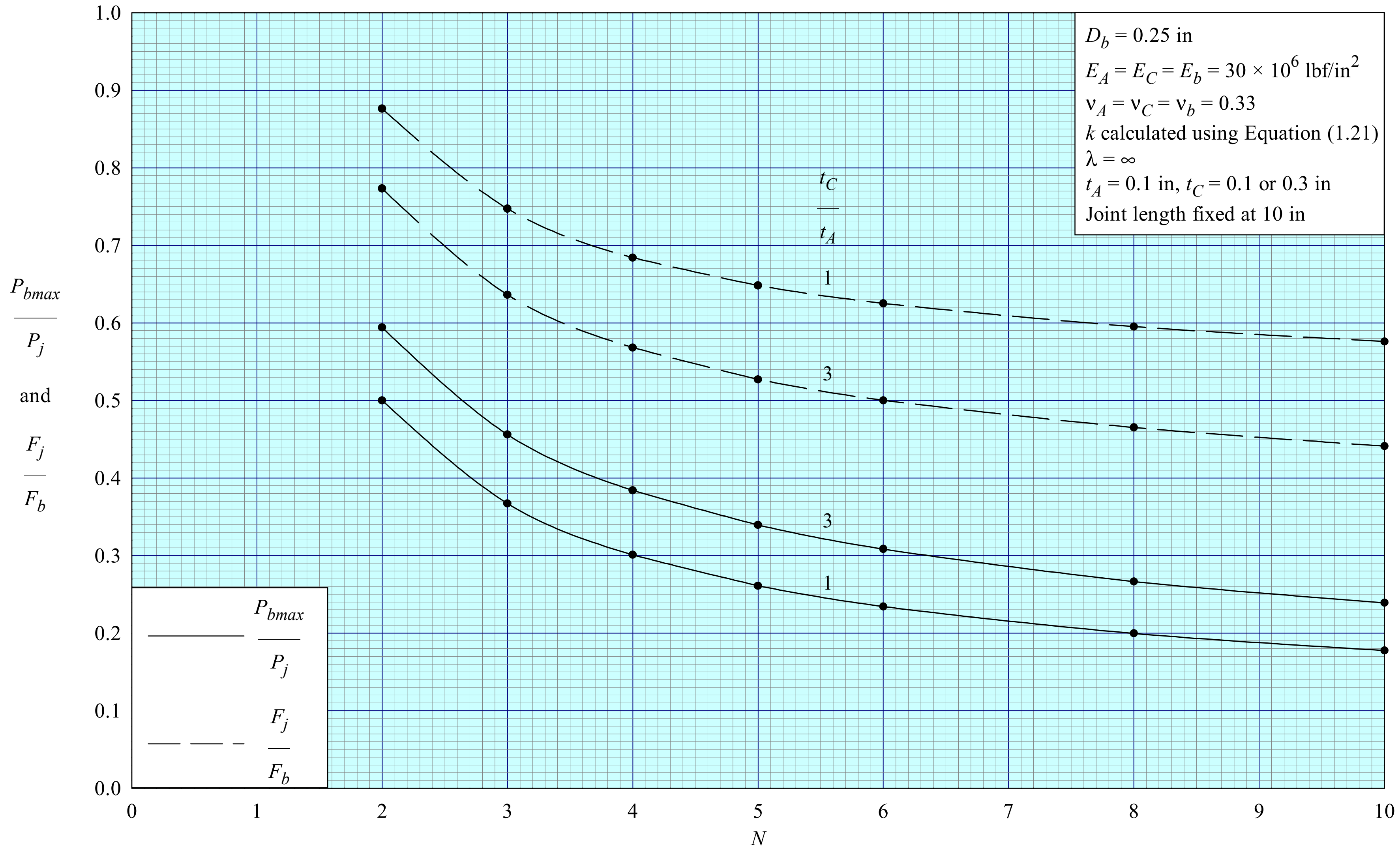


FIGURE 6 EFFECT OF VARYING *N* ON MAXIMUM BOLT LOADS AND FLEXIBILITIES FOR SINGLE LAP JOINTS OF CONSTANT LENGTH (SINGLE ROW OF BOLTS EQUALLY SPACED, JOINT PROPERTIES AND GEOMETRY CONSTANT ALONG LENGTH)

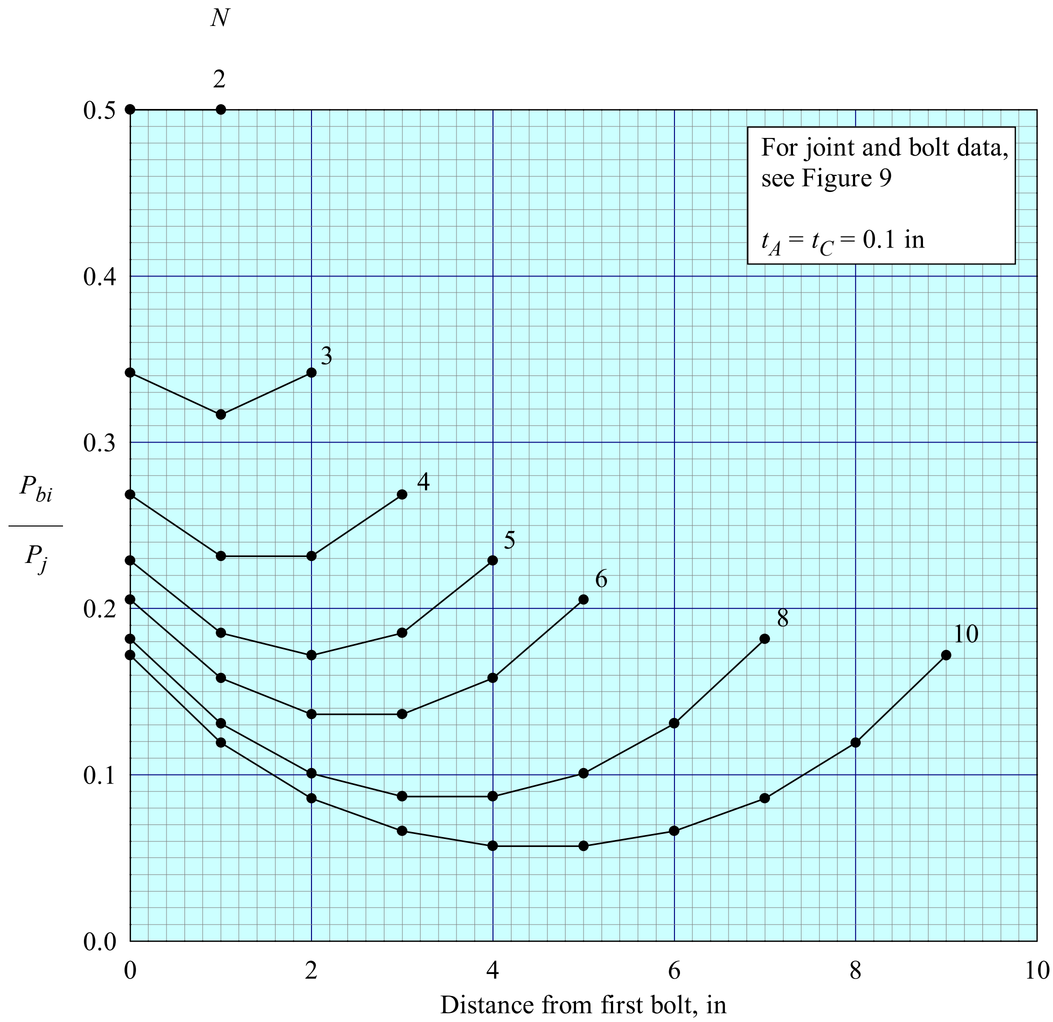


FIGURE 7 EFFECT OF VARYING *N* ON INDIVIDUAL BOLT LOADS IN A SINGLE LAP SYMMETRIC JOINT OF CONSTANT PITCH WITH JOINT LENGTH VARYING, LOADED IN TENSION

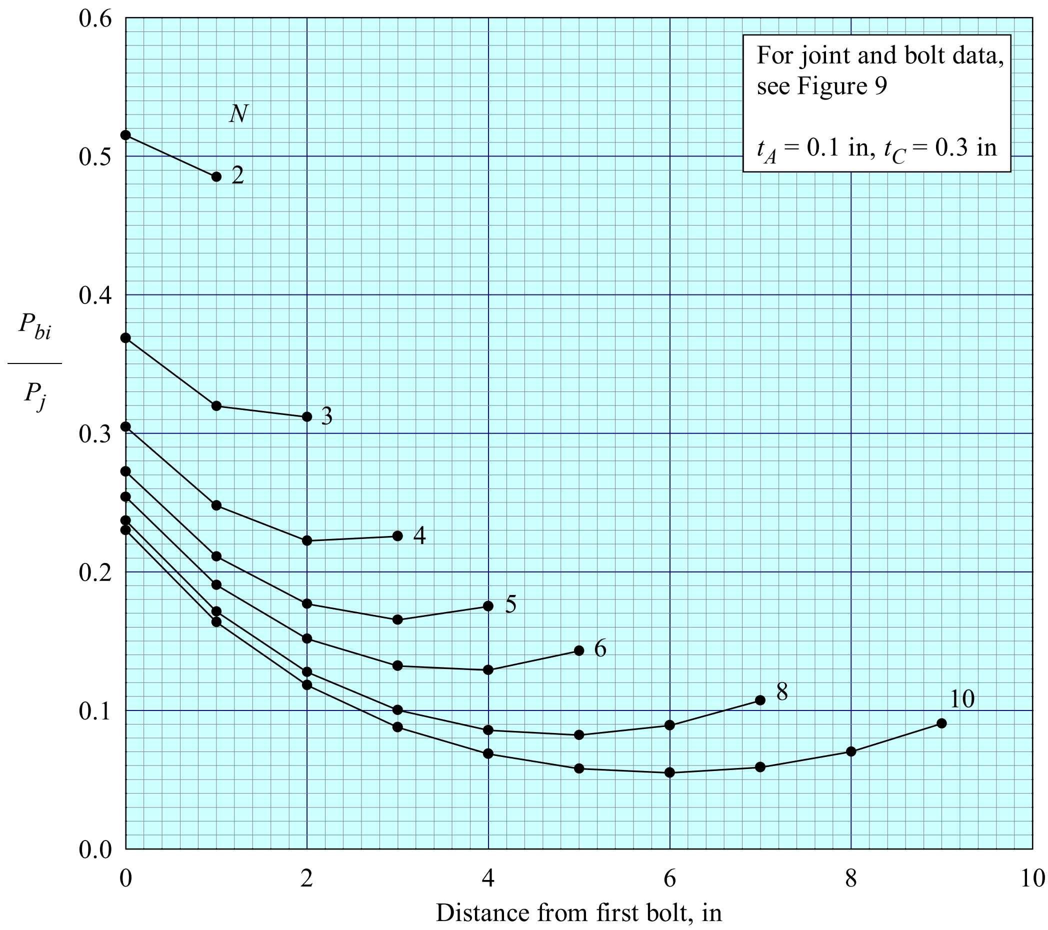


FIGURE 8 EFFECT OF VARYING *N* ON INDIVIDUAL BOLT LOADS IN A SINGLE LAP ASYMMETRIC JOINT OF CONSTANT PITCH WITH JOINT LENGTH VARYING, LOADED IN TENSION

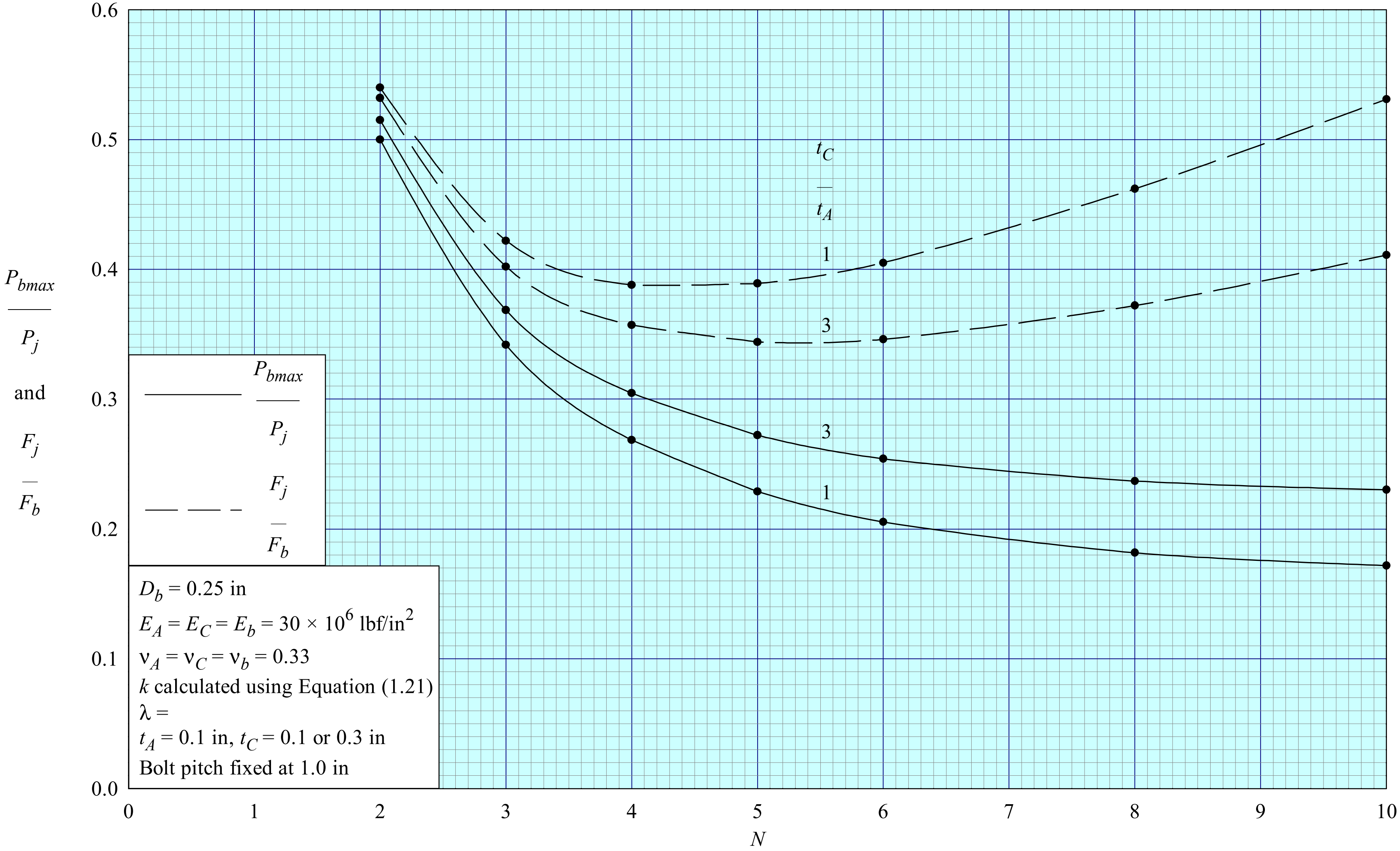


FIGURE 9 EFFECT OF VARYING *N* ON MAXIMUM BOLT LOADS AND FLEXIBILITIES FOR SINGLE LAP JOINTS OF FIXED PITCH AND VARYING JOINT LENGTH, SINGLE ROW OF BOLTS, JOINT PROPERTIES AND GEOMETRY CONSTANT ALONG LENGTH

2. ESDUpac A9812

2.1 INTRODUCTION TO THE PROGRAM

ESDUpac A9812 contains a Fortran program which accepts the properties and dimensions of the two plates and the bolts constituting a lap joint and calculates the individual bolt flexibilities and loads, the overall joint flexibility and its extension.

This Data Item relates to Version 1.0 of ESDUpac A9812V10 and Fortran program A9812V10.FOR. Both the ESDUpac number and the program number contain the version number encoded in the three characters V10. Throughout this Data Item the ESDUpac and its program are referred to by A9812 and the current version number is taken to apply. The Contents page of the Series Software Volume gives the current version number.

The program is available in the form of a Fortran source code and corresponding executable file. Both files can be found in ESDUscope (the CD version of the Series), on the internet version of the Series, and on disk in the Series Software Volume. Example input and output files are also provided.

Section 2.2 gives the scope and limitations of the program, Section 2.3 gives details of the input required by the program and Section 2.4 gives an explanation of the output, including all messages. Section 2.5 provides a selection of examples that illustrate the use of the program.

Every reasonable effort has been made to ensure that the program performs the intended calculations satisfactorily. If, however, any problems are encountered in its use, please inform ESDU and every effort will be made to overcome the difficulties. In common with other software producers, ESDU makes no representation as to the suitability or fitness of the program for a particular purpose and accepts no liability for any loss occasioned by any persons as a direct or indirect result of use of the program whether arising from negligence or otherwise. In no event shall ESDU or any individuals associated with the development of the program be liable for any damages, including loss of profit or consequential loss, arising out of or in connection with the program.

2.2 PROGRAM SCOPE AND LIMITATIONS

The program is capable of dealing with the following.

(i) Joints loaded in either tension or compression.

(ii) Joints fastened by a single bolt and multi-bolt joints with a line of up to twenty bolts aligned with the applied loading direction.

(iii) Joints between two plates of dissimilar isotropic materials.

(iv) Joints in which the pitch, thickness and width differ between each bolt. Joints that taper between bolts can be treated by introducing an effective width or thickness for the plate between adjacent bolts. Joints in which the plate(s) provide seatings for the nut and/or bolt head such that the distance between the bolt head and the nut differs from the plate thickness.

(v) Multi-bolt joints in which the bolts may all be different from one another (in terms of material and/or geometry).

(vi) Single lap joints. (Solutions for single lap joints may be used to determine the flexibility and bolt loads in symmetric double lap joints, see Section 1.4.4.)

(vii) Joints in which, if required, different values of the nut or bolt head rotational restraint may be assigned to each end of each bolt.

(viii) Joints in which the effective stiffnesses of the two plates in their support of the bolts may differ.

(ix) The determination of overall joint flexibility and extension under an applied load over the distance between the end bolts plus an arbitrarily chosen length beyond the end bolts over which the dimensions of the plates are taken to be constant. The arbitrary lengths at each end of the joint may be different.

(x) The direct input, where they are available, either from test or previous calculation, of values of bolt flexibility in each plate.

2.3 DATA INPUT REQUIREMENTS FOR THE FORTRAN PROGRAM

This section gives notes on the data input directly to the Fortran program. Table 2.1 gives a formal list of the data required and their order. Section 2.5 provides examples of input files for the Fortran program.

Prior to data input three lines are provided for run identification details. These lines may be left empty or can carry up to 72 alphanumeric characters each. The contents of these lines appear in the output.

2.3.1 Units

Following the three lines of text at the beginning of the input, two lines three characters wide are provided for the specification of units. The first line is for the specification of the unit of force, for example N or lbf, and the second line is for the specification of the unit of length, for example mm or in. The units are echoed in the output where they are quoted appropriately for all the output values. It is therefore essential to provide all the input data in a consistent set of units that agrees with the units declared. Whatever is entered in these two lines, the results will be in the same units as the input data and will be correct provided that the input data are in a coherent set of units. In the event that no units are declared, no units will appear in the output.

2.3.2 Loading, Number of Bolts and Material Properties of the Plates

The joint load is first required, positive for tension or negative for compression. A value must be provided even if the program is only being used to estimate the flexibility of a bolt. The joint load is used to calculate bolt loads and the overall extension using the joint flexibility. However, the sign of the load affects the plate flexibility calculation, see Section 1.4.1.4. Where the extension and bolt loads are not of interest the joint load can be conveniently set to unity. The number of bolts is required next; this should be between 1 and 20 inclusive.

The material properties of plates A and C are required, in order, together with the geometry of plate A prior to the first bolt and the geometry of plate C after the last bolt; the modulus of elasticity and Poisson’s ratio of each are required followed by their geometry. The lengths L_A and L_C are arbitrary and can be chosen as required and are used to define the limits of the joint. The plate geometries prior to the first bolt and after the last bolt are taken to be constant. If either L_A or L_C is set to zero then the respective width and thickness values are not required.

2.3.3 Geometry of Plates Between Bolts

For each pitch between the bolts the geometries of both plates A and C are required, in order, commencing with the pitch between bolts 1 and 2 and continuing to the pitch preceding the last bolt. Where the plates taper between bolts an effective value of width and/or thickness should be used.

2.3.4 Bolt Details

2.3.4.1 Bolt flexibilities known

If the bolt flexibilities are known from experiment or from previous calculations, then these values can be input directly. For each bolt, in order, the nominal diameter is required followed by the value of f_b in both plate A and plate C. These flexibilities are for the bolt in a single plate, see Section 1.3.3 and Sketch 1.3, and are combined within the program.

2.3.4.2 Bolt flexibilities to be calculated

If the bolt flexibilities are to be calculated then, for each bolt the following values are required. The nominal diameter, modulus of elasticity, Poisson's ratio and, for each plate, the plate thickness immediately under the bolt, the plate support stiffness and the rotational restraint. These properties are required, in order, commencing with the first bolt. Section 1.4.1.1 gives guidance on the value of the support stiffness k and, in the absence of other information, Equation (1.21) may be used, making sure that the value of D_{ref} used in the chosen system of units corresponds to 6.35 mm (0.25 in).

Note that if the single lap joint analysed by the program is being used to calculate the flexibility of a double lap joint, considerations of symmetry will require the bolt to have full rotational restraint in the middle of the joint. In these cases it is recommended that λ at this location is set to 1×10^{18}. The bolt head or nut on the outside of the joint can have a much lower value of λ.

The thicknesses of both plates A and C immediately under each bolt only differ from the thicknesses of the plates over the region between the bolts if the plate thickness tapers or otherwise varies, for example, by local spot-facing. In such cases the value at the bolt centre-line should be used with the bolt details.

If the flexibility of a bolt only is required, a single bolt joint can be analysed with L_A and L_C set to zero (in Entries 5 and 6 of Table 2.1).

Provision is made for use of either a default value of $A_{ps} = 0.1 D_b^2$ (see Entry 10a) or known values of A_{ps} to be supplied directly (see Entry 10b).

2.3.5 Input checks

The input data are checked by the program as described in Section 2.4.4. The checks performed can only identify errors of sign, size or logical errors that imply impossible designs. Additionally, Section 2.4.3 gives details of the warnings that are provided when the input data are contrary to common practice or are otherwise unusual. It is not possible to identify all impractical sets of input data and all input data sets should be carefully examined to ensure that they represent the practical joint intended. It is, for example, possible to produce a valid set of input data for a multi-bolt joint in which there is a step change in stiffness over one pitch only. In extreme cases this can result in one or more of the bolt loads acting in the opposite direction to that expected. Such results usually only arise in impractical designs.

TABLE 2.1 PROGRAM INPUT DATA

Entry order	*Entry*	*Enter the numerical values for the quantities indicated in free format. Each entry must be in the order shown below and made on a separate line with no blank line in between (except Entries 1 and 2). The values given for each entry on a single line must have a space or comma between them.*
1	Text	Enter run identification data, input file title or notes. Three lines 72 characters wide are allotted. Lines may be left blank.
2	Unit of force Unit of length	Enter details of the units used for the input data. Two lines three characters wide are allotted. Enter the unit of force on the first line followed by the unit of length on the second line. See Section 2.3.1 for more details. Lines may be left blank.
3	P_j	Enter the applied load, tensile positive, compressive negative. (Note that a value must be entered even if the program is only being run to determine a flexibility, see Section 2.3.2.)
4	N	Enter the number of bolts in the joint.
5	E_A ν_A L_A w_A t_A (if $L_A \neq 0$)	Enter the properties of plate A. Enter the modulus of elasticity followed, on the same line, by the Poisson's ratio. On the next line enter the length L_A (see Sketch 1.1). If $L_A \neq 0$ then enter, on the next line, the width and thickness of the length L_A.
6	E_C ν_C L_C w_C t_C (if $L_C \neq 0$)	Enter the properties of plate C. Enter the modulus of elasticity followed, on the same line, by the Poisson's ratio. On the next line enter the length L_C (see Sketch 1.1). If $L_C \neq 0$ then enter, on the next line, the width and thickness of the length L_C.
7	L_1 w_{A1} t_{A1} w_{C1} t_{C1} L_2 w_{A2} t_{A2} w_{C2} t_{C2} and so on up to L_{N-1} w_{AN-1} t_{AN-1} w_{CN-1} t_{CN-1}	For each pitch, commencing with the pitch between bolts 1 and 2 and progressing, in order, to the pitch between bolts $N-1$ and N, enter, on the same line, the following: the length of the pitch, the width of plate A, the thickness of plate A, the width of plate C and the thickness of plate C (see Sketch 1.1). No entries are required if $N = 1$.

TABLE 2.1 PROGRAM INPUT DATA (Continued)

Entry order	*Entry*	*Enter the numerical values for the quantities indicated in free format. Each entry must be in the order shown below and made on a separate line with no blank line in between (except Entries 1 and 2). The values given for each entry on a single line must have a space or comma between them.*
8a OR	0	If the bolt flexibilities are to be calculated then enter zero and proceed to Entry 9.
8b	1 D_{b1} f_{bA1} f_{bC} D_{b2} f_{bA2} f_{bC2} and so on up to D_{bN} f_{bAN} f_{bCN}	If the bolt flexibilities are known and are to be input then enter 1. On the following lines, for each bolt, commencing with bolt 1 and progressing, in order, to bolt N (see Sketch 1.1), enter the bolt diameter followed, on the same line, by the flexibilities in plates A and C. The input is then complete.
9	D_{b1} E_{b1} ν_{b1} t_{Ab1} t_{Cb1} k_{A1} k_{C1} λ_{A1} λ_{C1} D_{b2} E_{b2} ν_{b2} t_{Ab2} t_{Cb2} k_{A2} k_{C2} λ_{A2} λ_{C2} and so on up to D_{bN} E_{bN} ν_{bN} t_{AbN} t_{CbN} k_{AN} k_{CN} λ_{AN} λ_{CN}	If calculation of the bolt flexibilities is required (that is, if zero has been entered in Entry 8), on the following lines, for each bolt, commencing with bolt 1 and progressing, in order, to bolt N (see Sketch 1.1), enter, on the same line, the following: the bolt diameter, the modulus of elasticity and the Poisson's ratio of the bolt material, the thicknesses of plates A and C at the bolt centre-line, the effective stiffnesses of plates A and C (see Section 1.4.4), and the rotational restraint provided by the bolt head or nut at the surfaces of plates A and C.
10a OR	0	If the value of A_{ps} is to be taken as its default value of $0.1 D_b^2$ (see Section 1.4.1.3) then enter zero. The input is then complete.
10b	1 A_{psA1} A_{psC1} A_{psA2} A_{psC2} and so on up to A_{psAN} A_{psCN}	If values of A_{ps} other than the default value are to be used then enter 1. On the following lines enter the effective area over which shear acts for each bolt in each plate, commencing with bolt 1 and progressing, in order, to bolt N. The two values for each bolt must be on the same line, with the value for plate A entered first. The input is then complete.

2.4 PROGRAM OUTPUT

In all runs the program outputs a standard header (see the examples in Section 2.5) followed by the three lines of run identification details that were input.

2.4.1 Output of Input Data

All the input data are output. First is a statement of the units input for force and length. These units, or appropriate combinations of them, are output with all the data. If no units are input none will appear in the output.

The properties of both plates A and C are stated first and are followed by the joint geometry and bolt details. The geometry over the length L_A is then given followed by the details for bolt 1. The geometry of the pitch to the next bolt is then given, followed by the details of that bolt, and so on through all the bolts; finally, the geometry over the length L_C is given.

2.4.2 Interpretation of Successful Runs

In successful computations for joints loaded in tension the output first provides the calculated effective widths for the plates both between bolts and over lengths L_A and L_C. Effective widths are not calculated for joints loaded in compression since actual widths are used throughout the calculation (see Section 1.4.1.4). The following results are then given for each bolt:

(i) the shear load transferred by the bolt,

(ii) the flexibility of the bolt in each plate, f_b,

(iii) the flexibility of the bolt in the combined plates A and C, F_b,

and, for the complete joint (including the lengths L_A and L_C),

(a) the stiffness,

(b) the flexibility and

(c) the extension under the applied load.

All successful runs are terminated with the statement 'Computation completed, solution obtained.'.

2.4.3 Interpretation of Solutions With Warnings

If the program encounters unusual input data, it issues a warning message immediately after the list of input data and the computation is continued. A list of the warning messages is given in Table 2.2.

TABLE 2.2 INPUT DATA WARNING MESSAGES

No.	*Check*	*Message if check is violated*
101		Only one of the two units, force or length, has been specified. Therefore no units are appended throughout the output.
102	$L_A \geq \frac{D_{b1}}{2}$	The length LA is less than the radius of the first bolt.
103	$w_A \geq D_{b1}$	The width of the length LA is less than the diameter of the first bolt. This check is only performed if $L_A > 0$.
104	$L_C \geq \frac{D_{bN}}{2}$	The length LC is less than the radius of the last bolt.
105	$w_C \geq D_{bN}$	The width of the length LC is less than the diameter of the last bolt. This check is only performed if $L_C > 0$.
106	$L_A \geq l_{inf}$	The length LA is less than linf (see the definition of linf in the Data Item).
107	$D_{b1} \geq 0.1 w_A$	For the length LA, the bolt diameter is less than 0.1 of the plate width and the calculated effective width of the plate is out of range, see Section 1.4.1.4 of the Data Item.
108	$D_{b1} \leq 0.6 w_A$	For the length LA, the bolt diameter is greater than 0.6 of the plate width and the calculated effective width is out of range, see Section 1.4.1.4 of the Data Item.
109	$D_{bN} \geq 0.1 w_C$	For the length LC, the bolt diameter is less than 0.1 of the plate width and the calculated effective width of the plate is out of range, see Section 1.4.1.4 of the Data Item.
110	$D_{bN} \leq 0.6 w_C$	For the length LC, the bolt diameter is greater than 0.6 of the plate width and the calculated effective width is out of range, see Section 1.4.1.4 of the Data Item.
111	$L_C \geq l_{inf}$	The length LC is less than linf (see the definition of linf in the Data Item).
112	$0.1 \leq \frac{D_{bi}}{w_{Ai}} \leq 0.6$	The ratio of the diameter of bolt *i* to the width of plate A is outside the range 0.1 to 0.6 for which the equation for calculating the effective width of the plate is applicable.
113	$0.1 \leq \frac{D_{bi}}{w_{Ci}} \leq 0.6$	The ratio of the diameter of bolt *i* to the width of plate C is outside the range 0.1 to 0.6 for which the equation for calculating the effective width of the plate is applicable.

2.4.4 Failure to Provide a Solution Owing to Errors in the Input Data

Whenever the Fortran program is run, the checks given in Table 2.3 are performed and if any input values are found to be out of range the appropriate message(s) is issued and the program is terminated. For messages 201 to 203, only the message is output; in all the other cases, the input data are provided followed by the appropriate message(s) and the statement ‘Computation terminated, no solution obtained’.

TABLE 2.3 INPUT DATA CHECKS AND ERROR MESSAGES

No.	*Check*	*Message if check is violated*
201	$1 \le N \le 20$	The number of bolts specified should be in the range 1 to 20 inclusive.
202	0 or 1	The number indicating whether bolt flexibilities are known or are to be calculated should be either 0 or 1.
203	0 or 1	The number indicating whether Aps is to be taken at its default value or whether values are to be supplied should be either 0 or 1.
204	$P_j \ne 0$	The applied load should not be zero.
205	$E_A > 0$	The modulus of elasticity of plate A should be positive.
206	$0 \le \nu_A \le 0.5$	The Poisson`s ratio of plate A should be in the range 0 to 0.5 inclusive.
207	$L_A \ge 0$	The length LA should not be negative.
208	$w_A > 0$	The width of the length LA should be positive. This check is only performed if $L_A > 0$.
209	$t_A > 0$	The thickness of the length LA should be positive. This check is only performed if $L_A > 0$.
210	$E_C > 0$	The modulus of elasticity of plate C should be positive.
211	$0 \le \nu_C \le 0.5$	The Poisson`s ratio of plate C should be in the range 0 to 0.5 inclusive.
212	$L_C \ge 0$	The length LC should not be negative.
213	$w_C > 0$	The width of the length LC should be positive. This check is only performed if $L_C > 0$.
214	$t_C > 0$	The thickness of the length LC should be positive. This check is only performed if $L_C > 0$.
215	$f_{bAi} > 0$	The flexibility of bolt *i* in plate A should be positive.
216	$f_{bCi} > 0$	The flexibility of bolt *i* in plate C should be positive.
217	$L_i \ge \frac{D_{bi}}{2} + \frac{D_{bi+1}}{2}$	The pitch between bolts i and $i + 1$ is less than the sum of the radii of the adjacent bolts.
218	$w_{Ai} > 0$	The width of plate A between bolts i and $i + 1$ should be positive.
219	$t_{Ai} > 0$	The thickness of plate A between bolts i and $i + 1$ should be positive.
220	$w_{Ci} > 0$	The width of plate C between bolts i and $i + 1$ should be positive.
221	$t_{Ci} > 0$	The thickness of plate C between bolts i and $i + 1$ should be positive.
222	$D_{bi} > 0$	The diameter of bolt *i* should be positive.
223	$E_{bi} > 0$	The modulus of elasticity of bolt *i* should be positive.
224	$0 \le \nu_{bi} \le 0.5$	The Poisson`s ratio of bolt *i* should be in the range 0 to 0.5 inclusive.
225	$\lambda_{Ai} \ge 0$	The rotational stiffness of the nut or head of bolt *i* in plate A should not be negative.
226	$\lambda_{Ci} \ge 0$	The rotational stiffness of the nut or head of bolt *i* in plate C should not be negative.

TABLE 2.3 INPUT DATA CHECKS AND ERROR MESSAGES (Continued)

No.	*Check*	*Message if check is violated*
227	$t_{Abi} > 0$	The thickness of plate A under bolt *i* should be positive.
228	$t_{Cbi} > 0$	The thickness of plate C under bolt *i* should be positive.
229	$k_{Ai} > 0$	The effective stiffness of plate supporting bolt *i* in plate A should be positive.
230	$k_{Ci} > 0$	The effective stiffness of plate supporting bolt *i* in plate C should be positive.
231	$w_{Ai} \geq D_{bi}$ and $w_{Ai} \geq D_{bi+1}$	The width of plate A between bolts *i* and $i + 1$ is less than the diameter of bolt *i* or bolt $i + 1$.
232	$w_{Ci} \geq D_{bi}$ and $w_{Ci} \geq D_{bi+1}$	The width of plate C between bolts *i* and $i + 1$ is less than the diameter of bolt *i* or bolt $i + 1$.
233	$2D_{bi} < L_i$ and $2D_{bi+1} < L_i$	One of the bolt diameters is greater than or equal to half the pitch to an adjacent bolt.
234	$A_{psAi} \geq 0$	The effective area over which shear acts in plate A for bolt *i* should not be negative.
235	$A_{psCi} \geq 0$	The effective area over which shear acts in plate C for bolt *i* should not be negative.

2.4.5 Failure to Provide a Solution With Valid Input Data

In tests on the program with practical joints no failures of this kind have been encountered. The following failures might be encountered in extreme cases. In each case the program is terminated and the input printed out and no solution is possible.

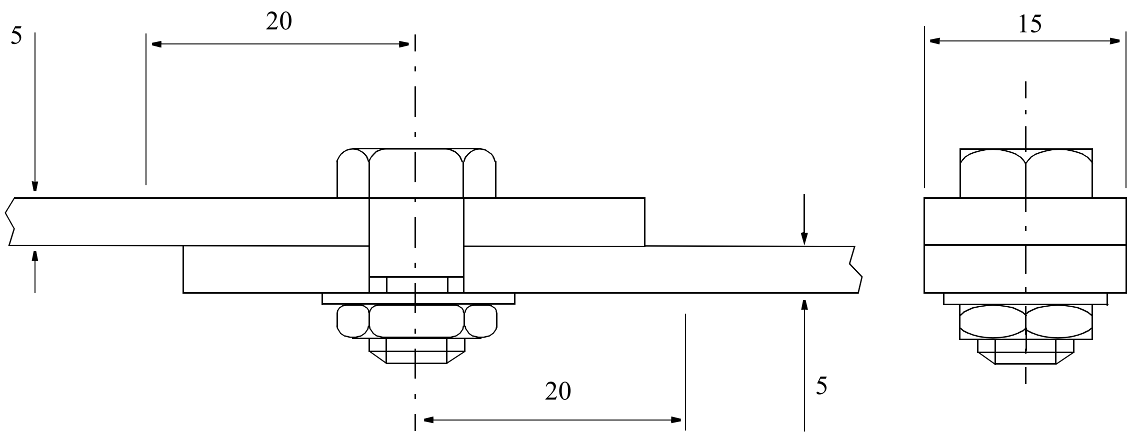
TABLE 2.4 ADDITIONAL ERROR MESSAGES

No.	*Message*
301	Calculation failed. Attempted division by near zero in solution of equations for either a bolt flexibility or the overall joint flexibility. Check input data.
302	Calculation failed. Exponentiation attempted which would exceed machine limit. A possible cause is an abnormally large value for the effective stiffness of the plate supporting the bolts. Check input data.
303	Calculation failed. Exponentiation attempted which would exceed machine limit. A possible cause is an abnormally large value of plate thickness. Check input data.

2.5 EXAMPLES

2.5.1 Example 1, Single Lap, Single-bolt Joint

It is required to determine the influence of bolt size on the flexibility of a single lap joint. The joint is shown in Sketch 2.1. Only 6 and 8 mm diameter bolts are practical options and both will be investigated. The joint is arbitrarily taken to extend 20 mm either side of the bolt centre-line.



Dimensions in millimetres

Sketch 2.1 Joint of Example 1

The properties of the two plates are identical and are as follows.

$$E_A = E_C = 200\ \text{GN/m}^2 \ \text{ and } \ \nu_A = \nu_C = 0.30\,.$$

The material properties of the bolts are also identical and are

$$E_b = 207\ \text{GN/m}^2 \quad \text{and} \quad \nu_b = 0.29.$$

The bolts will be torque tightened and the head restraint is expected to be effectively infinite. Accordingly, a value of $\lambda = 1 \times 10^{18}$ N mm/rad will be used for both assemblies.

The values of k_A and k_C are required and mean values may be estimated using Equation (1.21). Hence, for the 6 mm bolt.

$$k = 0.32 \times 200\ 000 \times \frac{6.0}{6.35} = 60\ 500\ \text{N/mm}^2.$$

A similar calculation for the 8 mm diameter bolt yields $k = 80\ 600$ N/mm^2.

Section 2.5.1.1 gives the input file for the program for the 6 mm bolt. Note that the units are described as N for newtons and mm for millimetres. Because of this, all the input values MUST be in these units and hence, for example, E_A is expressed as 200 000 N/mm^2. The input requires a load to be applied to the joint and a value of 1000 N has arbitrarily been used.

Section 2.5.1.2 gives the output file obtained using the 6 mm bolt. The results for the joint with a 6 mm bolt and those for a joint with an 8 mm bolt are summarised in the table below.

Bolt Diameter mm	*Flexibility*	
	Bolt 10^{-6} mm/N	*Complete Joint* 10^{-6} mm/N
6	9.437	12.57
8	7.934	11.55

The reduction in joint flexibility for an increase in bolt size is because, although the bolt flexibility decreases, the larger bolt holes increase the flexibility of the plates.

2.5.1.1 Input File for Joint with 6 mm Bolt

```
Example 1
Single lap, single-bolt joint
October 2001
N
mm
1000
1
200000  0.3
20
15      5
200000  0.3
20
15      5
0
6       207000  0.29    5       5       60500   60500   1E+18   1E+18
0
```

2.5.1.2 Output File for Joint with 6 mm Bolt

```
********************************************************************************
ESDU International plc
Program A9812

ESDUpac Number:     A9812v10
ESDUpac Title:      Flexibility of, and load distribution in,
                    multi-bolt lap joints subject to in-plane axial loads
Data Item Number:   98012
Data Item Title:    Flexibility of, and load distribution in,
                    multi-bolt lap joints subject to in-plane axial loads
ESDUpac Version:    1.0 October 2001

(See Data Item for full input/output specification and interpretation.)
********************************************************************************
Example 1
Single lap, single-bolt joint
October 2001
********************************************************************************
INPUT DATA
==========
Unit of force  N
Unit of length mm
```

```
Applied load (tensile +ve, comp. -ve) =    1000.     N

MATERIAL PROPERTIES OF PLATE A (commences before bolt 1  )
  Modulus of elasticity                =    .2000E+06 N/mm^2
  Poisson`s ratio                      =    .3000

MATERIAL PROPERTIES OF PLATE C (extends beyond bolt  1)
  Modulus of elasticity                =    .2000E+06 N/mm^2
  Poisson`s ratio                      =    .3000

JOINT GEOMETRY
==============
Before bolt number 1
  Length LA to bolt 1                  =    20.00     mm
  Width of length LA                   =    15.00     mm
  Thickness of length LA               =    5.000     mm

After bolt number  1
  Length LC after bolt  1              =    20.00     mm
  Width of length LC from bolt  1      =    15.00     mm
  Thickness of length LC from bolt  1 =    5.000     mm

BOLT DATA
=========
Mechanical Properties and Plate Thicknesses
-------------------------------------------
  Bolt      Diameter       Modulus     Poisson`s      Thickness Under Bolt
 Number                      of          Ratio         Plate A       Plate C
                          Elasticity
              mm            N/mm^2                      mm            mm

    1        6.000        .2070E+06     .2900          5.000         5.000

Effective Stiffnesses and Rotational Restraints
-----------------------------------------------
  Bolt          Effective Stiffness      Head Rotational Restraints
 Number           Supporting Bolt
                Plate A       Plate C     Plate A end    Plate C end
               N/mm^2        N/mm^2      N mm/rad       N mm/rad

    1        .6050E+05     .6050E+05     .1000E+19      .1000E+19

Default value for area of plates supporting the bolts
has been selected, that is, area Aps  = 0.1Db*Db  mm^2
===============================================================================
RESULTS
=======

Calculated Effective Widths of Plates
-------------------------------------

Plate A before bolt 1, length LA    =    12.76     mm

Plate C after bolt  1, length LC    =    12.76     mm
```

```
Bolt Loads and Flexibilities
----------------------------
  Bolt      Load       Flexibility   Flexibility      Total
 Number                    in            in        Flexibility
                        Plate A       Plate C     (Both Plates)
             N           mm/N          mm/N          mm/N

   1       1000.        .4719E-05     .4719E-05     .9437E-05

Total Joint Stiffness, Flexibility and Extension Between Reference Points
-------------------------------------------------------------------------
  Stiffness                            =    .7954E+05 N/mm
  Flexibility                          =    .1257E-04 mm/N
  Extension under applied load         =    .1257E-01 mm
*******************************************************************************
Computation completed, solution obtained.
```

2.5.2 Example 2, Multi-bolt, Tapered, Double Lap Joint

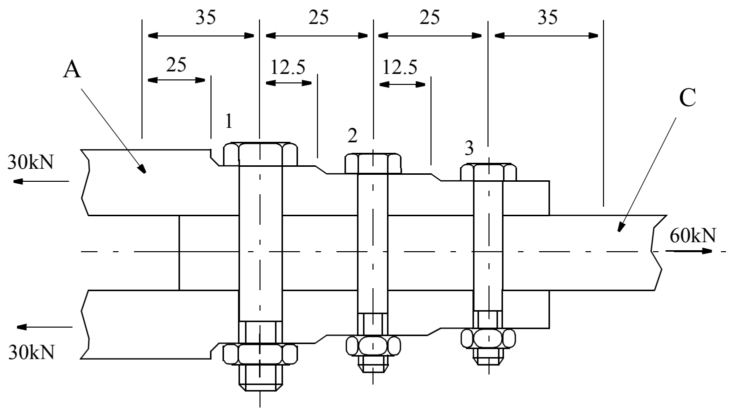
It is required to determine the flexibility, extension and bolt loads in a six-bolt double row tapered joint subject to a total load of 60 kN. The bolts are arranged in identical parallel rows of three bolts each as shown in Sketch 2.2. The material properties and bolt details are as follows.

Outer plates of forged material having $E = 72\ \text{GN/m}^2$ and $\nu = 0.33$.

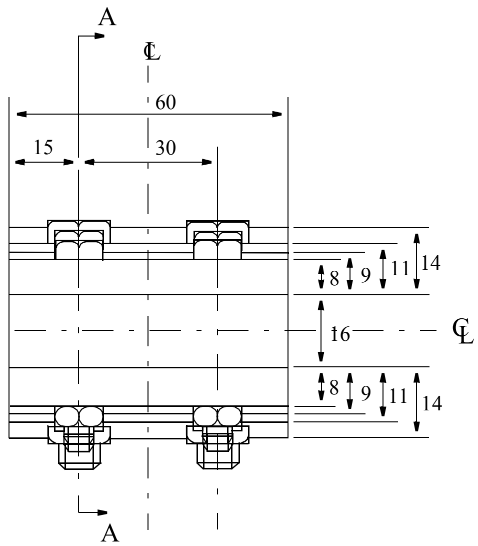
Inner plate having $E = 74\ \text{GN/m}^2$ and $\nu = 0.33$.

Bolts:

Number 1: diameter = 10 mm, head rotational restraint estimated at 10×10^6 N m/rad.
Numbers 2 and 3: diameter = 6 mm, head rotational restraint estimated at 3×10^6 N m/rad.
Material properties, all bolts: $E_b = 204\ \text{GN/m}^2$ and $\nu_b = 0.3$.



Sketch 2.2a Section A–A



Sketch 2.2b

Sketch 2.2 Joint of Example 2

As Sketch 2.2 shows, the joint is symmetric about both the horizontal and vertical centre-lines shown in Sketch 2.2b. It is therefore possible to analyse one quarter of the joint as a single lap joint subject to one quarter of the total load and then to calculate the total joint flexibility. Thus, the input file shown in Section 2.5.2.1 has been prepared for one quarter of the joint subject to a load of 15.0 kN.

The values of k_A and k_C are required for each bolt and these are estimated using Equation (1.21) expressing the bolt diameters and moduli in N and m. Thus, for bolt 1 in plate A,

$$k = 0.32 \times 72\ 000 \times \frac{0.010}{0.00635} = 36\ 300\ \text{MN/m}^2$$

and in plate C

$$k = 0.32 \times 74\ 000 \times \frac{0.010}{0.00635} = 37\ 300\ \text{MN/m}^2 .$$

For both bolts 2 and 3 in plate A

$$k = 0.32 \times 72\ 000 \times \frac{0.006}{0.00635} = 21\ 800\ \text{MN/m}^2$$

and in plate C

$$k = 0.32 \times 74\ 000 \times \frac{0.006}{0.00635} = 22\ 400\ \text{MN/m}^2 .$$

The head rotational restraints are provided. In the centre of the joint, symmetry considerations dictate that the bolt restraint is infinite. In these cases a value of 1×10^{18} MN m/rad is recommended. For the length L_A an average thickness over its 35 mm length is required. This is calculated as

$$\frac{35}{\frac{25}{14} + \frac{10}{11}} = 12.99\ \text{mm} \ \text{ or } 0.012\ 99\ \text{m} .$$

The units chosen will be MN and m. Thus, all the input will be provided using only these units. Section 2.5.2.1 gives the input file. Note that average values are used for the thicknesses of plate A between bolts 1 and 2 and 2 and 3.

Section 2.5.2.2 gives the output file obtained when using the input file of Section 2.5.2.1. The joint extension is found to be 0.1484×10^{-3} m. All four quarters of the joint will extend the same distance under their quarter of the total load. Therefore this extension is also the extension of the complete joint under the total load. Then, representing the flexibility in terms of the total applied load of 60 kN, the total joint flexibility is

$$F_j = \frac{0.1484 \times 10^{-3}}{0.060} = 0.00247\ \text{m/MN}.$$

The bolt loads are:

bolt 1, 10 mm diameter, 0.007 670 MN,
bolt 2, 6 mm diameter, 0.003 358 MN
and bolt 3, 6 mm diameter, 0.003 972 MN.

The bolt loads calculated are the shear loads transmitted across the faces between plates A and C and hence they sum to 0.015 MN. Each bolt transmits these loads across each interface giving a total load transmission of 0.06 MN or 60 kN.

2.5.2.1 Input File for Example 2

```
Example 2
Analysis of quarter of joint as a single lap joint
October 2001
MN
m
0.015
3
72000   0.33
0.035
0.030   0.01299
74000   0.33
0.035
0.030   0.008
0.025   0.030   0.010   0.030   0.008
0.025   0.030   0.0085  0.030   0.008
0
0.010   204000  0.3     0.011   0.008   36300   37300   10      1E+18
0.006   204000  0.3     0.009   0.008   21800   22400   3       1E+18
0.006   204000  0.3     0.008   0.008   21800   22400   3       1E+18
0
```

2.5.2.2 Output File for Example 2

```
******************************************************************************
ESDU International plc
Program A9812

ESDUpac Number:    A9812v10
ESDUpac Title:     Flexibility of, and load distribution in,
                   multi-bolt lap joints subject to in-plane axial loads
Data Item Number:  98012
Data Item Title:   Flexibility of, and load distribution in,
                   multi-bolt lap joints subject to in-plane axial loads
ESDUpac Version:   1.0 October 2001

(See Data Item for full input/output specification and interpretation.)
******************************************************************************
Example 2
Analysis of quarter of joint as a single lap joint
October 2001
******************************************************************************
INPUT DATA
==========
Unit of force  MN
Unit of length m

Applied load (tensile +ve, comp. -ve) =    .1500E-01 MN
```

MATERIAL PROPERTIES OF PLATE A (commences before bolt 1)

Modulus of elasticity	=	.7200E+05 MN/m^2
Poisson`s ratio	=	.3300

MATERIAL PROPERTIES OF PLATE C (extends beyond bolt 3)

Modulus of elasticity	=	.7400E+05 MN/m^2
Poisson`s ratio	=	.3300

JOINT GEOMETRY
==============

Before bolt number 1

Length LA to bolt 1	=	.3500E-01 m
Width of length LA	=	.3000E-01 m
Thickness of length LA	=	.1299E-01 m

Between Bolts	Pitch	Plate A Width	Plate A Thickness	Plate C Width	Plate C Thickness
	m	m	m	m	m
1 and 2	.2500E-01	.3000E-01	.1000E-01	.3000E-01	.8000E-02
2 and 3	.2500E-01	.3000E-01	.8500E-02	.3000E-01	.8000E-02

After bolt number 3

Length LC after bolt 3	=	.3500E-01 m
Width of length LC from bolt 3	=	.3000E-01 m
Thickness of length LC from bolt 3	=	.8000E-02 m

BOLT DATA
=========

Mechanical Properties and Plate Thicknesses

Bolt Number	Diameter	Modulus of Elasticity	Poisson`s Ratio	Thickness Under Bolt Plate A	Thickness Under Bolt Plate C
	m	MN/m^2		m	m
1	.1000E-01	.2040E+06	.3000	.1100E-01	.8000E-02
2	.6000E-02	.2040E+06	.3000	.9000E-02	.8000E-02
3	.6000E-02	.2040E+06	.3000	.8000E-02	.8000E-02

Effective Stiffnesses and Rotational Restraints

Bolt Number	Effective Stiffness Supporting Bolt Plate A	Effective Stiffness Supporting Bolt Plate C	Head Rotational Restraints Plate A end	Head Rotational Restraints Plate C end
	MN/m^2	MN/m^2	MN m/rad	MN m/rad
1	.3630E+05	.3730E+05	10.00	.1000E+19
2	.2180E+05	.2240E+05	3.000	.1000E+19
3	.2180E+05	.2240E+05	3.000	.1000E+19

Default value for area of plates supporting the bolts
has been selected, that is, area Aps = 0.1Db*Db m^2

==

```
RESULTS
=======

Calculated Effective Widths of Plates
-------------------------------------

Plate A before bolt 1, length LA    =    .2647E-01 m

 Between      Calculated Effective Width
  bolts          Plate A       Plate C
                   m             m

 1 and  2         .2491E-01     .2491E-01
 2 and  3         .2679E-01     .2679E-01

Plate C after bolt  3, length LC    =    .2866E-01 m

Bolt Loads and Flexibilities
----------------------------
  Bolt      Load       Flexibility  Flexibility      Total
 Number                    in           in         Flexibility
                         Plate A      Plate C     (Both Plates)
            MN           m/MN         m/MN          m/MN

   1       .7670E-02     .4263E-02    .4329E-02     .8592E-02
   2       .3358E-02     .1048E-01    .9972E-02     .2045E-01
   3       .3972E-02     .1017E-01    .9972E-02     .2014E-01

Total Joint Stiffness, Flexibility and Extension Between Reference Points
-------------------------------------------------------------------------
  Stiffness                           =    101.1     MN/m
  Flexibility                         =    .9895E-02 m/MN
  Extension under applied load        =    .1484E-03 m
*******************************************************************************
Computation completed, solution obtained.
```

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APPENDIX A TEST FOR EFFECTIVE STIFFNESS OF PLATE SUPPORTING BOLT, *k*

A1. INTRODUCTION

This appendix describes an experimental method of estimating the effective stiffness, k, of a plate supporting a bolt. The test method is described in detail in Section A2. In essence it consists of loading a specially manufactured double lap joint having relatively rigid outer plates. This design eliminates all overall joint eccentricity effects, minimises bolt bending effects and allows measurement of an extension, which arises principally from deformation of the material around the inner plate bolt hole. The relatively small plate extensions are calculated and subtracted from the measured extension and the resulting value is used to estimate k.

Values for k, and therefore tests for k, are particular to a bolt size and material and plate material, see Section 1.3 and Equation (1.13). Equation (1.21) indicates that k is linearly related to D_b. This simple relationship may be used to make an approximate correction to values of k for application to joints using bolts of a different diameter to that tested.

A2. THE TEST

Sketch A2.1 illustrates the test used in Derivation 6 to determine joint extensions from which the value of k is deduced. Apart from the basic geometrical and material properties of the bolt and plate, the following effects need to be considered.

(a) The bolt must only be tensioned sufficiently to remove all slack. It must not be tensioned to the point where significant load is transmitted between the plates by friction.

(b) The choice of outer plates that are very much stiffer than the centre specimen plate ensures that the displacements measured to determine k relate effectively only to the specimen plate.

(c) The plate through-the-thickness shear stiffness has an increasingly significant effect on the bolt stiffness as the plate thickness is increased. For computational purposes this effect is represented in the analysis by an empirically determined constant related to the bolt diameter. This effect is largely eliminated from the test for k by using a centre plate that is thin compared to the bolt diameter.

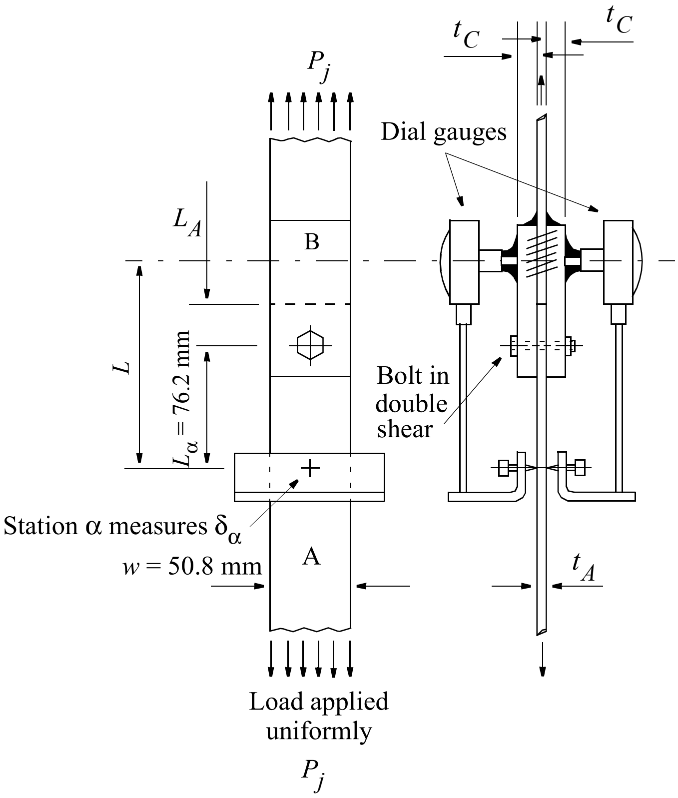
(d) By keeping the central test plate thin the effect of pin bending across the test section is minimised.

Details of the test used to establish a value of k for a 6.1 mm bolt were as follows.

Centre test plate	material	aluminium alloy BS L73	
	width	50.8 mm	(2.0 in)
	thickness, t_A	4.0 mm	(0.15 in)
Outer plates	material	mild steel	
	width	50.8 mm	(2.0 in)
	thickness, t_B	12.7 mm	(0.5 in)
Bolt	material	titanium alloy	
	diameter	6.1 mm	(0.24 in)
Nut	material	steel, torque tightened to 6 N m (53 lbf in)	

As shown in Sketch A2.1, gauges are placed on each side of the specimen to allow elimination of possible curvature effects. A uniform tensile load was applied to the plates. The gauges were first zeroed at 0.5 kN (112 lbf) and the load was then increased in increments to 15 kN (3370 lbf). Measurements of the extension at station α (see Sketch A2.1) were recorded at each load increment. The linear portions of the resulting load-extension curve were used to provide incremental values of load and extension for the calculation of k as described in Section A3.

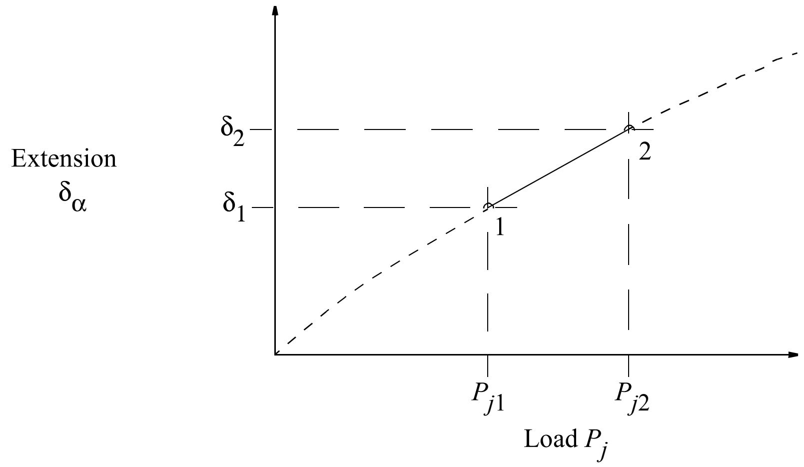
The test was also used to examine the effect of varying the free length of the centre plate, L_A, beyond the bolt. Free lengths of 12.7 mm (0.5 in), 25.4 mm (1.0 in) and 38.1 mm (1.5 in) gave no significant variation in the stiffnesses obtained.



Sketch A2.1

A3. DETERMINATION OF *k* FROM TEST DATA

Sketch A3.1 shows an idealised example of the load-extension curves obtained from the test described in Section A2.



Sketch A3.1

Values used to determine k are taken from the linear portion of the load-extension curve as indicated by points 1 and 2 in Sketch A3.1 above.

In the elastic range

$$F_o = F_p + F_b, \tag{A3.1}$$

where F_o is the observed flexibility of the double lap joint measured at station α,
F_p is the plain plate flexibility of the double lap joint at station α
and F_b is the bolt flexibility in the double lap joint.

The observed flexibility at station α is

$$F_o = \frac{\delta_2 - \delta_1}{P_2 - P_1} . \tag{A3.2}$$

The plain plate flexibility over length L is that of plate A over length L_α and that of plates C over the length $L - L_\alpha$. Thus,

$$F_p = \frac{L_\alpha}{w_A t_A E_A} + \frac{L - L_\alpha}{2 w_B t_B E_B} . \tag{A3.3}$$

(A3.4)

Substituting F_o and F_p from Equations (A3.2) and (A3.3) in (A3.1) gives

$$F_b = \left(\frac{\delta_2 - \delta_1}{P_{j2} - P_{j1}}\right) - \left(\frac{L_\alpha}{w_A t_A E_A} + \frac{L - L_\alpha}{2 w_B t_B E_B}\right). \tag{A3.5}$$

Since the test has been chosen to eliminate through-the-thickness shear effects

$$k = \frac{1}{F_b t_A} , \tag{A3.6}$$

which can be calculated using t_A and Equation (A3.5). Alternatively the program of Section 2 may be used to analyse the joint and by trial the value of k can be found that yields the experimentally determined value of F_o. The value will differ by a small amount from that of Equation (A3.6) since the program reduces the stiffness of the plates to account for the presence of the bolt holes.

ESDU Engineering Sciences Data Unit

98012

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98012

Flexibility of, and load distribution in, multi-bolt lap joints subject to in-plane axial loads
ESDU 98012

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ESDU 98012 introduces a Fortran program, ESDUpac A9812 provided both as source code and in compiled form, that computes the bolt loads and flexibilities, and joint extension and flexibility of a multi-bolt, single row, lap joint loaded in either axial tension or compression. The analysis allows for joints in which the bolts are of different sizes and are not at a regular pitch and in which the thickness and/or width of the joined plates vary between bolt pitches. The analysis may be applied to symmetric double lap joints and joints with multiple rows of bolts provided the rows are identical. The details of the idealisations adopted are explained, and worked examples illustrate the program input and output formats used. The program requires the joint geometry and material properties and the flexibilities of each of the bolts under shear loading. A theory for the calculation of these flexibilities is also described and may be used when they are not available. The program also requires the effective stiffnesses of the plates and an empirical relationship between effective stiffness and plate modulus and bolt diameter is given. However, it is suggested that, if possible, the effective stiffness of the plate to be used should be determined experimentally, and a suitable test procedure is provided. Detailed notes on the major parameters that influence the bolt loads and joint flexibility are included along with figures that illustrate those influences. The joint stiffness value calculated will be particularly useful to represent such a joint when using finite element analysis to determine the gross load distribution through a structure.

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