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PRELIMINARY INVESTIGATION OF THE LOADS CARRIED BY INDIVIDUAL BOLTS IN BOLTED JOINTS

By Manford B. Tate and Samuel J. Rosenfeld

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

A general solution is presented for the determination of loads carried by individual bolts in symmetrical butt joints. Expressions for bolt behavior are given by which the general solution may be readily adapted to the numerical calculation of bolt loading in joints made of any of several combinations of materials common to airplane construction, and an example is solved to illustrate the numerical procedure. All expressions are confined to the range of elastic action of joint components.

Tests were conducted in which the test specimens were made of 24S-T aluminum-alloy plates fastened by two or three $\frac{1}{4}$-inch alloy-steel bolts with the bolts in a single line in line with the applied load. Test results are given in the form of curves showing bolt-load histories through the elastic and yield ranges to joint failure. Empirically based principles are proposed to define the practical upper limit of elastic action of a joint subject to static loading and to obtain curves representing bolt action above this limit for three-bolt joints. With empirical data, such curves combined with analytical equations provide a means for the prediction of bolt loads at any joint load. Bolt-deflection curves and their relation to the general problem are also presented.

From the tests of three-bolt joints, agreement within about 10 percent was found between theoretical and experimental bolt loads within the elastic range. Although the bolts carried markedly unequal loads in the elastic range (as indicated by theory), it was found for such joints (containing not more than three bolts in the line of stress) that a process of bolt-load equalization took place beyond the limit of elastic action which for practical purposes caused the bolts to be loaded equally at

joint failure. Information is needed with respect to multirow joints, however, because in the elastic range the bolts in the first rows carry far greater loads than interior bolts and joint failure may occur before complete equalization of bolt loads is realized.

INTRODUCTION

In recent years the need for more rational means of design and analysis of connections has been emphasized by the exacting requirements of modern airplane construction. The methods of joint analysis are far more antiquated than those employed for other parts of the aircraft structure. Improved methods for predicting joint strength offer a means of reducing weight if they are adapted to make more efficient use of all connectors within a joint. From the production viewpoint, Jenkins (reference 1) has shown that approximately 50 percent of the total cost of the all-metal airplane frame is due to connecting the various components of the structure and that the cost of riveting and bolting constitutes between 80 and 90 percent of the total cost of connections. These conditions suggest a promising field for investigation.

The well-established methods for computing rivet or bolt loads are based on assumptions derived from ultimate-strength tests of a number of riveted joints. Rivets or bolts of the same size were thought to carry equal loads because the ultimate strength of the tested joint was approximately equal to the strength determined from the ultimate strength of a single rivet multiplied by the number of rivets in the joint. The fact that rivets or bolts in a structural joint do not generally carry equal loads in the elastic range was recognized as early as 1867 (reference 2). Batho (reference 3) demonstrated that a riveted joint is a statically indeterminate structural system and that the rivet loads may be obtained by the principle of least work. Hrennikoff (reference 2) developed equations for rivet loads for a limited number of joint arrangements from a consideration of the deformations of plates and rivets. Posner (reference 4) developed a general bolt- or rivet-load equation for lap joints based on the deformations of the plates in tension and in bearing under the bolts or rivets. Several investigators have made use of equations derived by means similar to those just mentioned but have obtained factors for rivet behavior by tests of various joint arrangements (references 1, 2, pp. 464-469, and 3). Some investigators have dealt with butt joints (references 2, pp. 464-469, and 5), some with lap joints (references 1 and 4), and some with both (references 3, 2, 5, and 6). A brief history of early investigations is given in reference 2, pp. 474-484.

The present paper deals with the problem of load distribution among the bolts of symmetrical butt joints. Tests were conducted to determine experimentally, both within and above the elastic range, the manner in which load was distributed among the various bolts. The test specimens were doubly symmetrical two- and three-bolt joints made of 24S-T aluminum-alloy plates joined by a single line of $\frac{1}{4}$-inch alloy-steel bolts. Reference to a joint having a certain number of bolts means that the total joint load is imposed on that number of bolts. Analytical expressions, based on elastic action of the joint components, are given whereby the bolt loads may be computed and the experimental and analytical results are compared. The important question of joint action above the limit of elastic behavior is most readily treated from the standpoint of empiricism. Principles, based on the test results, are suggested, but such principles can be extended for general application only when appropriate empirical data are available.

SYMBOLS

A cross-sectional area, square inches

C bolt constant, dependent upon elastic properties, geometric shape, dimensions, and manner of loading of bolts, and upon bearing properties and thickness of plates, inches per kip

D bolt diameter, inches

E Young's modulus, tension or compression, ksi

G shearing modulus of elasticity, ksi

I geometric moment of inertia, inches^4

K	plate constant for tension or compression loading, dependent upon geometric shape, dimensions, elastic properties of plates, and assumed stress distribution, inches per kip
L	length, inches
M	bending moment, inch-kips
N	number of bolts in transverse row
P	external applied load, kips
R	bolt load, kips
b	plate width, inches
p	pitch, inches
t	thickness, inches
w	uniform load on bolt per unit length, kips per inch
x	distance measured along bolt axis, inches
y	distance measured in plane of loading normal to bolt axis, inches
α	numerical factor for beams by which average shearing stress is multiplied in order to determine shearing stress at centroid of a cross section
δ	deflection of bolt, inches
Δ	deformation
ϵ	tensile strain
σ	direct stress, ksi
τ	shearing stress, ksi

Subscripts:

F	fixed-end
av	average

b bolt

bb bending of bolt

br bearing

bs shear of bolt

cr critical

i any transverse row of bolts

n last transverse row of bolts with reference to end of butt strap

p any plate, or main plate

s butt strap

Special combinations of symbols:

P_L measured internal load in lower main plate (see fig. 1) at a section where P_L should equal P, kips

P_s measured internal load in butt straps at center of joint, where P_s should equal P, kips

P_U measured internal load in upper main plate (see fig. 1) at a section where P_U should equal P, kips

$\sum_{1}^{i-1} R$ summation of all bolt loads from row 1 to row i, excluding row i

THEORY AND BASIC ASSUMPTIONS

Elastic behavior of joints.- A bolted joint is a statically indeterminate structural system and can be analyzed as such a system if certain conditions are known or assumed. The theoretical solution given in appendix A for the determination of the loads carried by bolts in symmetrical butt joints is based upon the following conditions:

(1) The ratio of stress to strain is constant

(2) The stress is uniformly distributed over the cross sections of main plates and butt straps

(3) The effect of friction is negligible

(4) The bolts fit the holes initially, and the material of the plates in the immediate vicinity of the holes is not damaged or stressed in making the holes or by inserting the bolts

(5) The relationship between bolt deflection and bolt load is linear in the elastic range

On the basis of these assumptions, it is found for symmetrical butt joints that the relationship between the loads on any two successive bolts in a single line of bolts is

$$R_{i+1} = \frac{C_i}{C_{i+1}}R_i + \frac{2K_p + K_s}{C_{i+1}}R_i - \frac{2K_p}{C_{i+1}}P + \frac{2K_p + K_s}{C_{i+1}}\sum_{1}^{i-1} R \qquad (1)$$

Equation (1) is used in the computation of bolt loads in appendix B.

Based on assumptions 1 and 2, the plate constant K may be stated as

$$K = \frac{p}{btE} \qquad (2)$$

The linear relation between bolt load and deflection (assumption 5) may be expressed in terms of the bolt constant C as

$$\delta = \frac{CR}{2} \qquad (3)$$

In the determination of C it is assumed that the bolt acts as a fixed-end beam with the bolt load R distributed uniformly along a length equal to the main-plate thickness. Acting in the opposite direction, the bolt load is uniformly distributed along two lengths, each equal to the butt-strap thickness. Bearing stress is computed in the conventional manner as bolt load divided by an area that is determined by projecting the bolt diameter on the plate thickness; and bearing deformations are expressed in terms of the compressive moduli of the materials and dimensions of the bolt and plates. From these considerations, C may be stated as follows for joints made of 24S-T plates with a butt-strap thickness of one-half the main-plate thickness $\left(t_s = \frac{t_p}{2}\right)$ and fastened with alloy-steel bolts:

$$C = \frac{8}{t_p E_{bb}} \left\{ 0.13 \left(\frac{t_p}{D}\right)^2 \left[2.12 + \left(\frac{t_p}{D}\right)^2\right] + 1.87 \right\} \quad (4)$$

For other symmetrical butt-joint arrangements, expressions for C are given in appendix A. (See equations A16 to A22.)

The third assumption (that the effect of friction is negligible) gives rise to a highly controversial point in the literature on riveted joints. It appears, at least in the design range common in civil-engineering practice, that a large part of the joint load is carried by friction in hot-riveted joints and in bolted joints if the bolts are drawn tight. Tests reported by Hill and Holt (reference 2, pp. 464-469) indicated, however, that friction is of little importance as a factor in the behavior of riveted joints. Epstein (reference 7) also conducted tests that indicated minor frictional effects in cold-riveted joints.

Apparently the fourth assumption (that the bolts fit the holes) would seldom be fulfilled in an actual joint, and departure from this assumption would be determined largely by fabrication methods. It should be remembered, however, that although the presence of numerous bolts in a joint makes the likelihood of errors from extraneous sources greater, the percentage deviation from the predicted theoretical bolt load will probably be less than in the case where only a small number of bolts make up the joint, because such errors are distributed among a larger number of bolts in the first instance than in the second. It may be anticipated, therefore, that the main features of the analysis will hold when connections are joined with several bolts and when good shop practices are used.

Further consideration of the second and fifth assumptions is made in the discussion of the analysis of test data, and the third and fourth assumptions were fulfilled insofar as practicable in the fabrication of test specimens.

Inelastic behavior of joints.- As load on a joint is increased, a load is reached at which yielding of the plates or of the bolts occurs. Whether yield takes place first in the plates or bolts or occurs simultaneously in both depends upon their relative dimensions and elastic properties. It is therefore possible for one component to act elastically and the other inelastically, but the yielding of any component constitutes the beginning of inelastic action of the joint as a whole. Interpretation of this viewpoint, however, should be practical and should not include yielding of small regions where there are stress concentrations when such yielding has no appreciable effect on the over-all elastic behavior of the joint. A part of the behavior of a joint may be predicted from an elastic theory, and empirical methods may be employed in the determination of the upper limit of elastic action and of joint behavior above this elastic limit. This upper limit of elastic action is termed the "critical bolt load" R_{cr} in this paper.

In the analysis of three-bolt joints in which the main plate and butt straps are of the same width and material with $t_s = \frac{t_p}{2}$ and are joined by bolts that are all of the same size and material, the following procedure may be used to predict the joint-load against the bolt-load (P-R) relationships throughout the elastic and yield ranges to joint failure. The procedure, however, has not been extended to include other joint arrangements because only two- and three-bolt joints were tested. For the elastic range, the P-R relationships may be established by means of equation (1). These relationships may be plotted and the experimentally determined value of R_{cr} plotted on the P-R curve for the most heavily loaded bolt, which is either end bolt for the case under consideration. The end bolts can be shown to support equal loads, and either one carries a greater load in the elastic range than the middle bolt. (See appendix C.) A straight line may be drawn connecting the points for R_{cr} and the average bolt load at failure, which is equal to the joint-failure load divided by 3; thus the P-R curve for either end bolt is completed. The end-bolt load R_1 may be computed for any joint load from equations of the straight lines obtained as outlined. Load R_2 on the middle bolt may be found at any given joint load as $R_2 = P - 2R_1$. The values of R_2, however, are of less importance than the greater loads R_1 on the end bolts and in many cases it is unnecessary to compute values of R_2.

TEST SPECIMENS AND PROCEDURES

Specimens and Apparatus

Specimens.- Materials common to aircraft construction were used for the test specimens, which were selected to provide a plate-thickness range sufficient to check the applicability of the theory. Six symmetrical butt-joint specimens - three of the two-bolt joints and three of three-bolt joints - were fabricated and tested. Each specimen was made symmetrical about its transverse center line. Such a condition provides duplicate test specimens, as the theoretical behavior of the part of the joint on one side of this center line is identical with that of the part on the other side of the line. The specimens were classified in two groups, A and B. It was decided to test two-bolt joints (group A) in order to procure information in regard to the reliability of the method for determining bolt loads from strain measurements and to secure additional information which might serve in the interpretation of data obtained from tests of three-bolt joints. The three-bolt joints (group B) were chosen to furnish an experimental check of the theory and to expedite testing and the analysis of data.

In all cases, the material of the plates was 24S-T aluminum alloy and the bolts were $\frac{1}{4}$-inch aircraft bolts, equivalent to those specified in reference 8 of heat-treated alloy steel with minimum ultimate tensile and shearing strengths of 125 ksi and 75 ksi, respectively.

Specimens A-1 and B-1 were of balanced design based on the usual assumption that load is divided equally among the bolts. The design stresses were 62 and 90 ksi for tension and bearing of the plates, respectively, and 75 ksi for shearing of the bolts. Although reference 8 permits a greater allowable tensile stress, it was considered advisable to use 62 ksi to attain the actual shear strength of the bolts. Specimen A-2 and B-2 were designed to fail in shear; and specimens A-3 and B-3, in tension. In every case, the butt-strap thickness was one-half that of the main plate. A width of $1\frac{1}{4}$ inches and a pitch of 2 inches were used for all specimens in order to accommodate the strain gages. A wring fit was used to fit the bolts in all specimens. In preference to washers, collars made of $\frac{1}{2}$-inch steel tubing were placed under the nuts to eliminate bearing of the plates on bolt threads. When the specimens were assembled, the nuts were first tightened to bring the plates together and then, in order to eliminate friction forces insofar as practicable, were loosened to cause firm contact between the bolt heads or collars and plates. The specimens are shown in figure 1 and their dimensions are given in table 1.

Apparatus.- Load was applied in tension by means of a hydraulic testing machine having 100-kip capacity and an accuracy within 0.5 percent. Wedge grips were used to apply load to specimen A-1, and the remaining specimens were gripped with Templin grips of 50-kip capacity.

Strain was measured by $\frac{1}{2}$-inch SR-4 electrical gages. With these gages, the error in strain measurement did not exceed 2 percent. An attempt was made to measure bolt deflections by means of micrometer microscopes but was abandoned because the instruments were not sufficiently precise to measure the small deflections that occurred in the elastic range. As load was applied, enlargement of the gap between main plates was measured with 1-inch Tuckerman optical strain gages. The arrangement of electrical strain gages is shown in figure 1 and the general test arrangement for a typical specimen is shown in figure 2.

Testing Procedure

The width and thickness of each plate were measured at several points along the length of the plate with a micrometer caliper of 0.0001-inch precision, and the bolt diameters were checked as a precaution against the use of appreciably irregular bolts.

Loading tests in the elastic range.- In the loading tests in the elastic range, load was applied in six or seven (usually equal) increments to a load approximately equal to 45 percent of the estimated ultimate load. The specimen was then unloaded with repetition of the increments used in the application of increasing load. This process was repeated twice, with the specimen thus subjected to three complete cycles of loading. Strain readings were made at each increment of load. This procedure was followed in testing all joints with the exception of specimen A-1, which was loaded directly to failure.

Loading tests to failure.- After the first phase of testing, the specimens were loaded to failure. Load was applied in 12 to 15 increments until failure occurred. Strain and the increase in width of the gap between main plates were observed at each load increment. Photographs of the fractured specimens are shown as figures 3 and 4.

Auxiliary tests.- Stress-strain data were secured from tests of standard tension specimens representing the plate components of the joints. The location of the tension specimens in relation to the plates from which they were obtained is shown in figure 1.

Shear tests of single bolts were conducted in order to evaluate a double-shear strength that would be representative of the bolts employed in joining the specimens of groups A and B. For these tests, the plates were of S.A.E. 6150 heat-treated steel and a wring fit was used to fit the bolts. The dimensions of the specimens are shown in table 1. As it was desired to compare separately determined bolt deflections with average values computed from the movement of the gap, three sets of deflection measurements were obtained in each of two tests. Movement of the gap was determined in the manner used to secure similar data for specimens of groups A and B. Deflection measurements for both bolts were obtained separately by placing 2-inch Tuckerman gages on opposite faces of a specimen with the fixed knife edge of a gage on the butt strap and with the lozenge on the main plate. Precautions were taken to ensure approximate parallelism between the gages and plate surfaces.

PRESENTATION AND DISCUSSION OF RESULTS

Determination of Bolt Loads and Deflections from Test Data

The experimental bolt loads were obtained by finding the loads in the butt straps at sections midway between bolts; the difference between loads at two adjacent sections was considered to be equal to the load on the intervening bolt. Butt-strap loads were computed from strain data; and for specimens A-1, A-2, B-1, and B-2 the loads were corrected for the effect of lateral bending moment in the butt straps, which acted in a plane normal to the plane of the straps. The lateral moment was induced by eccentricity of the resultant of the part of a bolt load that was transmitted to one butt strap. The curves of the butt-strap loads P_s are plotted in figures 5(a) to 15(a) to illustrate the effect of lateral bending and are shown in conjunction with the curves of joint load against bolt load because of the interrelationship of these curves owing to the use of P_s in the determination of correction factors. Calculation of butt-strap loads, the presence of lateral bending moment, and the correction procedure are explained in appendix D.

The methods used in the determination of bolt deflections are based on relative movements of the main plates and butt straps. Elongation of either the main plates or butt straps, depending upon the location of instruments used to measure joint movements, was included in measurements obtained during the tests. Deflections were computed by subtracting such elongation (considered to be PL/AE) from the test measurements. A more detailed explanation of the methods employed is given in appendix D.

Curves of Joint Load Against Bolt Load

The main results of this investigation are presented in the form of curves of joint load against bolt load shown in figures 5 to 15. These curves show the load history of each bolt and indicate the behavior that may be expected of bolts loaded under conditions similar to those of the tests.

Elastic behavior of test specimens.- Figures 5 to 9 show bolt-load values determined from the loading tests in the elastic range. The curves accompanying the plotted points represent experimental curves for the same bolts obtained from the tests to failure. During the testing of specimen A-3, the strain gages at the center of the joint on one butt strap became loosened. As a result, the values of P_s, R_2, and R_3 could not be determined; hence, only the curves for R_1 and R_4 are shown in figure 6. Replacement of the inoperative gages was made prior to the testing of specimen A-3 to failure. Inspection of figures 5 to 9 shows that there is good agreement among the repeated bolt loads and between these loads and the curves from tests to failure.

Figures 10 to 12 give the results of testing the specimens of group A to failure. Both theory (appendix C) and the conventional method of analysis (that is, the assumption of equal loads carried by the bolts) indicate that the P-R curves should be represented by the equation $R = 0.500P$. This curve is not shown, however, as it was considered more informative to give the experimental curves and their equations. In every case, the equation given for a curve applies to the initial straight-line portion of the curve. In general, it may be seen that deviations of 3 to 11 percent from an equal distribution of load to the bolts occurred in the two-bolt joints. The maximum deviation from equality of bolt loads occurred in the right end of specimen A-2, wherein the fourth bolt supported about 20 percent more load than indicated by either the elastic or the conventional analysis. It appears that differences between loads carried by the two bolts in one end of a joint were due to fabrication inequalities and variability of the properties of the bolts. Considerable variation of bolt characteristics was shown by results of the auxiliary shear tests; that is, bolts which were presumably identical and under the same loading conditions deflected amounts in the elastic range that differed by as much as 35 percent, and double-shear strengths were found to range from 5 to 32 percent greater than stipulated by the specification in reference 8. (See table 2.)

Figures 13 to 15 show bolt loads that were obtained from tests to failure of the specimens of group B and are plotted for comparison with analytical curves, which are shown only up to that load above which they no longer may be considered applicable. The analytical expressions given in the figures were obtained by use of equation (1); the manner in which they were determined and sample calculations are given in appendix C.

For the three-bolt joints, the P-R curves (figs. 13 to 15) clearly show the inequality of bolt loads within the elastic range. The end bolts (1, 3, 4, and 6) carried loads that differed from the analytically determined bolt loads by amounts ranging from 3 to 10 percent of the analytical values. In some instances differences of about 20 percent were found but are considered to be of little importance as they occurred at bolt loads below one-third of the critical bolt load R_{cr}. The middle bolts (2 and 5) supported loads that differed from the analytical bolt loads by amounts ranging from 5 to 20 percent of the analytical values. Loads on the middle bolts, however, were less than those on the end bolts in all cases and, consequently, are not of so much interest as the end-bolt loads. In the determination of values of R_1 and R_6, the probable experimental error is estimated to be about 5 percent, if the effects of lateral bending of the butt straps are excluded. Loads on bolts 2, 3, 4, and 5 were found indirectly and the probable experimental error was about 10 percent. Most of the discrepancies between experimental and analytical bolt loads therefore cannot be distinguished from experimental error.

Inelastic behavior of test specimens.- The upper limit of elastic action of a test specimen is marked in figures 10 to 15 as the critical bolt load R_{cr}. Above R_{cr} a joint as a whole is considered to behave inelastically although, at loads approaching and above R_{cr} and in some cases from R_{cr} to joint failure, there probably is a complex behavior with some components yielding in highly stressed regions and with continued elastic action of other components. The critical bolt loads R_{cr} and the empirical curves shown in figures 13 to 15 were derived from the experimental results and are explained in the following section.

The test results indicate that yielding of the plates in bearing under the most heavily loaded bolts, yielding of these bolts in shear and bending, or a combination of both caused a process of bolt-load equalization to take place between R_{cr} and joint failure which resulted in an approximately equal distribution of load among the bolts at joint failure. Joint behavior of this nature accounts for the findings of early investigators who made ultimate-strength tests of riveted joints and reported that load was about equally distributed among the rivets.

The equalization of bolt loads is best illustrated in figure 15, which shows the behavior of the three-bolt joint, specimen B-3. Comparison of measured bolt loads with values obtained from the empirical curves of figure 15 shows that the experimental bolt loads are within 4 percent of the empirical values with the exception of the P-R_2 curve, for which the maximum difference is about 8 percent. Although the failure of specimen B-3 was caused by failure of the butt straps in tension, it may be seen that the bolt loads were approximately equal at failure of the specimen. An equal distribution of load to the bolts at joint failure is also indicated by results of the tests in which specimen failure was due to shearing of the bolts. Specimens A-1, A-2, and B-2 failed in this manner. Average bolt loads at failure of these specimens agree closely and are also in agreement with most of the ultimate loads found in the auxiliary shear tests of single bolts. Values of the bolt loads just mentioned are given for comparison in table 3. For the bolts that failed in shear (3 and 4) in specimen A-2, final measurements were obtained at a joint load 40 pounds below the ultimate, and the bolt loads R_3 and R_4 were within 3 percent of equality when the final measurements were taken. (See fig. 11.) The equalization of bolt loads R_3 and R_4 above P_{cr} is also effectively shown in figure 11. Since bolts 1 and 2 in specimen A-1 failed in shear, it is of interest to note from figure 10 that extension of the curves for R_1 and R_2 to the joint-failure line indicates that these bolt loads were within 4 percent of equality at the time of failure. In specimens A-1 and A-2 the distribution of load to the bolts that did not fail (3 and 4 for specimen A-1, and 1 and 2 for specimen A-2) did not approach equality to the same extent as was the case for the bolts that failed. The curves for load on the bolts that did not fail (figs. 10 and 11) diverged from equality at failure of specimens A-1 and A-2 by about 13 and 9 percent, respectively. In view of the variable bolt characteristics pointed out in the preceding section, however, little importance is attached to this divergence. In general, specimen A-3 behaved as expected, and it appears from the curves in figure 12 that the bolt loads were approximately equal at failure of the joint.

In figures 13 and 14 (specimens B-1 and B-2, respectively), the points above P_{cr} show the effect of lateral bending of the butt straps to such an extent that no credence is given them as a true representation of the bolt-load relationships. Although these points were computed in the same manner as the others, the means of correction did not fully account for the bending at high loads; the points were plotted, as were the P_s-curves, to show the nature of errors arising from this source. The fact that failure of specimen B-2 was due to shear failure of bolts 1, 2, and 3, however, is forceful evidence that these bolts were about equally loaded at the time of failure, because the average bolt load at failure is in good agreement with similar bolt-load values found in other tests. (See table 3.) Specimen A-1 acted somewhat differently above R_{cr} than the other specimens, but it may be seen in figure 10 that the general tendency was toward equalization of bolt loads at joint failure. Attention is called to the test conditions: specimen A-1 was tested in wedge grips, and bending of the joint as a unit, revealed by strain gages on the main plates, existed to an undesirable extent. Perhaps averaging the strain measurements did not fully compensate for this effect and gave an indeterminable error in the computation of bolt loads. The use of Templin grips precluded bending of the entire specimen sufficiently to lend assurance of a negligible effect on the remaining specimens.

The fact that all bolts in the two-bolt specimens did not carry equal loads was probably due to inequalities in fabrication and variation in the bolts. Because these specimens contained only two bolts, the importance of such conditions was magnified in the two-bolt joints but was less disturbing in the three-bolt joints. For practical purposes, however, there was a uniform distribution of load among the bolts of each test specimen at failure.

Critical bolt loads and basis of empirical curves.- The general behavior of specimens A-1, A-2, and B-3 as depicted in figures 10, 11, and 15 suggests that the upper limit of elastic action of a joint subject to static loading can be termed the "critical bolt load" R_{cr}. The critical bolt load is dependent upon the factors that contribute in bringing about equalization of bolt loads, which are yielding of the plates in bearing under the most heavily loaded bolts and yielding of these bolts. Such yielding is dependent upon the mechanical properties, dimensions, geometric form, and manner of loading of the plates and bolts. From these considerations, the critical bolt load R_{cr} is defined as that bolt load at which yielding of the plates or bolt or a combination of both occurs to start the action of bolt-load equalization. The critical bolt load is found from an experimental curve as the value of R at the intersection of the straight-line portion of the lower part of the P-R curve with that of the upper part and is determined from the curve for the bolt that carried the greatest load when yielding occurred. The method is illustrated in figures 10(a) and 11(b). Evaluation of R_{cr} for purposes of design or analysis requires data in regard to the appropriate plate and bolt bearing strengths and shear strengths of bolts. For the test specimens, loads and stresses at R_{cr} and failure are given in table 3.

Empirical curves for the three-bolt joints are based on the observation that the general trend of the bolt loads above R_{cr} was toward equality at failure of the joints. The curves were obtained for the end bolts by drawing a straight line connecting the point representing R_{cr} with the point plotted for the average bolt load at joint failure. The point representing R_{cr} was determined as the intersection of the vertical line that locates R_{cr} along the R-axis with the analytical curve obtained from the elastic analysis. The average bolt load at failure was computed as the ultimate joint load divided by the number of bolts that supported the joint load. Curves for the middle bolts (2 and 5) were obtained from conditions of symmetry and equilibrium; that is, $R_2 = P - 2R_1$.

In order to determine empirical curves for specimens B-1 and B-2, data from the tests of specimens A-1 and A-2 were used. Because of the curvature of the upper parts of the P-R curves for specimens B-1 and B-2 (figs. 13 and 14), R_{cr} could not be determined for either specimen. The value of R_{cr} shown in figure 13 for specimen B-1 was computed from the average of bearing stresses calculated for the critical bolt loads of specimens A-1 and A-2. Inasmuch as the plates of specimen B-2 were of the same thickness as those of specimen A-2, the two specimens should have had the same value of R_{cr}; for this reason, the value shown in figure 14 is the same as the value obtained from the test results for specimen A-2. Empirical curves for specimens B-1 and B-2 were constructed with the values of R_{cr} thus found as the starting points and the average bolt loads at failure as the end points.

The bolt-load-equalization process probably starts before R_{cr} is reached; but R_{cr}, as used herein, provides a definable limit for the transition from formulas based on the assumption of elastic behavior to empirical expressions. Such empirical expressions are of practical interest as a basis for design at limit load, because such design generally comes within the range between R_{cr}

and failure and current design methods make no allowance for the unequal distribution of load among the bolts that exists within the greater part of this range. The process of bolt-load equalization is undoubtedly more complicated for joints containing more bolts in line than the specimens of the present tests. As a result, it is improbable that relationships for empirical curves above R_{cr} for multirow joints are as simple as those found in the present tests. It appears probable that failure of the initial bolts in a multirow joint may occur before the process of equalization is completed and may thus cause joint failure at a load appreciably less than the sum of the ultimate strengths of the individual bolts.

Curves of Bolt Load Against Deflection

Deflection of bolts in specimens of groups A and B.- Curves of bolt load plotted against deflection are presented in figure 16. The curves of figure 16(a) show average deflections δ_{av} of the two central bolts, one on each side of the gap between main plates, in each specimen of groups A and B. In plotting the curves of R against δ_{av}, the values of R were those computed from strain data and used to plot the P-R curves. Curves obtained by means of equation (3), $\delta = \frac{CR}{2}$, are plotted for comparison with the experimental curves; and deflections corresponding to the values of R_{cr} determined from the P-R curves are shown.

It may be seen in figure 16(a) for the specimens of groups A and B that, in general, bolt deflection increased rapidly after the critical bolt load R_{cr} was reached. Below R_{cr} there is excellent agreement between the experimental curves for specimens A-2 and B-2, which is in conformance with theory since both specimens were of the same thickness and fastened with bolts of the same size and material. There is good agreement, moreover, between the experimental points and the plot of equation (3), as the greatest difference for either specimen is about 10 percent of the corresponding analytical value. For specimen B-1, the experimental curve diverges from the curve of equation (3) between the origin and R_{cr} by a maximum of 45 percent and shows a divergence of about 30 percent at R_{cr}. Such disparity is not surprising, as the auxiliary tests show that presumably identical bolts may deflect amounts that differ by as much as 35 percent and the two methods used in the determination of δ_{av} yield results that may differ by approximately 15 percent. The experimental deflection at R_{cr} for specimen A-1 was about 36 percent greater than the value computed by means of equation (3); and between the origin and about $\frac{3}{4}R_{cr}$, the differences were between 10 and 20 percent.

The experimental relationships for specimens B-3 and A-3, as shown in figure 16(a), are represented by definitely curved lines. Apparently the straight line representing equation (3) is analogous to the secant line used in determinations of the secant modulus of elasticity for materials having nonlinear stress-strain curves. Up to a bolt load of one-fourth of the average bolt load at failure, the measured deflections were approximately 70 percent less than the analytical values; and at one-half the average bolt load at failure, about 45 percent less. The curve for specimen B-3 crosses the analytical curve at about 0.8 of the average bolt load at failure; and the curve for specimen A-3, at about 0.63.

Deflection of bolts in auxiliary (steel) specimens.- The bolt load against deflection (R-δ) relationships for bolts in two specimens for the auxiliary shear tests are shown in figure 16(b). From these relationships a comparison is made between two methods for the experimental determination of average bolt deflection, and deflection characteristics of bolts loaded under the same conditions are compared. The R-δ_1 and R-δ_2 curves show deflections that were determined separately for each bolt. Deflections δ_1 and δ_2 were averaged and are plotted for comparison with values of δ_{av}, which were computed from data for spreading of the gap between main plates; and curves obtained from equation (3) are shown for comparison with the experimental results.

In figure 16(b), the experimental deflections δ_1 of bolt 1 in specimen 1 and δ_2 and δ_{av} of the bolts in specimen 2 agree with values calculated by means of equation (3) within 3 percent below $R = 5$ kips. The measured deflections δ_2 and δ_{av} of the bolts in specimen 1 were approximately 25 percent less and the deflections δ_1 of bolt 1 in specimen 2 were about 35 percent greater than values of deflection determined from equation (3). The averages of δ_1 and δ_2 were about 15 percent less for specimen 1 and about 15 percent greater for specimen 2 than deflections computed by means of equation (3), and both sets of average deflections were from 10 to 15 percent less than corresponding values of δ_{av}, which were based on measurements of gap movement.

Discussion of results in relation to the R-δ curves.- It is seen from the deflection curves (fig. 16) that R_{cr}, as determined from the P-R curves, was the load carried by a bolt near the beginning of appreciable yield of the bolts or plates or both. It may also be noted that a greater rate of deflection of the bolts in the steel specimens occurred above a load of 5 kips whereas in the aluminum specimens this action always commenced at lower loads. Such action is due to differences in the bearing behavior of the two materials.

For the comparatively thin specimens, A-3 and B-3 $\left(\text{for which } \frac{D}{t_p} = 1.54 \text{ and } 1.33\text{, respectively}\right)$, the relationships between load and deflection were nonlinear; for the thick specimens, A-2 and B-2 $\left(\text{for which } \frac{D}{t_p} = 0.50\right)$, approximate linearity was shown to about 60 percent of the ultimate bolt load; and for the specimens of balanced design, A-1 and B-1 $\left(\text{for which } \frac{D}{t_p} = 0.80 \text{ and } 0.67\text{, respectively}\right)$, approximate linearity existed to about 50 percent of the average bolt load at failure. Probably the difference in behavior of bolts in the various specimens may be attributed to bearing action. The effects of bearing vary with the bearing properties and the relative dimensions of the bolts and plates, and bolt deflection in the thin specimens $\left(\frac{D}{t_p} = 1.33 \text{ and } 1.54\right)$ was largely dependent upon bearing action. The observed nonlinear relationship for deflection of the bolts in specimen B-3 explains, for the most part, the slightly curved shape of the P-R variation for specimen B-3 (fig. 15). In the calculation of bolt deflection by equation (3) the bearing terms in the expression for C have less influence on the results for a constant bolt size in specimens with thick plates $\left(\frac{D}{t_p} = 0.50 \text{ to } 0.80\right)$ than with thin plates $\left(\frac{D}{t_p} = 1.33 \text{ and } 1.54\right)$. It appears, furthermore, for the thick specimens $\left(\frac{D}{t_p} = 0.50 \text{ to } 0.80\right)$ that equation (3) furnishes as satisfactory an approximation of deflection as measured values, because of the variation in the bolts and uncertainty that evidently attends the experimental determination of bolt deflection. Also, despite the nonlinearity of R-δ curves for the thin specimens, the assumption of a linear relation between bolt load and deflection was satisfactory for use in conjunction with equation (1) to establish P-R curves for specimen B-3 $\left(\frac{D}{t_p} = 1.33\right)$.

Fortunately, the analytically determined bolt-load relationships are relatively insensitive to appreciable changes in magnitude of the bolt constant C or the plate constant K; nevertheless, further investigation of these factors is necessary. The effect of bearing has a large influence on the magnitude of C, and the present test results that are given as R-δ curves point to greater uncertainty of the adequacy of the bearing terms than other terms in the expression for C. In addition, further study of K is desirable, as short pitch may cause behavior that would make the actual bolt loads more dependent upon this factor than is indicated by present knowledge.

CONCLUSIONS

The following conclusions are drawn from the results of this investigation and apply to symmetrical butt joints made of 24S-T aluminum-alloy plates joined by two or three wring-fitted alloy-steel bolts of the same size with the bolts in a single line in the line of applied static load:

1. For joints in which the total load is imposed on three bolts, the bolt loads are not equal in the elastic range, as assumed in conventional analysis, and can be calculated within about 10 percent by means of the expressions presented in this paper.

2. Above the elastic range, a process of bolt-load equalization takes place as a result of yielding of the

plates in bearing, yielding of the bolts in shear and bending, or a combination of both; for practical purposes, this action causes the bolts to support equal loads when joint failure occurs.

3. Above the elastic range, also, the analytical curves can be extended in an empirical manner and this extension may be used to provide a basis for limit-load design.

4. For joints in which the total load is imposed on two bolts, the distribution of load to the bolts at the ultimate joint load is less affected by fabrication inequalities and variability of materials than is the distribution in the elastic range.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., December 20, 1945

APPENDIX A

DEVELOPMENT OF ANALYTICAL EXPRESSIONS USED IN DETERMINATION OF BOLT LOADS FOR ELASTIC BEHAVIOR OF SYMMETRICAL BUTT JOINTS

General Bolt-Load Relationship

Definitions and assumptions.- The type of bolted connection dealt with herein is termed a "symmetrical butt joint." In order to clarify the meaning of this phrase, the joint arrangement is defined by the following conditions:

(1) The butt straps must be of the same thickness and material. (A butt strap and the main plate may be of different materials and may have any thickness ratio.)

(2) The bolt pattern must be symmetrical about the longitudinal center line of the joint. (The pattern may be unsymmetrical about the transverse center line lying in the gap between main plates; such a case constitutes two separate problems in the determination of loads carried by bolts in the two halves of the joint.)

(3) Bolts in the same transverse row must be of the same size and material but need not be the same as those in any other row.

In the analysis of a joint as a statically indeterminate structure, there are certain conditions that must be known or assumed. For the present solution, the following assumptions are made:

(1) The ratio of stress to strain is constant.

(2) The stress is uniformly distributed over the cross-sections of main plates and butt straps.

(3) The effect of friction is negligible.

(4) The bolts fit the holes initially, and the material in the immediate vicinity of the holes is not damaged or stressed in making the holes or inserting the bolts.

(5) The relationship between bolt deflection and bolt load is linear in the elastic range and may be expressed as $\delta = \frac{CR}{2}$, in which C is a bolt constant to be determined subsequently.

Analysis of symmetrical butt joint fastened by bolts in a single line in line of applied load.- Briefly, solution of the problem consists of the following steps: After load is applied, a part of the joint between bolts i and $i + 1$ within the joint is considered (fig. 18), and the length $p + \Delta_p$ along the main plate between the two bolts is added to the deflection of bolt $i + 1$ and equated to the length $p + \Delta_s$ along the butt straps between the two bolts plus the deflection of bolt i. The deformations are expressed as functions of the load and deformation characteristics of the plates and bolts. The resulting equation is solved for the bolt load R_{i+1} in terms of the bolt load R_i, the joint load P, and the elastic constants of the plates and bolts.

It may be seen in figure 18 that

$$p + \Delta_p + \delta_{i+1} = p + \Delta_s + \delta_i$$

or

$$\delta_{i+1} = \delta_i - \Delta_p + \Delta_s \tag{A1}$$

From assumption (5),

$$\delta_{i+1} = \frac{C_{i+1}}{2} R_{i+1}$$

and

$$\delta_i = \frac{C_i}{2} R_i \tag{A2}$$

The load in the main plate between bolts i and $i + 1$ is equal to the joint load P minus the sum of the loads on all bolts $\sum_1^i R$ preceding the part of the joint under consideration; that is,

Load in main plates between bolts i and $i + 1 = P - \sum_1^i R$

The loads in the butt straps between bolts i and $i + 1$ are equal to the sum of the loads transmitted to the butt straps by all bolts preceding the section under consideration; and since there are two butt straps,

Load in one butt strap between bolts i and $i + 1 = \frac{1}{2} \sum_{1}^{i} R$

With these relations and the second assumption, the plate deformations may be written as

$$\Delta_p = \frac{p}{bt_pE} \left(P - \sum_{1}^{i} R\right)$$

$$\Delta_s = \frac{1}{2} \frac{p}{bt_sE} \sum_{1}^{i} R$$

Let

$$\frac{p}{bt_pE} = K_p$$

$$\frac{p}{bt_sE} = K_s$$

The plate deformations may then be written

$$\Delta_p = K_p \left(P - \sum_{1}^{i} R\right)$$

$$\Delta_s = \frac{K_s}{2} \sum_{1}^{i} R \qquad \text{(A3)}$$

Substituting expressions (A2) and (A3) into equation (A1) gives

$$\frac{C_{i+1}}{2} R_{i+1} = \frac{C_i}{2} R_i - K_p \left(P - \sum_{1}^{i} R\right) + \frac{K_s}{2} \sum_{1}^{i} R$$

Solve for R_{i+1}

$$R_{i+1} = \frac{C_i}{C_{i+1}} R_i + \frac{2K_p + K_s}{C_{i+1}} \sum_{1}^{i} R - \frac{2K_p}{C_{i+1}} P \qquad \text{(A4)}$$

Numerical work is facilitated by letting

$$\sum_{1}^{i} R = R_i + \sum_{1}^{i-1} R$$

Rewrite equation (A4)

$$R_{i+1} = \frac{C_i}{C_{i+1}} R_i + \frac{2K_p + K_s}{C_{i+1}} R_i - \frac{2K_p}{C_{i+1}} P + \frac{2K_p + K_s}{C_{i+1}} \sum_{1}^{i-1} R \qquad (A5)$$

Equation (A5) is the general relationship between the loads on any two successive bolts. In the form shown, this equation is readily adaptable to numerical calculation without obtaining general formulas for loads carried by individual bolts of the joint. If deemed preferable, equation (A5) may be used to determine general expressions for individual bolt loads. The numerical procedure is illustrated in appendix B, and general formulas for loads on the bolts of the three-bolt test specimens are shown in appendix C.

A case that occurs frequently is that in which the bolts are all of the same material and size and the butt straps are of the same material as the main plate with a thickness equal to one-half that of the main plate. Then

$$C_i = C_{i+1} = C$$

$$2K_p = K_s,$$

and

$$R_{i+1} = R_i + \frac{2K_s}{C} R_i - \frac{K_s}{C} P + \frac{2K_s}{C} \sum_{1}^{i-1} R \qquad (A6)$$

Equation (A6) applies to the specimens of the tests reported in this paper.

<u>Analysis of symmetrical butt joint fastened by bolts in several lines parallel to applied load.</u>- A solution of the general case illustrated in figure 19 may be obtained if, in addition to the assumptions made in the first section of this appendix, it is assumed that the bolts in any transverse row i are loaded equally. In a manner similar to that in which equation (A5) was obtained, it may be shown that

$$R_{i+1} = \frac{C_i}{C_{i+1}}R_i + \frac{N_i(2K_p + K_s)}{C_{i+1}}R_i - \frac{2K_p}{C_{i+1}}P + \frac{2K_p + K_s}{C_{i+1}}\sum_{1}^{i-1} R \qquad \text{(A7)}$$

It is probable that less accuracy would be realized in the application of equation (A7) in the analysis of joints of the type illustrated in figure 19 than in the analysis of joints of the type shown in figure 17. Posner (reference 4) developed a relationship for lap joints similar to equation (A7) from a consideration of plate deformations in tension and in bearing of the plates under the bolts or rivets. In the determination of bolt constants, Posner neglected the effects of shearing, bending, and bearing of the bolts. As a result, for any given bolt pattern and joint width, Posner's solution yields identical results for all bolt sizes or for all plate thicknesses when the thickness ratio of the lapped plates is constant. The solution contained herein, which is in agreement with the test results, shows that such a condition does not exist for butt joints.

Determination of Bolt Constant C

Factors affecting C.- In the development of the general bolt-load relationship, it was assumed that a linear relation exists between bolt deflection and bolt load in the elastic range. The relation is stated as

$$\delta = \frac{CR}{2} \qquad \text{(A8)}$$

From equation (A8) it may be seen that C is affected by the factors that influence deflection. These factors are shearing, bending, and bearing of the bolt; and, as C is used herein, the localized effect of bearing of the plates is included in the determination of C.

Since the bolt is loaded and acts in a highly complex manner, the deflection is not readily determined. A solution for C will be obtained by assuming the bolt to be a fixed-end beam loaded as shown in figure 20. As related to this assumption, it should be remembered that the theory of elasticity shows that the basic assumptions underlying conventional beam analysis are violated when such analysis is applied to this case. A more refined solution appears unwarranted, however, in view of the uncertainties introduced by the practical conditions of joint construction. Furthermore, the nature of the problem and present experimental results indicate that a highly exact determination of C is probably unnecessary. Expressions obtained on the basis of the foregoing and subsequent assumptions, however, require experimental checking over a wide range of joint arrangements before they may be considered generally acceptable.

Effect of shear, bending, and bearing of bolt.- The deflection caused by shear, bending, or bearing is determined separately and equated to an expression of the form of equation (A8) to obtain the part of C that may be attributed to each effect. Deflection is measured relative to a line that passes through the centroids of the end cross sections of the bolt, and shearing and bending deflections are found at the center of the span. The unit bearing deformation is defined as a percentage of the bolt diameter, and bearing stress is computed in the usual manner as R/tD. The bearing modulus of the bolt $E_{b_{br}}$ is assumed equal to the compressive modulus of the bolt material. It is then found for shear that

$$C_{bs} = \frac{\alpha(2t_s + t_p)}{4G_bA_b} \qquad \text{(A9)}$$

where α is a constant depending upon the shape of the cross section and is equal to $4/3$ for a circular section. Thus

$$C_{bs} = \frac{2t_s + t_p}{3G_bA_b} \qquad \text{(A10)}$$

For bending,

$$C_{bb} = \frac{8t_s^3 + 16t_s^2t_p + 8t_st_p^2 + t_p^3}{192E_{bb}I_b} \qquad \text{(A11)}$$

For bearing,

$$C_{b_{br}} = \frac{2t_s + t_p}{t_st_pE_{b_{br}}} \qquad \text{(A12)}$$

Effect of bearing of plate.- The assumption of a uniform distribution of stress in the plates gives satisfactory results when an average elongation is the quantity to be determined. Such an assumption, however, does not take into account the localized effect of bearing of the bolt on the plates. This effect is of greater importance when the bolt is of a harder material with appreciably greater bearing strength than the plate than when the plate is harder. This statement may be clarified by a consideration of the behavior of a bolt and a plate under bearing load. The material of the plate can flow outward at the edges of the hole and thereby permit further bearing deformation. This action produces a bulging of the plate under the bolt, an example of which may be seen by inspection of specimen A-1 in figure 3. The bulging induces a secondary effect by increasing the bearing area which in turn tends to provide greater resistance to bearing deformation. The material of the bolt is more confined than that of the plate; consequently, the bolt must deform more by a process of compaction than by flow of the contact surfaces. It follows that the two conditions represent different aspects of the bearing problem and that bearing of the plate is more critical when the material of the bolt is as hard as or harder than that of the plate, which is generally the prevailing condition in airplane structures. Epstein (reference 7) arrived at similar conclusions in regard to the bearing actions of the bolt and plate as a result of his investigation of bearing strength. Therefore, in the determination of plate deformations, provision must be made to include bearing deformation of the plates. Although bearing deformation is a function of the dimensions and elastic properties of the plates as well as the load, this deformation can be estimated more readily in terms of bolt behavior. For this reason, the resulting correction is applied to the bolt constant C rather than to the plate constants K_p and K_s.

The unit bearing deformation of a plate is defined as a percentage of the hole diameter, and the diameter of the hole is assumed equal to that of the bolt. Bearing stress is computed in the usual manner as R/tD. In the manner used in connection with shearing, bending, and bearing of the bolt, it is found that

$$C_{p_{br}} = \frac{1}{t_s E_{s_{br}}} + \frac{2}{t_p E_{p_{br}}} \qquad \text{(A13)}$$

where $E_{s_{br}}$ and $E_{p_{br}}$ are the bearing moduli of the plates, which are assumed equal to the compressive moduli of the plate materials in the calculation of C.

Combination of terms.- The bolt constant C may now be determined by adding expressions (A10), (A11), (A12), and (A13)

$$C = C_{bs} + C_{bb} + C_{b_{br}} + C_{p_{br}}$$

$$C = \frac{2t_s + t_p}{3G_bA_b} + \frac{8t_s^3 + 16t_s^2t_p + 8t_st_p^2 + t_p^3}{192E_{bb}I_b} + \frac{2t_s + t_p}{t_st_pE_{b_{br}}} + \frac{1}{t_sE_{s_{br}}} + \frac{2}{t_pE_{p_{br}}} \tag{A14}$$

When

$$t_s = t_p/2,$$

$$C = \frac{2t_p}{3G_bA_b} + \frac{5t_p^3}{96E_{bb}I_b} + \frac{4}{t_pE_{b_{br}}} + \frac{2}{t_pE_{s_{br}}} + \frac{2}{t_pE_{p_{br}}} \tag{A15}$$

Since

$$A_b = \pi D^2/4$$

and

$$I_b = \pi D^4/64$$

and if

$$\frac{E_{bb}}{G_b} = k_1$$

$$\frac{E_{bb}}{E_{b_{br}}} = k_2$$

$$\frac{E_{bb}}{E_{s_{br}}} = k_3$$

$$\frac{E_{b_b}}{E_{p_{br}}} = k_4$$

equation (A15) may be written

$$c = \frac{2}{t_p E_{b_b}} \left\{ \frac{4}{3\pi} \left(\frac{t_p}{D} \right)^2 \left[k_1 + \frac{5}{4} \left(\frac{t_p}{D} \right)^2 \right] + 2k_2 + k_3 + k_4 \right\} \quad \text{(A16)}$$

<u>Expressions for c for specific combinations of materials</u>.- The following expressions are limited to the case where $t_s = \frac{t_p}{2}$:

Case I. If average values of generally quoted moduli of the structural aluminum alloys 14S-T, 17S-T, 24S-T, 25S-T, and 75S-T are used, the values of k are within 1 percent of the following:

$$k_1 = 2.66$$

and

$$k_2 = k_3 = k_4 = 1$$

For any combination of bolts and plates of these materials, therefore,

$$c = \frac{8}{t_p E_{b_b}} \left\{ 0.13 \left(\frac{t_p}{D} \right)^2 \left[2.12 + \left(\frac{t_p}{D} \right)^2 \right] + 1 \right\} \quad \text{(A17)}$$

Case II. For steel plates and bolts,

$$E_{b_b} = E_{b_{br}} = E_{s_{br}} = E_{p_{br}} = 29,000 \text{ ksi}$$

$$G_b = 11,000 \text{ ksi}$$

$$k_1 = 2.64$$

and

$$k_2 = k_3 = k_4 = 1$$

Comparing these values of k with those for the aluminum alloys shows that equation (A20) also applies to the case of steel plates and bolts.

Case III. For aluminum plates and steel bolts,

$$k_1 = 2.64$$

$$k_2 = 1$$

and

$$k_3 = k_4 = \frac{29,000}{10,600} = 2.73$$

in which 10,600 ksi is an average value of the compressive moduli of the previously mentioned aluminum alloys. The expression for C is

$$C = \frac{8}{t_p E_{bb}} \left\{ 0.13 \left(\frac{t_p}{D} \right)^2 \left[2.12 + \left(\frac{t_p}{D} \right)^2 \right] + 1.87 \right\} \qquad \text{(A18)}$$

Case IV. For aluminum main plate, steel butt straps, and steel bolts,

$$k_1 = 2.64$$

$$k_2 = k_3 = 1$$

$$k_4 = 2.73$$

and

$$C = \frac{8}{t_p E_{bb}} \left\{ 0.13 \left(\frac{t_p}{D} \right)^2 \left[2.12 + \left(\frac{t_p}{D} \right)^2 \right] + 1.43 \right\} \qquad \text{(A19)}$$

Case V. For aluminum main plate, steel butt straps, and aluminum bolts,

$$k_1 = 2.66$$

$$k_2 = k_4 = 1$$

$$k_3 = 0.36$$

and

$$C = \frac{8}{t_p E_{bb}} \left\{ 0.13 \left(\frac{t_p}{D}\right)^2 \left[2.12 + \left(\frac{t_p}{D}\right)^2 \right] + 0.84 \right\} \qquad \text{(A20)}$$

The preceding expressions can be generally applied by replacing t_p with a hypothetical thickness equal to one-half the total thickness of the plates; that is,

$$t_{av} = \frac{2t_s + t_p}{2} \qquad \text{(A21)}$$

$$C_{av} = \frac{2}{t_{av} E_{bb}} \left\{ \frac{4}{3\pi} \left(\frac{t_{av}}{D}\right)^2 \left[k_1 + \frac{5}{4} \left(\frac{t_{av}}{D}\right)^2 \right] + 2k_2 + k_3 + k_4 \right\} \qquad \text{(A22)}$$

Because of the approximate nature of the expression for C, the further approximation inherent in equation (A22) is justified and values of C_{av} may be readily determined that do not differ too much from those calculated from equation (A14). In order to examine the differences involved, the extreme case of $t_s = t_p$ was chosen and the comparison is given in table 4. Table 4 shows that C_{av} is from 2 to 19 percent less than C for any value of D/t_p.

APPENDIX B

NUMERICAL EXAMPLE

In order to illustrate the application of equation (A5) in numerical calculations, solution of the following example is given. Consider a five-bolt joint (fig. 18) made up of the following components:

Steel bolts:

$$D = 1/4 \text{ inch}$$

$$E_{bb} = 29,000 \text{ ksi}$$

24S-T plates:

$$t_p = 5/16 \text{ in.}$$

$$t_s = 3/16 \text{ in.}$$

$$p = 1 \text{ in.}$$

$$b = 2 \text{ in.}$$

$$E = 10,500 \text{ ksi}$$

Since $C_{i+1} = C_i$, equation (A5) may be written, from equation (1),

$$R_{i+1} = R_i + \frac{2K_p + K_s}{C}R_i - \frac{2K_p}{C}P + \frac{2K_p + K_s}{C}\sum_{1}^{i-1} R \tag{B1}$$

$$K_s = \frac{p}{bt_sE} = \frac{1}{(2)(0.188)(10,500)} = \frac{1}{3940}$$

$$2K_p = \frac{2p}{bt_pE} = \frac{2}{(2)(0.313)(10,500)} = \frac{1}{3280} = \frac{1.2}{3940}$$

The bolt constant may be determined from equation (A18) by replacing t_p with t_{av}; then

$$C_{av} = \frac{8}{t_{av}E_{bb}}\left\{0.13\left(\frac{t_{av}}{D}\right)^2\left[2.12 + \left(\frac{t_{av}}{D}\right)^2\right] + 1.87\right\}$$

$$t_{av} = \frac{2t_s + t_p}{2} = \frac{2(0.188) + 0.313}{2} = 0.344$$

$$\frac{t_{av}}{D} = \frac{0.344}{0.25} = 1.375$$

and

$$C_{av} = \frac{1}{437}$$

$$\frac{2K_p + K_s}{C} = \frac{1.2 + 1}{3940} \times \frac{437}{1} = 0.244$$

$$\frac{2K_p}{C} = \frac{437}{3280} = 0.133$$

Starting with the second bolt, successive expressions for each unknown bolt load are written in terms of R_1 by means of equation (B1):

$$R_1 = R_1 = 1.000R_1$$

$$R_2 = 1.244R_1 - 0.133P = 1.244R_1 - 0.133P$$

$$R_3 = 1.244R_2 - 0.133P + 0.244R_1$$

$$= 1.244(1.244R_1 - 0.133P) - 0.133P + 0.244R_1$$

$$= 1.793R_1 - 0.299P$$

$$R_4 = 1.244R_3 - 0.133P + 0.244(R_1 + R_2)$$

$$= 1.244(1.793R_1 - 0.299P) - 0.133P + 0.244(2.244R_1 - 0.133P)$$

$$= 2.778R_1 - 0.537P$$

$$R_5 = 1.244R_4 - 0.133P + 0.244(R_1 + R_2 + R_3)$$

$$= 1.244(2.778R_1 - 0.537P) - 0.133P + 0.244(4.037R_1 - 0.432P)$$

$$= 4.440R_1 - 0.906P$$

Now,

$$P = \sum R = 11.255R_1 - 1.875P$$

and therefore

$$R_1 = \frac{2.875}{11.255} P = 0.256P$$

$$R_2 = 1.244 \times 0.256P - 0.133P = 0.185P$$

$$R_3 = 1.793 \times 0.256P - 0.299P = 0.159P$$

$$R_4 = 2.778 \times 0.256P - 0.537P = 0.173P$$

$$R_5 = 4.440 \times 0.256P - 0.906P = 0.228P$$

Arithmetical check: $P = \sum R = 1.001P$

In the conventional method of analysis, it is assumed that each bolt carries the same load, that is, $R = 0.200P$. Comparing the foregoing results with this value shows that the end bolts are overloaded and the interior bolts carry less load than they are usually considered to support. Thus,

$$R_1/R = 1.28$$
$$R_2/R = 0.93$$
$$R_3/R = 0.80$$
$$R_4/R = 0.87$$
$$R_5/R = 1.14$$

APPENDIX C

GENERAL EQUATIONS FOR AND CALCULATION OF BOLT LOADS FOR SPECIMENS OF GROUPS A AND B

Equation (A6) applies to the calculation of the loads carried by the individual bolts of specimens of groups A and B since $2K_p = K_s$ and the bolts are all of the same size and material. The expression is

$$R_{i+1} = R_i + \frac{2K_s}{C} R_i - \frac{K_s}{C} P + \frac{2K_s}{C} \sum_{1}^{i-1} R \qquad \text{(C1)}$$

General equations.- For any two-bolt joint when $2K_p = K_s$ and the bolts are of the same size and material, it can be shown that $R_1 = R_2 = P/2$. From equation (C1)

$$R_2 = R_1 + \frac{2K_s}{C} R_1 - \frac{K_s}{C} P \qquad \text{(C2)}$$

Since

$$P = \Sigma R = R_1 + R_2$$

$$P = R_1 + \left(R_1 + \frac{2K_s}{C} R_1 - \frac{K_s P}{C}\right)$$

and

$$P\left(\frac{C+K_s}{C}\right) = 2R_1\left(\frac{C+K_s}{C}\right)$$

therefore

$$R_1 = R_2 = P/2 \qquad \text{(C3)}$$

Equation (C3) applies to the three specimens of group A, since all fulfill the necessary conditions.

For the joints of group B, the following equations apply:
From symmetry

$$R_1 = R_3$$

$$P = \sum R = 2R_1 + R_2 \tag{C4}$$

Substituting equation (C2) into (C4) gives

$$P = 2R_1 + \left(R_1 + \frac{2K_s}{C} R_1 - \frac{K_s P}{C}\right)$$

and

$$R_1 = \left(\frac{C + K_s}{3C + 2K_s}\right) P \tag{C5}$$

Substitution of equation (C5) into (C4) gives

$$R_2 = \left(\frac{C}{3C + 2K_s}\right) P \tag{C6}$$

<u>Typical calculations for joint of group B, specimen B-1.</u>-

From table 1,

$$p = 2 \text{ in.}$$

$$b = 1\frac{1}{4} \text{ in.}$$

$$t_p = 0.374 \text{ in.}$$

$$E = 10,500 \text{ ksi}$$

For the bolts,

$$D = \frac{1}{4} \text{ in.}$$

$$E_{bb} = 29,000 \text{ ksi}$$

$$K_s = \frac{p}{bt_sE} = \frac{2}{(1.25)(0.187)(10,500)} = \frac{1}{1225}$$

$$\frac{t_p}{D} = 1.5$$

From equation (A18)

$$C = \frac{8}{t_pE_{bb}}\left\{0.13\left(\frac{t_p}{D}\right)^2\left[2.12 + \left(\frac{t_p}{D}\right)^2\right] + 1.87\right\}$$

Substituting in this expression the values of t_p, E_{bb}, and t_p/D gives

$$C = \frac{8}{(0.374)(29,000)}\left\{0.13\ (1.5)^2\left[2.12 + (1.5)^2\right] + 1.37\right\}$$

$$= \frac{1}{432} = \frac{2.84}{1225}$$

From equation (C5)

$$R_1 = \left(\frac{2.84 + 1}{32.84 + 2}\right)P$$

$$= \left(\frac{2.84}{10.52}\right)P = 0.365P$$

and from equation (C6)

$$R_2 = \left(\frac{2.84}{32.84 + 2}\right)P$$

$$= \left(\frac{2.84}{10.52}\right)P = 0.270P$$

The values of R for specimens B-2 and B-3 were found in like manner, and the elastic constants and ratios R/P for all specimens are shown in table 5.

APPENDIX D

METHODS OF ANALYSIS OF TEST DATA

Behavior of Specimens

Stress distribution in plates.- In the analysis given in appendix A, the stress in the plates is assumed to be uniformly distributed. The photoelastic studies of Coker and Filon (reference 9) and Frocht (reference 10), however, show that a nonuniform stress distribution exists in the plates of bolted joints. In the reduction of test data, plate loads were calculated on the assumption of a uniform distribution of stress; the stress was computed from an average strain, which was determined as the arithmetical average of three strains measured on the gage lines shown in figure 17. In order to study the manner in which the true stress distribution affected the actual bolt loads and the calculated plate loads, a brief discussion is given in connection with the observed strains.

It was observed that the strain distribution varied with load (fig. 17). At low loads the strain distribution was approximately uniform; but, as the joint load was increased, strains ϵ_1 measured on gage line 1 increased at a faster rate than the strains ϵ_2 measured on gage line 2, directly in line with the bolts. Although ϵ_1 increased more rapidly with load than ϵ_2 at all sections, the amount and rate of increase varied considerably from section to section of a specimen. All specimens exhibited similar behavior but it was somewhat more pronounced in specimens A-3 and B-3, from which the data plotted in figure 17 were obtained. The plotted points represent the averages of strains measured with all gages, which were located in similar positions on the butt straps.

Load-strain behavior of the type shown in figure 17 has been reported previously (references 5 and 11). In reference 5 several sets of stress diagrams for riveted and pin-connected joints are given, which indicate the same tendency and in some cases show that directly in line with the rivets the stress changes from tension at low loads to compression at higher loads. If an irregular strain distribution based on the diagrams of reference 5 is assumed, plate loads may be calculated and compared with the plate loads computed on the basis of average strain; the following comparisons are made on this basis.

The greatest percentage variation of measured strain occurred at section 2-2 of specimen A-3 at a joint load of 8.5 kips, where the center strain ϵ_2 was approximately 25 percent less than the average strain ϵ_{av}. The load in the butt straps at this section computed on the assumption of the irregular strain distribution is about 7 percent greater than the load computed on the basis of average strain. The two methods give loads at section 2-2 that differ by about 3 percent for specimen B-3 at a joint load of approximately two-thirds the ultimate and for specimen A-3 at about one-half the ultimate. At lower joint loads the differences are negligible. At section 1-1 the difference is less than 2 percent in all cases. At section 3-3 of specimen B-3 the maximum difference is 3 percent. For the remaining specimens the differences at corresponding sections are less than those just cited.

It may be concluded in regard to the specimens of these tests that, at the sections where strain measurements were made, the assumption of a uniform stress distribution provides a satisfactory means for the calculation of plate loads. The plate constants K_p and K_s, which are assumed equal to plate deformation per unit of load, are not necessarily determined with corresponding accuracy on the basis of the same assumption. The stress distribution undergoes a very considerable change from a section midway between adjacent bolts to a section through the bolt hole. High stress concentrations are present in the vicinity of the hole, which cause yielding of the material early in the load history of a joint and are largely responsible for the action illustrated in figure 17. To take such action into account theoretically would involve correction of the plate constants K_p and K_s, which would result in nonlinear curves of joint load againt bolt load (P-R) for all bolts of a joint. Such correction indicates that the first bolt carries a greater load than is at present determined by means of any proposed elastic theory. Previous investigators have found experimentally determined loads on the first bolt greater than those computed by means of an elastic theory (references 3, 5, and 6). It will be necessary, however, to secure more information about the stress distribution before revised values of K can be incorporated in the theory.

Lateral bending accompanying transfer of bolt loads to butt straps.- In the analysis given in appendix A, a bolt load is assumed to be distributed uniformly along the bolt. It has long been recognized, however, that the bolt load is distributed so that the resultant of the portion transmitted to one butt strap lies within the half-thickness of the butt strap adjacent to the main plate. As a result, a lateral bending moment acting in a plane normal to the plane of the butt strap is induced in the strap. This moment is resisted partly by flexural stiffness of the strap and partly by direct tension in the bolts. In these tests the nuts were loosened in order to minimize frictional effects and for this reason lateral bending was largely resisted by stiffness of the straps.

The presence of lateral bending moment in the butt straps has a negligible effect on the values of the bolt loads. Approximate calculations indicate that this lateral bending moment affects the load on the first bolt to an extent of the order of magnitude of 0.2 percent. The principal difficulty caused by lateral bending lies in the interpretation of strain data. Because of this bending effect, correction of calculated plate loads is necessary in some cases; this bending is explained in the following section in connection with those specimens for which correction was required.

The presence of lateral bending moment was confirmed experimentally by strain measurements taken on the outer surfaces of the butt straps at the centers of the joints. That moment existed in all cases except thin specimens may be verified by reference to table 6. In table 6 the test specimens are listed according to decreasing thickness of the plates as evidenced by increasing ratios of bolt diameter to butt-strap thickness D/t_s. The tabulated values are ratios of internal load to applied load. Internal loads were determined at three sections, at each of which it was known from conditions of equilibrium that the total applied load was resisted by the plates upon which strain measurements were made. Values of P_U and P_L were calculated from strain data obtained at sections 3 inches beyond the first bolt in the upper and lower main plates, respectively (fig. 1). Values of P_S were calculated from strain measurements taken on the butt straps at the centers of the joints. It may be noted from table 6 that P_S/P is less than P_U/P and P_L/P, in which case each of the last two always have the expected value of unity within 2 percent. Also P_S/P is influenced by the number of bolts fastening the plates and decreases from unity as D/t_S decreases. This behavior is attributed to more flexural resistance of the butt straps together with greater bending deflection of the bolts in the thicker specimens, which is accompanied by greater bolt-load eccentricity. Curves of P_S are shown in figures 5(a) and 7(a) to 15(a). It may be seen in figures 10(a) to 13(a) that P_S/P is constant, which is indicative of a constant bolt-load eccentricity, up to one-half of the ultimate load. The relationship ceases to be linear at higher loads, except for specimens A-3 and B-3. Evidently the butt-strap bending increases at a greater rate because of increasing bolt-load eccentricity attributable to the large bolt deflections that occur at high loads. Curves of P_U and P_L are not presented, as the strain behavior was fully in accord with that which would be predicted at sections where these values were determined. As the tests were in progress, it was observed that the effect of the eccentric location of the resultant bolt loads caused the free ends of the butt straps to move outward from the main plates. Movement was, of course, perceptible only at high loads. A similar behavior has been noted by previous investigators (reference 12).

In the usual types of bolted or riveted joints, it appears likely that bending of the straps would be relieved appreciably because of tension in the bolts or rivets, except at loads approaching the ultimate. As the bolts or rivets undergo large deformations at loads near the ultimate, they are unable to carry the tensile loads necessary to relieve bending.

Analysis of Test Data

Calculation of bolt loads from strain data.- In general, the load on a bolt was calculated as the difference between butt-strap loads at sections on each side of the bolt. The tensile modulus of elasticity used in the calculations was $E = 10,500$ ksi. Stress-strain curves plotted from tensile test data for coupons representing the plates were typical of 24S-T aluminum alloy; moduli of elasticity determined from these curves were within 2 percent of the recommended standard value of 10,500 ksi.

For specimens A-3 and B-3, the butt-strap loads were computed as the gross area of a butt strap times the average stress. The average stress was considered to be equal to the arithmetical average of three measured strains multiplied by the modulus of elasticity. The load on the first bolt R_1 was found by adding the butt-strap loads at section 2-2 for specimen A-3 and at section 3-3 for specimen B-3 (fig. 17). Load on the second bolt R_2 was found by subtracting R_1 from the sum of the butt-strap loads at section 1-1 for specimen A-3 and at section 2-2 for specimen B-3. For B-3, load on the third bolt R_3 was found by subtracting the quantity $R_1 + R_2$ from the sum of the butt-strap loads P_s at section 1-1. The remaining bolt loads R_4, R_5, and R_6 were determined in the same manner.

In the case of specimens A-1 and A-2, plate loads were determined also by this method. Lateral bending of the butt straps, however, necessitated correction of the values. From the method of strain measurement (by use of the gages on the outer surfaces of the straps), the magnitude of the bending moment could be evaluated only at the section where P_s was determined. It was assumed that the errors due to bending were proportional to $\frac{P_s}{P} - 1$, and correction was made by multiplying the butt-strap loads by P/P_s, after which the bolt loads were found in the manner used for specimen A-3. With respect to specimens B-1 and B-2, qualitative studies indicated lateral bending moments of the same sign at sections 1-1 and 3-3 (fig. 17) with moment of opposite sign at section 2-2. On this basis and the assumption that the errors were proportional to $\frac{P_s}{P} - 1$, the butt-strap loads were corrected by multiplying the loads at sections 1-1 and 3-3 by P/P_s and the loads at section 2-2 by P_s/P, after which the bolt loads were found in the manner used for specimen B-3. Although it was clear that at any one joint load the same correction factor did not apply at all sections, since the moment varied along the lengths of the butt straps, due consideration of the several factors involved in the behavior of all specimens indicated that the correction procedure was fairly adequate except at high loads for specimens B-1 and B-2. The curves of P_s are shown in conjunction with the bolt-load curves in figures 5(a) to 15(a) because of their interrelationship owing to the use of P_s in the determination of correction factors.

Calculation of bolt deflections.- In studying the load distribution in bolted or riveted joints a number of investigators have made use of the "loadslip" relationship or the deflection of the rivets or bolts (references 1, 2, 4, and 5). In the usual types of bolted joint, deflection of the bolts is not amenable to measurement, and the procedure to date has been to make indirect determinations on the sides of a joint by observation of the relative movement of the plates. Such methods, although approximate, are generally employed in the determination of bolt deflections; but the accuracy with which deflections are found cannot be stated with certainty. In the nomenclature of the present paper, the deflection relationship is

$$\delta = CR/2 \tag{D1}$$

The comparison of measured bolt deflections with values determined from equation (D1) furnishes a means for examination of the validity of the expression for the bolt constant C.

The deflection of bolts in specimens of groups A and B was determined from data obtained during the tests to failure by measurement of the spreading of the gap between main plates at the center of each joint. Total movement was computed as the average of measurements taken on both sides of a specimen. From this value, elongation of the butt straps between the two bolts on each side of the gap, considered to be PL/AE, was subtracted and the average deflection δ_{av} of the two bolts calculated as one-half the difference.

In order to compare the foregoing method for obtaining bolt deflections with a method by which the deflection of each bolt was separately determined, both methods were used in two of the auxiliary shear tests of bolts. The separately determined bolt deflections were obtained from measurements taken from butt strap to main plate on opposite faces of a specimen at the ends of the butt straps. Deflection was calculated as the average of the two values measured on opposite faces minus the elongation of the main plate in the length from the center line of the bolt to the strain-gage lozenge on the main plate.

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TABLE 1

ELEMENTS OF TEST JOINTS

Classification		Material: Plates	Material: Bolts (a)	Number of $\frac{1}{4}$-in. bolts per joint	Nominal dimensions: b/D	Nominal dimensions: D/t_p	Nominal dimensions: D/t_s	Measured dimensions: t_s (in.)	Measured dimensions: t_p (in.)	Measured dimensions: b_s (in.)	Measured dimensions: b_p (in.)	Measured dimensions: A_s (sq in.) (b)	Measured dimensions: A_p (sq in.) (b)	Remarks
Group A	A-1	24S-T	S.A.E. 2330	2	5	0.80	1.60	0.157	0.308	1.250	1.246	0.196	0.385	Balanced design
	A-2				5	.50	1.00	.251	.499	1.255	1.250	.315	.624	Joint designed to fail in shear
	A-3				5	1.54	3.09	.0826	.162	1.237	1.249	.1021	.203	Joint designed to fail in tension
Group B	B-1	24S-T	S.A.E. 2330	3	5	.67	1.33	.186	.374	1.253	1.247	.233	.467	Balanced design
	B-2				5	.50	1.00	.250	.501	1.253	1.251	.313	.626	Joint designed to fail in shear
	B-3				5	1.33	2.75	.0921	.188	1.253	1.253	.1154	.236	Joint designed to fail in tension
Auxiliary specimen	1	S.A.E. 6150	S.A.E. 2330	1	5	[c].59	[c]1.00	.250	.422	1.250	1.250	.313	.527	Joints designed to fail in shear
	2				5	[c].63	1.20	.208	.400	1.150	1.250	.260	.300	

[a]S.A.E. 2330 or equivalent.
[b]Gross area = bt.
[c]Determined from measured dimensions.

TABLE 2

ULTIMATE DOUBLE-SHEAR STRENGTHS OF $\frac{1}{4}$-INCH

HEAT-TREATED (125 ksi) ALLOY-STEEL BOLTS

Specimen	Bolt load, R (kips)	$\tau = \dfrac{R}{2\left(\dfrac{\pi D^2}{4}\right)}$ (ksi)	$\sigma_{br} = \dfrac{R}{tD}$ (ksi)	Remarks
Any	7.36	75.0	-----	Reference 8
Auxiliary 1	8.39	85.5	79.5	[1]Deflections measured (bolt 1, fig. 16)
	8.20	83.6	77.7	Deflections not measured.
	8.30	84.6	78.7	Do.
	7.78	79.3	73.8	Do.
Auxiliary 2	8.29	84.5	82.9	Deflections measured (bolt 1, fig. 16)
	9.74	99.3	97.4	Deflections measured (bolt 2, fig. 16)
	7.86	80.1	78.6	Deflections not measured.
A-1	7.98	81.3	103.8	Average of bolts 1 and 2 (fig. 16)
A-2	8.02	81.8	64.3	Average of two bolts 3 and 4 (fig. 16)
B-2	8.07	82.2	64.6	Average of bolts 1, 2 and 3 (fig. 16)

[1]Bolt 2 of auxiliary specimen 1 was not sheared; it was used to cause failure of bolt 2 of auxiliary specimen 2 and is therefore known to have a strength greater than 9.74 kips.

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TABLE 3

LOADS AND STRESSES AT R_{cr} AND FAILURE

Classification		Joint load at critical bolt load (kips)	Critical bolt load (kips) (a)	Stress at critical bolt load (ksi)			Joint load at failure (kips)	Average stress at failure of joint (ksi)			Type and location of failure
				Bearing	Shear	Tension (b)		Bearing	Shear	Tension (b)	
Group A	A-1	7.00	3.88	50.4	39.6	22.8	15.96	103.8	81.4	51.9	Shear; bolts 1 and 2
	A-2	8.00	4.80	38.4	48.9	16.0	16.04	64.3	81.7	32.1	Shear; bolts 3 and 4
	A-3	----------	---------	-------	-----	-------	10.62	131.2	54.1	67.1	Tension; at bolt 4, through net section of main plate
Group B	B-1	11.40	[c]4.16	44.7	42.3	30.6	23.40	84.1	79.6	62.7	Tension; at bolt 1, through net section of main plate
	B-2	13.56	[d]4.80	38.4	48.9	27.1	24.20	64.6	82.0	48.4	Shear; bolts 1, 2 and 3
	B-3	8.25	3.24	70.4	33.0	44.7	12.02	87.3	40.9	65.4	Tension; at bolt 3, through net section of butt straps

[a]Average of maximum bolt loads in upper and lower joints.

[b]Computed using net area equal 80 percent of gross area.

[c]Based on average of bearing stresses at R_{cr} for specimens A-1 and A-2.

[d]Determined from test of specimen A-2.

TABLE 4

COMPARISON OF C AND C_{av}

$\left[t_s = t_p\right]$

Case	Main plate (1)	Butt straps	Bolt	E_{bb} (ksi)	k_1	k_2	k_3	k_4	$\frac{C-C_{av}}{C} \times 100$	
									$D/t_p = 0$	$D/t_p = \infty$
I	A	A	A	10,500	2.66	1	1	1	2.3	11.1
II	S	S	S	29,000	2.64	1	1	1	2.3	11.1
III	A	A	S	29,000	2.64	1	2.73	2.73	2.3	11.1
IV	A	S	S	29,000	2.64	1	1	2.73	2.3	19.2
V	A	S	A	10,500	2.66	1	.36	1	2.3	16.4

[1]A refers to any of the aluminum alloys, 14S-T, 17S-T, 24S-T, 25S-T, and 75S-T.

S refers to steel.

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TABLE 5

BOLT AND PLATE CONSTANTS

AND ANALYTICAL BOLT LOADS

Specimens		D/t_p (1)	t_p	C	K_s	R_1/P	R_2/P
Group A	A-1	0.80	0.308	1/432	[2]------	0.500	0.500
	A-2	.50	.499	1/354	------	.500	.500
	A-3	1.52	.162	1/296	------	.500	.500
Group B	B-1	.67	.374	1/432	1/1225	.365	.270
	B-2	.50	.501	1/355	1/1640	.354	.292
	B-3	1.34	.188	1/329	1/607	.378	.244

[1]Based on measured dimensions
[2]Not required

TABLE 6

EFFECT OF BUTT-STRAP BENDING AS SHOWN BY

COMPARISON OF MEASURED INTERNAL LOADS

Specimen	$\frac{D}{t_s}$	$\frac{\text{Internal load}}{\text{Applied load}}$		
		$\frac{P_U}{P}$	$\frac{P_L}{P}$	$\frac{P_s}{P}$
A-2	1.00	1.000	0.995	0.897
A-1	1.60	.994	.984	.935
A-3	3.09	.983	1.000	.986
B-2	1.00	1.000	.994	.927
B-1	1.33	.995	.986	.965
B-3	2.75	1.015	1.016	.994

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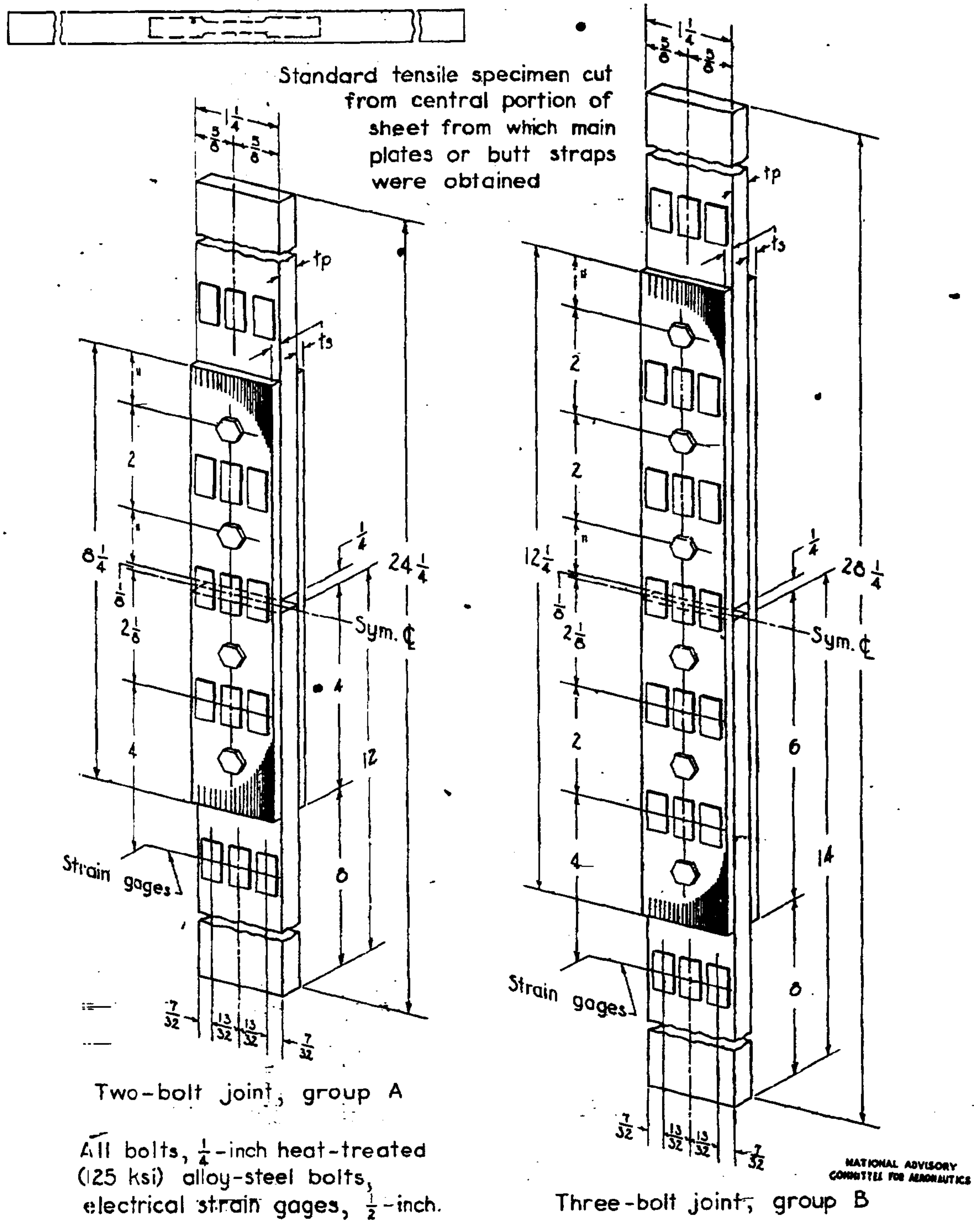


Figure 1. — Test specimens and arrangement of strain gages. (Arrangement of gages duplicated on opposite face of specimen.)

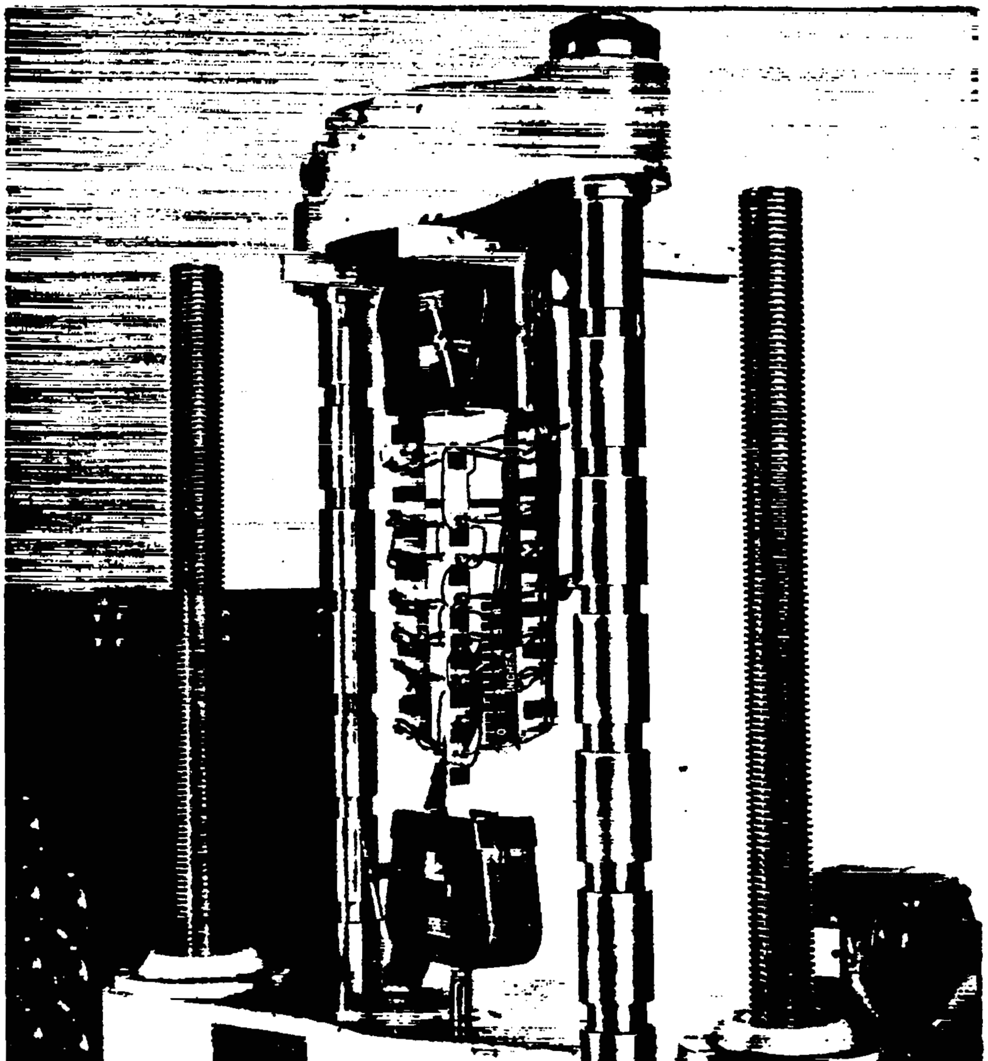


Figure 2.- General arrangement of specimen in testing machine.

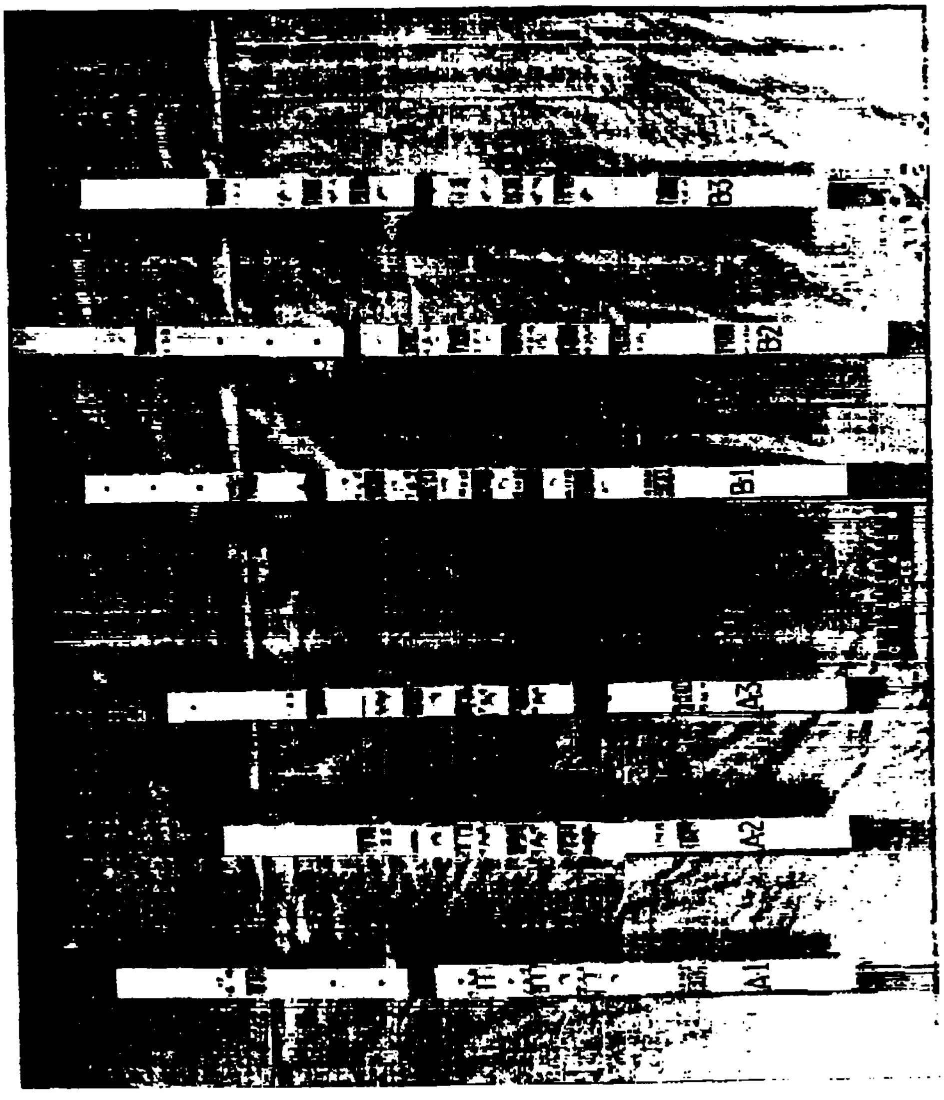


Figure 3.- Front view of fractured specimens.

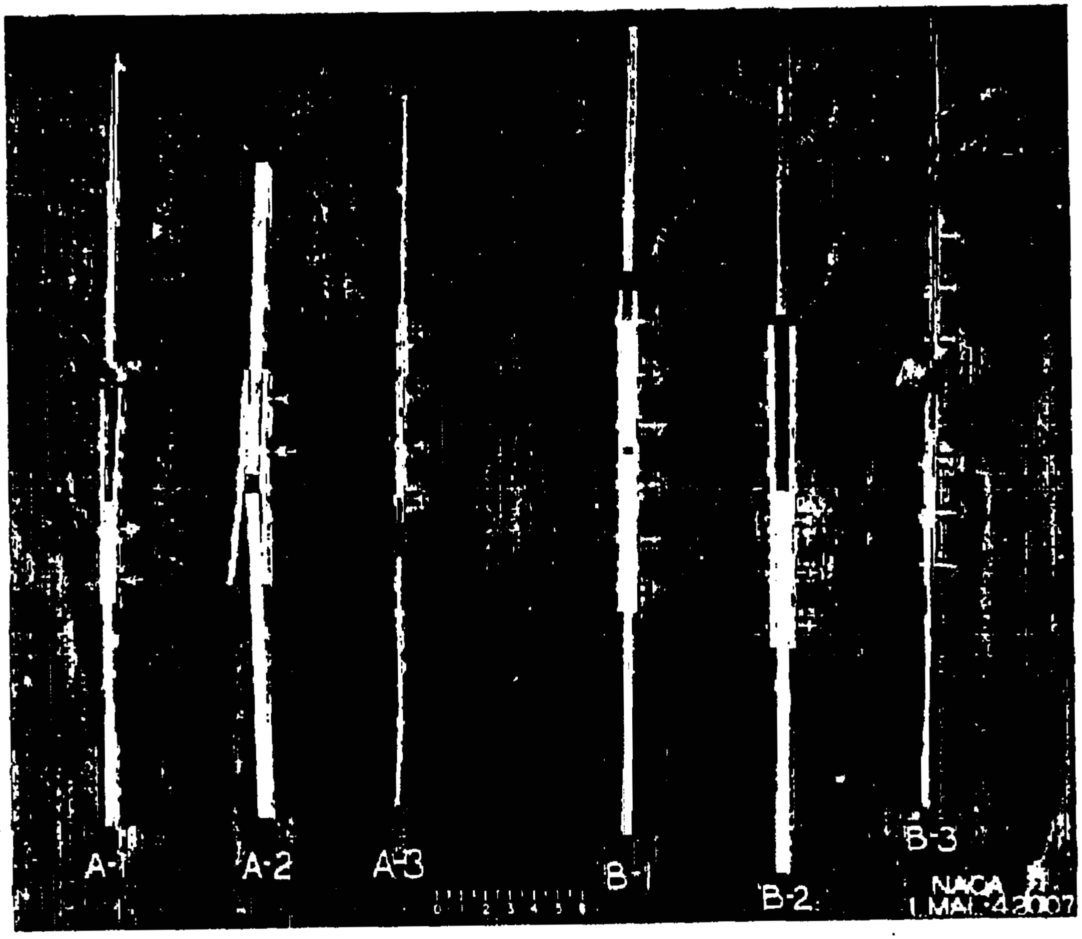
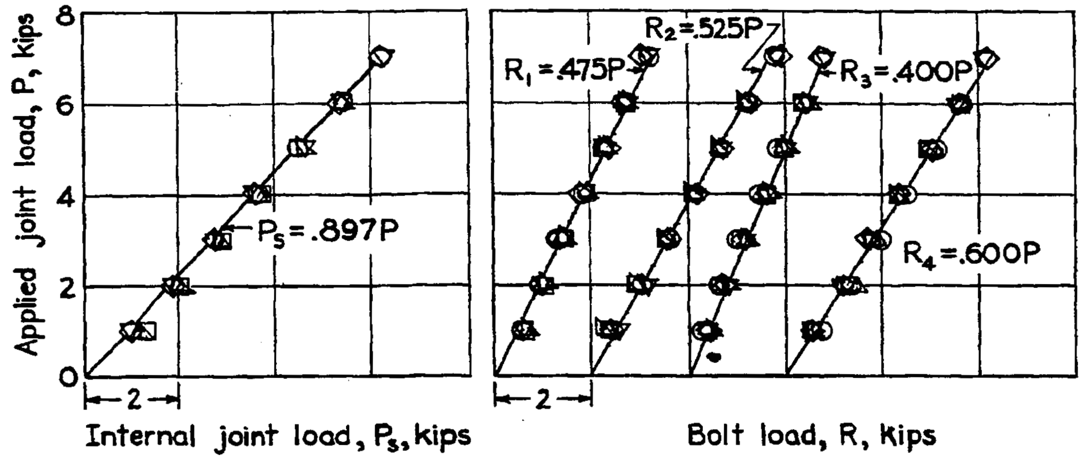


Figure 4.- Side view of fractured specimens.



(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads.

Figure 5. – Joint-load and bolt-load curves for loading of specimen A-2 in the elastic range.

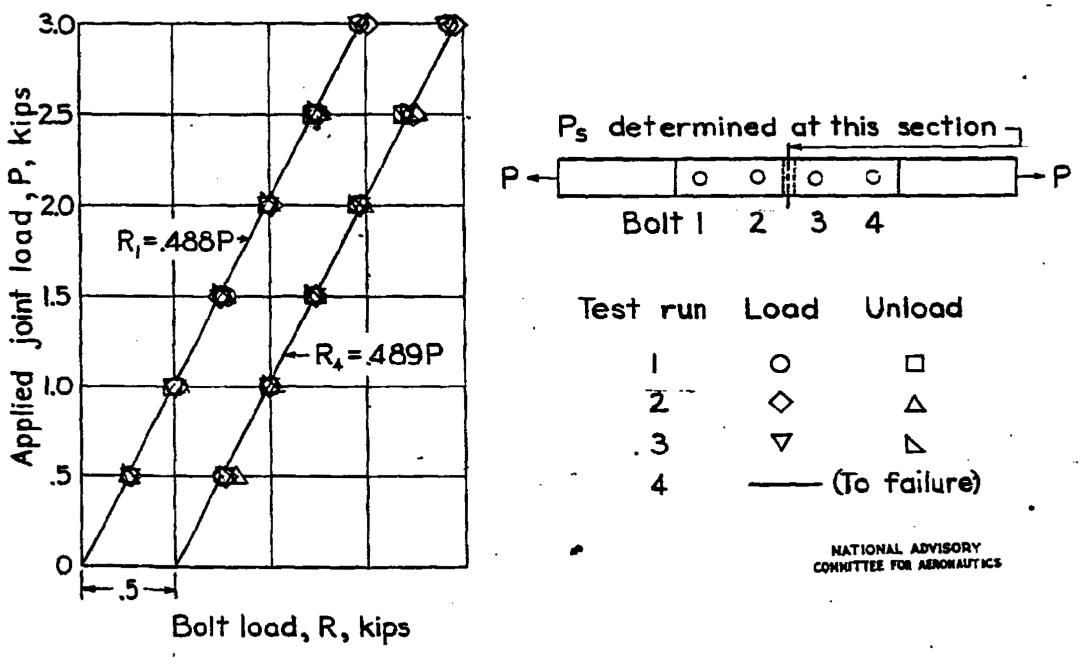


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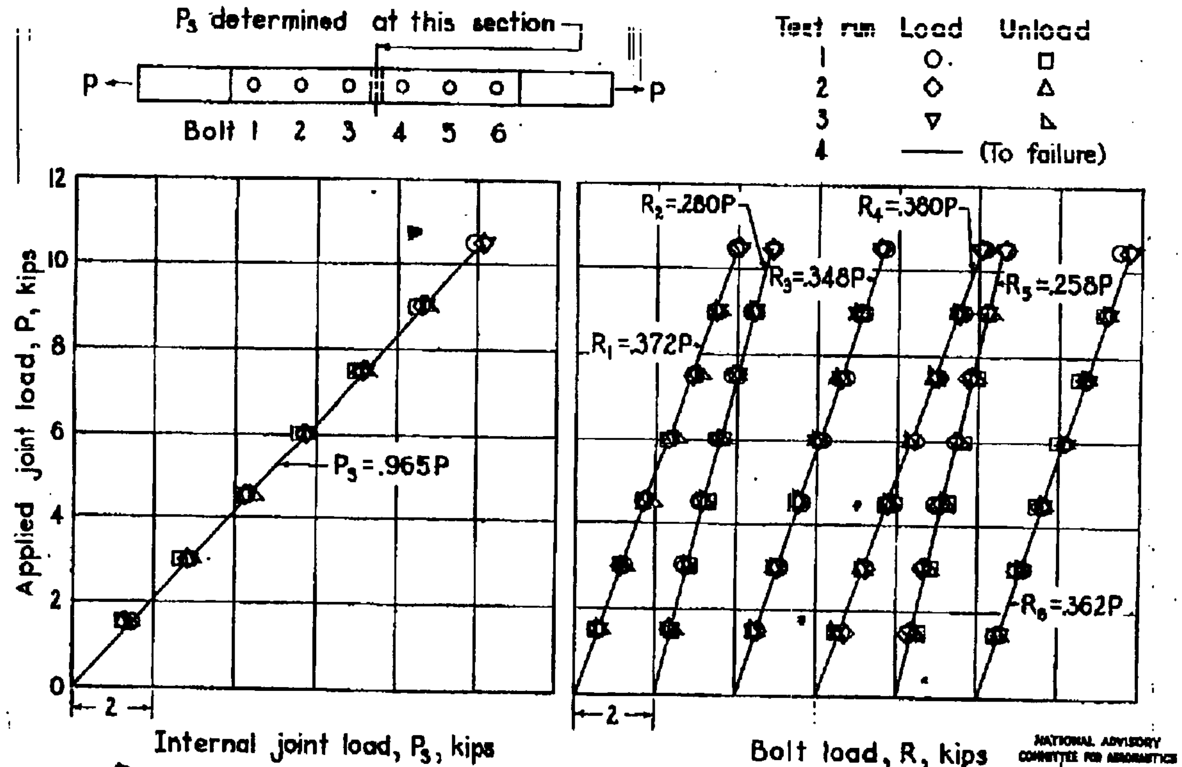
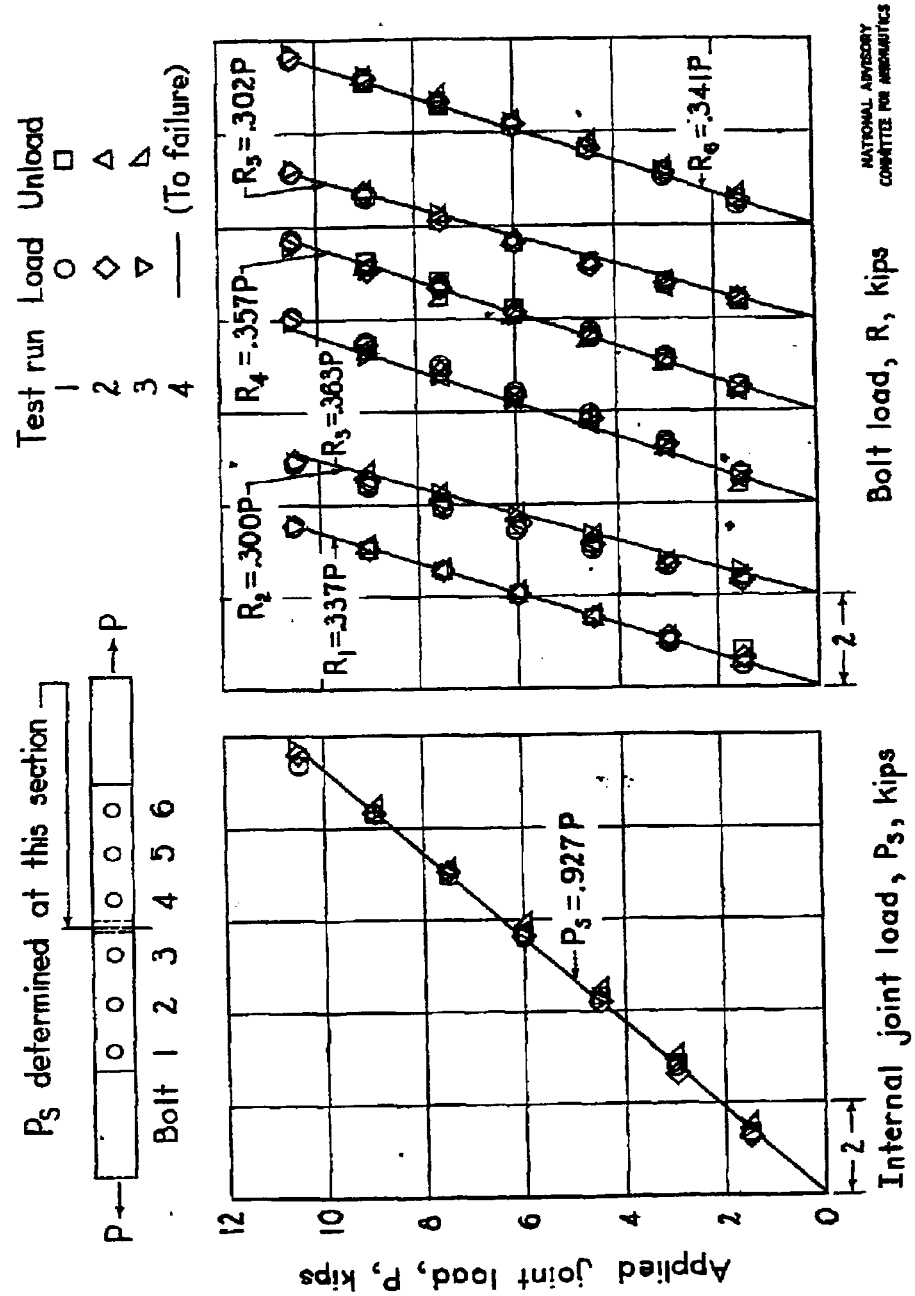
Figure 6.-Observed relationships between applied joint load and bolt loads for loading of specimen A-3 in the elastic range.



(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads.

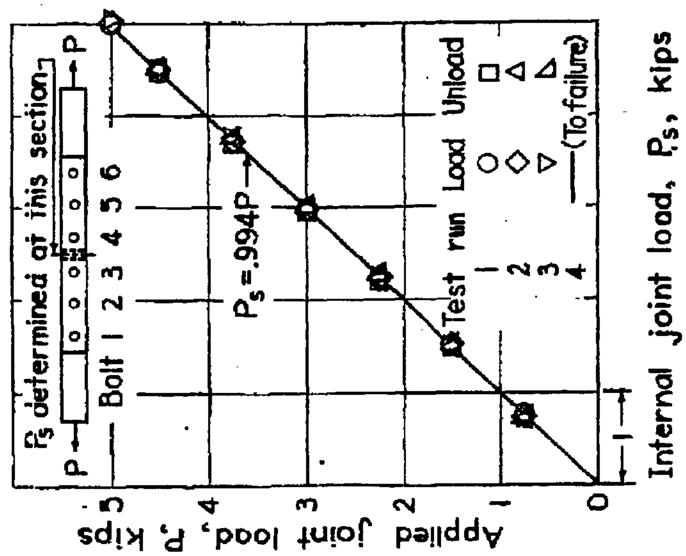
Figure 7. – Joint-load and bolt-load curves for loading of specimen B-1 in the elastic range.



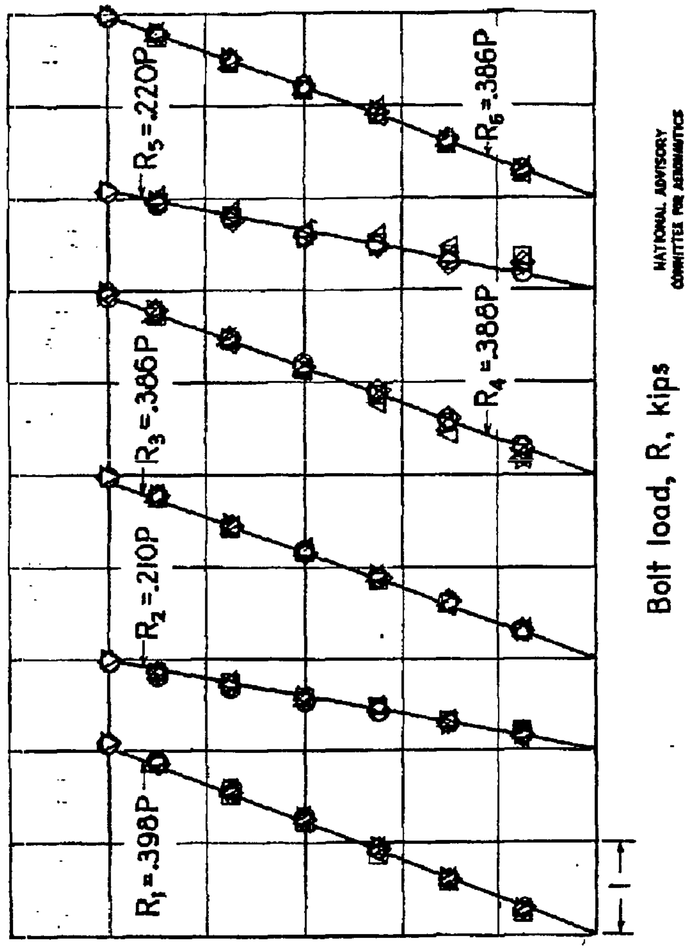
(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads.

Figure 8.- Joint-load and bolt-load curves for loading of specimen B-2 in the elastic range.

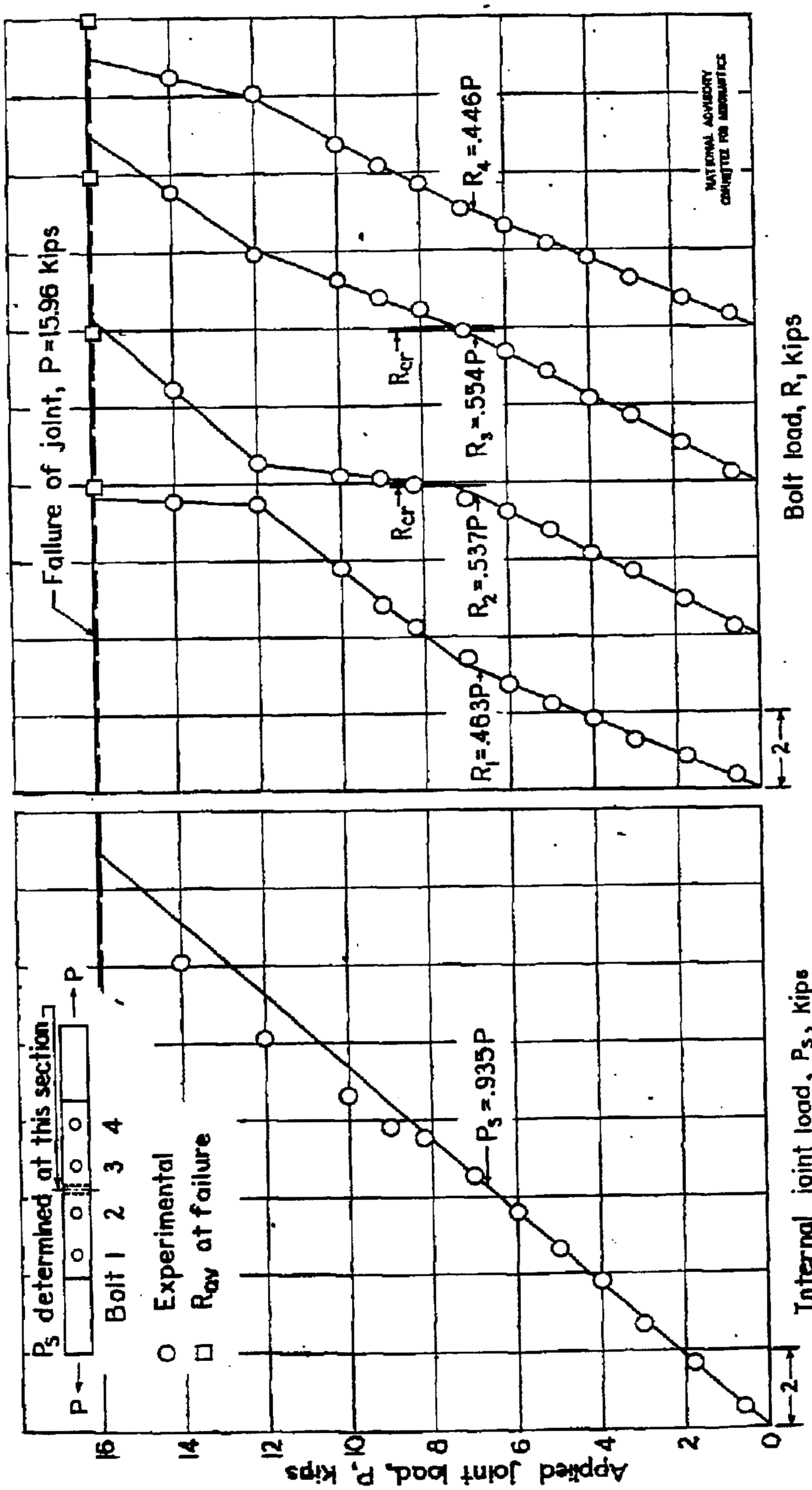


(a) Observed relationship between applied and internal joint loads.



(b) Observed relationships between applied joint load and bolt loads.

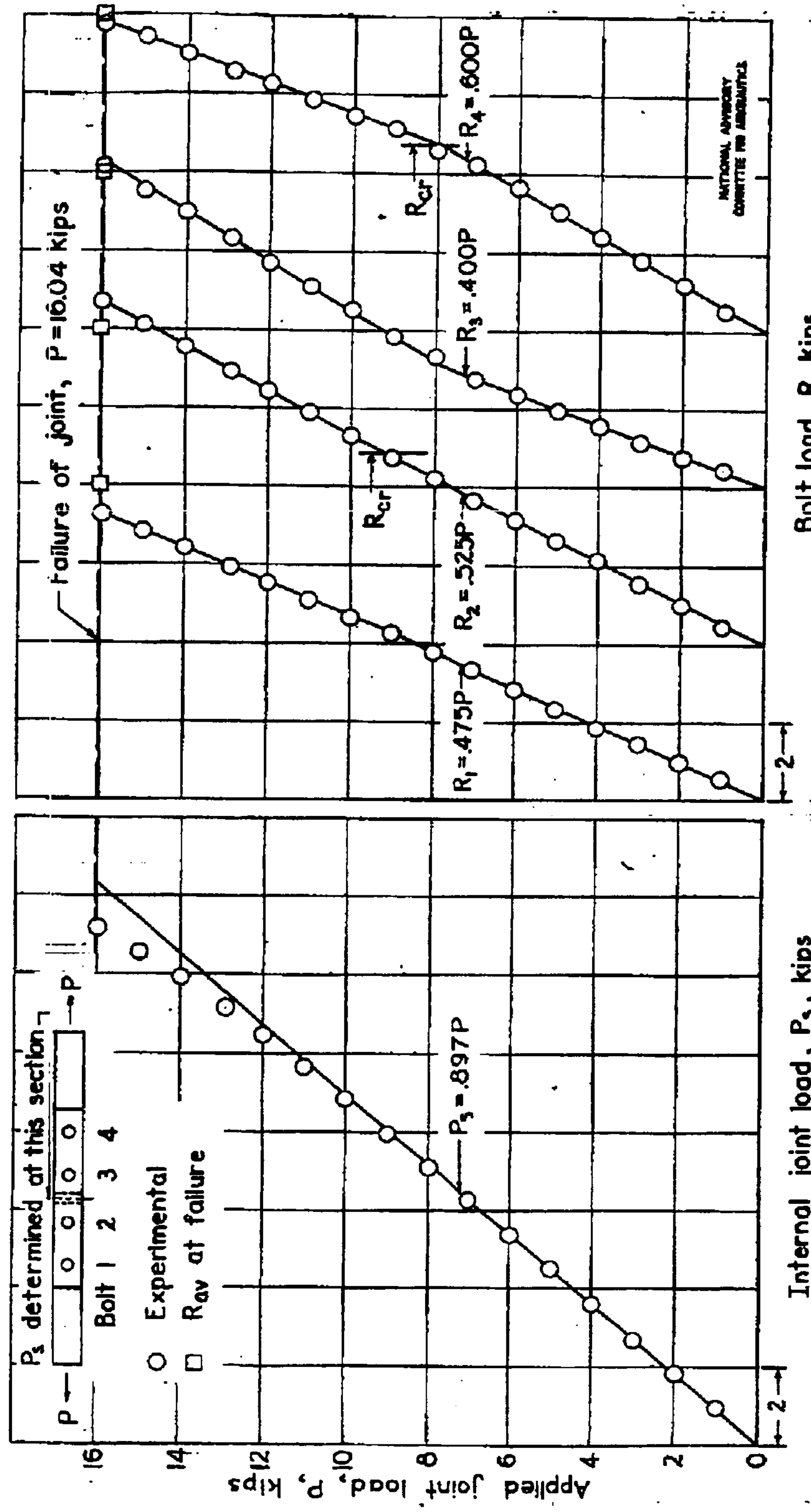
Figure 9. - Joint-load and bolt-load curves for loading of specimen B-3 in the elastic range.



(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads.

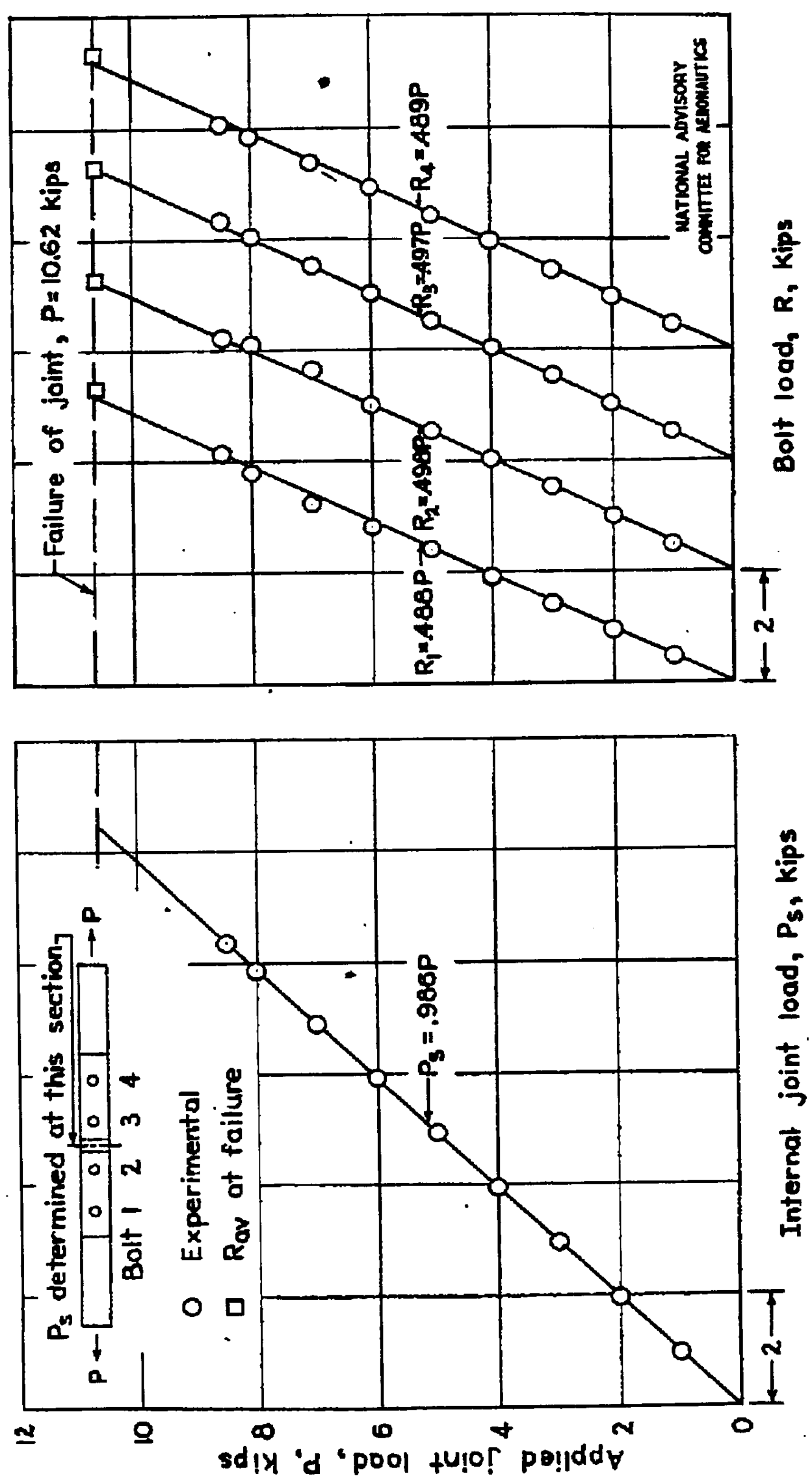
Figure 10a - Joint-load and bolt-load curves for specimen A-1 tested to failure.



(b) Observed relationships between applied joint load and bolt loads.

(c) Observed relationship between applied and internal joint loads.

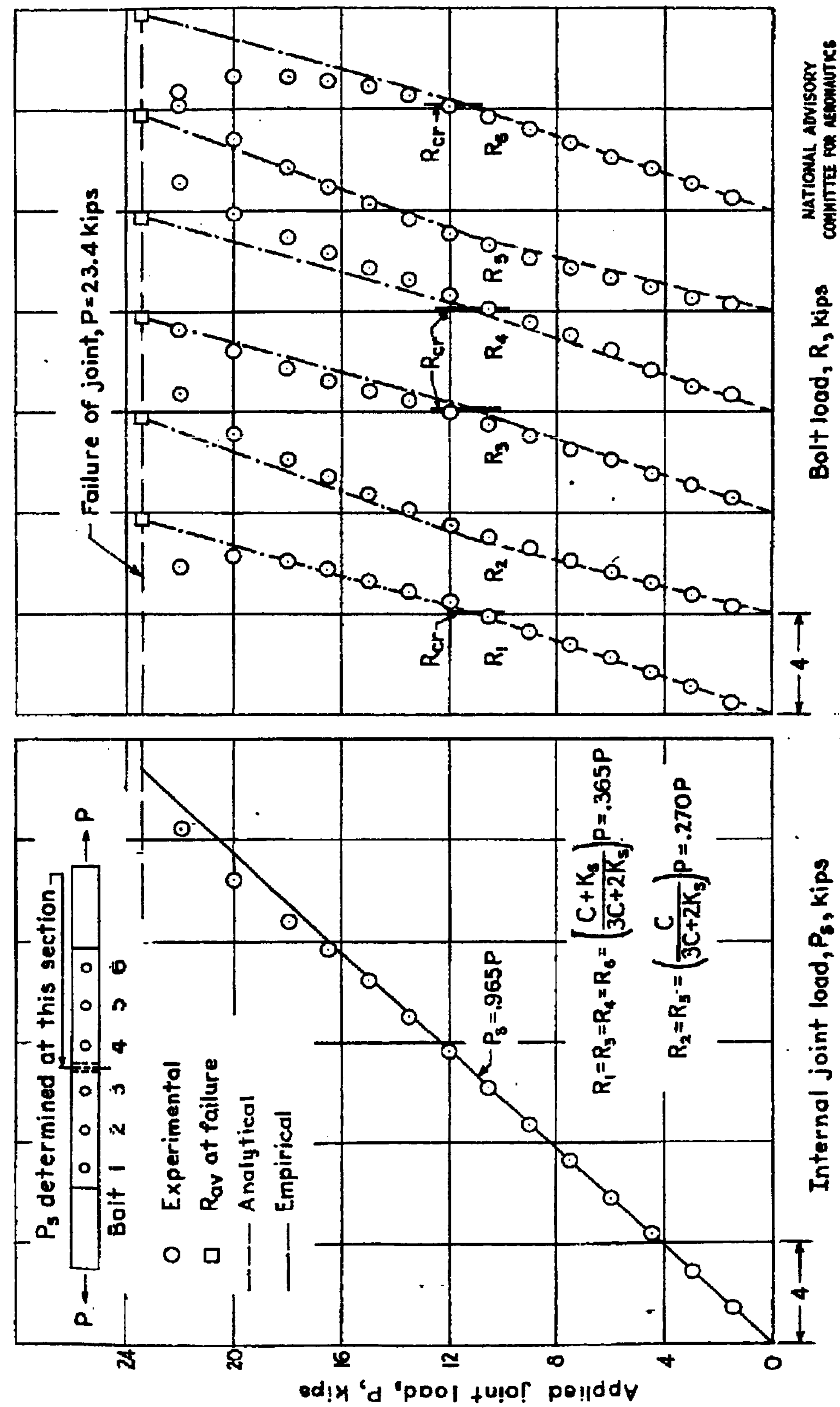
Figure 11. - Joint-load and bolt-load curves for specimen A-2 tested to failure.



(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads.

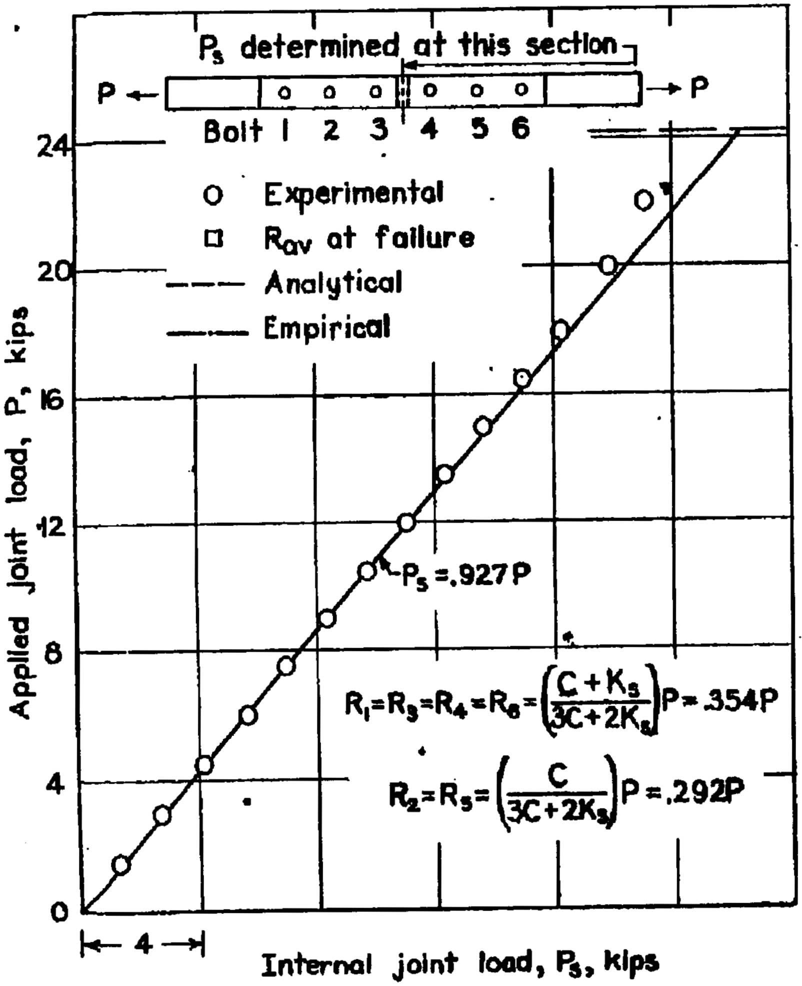
Figure 12.- Joint-load and bolt-load curves for specimen A-3 tested to failure.



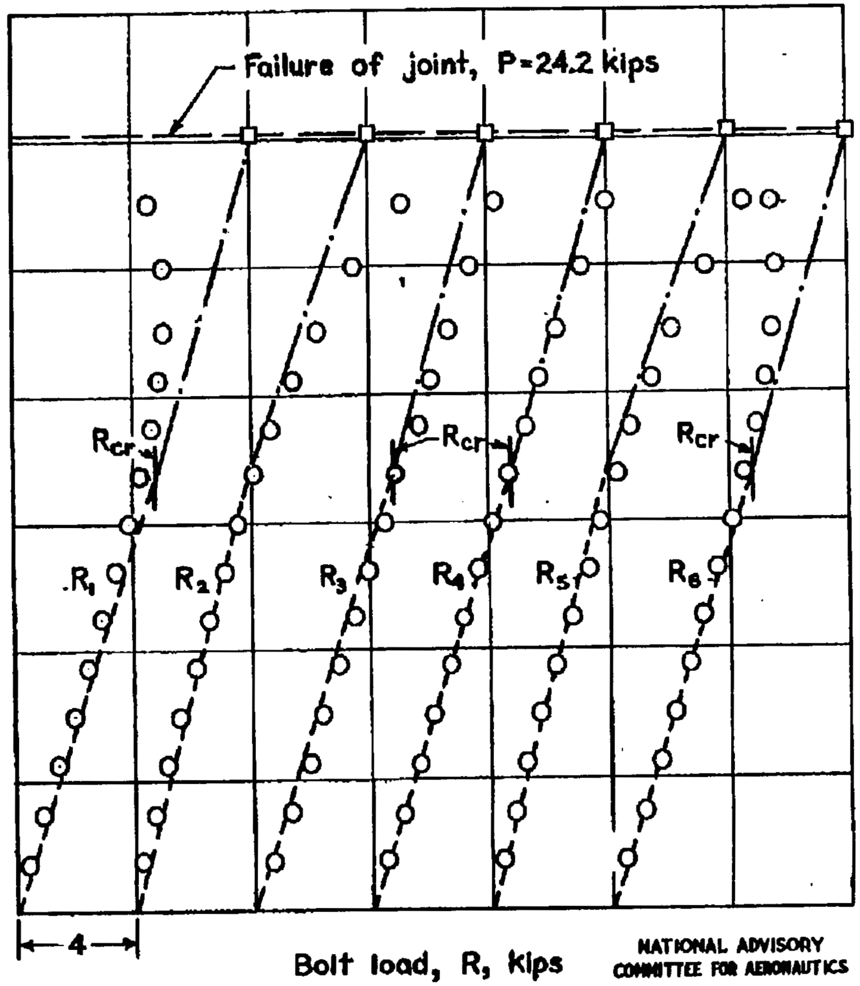
(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads and comparison with calculated values.

Figure 13. — Joint-load and bolt-load curves for specimen B-1 tested to failure.

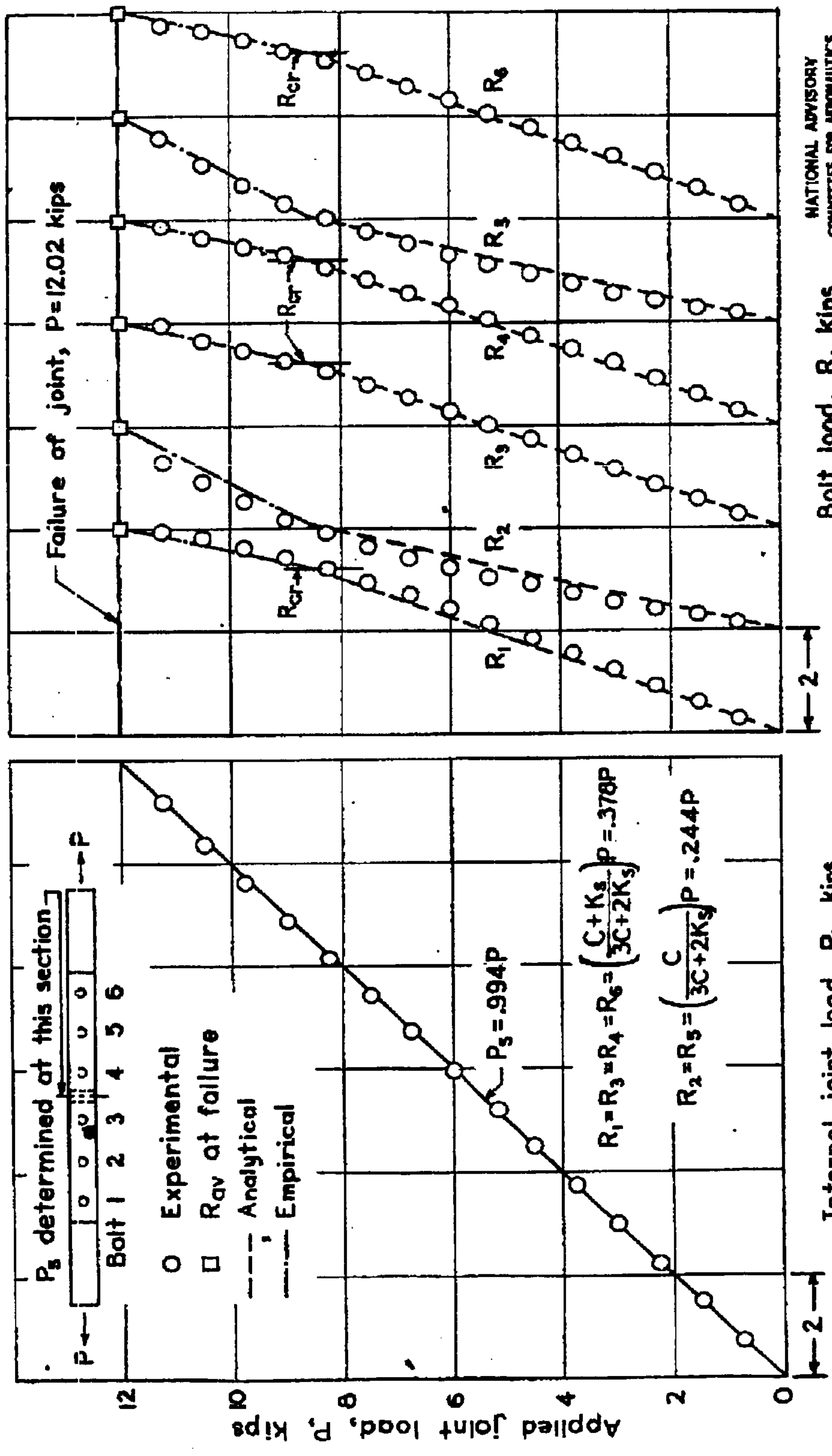


(a) Observed relationship between applied and internal joint loads.



(b) Observed relationships between applied joint load and bolt loads and comparison with calculated values.

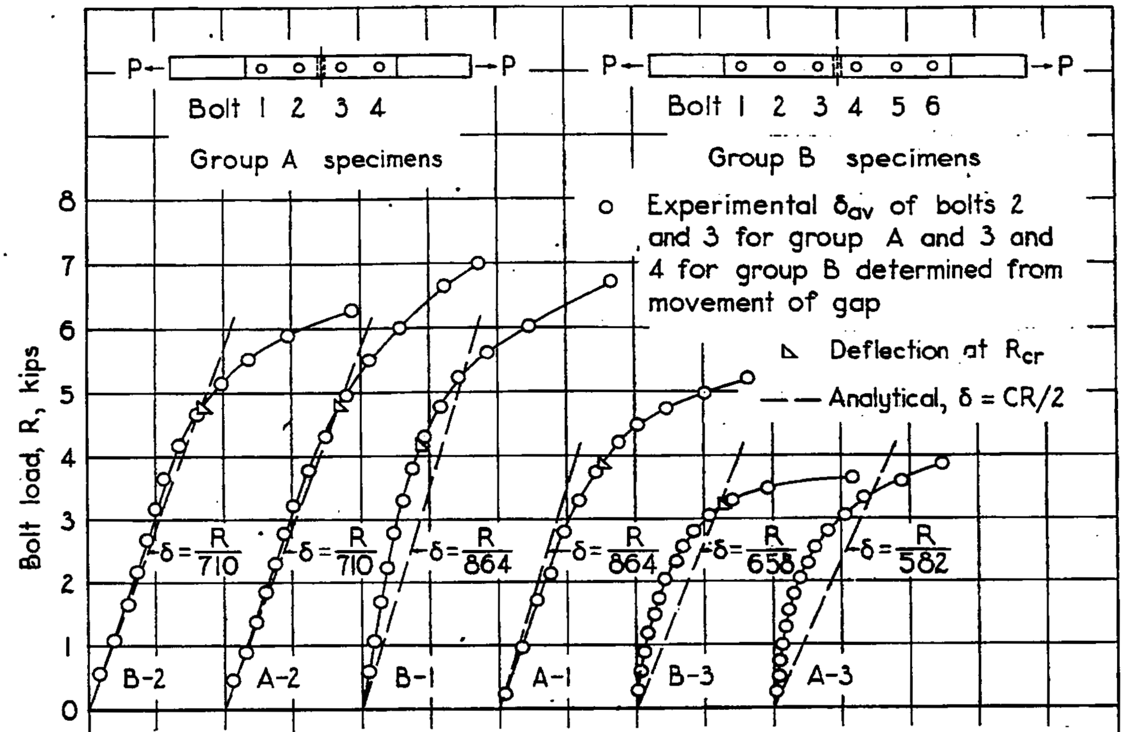
Figure 14.—Joint-load and bolt-load curves for specimen B-2 tested to failure.



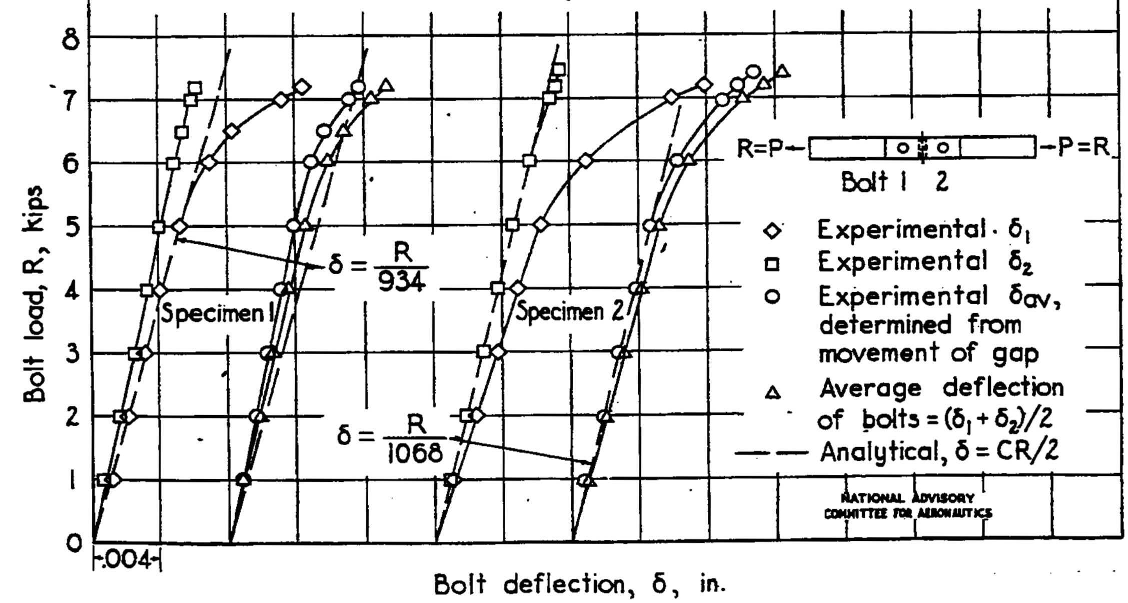
(a) Observed relationship between applied and internal joint loads.

(b) Observed relationships between applied joint load and bolt loads and comparison with calculated values.

Figure 15.– Joint-load and bolt-load curves for specimen B-3 tested to failure.



(a) Bolt deflections for specimens of groups A and B; 24S-T plates and heat-treated (125 ksi) alloy-steel bolts.



(b) Bolt deflections for auxiliary-shear-test specimens; S.A.E. 6150 steel plates and heat-treated (125 ksi) alloy-steel bolts.

Figure 16.– Comparison of experimental and analytical bolt deflections.

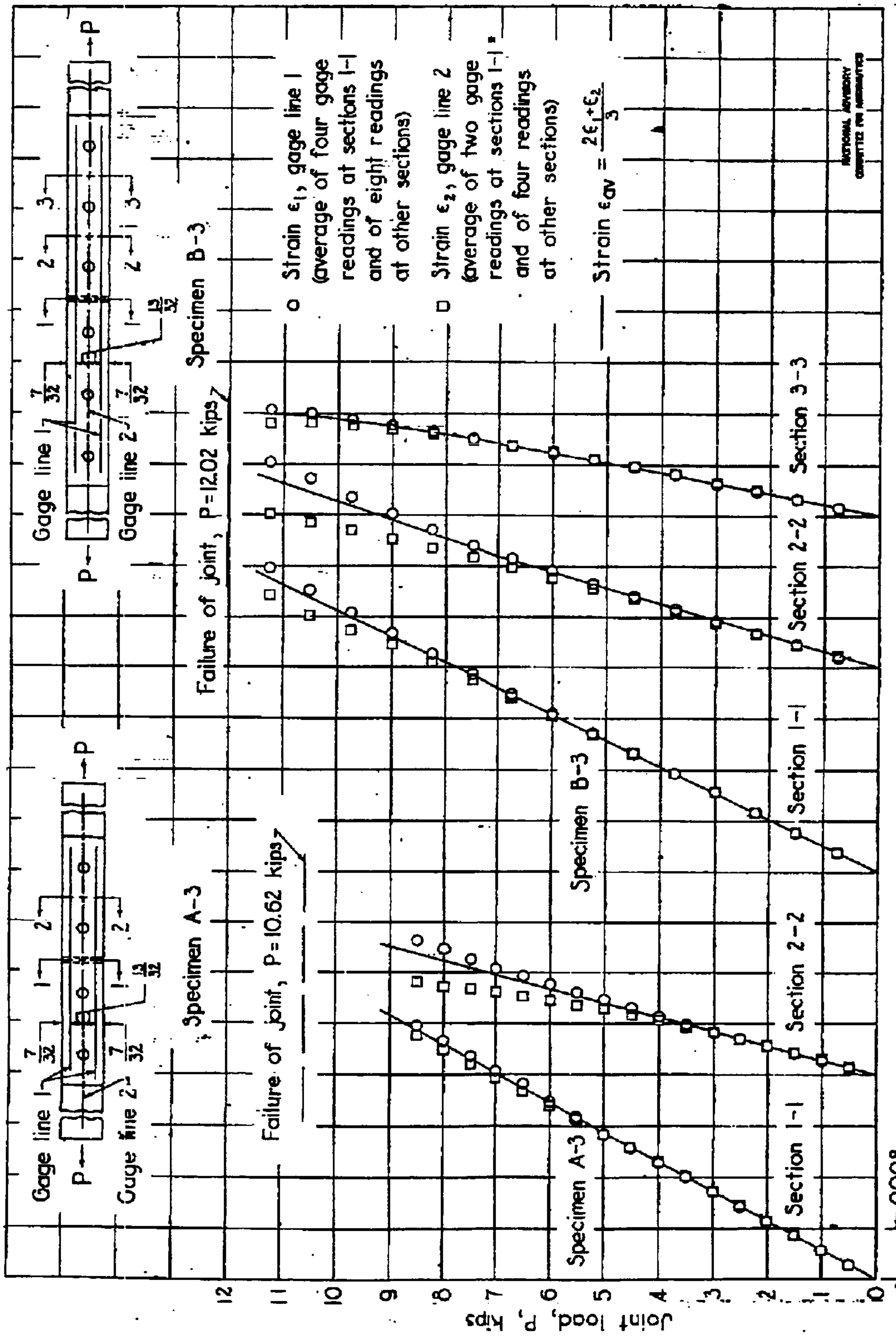


Figure 17.- Comparison of measured and average strains for typical specimens of groups A and B. (All strains measured on outer surface of butt straps)

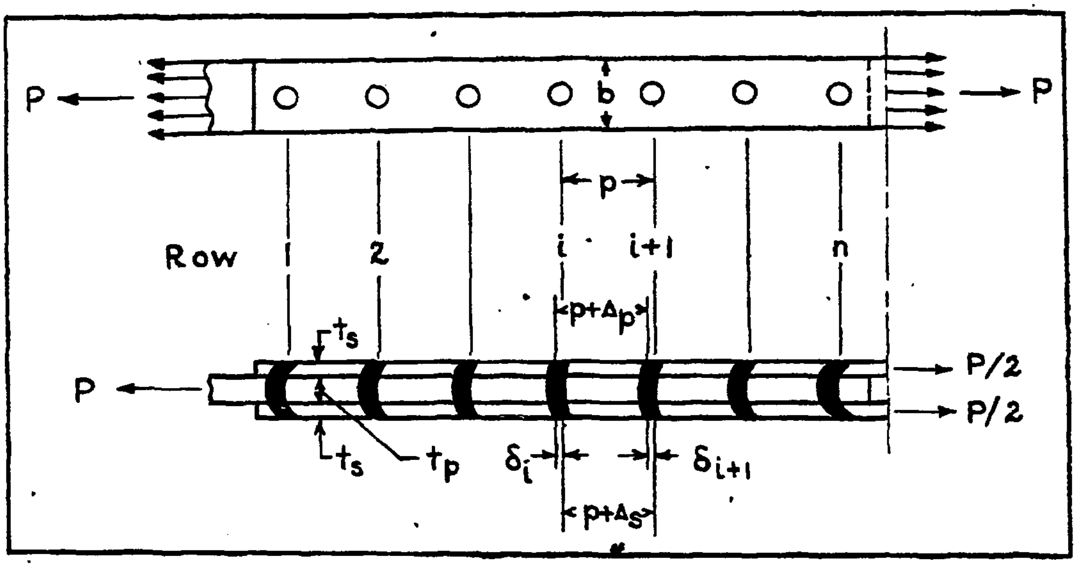


Figure 18.– Symmetrical butt joint with bolts in a single line in the line of applied load.

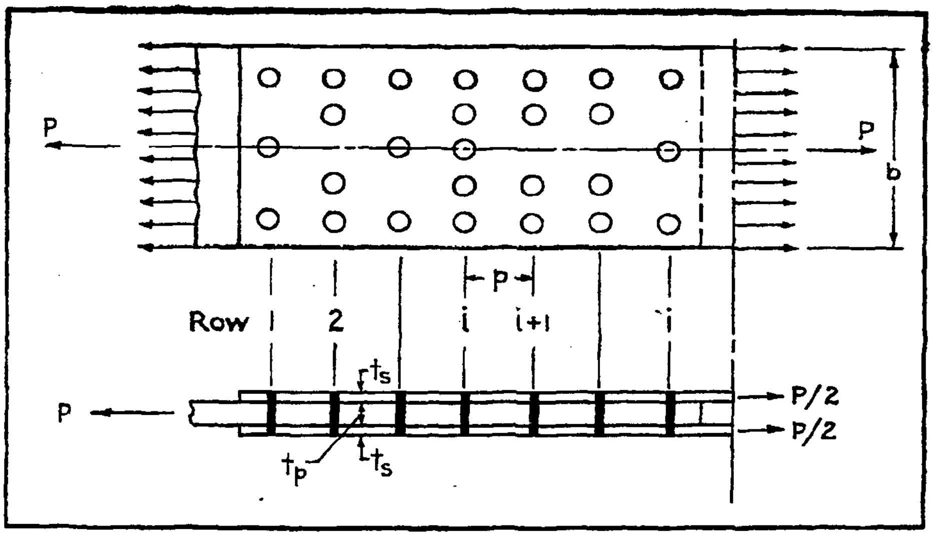


Figure 19.– Symmetrical butt joint with bolts in several lines parallel to applied load.

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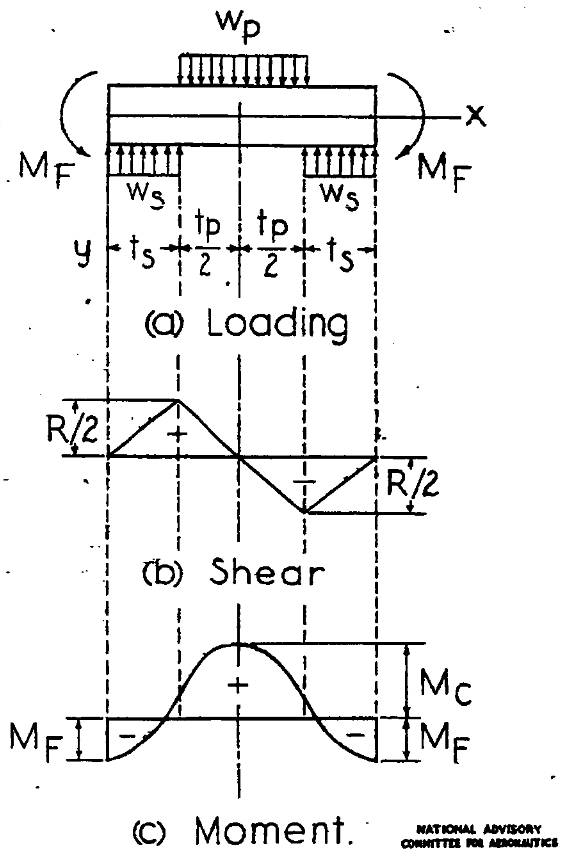

Figure 20. – Bolt loading, shear, and moment diagrams.