

A finite element method for calculating load distributions in bolted joint assemblies

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Abstract

Bolted joints are often the most critical parts with respect to fatigue life of structures. Therefore, it is important to analyze these components and the forces they are subjected to.

A one-dimensional finite element model of a bolted joint is created and implemented as a program module in the Saab software ‘DIM’, together with a complete graphical user interface allowing the user to generate the structure freely, and to apply both mechanical and thermal loads.

Available methods for calculating fastener flexibility are reviewed. The ones derived by Grumman, Huth and Barrois are implemented in the module, and can thus be used when defining a geometry representing a bolted joint assembly. Investigations have shown that it cannot be said that either method is generally better than the other. Calculated properties of interest include the fastener forces, plate bearing and bypass loads, and - for simpler geometries without thermal loads - the load distribution between rows of fasteners.

The program is fully functional and yields numerically accurate results for the most commonly used joints where fasteners connect two or three plates each. It has limited functionality on geometries with fasteners connecting four or more plates and for a certain loading combination also for three plates, due to the tilting of the fasteners not being accounted for in the model for these cases. Also, there is no explicit method available for finding an accurate value for the fastener flexibility for these, less common, joint structures.

Preface

This thesis work has been conducted at Saab Aeronautics via Linköping University during the spring of 2012, and marks the end of the authors studies for the Degree of Master of Science in Mechanical Engineering.

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Linköping, Sweden
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Chapter 1

Introduction

1.1 Background

1.1.1 Bolted joints

Bolted joints are widely used when connecting structural components in larger configurations. Information about the load distribution and fastener flexibility among fasteners in a joint is of interest in the design of lightweight structures, commonly occurring in the field of aeronautics. Aircraft structures, in particular the fuselage and wings, are often connected using bolted joints with various types of fasteners. A sketch of a bolted joint is shown in Figure 1.1.

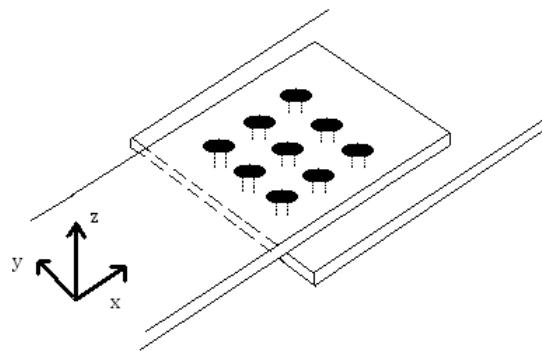


Figure 1.1: Example joint

The load distribution between the fasteners in the joint has a large impact on factors that affect the strength and fatigue life of the joint, such as bearing pressure and stress concentrations, and is therefore of interest when designing and sizing such a structure.

Fastener flexibility is a property of interest when calculating the load distribution in a joint. It is a measure of the fastener's influence on the flexibility of the joint, and has a large impact on load

distribution, as illustrated in Figure 1.2. A more thorough description of the fastener flexibility concept is given in Section 2.1.

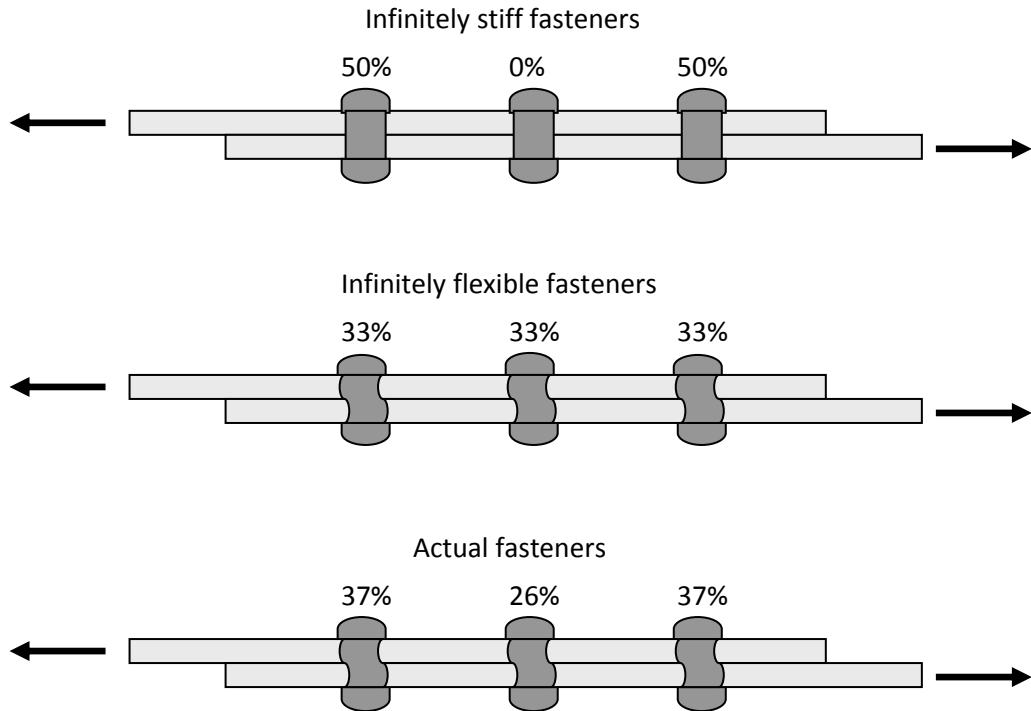


Figure 1.2: Cross-section of a joint; load distribution with varying fastener flexibility

1.1.2 Saab & DIM

Saab has developed both commercial and military aircraft, such as Saab 2000 and Saab 39 Gripen. Today, there are numerous methods and programs available at the company for analysis of structural components of metal and composites with respect to fatigue, buckling, and more. In order to make the dimensioning work more efficient and the analysis tools more user-friendly, the decision was made to develop the structural sizing tool ‘DIM’. It incorporates the total analysis chain from the global finite element analysis, via choice of elements and loads and various calculations, to writing a report. The DIM software is created in Matlab using object-oriented programming [11], and consists of a main program (‘core’), and so called modules. The modules perform various tasks such as defining geometries and loads, performing calculations, and report generation.

1.2 Objective

As a part of the development of DIM, a module for calculating fastener flexibility and load distribution between fasteners in a bolted joint is to be created. The module should be able to

handle applied forces and thermal loads on the joint, and it should be possible to define a joint geometry with an arbitrary number of connecting plates and fasteners. The module interface needs to be easily understood for a user familiar with calculating load distribution in joints, and give the user flexibility to define geometry, material parameters and in what way the calculation will be performed with respect to different kinds of methods. Input data and the result shall be presented as a user-defined, automatically generated, report.

The influence of fastener flexibility on load distribution between fasteners in a joint will be investigated, in order to assess the necessity for using different methods in calculating fastener flexibility.

Finally, the question will be investigated as to what value of the fastener flexibility, if inserted into the model being used in the module, yields displacements comparable to those found by experiments, and what a difference in flexibility or resulting displacement may be caused by.

1.3 Method

Initially, an investigation of methods for calculating fastener flexibility and load distribution will be made, whereupon a choice will be made as to which methods shall be implemented in the module and how the joint will be modeled.

The work is divided in two major parts, the first being the creation of the module itself, complete with graphical user interface (GUI) for user interaction in generating geometry, and assigning parameters and boundary conditions. The second part consists of implementation of calculations and the evaluation of these. This will be performed using parametric studies and comparisons with experiments. Calculations will be performed to verify the accuracy of the software and the model it utilizes.

Chapter 2

Problem definition

There are many ways that load distribution and fastener flexibility can be calculated in a bolted joint. How to go at hand in solving this problem depends, as in so many other engineering situations, mainly on its intended application; i.e. what results do we wish to obtain, and what simplifications are reasonable in order to obtain acceptable results.

2.1 Calculating fastener flexibility

2.1.1 Defining fastener flexibility

The fastener flexibility concept was introduced by Tate & Rosenfelt in 1946 [14], under the alias ‘bolt constant’, due to a desire to calculate load distribution in joints with multiple rows. It is defined by assuming a linear relationship between the displacement due to the presence of the fastener, and the load transfer. The fastener flexibility f can be written as

$$f = \frac{1}{k} = \frac{\delta}{P_{LT}} \quad (2.1)$$

where k is the fastener stiffness, P_{LT} the load transferred by the fastener (defined in Figure 2.1), and δ the contribution to the total displacement of the joint disregarding the elongation PL/EA of the plates. Thus, the fastener flexibility includes all phenomena that affect the flexibility of the joint (apart from plate flexibility) such as fastener deformation, fastener tilt, and deformation of fastener holes. In determining the fastener flexibility experimentally, there are several approaches, of which a few are described here.

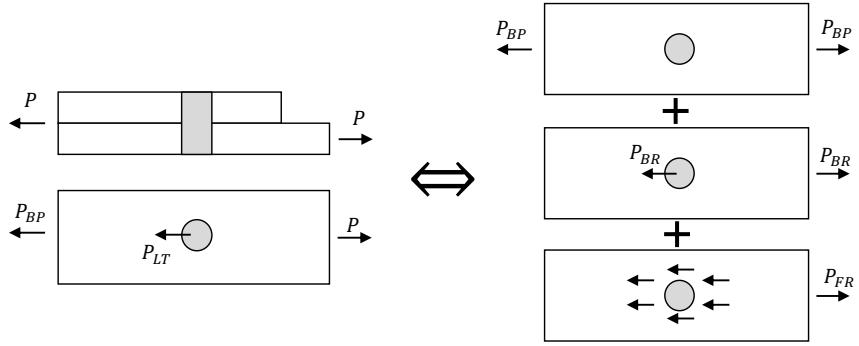


Figure 2.1: Forces acting on a joint: transferred load (P_{LT}), bypassing force (P_{BP}), bearing force (P_{BR}), frictional force (P_{FR})

Jarfäll [10] measured the gap g of Figure 2.2 for the applied force $2P$.

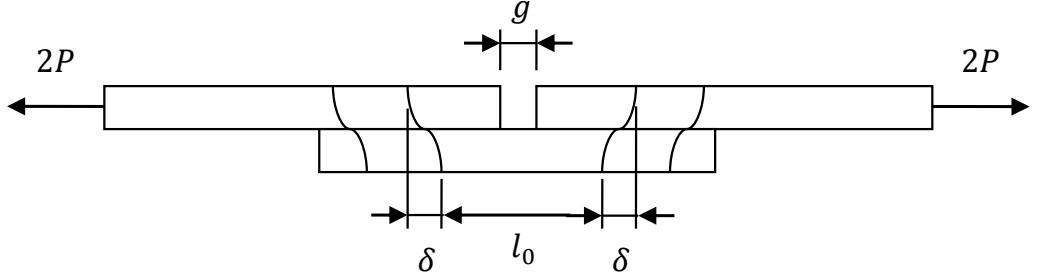


Figure 2.2: Finding fastener flexibility (Jarfäll)

The gap g relates to δ as

$$\Delta g = \Delta l_0 + 2\delta \quad (2.2)$$

This yields

$$\frac{\partial g}{\partial P} = \frac{2l_0}{AE} + 2f \quad (2.3)$$

and the fastener flexibility becomes

$$f = \frac{1}{2} \frac{\partial g}{\partial P} - \frac{l_0}{AE} \quad (2.4)$$

Huth [8] performed measurements on the total displacement Δl_{tot} between points A and B of the single shear geometry (see Section 2.1.2) with two fasteners in Figure 2.3, thus yielding average values of δ .

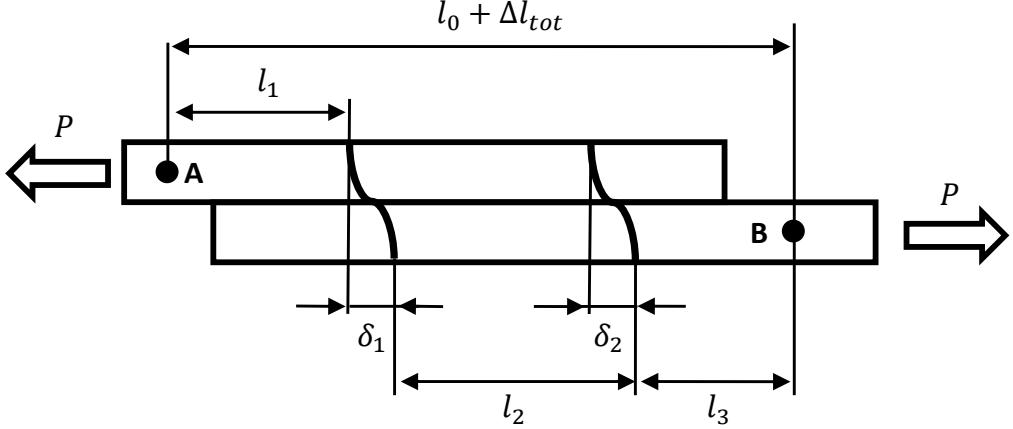


Figure 2.3: Finding fastener flexibility (Huth, single shear)

The total displacement is written as

$$\Delta l_{tot} = \frac{\delta_1 + \delta_2}{2} + \Delta l_1 + \Delta l_2 + \Delta l_3 \quad (2.5)$$

From this, δ becomes

$$\delta = \frac{\delta_1 + \delta_2}{2} = \Delta l_{tot} - \Delta l_{elast} \quad (2.6)$$

where, with the plate width w , thickness t , and Young's modulus E ,

$$\Delta l_{elast} = \frac{P}{t_1 w E_1} \left(l_1 + \frac{l_2}{\left(\frac{t_2}{t_1} \frac{E_2}{E_1} \right)} + \frac{l_3}{\left(1 + \frac{t_2}{t_1} \frac{E_2}{E_1} \right)} \right) \quad (2.7)$$

and the fastener flexibility is

$$f = \frac{1}{2} \frac{(\delta_1 + \delta_2)}{P/2} = \frac{\delta_1 + \delta_2}{P} \quad (2.8)$$

For the double shear geometry (Section 2.1.2) in Figure 2.4, Huth [8] obtained the fastener flexibility by measuring the total displacement between points A and B , which is written as

$$\Delta l_{tot} = \delta + \Delta l_1 + \Delta l_2 \quad (2.9)$$

From this, δ becomes

$$\delta = \Delta l_{tot} - (\Delta l_1 + \Delta l_2) = \Delta l_{tot} - \Delta l_{elast} \quad (2.10)$$

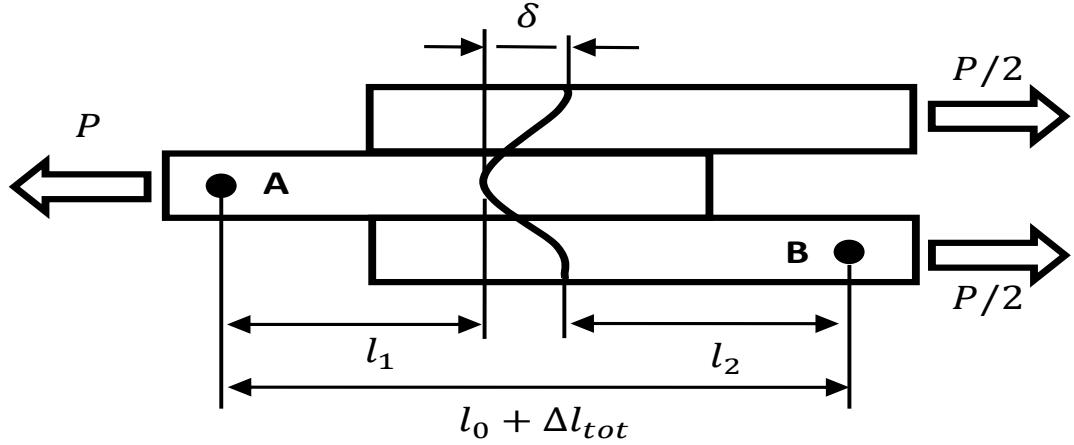


Figure 2.4: Finding fastener flexibility (Huth, double shear)

where

$$\Delta l_{elast} = \frac{P}{w} \left(\frac{l_1}{t_1 E_1} + \frac{l_2}{2t_2 E_2} \right) \quad (2.11)$$

The fastener flexibility is then found as

$$f = \frac{\delta}{P} \quad (2.12)$$

The relationship between force and displacement is in reality non-linear, and therefore there are several ways to identify a fastener flexibility (as a constant) from experimental data. Jarfall [10] describes some of these methods thoroughly. The way that is probably most representative when striving for an elastic model to describe the behavior of a joint, is the Jarfall alternative d , which was also used by Huth. Figure 2.5 shows a sketch of the characteristic behavior of a joint when subjected to cyclically increasing load, where also the fastener flexibility as obtained by Huth is indicated.

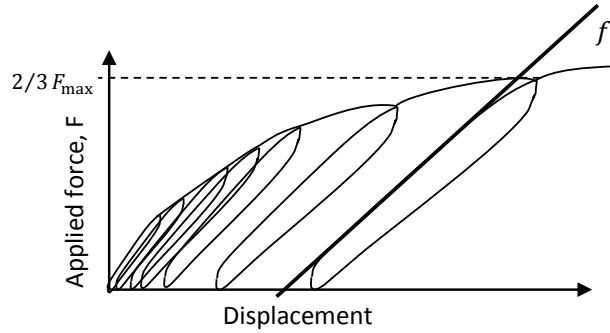


Figure 2.5: Example of measured fastener flexibility

2.1.2 Overview of methods

As seen, there are several ways to find the fastener flexibility of Eq. (2.1) experimentally. Many have attempted - via testing on geometries with varying parameters - to create methods for describing the joint behavior by calculating the fastener flexibility as a function of these parameters. These include empirical formulas derived from specific types of joints and materials by Grumman [10], Huth [8], Boeing [10], Douglas [10], Tate & Rosenfeld [9] and others, using an analytical approach such as methods by Barrois [2] and ESDU [5]. The great variety of available methods is due to the fact that they have been derived using different simplifications and/or that they apply to specific materials or specific types of joints.

Things that affect the joint behavior include bolt pre-tension, fastener fit (hole clearance), hole surface quality, type of fastener (countersunk, rivets, bolts), surface quality including coatings or sealants and more.

Two common configurations occur when referring to joints and fastener flexibility, namely single shear and double shear loaded fasteners, illustrated in Figure 2.6.

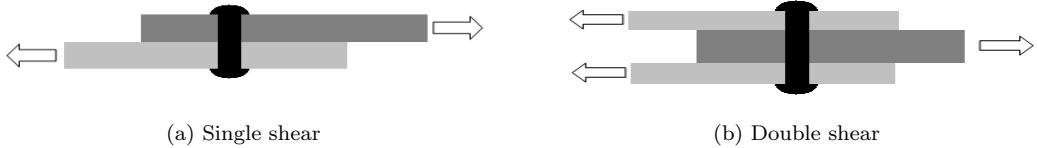


Figure 2.6: Types of shear

In the case of single shear, another physical phenomenon presents itself due to the fastener tilting under that kind of load, called secondary bending. Even with the external load being free from bending moment, the tilting of the fastener that occurs in single shear induces bending in the joint which has a high impact on fatigue life of joints.

The methods presented by Grumman, Huth, and Barrois are frequently used at Saab and are therefore to be implemented in the program module.

2.1.3 Grumman [10]

The Grumman equation is an empirically derived formula that was presented by the Grumman Aerospace Corporation and was used during the development of the Saab 37 Viggen aircraft, and the fastener flexibility is given by

$$f = \frac{(t_1 + t_2)^2}{E_f d} + 3.72 \cdot \left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right) \quad (2.13)$$

where E_f and d are the Young's modulus and diameter of the fastener, respectively.

The conditions under which the testing was performed, that eventually lead up to the Grumman formula, is unclear. Nordin [12] claims it was derived for metallic materials, for which both bolts and rivets can be used in joining plates. It was however used during the development of a composite component for the Viggen aircraft [3], which are usually not joined by rivets. The formula does however not account for fastener tightening, hole clearance, and whether the fastener is countersunk or not [12].

2.1.4 Huth [9]

Based on extensive testing on different types of joints and materials, a formula for fastener flexibility was fitted to load-displacement curves as

$$f = \left(\frac{t_1 + t_2}{2d} \right)^a \frac{b}{n} \left(\frac{1}{t_1 E_1} + \frac{1}{nt_2 E_2} + \frac{1}{2t_1 E_f} + \frac{1}{2nt_2 E_f} \right) \quad (2.14)$$

where a , b and n are parameters defining the joint type as seen in Table 2.1.

| | |
|------------------------------|--------------------|
| Single shear | $n = 1$ |
| Double shear | $n = 2$ |
| Bolted metallic joints | $a = 2/3, b = 3.0$ |
| Riveted metallic joints | $a = 2/5, b = 2.2$ |
| Bolted graphite/epoxy joints | $a = 2/3, b = 4.2$ |

Table 2.1: Huth parameters

The Huth formula is derived with a single-spring assumption, for single and double shear alike. This assumption is discussed further in Section 2.3.1.

2.1.5 Barrois [2]

The method by Barrois was developed using an analytical approach by modeling the fastener as a beam on an elastic foundation, taking into account bending and shearing deflections of the fastener. The assumption is made that there is a linear relation between the deflection of the fastener and the applied load. Also, it is assumed there is no clearance between fastener and foundation. Both single shear and double shear loaded fastener installations are handled.

In the derivation it is assumed that the joined plates are of the same material. Finally, two different boundary conditions are applied at the fastener ends, yielding several ways of using Barrois' method ('variants'). These boundary conditions are: clamped fastener heads (bolts) and free fastener heads (pins). Barrois uses a single-spring assumption, similar to Huth. Also, in calculating load distribution, Barrois attempts to take into account holes in plates, see Section 2.3.1.

The Barrois derivation of the fastener flexibility is quite extensive and not reproduced in detail in this report. The interested reader may find a detailed description of the method by Barrois in Reference [2].

2.2 Calculating the load distribution

2.2.1 Overview of models

A common method for modeling a flexible joint assembly including several rows of fastener elements is by representing fasteners and fastened components as springs. The method, in which the spring constant for the fasteners is the inverse of the fastener flexibility (i.e. fastener stiffness), can be illustrated as seen in Figure 2.7.

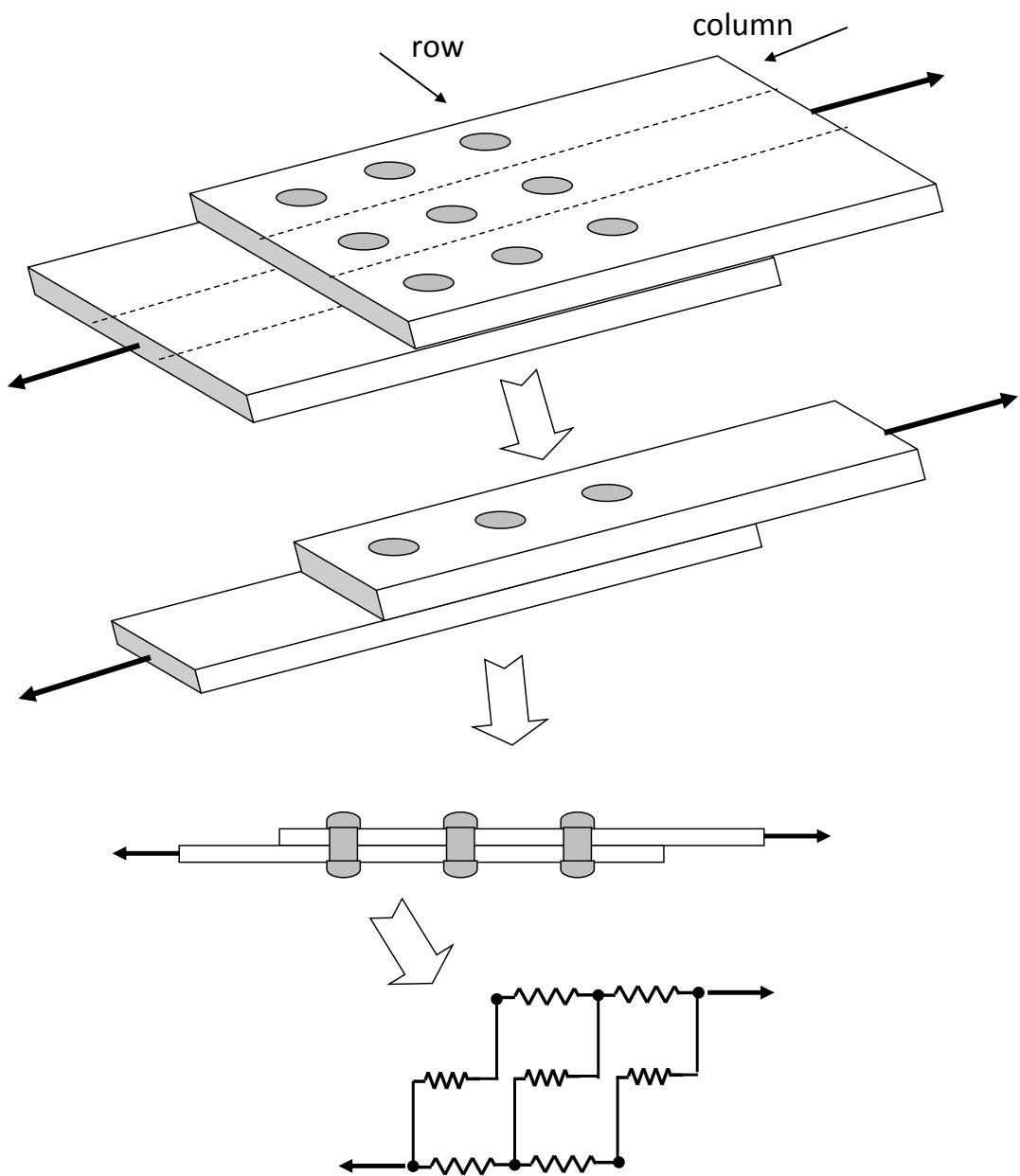


Figure 2.7: Common 1D-model of a joint

The model is one-dimensional, and can thus only take into account variations in the longitudinal direction ('column'-direction of Figure 2.7). Bending is omitted in this model. Applying this model to an entire joint assembly therefore assumes that there is no variation between fastener columns, and the result is the load distribution per row of fasteners. This model is attractive due to its simple nature and low calculation costs. Also, it is very common that the joints in airplane fuselage consists of a set of columns of equal fasteners, which in most cases are subjected to forces mainly in the longitudinal direction. Even so, it is a common engineering approach to assume that shear forces have the same effect on load distribution as longitudinal forces (see Section 5.4.1 for a description of this approach). Therefore, this one-dimensional model approximation is often applicable.

Using a two-dimensional model, one can take into account variations in bolt properties between fastener columns and handle situations where, for example, a large joint assembly has been reinforced at some point with additional plates and fasteners of different dimensions. This kind of model could also more accurately account for forces in the transverse direction, in cases where these are prominent.

With a three-dimensional model, the entire joint could be modeled in detail, giving a more realistic model. However, to use a detailed 3D FE-model of a large joint assembly would require tremendously high calculation costs compared to the other available methods, and still there are factors that are very difficult to take into account; such as friction, hole clearance and local plastic deformations.

At Saab, the need for calculations where column-wise variations occur is deemed to be low. Only a few of the joints on e.g. the fighter aircraft Saab 39 Gripen are such that a one-dimensional model is not sufficient. These joints are today instead dealt with using commercial finite element software. Thus, for the purposes of this work, it is deemed that a one-dimensional model is sufficient and this is therefore used in further calculations and implemented in the developed program module.

2.2.2 Detailed description of the finite element model

The guideline of the model is that of the one-dimensional model in the previous section. However, in order to be able to handle temperature variations and to spare the user from having to calculate the stiffnesses in the joined elements (plates), these are modeled as bars (Figure 2.8).

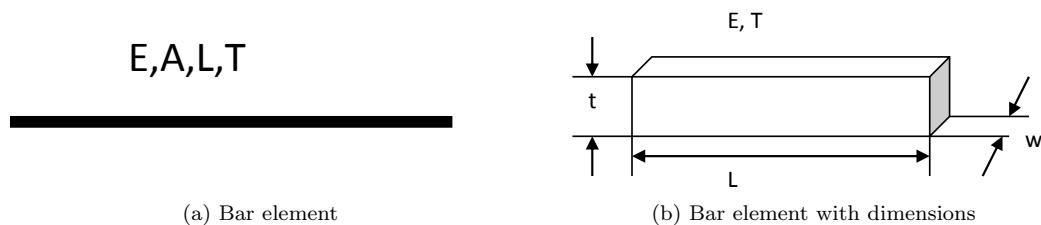


Figure 2.8: The bar element

The principle of the finite element model together with the system of element and nodal numbering is shown in Figure 2.9.

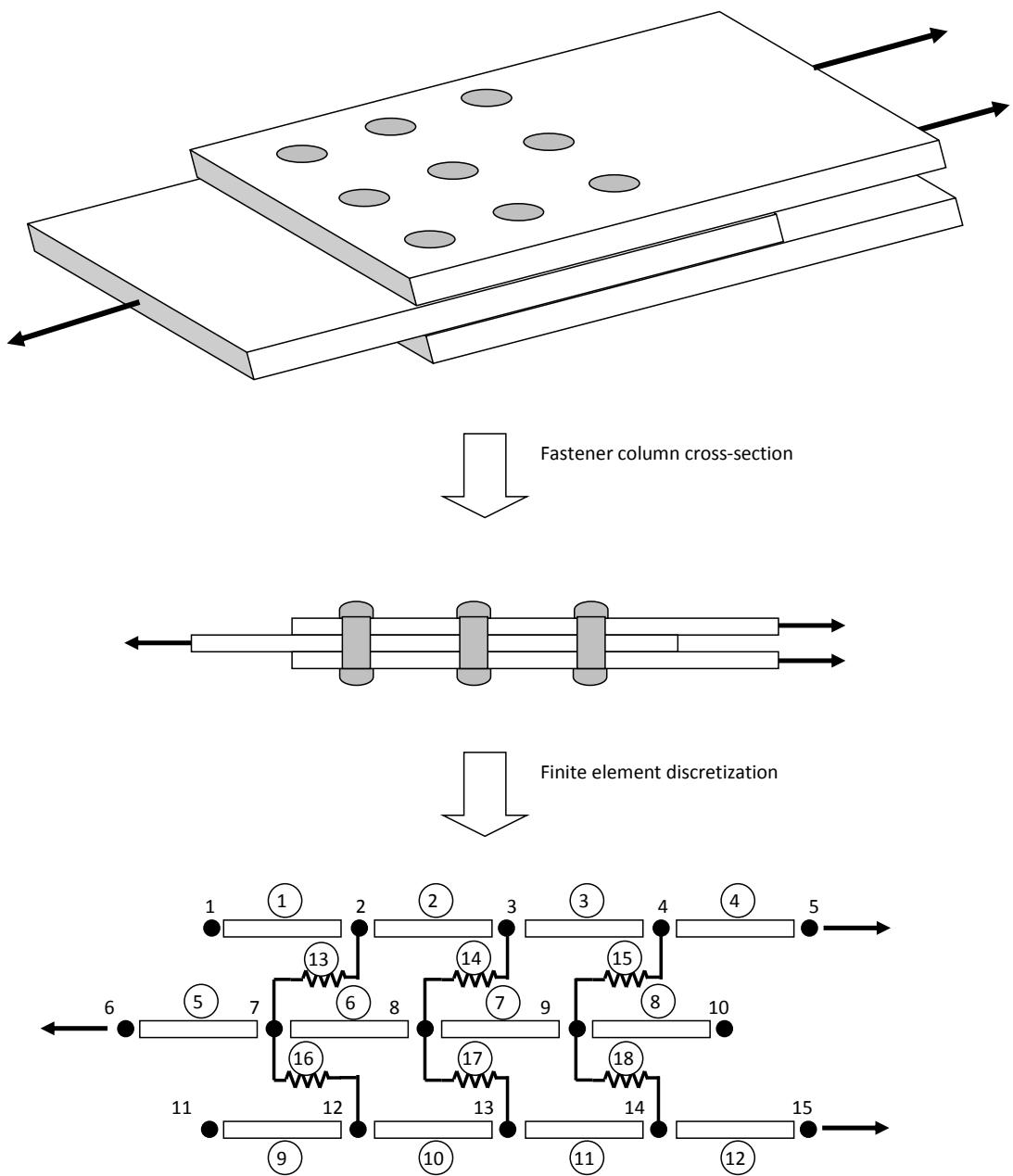


Figure 2.9: Example of the one-dimensional joint model used for this work

The calculation on the finite element model is conducted in the following steps:

1. Calculate initial stresses
2. Calculate initial nodal forces
3. Assemble global stiffness matrix
4. Enforce boundary conditions (constrained motion)
5. Calculate nodal displacements
6. Calculate element forces

Using the method of direct equilibrium, a relation between nodal forces and displacements can be obtained with parameters shown in Figure 2.10.

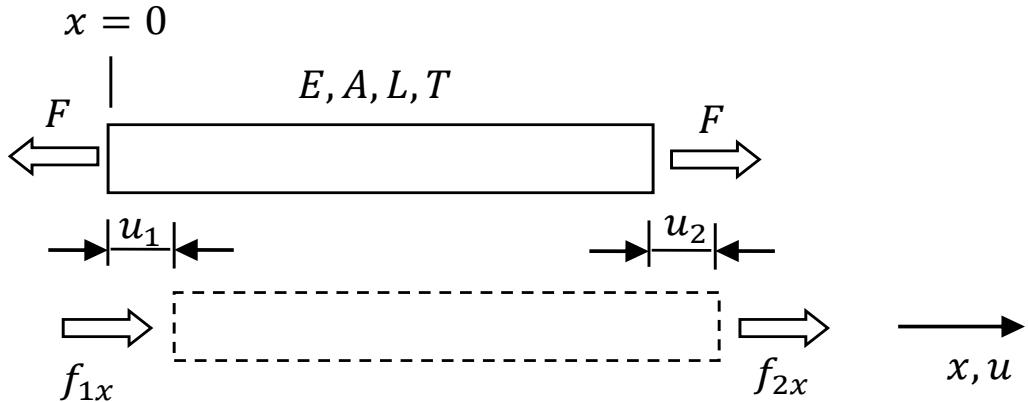


Figure 2.10: Tension loaded 1D bar element

The displacement of the nodes is written as

$$u = [N_1 \quad N_2] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (2.15)$$

where $N_1 = 1 - x/L$ and $N_2 = x/L$ are the shape functions of the bar element, describing its linear displacement behavior.

With σ_x being the element stress and ε the strain, the relation between nodal forces and displacement is obtained in the following way

$$F = A\sigma_x \quad (2.16)$$

$$\sigma_x = E\varepsilon \quad (2.17)$$

$$\varepsilon = \frac{\delta}{L} = \frac{u_2 - u_1}{L} \quad (2.18)$$

$$f_1 = -F = AE \left(\frac{u_1 - u_2}{L} \right) \quad (2.19)$$

$$f_2 = F = AE \left(\frac{u_2 - u_1}{L} \right) \quad (2.20)$$

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = [\mathbf{k}^e] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (2.21)$$

where \mathbf{k}^e is the element stiffness matrix. Displacement calculation for spring elements is conducted in a similar fashion, replacing the term EA/L by the spring constant k .

The typical approach when dealing with thermal loads is to calculate the initial stresses that arise from the element temperature difference relative to a set reference temperature (here: 0), when all displacements are prohibited, see for example Ref. [4]. The initial nodal forces are then obtained by superposing the resulting nodal forces due to the temperature field with mechanical loads. The element initial stress is calculated as

$$\sigma_0^e = -E\alpha\Delta T \quad (2.22)$$

where α is the thermal expansion coefficient and ΔT is the element (average) temperature difference.

The element stiffness matrices are then assembled into the global stiffness matrix $[\mathbf{K}]$. For all elements N_{els} , each element stiffness matrix is added to the global stiffness matrix according to Eq. (2.23).

$$[\mathbf{K}] = \sum_{n=1}^{N_{els}} [\mathbf{k}^e]_n \quad (2.23)$$

$$\text{where } [\mathbf{k}^e]_n = \begin{pmatrix} u_i/u_1 & u_j/u_2 \\ u_i/u_1 & \begin{pmatrix} : & : \\ \vdots & \vdots \\ k_n & -k_n \\ \vdots & \vdots \\ -k_n & k_n \\ \vdots & \vdots \end{pmatrix} \\ u_j/u_2 & \begin{pmatrix} : & : \\ \vdots & \vdots \\ -k_n & k_n \\ \vdots & \vdots \end{pmatrix} \end{pmatrix} \quad (2.24)$$

where u_1 and u_2 are the displacements of nodes 1 and 2 of element n , respectively, placed in the global stiffness matrix on rows and columns corresponding to global element numbers i and j (circled numbers in Figure 2.9).

The relation between nodal forces and displacements is written

$$\{\mathbf{P}\} = [\mathbf{K}]\{\mathbf{D}\} \quad (2.25)$$

where $\{\mathbf{P}\}$ is the nodal force vector and $\{\mathbf{D}\}$ the nodal displacement vector.

After imposing boundary conditions preventing rigid body motion, $[\mathbf{K}]$ becomes non-singular and Eq. (2.25) gives a unique solution. In the model shown in Figure 2.9, this is done by constraining nodes from motion in the x -direction. Because the model is one-dimensional and motion is only possible in one direction, it is sufficient to lock one node in the model. Imposing of nodal

constraints is done by removing rows and columns of the terms in Eq. (2.25) corresponding to the constrained node, thus making the problem possible to solve using e.g. Gauss elimination.

With nodal displacements calculated, the element stresses can be calculated for element n as

$$\sigma_n = \frac{E_n(u_2^n - u_1^n)}{L_n} + \sigma_0^n \quad (2.26)$$

and element forces are thus

$$F_n = \sigma_n A_n \quad (2.27)$$

2.3 Matching methods to model

2.3.1 Single spring assumption

All the methods of calculating fastener flexibility of Section 2.1 are based on a single spring assumption, meaning that the calculated result yields a total flexibility of the fastener. As apparent in the model of Figure 2.9, for the case of double shear, the fastener is represented by two spring elements, see Figure 2.11. This means that in order to use the flexibility calculation methods accurately, the resulting flexibility from the methods needs to be scaled by a factor of 2 according to Eq. (2.28) before inserted into the model (for the case of double shear). The described methods that this concerns are the ones by Huth and Barrois, since the Grumman formula was derived for single shear only.

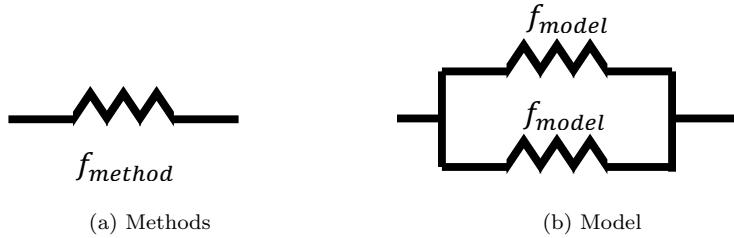


Figure 2.11: Representation of a fastener in double shear

$$f_{model} = 2 \cdot f_{method} \Leftrightarrow k_{model} = \frac{k_{method}}{2} \quad (2.28)$$

2.3.2 Influence of holes in calculating plate elongation

When calculating load distribution using the method by Barrois [2], an attempt is made to take into account the fact that there are holes in the plates (due to present fasteners), which in reality yields a larger compliance of the plates. This is done by scaling the theoretical plate elongation by an empirical factor α , see Eq. (2.32).

The theoretical plate elongation is

$$\Lambda = \frac{L}{EA} P \quad (2.29)$$

Barrois [2] suggested the empirical factor

$$\alpha = 1 + \frac{d}{L} \left[\left(\frac{1.3}{1 - \lambda} \right) - 1 \right] \quad (2.30)$$

where d is the hole diameter and

$$\lambda = \frac{w}{d} \quad (2.31)$$

with w being the plate strip width, i.e. the plate width divided by the number of fastener columns in the structure.

Thus, the plate elongation according to Barrois becomes

$$\Lambda_{\text{Barrois}} = \frac{L}{EA} P \alpha \quad (2.32)$$

The resulting plate stiffness as a function of the hole diameter is shown in Figure 2.12.

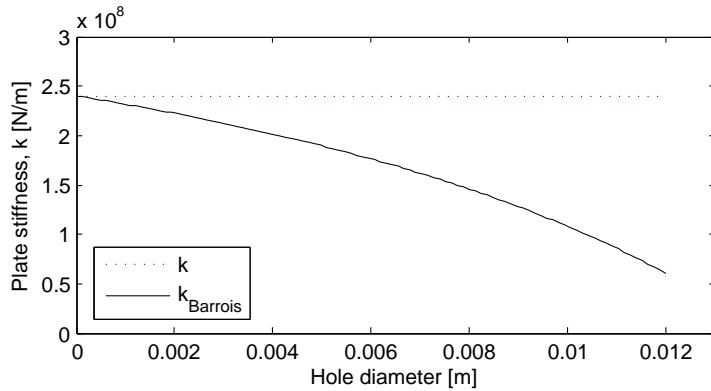


Figure 2.12: Plate stiffness k as a function of the hole diameter according to Barrois

Using α in the way Barrois intended requires that the fastener hole diameters on either side of each element are the same, due to α being a function of the diameter. For calculations on structures with varying fastener diameters, it is recommended to use either Grumman or Huth for single shear, and Huth only for double shear loaded fastener structures. Note that the methods by Huth and Grumman implicitly have taken this effect into account, since these formulas are empirically derived on fastened plates with actual holes.

It is recommended to use this term when using Barrios for fastener flexibility since this method is based on using it, and not for the other methods since they were not. If fastener flexibility is inserted manually, the choice is up to the user depending on how the flexibility was obtained.

2.3.3 Adjusting parameters to fastener sites

The calculation of fastener flexibility is performed using parameters at the fastener sites. Thus, if the plates have varying width or thickness, the parameters need to be adjusted since the model of Figure 2.9 accepts the average values of the parameters of the plate elements at either side of the fastener and not at the fastener sites. This is done by means of linear interpolation, using the parameters of Figure 2.13 and the graph of Figure 2.14.

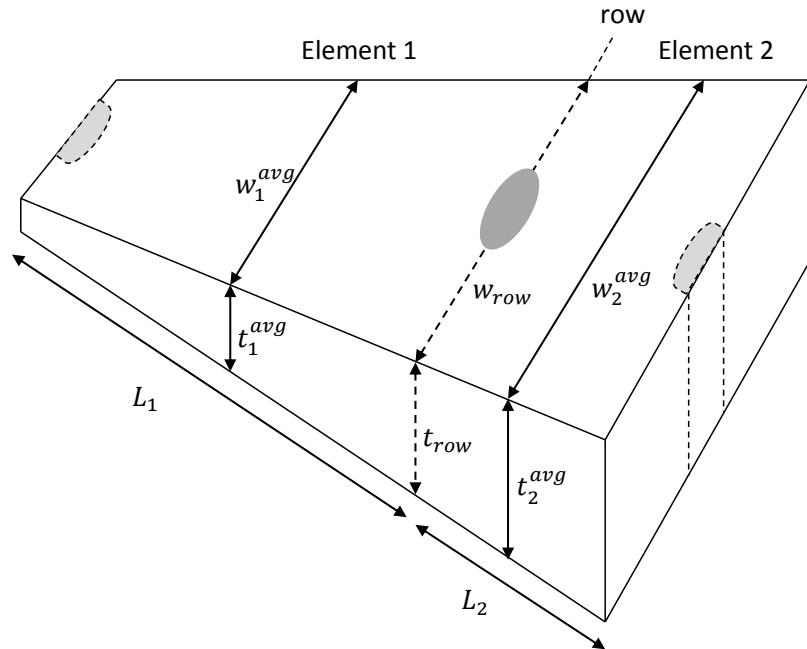


Figure 2.13: Interpolation of parameters

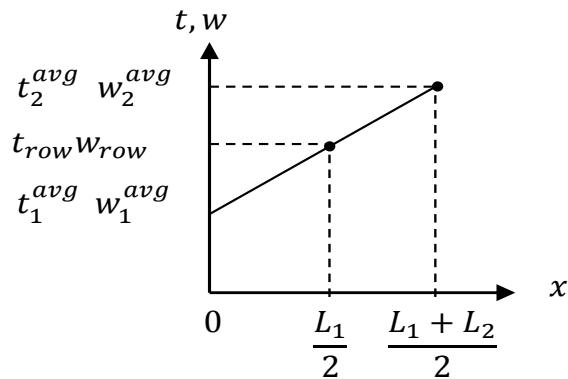


Figure 2.14: Interpolation procedure

The thickness is

$$t = \frac{t_2^{avg} - t_1^{avg}}{(L_2 + L_1)/2} x + t_1^{avg} \quad (2.33)$$

This yields the thickness at the faster site t_{row} as

$$t_{row} = \frac{t_2^{avg} - t_1^{avg}}{(L_2 + L_1)} L_1 + t_1^{avg} \quad (2.34)$$

and similarly, the width

$$w_{row} = \frac{w_2^{avg} - w_1^{avg}}{(L_2 + L_1)} L_1 + w_1^{avg} \quad (2.35)$$

It should be noted that this procedure is an attempt to allow for continuous changes in width and thickness as in Figure 2.13, but that the calculation actually interprets this as shown in Figure 2.15.

An example of a calculation that uses this theory is given in Section 5.3.

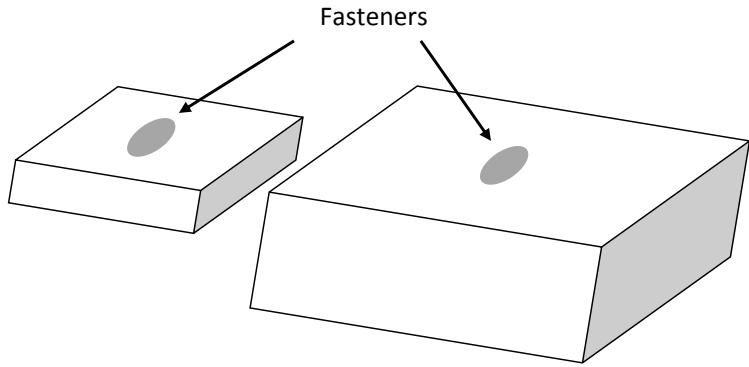


Figure 2.15: Calculation interpretation of parameter interpolation

2.3.4 Stiffness error from using step-wise varying plate dimensions

As stated in Section 2.3.3, the plates in the model assume discrete values for thickness and width of the plate elements. If these parameters actually vary, e.g. linearly as in Figure 2.13, then there will be an error in the calculated plate stiffness k_{avg} in comparison with the actual theoretical stiffness k_{true} .

With an element as in Figure 2.16, the calculated stiffness becomes

$$k_{\text{avg}} = \frac{1}{2} \frac{E(A_1 + A_2)}{L} = \frac{EA_{\text{avg}}}{L} \quad (2.36)$$

whereas, with $A(x) = A_1 + (A_2 - A_1) \cdot x/L$, and E and applied force P constant over the element length, the theoretical plate stiffness is obtained from

$$\begin{aligned} \delta_{\text{true}} &= \int_0^L \varepsilon(x) dx = \int_0^L \frac{\sigma(x)}{E} dx = \int_0^L \frac{P}{EA(x)} dx = \frac{P}{E} \int_0^L \frac{dx}{A(x)} \\ &= \frac{P}{E} \int_0^L \frac{dx}{A_1 + \frac{A_2 - A_1}{L}x} = \frac{P}{E \cdot \frac{A_2 - A_1}{L}} \left[\ln \left(A_1 + \frac{A_2 - A_1}{L}x \right) \right]_0^L \\ &= \frac{PL}{E(A_2 - A_1)} (\ln A_2 - \ln A_1) \end{aligned} \quad (2.37)$$

which gives

$$k_{\text{true}} = \frac{P}{\delta_{\text{true}}} = \frac{E(A_2 - A_1)}{L(\ln A_2 - \ln A_1)} \quad (2.38)$$

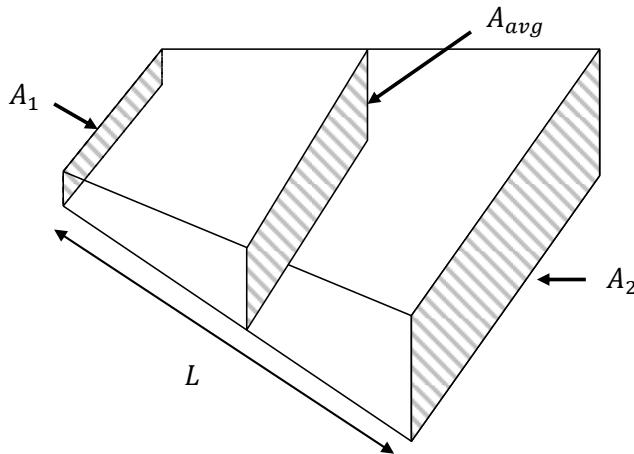


Figure 2.16: Plate element with linearly varying dimensions

With, for example, $E = 72$ GPa, $L = 30$ mm and $A_2 = 100$ mm 2 , the element stiffness values k_{avg} and k_{true} as a function of the ratio A_1/A_2 becomes as displayed in Figure 2.17a, and the relative error is seen in Figure 2.17b.

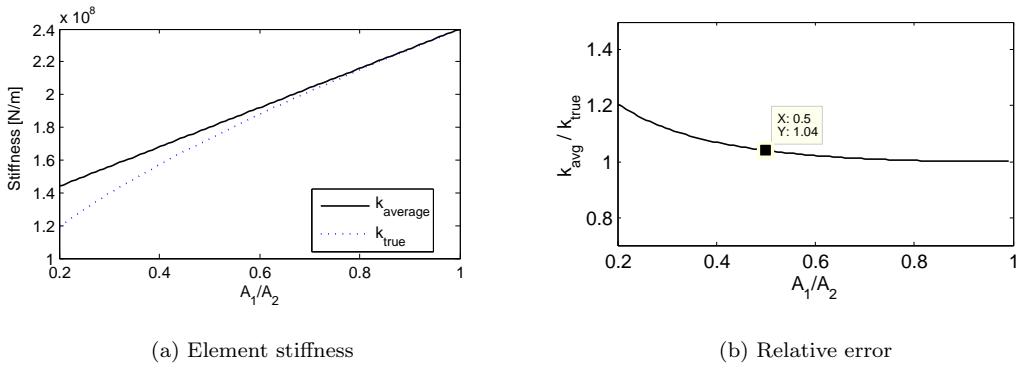


Figure 2.17: Discretization error on element stiffness

As seen by the relative error in Figure 2.17b, the error is very small for an area ratio down to about 0.5 (about 4% error), but then increases exponentially. Thus, for plates that have rapidly changing dimensions it is recommended to divide all plate elements that have a A_1/A_2 -ratio below 0.5 (or a higher value, if higher accuracy is desired) into several elements, such that all elements have a sufficiently low stiffness error.

Chapter 3

Effect of fastener flexibility on load distribution

That fastener flexibility is a factor that affects the load distribution in a bolted joint assembly is a fact that has been known for a long time. What can be interesting to discuss however, is how much the difference in flexibility from the different calculation methods (Huth, Grumman, and Barrois) impacts the load distribution, and consequently the question arises: Is it really necessary to use several different methods for calculating the fastener flexibility?

In order to address this question, the load distribution as a function of the fastener flexibility will be calculated in a simple joint geometry. Dimensions and parameters of the joint that affect the fastener stiffness calculations will be varied. The parameters that are common for the three methods of interest are the thicknesses of the plates, diameter of the fasteners, and the Young's modulus of the plates and the fasteners. To get a reasonable comparison between the methods, the proper version of each method needs to be used depending on what assumption they have been derived from.

How much do the results differ if an “incorrect” method is being used, for example if the user applies the double shear version of a method on a geometry that actually has single shear loaded fasteners.

3.1 Comparative setup

The Grumman formula was derived for single shear only and both Huth and Barrois support this kind of geometry. Therefore, this type will be used. The three-row geometry as seen in Figure 3.1 will be used during calculations. The examined cases are: composite and metallic plates, thick and thin plates, and thick and thin fasteners, with parameters varying according to Table 3.1.

The method by Barrois was derived under the assumption that the different plates had the same Young's modulus. Therefore, both plates will be of the same material. Plate strip width w , and plate element length L , is set to be 5 times the fastener diameter, and fastener Young's modulus E is set to 200 GPa (steel).

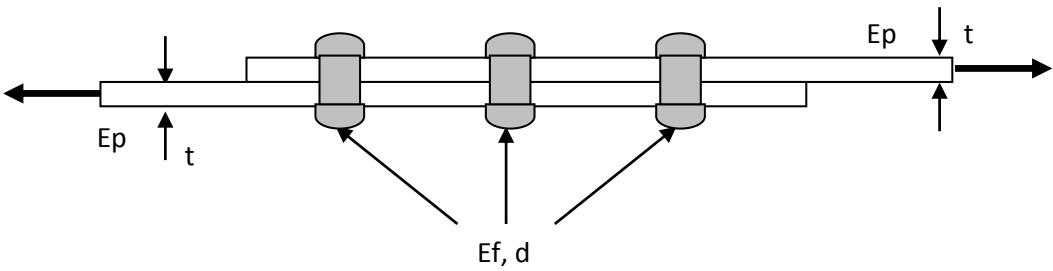


Figure 3.1: Three-row joint

For fastener flexibility calculation using Grumman's formula (Eq. 2.13), all parameters are defined in Figure 3.1. In the case of Huth (Eq. 2.14), $n = 1$ initially because of the geometry being of single shear.

Since the Grumman formula was used at Saab for bolted joints in composites, the comparable version of Huth is here said to be the case of bolted graphite/epoxy joint (see Table 2.1), and for Barrois the clamped head case (see Section 2.1.5).

| | E_p [MPa] | t [mm] | d [mm] |
|-----------|-------------|----------|----------|
| Case I | 69000 | 5 | 6 |
| Case II | 69000 | 5 | 12 |
| Case III | 69000 | 10 | 6 |
| Case IV | 69000 | 10 | 12 |
| Case V | 45000 | 5 | 6 |
| Case VI | 45000 | 5 | 12 |
| Case VII | 45000 | 10 | 6 |
| Case VIII | 45000 | 10 | 12 |

Table 3.1: Parameter variations for the different test cases

3.2 Using all method variations of calculating fastener flexibility

The program module will support the use of any variant of any method on any geometry - provided that sufficient parameters are defined. The Grumman formula for instance, is not supposed to be used in the case of double shear, and therefore it is relevant to investigate how much the results would differ if the user was to apply a method (or variant of a method) on a structure that is was not created for. The same geometry as defined above for Case I will be used, with method variations as described in Table 3.2.

| Method | Variation |
|-------------|--------------------------------|
| Huth I | Single shear, bolted metallic |
| Huth II | Single shear, riveted metallic |
| Huth III | Single shear, bolted graphite |
| Huth IV | Double shear, bolted metallic |
| Huth V | Double shear, riveted metallic |
| Huth VI | Double shear, bolted graphite |
| Barrois I | Single shear, clamped heads |
| Barrois II | Single shear, free heads |
| Barrois III | Double shear, clamped heads |
| Barrois IV | Double shear, free heads |

Table 3.2: Method variations

3.3 Results

Calculated results for the different cases can be seen in Figures 3.2a through 3.3d, where the calculated stiffnesses using the different methods are indicated, thus showing the resulting difference in load distribution using the different methods. An example of how the geometry affects the load distribution in the joint as a function of the fastener stiffness is shown in Figure 3.4. Fastener stiffness and the resulting row load distribution is displayed in Table 3.3 and Figure 3.5. Figure 3.6 shows the difference in resulting load distribution between variations of methods for the geometry of Case I. It should be noted that the stiffnesses are the same as inserted into the model, and does therefore for the double shear variations not directly coincide with the results from the methods, as described in Section 2.3.1.

| | Fastener stiffness [kN/mm] | | | Row 1 & 3 [%] | | | Row 2 [%] | | |
|-----------|----------------------------|------|-------|---------------|------|-------|-----------|------|-------|
| | Grum. | Huth | Barr. | Grum. | Huth | Barr. | Grum. | Huth | Barr. |
| Case I | 41.9 | 39.6 | 115.6 | 34.6 | 34.5 | 36.4 | 30.8 | 31.0 | 27.2 |
| Case II | 45.8 | 62.8 | 173.2 | 34.7 | 35.2 | 37.5 | 30.6 | 29.6 | 25.0 |
| Case III | 49.9 | 49.8 | 90.9 | 34.1 | 34.1 | 34.7 | 31.8 | 31.8 | 30.6 |
| Case IV | 83.8 | 79.1 | 231.1 | 34.6 | 34.6 | 36.3 | 30.8 | 30.8 | 27.4 |
| Case V | 28.3 | 27.2 | 89.7 | 34.7 | 34.6 | 36.8 | 30.6 | 30.8 | 26.4 |
| Case VI | 30.0 | 43.2 | 121.5 | 34.7 | 35.2 | 37.7 | 30.6 | 29.6 | 24.6 |
| Case VII | 38.8 | 34.2 | 72.1 | 34.2 | 34.1 | 35.0 | 31.6 | 31.8 | 30.0 |
| Case VIII | 56.5 | 54.4 | 179.3 | 34.6 | 34.6 | 36.7 | 30.8 | 30.8 | 26.6 |

Table 3.3: Fastener stiffness and row load distribution for the different cases

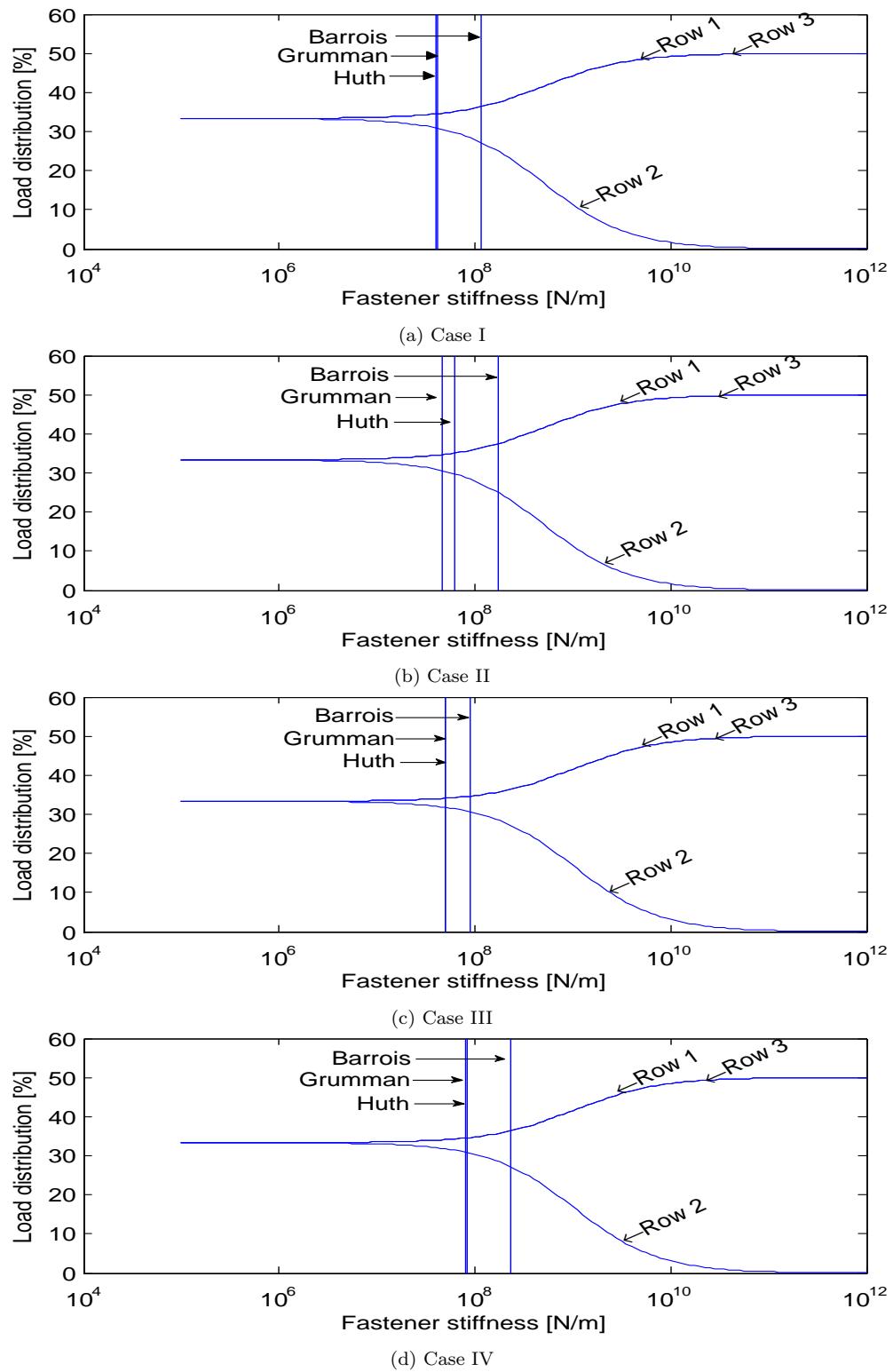


Figure 3.2: Fastener stiffness impact on load distribution, cases I-IV

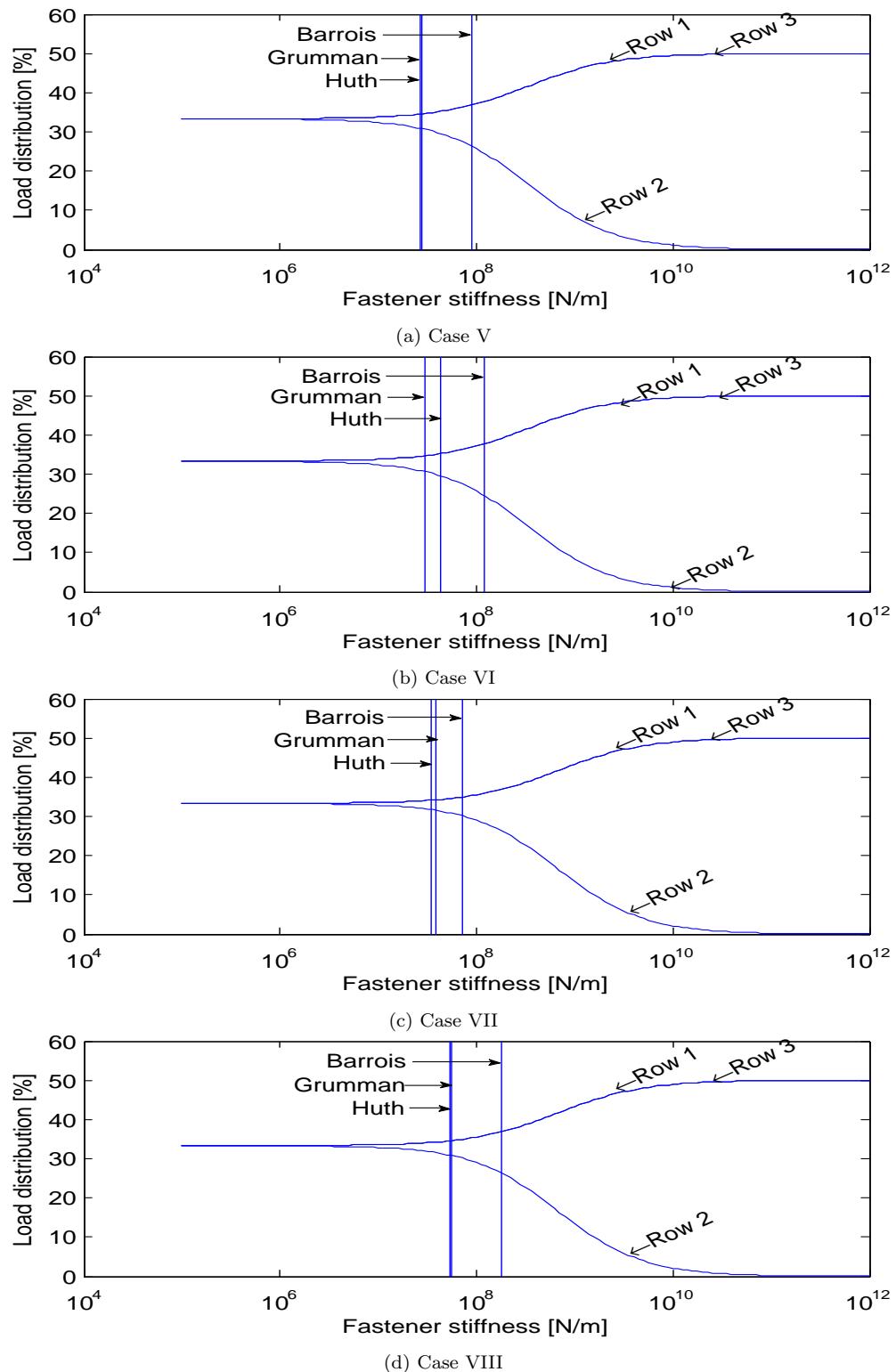


Figure 3.3: Fastener stiffness impact on load distribution, cases V-VIII

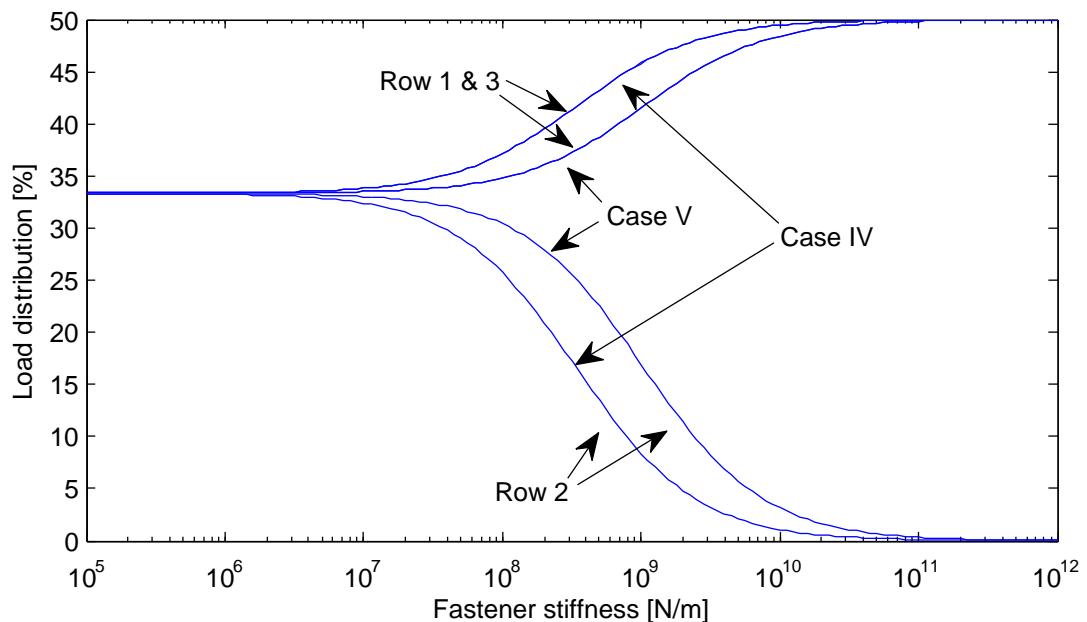


Figure 3.4: Influence of geometry on load distribution as a function of fastener stiffness

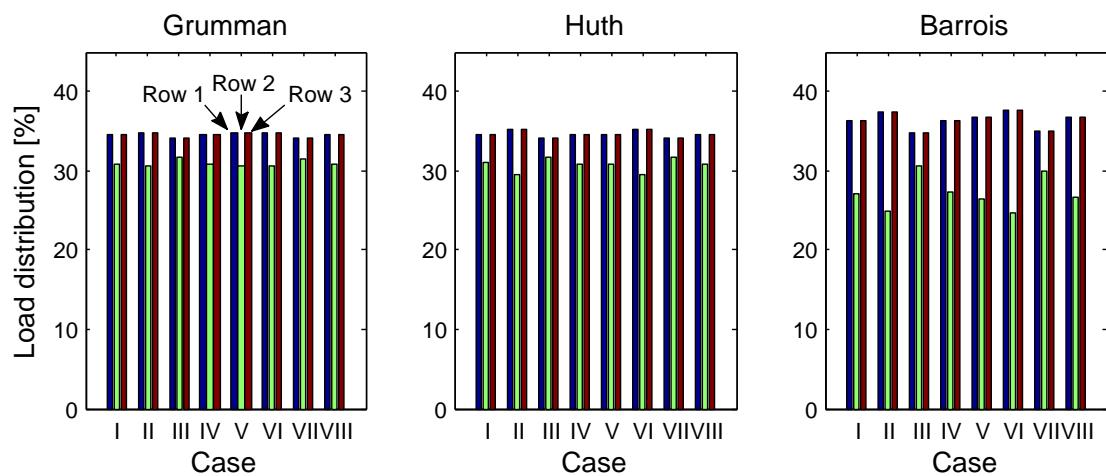


Figure 3.5: Row load distribution for the different cases

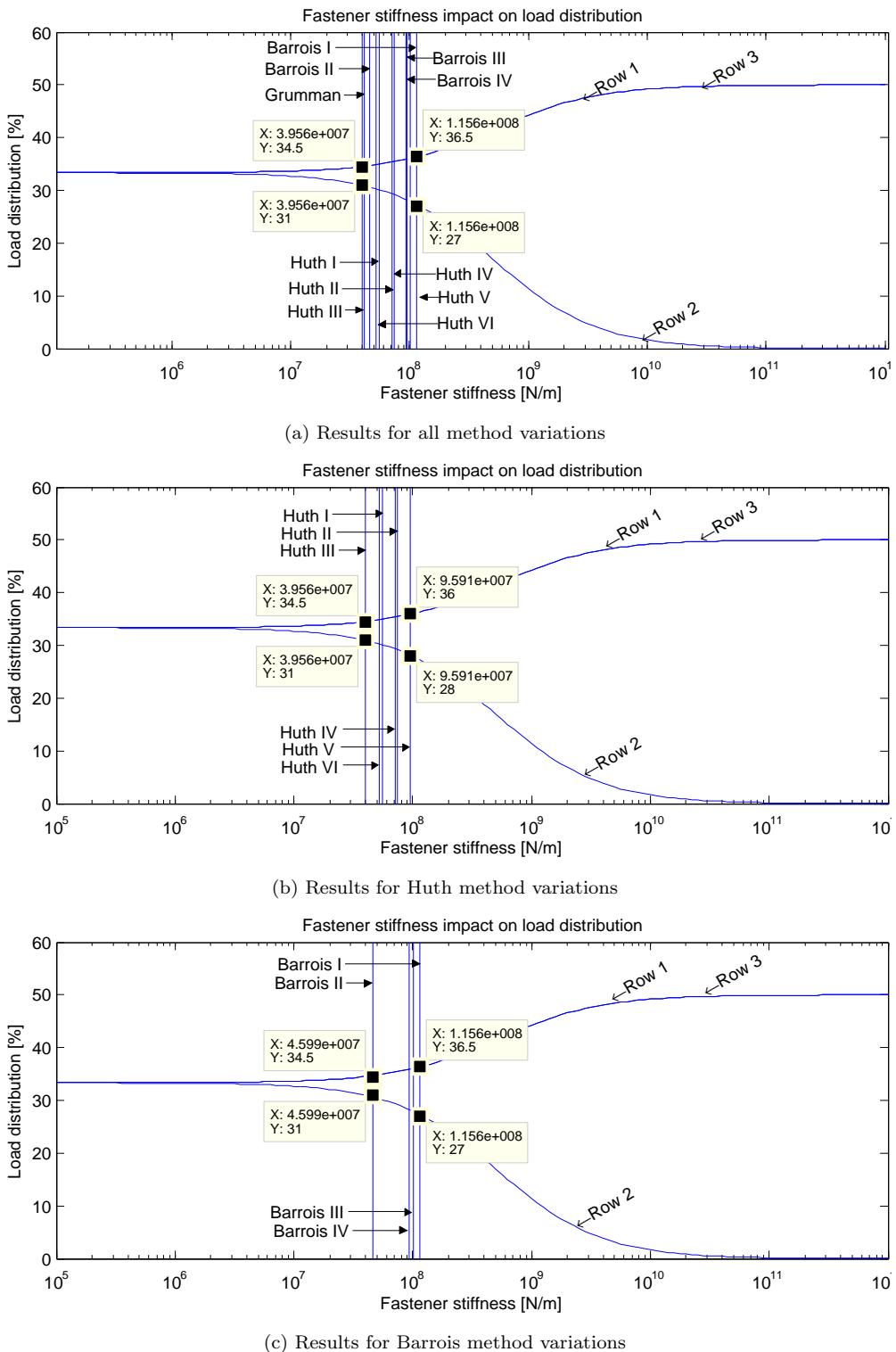


Figure 3.6: Result for the different method variations

3.4 Discussion

Looking at Figures 3.2 and 3.3 of the load distribution variation over the fastener stiffness, it is clear that each geometry case have a span of fastener stiffnesses that significantly affects the load distribution. For all cases, the results for Grumman and Huth lie in the beginning of this region where a small change in fastener flexibility has a low effect on the load distribution, thus yielding quite similar load distribution values for these cases. The method of Barrois, however, gives fastener stiffness values significantly higher than the others (see Table 3.3); up to about four times the stiffness obtained using the methods by Grumman or Huth. The load distribution variation between fastener rows is therefore greater with Barrois. However, the difference is no more than a few percent in the load distribution between the rows of fasteners.

The resulting load distributions, as seen in Figures 3.6, show large fluctuations between the different variations of the methods for calculating fastener flexibility. It can be seen that the double shear methods generally yield larger load distribution variation between the fastener rows, which is reasonable since double shear loaded fasteners usually are stiffer than single shear loaded fasteners. Thus, the method parameter that affects load distribution most appears to be if the fasteners are subjected to single or double shear loading. An exception to this pattern is the Barrois II variant (single shear, clamped heads), which is significantly stiffer than any other single shear variant. This is likely due to the assumption by Barrois that the fastener heads are *perfectly* clamped, which is physically unreasonable. Even well-tightened bolts have some ability to tilt and bend due to for example local deformations.

Due to the many assumptions and simplifications used in the method by Barrois, and that Grumman and Huth have been obtained from actual testing on specific types of joints, a qualified guess is that Grumman and Huth yields far more accurate results for structures similar to those they were derived for, than Barrois. It is however highly possible that the more general, analytical approach by Barrois can give better results for more arbitrary structures.

From the results obtained in this investigation, it appears as though the method of choice for calculating fastener flexibility is not very significant. Considering the simplifications in that the model is one-dimensional, and thus that it cannot for certain accurately include forces in more than one direction and cannot handle for example an offset between fastener rows (which is quite common in bolted joint assemblies). Furthermore, since the model does not in itself include phenomena such as friction and non-linear behavior, the few percent that differ in load distribution between the methods appears quite negligible.

As seen in Figure 3.4, the relationship between fastener flexibility and load distribution is greatly dependent on the geometry. For an arbitrary geometry, it is therefore very difficult to say how much the difference in calculated flexibilities, from the different methods, would impact the load distribution. Thus it cannot be said - in general - whether the method of choice for calculating fastener flexibility is significant or not.

It is essential to notice though, that none of the calculated fastener stiffnesses can be said to be neither accurate nor false, since each of the formulas used have been designed using individually specific geometries, parameters and simplifications as described in Section 2.1. In order to get the results that would be likely to be in closest agreement with reality, it is recommended that the user attempts to notice which of the methods have been derived using a structure with similar properties as the one being used, and therefore would correspond best to the current situation.

If a fitting method cannot be easily identified for the geometry in question, and the fastener flexibility is not known beforehand, it is recommended to try and obtain the fastener flexibility in another way. If the user has access to proper finite element software, it could be a good idea to consult the work by Gunbring [6], in which methods for obtaining fastener flexibilities from 3D models are described.

Chapter 4

Finding fastener flexibility from experimental displacements

As previously discussed, the different methods for calculating fastener flexibility yield varying results because of their individual origin of derivation. In order to make an assessment of the validity of the model and the implemented methods of Sections 2.1 and 2.2, a comparison with experimental data will be made.

From the attained fastener displacements during experiments of tension loaded bolt joint assemblies it is possible to acquire the fastener stiffness that, inserted into the model of Figure 2.9, yield a similar displacement of the fasteners. Comparing this stiffness to that obtained using the implemented methods, can give a hint of the validity of the model.

4.1 Experimental displacements

In 1982, Sjöström [13] measured displacements for different geometries and load cases according to Figure 4.1. Measurements were conducted on the bolt row denoted with an asterisk (*) for the different geometries in Figure 4.1. For the geometries 1, 2, 3 and 5; forces and displacements where recorded according to Figure 4.2 and Table 4.1b with parameters as defined in Table 4.1a.

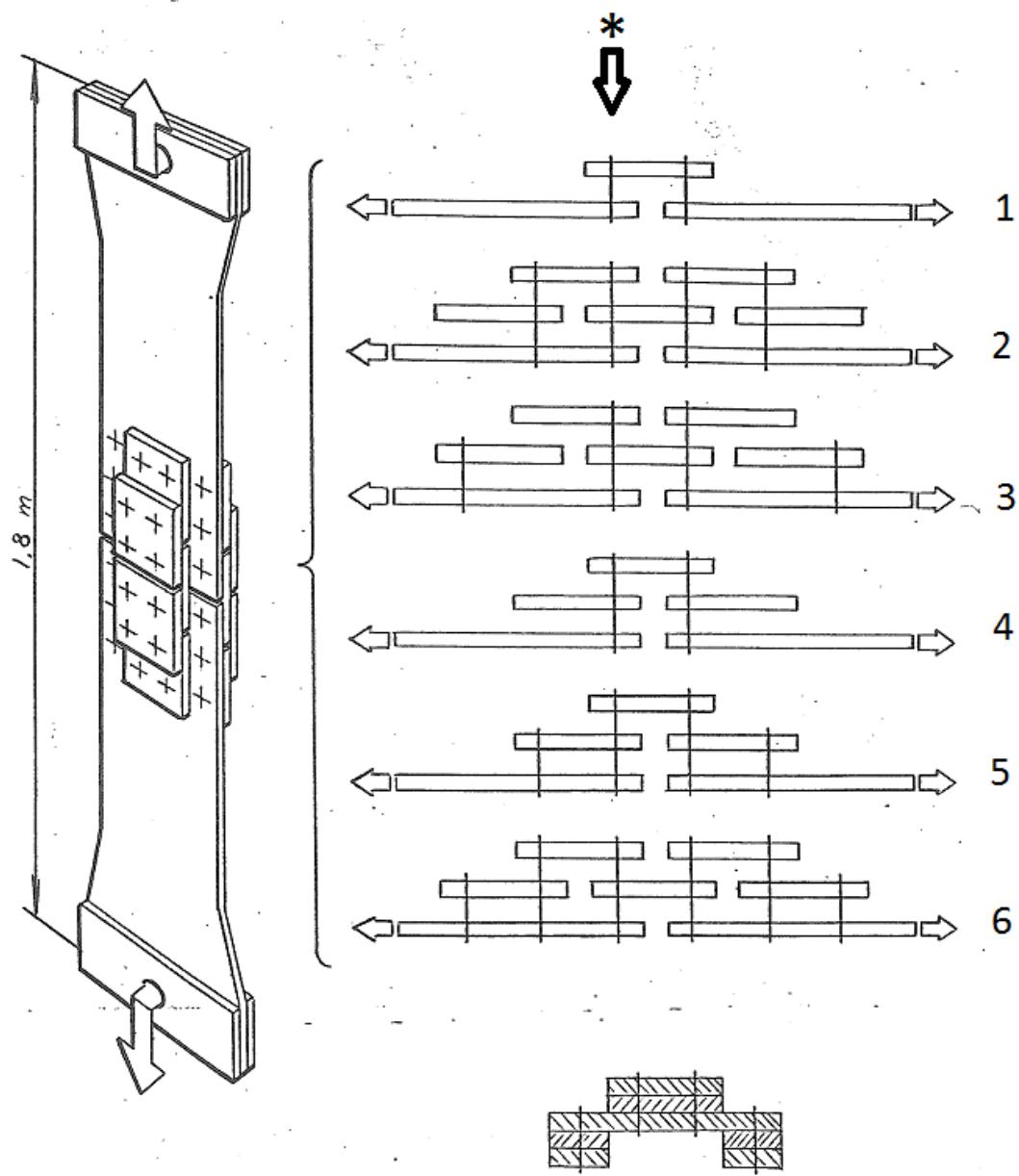
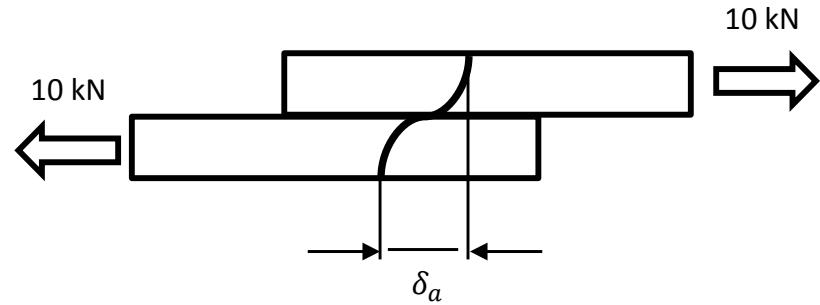
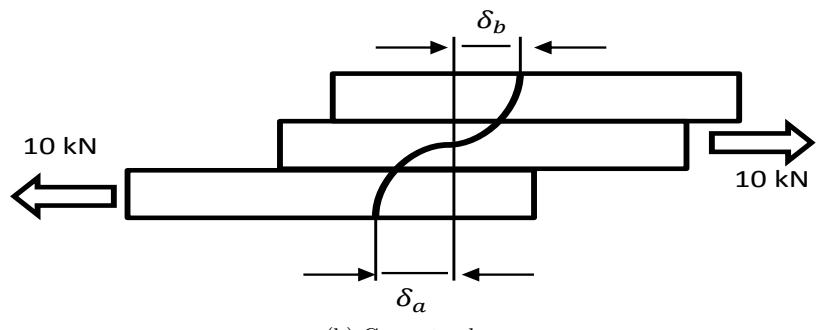


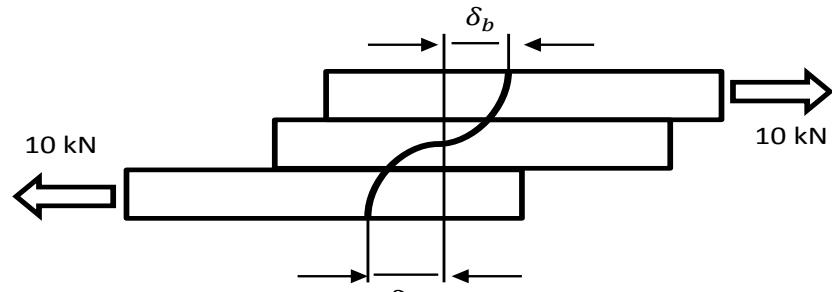
Figure 4.1: Experimental setup by Sjöström, picture from Sjöström [13]



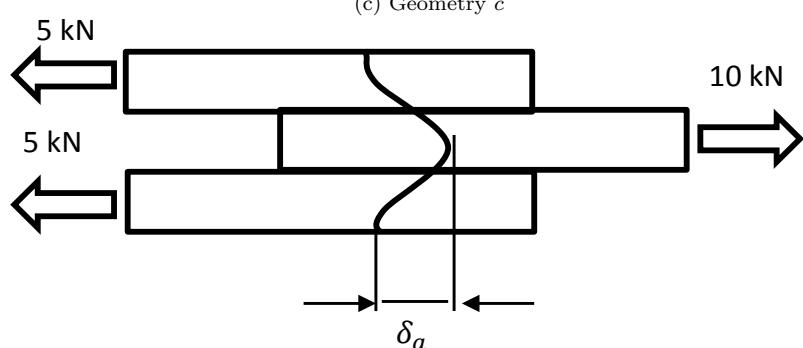
(a) Geometry *a*



(b) Geometry *b*



(c) Geometry *c*



(d) Geometry *d*

Figure 4.2: Sjöström [13] measurements

| Parameter | Value |
|------------------------|---------|
| E_{plates} | 71 GPa |
| $E_{\text{fasteners}}$ | 206 GPa |
| d | 12 mm |
| t | 8 mm |
| w | 48 mm |

(a) Parameters

| Geometry | δ_a [mm] | δ_b [mm] |
|----------|-----------------|-----------------|
| a | 0.138 | - |
| b | 0.077 | 0.027 |
| c | 0.104 | 0.104 |
| d | 0.025 | - |

(b) Displacements

Table 4.1: Parameters used, and displacements obtained by Sjöström

4.2 Results

Fastener stiffnesses that - inserted in the model of Figure 2.9 - yield displacements similar to those found by Sjöström (Table 4.1b) are found in Table 4.2. Results from using the different methods of calculating fastener stiffness that would correspond best to each of the four Sjöström geometries are presented in Table 4.3. The variations that were deemed most fitting were for Huth the bolted metallic and for Barrois the clamped head setup, with single shear for geometries $a-c$ and double shear for geometry d .

| Geometry | k [kN/mm] | δ_a [mm] | δ_b [mm] |
|----------|-------------|-----------------|-----------------|
| a | 72.5 | 0.138 | - |
| b | 130 | 0.077 | 0 |
| c | 95 | 0.105 | 0.105 |
| d | 200 | 0.025 | - |

Table 4.2: Displacements for the Sjöström geometries using the model of Figure 2.9

| Geometry | Method | k [kN/mm] | δ_a [mm] | δ_b [mm] |
|----------|---------|-------------|-----------------|-----------------|
| a | Grumman | 72.5 | 0.138 | - |
| | Huth | 106 | 0.095 | - |
| | Barrois | 221 | 0.045 | - |
| b | Grumman | 72.5 | 0.138 | 0 |
| | Huth | 106 | 0.095 | 0 |
| | Barrois | 221 | 0.045 | 0 |
| c | Grumman | 72.5 | 0.138 | 0.138 |
| | Huth | 106 | 0.095 | 0.095 |
| | Barrois | 221 | 0.045 | 0.045 |
| d | Grumman | 72.5 | 0.138 | - |
| | Huth | 141 | 0.095 | - |
| | Barrois | 174 | 0.045 | - |

Table 4.3: Displacements for the Sjöström geometries using the different fastener flexibility methods

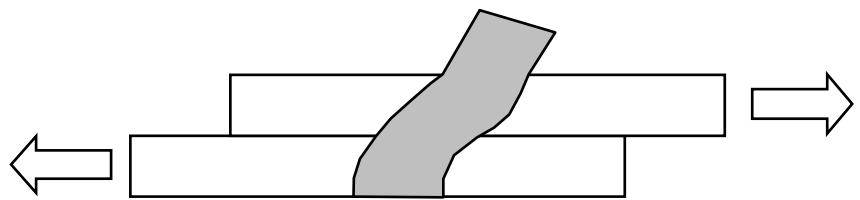
4.3 Discussion

From Tables 4.2 and 4.3 we see that the resulting displacement using the Grumman formula is in excellent agreement with the measured displacements for the simple two-plate single shear geometry *a*. For geometry *c*, Huth yields a displacement closest to measurements, and for the double shear geometry *d*, Barrois is in closest agreement with experimental values. Not surprisingly, using the Grumman formula on the latter geometry gave the worst result (since Grumman is a method developed for single shear only). Note that the stiffness values for Huth and Barrois on this geometry are as inserted into the model, and thus only half the value as that obtained from the formulas.

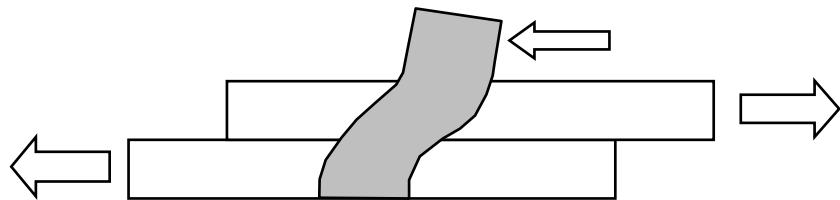
From the results for the geometries *a*, *c*, and *d*, it cannot be said that either of the methods is more accurate than the other due to the fact that each method yields a displacement in closest agreement with measurements for one of the geometries.

A striking result for these tests when looking at Tables 4.2 and 4.3 is that for geometry *b* the model does not yield any displacement δ_b as present from measurements. As Sjöström [13] concluded, this displacement of the upper plate and the increased stiffness of the structure is due to the fastener tilt. The upper plate obstructs the tilting, thus increasing the stiffness. An attempt to illustrate this is displayed in Figure 4.3. This has a major impact on the validity of the model - being that for an arbitrary structure, it cannot be claimed that the model yields realistic results. The reason that the model cannot take this fastener tilt into account is that it divides the fastener into elements that are treated individually, i.e. there is no realistic coupling between one part of the fastener and another for the fastener tilt.

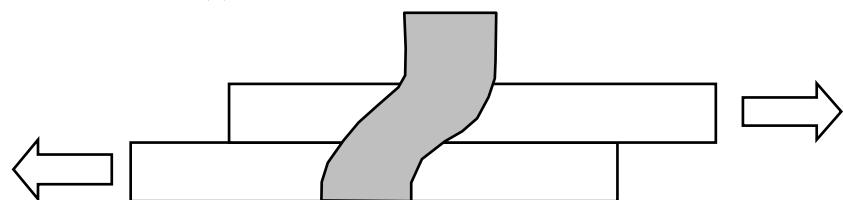
There are ways that this fastener tilt can be taken into account. Sjöström [13] for instance, suggested a model where a torsion spring simulates this tilting, see Figure 4.4. This approach was only investigated for the loading conditions of the three-layer geometry *b*. Sjöström's model could be expanded to any amount of joined plates. This would however require an increasing amount of testing with increasing number of plates, since for each added plate there are new load combinations that the fastener can be subjected to; each of these cases yielding different behavior of the fastener.



(a) Fastener reaction without upper plate



(b) Fastener reaction with upper plate present



(c) Fastener reaction with upper plate in model

Figure 4.3: Fastener tilt

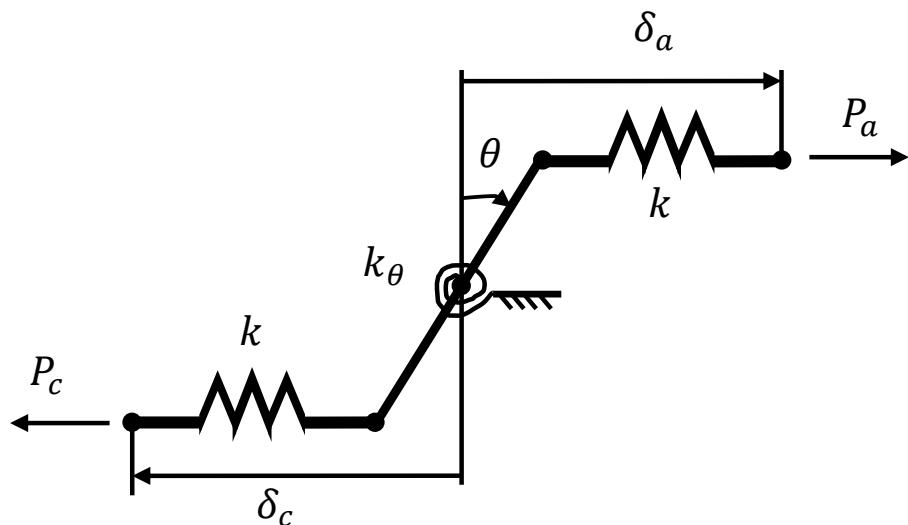


Figure 4.4: Model with a torsion spring

Chapter 5

Dimensioning tool DIM

5.1 DIM

As previously described, DIM is a structural sizing tool developed by Saab that incorporates the total analysis chain from global finite element analysis, through analyses with respect to for example buckling, fatigue and load distributions, to writing a report.

Currently, the version of DIM under development has the layout of Figure 5.1.

A new version of DIM is under development, with the goal of simplifying the ways of implementing new modules. Thus, it should be enough with only a basic knowledge of Matlab coding in order to create new modules, and little understanding of how DIM itself works should be required. In DIM, geometries and calculations are separate classes, allowing for several kinds of calculations applying to a specific geometry - and vice versa. The geometry module created for this thesis work is named BoltedJoint1D, and the calculation is called BoltDistribution1D.

It should be pointed out that since DIM is under constant development, the created module may be subject to change.

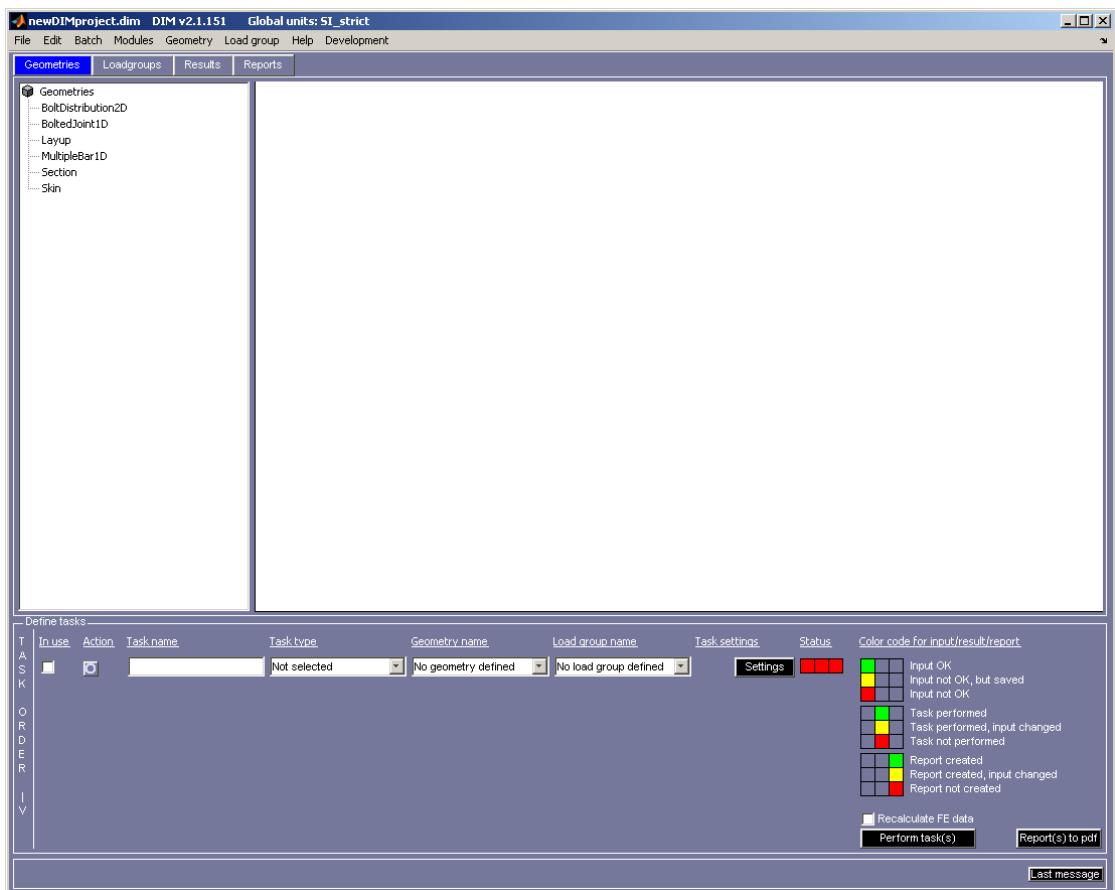


Figure 5.1: Starting window of DIM as of June 5, 2012

5.2 The developed program module

The main point of the module is to be able to perform calculations on bolted joint assemblies using the methods by Grumman, Huth and Barrois as described in Chapter 2, and to obtain results that are in accordance with these methods.

5.2.1 Geometry module description

Upon start of the BoltedJoint1D geometry module, a simple default geometry is generated. The starting window is seen in Figure 5.2. It is divided logically into five parts, or panels, namely

1. Geometry layout panel
2. Parameters panel
3. Tables panel
4. Plot panel
5. Information panel

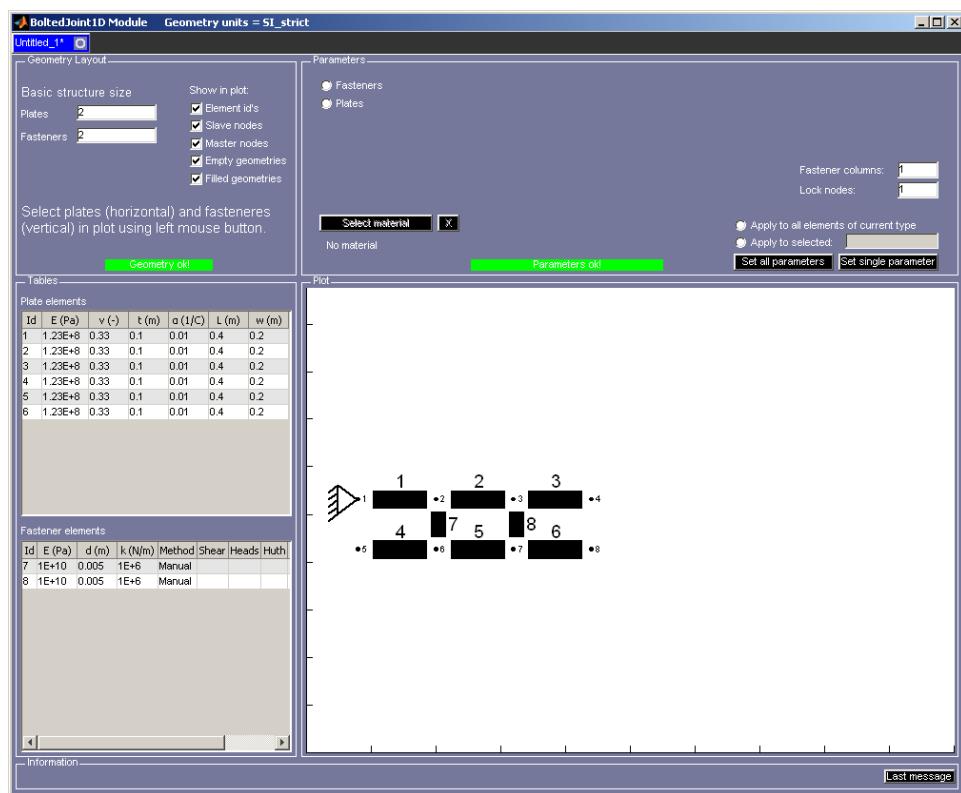


Figure 5.2: Starting window of BoltedJoint1D geometry module

Geometry layout panel

The geometry layout panel is the startoff-point when defining a new geometry; as the ‘Basic structure size’ determines the size of the structure that the user will be able to work with or, simply put, the number of rectangles in the plot panel, see Figures 5.2 and 5.3. Here, the user also has the option to determine what items of the geometry should be displayed in the plot window.

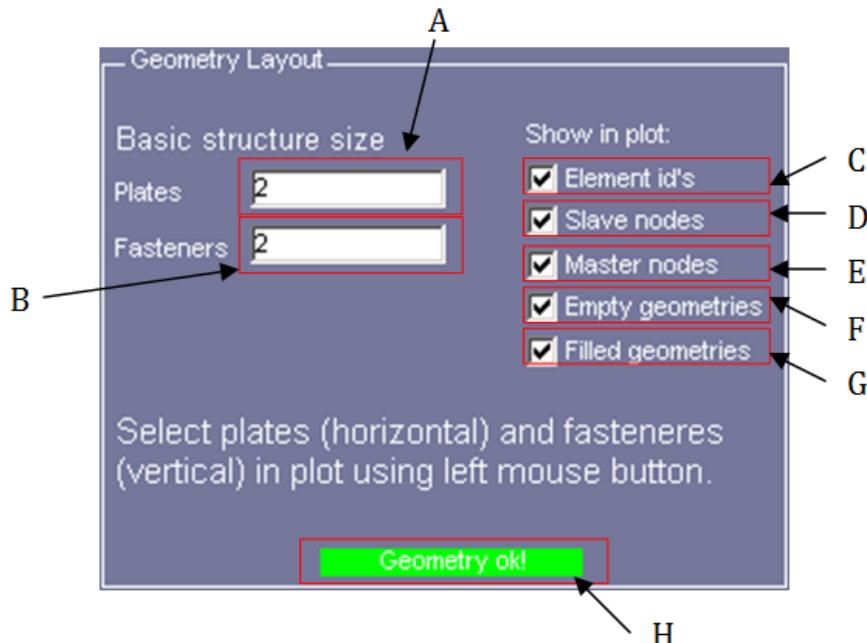


Figure 5.3: Geometry layout panel

The dynamic objects of this panel (objects that the user in one way or the other can change, marked in Figure 5.3) and their functions are

- Define number of plate layers that the user can create the structure from. Accepts as input an integer or numerical expression that yields an integer.
- Define number of fastener columns that the user can create the structure from. Accepts as input an integer or numerical expression that yields an integer.
- Hide/show element numbers of the geometry in the plot panel.
- Hide/show the nodes of the geometry that are not allowed to be locked and no forces can be applied to in the plot panel.
- Hide/show the nodes of the geometry that can be locked and that forces may be applied to in the plot panel.
- Hide/show the parts of the structure that has not been defined as part of the geometry in the plot panel.
- Hide/show the parts of the structure that has been defined as part of the geometry in the plot panel.

- H) Display geometry layout status. If the defined geometry (in the plot window) fulfills requirements (see Section 5.2.6), this is displayed in green and if not, this is displayed in red, as seen in Figure 5.4.



Figure 5.4: Displaying geometry layout status

Parameters panel

The next step after defining the available size of the structure is defining the geometry. This is described in the ‘Plot panel’-section. Assuming that a geometry has been defined, the next step is defining the parameters of the geometry, which is done in the parameter panel. Initially, its structure is as displayed in Figure 5.5.

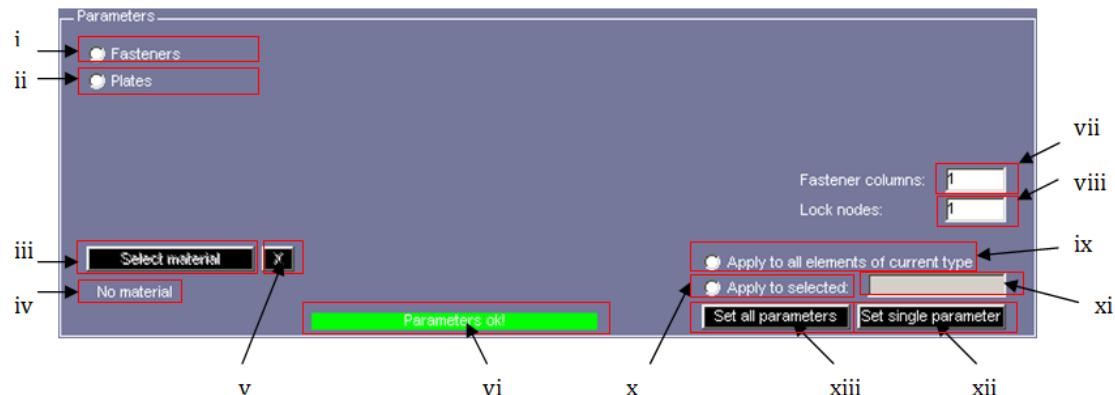


Figure 5.5: Parameter panel

The dynamic objects of this window (objects that the user in one way or the other can change, highlighted in Figure 5.5) and their functions are

- i) Define parameters for fastener elements.
- ii) Define parameters for plate elements.
- iii) Select from available materials.
- iv) Display the name of the material, if one has been chosen.
- v) Remove chosen material.
- vi) Show the status of the defined parameters. If the parameters defined fulfill parameter requirements (see Section 5.2.7), this is displayed in green and if not, this is displayed in red, see Figure 5.6.

- vii) Define number of fastener columns in the structure.
- viii) Select which node(s) of the geometry in the plot panel that should be constrained from motion. Required input is an integer or an array of integers, which can be written as would any numerical array in Matlab. Example,
input: 1, 3:7 9
resulting element array: [1 3 4 5 6 7 9]
- ix) Apply defined parameters to all elements of current type (plate/fastener).
- x) Apply defined parameters to the elements defined in xi).
- xi) Define element(s) that parameters will be assigned to. Required input is an integer or an array of integers, which can be written as would any numerical array in Matlab (see viii)
- xii) Assign most recently changed parameter (indicated in light blue color, see Figure 5.7) to the selected elements as defined by ix), or x) and xi).
- xiii) Assign all parameters to the selected elements as defined by ix), or x) and xi).



Figure 5.6: Displaying parameter status

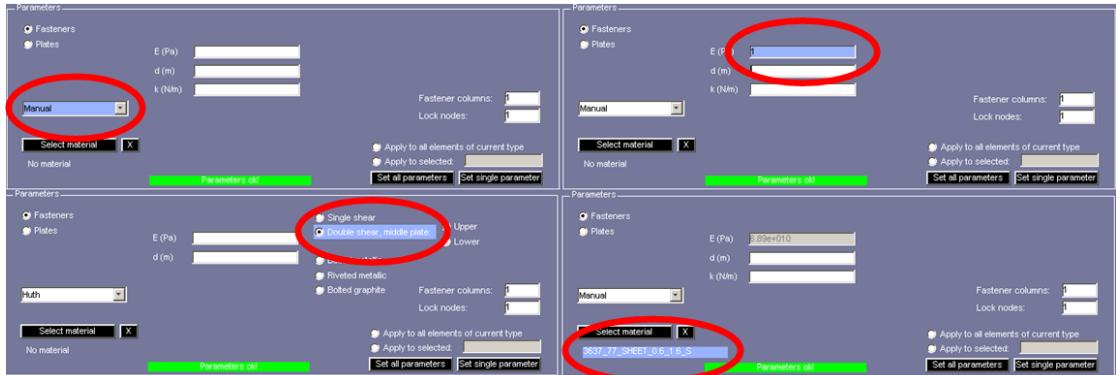


Figure 5.7: Displaying latest changed parameter

Upon selection of fasteners using i), the user is allowed to select with which method the fastener flexibility shall be calculated (see Figure 5.8). Following this, various options are displayed depending on the method of choice as shown in Figure 5.9.

- xiv) Select fastener flexibility calculation method. Currently, the available methods are: Manual, Grumman, Huth, and Barrois.
- xv) Define Young's modulus for fastener element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.

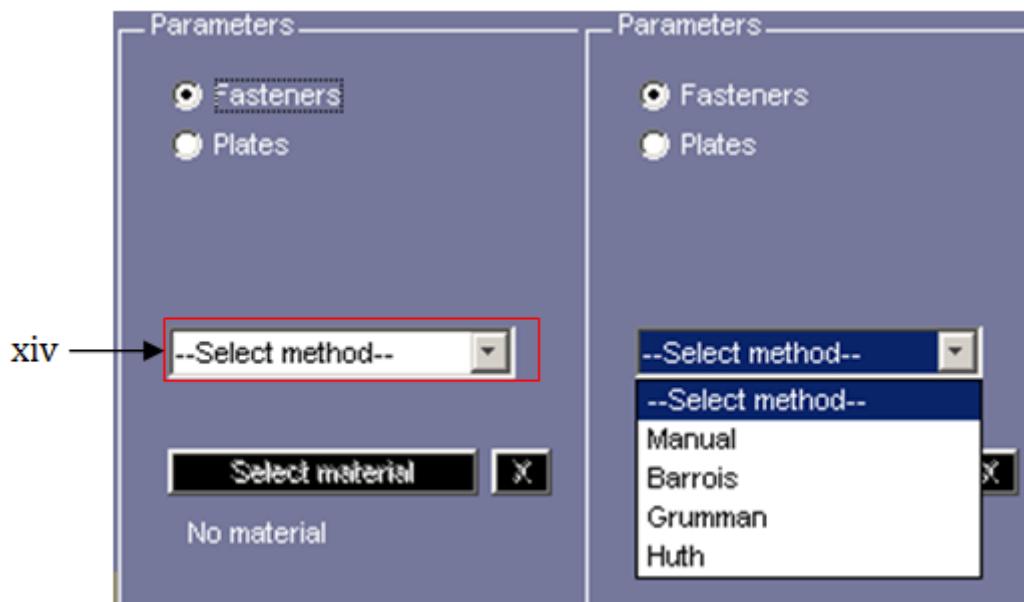


Figure 5.8: Choosing method for calculating fastener flexibility

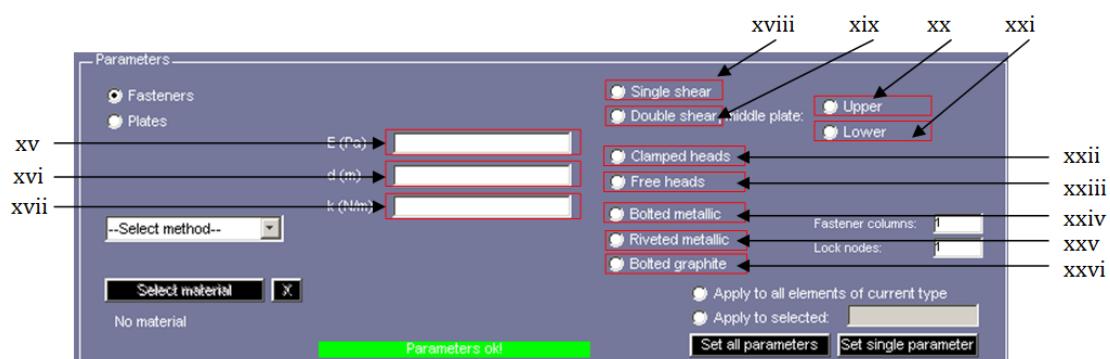


Figure 5.9: Parameter panel with all fastener dynamic objects visible

- xvi) Define diameter of fastener element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.
- xvii) Define stiffness of fastener element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.
- xviii) Select single shear configuration. Visible if any of the methods Huth or Barrois has been chosen.
- xix) Select double shear configuration. Visible if any of the methods Huth or Barrois has been chosen.
- xx) Define upper plate relative to the fastener element(s) to be the middle plate in double shear. Visible if double shear is selected.
- xi) Define lower plate relative to the fastener element(s) to be the middle plate in double shear. Visible if double shear is selected.
- xxii) Select clamped fastener heads. Visible if Barrois' method is chosen.
- xxiii) Select simply supported (free) fastener heads. Visible if Barrois' method is chosen.
- xxiv) Select joint type to be bolted metallic. Visible if Huth's method is chosen.
- xxv) Select joint type to be riveted metallic. Visible if Huth's method is chosen.
- xxvi) Select joint type to be bolted graphite/epoxy. Visible if Huth's method is chosen.

Upon selection of plates using ii), the parameter panel is displayed as seen in Figure 5.10.

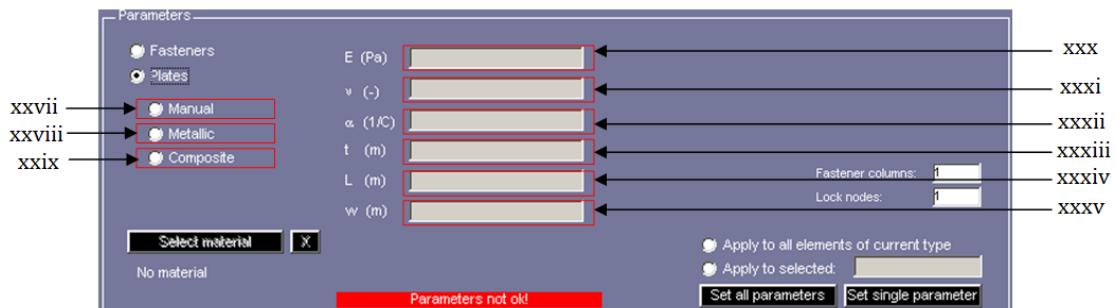


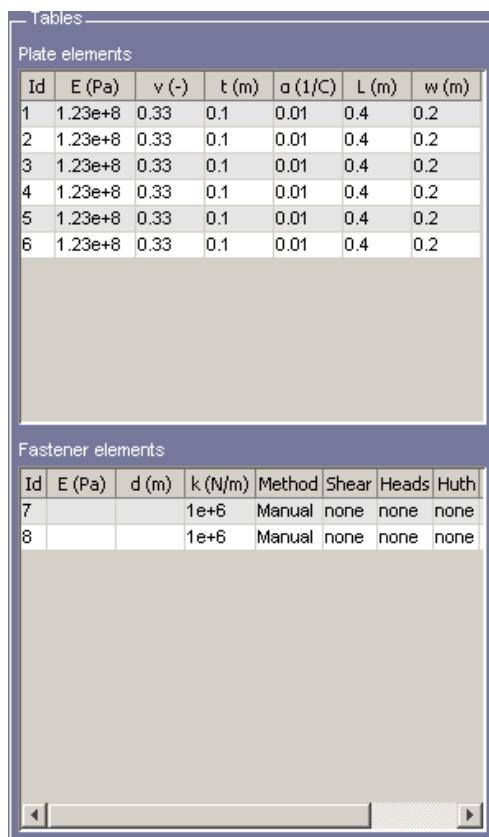
Figure 5.10: Parameter panel with all plate dynamic objects visible

- xxvii) Select to input plate parameters manually.
- xxviii) Select plate material to be metallic (chosen from 'Select material', iii)
- xxix) Select plate material to be composite (chosen from 'Select material', iii)
- xxx) Define Young's modulus of plate element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.

- xxxii) Define Poisson's ratio of plate element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.
- xxxiii) Define coefficient of thermal expansion of plate element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.
- xxxiv) Define thickness of plate element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.
- xxxv) Define length of plate element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.
- xxxvi) Define width of plate element(s). Accepts any numerical input or numerical expression that can be evaluated by Matlab.

Tables panel

The Tables panel of Figure 5.11 is used only to give overview of the assigned parameters, to help the user in keeping track on which parameters have been assigned to what element and which parameters still need to be defined. As such, no object in this panel is editable, however the values update simultaneously with the assignment of parameters and it is possible to select the values in the tables, copy them and insert into text-documents, if desired.



The screenshot shows the 'Tables' panel with two tables:

| Plate elements | | | | | | |
|----------------|---------|-------|-------|----------------|-------|-------|
| Id | E (Pa) | v (-) | t (m) | α (1/C) | L (m) | w (m) |
| 1 | 1.23e+8 | 0.33 | 0.1 | 0.01 | 0.4 | 0.2 |
| 2 | 1.23e+8 | 0.33 | 0.1 | 0.01 | 0.4 | 0.2 |
| 3 | 1.23e+8 | 0.33 | 0.1 | 0.01 | 0.4 | 0.2 |
| 4 | 1.23e+8 | 0.33 | 0.1 | 0.01 | 0.4 | 0.2 |
| 5 | 1.23e+8 | 0.33 | 0.1 | 0.01 | 0.4 | 0.2 |
| 6 | 1.23e+8 | 0.33 | 0.1 | 0.01 | 0.4 | 0.2 |

| Fastener elements | | | | | | | |
|-------------------|--------|-------|---------|--------|-------|-------|------|
| Id | E (Pa) | d (m) | k (N/m) | Method | Shear | Heads | Huth |
| 7 | | | 1e+6 | Manual | none | none | none |
| 8 | | | 1e+6 | Manual | none | none | none |

Figure 5.11: Tables panel

Plot panel

As seen in Figure 5.12, the Plot panel is used to display the structure. Filled black rectangles are elements part of the chosen geometry, whereas empty rectangles with black outlining are not. Left-clicking on a rectangle toggles it between being part of the geometry or not. Large and small numbers are element and nodal numbers, respectively. When nodes for locking have been chosen, this is displayed, in this case for node 1.

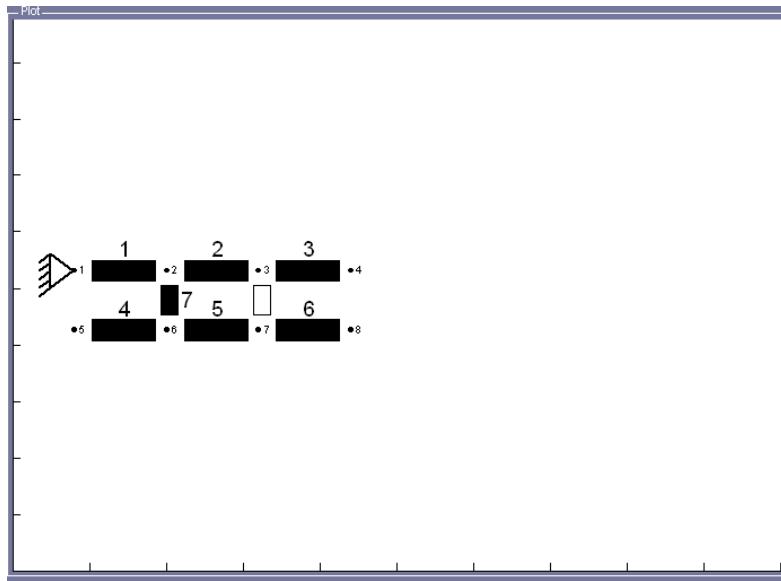


Figure 5.12: Plot panel

Information panel

The Information panel is used to display messages to the user (error, warning, information) in order to convey information regarding the status of the task at hand, see Figure 5.13.



Figure 5.13: Information panel

5.2.2 Load group module description

There are different load group modules for different kinds of geometries and tasks found under the 'Load group'-tab in Figure 5.1. Two can be used together with BoltedJoint1D; the one for force in the x -direction (Figure 5.14), and the one for temperature differences (Figure 5.15). Forces may only be applied to nodes at the ends of the plates, and temperature loads to plate elements.

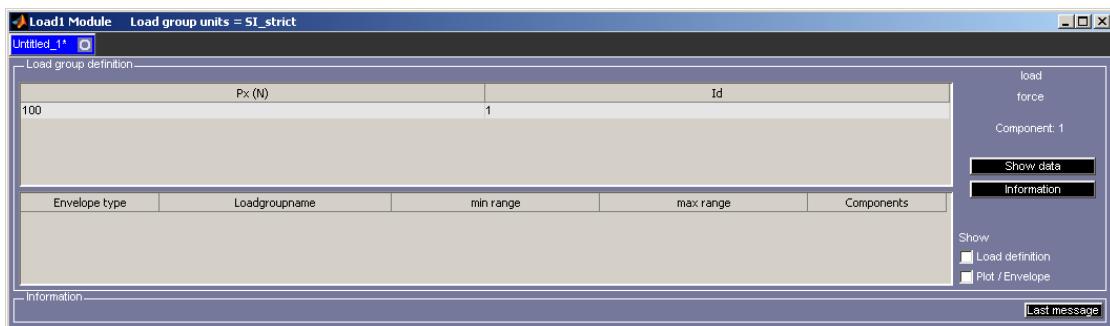


Figure 5.14: Load group module for force in the x-direction

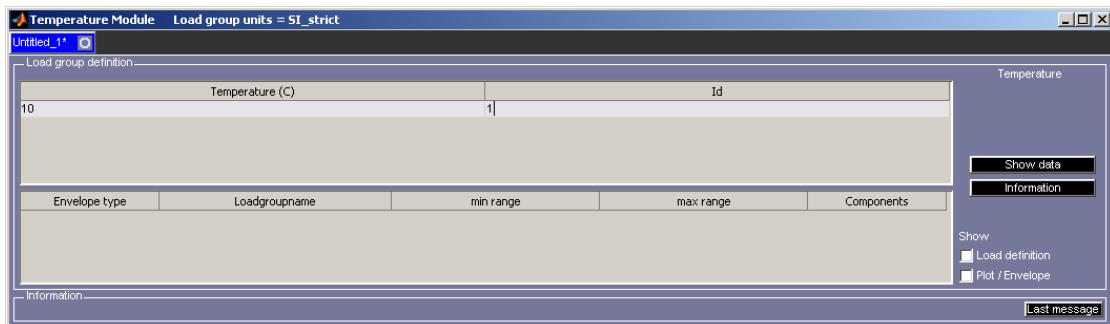


Figure 5.15: Load group module for temperature differences

5.2.3 Connecting loads to geometry

When geometry and suitable load groups have been defined, a task must be defined. This is done in the ‘Define tasks’ panel of Figure 5.1. If the task type is chosen to be BoltDistribution1D (which is currently the only available task type for the BoltedJoint1D geometry), a button appears beneath ‘Load group name’ that allows the user to connect the geometry to the loads as seen in Figure 5.16.

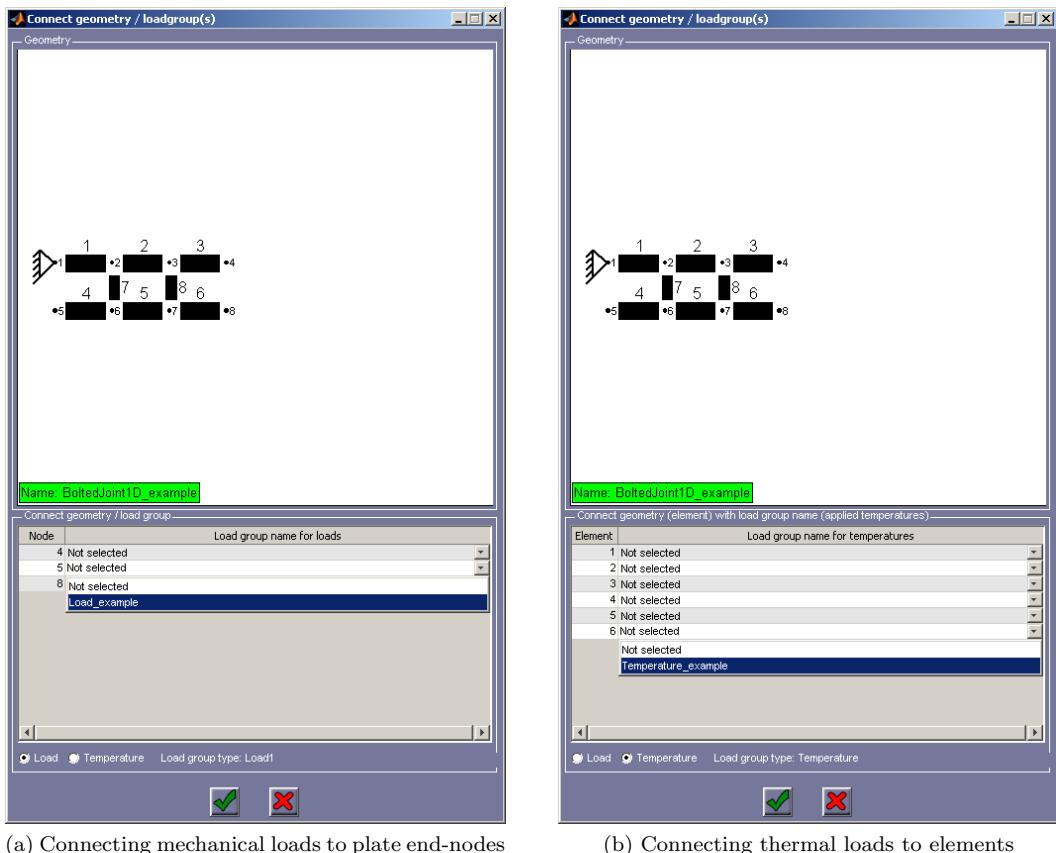


Figure 5.16: Geometry-load connection

5.2.4 Calculation

Calculated properties

The calculation yields the following results

1. Fastener flexibilities
2. Initial nodal forces
3. Nodal displacements
4. Element forces
5. Transfer loads
6. Bearing loads
7. Bypass loads
8. Load distribution (only for simple geometries)

The properties 1 through 4 are directly obtained from the finite element calculation. The transfer loads are simply the forces of the fastener elements. The bearing and bypass loads, defined in Figure 2.1, are calculated according to Eqs. (5.1) and (5.2), with forces acting at a plate-fastener conjunction according to Figure 5.17. The transfer loads and bypass loads are both used during analysis of composite laminates in bolted joints.

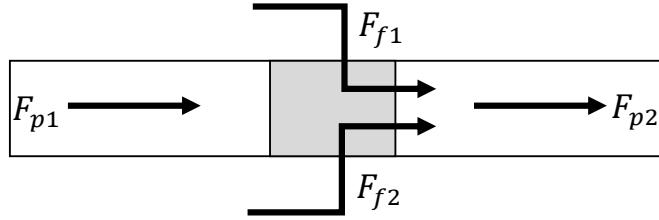


Figure 5.17: Bearing and bypass loads at a plate-fastener conjunction

$$F_{BR} = F_{f1} + F_{f2} = F_{p2} - F_{p1} \quad (5.1)$$

$$F_{BP} = \begin{cases} F_{p2}, & F_{p1} > F_{p2} \\ F_{p1}, & F_{p1} \leq F_{p2} \end{cases} \quad (5.2)$$

Load distribution between fastener rows (percent of load per row) is only calculated for simple single and double shear geometries that are not subject to thermal loads. The reason that load distribution is not always calculated is that the concept becomes obscure for complex geometries with complex loading conditions. Normally, load distribution is calculated as the percentage of the total applied load that is transferred for each bolt row. For thermal loads for instance, it is unclear what the total applied load should be taken to be.

Strip calculation

The module takes as input the width of the plates, and the number of fastener columns. However, during the calculation the structure is split up into strips (one for each fastener column), and the calculation is performed on a strip of the structure. Thus, also the obtained results are for each strip of the structure. The bearing load at a fastener-plate conjunction for example, is critical in determining fatigue life. The **total** bearing load of a row of fasteners doesn't say much about this as opposed to the bearing load at each fastener, and this is why the calculations are performed on joint strips. It can be noted that strip-wise calculation has no effect on the load distribution.

Time consumption

The module may have to deal with a large number of loads, resulting in many iterations of calculating the FE-code. In order to reduce time consumption when dealing with a large number of loads, a special methodology is used. Firstly, a unit FE-calculation is performed at every position (node/element) where loads (mechanical/thermal) are applied. This means that the load in that position is set to 'unit' (1), upon which the FE-calculation is performed. Following this, each calculated unit result is scaled at each position using the magnitude of the load in that position for that specific load group. Finally, the scaled results are added together yielding a result for that particular load group. Thus, only one single iteration of the FE-calculation must be performed per position where load is applied, whereas with regular straightforward calculation, this must be done once for each load case. This method is possible to use due to the problem being linear.

Calculations has been performed on a geometry similar to the one in Figure 5.25 and the recorded time consumption is shown in Table 5.1.

| Number of loads | Unit calculation | Regular calculation |
|-----------------|------------------|---------------------|
| 1 | 0.018 s | 0.0038 s |
| 10 | 0.018 s | 0.0070 s |
| 100 | 0.019 s | 0.033 s |
| 1000 | 0.033 s | 0.31 s |
| 10000 | 0.17 s | 2.9 s |

Table 5.1: Time consumption with increasing number of loads

As seen in Table 5.1, the time consumption using regular, straightforward calculations increases with a much higher rate with increasing number of loads. Even though the unit calculation works slower for a few load groups, the time consumption for this is negligible and the difference in time consumption between the two approaches would be highly noticeable for a large number of load groups, a pattern clearly seen in Figure 5.18. The separation point where unit calculations become faster is for about 45 loads, at which the calculation time is only about 20 milliseconds. It should be noted that these time recordings are for the FE-calculation only.

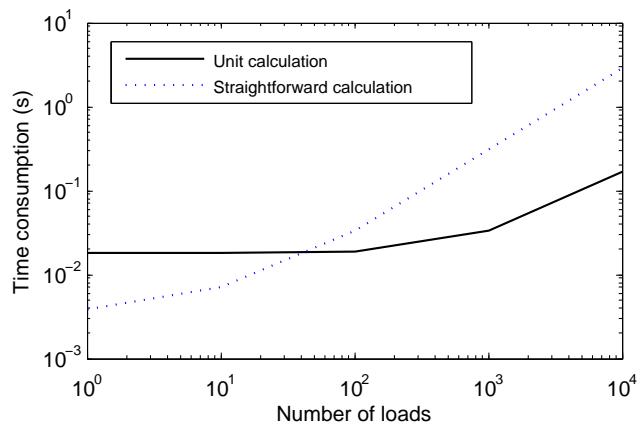


Figure 5.18: Increasing time consumption with increasing number of loads

5.2.5 Displaying calculated results

Following a successful calculation, the user can acquire a report containing information about the geometry, calculation, used load groups, and the calculated results. What this report contains depends on how the user has defined the report settings in DIM. An example of a report is shown in Figures 5.19 through 5.22.

```
CALCULATION
Name:      Testcalculation
Purpose:   Load distribution in a bolted joint.
Units:     SI_strict
Date:     04-Jun-2012 13:28:29
=====
```

Figure 5.19: Results - calculation definition

```

GEOMETRY

General
-----
Name: Testgeometry
Type: BoltedJoint1D
Units: m, N/m, N/m2, 1/C

Description
-----
Define geometry and parameters for a joint with symmetrical fastener columns.
Fastener material can be defined manually or by choosing from the metallic material library.
Plate material can be defined manually or by choosing from metallic and/or composite material libraries.
Fastener flexibility inserted manually, or calculated using any of the methods:
- Crumman
- Huth
- Barrois
At least one node must be locked to prevent rigid body motion.
-----
Locked node(s): 1
-----
Sketch:
nodes in parentheses () and element ids in brackets []
(1)-[1]-(2)      (2)-[2]-(3)      (3)-[3]-(4)
          |          |
          [7]          [8]
          |
(5)-[4]-(6)      (6)-[5]-(7)      (7)-[6]-(8)
-----
Plate elements:
Id   E alpha t L w nu
1 1.2E+8  0.01 0.1 0.4 0.2 0.33
2 1.2E+8  0.01 0.1 0.4 0.2 0.33
3 1.2E+8  0.01 0.1 0.4 0.2 0.33
4 1.2E+8  0.01 0.1 0.4 0.2 0.33
5 1.2E+8  0.01 0.1 0.4 0.2 0.33
6 1.2E+8  0.01 0.1 0.4 0.2 0.33
-----
Fastener elements:
Id   E   d   k Method
7 1E+10 0.005 1E+6 Manual
8 1E+10 0.005 1E+6 Manual
=====
```

Figure 5.20: Results - geometry definition

```

Loadgroups used: Testforce Testtemperature
-----
LOAD GROUP
Load group name: Testforce
Load group type: Loadl
Load group units: N

Load definition
-----
Applied loads in N
in directions: 1

Applied loads
-----
Id Px
(N)
1 368 >

min 368
Id 1
max 368
Id 1
=====

```

Figure 5.21: Results - load definition

```

LOADS & RESULTS:

Load id: 1
Px-(8) T-[1] T-[2] T-[3]
(N) (C) (C) (C)
368 15 15 15

Element forces:
Element: 1 2 3 4 5 6 7 8
Force (N): 368 -2.56E+4 -8.03E-11 -1.21E-10 2.6E+4 > 368 2.6E+4 -2.56E+4

Bearing loads:
Node: 1 2 3 4 5 6 7 8
Load (N): 0 -2.56E+4 2.56E+4 0 0 2.6E+4 -2.56E+4 0

Bypass loads:
Node: 1 2 3 4 5 6 7 8
Load (N): 0 -2.56E+4 -2.56E+4 0 0 -1.21E-10 368 0

Load distribution between fastener rows:
Load distribution not calculated for this joint configuration.
=====
```

Figure 5.22: Results - calculated results

5.2.6 Geometry requirements

When defining the geometry using the Plot panel, there are three requirements that must be fulfilled in order to be able to perform calculations on the structure.

1. There must be at least one fastener in the structure.
2. All fastener elements must be connected to plate elements on all sides, see Figure 5.23.
3. The geometry must be **one** single structure.

The second requirement implies that no gap is allowed where two (or more) plates are connected by a fastener (see Figure 5.24).

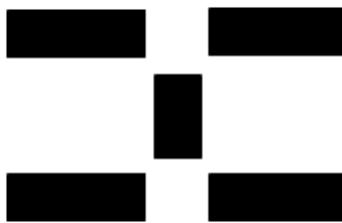


Figure 5.23: Geometry requirement 2

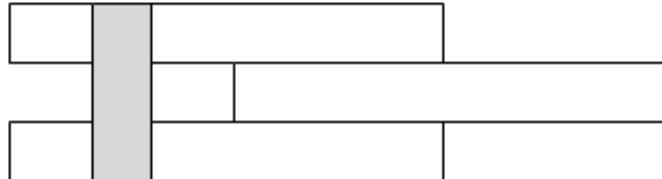


Figure 5.24: Gap between fastened plates - not allowed

5.2.7 Parameter requirements

In order to be able to perform calculations, sufficient parameters and conditions must be defined for the geometry. The following requirements apply:

1. All material parameters and dimensions must be defined and within predefined limits
2. For fasteners:
 - Stiffness k must be greater than a certain k_{\min} to avoid errors in the calculations
 - If Barrois' method is selected, the type of shear and fastener end conditions must be defined

- If Huth's method is selected, the type of shear and the variant of the method must be defined
 - If double shear is selected, the middle plate must be defined
3. All elements of a plate must have the same material parameters
 4. All elements of a fastener must have the same Young's modulus and diameter
 5. The ratio $\lambda = d/w_{\text{strip}}$ must be less than a certain λ_{\max}
 6. It is not allowed to mix the fastener flexibility methods in a geometry, with the exception of mixing manually inserted flexibility together with **one** of the methods

There are also some conditions that do not yield errors, but are not recommended. These yield warnings to the user, and the conditions are

1. A single shear method has been used on a fastener connecting more than two plates
2. One of the methods Grumman, Huth or Barrois has been used on a fastener connecting more than three plates
3. A fastener connects more than two plates (may yield fastener tilt depending on loading conditions)

This author may not have been able to foresee all possibilities of generating an incorrect structure, and thus the lists of requirements may be subject to change.

5.2.8 Limitations

The module gives the user flexibility when applying parameters and methods, which means that caution needs to be taken in order to use it 'correctly', i.e. in accordance with how the model and methods are defined.

The model, as described in Section 2.2.2, is able to handle complex one-dimensional joint geometries. Theoretically these can be of any size and shape, limited only by the power of the computer that the program is run on. The user of the program must however be aware that the results from using the program on a non-standardized geometry may be unreliable. Using any of the fastener flexibility calculation methods on a type of structure (or part of a structure) that it was not defined for, may yield inaccurate results.

Following the discussion in Section 4.3, the user also needs to be observant when generating structures where three or more plates are connected by single bolts, since the model in that case may or may not be able to take into account resulting forces and displacements due to fastener tilt, depending on the applied forces. Even so, the fastener flexibilities when connecting more than three plates is not to be calculated using the methods introduced in this report, since these are only based on single and double shear loaded fastener installations. Instead, fastener flexibility in this case needs to be obtained elsewhere and inserted manually.

5.2.9 Short user guide

1. Define geometry

- (a) Open BoltedJoint1D module.
- (b) Select the needed number of plate layers and fastener rows in the Geometry layout panel. Accept changes if inquired.
- (c) Generate desired geometry by left-clicking the elements in the Plot panel. The geometry is acceptable when ‘Geometry ok!’ is displayed in green in the layout panel.
- (d) Define parameters and lock node(s) using the Parameter panel for all the elements of the chosen geometry. Parameter settings are acceptable when ‘Parameters ok!’ is displayed in green in the Parameter panel.
- (e) Check that the assigned parameters are as desired using the Table panel.
- (f) Save the geometry.

2. Define loads

- (a) Define load groups needed for calculation.
- (b) Save load groups.

3. Define load-geometry connection

- (a) Select geometry to be included in the calculation.
- (b) Press ‘Select load group(s)’ button.
- (c) Apply force load groups to nodes and temperature load groups to elements.

4. Perform calculation

- (a) Press ‘Perform task(s)’.

5. Display results

- (a) Select ‘Results’-flap.
- (b) Right-click on desired object in the tree (displayed with its defined task name) and choose ‘View results’.
- (c) The results are displayed in accordance with the result settings.

5.3 Comparing previous calculations

In order to verify the calculations, and to show the users of the module how to provide input in comparison with previous software, some comparative tests have been performed.

5.3.1 Grumman

The Saab-software BH014 [7] is used for calculating fastener loads in a single shear geometry. It provides the Grumman formula, but the user is obliged to calculate the fastener stiffness and insert it manually. The finite element model of BH014 is similar to the one used in the module (see Figure 2.9), but differs in its highly limited allowed geometry. BH014 calculates only element forces. An example calculation follows.

Geometry

The geometry used is displayed in Figure 5.25, with parameters as used in BH014 in Table 5.2 and for BoltedJoint1D in Table 5.3.

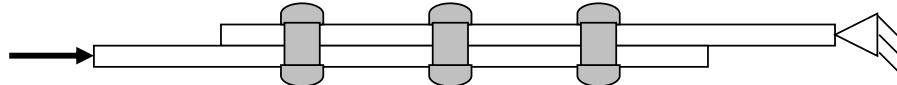


Figure 5.25: Geometry for the BH014 calculation example

| Parameter | Element/node* | Value | Unit |
|------------|-------------------|----------------------|-----------|
| EA/L | 2,5,8 | 150000 | N/mm |
| EA/L | 11 | 153000 | N/mm |
| EA/L | 1,4,7,10 | 120000 | N/mm |
| α | 1,2,4,5,7,8,10,11 | $0.27 \cdot 10^{-4}$ | K $^{-1}$ |
| L | 1,2,4,5,7,8,10,11 | 15 | mm |
| k | 3,6,9 | 7500 | N/mm |
| ΔT | 2,5,8,11 | 100 | K |
| ΔT | 1,4,7,10 | 0 | K |
| P | 1 | -1000 | N |

Table 5.2: Parameters for BH014

*Numbering according to BH014

| Parameter | Element/node* | Value | Unit |
|------------|---------------|---------------------|-----------|
| E | 1-3 | $2.25 \cdot 10^6$ | Pa |
| E | 4 | $2.295 \cdot 10^6$ | Pa |
| E | 5-8 | $1.8 \cdot 10^6$ | Pa |
| w | 1-8 | 1 | m |
| t | 1-8 | 1 | m |
| α | 1-8 | $2.7 \cdot 10^{-5}$ | K $^{-1}$ |
| L | 1-8 | 0.015 | m |
| k | 9-11 | $7.5 \cdot 10^6$ | N/m |
| ΔT | 1-4 | 100 | K |
| ΔT | 5-8 | 0 | K |
| P | 6 | -1000 | N |

Table 5.3: Parameters for BoltedJoint1D

*Numbering according to BoltedJoint1D

Results

Calculated loads in the elements of the structure are shown in Table 5.4.

| Element | Force [N] | |
|---------|-----------|--------|
| | BH014 | Module |
| 1 | 0 | 0 |
| 2 | 71.9 | 71.9 |
| 3 | 393 | 393 |
| 4 | 1000 | 1000 |
| 5 | 1000 | 1000 |
| 6 | 928 | 928 |
| 7 | 607 | 607 |
| 8 | 0 | 0 |
| 9 | -71.9 | -71.9 |
| 10 | -321 | -321 |
| 11 | -607 | -607 |

Table 5.4: Calculated element loads

5.3.2 Huth

In the article by Huth [9], measurements are conducted on the load distribution between rows of joints on several geometries and compared to calculated results using the Huth's formula for fastener flexibility, Eq. (2.14).

Geometries

Geometries used in the comparison with the calculations by Huth are shown in Figure 5.26 and the corresponding model in BoltedJoint1D in Figure 5.27. Dimensions are found in Figure 5.26 and the material used for the plates are Aluminum 2024T3 and for the fasteners Titanium alloy Ti 6Al 4V with material parameters according to Table 5.5.

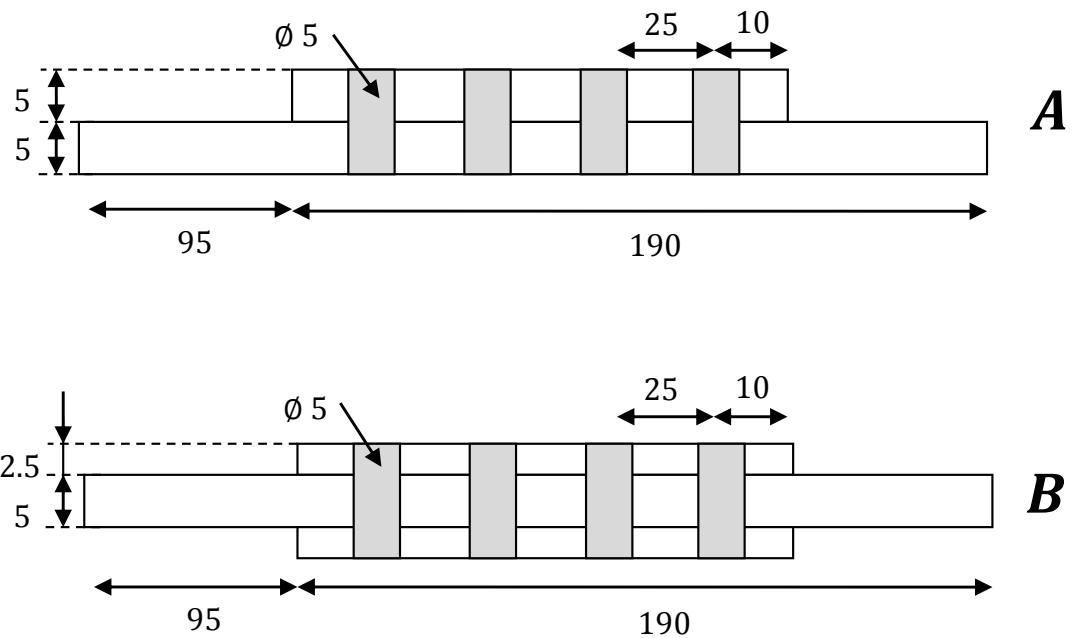
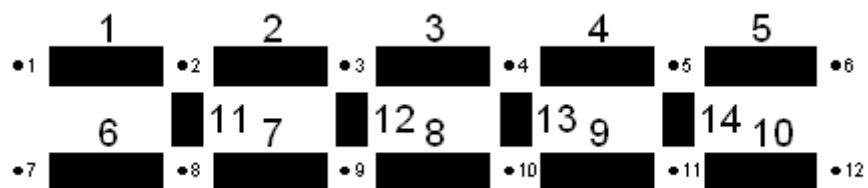
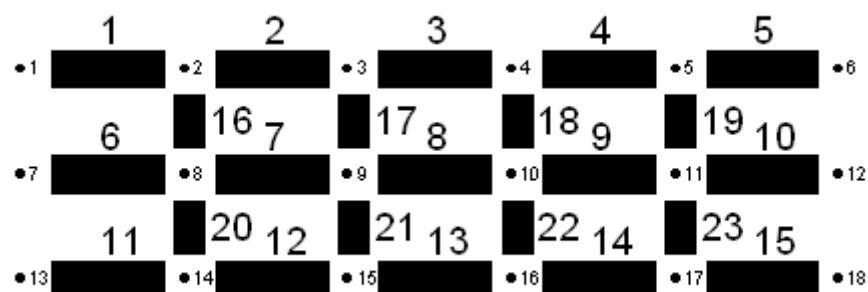


Figure 5.26: Geometries for the Huth calculation example (plate width is 25 mm)



(a) Model for the Huth geometry A



(b) Model for the Huth geometry B

Figure 5.27: Geometries for the Huth calculation examples using BoltedJoint1D

| Material | Parameter | Value |
|-----------|-----------|----------|
| 2024T3 | E | 73.1 GPa |
| | ν | 0.33 |
| Ti 6Al 4V | E | 110 GPa |
| | ν | 0.33 |

Table 5.5: Material used in Huth joints

Results

Results given in the article by Huth is displayed in Figure 5.28 together with calculated results using the program module, with calculated numerical values in Table 5.6.

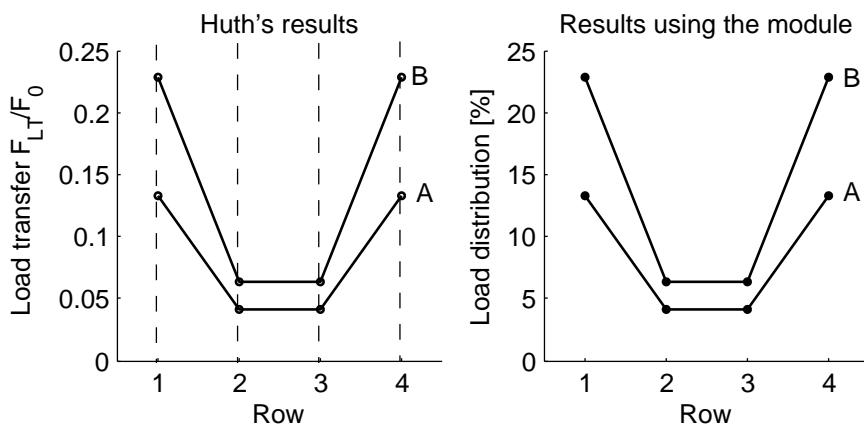


Figure 5.28: Results for Huth comparative calculation

| Row | Load distribution | |
|-----|-------------------|--------|
| | A | B |
| 1 | 13.3 % | 22.8 % |
| 2 | 4.1 % | 6.3 % |
| 3 | 4.1 % | 6.3 % |
| 4 | 13.3 % | 22.8 % |

Table 5.6: Calculated row load distribution using the module

5.3.3 Barrois

The method presented by Barrois (described in Section 2.1.5) is used in the Saab-software BARROIS [1], both for fastener flexibility and load distribution. Due to the Barrois method being more complex than the others and that BARROIS allows for variations in plate widths and thickness, more cases will be tested and compared with the BARROIS-software. Since input in BARROIS is for the values at the bolt rows, and for BoltedJoint1D values of the elements, the

input where dimensions vary (width/thickness) is adjusted such that when linearly interpolated, it matches BARROIS, see Section 2.3.3.

In calculating the load distribution with Barrois' method (and the BARROIS software), account is taken to the holes in the plates when calculating plate elongation using an empirical factor, see Section 2.3.2. This factor can be chosen to be included in the calculations or not (by changing the calculation settings), depending on the user's preference.

The BARROIS software yields fastener flexibilities, row load distributions and stress concentrations at the plate-fastener conjunctions.

Geometries

The geometries used are shown in Figure 5.29 with corresponding BoltedJoint1D models in Figures 5.30 and 5.31, which show that the number of fastener columns to insert into the module is 3. Parameters for the different Barrois calculation examples are found in Tables 5.7 and 5.8. As a comparison, the input for Geometry 1 in BARROIS is displayed in Figure 5.32.

| Parameter | Value |
|-----------|--------|
| E | 71 GPa |
| L | 25 mm |
| d | 6 mm |

Table 5.7: Common parameters

| Element | Geometry 1 | | Geometry 2 | | Geometry 3 | | Geometry 4 | |
|---------|------------|--------|------------|--------|------------|--------|------------|--------|
| | t [mm] | w [mm] |
| 1 | 4 | 75 | 4.375 | 75 | 4.375 | 87.5 | 4 | 75 |
| 2 | 4 | 75 | 3.625 | 75 | 3.625 | 82.5 | 4 | 75 |
| 3 | 4 | 75 | 2.875 | 75 | 2.875 | 77.5 | 4 | 75 |
| 4 | 4 | 75 | 2.125 | 75 | 2.125 | 72.5 | 4 | 75 |
| 5 | 4 | 75 | 1.375 | 75 | 1.375 | 67.5 | 4 | 75 |
| 6 | 4 | 75 | 0.625 | 75 | 0.625 | 62.5 | 4 | 75 |
| 7 | 4 | 75 | 0.625 | 75 | 0.625 | 62.5 | 4 | 75 |
| 8 | 4 | 75 | 1.375 | 75 | 1.375 | 67.5 | 4 | 75 |
| 9 | 4 | 75 | 2.125 | 75 | 2.125 | 72.5 | 4 | 75 |
| 10 | 4 | 75 | 2.875 | 75 | 2.875 | 77.5 | 4 | 75 |
| 11 | 4 | 75 | 3.625 | 75 | 3.625 | 82.5 | 4 | 75 |
| 12 | 4 | 75 | 4.375 | 75 | 4.375 | 87.5 | 4 | 75 |
| 13 | - | - | - | - | - | - | 4 | 75 |
| 14 | - | - | - | - | - | - | 4 | 75 |
| 15 | - | - | - | - | - | - | 4 | 75 |
| 16 | - | - | - | - | - | - | 4 | 75 |
| 17 | - | - | - | - | - | - | 4 | 75 |

Table 5.8: Parameters that vary between the geometries

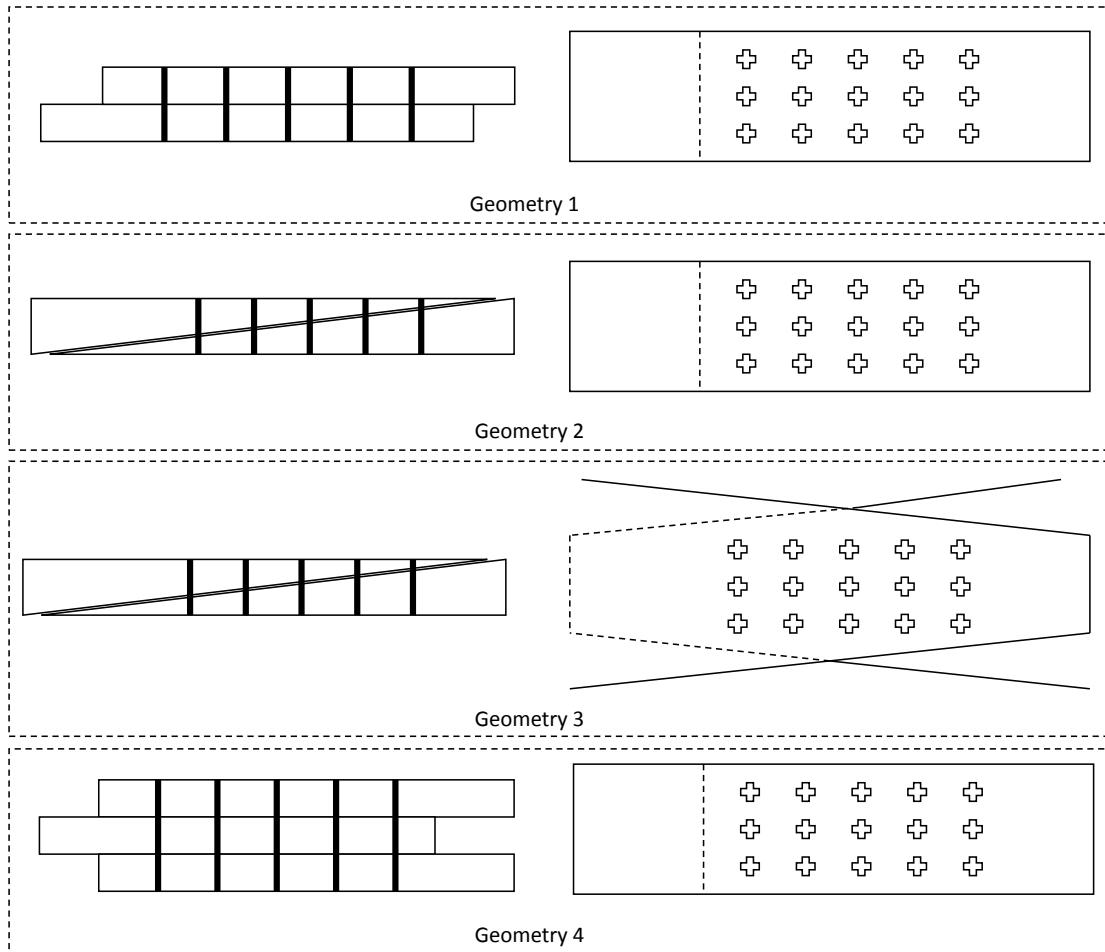


Figure 5.29: Geometries for comparison with BARROIS software

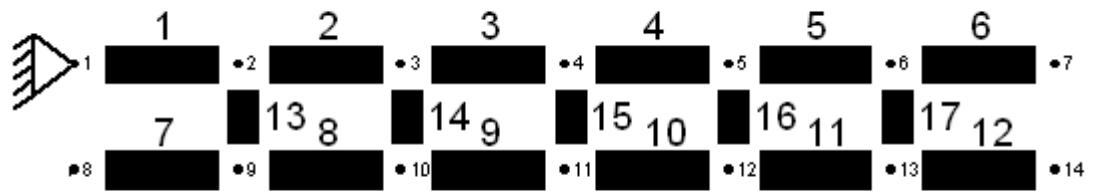


Figure 5.30: Model for geometries 1-3

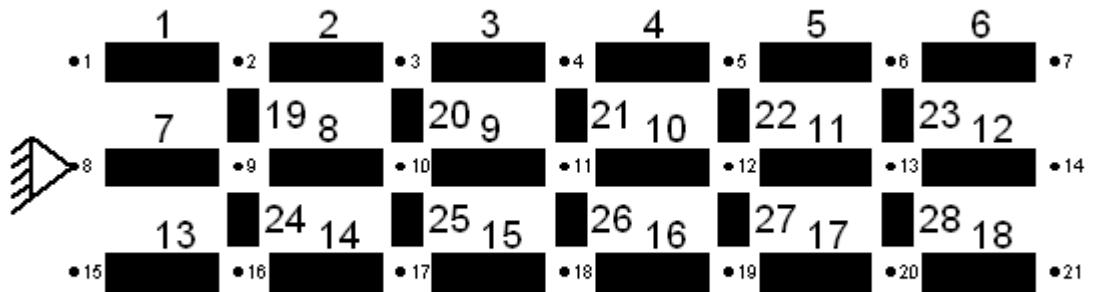


Figure 5.31: Model for geometry 4

```

Input file GEOMETRY1:COM

HEADER
HEAD1=Example of command and result file
HEAD2=Single shear loaded lap joint
HEAD3=Geometry type 1
CREA=TUKHB14/LS
FILE=GEOMETRY1.LIS
EXIT
EP 71000
FIX 1
ROW1 4.00 4.00 75. 75. 25. 6. 100 2.40 71000. 3
ROW2 4.00 4.00 75. 75. 25. 6. 100 2.40 71000. 3
ROW3 4.00 4.00 75. 75. 25. 6. 100 2.40 71000. 3
ROW4 4.00 4.00 75. 75. 25. 6. 100 2.40 71000. 3
ROWS 4.00 4.00 75. 75. 25. 6. 100 2.40 71000. 3
TYPE 1
RUN
STOP

```

Figure 5.32: BARROIS input for Geometry 1

Results

Results of the calculations are shown in Tables 5.9 through 5.12. The difference in results when not taking holes in plates into account when calculating plate elongation is seen in Figure 5.33.

| | Clamped heads | | | | Free heads | | | |
|-----|---------------|---------|----------|---------|------------|---------|----------|---------|
| | k [kN/mm] | | Load [%] | | k [kN/mm] | | Load [%] | |
| Row | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS |
| 1 | 65.86 | 65.85 | 27.88 | 27.88 | 28.89 | 28.90 | 24.08 | 24.08 |
| 2 | 65.86 | 65.85 | 15.88 | 15.88 | 28.89 | 28.90 | 17.91 | 17.91 |
| 3 | 65.86 | 65.85 | 12.49 | 12.49 | 28.89 | 28.90 | 16.01 | 16.01 |
| 4 | 65.86 | 65.85 | 15.88 | 15.88 | 28.89 | 28.90 | 17.91 | 17.91 |
| 5 | 65.86 | 65.85 | 27.88 | 27.88 | 28.89 | 28.90 | 24.08 | 24.08 |

Table 5.9: Results for geometry 1

| | Clamped heads | | | | Free heads | | | |
|-----|---------------|---------|----------|---------|------------|---------|----------|---------|
| | k [kN/mm] | | Load [%] | | k [kN/mm] | | Load [%] | |
| Row | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS |
| 1 | 42.35 | 42.35 | 20.19 | 20.19 | 24.27 | 24.27 | 23.12 | 23.12 |
| 2 | 56.20 | 56.20 | 20.01 | 20.01 | 21.27 | 21.28 | 18.72 | 18.72 |
| 3 | 60.29 | 60.29 | 19.59 | 19.59 | 18.75 | 18.76 | 16.33 | 16.33 |
| 4 | 56.20 | 56.20 | 20.01 | 20.01 | 21.27 | 21.28 | 18.72 | 18.72 |
| 5 | 42.35 | 42.35 | 20.19 | 20.19 | 24.27 | 24.27 | 23.12 | 23.12 |

Table 5.10: Results for geometry 2

| | Clamped heads | | | | Free heads | | | |
|-----|---------------|---------|----------|---------|------------|---------|----------|---------|
| | k [kN/mm] | | Load [%] | | k [kN/mm] | | Load [%] | |
| Row | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS |
| 1 | 41.21 | 41.23 | 18.64 | 18.67 | 24.06 | 24.04 | 22.21 | 22.23 |
| 2 | 55.86 | 55.88 | 20.72 | 20.71 | 21.26 | 21.23 | 19.23 | 19.21 |
| 3 | 60.29 | 60.24 | 21.27 | 21.24 | 18.75 | 18.76 | 17.14 | 17.12 |
| 4 | 55.86 | 55.88 | 20.72 | 20.71 | 21.26 | 21.23 | 19.23 | 19.21 |
| 5 | 41.21 | 41.23 | 18.64 | 18.67 | 24.06 | 24.04 | 22.21 | 22.23 |

Table 5.11: Results for geometry 3

| | Clamped heads | | | | Free heads | | | |
|-----|---------------|---------|----------|---------|------------|---------|----------|---------|
| | k [kN/mm] | | Load [%] | | k [kN/mm] | | Load [%] | |
| Row | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS | DIM | BARROIS |
| 1 | 60.11 | 60.10 | 38.79 | 38.79 | 55.17 | 55.16 | 37.94 | 37.94 |
| 2 | 60.11 | 60.10 | 18.07 | 18.07 | 55.17 | 55.16 | 18.35 | 18.35 |
| 3 | 60.11 | 60.10 | 10.79 | 10.79 | 55.17 | 55.16 | 11.27 | 11.27 |
| 4 | 60.11 | 60.10 | 11.52 | 11.52 | 55.17 | 55.16 | 11.87 | 11.87 |
| 5 | 60.11 | 60.10 | 20.82 | 20.82 | 55.17 | 55.16 | 20.57 | 20.57 |

Table 5.12: Results for geometry 4

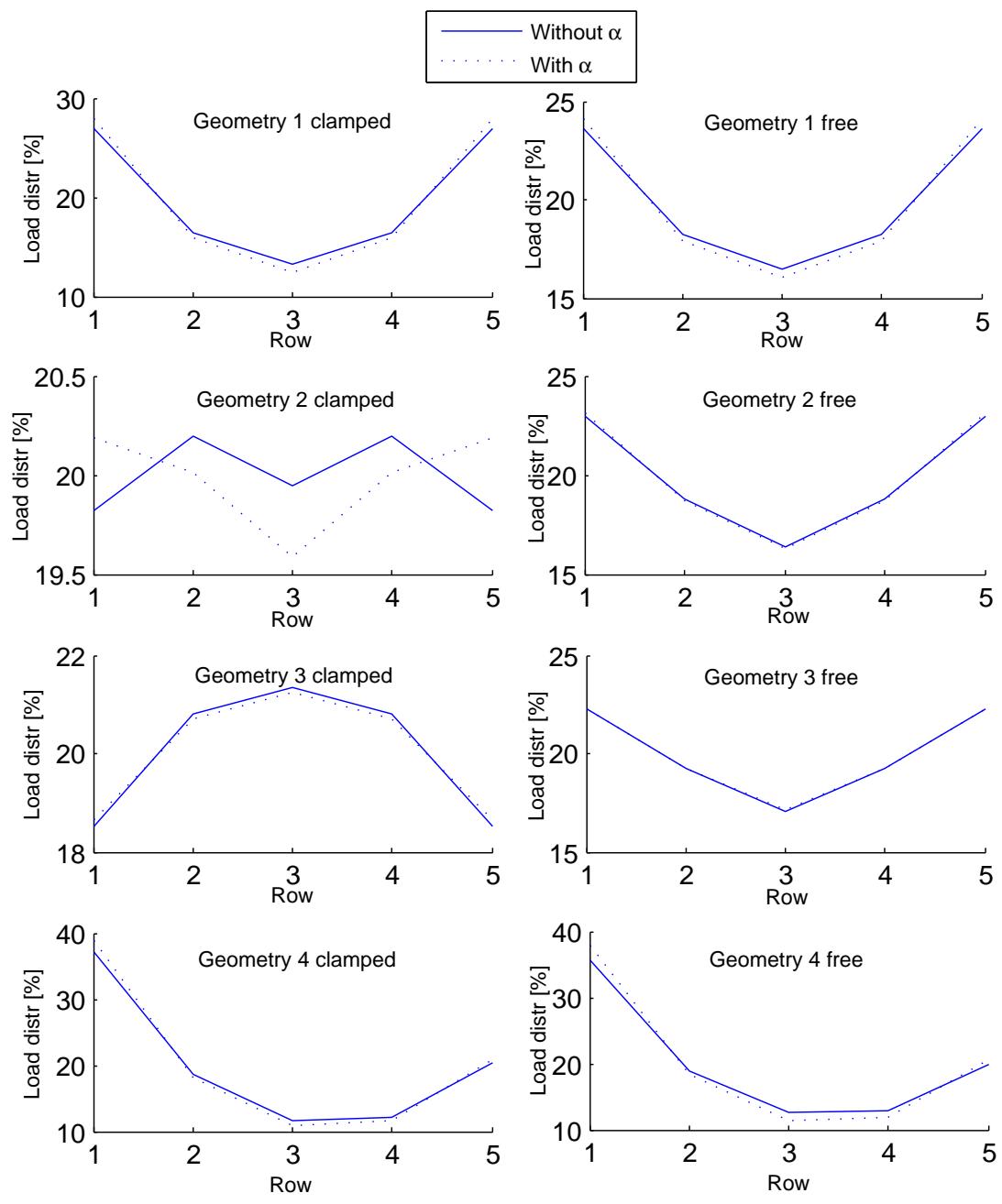


Figure 5.33: Comparative results with or without taking hole influence on plate elongation into account

5.4 Matching calculations to model

5.4.1 Including shear loads in a one-dimensional model

A very common joint in e.g. the Saab 340 and Saab 2000 aircrafts is the splice seen in Figure 5.34 and its placement inside the fuselage is shown in Figure 5.35. In this joint, both normal and shear forces are usually present.

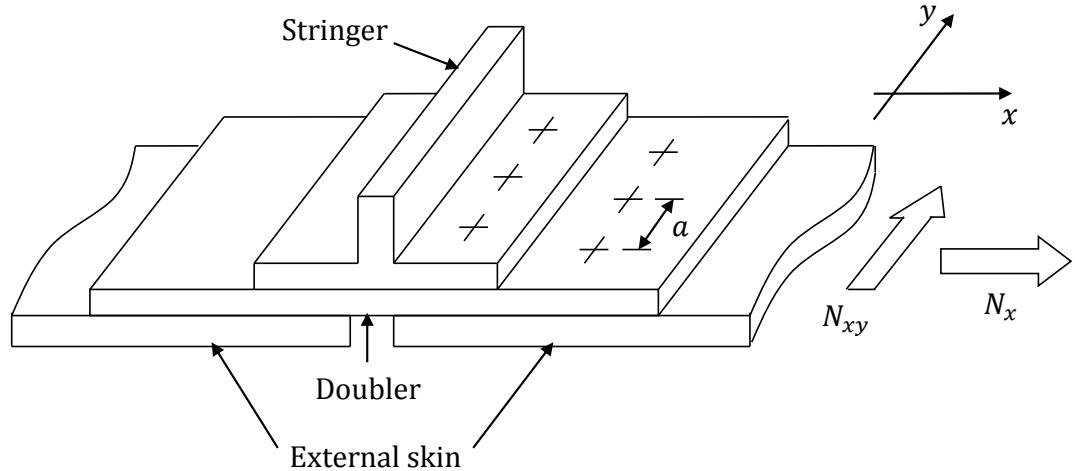


Figure 5.34: Common splice assembly

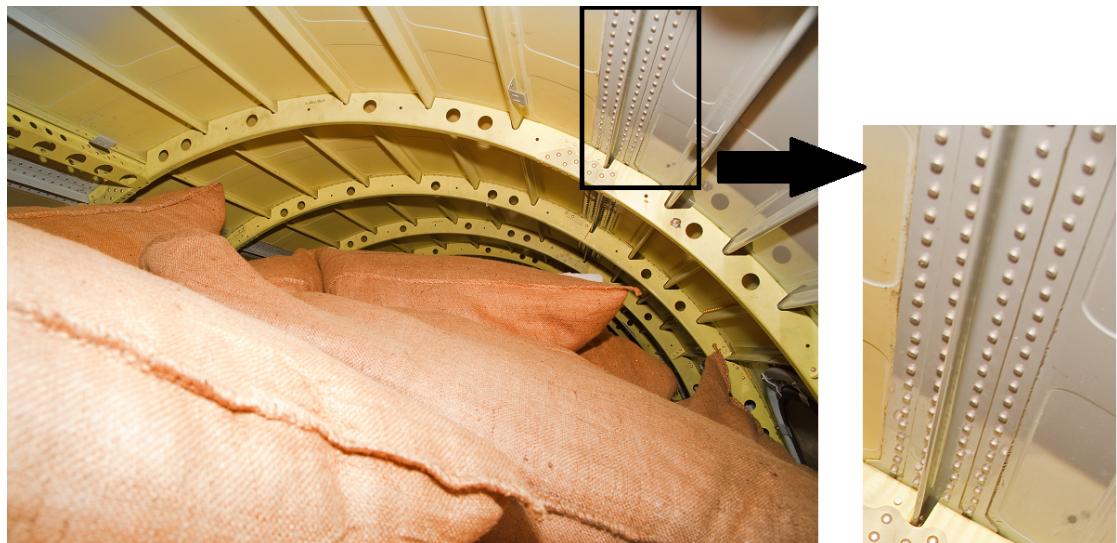


Figure 5.35: Splice position inside the Saab 2000 fuselage

| | | |
|------------------|------------|-------|
| Normal load: | N_x | [N/m] |
| Shear load: | N_{xy} | [N/m] |
| Fastener pitch: | a | [m] |
| Load/pitch in x: | $N_x a$ | [N] |
| Load/pitch in y: | $N_{xy} a$ | [N] |

Table 5.13: Splice parameters

A common engineering simplification is to assume that the load distribution between fastener rows are the same for both normal and shear load. Thus, the resulting force to be used in the one-dimensional model becomes

$$P = \sqrt{N_x^2 a^2 + N_{xy}^2 a^2} = \sqrt{N_x^2 + N_{xy}^2} \cdot a \quad (5.3)$$

A model representing this splice in BoltedJoint1D could be defined as shown in Figure 5.36 (or, by using symmetry, cutting elements 3 and 9 and modeling half of the structure).

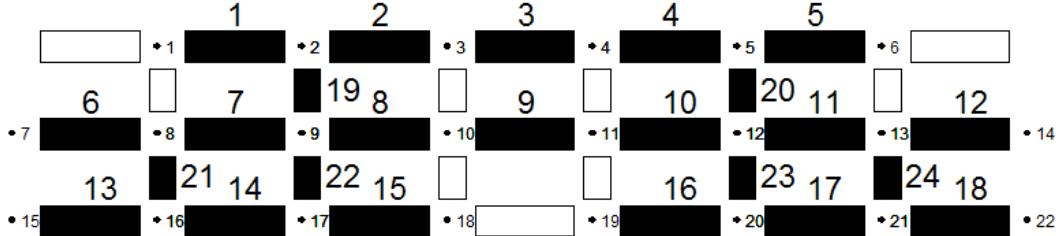


Figure 5.36: Splice model

5.4.2 Step-wise dimension variations

There are joints in which the width or thickness vary in steps between fastener sites. An example of the latter is shown in Figure 5.37. For this example, the parts of the plates between the fasteners must be divided into several elements in order to accurately take into account these dimensional variations. The model of Figure 5.37 could for example look like the model of Figure 5.38 with parameters according to Table 5.14.

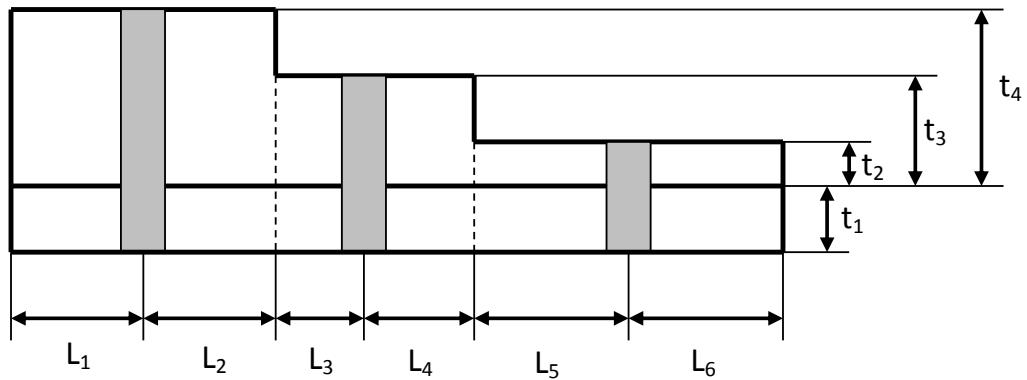


Figure 5.37: Structure with step-wise thickness variation



Figure 5.38: Model of the structure in Figure 5.37

| Parameter | Elements |
|-----------|----------------|
| L_1 | 1,7 |
| L_2 | 2,8 |
| L_3 | 3,9 |
| L_4 | 4,10 |
| L_5 | 5,11 |
| L_6 | 6,12 |
| t_1 | 7,8,9,10,11,12 |
| t_2 | 5,6 |
| t_3 | 3,4 |
| t_4 | 1,2 |

Table 5.14: Parameters for the structure in Figure 5.37

5.4.3 Complex structures

As discussed in Section 4.3, the model used in module cannot accurately reproduce any arbitrary one-dimensional joint structure. This does not mean that the model is useless for all complex structures. Take the model in Figure 5.39 for instance. This model could - depending on how loads are applied - be used for the structure it represents as accurately as the simpler single or double shear geometries calculated on in Section 5.3. Assuming that all fasteners are subject to either single shear or double shear loading (again, this depends on the applied loads), the fasteners of elements 33,35 and 37,38 could be identified as double shear, and fasteners 34, 36, 39 and 40 as single shear loaded fasteners. Thus, an applicable variant of the formulas for fastener flexibility can be used, and the calculation is likely to yield reasonable results.

As stated, the validity of using this model on this structure depends greatly on the way that loads are applied. If for example loads are applied at nodes 9 and 17 only, then the same problem as found on Geometry *b* in Section 4.3 arises.

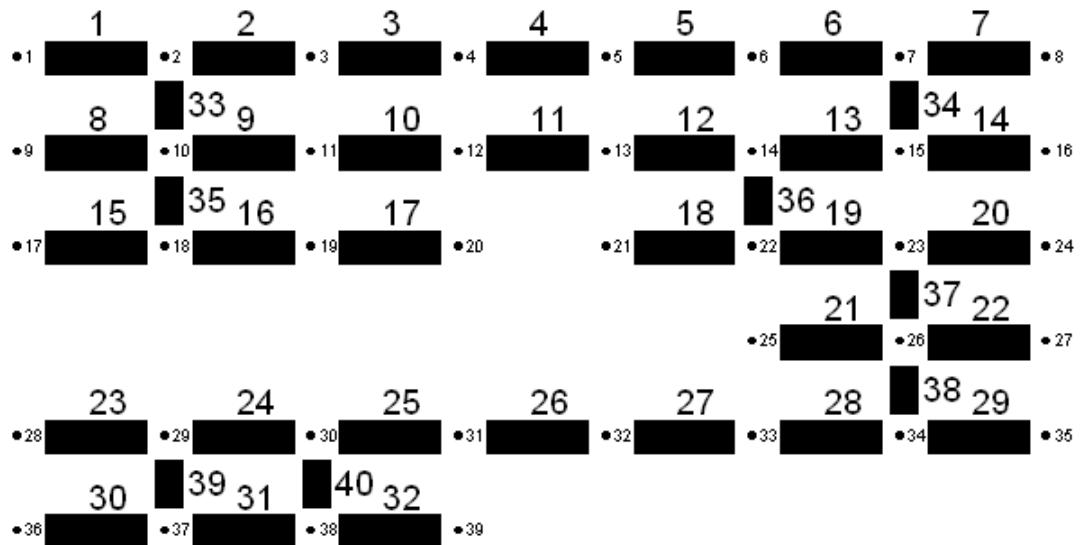


Figure 5.39: Complex structure

5.4.4 Regarding symmetry-modeling of double shear joints

Historically, symmetrical double shear joints have often been modeled by splitting the structure along its symmetry-line and performing calculations on that half-structure (Figure 5.40). This allows for the double shear formulas by Huth and Barrois to be applied directly, without modification, into the calculations. However, in the model here (Section 2.2.2), the flexibility is reshaped to fit the model according to Section 2.3.1. Therefore it is **not** possible to apply the cut symmetry-model in the module. Besides, this kind of symmetry-modeling is only possible for joints that actually are symmetric, which limits the versatility of that approach significantly.

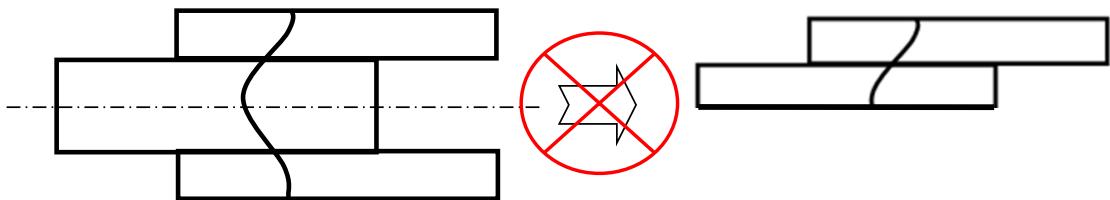


Figure 5.40: Modeling double shear loaded fasteners using symmetry

5.5 Discussion

A complete graphical user interface for defining geometry, parameters, methods for calculating fastener flexibility and defining and applying loads to a bolted joint assembly is available. The parameters of the structure must be well-defined and within limits. The calculations yield results with respect to for example bearing and bypass loads for a *strip* of the defined structure (see Section 5.2.4). The load distribution between fastener rows is only calculated for structures where this property is well-defined. Warnings are given to the user if the structure is such that the underlying model may be insufficient in how it describes the behavior of the bolted joint assembly.

The effect of taking into account holes in plates in calculating plate elongation yields up to a few percent in difference in load distribution as opposed to not including this effect (Figure 5.33). Thus, this term should be included for the case that Barrois method is used for calculating fastener flexibility, and perhaps also when inserting the flexibility manually, depending on the way that this was obtained.

It is often assumed that shear forces can be included in the one-dimensional model, under the assumption that shear forces have the same impact on load distribution as do normal forces (Section 5.4.1). The validity of this approach can be discussed, and an investigation on this topic may be necessary.

The obtained numerical results of Section 5.3 are in excellent agreement with similar software (BH014 [7] and BARROIS [1]) and calculations by Huth [9].

Following a successful calculation it is possible to get a complete report, including geometry definition, parameters, used methods, applied loads and results.

Chapter 6

Discussion

A complete graphical user interface for defining geometry, parameters, methods for calculating fastener flexibility and defining and applying loads to a bolted joint assembly is available. The results obtained by the module are in excellent agreement with results from other sources (see Section 5.3), for single and double shear fastener installations.

Barrois generally yields much larger values for the fastener stiffnesses, and thus also larger variations in load distribution between fastener rows than the methods by Grumman and Huth, which yield quite similar results (at least for single shear geometries). Following the discussions in Sections 3.4 and 4.3, it cannot be claimed that either method is preferred over the other, generally. Also, for the investigations conducted here, the resulting load distributions are not significantly affected by which of the methods Grumman, Huth and Barrois is being used (as long as the proper variant is applied). It is however a good idea to attempt to identify which of the methods is best suited for the geometry at hand; since the results are likely to be more accurate the closer the structural configuration is to the configurations that the methods were derived from.

The fastener flexibility for fasteners connecting more than three plates is not possible to calculate (accurately) using any of the implemented methods. The methods can of course be used anyways, but the results are in that case questionable. It is instead suggested that the user attempts to find a more accurate flexibility, either by consulting Gunbring [6] or by making assumptions on the fastener behavior in relation to the flexibility obtained from single shear or double shear, and then inserting this flexibility manually in the program.

Furthermore, the implemented model cannot account for the effect of the fastener's tilting in larger structures due to the lack of coupling between the different elements of the fastener, as described in Section 4.3. This effect could be taken into account by implementing a model such as the one in Figure 4.4, but to obtain accurate fastener behavior for an arbitrary geometry with arbitrary loading conditions would require an infinite amount of testing. This is of course not viable, but there have already been tests performed on the three-plate geometry with loading conditions according to Figure 4.2b. This case could thus quite easily be accounted for, and if other kinds of geometries with specific loading conditions are of special interest, these could be tested upon and also implemented in the model.

Fastener tilt is present also in simple single shear loaded fastener configurations, and the reason why accurate values are obtained for these cases is that fastener tilt is accounted for when using the methods by Grumman, Huth, and Barrois for single shear. Since only one spring is used for representing the fastener in the model, no coupling between fastener elements is needed.

It must be emphasized that the obtained results generally are rough approximations compared with true joint behavior. The one-dimensional model in itself is a huge simplification, in that it can only definitely accurately account for forces in one direction (even though it is often assumed that also shear forces can be included, see Section 5.4.1), and does not accurately handle joints where for example fastener rows are in offset to each other, which is quite common in bolted joint assemblies. In addition, the load distribution is affected by many parameters, such as friction, number of load cycles and plastic deformations, to name a few. Due to the fact that the methods have been derived for specific fastener installations with different setups of these parameters, they are only certainly accurate for those specific setups.

On another note, it can be said that the author has put more focus on implementing and analyzing the methods than in optimizing the source code. It can however handle reasonably sized structures with thousands of loads without significant delay.

6.1 Recommendations

The following should be viewed upon as a summary of suggestions to the user of the created module, and not as fixed restrictions.

- Use the fastener flexibility method & variant that is best suited for the geometry. If the fastener flexibility is unknown, and none of the methods presented in this report is suitable for the calculation, it is recommended to find this flexibility elsewhere, for example by consulting the work by Gunbring [6].
- If the plates in the joints have varying thickness or width, it is recommended to divide the structure into a model with sufficient number of plate elements to reduce the error. For structures with step-wise changing dimensions, it is sufficient to split the plate at each site where the dimensional change occurs. For structures with linearly changing dimensions, the plate needs to be split into elements such that each element has a A_1/A_2 , $A_1 < A_2$ ratio of no less than 0.5, which gives a maximum error of about 4% on the plate stiffness.
- Do not use the method by Barrois if plates of different materials are used in the structure.
- Do not model symmetric double shear fastener installations by cutting in the horizontal plane, unless fastener flexibility is to be inserted manually.
- Always define all parameters accurately for the created geometries using BoltedJoint1D. Remember that the saved geometries may be used by others, under different loading conditions than the specific calculation it was originally intended for.
- If precise and accurate values of load distributions are needed, testing on the geometry needs to be done.

Chapter 7

Further work

Found below are suggestions on what the created module and its underlying calculations could be extended to.

- Implementation of a model that takes into account fastener tilt in non-trivial structures. For structures or load conditions other than the one in Figure 4.2b, testing would have to be performed to be able to find suitable ways of calculating the fastener flexibilities.
- Calculation of stress concentrations, as is done in for example the BARROIS [1] software.
- Investigation of the validity in the assumption that the load distribution is the same for shear forces as for normal forces in a joint.
- Investigation of the effect of varying fastener diameters between fastener rows on the implemented methods and model.
- Program efficiency can be improved. An experienced programmer could probably increase the program efficiency significantly, if this is deemed necessary.

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