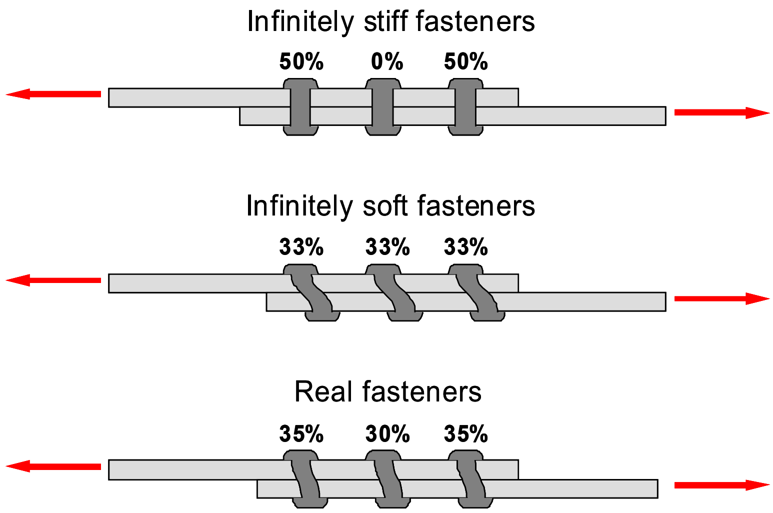
The fastener flexibility is a measure of the influence of fasteners (rivets, bolts, etc.) on the flexibility of the whole joints. It plays an important role when considering the factors influencing the strength level and fatigue life of an aircraft joint.

The flexibility can be defined as follows:

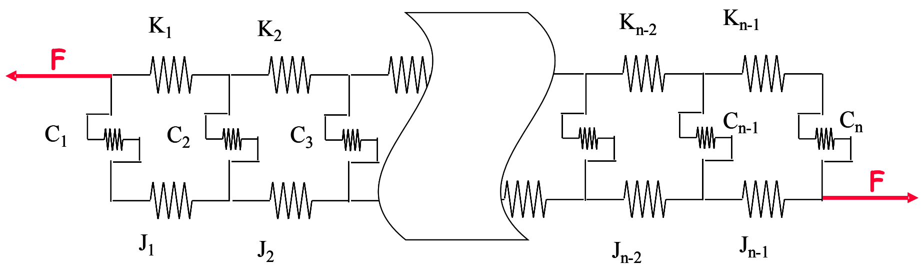
$$flexibility\,(f) = \frac{1}{Stiffness\,(K)} = \frac{\Delta l}{F}$$

in which F refers to the external force and Δl to the deflection of the joint due to the fastening (in other words: the deflection of the joint around the fastener excluding the normal extension of the sheet material).

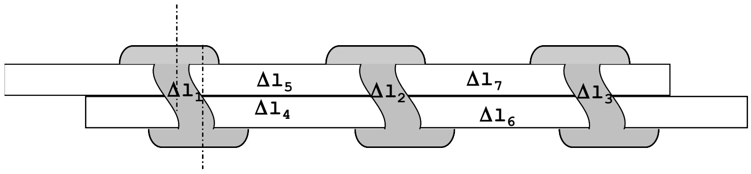
In terms of load transfer and deformation, the fasteners stiffness (flexibility) determines the way it is transferred from one component to another, and choose the right value is an important factor in the results of a joint analysis:



The basic approach to modelize a flexible joint is representing the fasteners and the fastened components by springs, with the corresponding flexibility as follow:



with: K_i = stiffness (flexibility) inter fastener upper plate
J_i = stiffness (flexibility) inter fastener lower plate
C_i = fasteners stiffness (flexibility)



in that way some equations of compatibility must be satisfied by the system plates-fasteners with coherent deformations.

There are several formulations for the fasteners flexibility. The most important existing and recognized formulas are, with the parameters defined as follow:

Configuration: d = Hole diameter
t = Plate thickness
n = Single o double shear switch
Material: E = Young modulus
ν = Poisson ratio
Indices: 1 = Plate 1 (central one in double shear)
2 = Plate 2 (outer ones in double shear)
f = fastener

- **Swift (Douglas)**

$$f = \frac{5}{dE_f} + 0.8\left(\frac{1}{t_1E_1} + \frac{1}{t_2E_2}\right)$$

- **Tate & Rosenfeld**

$$f = \frac{1}{E_ft_1} + \frac{1}{E_ft_2} + \frac{1}{E_1t_1} + \frac{1}{E_2t_2} +$$

$$+\frac{32}{9E_f\pi d^2} \cdot (1+\nu_f)(t_1+t_2) + \frac{8}{5E_f\pi d^4} \cdot (t_1^3 + 5t_1^2t_2 + 5t_1t_2^2 + t_2^3)$$

- **Boeing**

$$f = \frac{2^{\left(\frac{t_1}{d}\right)^{0.85}}}{t_1}\left(\frac{1}{E_1} + \frac{3}{8E_f}\right) + \frac{2^{\left(\frac{t_2}{d}\right)^{0.85}}}{t_2}\left(\frac{1}{E_2} + \frac{3}{8E_f}\right)$$

- **Huth, Schwarmann (Airbus)**

$$f = \left(\frac{t_1+t_2}{2d}\right)^a \frac{b}{n}\left(\frac{1}{t_1E_1} + \frac{1}{nt_2E_2} + \frac{1}{2t_1E_f} + \frac{1}{2nt_2E_f}\right)$$

a and b depends on the type of joint, $n=1$ for single shear and 2 for double shear

Type	a	b
bolted metallic	2/3	3.0
riveted metallic	2/5	2.2
bolted graphite/epoxy	2/3	4.2

- **Vought**

$$f = 56\left(\epsilon\left(\frac{t_1}{d}\right)\frac{1}{t_1} + \epsilon\left(\frac{t_2}{d}\right)\frac{1}{t_2}\right)$$

In which $e=1$ for $t/d<0.65$ and $e=1.29\ t/d$ for $t/d>0.9$. This version of the formula only applies to aluminium sheets joined by steel fasteners.

- **Grumman**

$$f = \frac{(t_1+t_2)^2}{E_f d^3} + 3.7\left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right)$$

- **Delft University**

$$f = \left(\left(\frac{2845}{E_{L_1} t_1} + \frac{2845}{E_{L_2} t_2}\right) + c_f \cdot \left(\left(\frac{500}{E_f t_1} + \frac{1000}{E_{ST_1} t_1}\right)\left(\frac{t_1}{d}\right)^2 + \left(\frac{500}{E_f t_2} + \frac{1000}{E_{ST_2} t_2}\right)\left(\frac{t_2}{d}\right)^2\right)\right) \cdot$$

$$\cdot \left(\frac{d_{head}}{d}\right)^{-0.34} \cdot \left(\frac{s}{d}\right)^{-0.5} \cdot \left(\frac{p}{d}\right)^{0.34} \cdot e^{0.3r}$$

$c_f = 1$ for normal aluminium rivets, 8.2 for countersink aluminium rivets and 13.1 for titanium Hi-locks. (E in GPa, t and d in mm)

In general, the formulas follow similar trends while varying the parameters, although there is a very large spread in the absolute values.