Methods for flexibility determination of bolted joints: empirical formula review

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Methods for flexibility determination of bolted joints: empirical formula review

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Abstract. In this work, a comparative analysis of existing methods for determining the compliance of mechanical joints such as 'composite-composite' and 'composite-metal' is performed. Polymer composite materials are widely used in modern aircraft industry. As a result, it is necessary to take into account the characteristics of joints of composite aggregates with composite and metallic ones. Considering the compliance of connections in the global finite element models of structural units of aircraft allows increasing the accuracy of calculations. The use of empirical formulae can significantly reduce time and labour costs in calculating the compliance of bolted connections for use in global finite element models. In this paper we review and analyse the existing empirical dependencies. Calculation of rigidity and compliance of single-shear 'composite-composite' and 'composite-metal' joints by finite element method for small, medium and large membrane thicknesses is carried out, and the results are compared with the calculations using empirical formulae. As a result of the analysis for medium and small thicknesses it is proposed to determine the value of bolted joint flexibility by the empirical formula Boeing 1, and for large thicknesses of connecting membranes it is proposed to use empirical formula Huth.

1. Introduction
The design of the aircraft must meet the requirements for strength and durability, as well as to have minimum weight. Wide use of composite materials allows improving the design characteristics significantly. However, the methods of connecting the structural elements still require careful study [1]. Despite the possibility of using cohesive joints in aircraft structures, mechanical joints, in particular bolted and riveted joints, are widely used. The disadvantages of such joints can be attributed to a large mass, but among the advantages there are high strength and less exposure to moisture.

There are several approaches to the calculation of the bolted joints. The spring method is considered traditional one, since it has long been mastered and gives quite accurate results, but it becomes time-consuming in the composite structures calculation [2,3]. The use of Finite Element Method in the calculation of bolted joints allows you to obtain results with a high degree of accuracy. In paper [4], close attention is paid to bolted joints of composite structures. However, large number of joints with different thicknesses requires a significant amount of time to prepare and calculate the global FEM (Finite Element Method) model. At the same time, the use of empirical formulae can significantly speed up the process, but the reliability of the obtained results is questionable [5].

In engineering, it is common to neglect the influence of joints during the development of global finite element models. This approach reduces the time required to prepare the model and calculate it, but it also reduces the accuracy of calculations. In paper [2] the influence of the bolted joint compliance on the Finite Element model of the aircraft wingbox is presented. The displacements obtained in the wingbox simulation, where the influence of the bolted joints compliance is taken into account, increased by 4.6% compared to the displacements for the model with the use of rigid connection of parts. When calculating aircraft units, such as wingox, the compliance of bolted joints significantly affects the stress-strain state, so it is necessary to take into account the rigidity characteristics of such joints.

Compliance of fastening elements (rivets, bolts, etc.) is a measure of their influence on the stiffness of connections. Recently, the use of finite element models to calculate the compliance of joints has proved relevant, but in engineering practice, this approach may be less effective [6].

For large areas of the connected structural elements of the aircraft in the work [2] it is proposed to use the simulation of the bolted joints rigidity using a cohesive contact. However, the calculation of the stiffness parameters by local modeling is extremely time-consuming. The main task of the work is to reduce the labor costs and computing power of the computer by simplifying the calculations of the compliance of bolted joints with different geometrical and mechanical properties. To complete this task it is necessary to determine the field of empirical formulae application for the structural elements joints of medium and large thicknesses made of metals and composite materials. It is shown that some empirical formulae are valid for certain pairs of thicknesses and materials. When changing the geometric and mechanical properties of a single-shear bolted joint, the same formulae give a large error compared to the calculation of a mathematical model that takes into account the boundary conditions, mechanical properties and geometric parameters of the samples.

2. Methods and calculation

Experimental compliance can be determined by the following method [7]. The scheme is given in the figure 1. The equation is given below (1):
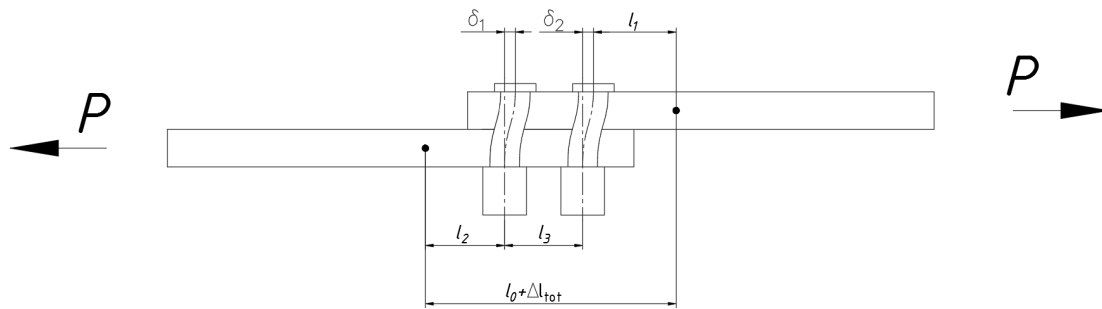

Figure 1. Single-shear joint.

$$\delta = \frac{\delta_1 + \delta_2}{2} = \Delta l_{tot} - \Delta l_{elast}, \Delta l_{elast} = \frac{P}{t_1 w E_1}\left(l_1 + \frac{l_2}{\frac{t_2}{t_1} \cdot \frac{E_2}{E_1}} + \frac{l_3}{1 + \frac{t_2}{t_1} \cdot \frac{E_2}{E_1}} \right),$$

$$\left.\begin{array}{l} C_1 = \dfrac{\delta_1}{P/2} \\[2mm] C_2 = \dfrac{\delta_2}{P/2} \end{array}\right\} C_1 + C_2 = \frac{\delta_1 + \delta_2}{2}, C = \frac{1}{2}\left(C_1 + C_2\right) = \frac{1}{2} \cdot \frac{\delta_1 + \delta_2}{P/2} = \frac{\delta_1 + \delta_2}{P} \qquad (1)$$

The accuracy of the model depends on many factors such as determining material properties, friction coefficient, contact type formulation and pretensioning, etc. Thanks to the accumulated experimental basis, analytical formulae have been derived to determine the compliance.

2.1. Huth Formula (1984)

Linear models are used for analytical calculations. The Huth equations (2) are used for the several types of joints. This formula can be used for both single- and double-shear joints [8,9].

$$C = \left(\frac{t_2 + t_2}{2d}\right)^a \times \frac{b}{n}\left(\frac{1}{t_1 E_1} + \frac{1}{nt_2 E_2} + \frac{1}{2t_1 E_3} + \frac{1}{2nt_2 E_3}\right) \tag{2}$$

where E_1 – upper membrane modulus of elasticity, t_1 – shell thickness, E_2 – lower membrane modulus, t_2 – spar thickness, E_3 – bolt modulus of elasticity, d – bolt diameter, a,b,n – joint-related coefficients (table 1):

Table 1. Coefficient values for different types of joints.

Type of joint	Coefficient value
Single shear joint	n=1
Double shear joint	n=2
Metal bolted joint	a=2/3, b=3
Metal joint with rivets	a=2/5, b=2.2
Composite bolted joint	a=2/3, b=4.2

2.2. Tate & Rosenfeld Formula

This equation (3) applies to a double-shear joint [10,11].

$$C = \left(\frac{8t_2{}^3 + 16t_2{}^2 t_1 + 8t_2 t_1{}^2 + t_1{}^3}{192 E_3 I_3} + \frac{2t_2 + t_1}{3G_3 A_3} + \frac{2t_2 + t_1}{t_2 t_1 E_3} + \frac{1}{t_2 E_2} + \frac{2}{t_1 E_1}\right) \tag{3}$$

where: I_3 – bolt moment of inertia, A_3 – bolt cross-sectional area

2.3. Grumman Formula

The Grumman equation (4) is an empirically derived formula provided by the Grumman Aerospace Corporation and used to develop the Saab 37 Viggen [12,13].

$$C = \frac{(t_1 + t_2)^2}{E_3 d} + 3.72\left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2}\right) \tag{4}$$

2.4. Swift Formula (Douglas Aircraft Company)

This equation (5) applies to a single-shear joint [9].

$$C = \frac{1}{E_3 d}\left[A + Bd\left(\frac{1}{t_1} + \frac{1}{t_2}\right)\right] \tag{5}$$

where: $A = 5$, $B = 0.8$.

2.5. Boeing Formula (Boeing Aircraft Company)

Boeing 1 equation (6) [1,5]:

$$C = \left(\frac{t_2^3 + 5t_2^2 t_1 + 5t_2 t_1^2 + t_1^3}{40E_3 I_3} + \frac{4(t_2 + t_1)}{5G_3 A_3} + \frac{t_2 + t_1}{t_2 t_1 E_3} + \frac{1}{t_2 E_2} + \frac{2}{t_1 E_1} \right) \tag{6}$$

Boeing 2 equation (7) [1]:

$$C = \frac{2^{\left(t_1 / d\right)^{0.85}}}{t_1} \left(\frac{1}{E_1} + \frac{3}{8E_3} \right) + \frac{2^{\left(t_2 / d\right)^{0.85}}}{t_2} \left(\frac{1}{E_2} + \frac{3}{8E_3} \right) \tag{7}$$

2.6. Change in compliance of joined specimens as a function of thickness

Comparison of the presented analytical formulae was given on the example of 'composite-composite' and 'composite-metal' joints.

The joints are made of typical materials for the aviation industry: composite material consisting of epoxy resin as a matrix and carbon fibre as a reinforcing agent, as well as aluminum alloy. Characteristics of joints are presented in tables 2-3.

Table 2. Mechanical characteristics of composite-metal joint and of composite-composite joint.

Bolt modulus of elasticity, MPa	Bolt shear modulus, MPa	Lower metal membrane modulus of elasticity, MPa	Upper composite membrane E1, MPa
Mechanical characteristics of composite-metal joint			
210,000	80,000	70,000	77,648
Mechanical characteristics of composite-composite joint			
210,000	80,000	30,568	77,648

Table 3. Geometric characteristics of a bolt in a joint.

Diameter, mm	Bolt cross-sectional area, mm²	Bolt moment of inertia, mm⁴
7.1	39.592	124.74

For all the formulae, the dependencies of the joint's compliance on the thickness of the membrane were presented. The results of the comparison of the two types of joints are shown in figure 2.
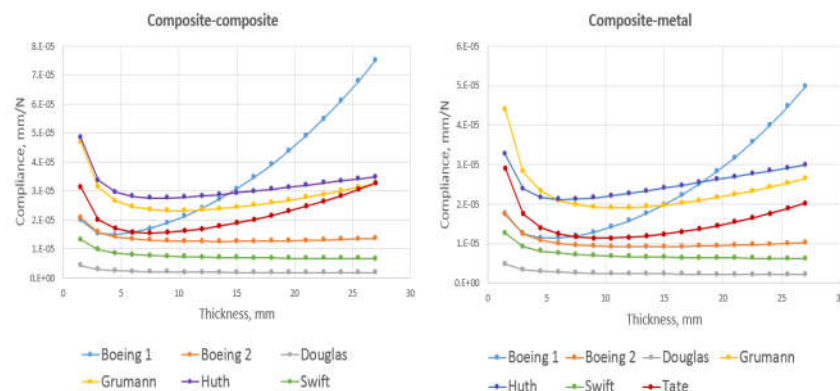

Figure 2. Results of formula comparison for joints: 'composite-metal' (a) and 'composite-composite' (b).

3. Results and discussion

3.1. Description of the finite element model (FEM) for the calculation of the bolted joints compliance
To calculate the stiffness and strength of composite-composite and composite-metal joints of aircraft structural elements, such as fuselage, wing or empennage, it is necessary to develop three-dimensional local finite element models that take into account the initial geometry, elastic and plastic properties of the joints' elements, bolt tightening forces and friction effects.

Further calculation of compliance of bolted joints of 'composite-composite' type will be carried out on an example of fastening of wingbox skin with a spar made of polymeric composite materials (PCM), and joints of 'composite-metal' type on an example of fastening of wingbox skin made of PCM, with metal ribs (figure 3).

To consider the volumetric stress state in the vicinity of the bolted joint, the joint components are modelled using 8-node volumetric finite elements (FE). Skin, spars and ribs are modeled with an average FE size of 0.3 mm in the bolt contact zone and 2 mm closer to the sealing zone. Bolt is also simulated by 8-node FE with an average size of 0.5 mm [14,15].
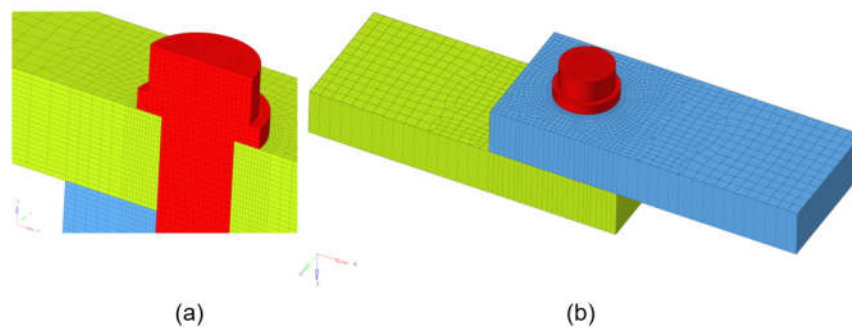

Figure 3. Local FEM: contact zone of the bolt with the membranes (a);
general view of the FEM (b).

Elements made of PCM are simulated with consideration of anisotropic properties of the material. Material properties are determined from the given characteristics of the package, depending on the laying direction and the number of layers [16].

To simulate the interactions between the components of computational models, the method of contact surface discretization 'Surface-to-Surface' is used, in which sets of contact nodes are interpolated by spline surfaces. The penalty method is used to calculate contact forces and displacements. The friction coefficient of the contact surfaces is 0.3.

For the analysis of bolt tightening force influence on rigidity and durability of connections the tightening is simulated by application of a tensile load in section of a bolt on border of contact of a nut and a plate. The 'Pretension Load' tool [17] is used for modelling the bolt preload. According to GOST 1759.4-87 (ISO 898/1-78) for bolt M7 of strength class 8.8 pretension load value is 12,600 N.

3.1.1. Boundary conditions. Stretching force is applied to the upper membrane end by the means of the RBE3 element. For each thickness pair, calculations with tensile forces of 1000 N, 5000 N, 10,000 N, 15,000 N, 20,000 N and 25,000 N are given [18-20]. Restrictions on the displacement along the OZ axis are imposed on the upper membrane ends. Restrictions on the displacement along OX, OY and OZ axes are imposed on the lower membrane ends. Upper and lower membranes have displacement restricted along the OY axis at the side faces. Typical results of calculation may be seen in the figure 4.
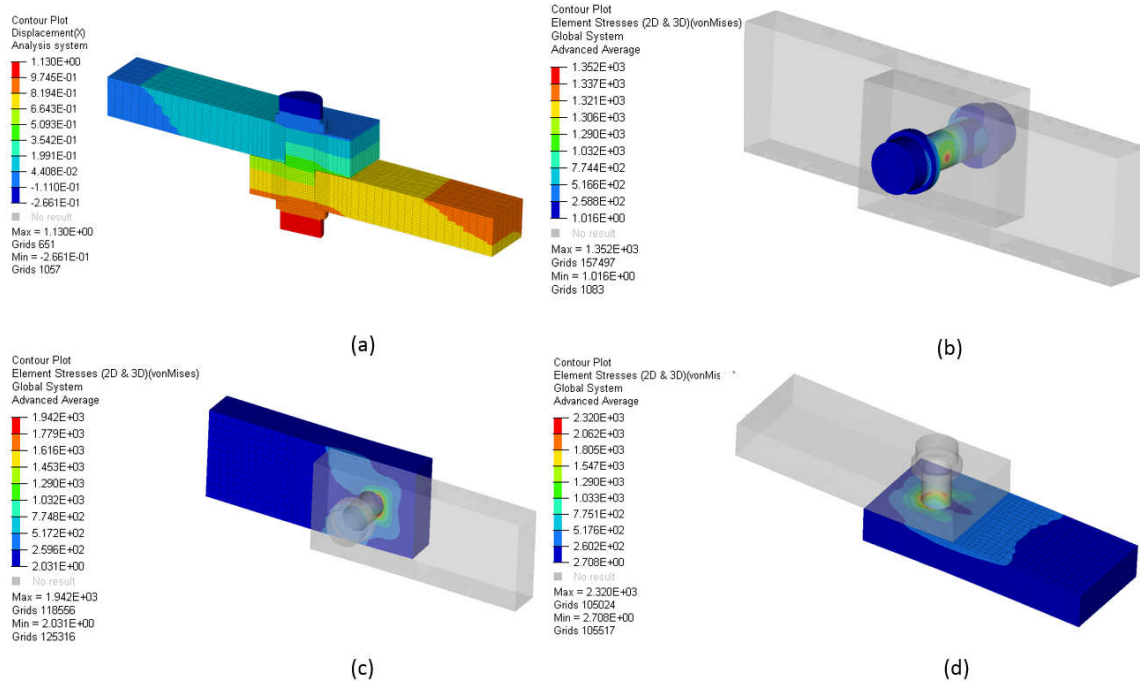
(a)   (b)

(c)   (d)

Figure 4. Typical FEM results: displacement along the axis of OX, mm (deformed state scale - 1 to 1) (a); olt Von Mises stresses, MPa (b); lower membrane Von Mises stresses, MPa (c); upper membrane Von Mises stresses, MPa (d).

3.1.2. Analysis results. For the analysis of the deformed state of joints figure 5 shows the distribution of displacement and stress fields after the application of pre-tensioned bolts and tensile force. For single-shear bolted joints, the bolted joint compliance is calculated by equation (8):

$$C_b\left(P\right) = \frac{d_{U_b}\left(P\right)}{d_P} \tag{8}$$

where C_b – bolted joint compliance, d_{U_b} – bolt displacement in the direction of applied load, P – applied tensile force [21].
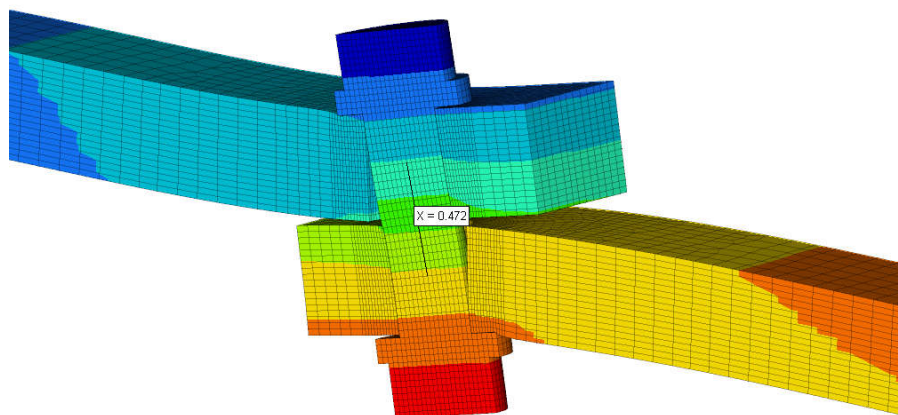

Figure 5. Bolt displacement in the direction of applied load (scale factor 5:1).

Calculation of rigidity and compliance of single-shear 'composite-metal' and 'composite-composite' joints was carried out for small, medium and large thicknesses of the membranes to be connected and compared with the calculations using empirical formulas. Below you can see the calculated load-displacement curves used to determine the joint compliance on an example of a skin (9 mm) - spar (9 mm) joint.
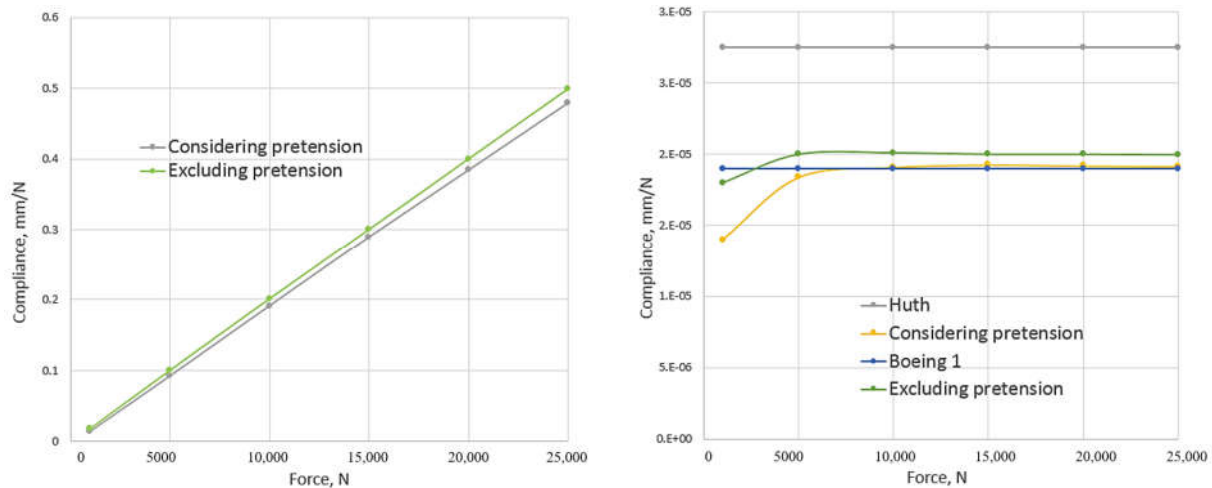

Figure 6. Force-displacement and force-compliance for small and medium-sized membranes.

From the load-displacement curve, you can see that the displacement is linear. At a certain tensile force, the load-compliance curve of the bolted joint becomes invariable.
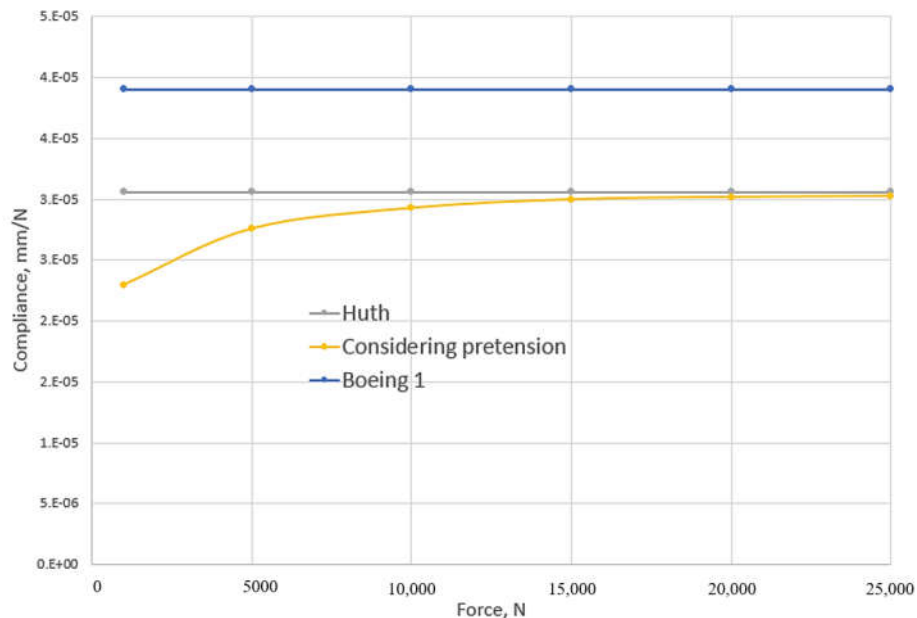

Figure 7. Force-compliance curve for high thickness membranes.

As can be seen in figures 6 and 7 at certain thicknesses of the connecting membranes, the value of compliance by the empirical formula Boeing 1 begins to increase and, determined by the finite element method compliance of the bolted joint, begins to approach the value of the empirical formula Huth.

Table 4 show the values of mean errors of single-shear boltjoints of 'composite-metal' and 'composite-composite' types of joints between the finite element method calculations and empirical formulae.

Table 4. Value of average error of 'composite-metal' and 'composite-composite' types of joints.

	composite-composite		composite-metal	
	Huth	**Boeing 1**	**Huth**	**Boeing 1**
Large membrane thicknesses, %	3.09	40.35	7.65	36.55
Small and medium thicknesses membranes, %	30.27	3.57	28.87	7.79

4. Conclusions

The improved technique presented in this paper allows us to take into account the mutual movements of structural elements without significant complications of the global FEM model. It is achieved by modelling the bolted joints of different wingbox zones by the means of cohesive contacts using the rigidity parameters of the joint elements.

Depending on the properties of materials and thicknesses of connected elements in the calculation of the compliance of single-shear bolted joints, it is proposed to build a thickness-compliance curve according to the empirical dependencies of Huth and Boeing 1, as shown in figure 2. Having determined the intersection point of the curves, you can calculate small and average thicknesses of the samples to be joined (thicknesses up to the intersection point of the curves) and large thicknesses (thicknesses after curves crossing).

For medium and small thicknesses it is proposed to determine the value of bolted joint flexibility by the empirical formula Boeing 1, and for large thicknesses of connecting membranes it is proposed to use the empirical formula Huth.

The method proposed above takes into account the results obtained in course of the carried out work and allows you to significantly reduce labour costs for determining the rigidity of bolted joints of all structural elements of the aircraft.

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