I B. Tech., Even Semester :: A.Y. 2024-25

Linear Algebra & Calculus for Engineers (23MT1001)

CO-1 TUTORIAL PROBLEMS

Tutorial-1

1. Determine rank of a matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

2. A florist is making **5** identical bridesmaid bouquets for a wedding. She has **$610** to spend (including tax) and wants **24** flowers for each bouquet. Roses cost **$6** each, tulips cost **$4** each, and lilies cost **$3** each. She wants to have twice as many roses as the other **2** flowers combined in each bouquet. Obtain the rank of the coefficient matrix of the phenomena.
3. Rohan invests a total of $10,000 in three accounts, one paying 5% interest, another paying 8% interest, and the third paying 9% interest. The annual interest earned on the three investments last year was $770. The amount invested at 9% was twice the amount invested at 5%. Determine the coefficient Matrix and determine the amount invested in each account by Gauss-elimination Method.
4. Using Gauss elimination method, solve $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$.
5. The cost of a ticket to the circus is $25.00 for children and $50.00 for adults. On a certain day, attendance at the circus is 2,000 and the total gate revenue is $70,000. Use Gauss-elimination method to determine number of children and adults bought tickets?
6. A DC circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for current flow in milli amperes:
   $10I_1 + I_2 + I_3 = 12$; $2I_1 + 10I_2 + I_3 = 13$; $I_1 + I_2 + 5I_3 = 7$; Apply Gauss elimination method to solve for I_1, I_2 and I_3
7. An Industrial outlet can produce three types of cloths P, Q and R. Three different kinds of wool are required for it, say red, green and blue wool. One unit of type 'P' cloth needs 2 yards of red wool, 8 yards of green and one yard of blue wool; one unit length of type 'Q' cloth needs one yard of red, 3 yards of green and 5 yards of blue wool; one unit length of type 'R' cloth needs 3 yards red, 1 yard of green and three yards of blue wool.

The firm has only a stock of 6 yards red, 12 yards green and 9 yards of blue wool. Apply LU Decomposition method to determine the number of units of cloth P, Q and R if total stock is used.

Tutorial -2

1. Find all eigenvalues and corresponding eigenvectors for the matrix A if A =. $\begin{bmatrix}1 & 1 & 3\\1 & 5 & 1\\3 & 1 & 1\end{bmatrix}$
2. Determine the Eigen values and Eigen vectors of the matrix A =. $\begin{bmatrix}2 & -3 & 0\\2 & -5 & 0\\0 & 0 & 3\end{bmatrix}$
3. Verify the system dX/dt=AX is stable or not where $\begin{bmatrix}1 & -2\\-5 & 4\end{bmatrix}$ also find the Eigen vectors.
4. Verify the system dX/dt=AX is stable or not where A = $\begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix}$
5. Verify the system dX/dt=AX is stable or not where $\begin{bmatrix}1 & 0\\5 & 7\end{bmatrix}$ also find the Eigen vectors.
6. Find the eigen values of the matrix $A=\begin{bmatrix}3 & 1 & 4\\0 & 2 & 6\\0 & 0 & 5\end{bmatrix}$ and find the Eigen values of A^{-1}, also verify the sum and product of eigen values is same as trace and determinant of matrix A.
7. If the product of two eigen values of the matrix $\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}$ is 16, then find the third eigen value.

Tutorial-3

1. Check whether the matrix A=$\begin{bmatrix}2 & 3\\3 & 2\end{bmatrix}$ is diagonalizable or not.
2. Find the Modal matrix P, which transforms $A=\begin{bmatrix}8 & -8 & -2\\4 & -3 & -2\\3 & -4 & 1\end{bmatrix}$ to the diagonal form.
3. Identify the nature of the quadratic form $3x^2+5y^2+3z^2+2xy+2yz+2zx$.
4. Reduce the Q.F. $x_1^2+4x_2^2+x_3^2-4x_1x_2+2x_1x_3-4x_2x_3$ to the canonical form and hence determine its rank, index, signature and nature.
5. Reduce the Q.F. $x^2+2y^2-3z^2$ to the sum of squares form and hence determine its rank, index, signature and nature.