SECTION-C

1.A 50μF capacitor is connected across a 230V, 50Hz supply. Calculate (a) the reactance offered by the capacitor (b) the maximum current (c) the r.m.s. value of the current drawn by the capacitor (d) the maximum energy stored in the capactitor and (e) plot the current and voltage waveforms.

given that $C = 50\mu F$, $f = 50Hz$ $\therefore \omega = 2\pi f = 100\pi$ and $V_m = 230V$

Capacitive reactance $X_c = \frac{1}{\omega C} = \frac{1}{100\pi \times 50\times 10^{-6}} = 63.66\Omega$

The capacitive reactance is $-jX_C = -j63.66 = 63.66\angle -90°$

The maximum current $I_m = \frac{V_m}{|X_C|} = \frac{230}{63.66} = 3.613A$

The rms value of the current $= \frac{I_m}{\sqrt{2}} = 0.707\times 3.613 = 2.554A$

The maximum energy stored $E = \frac{1}{2}CV_m^2 = \frac{1}{2}\times 50\times 10^{-6}\times 230^2$

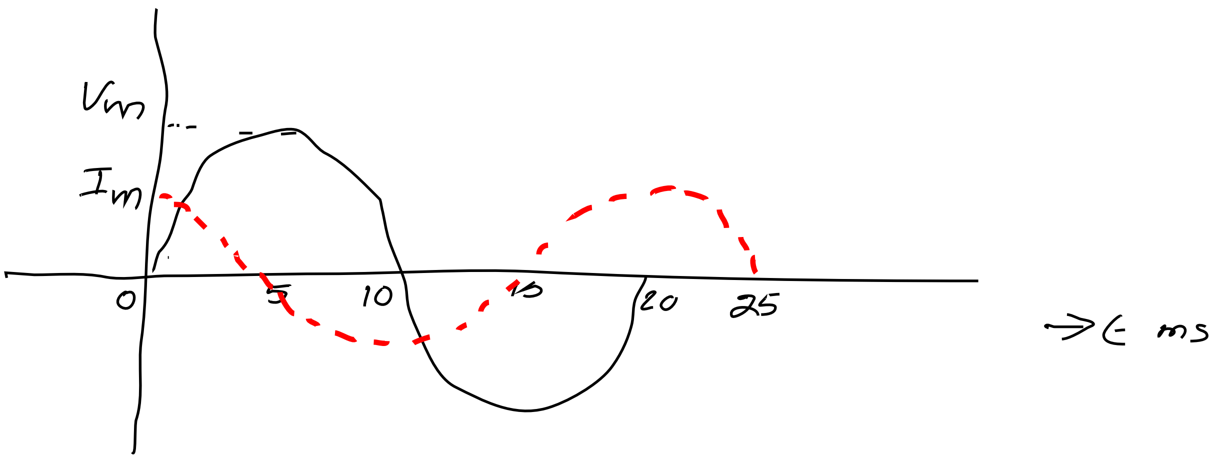
$= 1.323$ Joules

Here $V = 230 \sin 100\pi t$

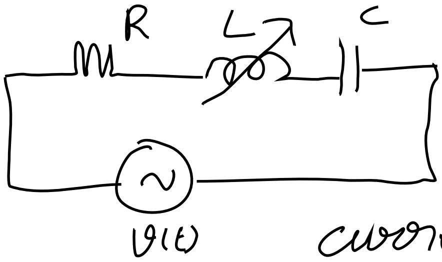
$V = 230\angle -90°$; $I = \frac{230\angle -90°}{63.66\angle -90°} = 3.613\angle 0°$

$\therefore i = 3.613 \cos 100\pi t = 3.613 \sin(100\pi t + 90°)$

Here $f = 50Hz$ $\therefore T = \frac{1}{50} = 20ms$



2.For a series R-L-C circuit the inductor is variable. Source voltage is 283sin100πt. Maximum current obtainable by varying the inductance is 0.314 A and the voltage across the capacitor then is 300V. Find the circuit element values, resonance frequency, Q-factor and bandwidth of the circuit.



Here $v(t) = 283\sin 100\pi t$ V

$\omega = 100\pi$.

Since 'L' varied at one instant current is maximum means, the circuit is in resonance and the resonance frequency $\omega_0 = \omega$

$\Rightarrow \omega_0 = 100\pi \Rightarrow f_0 = \frac{100}{2\pi} = 50\,Hz$

at resonance current is maximum

$I = I_0 = \frac{V_m}{|Z|}$. Here $Z = R + jX$

where $X = 0 \Rightarrow X_L = X_C$

$\therefore I_0 = \frac{V_m}{R} = \frac{283}{R} \Rightarrow R = \frac{283}{0.314} = 901.27\Omega$

at resonance $|jX_L| = |-jX_C|$

and $|V_L| = |V_C| = 300$

but $|V_L| = |jX_L I_0| = \omega_0 L I_0 = 300$

$\therefore L = \frac{300}{\omega_0 I_0} = \frac{300}{100\pi \times 0.314} = 3.041\,H$

but $|V_C| = |-jX_C I_0| = \frac{I_0}{\omega_0 C} = 300$

$\Rightarrow C = \frac{I_0}{300\omega_0} = \frac{0.314}{300 \times 100\pi} = 3.332\,\mu F$

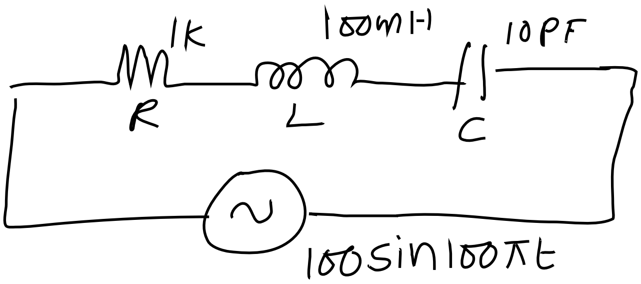
The Q-factor $= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}} = 1.06$

Bandwidth $\Delta f = \frac{R}{2\pi L} = 47.17\,Hz$

or $\Delta f = \frac{f_0}{Q} = \frac{50}{1.06} = 47.17\,Hz$

10μF

3.A series circuit with R = 1kΩ, L = 100 mH and C = 10μF is supplied with 100V,50 Hz. Determine the impedance, current, power factor, resonance frequency, Q-factor and bandwidth of the circuit.



Here $jX_L = j\omega L$

$= j100\pi \times 100 \times 10^{-3}$

$= j10\pi\ \Omega = 10\pi\angle 90^\circ$

$$-jX_C = \frac{-j}{\omega C} = \frac{-j}{100\pi \times 10 \times 10^{-6}} = -j318.31\ \Omega$$

The impedance $Z = R + jX_L - jX_C = 1000 + j10\pi - j318.31$

$= 1000 - j286.9\ \Omega = 1040\angle -16^\circ$

at resonance $Z = R = 1000\Omega$

current $I_0 = \dfrac{V_m}{R} = \dfrac{100}{1000} = 0.1A$

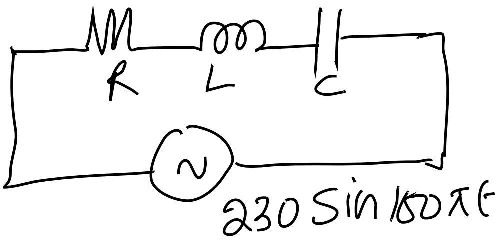
Power factor $PF = \cos 0^\circ = 1$

resonance frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 159.15Hz$

Q-factor $= \dfrac{\omega_0 L}{R} = \dfrac{2\pi \times 159.15 \times 100 \times 10^{-3}}{1000} \simeq 0.1$

Bandwidth $\Delta f = \dfrac{R}{2\pi L} = 1591.5Hz = \dfrac{f_0}{Q}$

4.A series circuit with R = 100Ω, L = 10 mH and C = 10 μF is supplied with 230V,50 Hz. Determine the impedance, current, power factor resonance frequency, Q-factor and bandwidth of the circuit.



$$jX_L = j\omega L = j\,100\pi \times 10 \times 10^{-3}$$

$$= j\pi\,\Omega = \pi\angle 90^\circ\,\Omega$$

$$-jX_C = \frac{-j}{\omega C} = \frac{-j}{100\pi \times 10 \times 10^{-6}}$$

$$= -j\,318.3\,\Omega = 318.3\angle -90^\circ\,\Omega$$

$$\therefore Z = R + jX$$

$$= 100 + j3.14 - j318.3 = 100 - j315.17 = 330.65\angle -72.4^\circ$$

The impedance at resonance $Z = R$ $\quad \because X = 0$

$$\therefore Z = 100\Omega \;;\; \text{current } I_0 = \frac{V_m}{|Z|} = \frac{V_m}{R} = \frac{230}{100} = 2.3A$$

$$\text{power factor} = \cos 0^\circ = 1$$

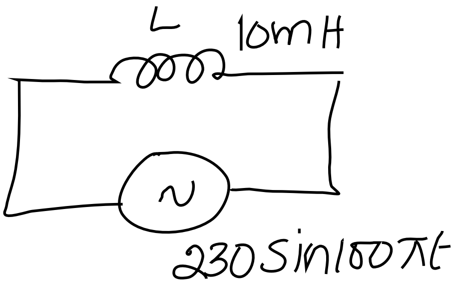
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10\times10^{-3}\times10\times10^{-6}}} = 503.3\,Hz$$

$$Q\text{-factor }(Q) = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi\times503.3\times10\times10^{-3}}{100}$$

$$= 0.316$$

$$\text{Bandwidth } \Delta f = \frac{R}{2\pi L} = \frac{f_0}{Q} = \frac{100}{2\pi\times10\times10^{-3}} = 1591.5\,Hz$$

5. A 10mH inductor is connected across a 230V, 50Hz supply. Calculate (a) the reactance offered by the inductor (b) the maximum current and (c) the rms value of the current drawn by the inductor (d) the maximum energy stored in the inductor and (e) plot the current and voltage waveform.



The reactance JX_L

$$= J\omega L$$

$$= J\,100\pi \times 10 \times 10^{-3}$$

$$= J\pi\,\Omega = \pi\angle 90^\circ$$

$$v = 230 \sin 100\pi t$$

$$\therefore V = 230\angle 90^\circ$$

Hence current $I_m = \dfrac{V}{JX_L}$

$$= \frac{230\angle -90^\circ}{\pi\angle 90^\circ} = 73.21\angle -180^\circ\ A$$

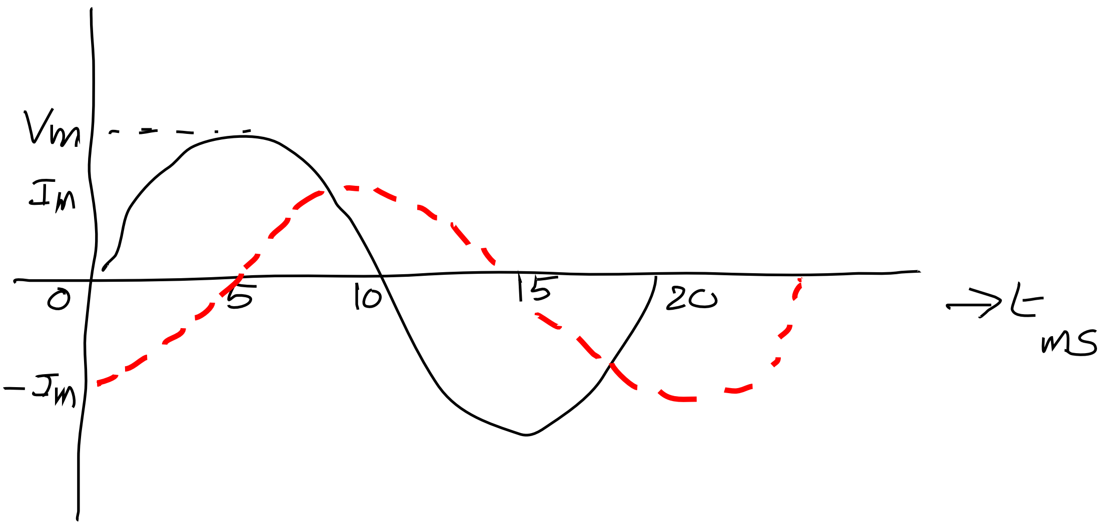
i.e $i = 73.21 \sin(100\pi t - 90^\circ)$

The rms current $= \dfrac{I_m}{\sqrt{2}} = 73.21 \times 0.707 = 51.76\,A$

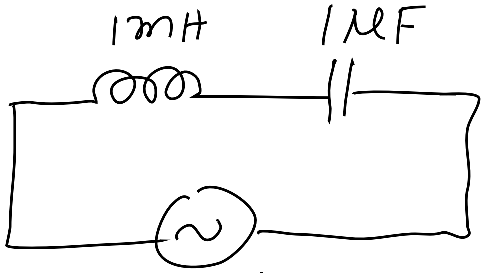
The maximum Energy stored in the inductor

is $E = \dfrac{1}{2} L I_m^2 = \dfrac{1}{2} \times 10 \times 10^{-3} \times 73.21^2$

$$= 26.8 \text{ Joules}$$



6. A 1mH inductor and a 1μFcapacitor are connected in series with a 230V, 50Hz supply. Calculate (a) the reactance offered by the inductor (b) the maximum current and (c) the rms value of the current drawn by the inductor (d) the maximum energy stored in the inductor and (e) plot the current and voltage waveform.



Here $\omega = 100\pi$

$v = 230 \sin 100\pi t$

$V = 230 \angle -90^\circ$

inductive reactance $jX_L = j\omega L = j\,100\pi \times 1 \times 10^{-3}$

$= j0.314\ \Omega = 0.314 \angle 90^\circ$

capacitive reactance $-jX_C = \dfrac{-j}{\omega C} = \dfrac{-j}{100\pi \times 10^{-6}}$

$= -j3183\ \Omega = 3183 \angle -90^\circ\ \Omega$

Now $Z = jX_L + jX_C = j0.314 - j3183$

$= -j3182.7\ \Omega = 3182.7 \angle -90^\circ\ \Omega$

Hence current $= \dfrac{V}{Z} = \dfrac{230 \angle -90^\circ}{3182.7 \angle -90^\circ} = 72.3 \angle 0^\circ$ mA

Hence $i = 72.3 \sin(100\pi t + 90^\circ)$ mA

maximum current in the inductor $= 72.3$ mA

rms current $= 72.3 \times 0.707 = 51$ mA

maximum energy stored in the inductor is

$E = \frac{1}{2} L I_m^2 = \frac{1}{2} \times 10^{-3} (72.3 \times 10^{-3})^2$

$= 2.613\ \mu$ Joules