# Problems and Solutions on Energy and Power

1 Determine the values of $P_\infty$ and $E_\infty$ for each of the following signals:

**(a)** $x_1(t) = e^{-2t}u(t)$ **(b)** $x_2(t) = e^{j(2t+\pi/4)}$ **(c)** $x_3(t) = \cos(t)$

**Ans:** **(a)** $x_1(t) = e^{-2t}u(t)$ $\quad E_\infty = \int_0^\infty e^{-4t}dt = \frac{1}{4}$, $P_\infty = 0$, because $E_\infty < \infty$

**(b)** $x_2(t) = e^{j(2t+\frac{\pi}{4})}$, $|x_2(t)| = 1$. Therefore, $E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} dt = \infty$,

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_2(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} dt = \lim_{T\to\infty} 1 = 1$$

**(c)** $x_3(t) = \cos(t)$. Therefore, $E_\infty = \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \int_{-\infty}^{\infty} \cos^2(t) dt = \infty$,

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2(t)dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left(\frac{1+\cos(2t)}{2}\right) dt = \frac{1}{2}$$

2. Consider a continuous time signal $x(t) = ¢(t+2) - ¢(t-2)$. Find the energy of a signal

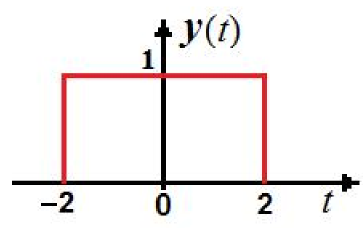
$$y(t) = \int_{-\infty}^{t} x(^2)\, d^2 .$$

**Ans:** We know that $u(t) = \int_{-\infty}^{t} ¢(t)\, dt$

Then $y(t) = \int_{-\infty}^{t} x(^2)\, d^2 = \int_{-\infty}^{t} \{¢(t+2) - ¢(t-2)\}\, dt$

$$= \int_{-\infty}^{t} ¢(t+2)\, dt - \int_{-\infty}^{t} ¢(t-2)\, dt$$

$$= u(t+2) - u(t-2)$$




This signal is shown in figure. Since it is pulse signal it is an energy signal.

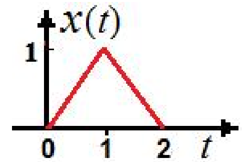
Therefore the energy of the signal $y(t)$, $E_y = \int_{-\infty}^{\infty} |y(t)|^2\, dt = \int_{-2}^{2} 1\, dt = 4$ Jouls.

3. Consider a signal represented by

$$x(t)=\begin{cases} t, & 0\le t\le 1 \\ 2-t, & 1\le t\le 2 \\ 0, & Otherwise \end{cases}$$

Classify whether this signal is power or energy signal, and find its value. Suppose now this signal is shifted right side by 2 units (i.e., $x(t-2)$). Find the new signal for energy or power?. Are there any changes in the value of power or energy calculated? Give the reasons.

**Solution:** Since the given signal is a finite duration signal, it is referred to as energy signal. Then the energy is computed as below.

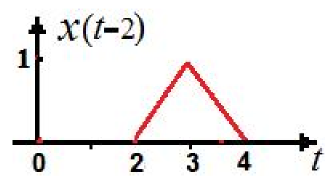



$$E=\int_{-\infty}^{\infty}|x(t)|^2\,dt=\int_0^1 t^2\,dt+\int_1^2(2-t)^2\,dt=\int_0^1 t^2\,dt+\int_1^2(4-4t+t^2)\,dt$$

$$=\left[\frac{t^3}{3}\right]_0^1+\left[4t-4\frac{t^2}{2}+\frac{t^3}{3}\right]_1^2=\left[\frac{1}{3}\right]+\left[4(1-0)-2(4-1)+\frac{8-1}{3}\right]=\frac{2}{3}\text{ Jouls}$$

If the signal is shifted by 2 units right then he shifted signal is represented by

$$x(t-2)=\begin{cases} t-2, & 2\le t\le 3 \\ 4-t, & 3\le t\le 4 \\ 0, & Otherwise \end{cases}$$




Now the energy of the shifted signal is

$$E_{shifted}=\int_{-\infty}^{\infty}|x(t-2)|^2\,dt=\int_2^3(t-2)^2\,dt+\int_3^4(4-t)^2\,dt=\int_2^3(t^2-4t+4)\,dt+\int_3^4(16-8t+t^2)\,dt$$

$$=\left[\frac{t^3}{3}-4\frac{t^2}{2}+4t\right]_2^3+\left[16t-8\frac{t^2}{2}+\frac{t^3}{3}\right]_3^4=\left[\frac{27-8}{3}-2(9-4)+4(3-2)\right]+\left[16(4-3)-4(16-9)+\frac{64-27}{3}\right]=\frac{2}{3}\text{ Jouls}$$

The energy content will not change even though the signal is shifted.

4. Determine the power for each of the following signals.

(i) $5+10\cos(100\text{®}t+\text{®}/3)$ (ii) $10\cos(100\text{®}t+\text{®}/3)+16\sin(100\text{®}t+\text{®}/5)$

(iii) $(10+2\sin 3t)\cos 10t$ (iv) $x(t)=\sin t\,u(t)$ (v) $e^{jat}\cos\Omega_0 t$ (vi) $u(t)$ (vii) $r(t)$

(viii) $x(t)=e^{-at}$, $a>0$, for $t\ge 0$ and zero otherwise.

**Ans:**

(i) Given that $x(t)=5+10\cos(100\text{®}t+\text{®}/3)$

Average Power $P=\lim\limits_{T\to\infty}\dfrac{1}{2T}\displaystyle\int_{-T}^{T}|x(t)|^2dt$

$$= 5^2 + \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|10\cos(100\text{®}t + \tfrac{\text{®}}{3})\right|^2 dt = 25 + \frac{10^2}{2} = 75W$$

(ii) Given that $x(t) = 10\cos(100\text{®}t + \text{®}/3) + 16\sin(100\text{®}t + \text{®}/5)$

Average Power $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

We know that the power of the signal $A\cos(\Omega_0 t + \bar{\ })$ is $A^2/2$ Watts

$$\text{Power } P = \frac{10^2}{2} + \frac{16^2}{2} = 178W$$

(iii) Given that $x(t) = (10 + 2\sin 3t)\cos 10t$

$$= 10\cos 10t + 2\sin 3t \cos 10t$$
$$= 10\cos 10t + \sin 13t - \sin 7t$$

$$\text{Power } P = \frac{10^2}{2} + \frac{1^2}{2} + \frac{1^2}{2} = 51W$$

(iv) Given that $x(t) = \sin t\, u(t)$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |\sin t|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} \frac{(1-\cos 2t)}{2} dt = 0.25 \text{ Watts}$$

(v) Given that $x(t) = e^{jat}\cos\Omega_0 t = \frac{1}{2}\left[e^{j(a+\Omega_0)t} + e^{j(a-\Omega_0)t}\right]$

Power $P = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}W$

(vi) Given that $x(t) = u(t)$. The power of the signal is computed as below.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |u(t)|^2 dt = [t]_{-\infty}^{\infty} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |u(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} 1^2 dt = \frac{1}{2} \text{ Watts}$$

(vii) Given that $x(t) = r(t)$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |r(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |tu(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |t|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \left[\frac{|t|^3}{3}\right]_0^T = \infty$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left|5 + 10\cos(100\text{®}t + \tfrac{\text{®}}{3})\right|^2 dt = \infty$$

Hence $x(t)=r(t)$ is neither energy nor power signal

(viii) Given that $x(t)=e^{-at}, a>0$, for $t \geq 0$ and zero otherwise.

$$E=\int_{-\infty}^{\infty}|x(t)|^2 dt=\int_{-\infty}^{\infty}\left|e^{-at}u(t)\right|^2 dt=\int_{0}^{\infty}e^{-2at}dt=\left[\frac{e^{-2at}}{-2a}\right]_0^{\infty}=\frac{1}{2a}$$

$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2 dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left|e^{-at}u(t)\right|^2 dt$$

$$=\lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T}e^{-2at}dt=\lim_{T\to\infty}\frac{1}{2T}\left[\frac{e^{-2at}}{-2a}\right]_0^{T}=\lim_{T\to\infty}\frac{1}{2T}\left[\frac{e^{-2aT}-1}{-2a}\right]=0$$

5. Find the power of a periodic signal $x(t)=4\cos(300\circledR t-\circledR/6)$

Let (i) $y(t)=x(2t)$ and (ii) $z(t)=x(t/2)$. Test whether these signals are an energy or power signal, and find its value.

**Solution.** The given signal $x(t)=4\cos(300\circledR t-\circledR/6)$ is a periodic signal and hence power signal. Power of the given periodic signal is computed as below.

$$P=\frac{1}{T}\int_0^T |x(t)|^2\, dt=\frac{1}{T}\int_0^T |4\cos(300\circledR t-\frac{\circledR}{6})|^2\, dt=\frac{1}{T}\int_0^T 16\cos^2(300\circledR t-\frac{\circledR}{6})dt$$

$$=\frac{1}{T}\int_0^T\frac{16}{2}\left[1-\cos 2(300\circledR t-\frac{\circledR}{6})\right]dt=\frac{8}{T}\int_0^T dt-\int_0^T\cos 2(300\circledR t-\frac{\circledR}{6})\,dt$$

$$=\frac{8}{T}[t]_0^T=\frac{8}{T}T=8\text{ Watts}$$

Energy is computed as below:

$$E=\int_{-\infty}^{\infty}|x(t)|^2\, dt=\int_{-\infty}^{\infty}|4\cos(300\circledR t-\frac{\circledR}{6})|^2\, dt=\int_{-\infty}^{\infty}16\cos^2(300\circledR t-\frac{\circledR}{6})dt$$

$$=\int_{-\infty}^{\infty}\frac{16}{2}\left[1-\cos 2(300\circledR t-\frac{\circledR}{6})\right]dt=8\int_{-\infty}^{\infty}dt-8\int_{-\infty}^{\infty}\cos(600\circledR t-\frac{\circledR}{3})\,dt$$

$$=\infty$$

For any sinusoidal signal any phase or any frequency the average power always equal to $A^2/2$. Where A is peak value.

(i) $y(t)=x(2t)$ is the compressed version of $x(t)$. That is $y(t)=x(2t)=4\cos(600\circledR t-\frac{\circledR}{6})$.

(ii) $z(t)=x(t/2)$ is the expanded version of $x(t)$. That is $z(t)=x\left(\frac{t}{2}\right)=4\cos(150\circledR t-\frac{\circledR}{6})$.

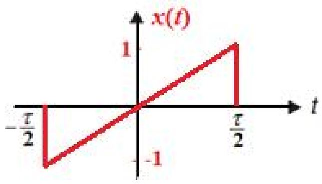
These two are also periodic and power signals. The power completely depends only on peak value, but not on the frequency and phase components. Here the peak values are not changed in both cases. Hence the power is same as 8 Watts.

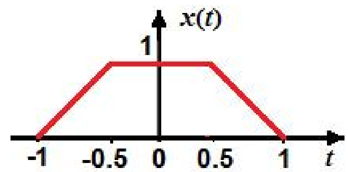
6. Find the power / energy of the following signals

(a) A rectangular pulse having unit height and unit width centered at origin.

(b) A triangular pulse having unit height and unit width centered at origin.

(c) Signals shown below:

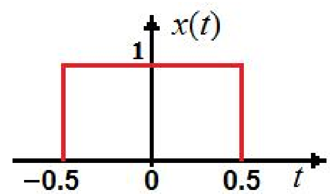



**Solution.**

(a) Rectangular pulse is a finite duration signal. Hence it is a energy signal

A rectangular pulse having unit height and unit width centered at origin is shown in below figure.




The graphical representation of given signal is shown in figure.

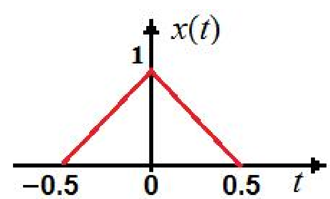
The mathematical expression is $x(t)=\begin{cases}1, -0.5<t<0.5\\ 0, \quad Elsewhere\end{cases}$

Since it is pulse signal it is energy signal and is computed below

The energy $E=\int_{-\infty}^{\infty}|x(t)|^2\,dt=\int_{-0.5}^{0.5}1\,dt=1\text{ Joul}$

(b) The graphical representation of given signal is shown in figure.

The mathematical expression is $x(t)=\begin{cases}2t+1, & -0.5\le t\le 0\\ -2t+1, & 0\le t\le 0.5\\ 0, & Otherwise\end{cases}$




Since it is pulse signal it is energy signal and is computed below

$$E=\int_{-\infty}^{\infty}|x(t)|^2\,dt=\int_{-0.5}^{0}|2t+1|^2\,dt+\int_{0}^{0.5}|-2t+1|^2\,dt$$

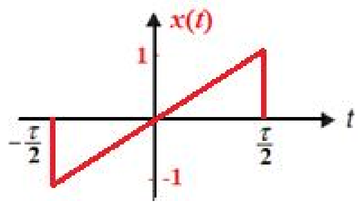
$$=\int_{-0.5}^{0}(4t^2+4t+1)\,dt+\int_{0}^{0.5}(4t^2-4t+1)\,dt$$

$$=\left[4\frac{t^3}{3}+4\frac{t^2}{2}+t\right]_{-0.5}^{0}+\left[4\frac{t^3}{3}-4\frac{t^2}{2}+t\right]_{0}^{0.5}$$

$$=\left[4\frac{0.125}{3}+4\frac{-0.25}{2}+0.5\right]+\left[4\frac{0.125}{3}-4\frac{0.25}{2}+0.5\right]=\frac{1}{3}\text{ Jouls}$$

(c) The given both signals are finite duration signals. Hence they are energy signals

(i)



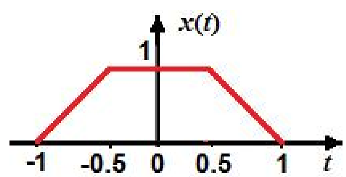

The given signal in mathematical form is represented by $x(t) = \begin{cases} \frac{2t}{2}, -\frac{2}{2} \le t \le \frac{2}{2} \\ 0, elsewhere \end{cases}$

This signal is a pulse signal, it is an aperiodic signal. Then the energy of the signal is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-2/2}^{2/2} |\tfrac{2t}{2}|^2 \, dt = \frac{4}{2\,2} \int_{-2/2}^{2/2} |t|^2 \, dt$$

$$= \frac{4}{2\,2} \int_{-2/2}^{2/2} |t|^2 \, dt = \frac{4}{2\,2} \left[ \frac{t^3}{3} \right]_{-2/2}^{2/2} = \frac{2}{3} \text{Jouls}$$

(ii) This is also a finite duration signal. Hence it is a energy signal.



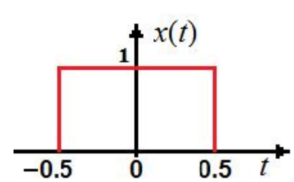

The mathematical equation for given signal is

$$x(t) = \begin{cases} 2(t+1), & -1 \le t \le -\frac{1}{2} \\ 1, & -\frac{1}{2} \le t \le \frac{1}{2} \\ -2(t-1), & \frac{1}{2} \le t \le 1 \end{cases}$$

Then the energy of the signal is

$$E = \int_{-1}^{-1/2} [2(t+1)]^2 \, dt + \int_{-1/2}^{1/2} 1^2 \, dt + \int_{1/2}^{1} [-2(t-1)]^2 \, dt = \frac{4}{3} \text{ Jouls}$$

7. Consider a rectangular pulse $x(t)$ having unit height and unit width centered at origin. Suppose this signal is undergone the operations of compression $y(t) = x(2t)$ and expansion $z(t) = x\left(\frac{t}{2}\right)$. Find the energies of these signals and comment.

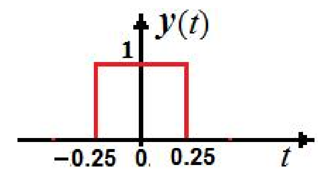


**Ans:** The graphical representation of given signal $x(t)$ is shown in figure.

The mathematical expression is $x(t) = \begin{cases} 1, -0.5 < t < 0.5 \\ 0, \quad Elsewhere \end{cases}$

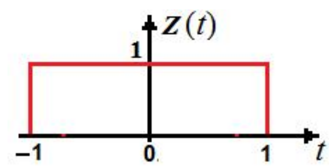
Since it is pulse signal it is an energy signal and is computed below

The energy $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-0.5}^{0.5} 1 \, dt = 1 \text{ Joul}$




(i) $y(t) = x(2t)$: The signal is shown in figure.

Then the energy $E_y = \int_{-\infty}^{\infty} |y(t)|^2 \, dt = \int_{-0.25}^{0.25} 1 \, dt = 0.5 \text{ Jouls}$




(ii) $z(t) = x\left(\frac{t}{2}\right)$: The signal is shown in figure

Then the energy $E_z = \int_{-\infty}^{\infty} |y(t)|^2 \, dt = \int_{-1}^{1} 1 \, dt = 2 \text{ Jouls}$

8. Justify the following signals having the same energy content.