KONERU LAKSHMAIAH EDUCATION FOUNDATION
(DEEMED TO BE UNIVERSITY)
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I/IV B. Tech. Even Semester :: A.Y. 2024-25
Linear Algebra & Calculus for Engineers (23MT1001)
CO-3- Classroom delivery problems
SESSION-16 &17
Beta and Gamma functions and Its Application

1. Define gamma function and discuss its properties (without proof).
2. Determine the following:
   (a) $\Gamma(4.5)$ (b) $\Gamma(-3.5)$ (c) $\Gamma(7)$ (d) $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$
3. Evaluate $\int_0^\infty x^5 e^{-x}\,dx$ using gamma function
4. Evaluate $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$, $n > 0$.
5. Define beta function and discuss its properties.
6. Evaluate $\int_0^1 \frac{x}{\sqrt{1-x^5}}\,dx$ in terms of beta function.
7. State the relation between beta and gamma function.
8. Use beta and gamma function to evaluate $\int_0^1 x^4(1-x)^3\,dx$
9. Evaluate $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}\,dx$ using beta-gamma functions.
10. Use beta and gamma function to evaluate $\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$
11. Evaluate $\int_0^{\pi/2} \sin^5\theta\,\cos^{7/2}\theta\,d\theta$
12. Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\,d\theta$

SESSION-18

Double integrals in Cartesian and polar coordinates with applications

1. Introduction to double integrals.
2. Evaluate the integral $\int_0^1\int_1^2 (x+y)\,dy\,dx$

3. Evaluate the integral $\int_0^4 \int_0^{\sqrt{4-x^2}} (xy)\, dy\, dx$
4. Determine the area enclosed by the parabolas $y = 4x^2$ *and* $x = 4y^2$
5. Evaluate $\iint_C xy\, dx\, dy$ where C is the region bounded by y-axis, ordinate y=2a and the curve $y^2 = 4ax$.
6. Evaluate the integral $\iint r \sin\theta \; dr\, d\theta$ over the cardioid $r = a(1-\cos\theta)$ about the initial line.
7. Calculate $\iint r^3\, dr\, d\theta$ over the area included between the circles $r = 4sin\theta$ and $r = 6sin\theta$.
8. Evaluate $\iint_R r^2 \sin\theta\, dr\, d\theta$ where R is the semi-circle $r = 2a\cos\theta$ above the initial line.

SESSION-19

Change to polar coordinates

1. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing to polar co-ordinates.
2. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dxdy$ by changing to polar co-ordinates
3. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ by changing to polar co-ordinates
4. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2 + y^2} dxdy$ by changing to polar co-ordinates

SESSION-20

Change of order of integrations

Change the order of integration and hence evaluate the following:

1. $\int_0^a \int_y^a \frac{4x}{x^2 + y^2}\, dx\, dy$

2. $\int_0^1 \int_{e^x}^e \frac{dx\, dy}{2\log y}$

3. $\int_0^1 \int_{\sqrt{y}}^1 (xy + \sin(x^4))\, dx\, dy$

4. $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\; dx$

5. $\int_{-a}^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)\, dx\; dy$

SESSION-21
Evolution of Triple integrals

1. Evaluate $\int_{x=0}^{1} \int_{y=0}^{2} \int_{z=0}^{3} x\ y^2 z\ dx\,dy\,dz$

2. Evaluate $\int_0^a \int_0^b \int_0^c xy^2\ dx\,dy\,dz$

3. Evaluate $\int_{x=0}^{x=1} \int_{y=0}^{y=x} \int_{z=0}^{z=x+y} xyz\,dz\,dy\,dx$

4. Evaluate $\int_{x=0}^{x=a} \int_{y=0}^{y=x} \int_{z=1}^{z=x+y} e^{x+y+z}\ dz\,dy\,dx$

5. Evaluate the following integral. $\iiint_B 8xyz\,dV$, where

   $B: 2 \le x \le 3, 1 \le y \le 2\ and\ 0 \le z \le 1$

6. Evaluate the triple integral $\int_1^e \int_1^{logy} \int_1^{e^x} logzdzdxdy$

SESSION-22
Applications of triple integrals

1. Calculate the volume of the solid bound by the planes $x = 0, y = 0, z = 0\, and\ x + y + z = 1$.

2. Determine the volume of the region bounded by the plane $2x + y + z = 3$ that lies in the first octant

3. Evaluate $\iiint (x + y + z)\,dx\,dy\,dz$ bounded by the planes $x = 0, y = 0, z = 0\, and\ x + y + z = 4$

4. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 4$