KONERU LAKSHMAIAH EDUCATION FOUNDATION

(Deemed to be University, Estd. u/s. 3 of UGC Act 1956)

I/IV B. Tech. Even Semester :: A.Y. 2024-25

Linear Algebra & Calculus for Engineers (23MT1001)

CO-2 CLASSROOM DELIVARY PROBLEMS

Session-9: Partial derivatives

1. Find all the first and second order partial derivatives of the function $f(x,y)=x^2y+\sin x+\cos y$ and prove that $f_{xy}=f_{yx}$.
2. Determine all the first and second order partial derivatives of $f(x,y)=x^3+y^4+4ax^2y$ at the point (1, 1).
3. Determine all the first and second order partial derivatives of $u=x^2\cos y-e^{4x}+\log y$ at the point (1, $\pi/4$).
4. Compute all the first and second order partial derivatives of $f(x,y)=3e^{xy}+4x^2-y^3+5$
5. Calculate all the first and second order partial derivatives of $f(x,y)=7\sin(2x+y)+6\cos(x-y)$.
6. Verify $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$ for the function $u=\sin^{-1}\left(\frac{x}{y}\right)$.
7. Find all the first and second order partial derivatives of $f(x,y)=\cos(2x)-x^2e^{5y}+3y^2$ and prove that $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$.

Session-10: Total derivatives and Jacobian

1. Find the value of $\frac{du}{dt}$ given $u=y^2-4ax, x=at^2, y=2at$
2. Given that $u=\cos\left(\frac{x}{y}\right)$, and $x=e^t$, $y=t^2$ then calculate the total derivative $\frac{du}{dt}$.
3. Find total derivative $\frac{du}{dt}$, where $u=x^2+y^2+z^2, x=e^{2t}, y=\sin 3t, z=\cos 3t$.
4. Calculate $\frac{\partial(u,v)}{\partial(x,y)}$ for the function $u=x^2-2y, v=3x+y$.
5. Given $x=r\cos\theta, y=r\sin\theta$ Find the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$.
6. If $u=x^2-2y, v=x+2y+z^2, w=x-2y^2+3z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,0,0).

Session-11: Taylor's and Maclaurin's series expansions for functions of two variables

1. Express the Taylor's series expansion for $f(x, y) = x^2y + 6y + 5x - 2$ in powers of $(x+1)$ and $(y-2)$ up-to second degree.
2. Expand $f(x,y) = \sin x \cos y$ in powers of 'x' and 'y' up to the terms of second degree.
3. Expand the function $f(x, y) = e^x \log(y)$ in terms of 'x-1' and 'y-1' up to the terms of second degree.
4. Expand the function $f(x, y) = \cos(xy)$ in the neighborhoods of $(1, -\frac{\pi}{2})$ up to second degree by Taylor's series.

Session- 12 & 13: Maxima and Minima for functions of two variables and Lagrange's Multipliers method

1. Determine the maxima and minima of $f(x,y) = x^2 + y^2 + 6x + 12$
2. Identify the maximum and minimum value of the function: $f(x,y) = 2x^2 + 2xy + 2y^2 - 6x$
3. Identify minimum values of $x^2 + y^2 + z^2$, given that $xyz = 27$
4. Find the dimensions of the rectangular box, open at the top, of maximum capacity where surface is 432 sq.cm..
5. Divide 36 into 3 parts such that the product of the first square of the second and cube of the third is maximum.

Session-14: Solving of Second and higher order differential equations

1. Solve the differential equation $\frac{d^2y}{dx^2} - 9y = 0$.
2. Solve the DE $y'' + y' - 2y = 0, \text{with } y(0) = 4 \text{ and } y'(0) = 1$.
3. Solve the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{2x}$
4. Solve the differential equation $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = \sin x$
5. Solve the DE $(D^2 + 4)y = \sin 2x$.

Session-15: Modeling an Engineering Problem as a Second Order Ordinary Differential Equation

1. Compute the charge on the capacitor in an LRC series circuit at t when inductance 3H, resistance 15Ω, capacitance (1/12) F, E(t)= 0 V, q (0) =1C, and i (0) =2 A.
2. Determine the charge on the capacitor in an LC series circuit at t=2sec when inductance 1 H, resistance and capacitance 1/25 F, E(t)= Sin 3t V, q(0) =1 C, and i(0) =0 A.
3. The motion of a mass spring system without damping is described by the initial value problem $\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=e^{2t}, x(0)=0, x'(0)=2$, where x is the distance of the mass from the equilibrium position, downward being taken as positive direction. Calculate the displacement of the motion.
4. The motion of a mass spring system without damping is described by the initial value problem $\frac{d^2x}{dt^2}+16x=\cos 2t, x(0)=0, x'(0)=0.$, where x is the distance of the mass from the equilibrium position, downward being taken as positive direction. Identify the displacement of the motion.

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CO-2

Tutorial-4

1. **Compute all first and second order partial derivatives of $f(x,y)=e^{xy}+3x^2-5y^3$ and verify $f_{xy}=f_{yx}$.**
2. **Compute the first and second order partial derivatives of $z=x^2y^3+\sin x\cos y$.**
3. **Verify $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$ for the function $u=\tan^{-1}\left(\frac{x}{y}\right)$.**
4. **Given $u=e^x\cos y$, $x=t^2+1, y=2t$ then find the total derivative $\frac{du}{dt}$.**
5. **Given $u=\log(x+y+z)$, $x=e^t, y=\sin t, z=\cos t$ then find the total derivative $\frac{du}{dt}$.**
6. **Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ of following functions:**

   (a). $u=x^2-2y,\ v=5x+7y$ (b). $u=x(1-y),\ v=xy$
7. **Find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ of $u=\frac{yz}{x}, v=\frac{xz}{y}, w=\frac{xy}{z}$.**

Tutorial-5

1. Apply Taylor's series to expand f(x, y)=$x^3+2xy+y^3$ in powers of (x+1)and (y+2) up-to second degree terms.
2. Applying Taylor's series expansion expand the function f(x, y)= e^xsiny at (-1,π/4) up to the terms of second degree.
3. Examine the maximum and minimum for the function $f(x,y)=x^3+y^3-63(x+y)+12xy$.
4. The sum of three numbers is constant. Prove that their product is maximum when they are equal.
5. Evaluate minimum values of $x^2+y^2+z^2$, given that ax + by + cz=p.
6. A rectangular box open at the top is to have a volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction
7. The temperature T at any point (x, y, z) in space is $T=k\,xyz^2$.Find the highest temperature on the surface of the unit sphere of $x^2+y^2+z^2=1$.

Tutorial-6

1. Solve the DE $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0.$
2. Solve the DE $y'''+4y''+4y'=0.$
3. Determine the solution of the initial value problem $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=2$, given that $y(0)=0,\ y'(0)=1.$
4. Determine the charge on the capacitor in an LRC series circuit at t when inductance 1 H, resistance 4Ω, capacitance 0.25 F, E(t) = 0 V, q(0) =5 C, and i(0) =0 A.
5. Determine the charge on the capacitor in an LC series circuit at t when inductance 1 H, capacitance 1F, $E(t)=e^t$ V, q(0) = 2 C, i(0) = 0 A.
6. The motion of a mass spring system without damping is described by the initial value problem $\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=e^{4t}, x(0)=0, x'(0)=1$, where x is the distance of the mass from the equilibrium position, downward being taken as positive direction. Calculate the displacement of the motion.
7. The motion of a mass spring system without damping is described by the initial value problem $\frac{d^2x}{dt^2}+4x=\cos 3t, x(0)=0, x'(0)=0.$, where x is the distance of the mass from the equilibrium position, downward being taken as positive direction. Identify the displacement of the motion.

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CO-2 Home Assignment Problems

1. Compute the first and second order partial derivatives of $f(x,y)=\sin(xy)+x^2\log_e y$
2. Given $x=r\cos\theta, y=r\sin\theta$ Find the value of $\dfrac{\partial(x,y)}{\partial(r,\theta)}$.
3. Apply Taylor's series to expand $f(x, y)=x^2+xy+y^2$ in powers of (x-1) and (y-2).
4. Determine the maxima and minima of $f(x,y)=2x+2y-2xy-2x^2-y^2$.
5. Given $x+y+z=a$ find the maximum of $x^m y^n z^p$
6. Show that if the perimeter of a triangle is constant, the triangle has maximum area when it is equilateral.
7. Determine the solution of the initial value problem $(D^2-5D+6)y=e^{4x}$, given that $y(0)=0,\ y'(0)=1$.
8. Determine charge q and current i in the LCR circuit with inductance 1H, resistance 12 ohms, capacitance (1/35)F, E(t) =0.
9. Determine the charge and current in an LCR series circuit when inductance 3 H, resistance 6 ohms capacitance 1/3 F, and E (t) =sin2t.
10. The motion of a mass spring system without damping is described by the initial value problem $\dfrac{d^2x}{dt^2}+8\dfrac{dx}{dt}+16x=\cos 3t, x(0)=0, x'(0)=1$ where x is the distance of the mass from the equilibrium position, downward being taken as positive direction. Calculate the displacement of the motion.