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| 1.A. | Explain the random process concept with example. |
| 1.B. | <p>Draw</p> <p>A fair coin is tossed. If heads come up, a sine wave $x_1(t) = \sin(5\pi t)$ is sent. If tails come up, then a ramp $x_2(t) = t$ is sent. The resulting random process $x(t)$ is an ensemble of two realizations, a sine wave, and a ramp signal. Describe and draw the ensemble. The sample space S is discrete.</p> |
| 1.C. | Explain the continuous random sequence. |
| 1.D. | Find the $X(\omega)$, where $x(t) = \sin(\omega_0 t)$ |
| 1.E. | Write the difference between autocorrelation and cross-correlation. |
| 1.F. | Define the mean value of the system response with mathematical expression. |
| 2. | Answer All Question |
| 2.A. | <p>Examine</p> <p>Examine whether the random process $x(t) = A \cos(\omega_o t + \theta)$ is a wide sense stationary if A and ω_o are constants and θ is uniformly distributed random variable in $(0, 2\pi)$</p> |
| 2.B. | <p>Show that</p> <p>Consider the discrete-time random process $\{X(n), n \in \mathbb{Z} \dots\}$, in which the $X(n)$'s are i.i.d. with CDF $F_{X(n)}(x) = F(x)$. Show that this is a (strict-sense) stationary process.</p> |
| 2.C. | Explain the properties of the Power Density Spectrum. |
| 2.D. | Derive the mean square value of system response $Y(t)$. |

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| 3.A. | Explain the Gaussian Random Process with mathematical expression. |
| 3.B. | <p>Show that</p> <p>Consider the random process $\{X(t), t \in \mathbb{R}\}$ defined as</p> $X(t) = \cos(t + U),$ <p>where $U \sim \text{Uniform}(0, 2\pi)$. Show that $X(t)$ is a WSS process.</p> |
| 4. | Answer All Question |
| 4.A. | Define the time average of a random process and time autocorrelation function with mathematical expression. |
| 4.B. | <p>Show That</p> <p>Consider a random process $X(t)$ and its derivative, $X'(t) = \frac{d}{dt}X(t)$. Assuming that the derivatives are well-defined, show that</p> $R_{XX'}(t_1, t_2) = \frac{\partial}{\partial t_2} R_X(t_1, t_2).$ |

5.A. Explain the significance of spectral characteristics in signal processing, system analysis, and predictive modelling.

Find the Value

Let $X(t)$ be a continuous-time WSS process with mean $\mu_X = 1$ and

$$R_X(\tau) = \begin{cases} 3 - |\tau| & -2 \leq \tau \leq 2 \\ 1 & \text{otherwise} \end{cases}$$

a. Find the expected power in $X(t)$.

b. Find $E \left[\left(X(1) + X(2) + X(3) \right)^2 \right]$.

6.A. Summarize the key concepts of random signal response in linear systems.

Find The Value

Let $X(t)$ be a white Gaussian noise with $S_X(f) = \frac{N_0}{2}$. Assume that $X(t)$ is input to an LTI system with

$$h(t) = e^{-t}u(t).$$

6.B.

Let $Y(t)$ be the output.

- a. Find $S_Y(f)$.
- b. Find $R_Y(\tau)$.
- c. Find $E[Y(t)^2]$.