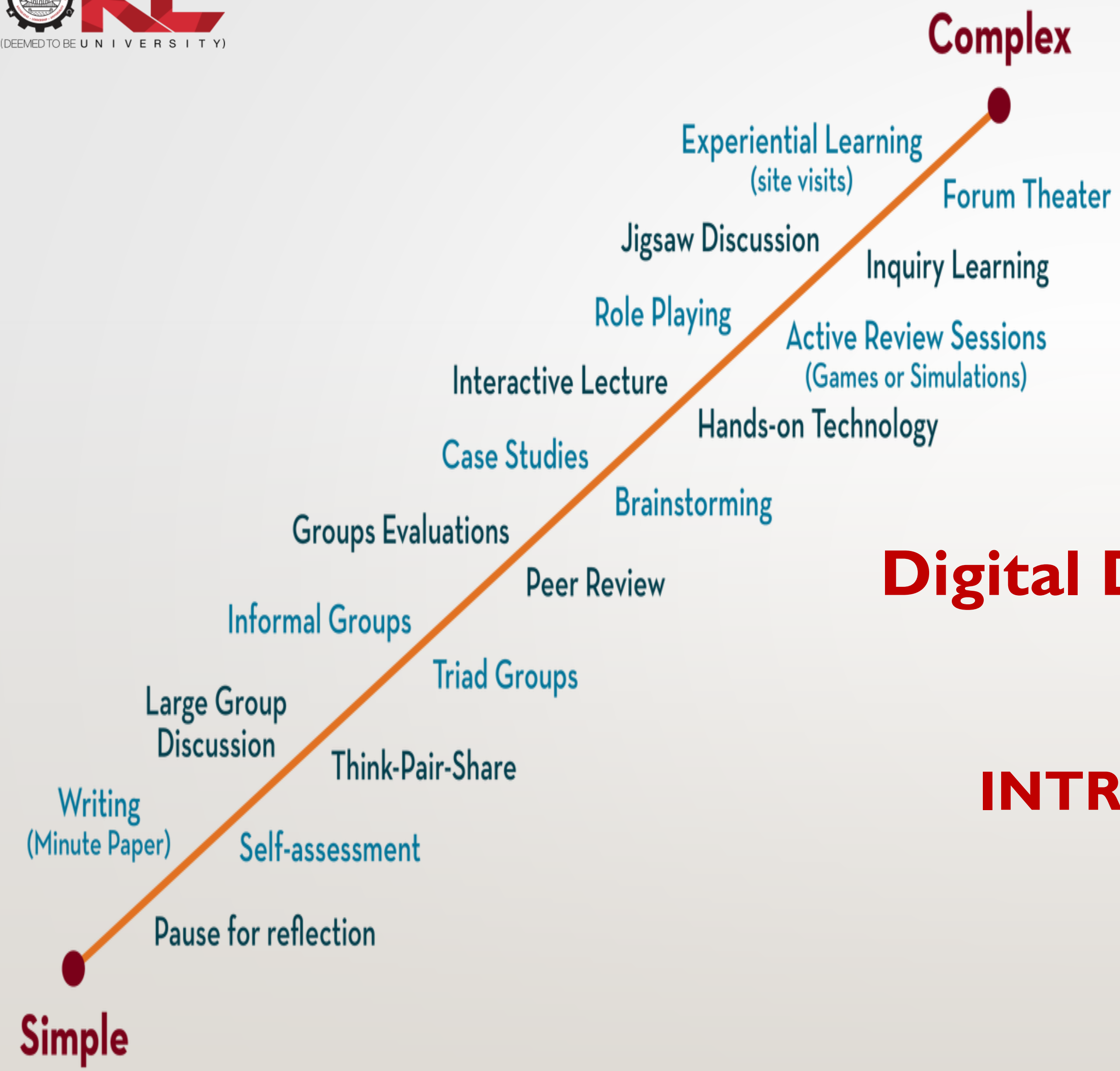

Department of BES-II

Digital Design and Computer Architecture
23EC1202

Topic:
INTRODUCTION, BOOLEAN ALGEBRA

Session No: 02

AIM OF THE SESSION

To familiarize students with the basic concept of Boolean Algebra laws and optimization methods.

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Develop a comprehensive understanding of Boolean algebra, including its fundamental operations, laws, and theorems, to analyze and simplify complex logical expressions and circuits.
2. Acquire the skills to apply Boolean algebra in the design and optimization of digital circuits, fostering the ability to manipulate logical symbols and equations to achieve desired logical outcomes efficiently.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Demonstrate the ability to manipulate and simplify logical expressions using Boolean algebra, showcasing proficiency in applying fundamental operations, laws, and theorems.
2. Acquire the skills to design and optimize digital circuits using Boolean algebra, leading to efficient and streamlined logical structures in various applications.

Boolean Algebra

- Boolean variable is used to represent the voltage level present on a wire or the input/output terminal of a circuit. At any time, it takes either logic 0 or logic 1 and may be designated as (A, B, C) or (X,Y, Z) for three variables.
- Boolean Algebra is a means for expressing the relation between inputs and outputs of a logic circuit. It can be used to analyze a logic circuit and express its operation mathematically.
- **Advantages of Boolean Algebra over ordinary Algebra:**
  - Easy to understand or use as only two binary values are present.
  - No fractions, decimals, negative numbers, roots, logarithms, etc.
  - Contains only three functions AND, OR and NOT.

BOOLEAN ALGEBRA PRINCIPLES

Boolean Algebra Principles

Boolean algebra defines laws and theorems which can be used for simplification of logic functions.

Complement of a variable

The complement of a variable is also known as its "negation" or "inverse."

It is denoted by placing a bar (') or an overline over the variable.

Example: For a variable " A ", the complement is " $\bar{A}$ " .

Single variable theorems

$a)\ A + 0 = 0 + A = A$ $\quad b)\ A.1 = 1.A = A$ $\quad c)\ A + \bar{A} = 1$ $\quad d)\ A.\bar{A} = 0$

$e)\ A + A = A$ $\quad f)\ A.A = A$ $\quad g)\ A + 1 = 1$ $\quad h)\ A.0 = 0$

BOOLEAN ALGEBRA THEOREMS - MULTIVARIABLE

$a)\ A + B = B + A$

$b)\ A . B = B . A$

$c)\ A.(B + C) = A . B + A . C$

$d)\ A + (B . C) = (A + B).(A + C)$

$e)\ A + A . B = A$

$f)\ \bar{A} + A . B = \bar{A} + B$

DeMorgan's Theorems

Theorem 1: $\overline{A . B} = \overline{A} + \overline{B}$

Theorem 2: $\overline{A + B} = \overline{A} . \overline{B}$

A	B	A . B	$\overline{A . B}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

A	B	A + B	$\overline{A + B}$	$\overline{A}$	$\overline{B}$	$\overline{A} . \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Boolean Properties

Name of the Property	AND	OR
Commutative	$A\,.\,B = B\,.\,A$	$A + B = B + A$
Associative	$(A\,.\,B)\,.\,C = A\,.\,(B\,.\,C)$	$(A + B) + C = A + (B + C)$
Distributive	$A\,.\,(B + C) = (A\,.\,B) + (A\,.\,C)$	$A + (B\,.\,C) = (A + B)\,.\,(A + C)$
Identity	$A\,.\,1 = A$	$A + 0 = A$
Complement	$A\,.\,(\bar{A}) = 0$	$A\,.+(\bar{A}) = 1$
De Morgan's law	$\overline{A\,.\,B} = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A}\,.\,\bar{B}$

Example - Complement of a Function

Given

$$F(X,Y,Z) = (XY) + (\overline{X}Z) + (Y\overline{Z})$$

Complement of the function

$$\overline{F}(X,Y,Z) = \overline{(XY) + (\overline{X}Z) + (Y\overline{Z})}$$

$$= \overline{(XY)}\,\overline{(\overline{X}Z)}\,\overline{(Y\overline{Z})}$$

$$= (\overline{X}+\overline{Y})(X+\overline{Z})(\overline{Y}+Z)$$

Examples on Boolean Simplification

$$
\begin{aligned}
(A+B)(A+C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1 + C + B) + BC \\
&= A \cdot 1 + BC \\
&= A + BC
\end{aligned}
$$

$$
\begin{aligned}
A\overline{B}D + A\overline{B}\,\overline{D} = \quad & A\overline{B}\left(D + \overline{D}\right) \\
& A\overline{B}\,(1) \\
& A\overline{B}
\end{aligned}
$$

$$
\begin{aligned}
Y &= AB + A(B + C) + B(B + C) \\
&= AB + AB + AC + BB + BC && \textit{distributive law} \\
&= AB + AB + AC + B + BC && \textit{idempotency theorem} \\
&= AB + AC + B + BC && B + BC = B \\
&= AB + AC + B && AB + B = B \\
&= B + AC
\end{aligned}
$$

TERMINAL QUESTIONS

Short answer questions:

1. Apply De Morgan's theorems to simplify the expression: $\overline{(A+B).(C+D)}$
2. Simplify the expression F = A B D+A B' using Boolean identities.
3. Apply De Morgan's theorem to find the complement of the function F = ABC+A'BC'+BC.
4. Reduce A (A + B) to the least number of terms.

Long answer questions:

1. Simplify the following expressions using boolean laws.

$$F = A.B + A.(C.D + C.\overline{D}) \qquad F = A.B.C + \bar{A} + A.C.\bar{B}$$

SELF-ASSESSMENT QUESTIONS

1. What is the result of the expression $A \cdot B + C = A \cdot B' + C'$ when $A=1, B=0, C=1$?

(a) 0
(b) 1
(c) A
(d) B

2. Which of the following represents the NOR operation?

(a) A . B
(b) A+B
(c) A' . B'
(d) A'+B'

SELF-ASSESSMENT QUESTIONS

3. Which Boolean Algebra law states that changing the order of operands in an AND operation does not affect the result?

(a) Identity law
(b) Commutative law
(c) Distributive law
(d) Absorption law

4. De Morgan's Law states the relationship between:

(a) AND and OR operations
(b) OR and NOT operations
(c) XOR and NOR operations
(d) NAND and NOR operations

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Computer System Architecture by M. Moris Mano
2. Fundamentals of Digital Logic with Verilog HDL by Stephen Brown and ZvonkoVranesic

Sites and Web links:

1. https://www.javatpoint.com/boolean-algebra-in-digital-electronics
2. https://www.geeksforgeeks.org/basics-of-boolean-algebra-in-digital-electronics/

THANK YOU

(DEEMED TO BE UNIVERSITY)

Team – Digital Design & Computer Architecture