Signals and Communication Systems

Properties of FT

Properties of FT…

Time Reversal Property

$$\text{If} \quad x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$$

$$\text{Then} \quad x(-t) \xleftrightarrow{\text{F.T}} X(-j\Omega)$$

Find the FT of signal $x(-t)$ where $x(t) = e^{-3t}u(t)$

Ans: We know that $e^{-3t}u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{3+j\Omega}$

By Time reversal property

$$e^{3t}u(-t) \xleftrightarrow{\text{F.T}} \frac{1}{3-j\Omega}$$

Then the required FT of $x(-t)$ is $\dfrac{1}{3-j\Omega}$

Find the time domain signal, if $Y(j\Omega) = \dfrac{2}{2-j\Omega}$

Find the FT of $x(t) = e^{-a|t|}$

Ans: The given signal is represented by

$$x(t) = e^{-a|t|} = e^{at}u(-t) + e^{-at}u(t)$$

We know that $e^{-at}u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{a + j\Omega}$

By Time reversal property $e^{at}u(-t) \xleftrightarrow{\text{F.T}} \dfrac{1}{a - j\Omega}$

Then $X(j\Omega) = \dfrac{1}{a + j\Omega} + \dfrac{1}{a - j\Omega} = \dfrac{2a}{a^2 - \Omega^2}$

Time reversal

Find the inverse FT of signals

(a) $X(j\Omega) = e^{-a\Omega}u(\Omega)$ (b) $X(j\Omega) = e^{a\Omega}u(-\Omega)$

Ans: We Know that $e^{-at}u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{a+j\Omega}$

Then by duality, $\dfrac{1}{a+j\,t} \xleftrightarrow{\text{F.T}} 2\pi\, e^{a\,\Omega}u(-j\Omega)$

By time reversal $\dfrac{1}{a-j\,t} \xleftrightarrow{\text{F.T}} 2\pi\, e^{-a\,\Omega}u(j\Omega)$

Time Differentiating Property:

If $x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$

Then $\frac{d}{dt}x(t) \xleftrightarrow{\text{F.T}} j\Omega\, X(j\Omega)$

and $\frac{d^2}{dt^2}x(t) \xleftrightarrow{\text{F.T}} (j\Omega)^2\, X(j\Omega)$

In general $\frac{d^n}{dt^n}x(t) \xleftrightarrow{\text{F.T}} (j\Omega)^n\, X(j\Omega)$

Consider a signal $x(t) = e^{-at}u(t)$

Find the FT of signal $\dfrac{d}{dt}x(t)$

We know that $e^{-at}u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{a+j\Omega}$

By Time differentiation property,

$$\frac{d}{dt}\left\{e^{-at}u(t)\right\} \xleftrightarrow{\text{F.T}} j\Omega\frac{1}{a+j\Omega}$$

$$\frac{d^2}{dt^2}\left\{e^{-at}u(t)\right\} \xleftrightarrow{\text{F.T}} (j\Omega)^2\frac{1}{a+j\Omega}$$

Find $x(t)$, *if* $X(j\Omega) = \dfrac{j\Omega}{a + j\Omega}$

Ans: Method1: Using Time differentiation property

We know that $\dfrac{1}{a + j\Omega} \xleftarrow{\text{F.T}} \xrightarrow{} e^{-at}u(t)$

By Time differentiation property, $j\Omega \dfrac{1}{a + j\Omega} \xleftarrow{\text{F.T}} \xrightarrow{} \dfrac{d}{dt}\left\{e^{-at}u(t)\right\}$

$$= \delta(t) - a\, e^{-at}u(t)$$

Ans: Method2: General

The given signal is represented by $X(j\Omega) = \dfrac{j\Omega}{a + j\Omega} = \dfrac{j\Omega + a - a}{a + j\Omega} = 1 - a\dfrac{1}{a + j\Omega}$

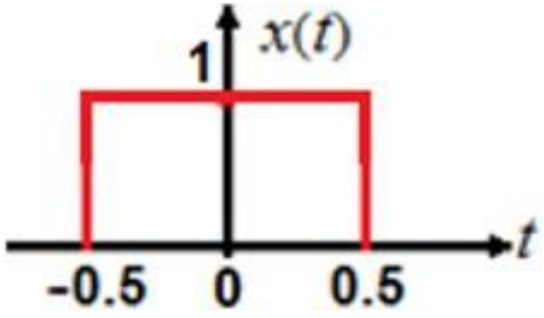
Then by inverse FT, we get $x(t) = \delta(t) - a\, e^{-at}u(t)$

Find the time domain signal, if $Z(j\Omega) = \dfrac{3 - j\Omega}{3 + j\Omega}$

Find the F.T. of a signal shown in figure

Ans: The given signal is represented by

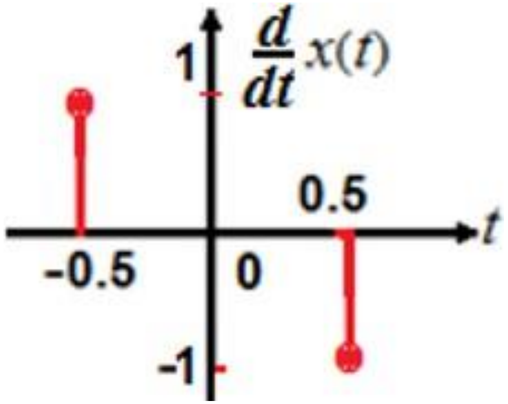
$$x(t) = u(t+0.5) - u(t-0.5)$$



By taking differentiation on both sides of the equation,

we get $\dfrac{d}{dt}x(t) = \dfrac{d}{dt}\{u(t+0.5) - u(t-0.5)\}$

$= \delta(t+0.5) - \delta(t-0.5)$ as shown in figure.



By taking FT on both sides of the equation, we get

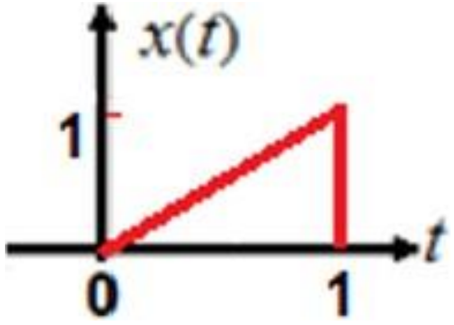
$$j\Omega\, X(j\Omega) = e^{j0.5\Omega} - e^{-j0.5\Omega}$$

$$= j2\sin 0.5\,\Omega = j2\sin \Omega/2$$

On simplification, we obtain

$$X(j\Omega) = \frac{2}{\Omega}\sin \Omega/2 = \frac{\sin \Omega/2}{\Omega/2} = Sa\,(\Omega/2)$$

Exercise: Find the F.T. of a signal shown in figure



Hint: Perform double differentiation

Frequency Differentiating Property

$$\text{If} \quad x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$$

$$\text{Then} \quad t\,x(t) \xleftrightarrow{\text{F.T}} j\frac{d}{d\Omega}X(j\Omega)$$

Find the F.T. of $x(t) = t$

Ans: We know that $1 \xleftrightarrow{\text{F.T}} 2\pi\,\delta(\Omega)$

Then by Frequency differentiation property,

$$1.t \xleftrightarrow{\text{F.T}} j\frac{d}{d\Omega}X(j\Omega) = j\frac{d}{d\Omega}2\pi\,\delta(\Omega) = j2\pi\frac{d}{d\Omega}\delta(\Omega)$$

Find the FT of a signal $x(t) = t\, e^{-at} u(t)$

Ans: We know that $e^{-at} u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{a + j\Omega}$

Then by Frequency differentiation property

$$t\, e^{-at} u(t) \xleftrightarrow{\text{F.T}} j \frac{d}{d\Omega}\left(\frac{1}{a + j\Omega}\right) = \frac{1}{(a + j\Omega)^2}$$

Time Scaling Property:

$$\text{If} \quad x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$$

$$\text{Then} \quad x(at) \xleftrightarrow{\text{F.T}} \frac{1}{|a|} X\left(j\frac{\Omega}{|a|}\right)$$

Find the F.T. of $x(t) = \delta(3t)$

Ans: We know that $\delta(t) \xleftrightarrow{\text{F.T}} 1, \text{ for all } '\Omega'$

Then by scaling property, $\delta(3t) \xleftrightarrow{\text{F.T}} \frac{1}{|3|} 1 = \frac{1}{3}$

Time Convolution Property:

If $x_1(t) \overset{\text{F.T}}{\longleftrightarrow} X_1(j\Omega)$

and $x_2(t) \overset{\text{F.T}}{\longleftrightarrow} X_2(j\Omega)$

Then $x(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) * x_2(t-\tau)\, d\tau \overset{\text{F.T}}{\longleftrightarrow} X(j\Omega) = X_1(j\Omega) X_2(j\Omega)$

Show that $y(t) = x(t) * \delta(t) = x(t)$

Ans: We know that $x(t) \overset{\text{F.T}}{\longleftrightarrow} X(j\Omega)$,

and $\delta(t) \overset{\text{F.T}}{\longleftrightarrow} 1$

$$y(t) = x(t) * \delta(t) \overset{\text{F.T}}{\longleftrightarrow} Y(j\Omega) = X(j\Omega).1 = X(j\Omega)$$

By taking the inverse FT, $y(t) = x(t)$

Find the response of the system if its impulse response is

$h(t)=\delta(t-2)$ and excited by $x(t)=e^{-at}u(t)$

Ans: We know that $x(t)=e^{-at}u(t) \xleftrightarrow{\text{F.T}} X(j\Omega)=\dfrac{1}{a+j\Omega}$, and

By Time shifting property, $h(t)=\delta(t-2) \xleftrightarrow{\text{F.T}} H(j\Omega)=e^{-j2\Omega}$

Then by convolution theorem,

$$y(t)=x(t)*h(t) \xleftrightarrow{\text{F.T}} Y(j\Omega)=X(j\Omega)H(j\Omega)$$

Therefore $Y(j\Omega)=X(j\Omega)H(j\Omega)=\dfrac{1}{a+j\Omega}e^{-j2\Omega}$

We know that $\dfrac{1}{a+j\Omega} \xleftrightarrow{\text{F.T}} e^{-at}u(t)$

By Time shifting property, $\dfrac{e^{-j2\Omega}}{a+j\Omega} \xleftrightarrow{\text{F.T}} e^{-a(t-2)}u(t-2)$

Then the response of the system $y(t)=e^{-a(t-2)}u(t-2)$

1. Find the FT of a signal $x(t) = \{e^{-3t}u(t) * \delta(t-1)\}$ using properties of FT.

2. Find the convolution between $x(t) = e^{-\alpha t}u(t)$ and $h(t) = e^{-\beta t}u(t)$ using time convolution property of FT.

Frequency Convolution Theorem

$$\text{If} \quad x_1(t) \xleftrightarrow{\text{F.T}} X_1(j\Omega)$$

$$\text{and} \quad x_2(t) \xleftrightarrow{\text{F.T}} X_2(j\Omega)$$

$$\text{Then} \quad x(t) = x_1(t)x_2(t) \xleftrightarrow{\text{F.T}} X(j\Omega) = \frac{1}{2\pi} X_1(j\Omega) * X_2(j\Omega)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\lambda) X_2(t-\lambda)\, d\lambda$$

Parseval's Energy Theorem

$$\int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\Omega)|^2 \, \mathrm{d}\Omega$$

Energy in Time Domain

Energy in Frequency domain

End