# I/IV B. Tech. Even Semester : A.Y.2024-25

# Linear Algebra & Calculus for Engineers (23MT1001)

**CO-1 CLASSROOM DELIVARY PROBLEMS**

## Session-1

Discuss course handout and introduction of matrices.

## Session-2 (Introduction of Matrices, Rank and Echelon form)

1. Reduce the following matrices in to Echelon form and hence obtain its rank.

(i) $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & 4\\ 3 & 5 & 7\end{bmatrix}$ (ii) $A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$ (iii) $A=\begin{bmatrix}0 & 4 & 3 & 2 & 1\\ 0 & -2 & -3 & -1 & 4\\ 0 & 6 & 7 & 2 & 9\\ 0 & 8 & 3 & 6 & 6\end{bmatrix}$

2. Two containers contain a water of different temperatures. If we mix 240 g of water from the first container with 260 g of water from the second container, the resulting water temperature will be 52°C. If we mix 180 g of water from the first container with 120 g of water from the second container, the resulting water temperature will be 46°C. Model this phenomenon into linear system of equations and hence determine the rank of the coefficient matrix?
3. Obtain rank of the coefficient matrix for the following system of equations
$2x+3y+z=6,\ 4x+5y+z=10,\ \ x+y+3z=5$
4. Whether the vectors [1 -2 3], [-2 4 6], [3 -6 9] are coplanar or not?

## Session-3 (Gauss-elimination method)

1. Solve the following system of equations using Gauss elimination method:
2x+y+z=10; 3x+2y+3z =18; and x+4y+9z =16.
2. The upward velocity $v(t)$ of a rocket at time t is approximated by $v(t)=at^2+bt+c$, $0\le t\le 100$ where a, b, and c are constants. It has been found that the velocity at times

t=3, t=6, and t = 9 seconds are 64, 133, and 208 miles per second respectively. Determine the velocity at time t = 15 seconds using Gaussian elimination method.

3. A DC circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for current flow in milli amperes: $3i_1 + 2i_2 - i_3 = 4,\ 4i_1 - 3i_2 + 2i_3 = 3,\ i_1 + 2i_2 + 3i_3 = 6$. Apply Gauss elimination method to solve for $i_1$, $i_2$ and $i_3$.
4. A firm can produce three types of cloths A, B and C. Three kinds of wool are required for it, say red, green and blue wool. One unit of type 'A' cloth needs 2 yards of red wool, 8 yards of green and one yard of blue wool; one unit length of type 'B' cloth needs one yard of red, 3 yards of green and 5 yards of blue wool; one unit length of type 'C' cloth needs 6 yards red, 2 yards of green and one yard of blue wool. The firm has only a stock of 9 yards red, 13 yards green and 7 yards of blue wool. If total stock is used, then write mathematical formulation of the problem, to determine the number of units produce three types of cloths A, B and C.

## Session-4 (LU- Decomposition Method)

1. Apply LU decomposition method, find Lower and Upper triangular matrices and hence solve the equations 3x+2y+7z = 4; 2x+3y+z = 5; 3x+4y+z = 7.

2. Print India Ltd. wishes to produce three types of souvenirs: types A, B, and C. To manufacture a type-A souvenir requires 2 minutes on machine-I, 1 minute on machine-II, and 2 minutes on machine-III. A type-B souvenir requires 1 minute on machine-I, 3 minutes on machine-II, and 1 minute on machine-III. A type-C souvenir requires 1 minute on machine-I and 2 minutes on machines-II and 2 minute on machine-III. There are 180 min available on machine-I, 300 min available on machine-II, and 240 min available on machine-III for processing the order. How many souvenirs of each type should Ace Novelty make to use all of the available time, write the mathematical formulation to the problem and solve it by using LU decomposition method.

3. The tensions, $T_1$, $T_2$ *and* $T_3$ in a simple framework are given by the equations $T_1 + 2T_2 + 4T_3 = 3,\ 5T_1 + 5T_2 + 5T_3 = 7,\ 4T_1 + 2T_3 = 4$. Determine $T_1$, $T_2$ *and* $T_3$ using LU decomposition method. (T1=19/25, T2=4/25, T3=12/25)

## Session-5 (Eigen values and Eigen vectors)

1. Determine the Eigen values and Eigen vectors of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

2. Determine the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

3. Find all eigenvalues and corresponding eigenvectors for the matrix A if $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

4. Determine the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & -1 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{pmatrix}$

5. If two eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are 1 and 5 then determine the third Eigen value. Hence find eigen values of $A^4$, $A^T$ and $A^{-1}$.

## Session-6 (Applications of Eigen values and Eigen vectors, stability analysis)

1. Verify the system $\frac{dX}{dt} = AX$ is stable or not where $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ also find the Eigen vectors.
2. Verify the system $\frac{dX}{dt} = AX$ is stable or not where $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ also find the Eigen vectors.
3. Verify the system $\frac{dX}{dt} = AX$ is stable or not where $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$
4. Verify the system $\frac{dX}{dt} = AX$ is stable or not where $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

## Session-7 (Diagonalization of Matrices)

1. Check whether the matrix is diagonalizable or not, $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. If diagonalizable then find $A^4$.
2. Diagonalize the matrix $A = \begin{bmatrix} -2 & 5 \\ -1 & 4 \end{bmatrix}$ and hence find $A^5$.
3. Determine the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ is diagonalizable or not?

## Session-8 (Quadratic forms and their Canonical forms, Nature of a quadratic forms)

1. Reduce the quadratic form $x^2 + 4y^2 + z^2 - 4xy + 2zx - 4yz$ into a sum of squares form and hence find its rank, index, signature and nature.
2. Determine canonical form, rank, index, signature and nature of the quadratic form $2x^2 + 2y^2 + 2z^2 + 2yz$.
3. Reduce the given Quadratic form $Q \equiv 2xy + 2xz - 2yz$ into $X^TAX$ form and also determine canonical form, rank, index, signature and nature of the quadratic form.