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EDUCATION FOUNDATION**  
(Deemed to be University, Estd. u/s. 3 of UGC Act 1956)

**I/IV B. Tech. Even Semester :: A.Y. 2024-25**

**Linear Algebra & Calculus for Engineers (23MT1001)**

**CO-2**

### **Tutorial-4**

1. Compute all first and second order partial derivatives of  $f(x, y) = e^{xy} + 3x^2 - 5y^3$  and verify  $f_{xy} = f_{yx}$ .
2. Compute the first and second order partial derivatives of  $z = x^2y^3 + \sin x \cos y$ .
3. Verify  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$  for the function  $u = \tan^{-1}\left(\frac{x}{y}\right)$ .
4. Given  $u = e^x \cos y$ ,  $x = t^2 + 1$ ,  $y = 2t$  then find the total derivative  $\frac{du}{dt}$ .
5. Given  $u = \log(x + y + z)$ ,  $x = e^t$ ,  $y = \sin t$ ,  $z = \cos t$  then find the total derivative  $\frac{du}{dt}$ .
6. Find the Jacobian  $\frac{\partial(u, v)}{\partial(x, y)}$  of following functions:  
(a).  $u = x^2 - 2y$ ,  $v = 5x + 7y$                       (b).  $u = x(1 - y)$ ,  $v = xy$
7. Find the Jacobian  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$  of  $u = \frac{yz}{x}$ ,  $v = \frac{xz}{y}$ ,  $w = \frac{xy}{z}$ .

### **Tutorial-5**

1. Apply Taylor's series to expand  $f(x, y) = x^3 + 2xy + y^3$  in powers of  $(x+1)$  and  $(y+2)$  up-to second degree terms.
2. Applying Taylor's series expansion expand the function  $f(x, y) = e^x \sin y$  at  $(-1, \pi/4)$  up to the terms of second degree.
3. Examine the maximum and minimum for the function  $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ .
4. The sum of three numbers is constant. Prove that their product is maximum when they are equal.
5. Evaluate minimum values of  $x^2 + y^2 + z^2$ , given that  $ax + by + cz = p$ .
6. A rectangular box open at the top is to have a volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction
7. The temperature  $T$  at any point  $(x, y, z)$  in space is  $T = kxyz^2$ . Find the highest temperature on the surface of the unit sphere of  $x^2 + y^2 + z^2 = 1$ .

## Tutorial-6

1. Solve the DE  $\frac{d^3 y}{dx^3} - 9\frac{d^2 y}{dx^2} + 23\frac{dy}{dx} - 15y = 0$ .
2. Solve the DE  $y''' + 4y'' + 4y' = 0$ .
3. Determine the solution of the initial value problem  $\frac{d^2 y}{dx^2} - 8\frac{dy}{dx} + 15y = 2$ , given that  $y(0) = 0$ ,  $y'(0) = 1$ .
4. Determine the charge on the capacitor in an LRC series circuit at  $t$  when inductance 1 H, resistance  $4\Omega$ , capacitance 0.25 F,  $E(t) = 0$  V,  $q(0) = 5$  C, and  $i(0) = 0$  A.
5. Determine the charge on the capacitor in an LC series circuit at  $t$  when inductance 1 H, capacitance 1F,  $E(t) = e^t$  V,  $q(0) = 2$  C,  $i(0) = 0$  A.
6. The motion of a mass spring system without damping is described by the initial value problem  $\frac{d^2 x}{dt^2} + 3\frac{dx}{dt} + 2x = e^{4t}$ ,  $x(0) = 0$ ,  $x'(0) = 1$ , where  $x$  is the distance of the mass from the equilibrium position, downward being taken as positive direction. Calculate the displacement of the motion.
7. The motion of a mass spring system without damping is described by the initial value problem  $\frac{d^2 x}{dt^2} + 4x = \cos 3t$ ,  $x(0) = 0$ ,  $x'(0) = 0$ , where  $x$  is the distance of the mass from the equilibrium position, downward being taken as positive direction. Identify the displacement of the motion.