

1. Show that the random process $X(t) = 100 \sin(\omega t + \theta)$ is first order stationary, if it is assumed that ω is constant and θ is uniformly distributed in $(0, 2\pi)$.
2. Consider the random process $X(t) = A \cos(\omega t + \varphi)$, where ω is a random variable with density functions $f(\omega)$ and φ a random variable uniform in the interval $(-\pi, \pi)$ and independent of ω , prove that $X(t)$ is a first order stationary with zero means.
3. Consider the process $X(t) = 10 \sin(200t + \varphi)$, where φ is uniformly distributed in the interval $(-\pi, \pi)$. Check whether the process is stationary or not.
4. Give an example of stationary random process and justify your claim.
5. If $X(t) = Y \cos \omega t + Z \sin \omega t$ where Y and Z are two independent normal random variables with $E[Y] = E[Z] = 0$, $E[Y^2] = E[Z^2] = \sigma^2$ and ω is a constant. Prove that $X(t)$ is a SSS of order 2.
6. Show that the random process $X(t) = A \sin(\omega t + \varphi)$, where A and ω are constants, φ is a random variable uniformly distributed in $(0, 2\pi)$ is WSS.
7. Show that the random process $X(t) = 100 \sin(\omega t + \varphi)$ is WSS, where ω is a constant and φ is uniformly distributed in $(0, 2\pi)$
8. If $X(t) = \cos(\lambda t + y)$ where y is uniformly distributed random variable in $(-\pi, \pi)$. Show that $X(t)$ is WSS.
9. If $X(t) = U \cos t + V \sin t$ where U and V are independent random variables each of which assumes the values -2 and 1 with probabilities $1/3$ and $2/3$ respectively. Show that $X(t)$ is covariance stationary (WSS).
10. Show that the random process $X(t) = A \cos t + B \sin t$, $-\infty < t < \infty$, is a WSS process, where A and B are independent random variables each of which has a value -2 with probability $1/3$ and a value 1 with probability $2/3$.
11. Let $X(t) = Y \cos t + Z \sin t$ for all t , where Y and Z are independent binary random variables each of which assumes the values -1 and 1 which are equally likely, then prove that $X(t)$ is WSS.
12. Consider a random process $X(t) = A \cos(50t + \varphi)$, where A and φ are independent random variables. A is a random variable with zero mean and variance 1. φ is uniformly distributed in $(-\pi, \pi)$ show that $X(t)$ is WSS.
13. If $X(t) = 2 \cos(5t + \theta)$, $Y(t) = 5 \cos(5t + \theta)$, where θ a random variable uniformly distributed in $(0, 2\pi)$, prove that the process $X(t)$ and $Y(t)$ are jointly WSS.
14. Consider a random process $Z(t) = X_1 \cos(\omega_0 t) - X_2 \sin(\omega_0 t)$, where X_1 and X_2 are independent Gaussian random variables with zero mean and variance σ^2 . Find $E[Z]$ and $E[Z^2]$
15. The random process $X(t) = \sin(\omega t + y)$, where y is a random variable uniformly distributed in $(0, 2\pi)$. Prove that

$$C(t_1, t_2) = R(t_1, t_2) = \frac{1}{2} \cos \omega (t_1 - t_2).$$

16. If $X(t) = R \cos(\omega t + \varphi)$ where R and φ are independent random variables with $E[R] = 2$ and $V(R) = 6$ and φ is uniformly distributed in $(-\pi, \pi)$. Prove that $X(t)$ is WSS process.
17. Consider the random process $X(t) = \cos(t + \varphi)$ where φ is a random variable with density function $f(\varphi) = 1/\pi$, $-\pi/2 < \varphi < \pi/2$, check whether or not the process is wide-sense stationary.
18. Let $X(t)$ be a WSS process with auto correlation $R_{XX}(\tau) = A e^{-\alpha |\tau|}$. Find the second moment of the random variable $Y = X(5) - X(2)$.
19. Consider the random process $X(t)$ with $X(t) = A \cos(\omega^2 t + \theta)$ where θ is uniformly distributed in $(-\pi, \pi)$. Prove that $X(t)$ is correlation ergodic.
20. If the autocorrelation function of a stationary Gaussian process $X(t)$ is $R(t) = 10 e^{-|\tau|}$, prove that $X(t)$ is ergodic both mean and correlation.

21. A random process $X(t)$ has sample values $x_1(t) = 5$, $x_2(t) = 3$, $x_3(t) = -1$, $x_4(t) = -3$, $x_6(t) = -5$. Find the mean and variance of the process. Is the process ergodic in the mean.
22. If the customers arrive at a bank according to a poisson process with a mean rate of 2 per minute, find the probability that, during an 1- minute interval no customer arrives.