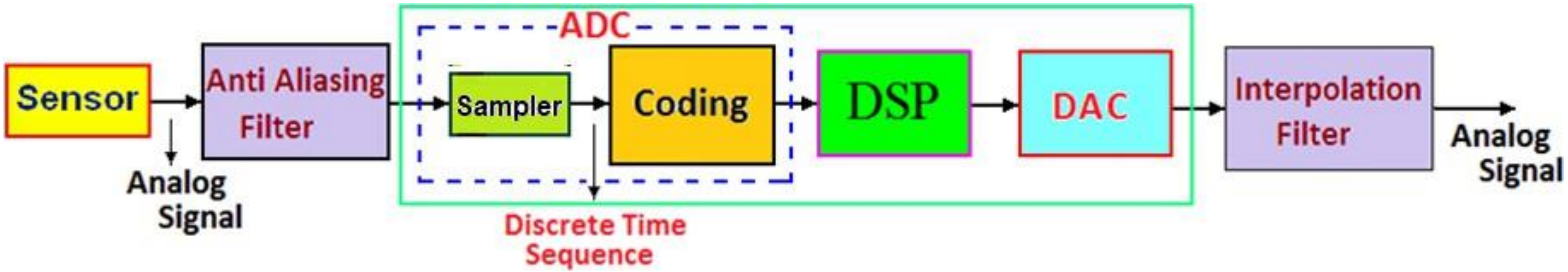
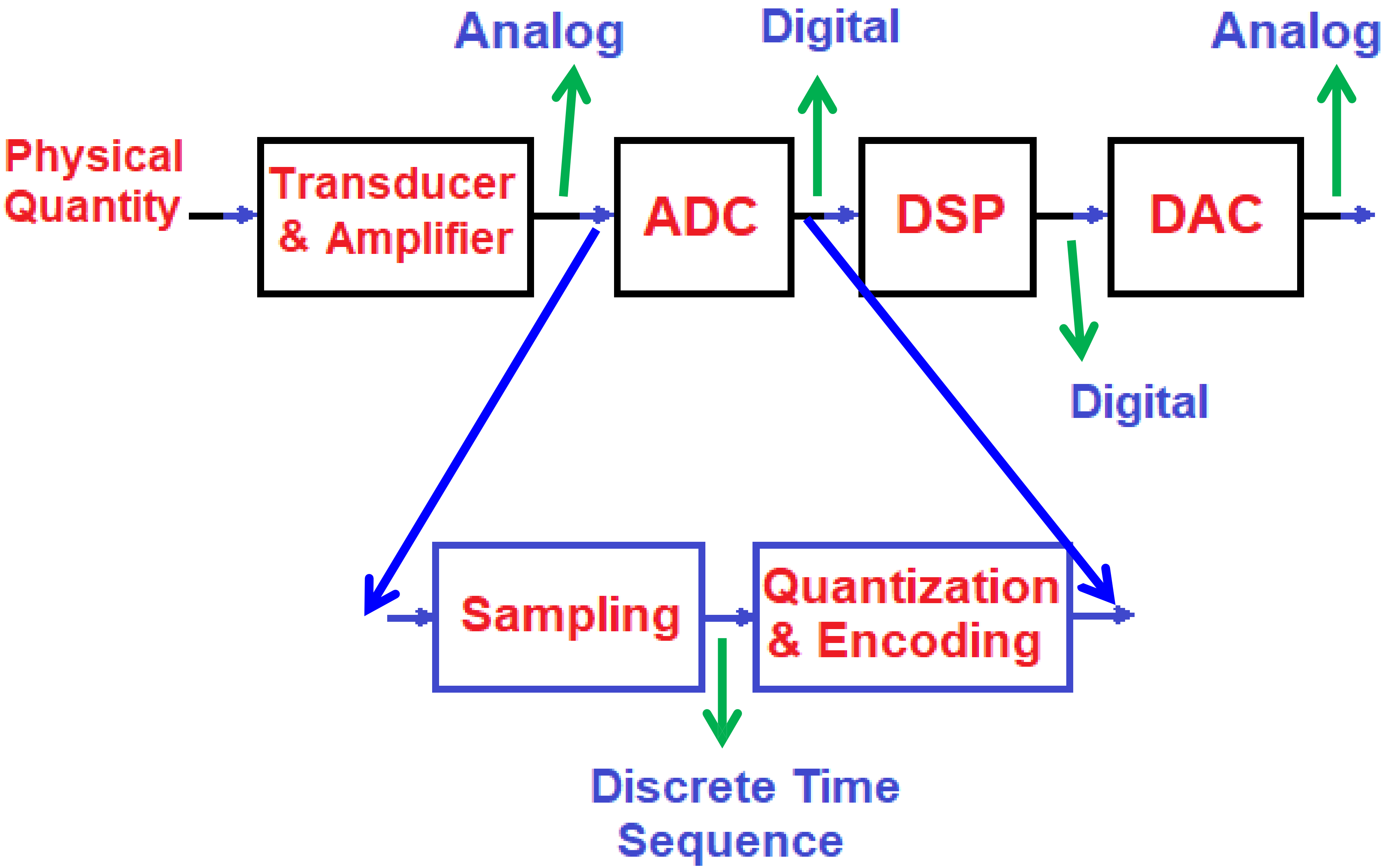
Signals and Communication Systems

Sampling and Reconstruction

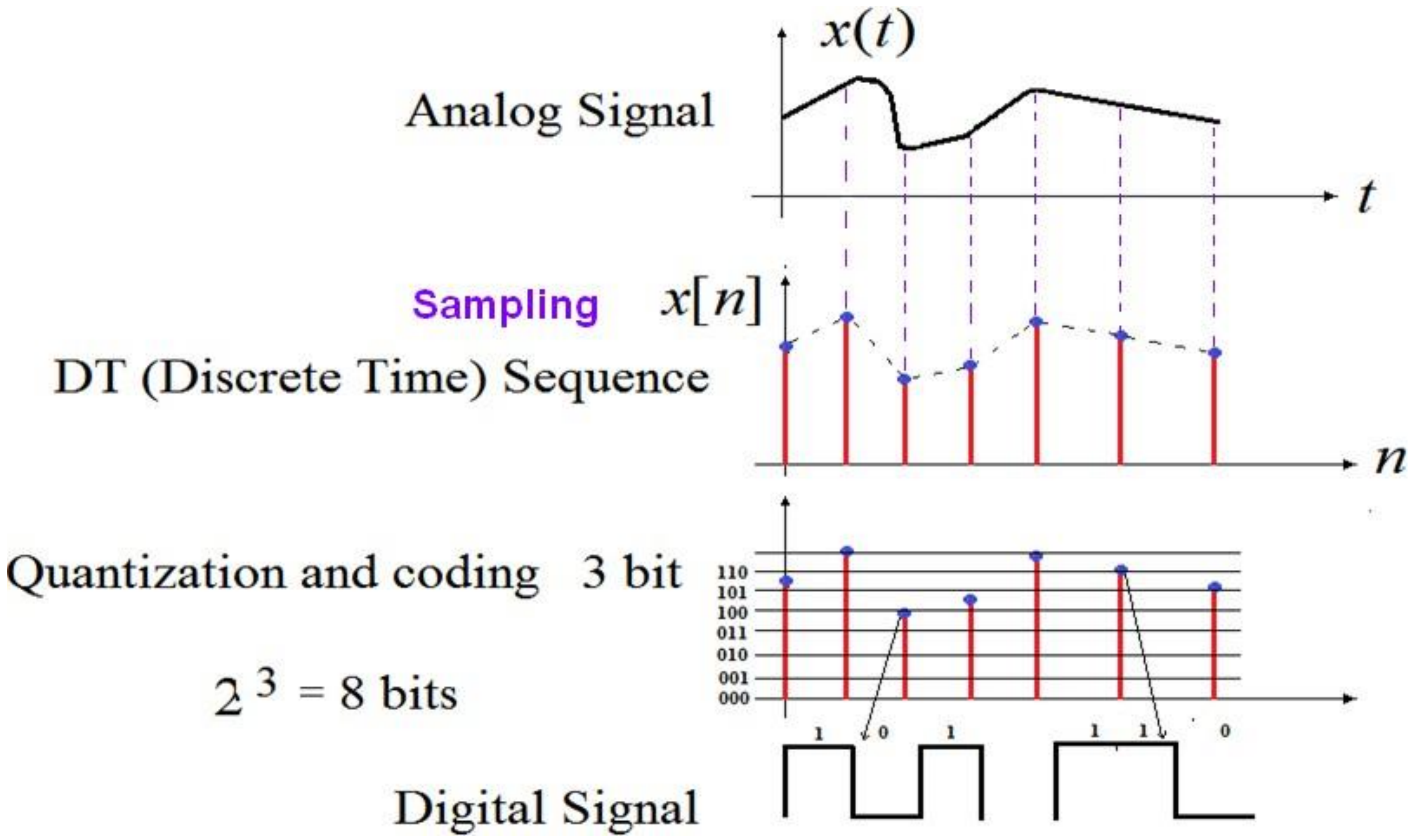
Sampling Theorem

Digital Signal Processing System

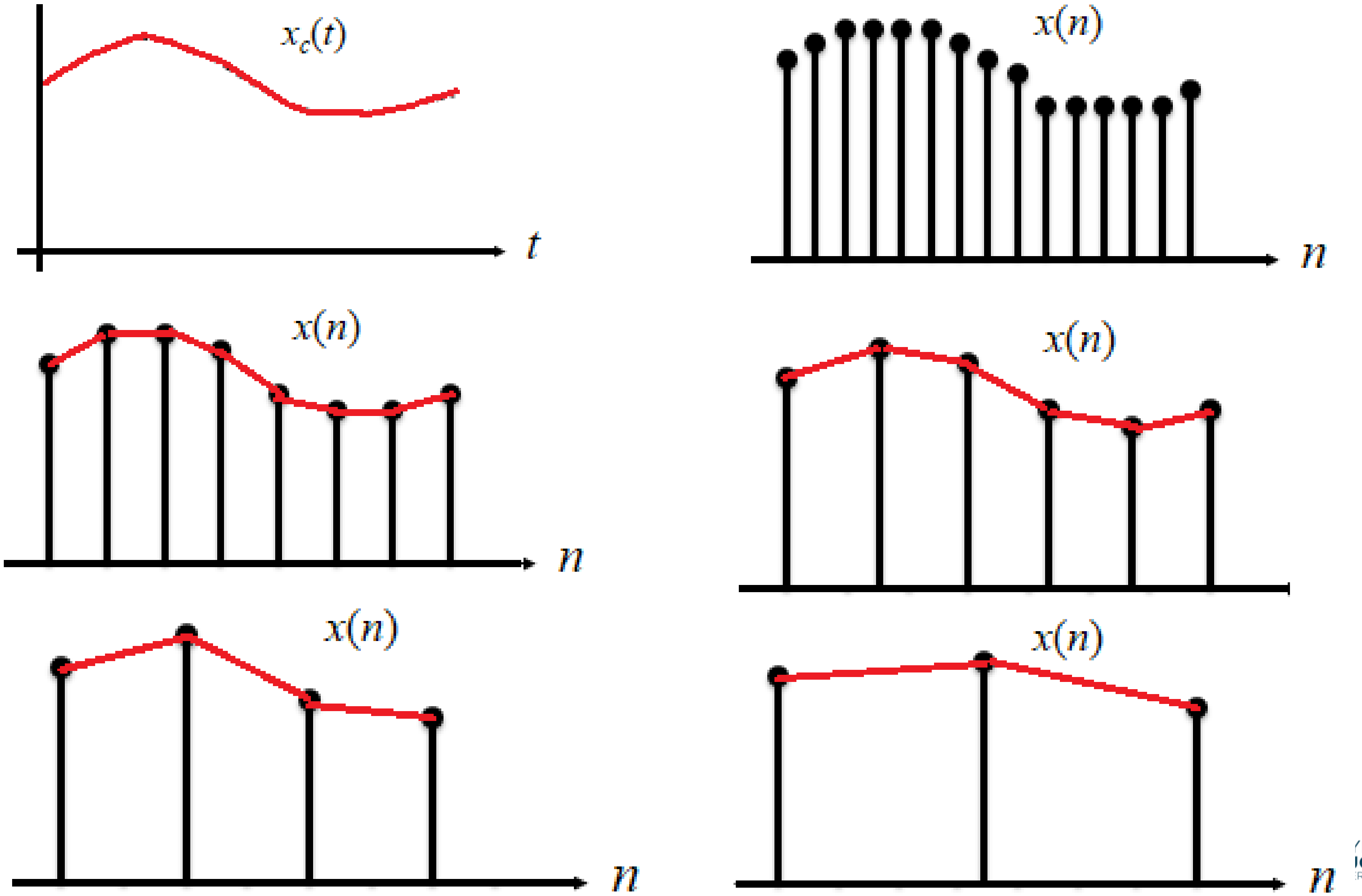


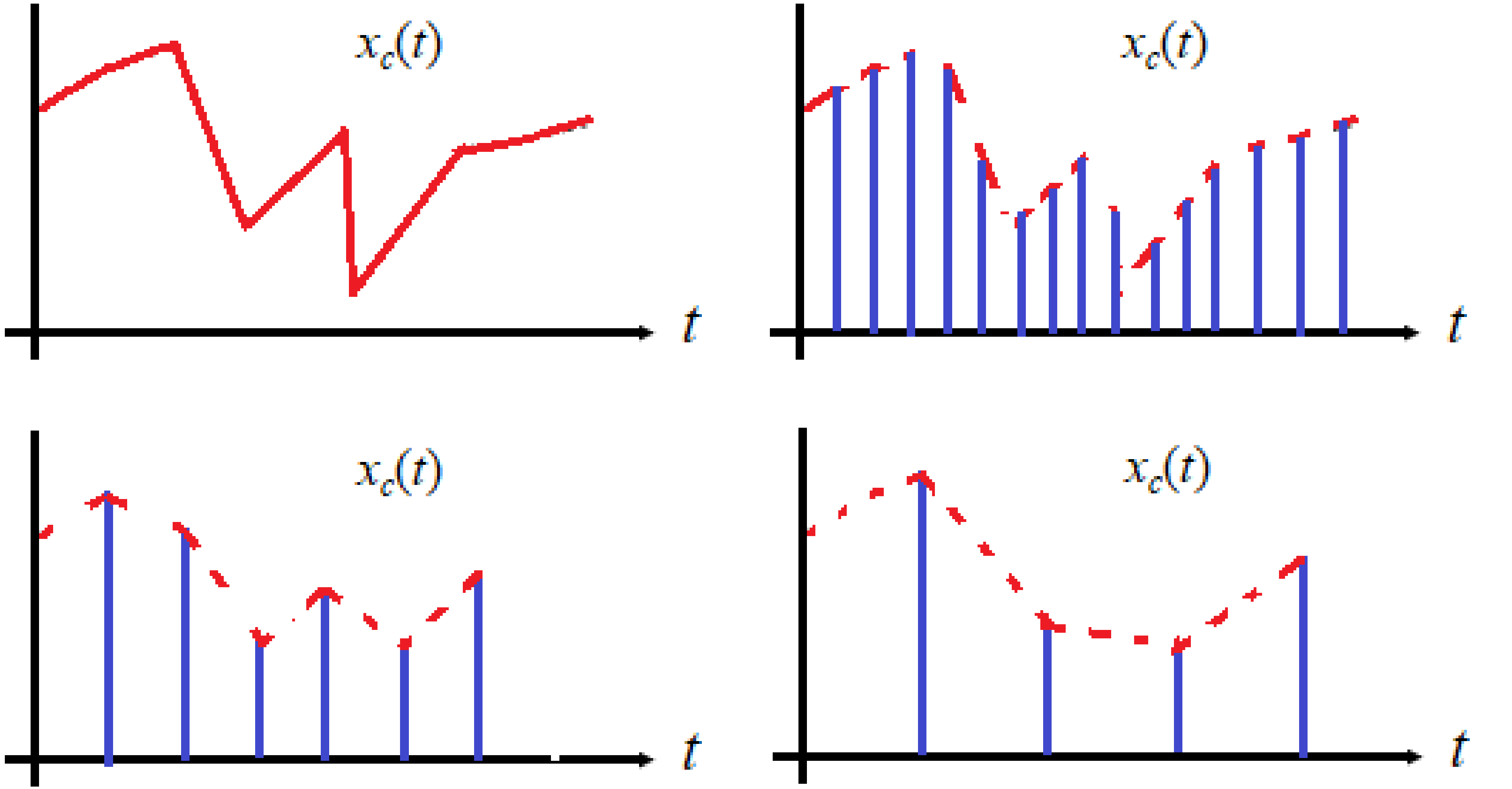

Analog
Digital
Analog
Physical Quantity
Transducer & Amplifier
ADC
DSP
DAC
Digital
Sampling
Quantization & Encoding
Discrete Time Sequence

Analog, Discrete Time and Digital Signals



How many samples to be taken for given a signal,so that the original signal can recovered exactly? Is there any rule?





How many samples to be taken for given a signal,so that the original signal can recovered exactly?

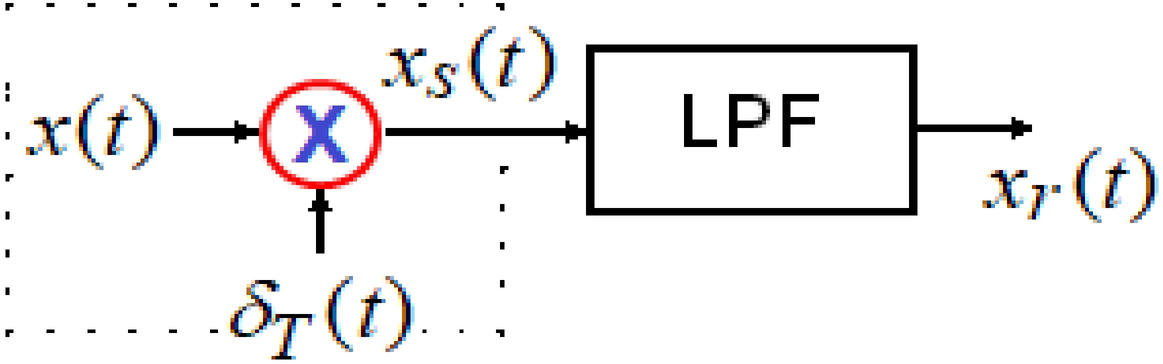
Is there any rule?

Uniform Sampling Theorem

The sampling theorem is significant in communication systems because it provides the basis for transmitting analog signals by use of digital techniques.

Time domain Statement: A band limited signal having no frequency components higher than F_m Hz may be completely recovered from the knowledge of its samples taken at the rate of at least $2F_m$ samples per second.

Frequency domain Statement: A band limited signal having no frequency components higher than F_m Hz is completely described by its sample values at uniform intervals less than or equal to $1/2F_m$ seconds apart.

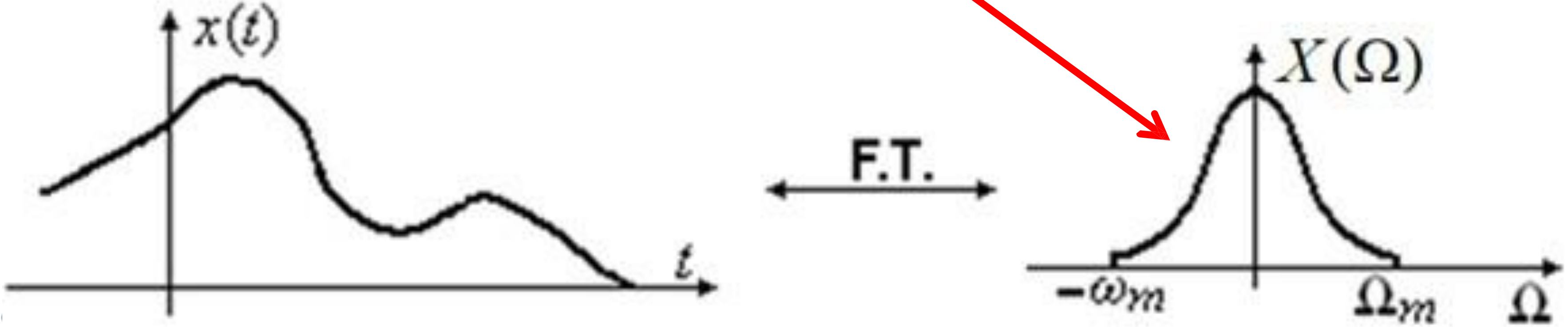


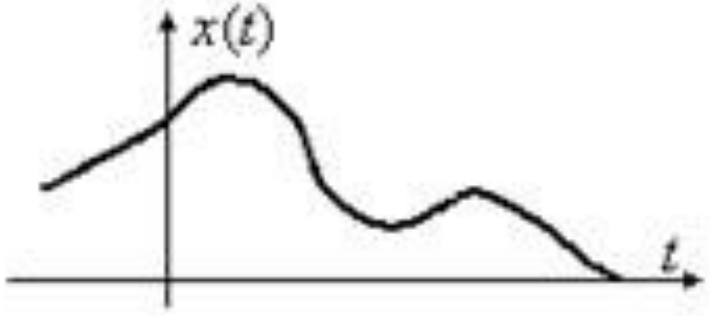
Sampling **Reconstruction**

Consider a band limited signal $x(t)$ having no frequency components beyond F_m Hz,

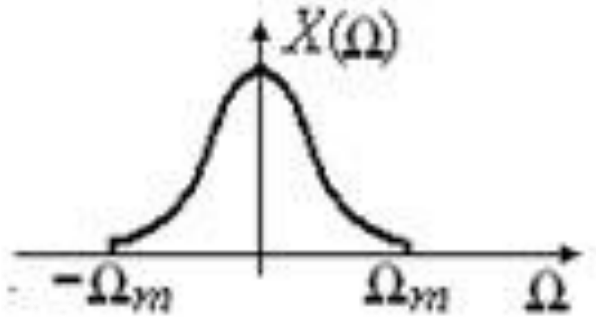
i.e., $X(\Omega) = 0$ for $|\Omega| > \Omega_m$,

where $\Omega_m = 2\pi F_m$.

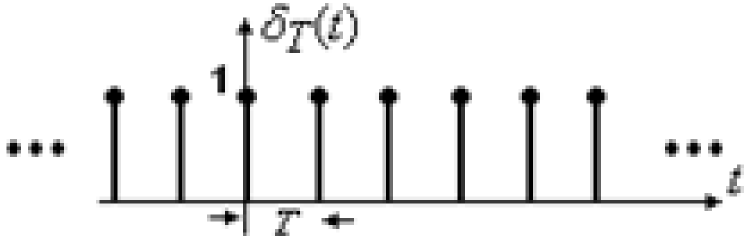




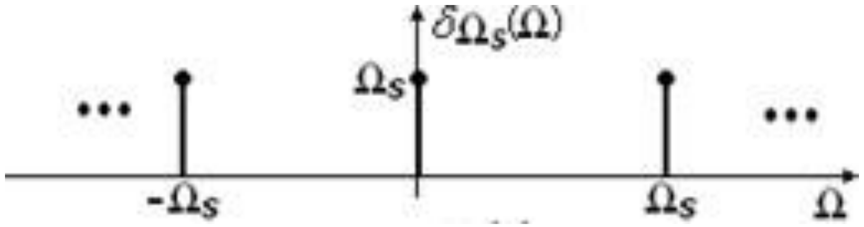
$\xleftrightarrow{\text{F.T.}}$



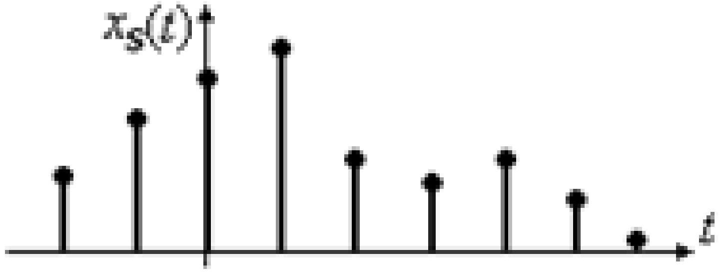
$$\delta_T(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT) \xleftrightarrow{\text{FT}} \delta_{\Omega_s}(j\Omega)=\Omega_s\sum_{k=-\infty}^{\infty}\delta(\Omega-k\Omega_s)$$



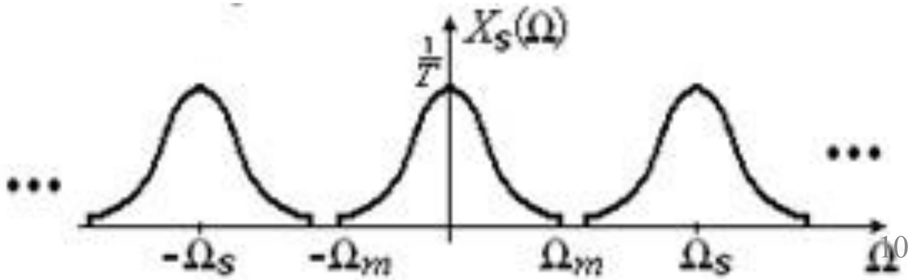
$\xleftrightarrow{\text{F.T.}}$

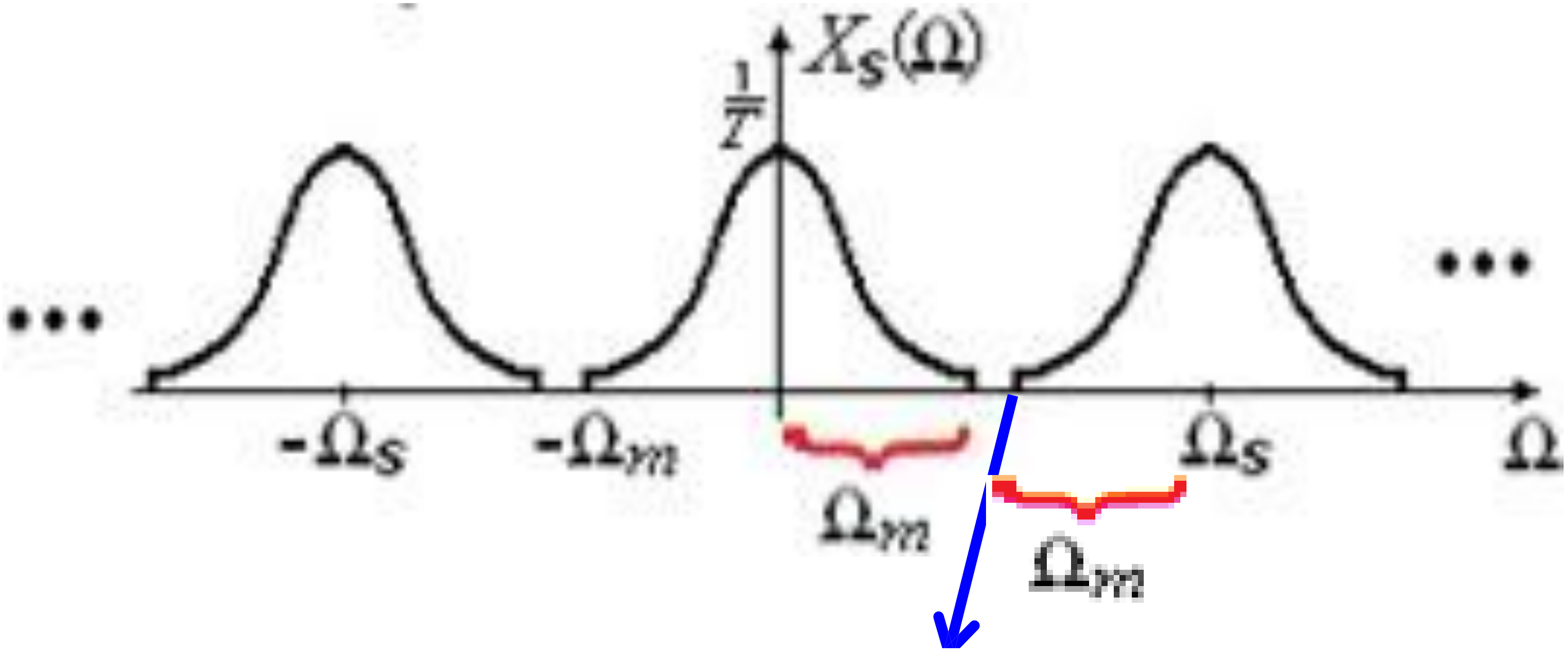


$$\sum_{n=-\infty}^{\infty} x_n\,\delta(t-nT) \xleftrightarrow{\text{FT}} \frac{1}{T}X_s(j\Omega)*\sum_{k=-\infty}^{\infty}\delta(\Omega-k\Omega_s)$$



$\xleftrightarrow{\text{F.T.}}$

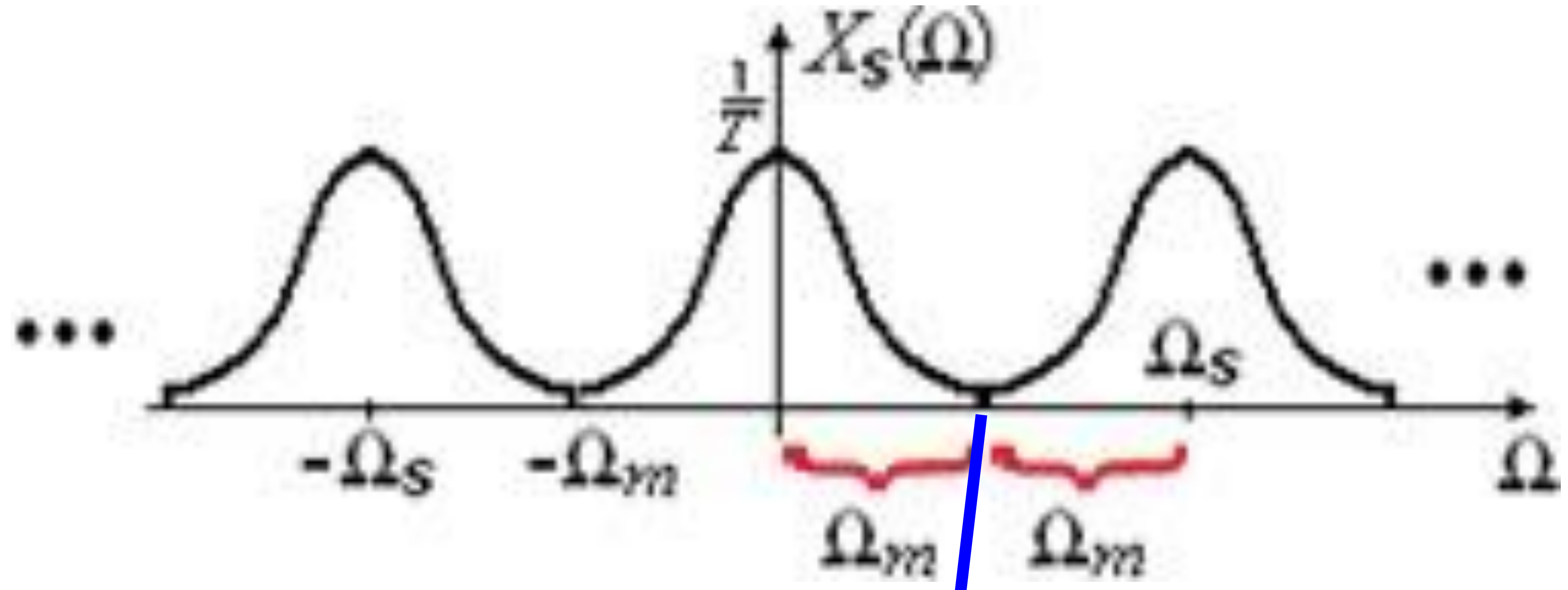




$$\Omega_s - \Omega_m > \Omega_m$$

$$\Rightarrow \Omega_s > 2\Omega_m$$

$$or \quad F_s > 2F_m$$

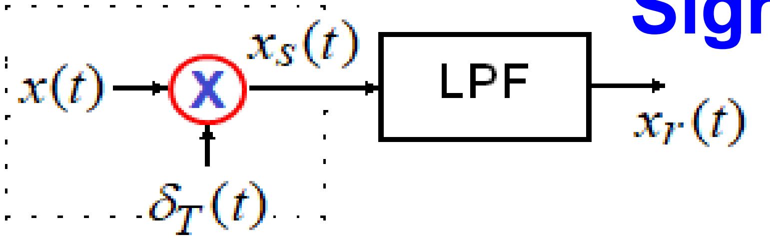


$$\Omega_s - \Omega_m$$

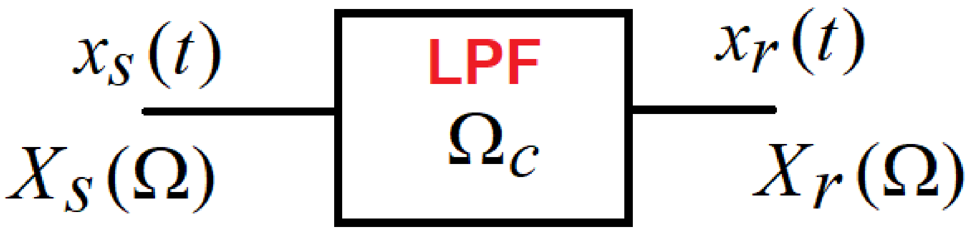
$$\Omega_s = 2\Omega_m \quad \text{or} \quad F_s = 2F_m$$ **Nyquist Rate**

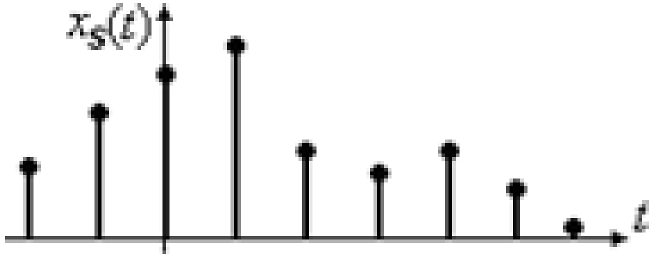
$$T_s = \frac{1}{2F_m}$$ **Nyquist Interval**

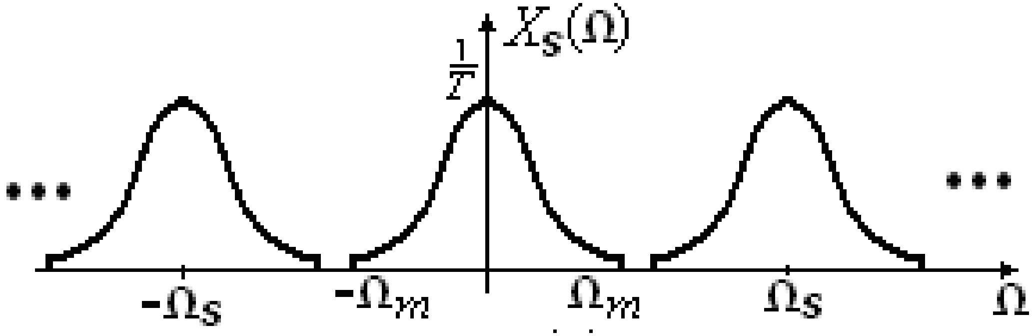
Signal Recovery from its samples

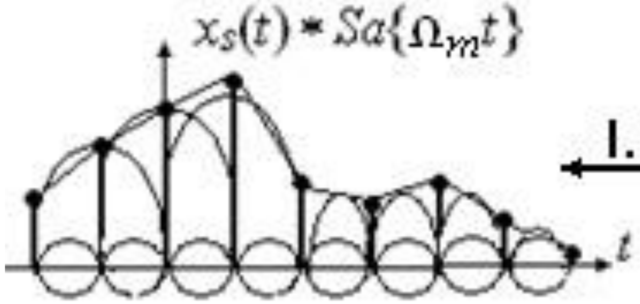


Sampling **Reconstruction**

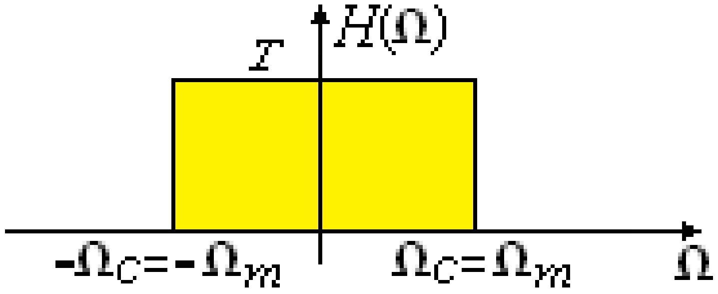


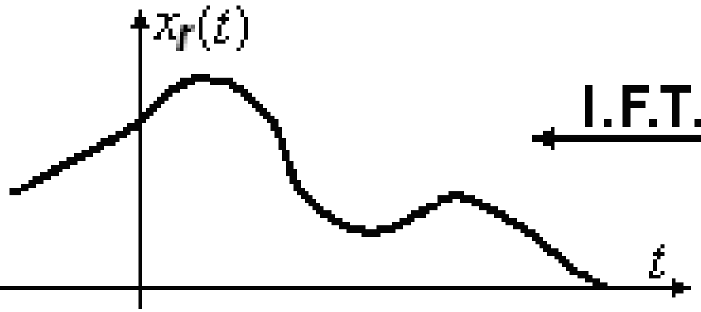






I.F.T.





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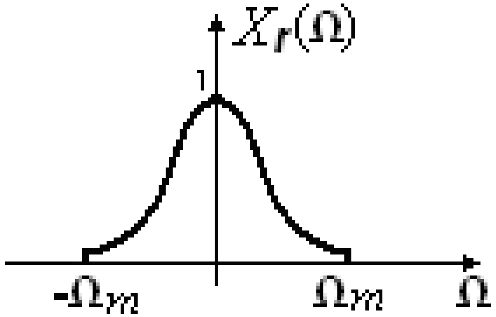
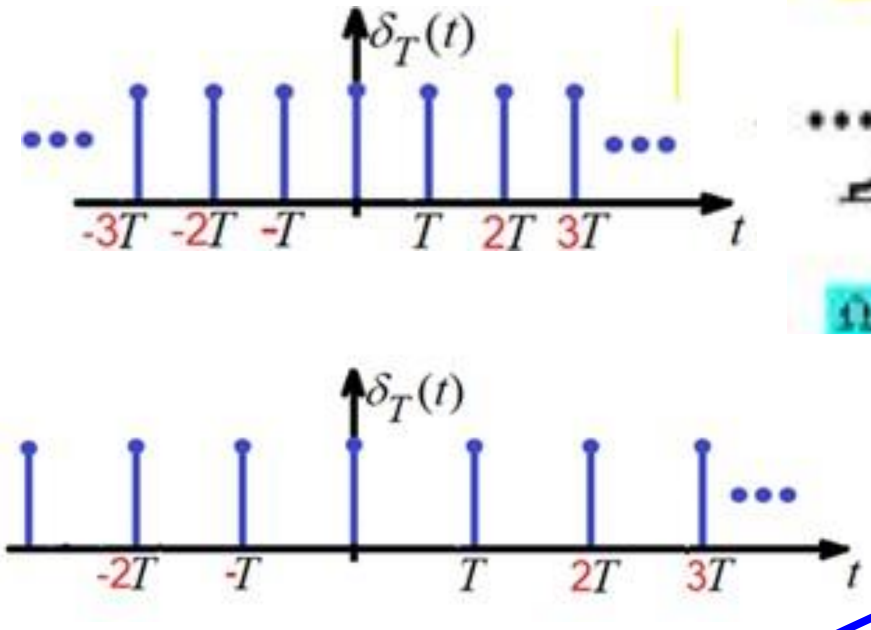
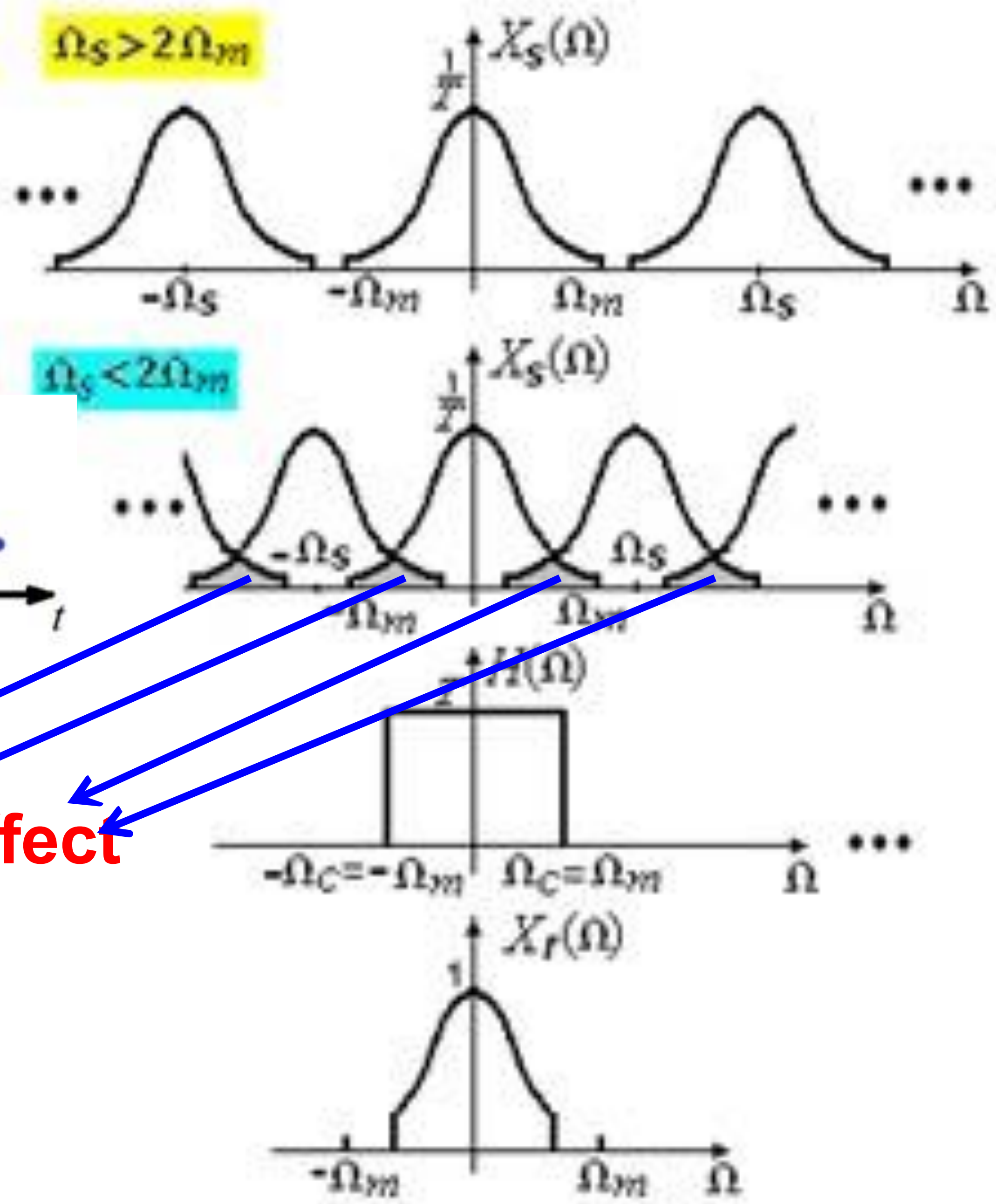


Under Sampling

Under Sampling

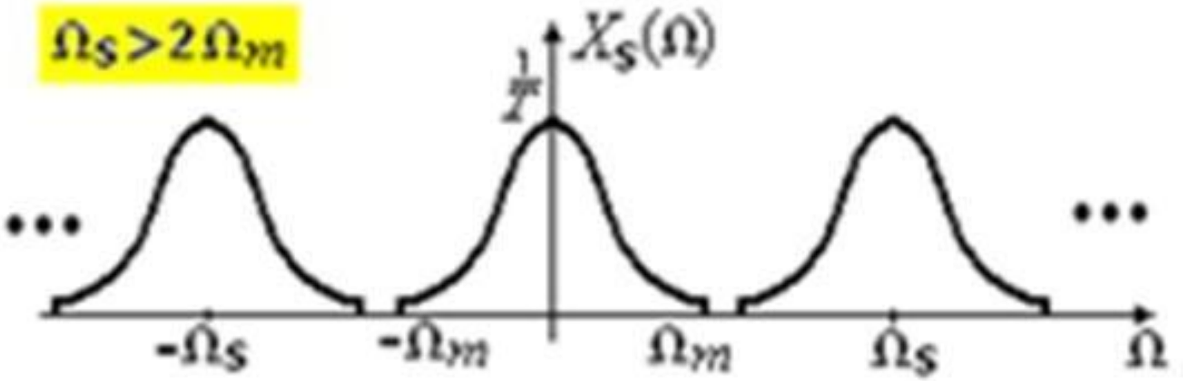




Aliasing effect / frequency folding effect

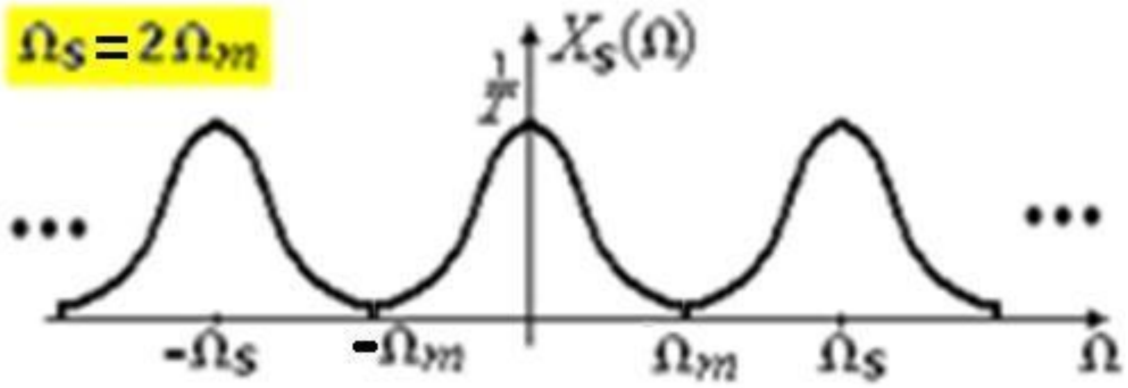
Over Sampling

$$F_s > 2F_m$$



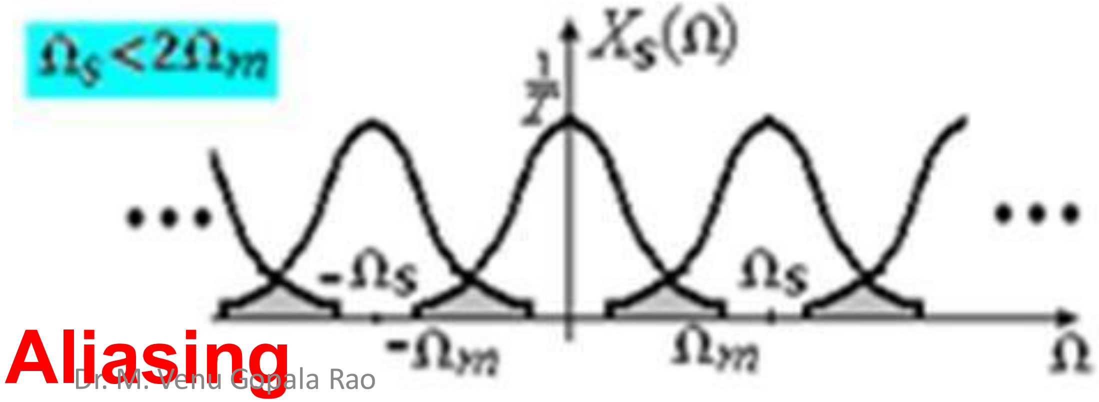
Critical Sampling

$$F_s = 2F_m$$



Under Sampling

$$F_s < 2F_m$$



Aliasing

Uniform Sampling Theorem

Time domain Statement: A band limited signal having no frequency components higher than F_m Hz may be completely recovered from the knowledge of its samples taken at the rate of at least 2 F_m samples per second.

$$F_s = 2F_m$$

In practice $F_s \geq 2F_m$

Frequency domain Statement: A band limited signal having no frequency components higher than F_m Hz is completely described by its sample values at uniform intervals less than or equal to 1 / 2 F_m seconds apart.

$$T_s = \frac{1}{2F_m}$$

In practice $T_s \leq \frac{1}{2F_m}$

Problems Discussion

Ex1. Consider two analog signals:

$$(i)\ x(t) = 3\cos 50\pi t, \quad (ii)\ y(t) = 10\sin 300\pi t.$$

Find the minimum sampling rate / Nyquist rate of the above signals

Ans: (i) Given that $x(t) = 3\cos 50\pi t.$

Then the frequency of the signal is $F_m = 25 \text{ Hz}$

Hence Nyquist rate $F_s = 2F_m = 50 \text{ sam/sec}$

(ii) The frequency of the signal is $F_m = 150 \text{ Hz}$

Hence Nyquist rate $F_s = 2F_m = 300 \text{ sam/sec}$

Ex2. Consider two analog signals:

$$(i)\ x(t) = 3\cos 50\pi t, \quad (ii)\ y(t) = 10\sin 300\pi t$$

Find the sampling rate and sampling interval for the following cases:

$$(a)\ z(t) = 2 + y(t), \qquad (b)\ r(t) = x^2(t)$$

$$(c)\ p(t) = x(t) + y(t), (d)\ q(t) = x(t)y(t)$$

$$(a)\ z(t) = 2 + y(t) = 2 + 10\sin 300\pi t$$

Ans : Highest frequency is $F_m = 150$ Hz

$$\therefore F_s = 300 \text{ sam/sec}$$

$(b)\ r(t) = x^2(t)$: $Ans : r(t) = 4.5\ (1 + \cos 100\pi t).$

Highest frequency is $F_m = 50$ Hz

$$\therefore F_s = 100 \text{ sam/sec}$$

$$(c)\ p(t) = x(t) + y(t):$$

$$Ans\ \ p(t) = 3\cos 50\pi t + 10\sin 300\pi t$$

$$:\text{Highest frequency is}\quad F_m = 150\text{ Hz}$$

$$\therefore F_s\ = 300\text{ sam/sec}$$

$$(d)\ q(t) = x(t)y(t):$$

$$Ans: q(t) = 3\cos 50\pi t\ \text{x}\ 10\sin 300\pi t$$

$$= 15\sin 350\pi t + \sin 250\pi t$$

$$\text{Highest frequency is } F_m = 175\text{ Hz}$$

$$\therefore F_s\ = 350\text{ sam/sec}$$

Determine the Nyquist rate corresponding to each of the following analog signals.

(a) $x(t) = 3\sin 100\pi t - \cos 100\pi t$

(b) $x(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + \cos 12000\pi t$

(c) $x(t) = 1 + \cos 2000\pi t + \sin 4000\pi t$

(d) $x(t) = -10\sin 40\pi t \cos 300\pi t$

(e) $x(t) = \dfrac{\sin 4000\pi t}{\pi t}$

(f) $x(t) = \left[\dfrac{\sin 4000\pi t}{\pi t}\right]^2$

(g) $x(t) = \mathrm{sinc}(100\pi t)$

(h) $x(t) = \mathrm{sinc}^3(1000\pi t)$

An analog signal $x(t)$ having unit amplitude is band limited to 1280 Hz. Determine the Nyquist rate and Nyquist interval for the following cases:

(i) $x(t)$

(ii) The frequency of signal $x(t)$ is doubled.

(iii) The amplitude of signal $x(t)$ is doubled.

(iv) $y(t) = 2x(t)\, cos\, 3400\, \pi t$.

(v) Two signals with band limited to 500 Hz and 2kHz are added to $x(t)$. The resultant signal may be referred to as $z(t)$.

(vi) Suppose now the signal with frequency 1280 Hz is removed from $z(t)$.

End