AY-2025-2026
ODD SEM

Department of ECE

ANALOG ELECTRONIC CIRCUIT DESIGN
23EC2104

Topic:

SMALL-SIGNAL OPERATION AND MODELS

Session - 07

SESSION CONTENT

- **Trans-conductance**
- **Input Resistance at the Base**
- **Input Resistance at the Emitter**
- **Voltage Gain**
- **Separating the Signal and the DC Quantities**

- **Hybrid-π Model**
- **T Model**

AIM OF THE SESSION

To understand and analyze the small-signal behavior of Bipolar Junction Transistors (BJTs) using equivalent models for AC signal analysis.

INSTRUCTIONAL OBJECTIVES

The Session is designed to:

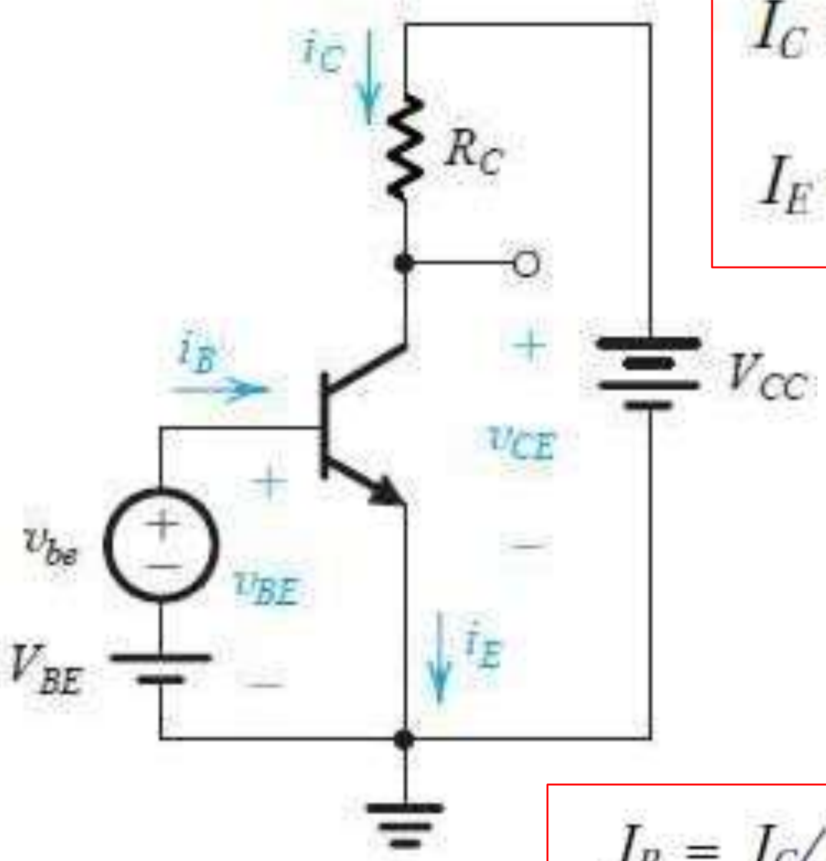
1. Define small-signal operation in BJTs and its purpose.
2. Explain the significance of small-signal models in amplifier analysis.
3. Describe the components of Hybrid-π and T-models.

LEARNING OUTCOMES

At the end of this session, learners will be able to:

1. Apply small-signal models for analyzing BJT amplifier circuits.
2. Derive and use small-signal parameters from DC operating points.
3. Choose the appropriate model (Hybrid-π or T-model) for different applications.

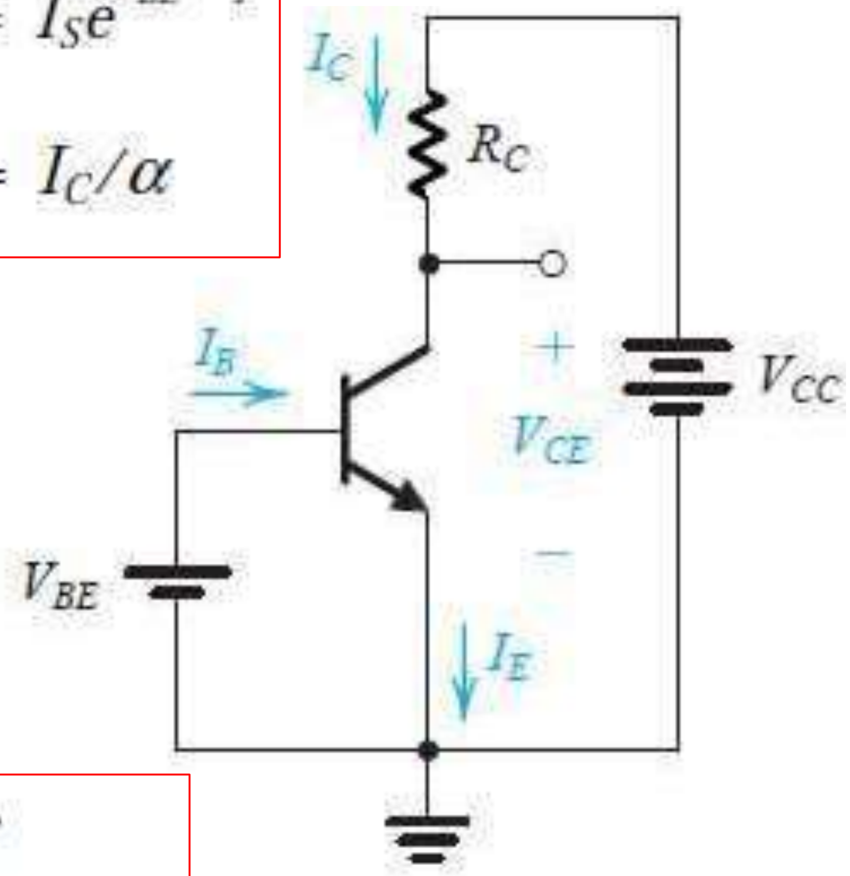
Small-Signal Operation and Models



(a)

$$I_C = I_S e^{V_{BE}/V_T}$$

$$I_E = I_C/\alpha$$



(b)

$$I_B = I_C/\beta$$

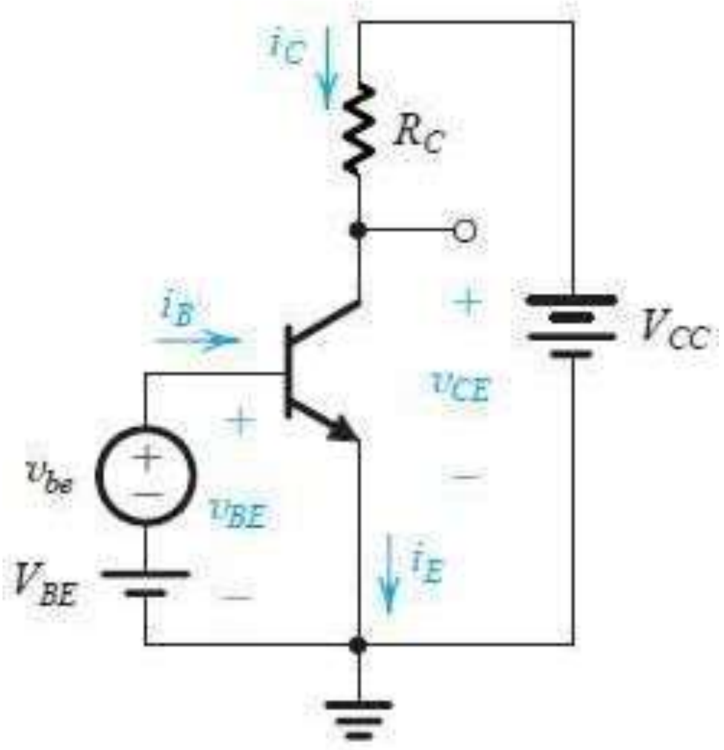
$$V_{CE} = V_{CC} - I_C R_C$$

Transconductance

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

vBE = VBE + vbe-----(1)

$$i_C = I_S e^{v_{BE}/V_T} = I_S e^{(V_{BE}+v_{be})/V_T}$$
$$= I_S e^{V_{BE}/V_T} e^{v_{be}/V_T}$$



(a)

$$i_C = I_C e^{v_{be}/V_T}$$ -----(2)

If *vbe* < *VT*,

$$e^{\frac{v_{be}}{V_T}} = 1 + \left(\frac{v_{be}}{V_T}\right) + \frac{\left(\frac{v_{be}}{V_T}\right)}{2!} + \frac{\left(\frac{v_{be}}{V_T}\right)}{3!} + \cdots$$

$$i_C \simeq I_C\left(1 + \frac{v_{be}}{V_T}\right)$$ -----(3)

Transconductance

$$e^{\frac{v_{be}}{V_T}} = 1 + \left(\frac{v_{be}}{V_T}\right) + \frac{\left(\frac{v_{be}}{V_T}\right)^2}{2!} + \frac{\left(\frac{v_{be}}{V_T}\right)^3}{3!} + \cdots \qquad \text{-----(3)}$$

This approximation, which is valid only for *vbe less than approximately 10 mV, is* referred to as the **small-signal approximation.**

$$i_C \simeq I_C\left(1 + \frac{v_{be}}{V_T}\right) \Rightarrow i_C = I_C + \frac{I_C}{V_T} v_{be} \qquad \text{-----(4)}$$

Thus the collector current is composed of the dc bias value *IC and a signal component ic ,*

$$i_c = \frac{I_C}{V_T} v_{be}$$

Transconductance

$$i_c = \frac{I_C}{V_T} v_{be}$$

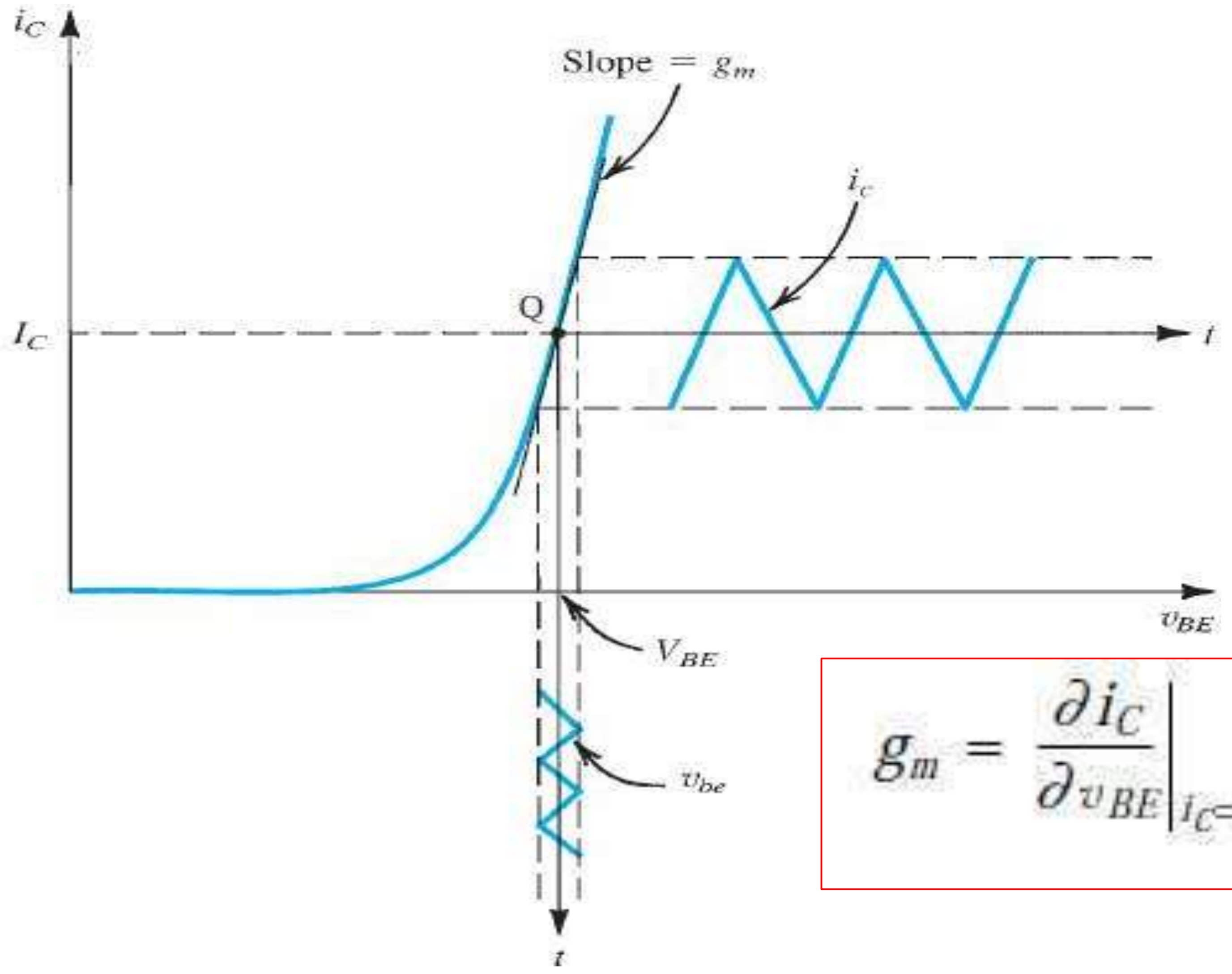
This equation relates the signal current in the collector to the corresponding base–emitter signal voltage. It can be rewritten as

$$i_c = g_m v_{be}$$

$$g_m = \frac{I_C}{V_T}$$

$$g_m = \left. \frac{\partial i_C}{\partial v_{BE}} \right|_{i_C = I_C}$$

Transconductance



$$\boxed{g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C}}$$

Input Resistance at the Base

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta}\frac{I_C}{V_T} v_{be}$$

Thus,

$$i_B = I_B + i_b$$

where *IB is equal Ic/β to and the signal component ib is given by*

$$i_b = \frac{1}{\beta}\frac{I_C}{V_T} v_{be}$$

Substituting for IC/VT by *gm gives*

$$i_b = \frac{g_m}{\beta} v_{be}$$

The small-signal input resistance between base and emitter, *looking into the base, is denoted* by ***rπ*** *and is defined as*

$$r_\pi \equiv \frac{v_{be}}{i_b}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$r_\pi = \frac{V_T}{I_B}$$

Substitute gm and Replace Ic/β by IB

Input Resistance at the Emitter

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

Thus,

$$i_E = I_E + i_e$$

where *IE is equal to IC /α and the signal current ie is given by*

$$i_e = \frac{i_c}{\alpha} = \frac{I_C}{\alpha V_T} v_{be} = \frac{I_E}{V_T} v_{be}$$

If we denote the **small-signal resistance** between **base** and **emitter** *looking into the emitter by* ***re****, it can be defined as*

$$r_e \equiv \frac{v_{be}}{i_e}$$

$$r_e = \frac{V_T}{I_E}$$

$$r_e = \frac{\alpha}{g_m} \simeq \frac{1}{g_m}$$

Input Resistance at the Emitter

The relationship between ***rπ*** *and* ***re*** *can be found by combining their respective definitions*

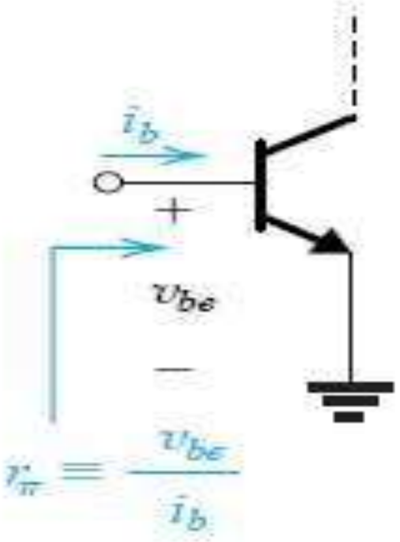
$$\boxed{v_{be} = i_b r_\pi = i_e r_e}$$

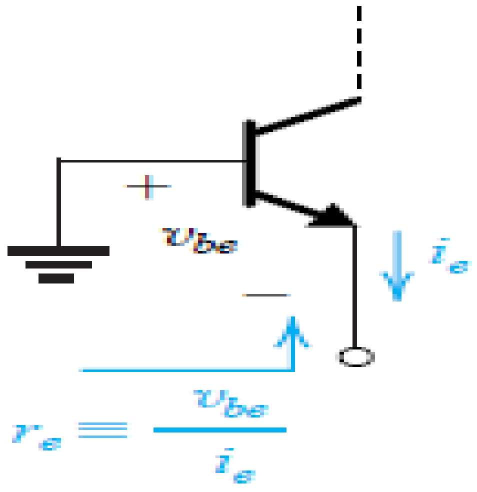
Thus,

$$\boxed{r_\pi = (i_e / i_b) r_e}$$

which yields

$$\boxed{r_\pi = (\beta + 1) r_e}$$





Voltage Gain

$$
\begin{aligned}
v_{CE} &= V_{CC} - i_C R_C \\
&= V_{CC} - (I_C + i_c) R_C \\
&= (V_{CC} - I_C R_C) - i_c R_C \\
&= V_{CE} - i_c R_C
\end{aligned}
$$

Here the quantity *VCE is the dc bias voltage at the collector, and the signal voltage is given by*

$$v_{ce} = -i_c R_C = -g_m v_{be} R_C$$

$$= (-g_m R_C) v_{be}$$

Thus the voltage gain of this amplifier *Av is*

$$A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$A_v = -\frac{I_C R_C}{V_T}$$

Small Signal Operation

Transconductance

$$g_m = \frac{I_C}{V_T}$$

$$g_m = \frac{i_c}{V_{be}}$$

$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C}$$

Input Resistance at the Base

$$i_B = I_B + i_b$$

$$r_\pi = \frac{V_T}{I_B}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$r_\pi = (\beta + 1) r_e$$

Input Resistance at the Emitter

$$i_E = I_E + i_e$$

$$r_e = \frac{V_T}{I_E}$$

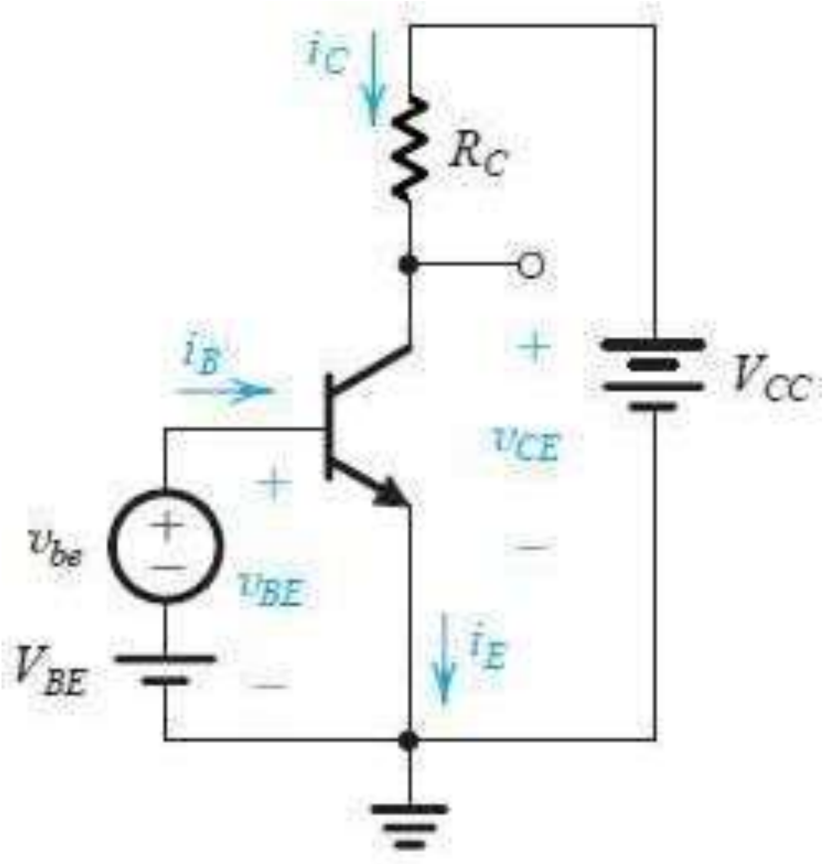
$$r_e = \frac{\alpha}{g_m} \simeq \frac{1}{g_m}$$

Voltage Gain

$$A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$A_v = -\frac{I_C R_C}{V_T}$$

Separating the Signal and the DC Quantities

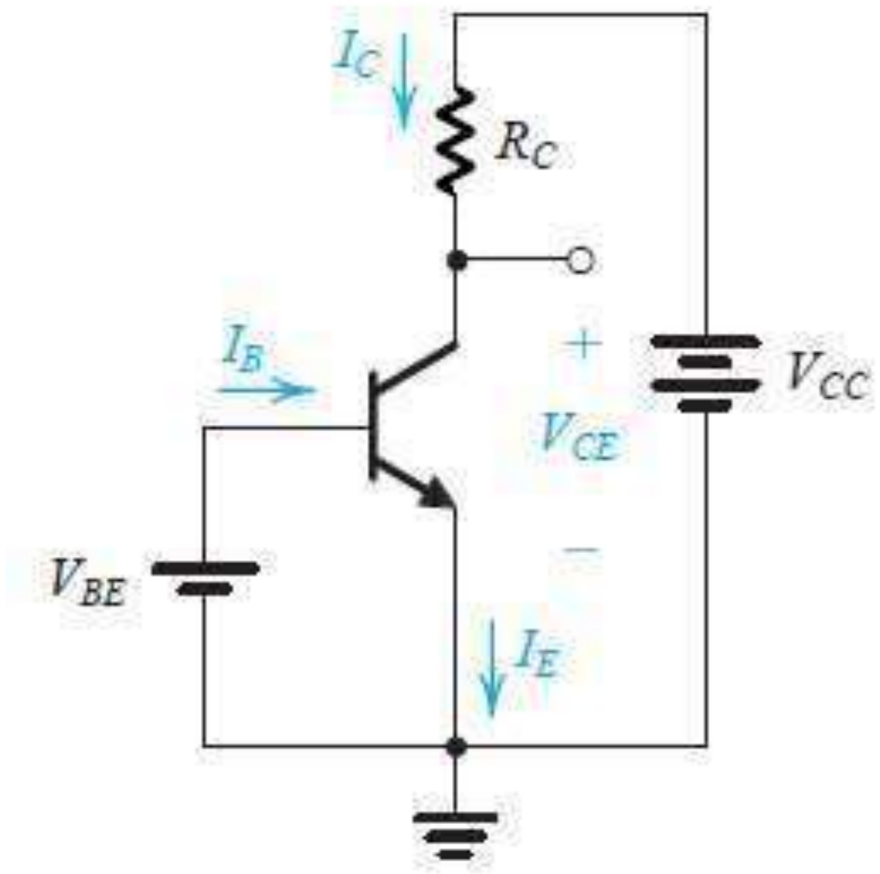


(a)

❖Every current and voltage in the amplifier circuit of Fig.(a) is composed of two components: a dc component and a signal component

❖For instance, *vBE = VBE + vbe, iC = IC + ic, and so on*

Separating the Signal and the DC Quantities



(b)

The dc components are determined from the dc circuit given in Fig. 6.36(b) and from the relationships imposed by the transistor.

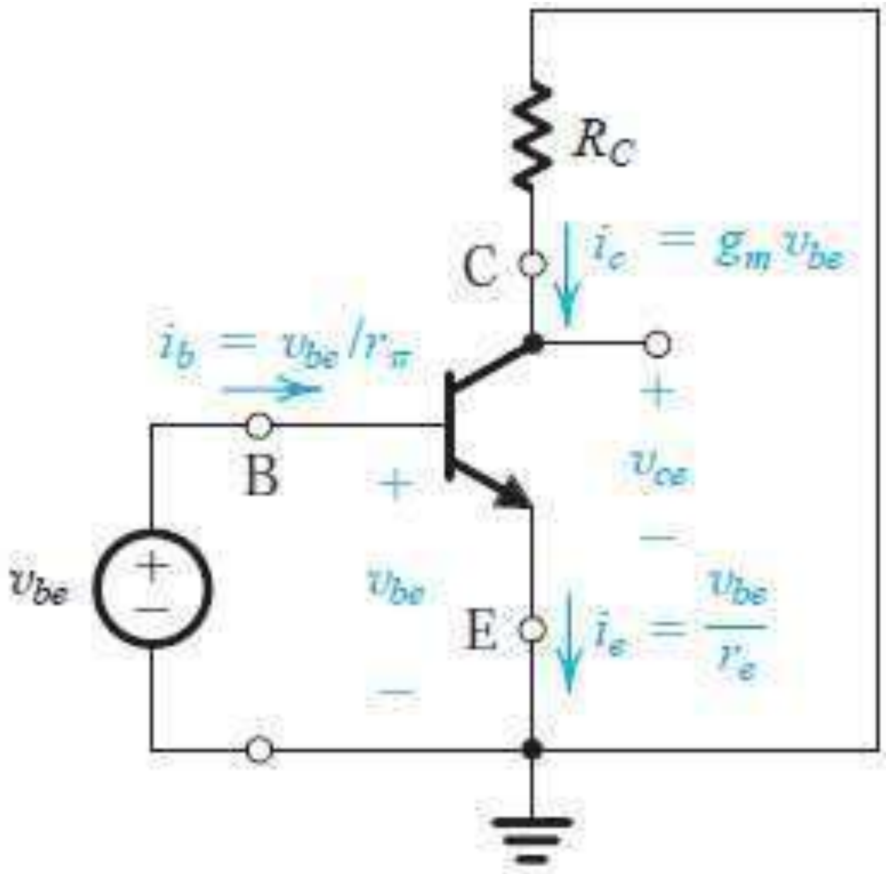
$$I_C = I_S e^{V_{BE}/V_T}$$

$$I_E = I_C/\alpha$$

$$I_B = I_C/\beta$$

$$V_{CE} = V_{CC} - I_C R_C$$

Separating the Signal and the DC Quantities

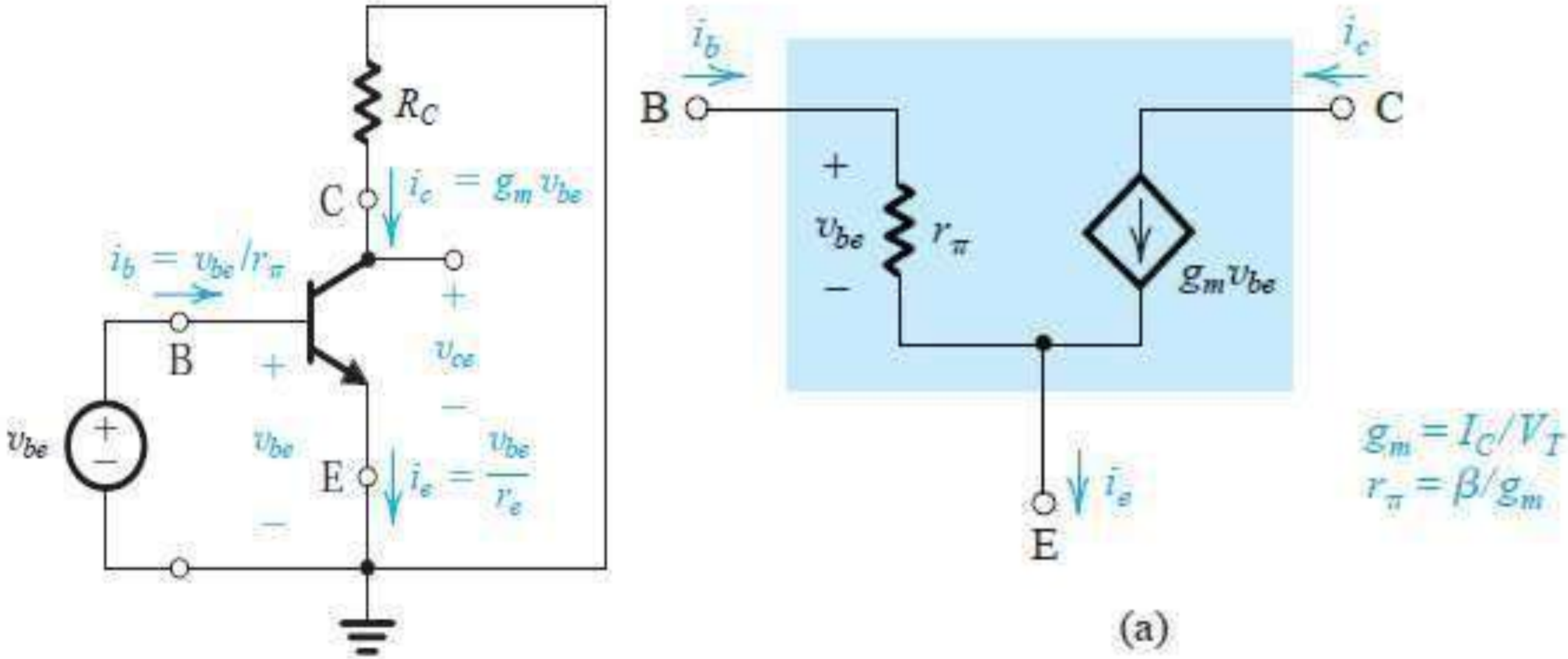


❖On the other hand, a representation of the signal operation of the BJT can be obtained by eliminating the dc sources, as shown in Fig.

❖Expressions for the current elements ***(ic, ib, and ie)*** *obtained* when a **small signal** ***vbe*** *is applied.*

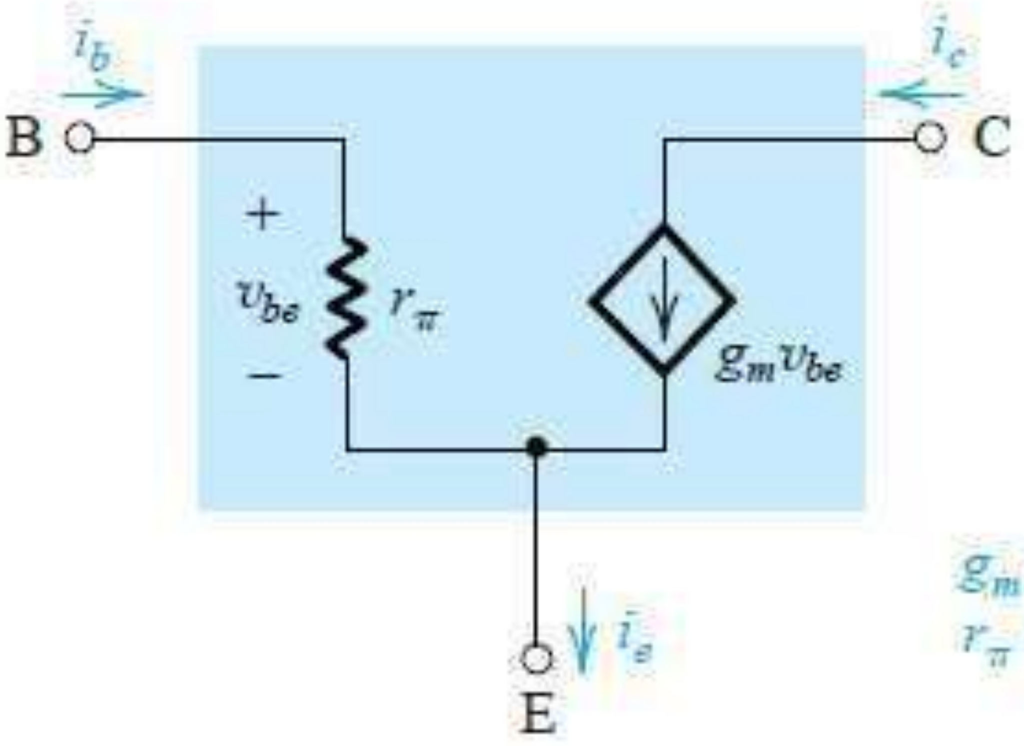
❖Small-Signal Circuit Model.

HYBRID π MODEL



This model represents the BJT as a **voltage controlled current source** and explicitly includes the input resistance looking into the base , *rπ*

HYBRID π MODEL



(a)

$g_m = I_C/V_T$
$r_\pi = \beta/g_m$

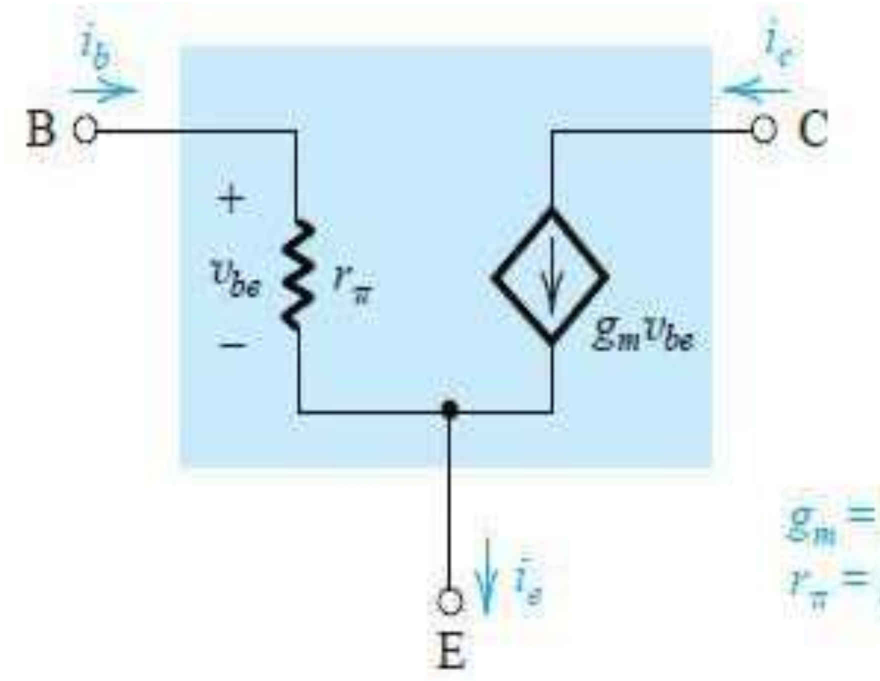
In this model

$$\boxed{\begin{aligned} i_c &= g_m v_{be} \\ i_b &= \frac{v_{be}}{r_\pi} \end{aligned}}$$

HYBRID π MODEL

In this model

Ie=ic+ib



(a)

$g_m = I_C/V_T$

$r_\pi = \beta/g_m$

$$i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}(1+g_m r_\pi)$$

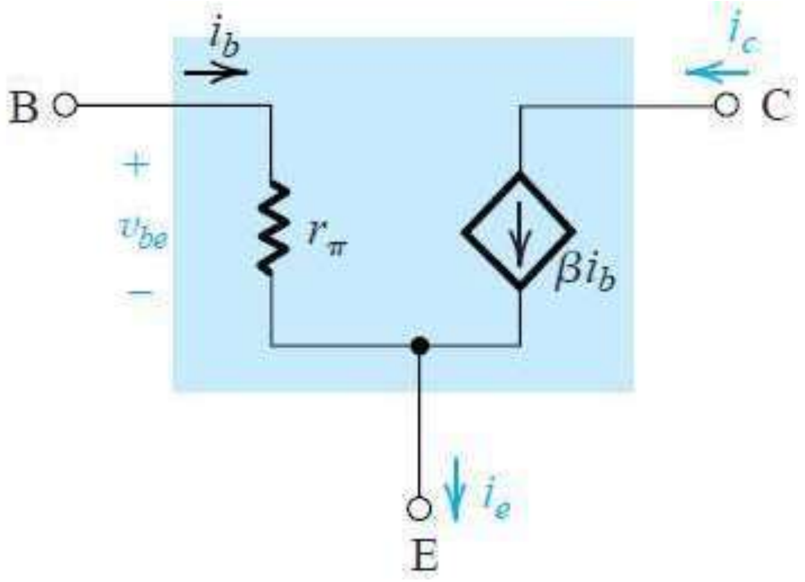
But $g_m r_\pi = \beta$;

$$= \frac{v_{be}}{r_\pi}(1+\beta) = v_{be} \Big/ \left(\frac{r_\pi}{1+\beta}\right)$$

$$= v_{be}/r_e$$

HYBRID π MODEL

Slightly different model can be obtained by expressing the **current of the controlled source** **($g_m v_{be}$)** *interms of base current* ***i_b***

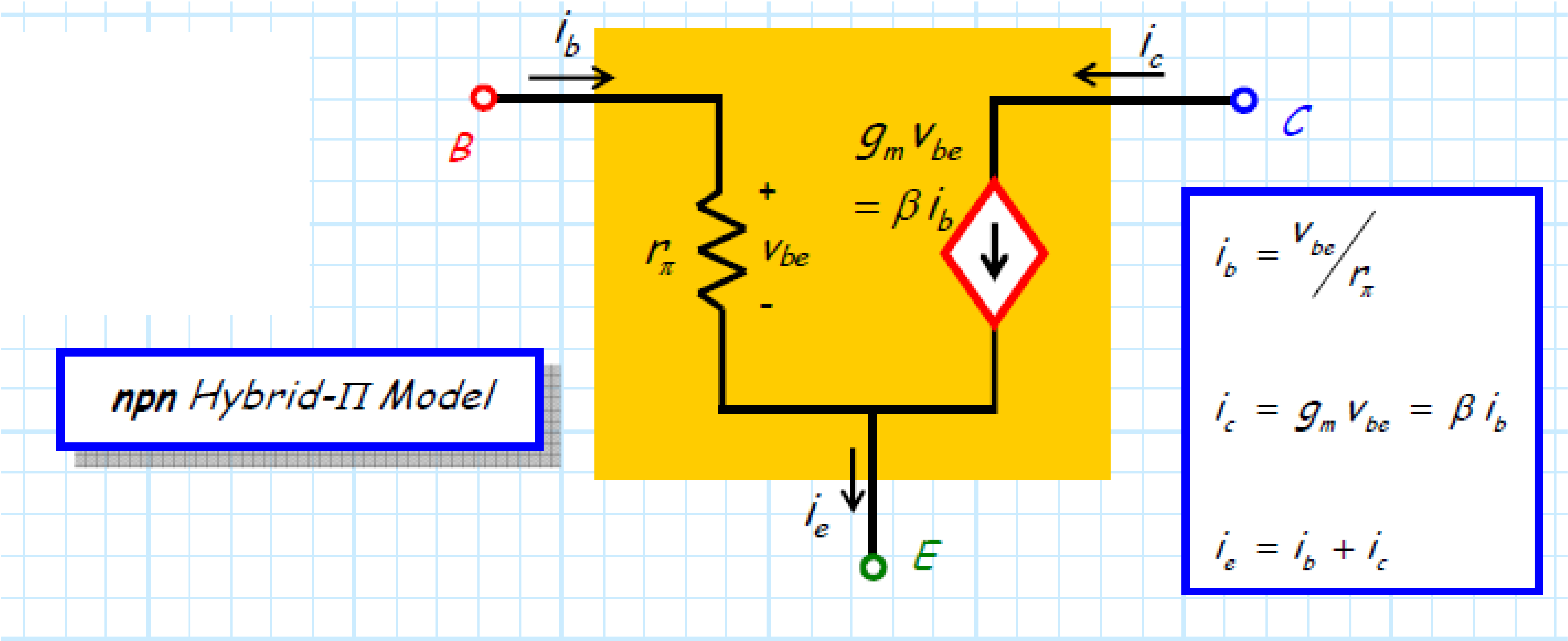


$$g_m v_{be} = g_m(i_b r_\pi)$$
$$= (g_m r_\pi) i_b = \beta i_b$$

Equivalent circuit model is

Here the transistor is represented as a **current-controlled current source**, with the control current being ***ib***.

HYBRID π MODEL

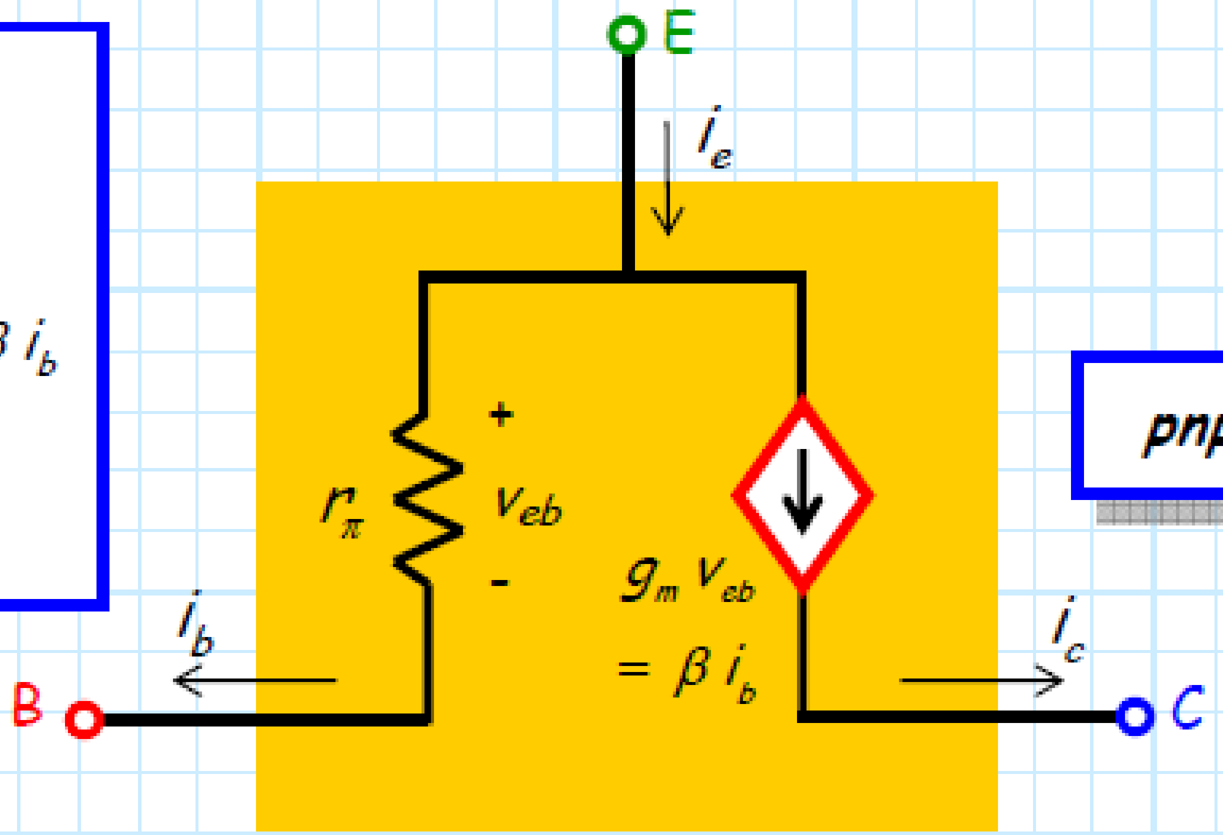


HYBRID π MODEL

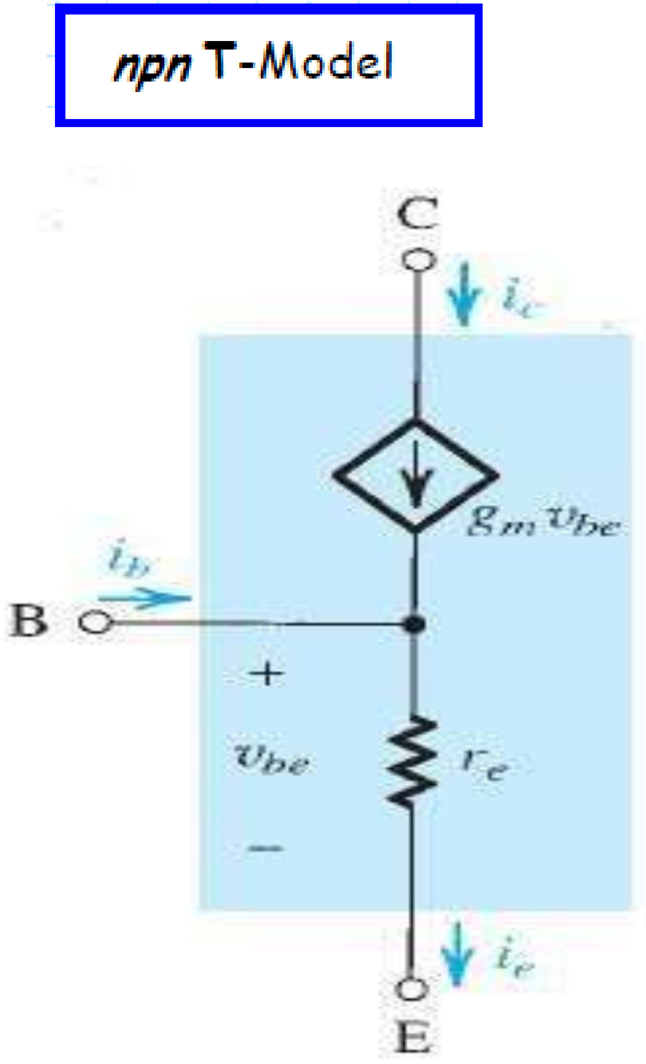
$$i_b = {v_{eb}}/{r_\pi}$$

$$i_c = g_m v_{eb} = \beta\, i_b$$

$$i_e = i_b + i_c$$



pnp **Hybrid-Π Model**



(a)

T Model

Ie=ic+ib

ib=ie-ic

$$i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e)$$

$$= \frac{v_{be}}{r_e}(1-\alpha) = \frac{v_{be}}{r_e}(1 - \frac{\beta}{\beta+1})$$

$$= \frac{v_{be}}{(\beta+1)r_e} = \frac{v_{be}}{r_\pi}$$

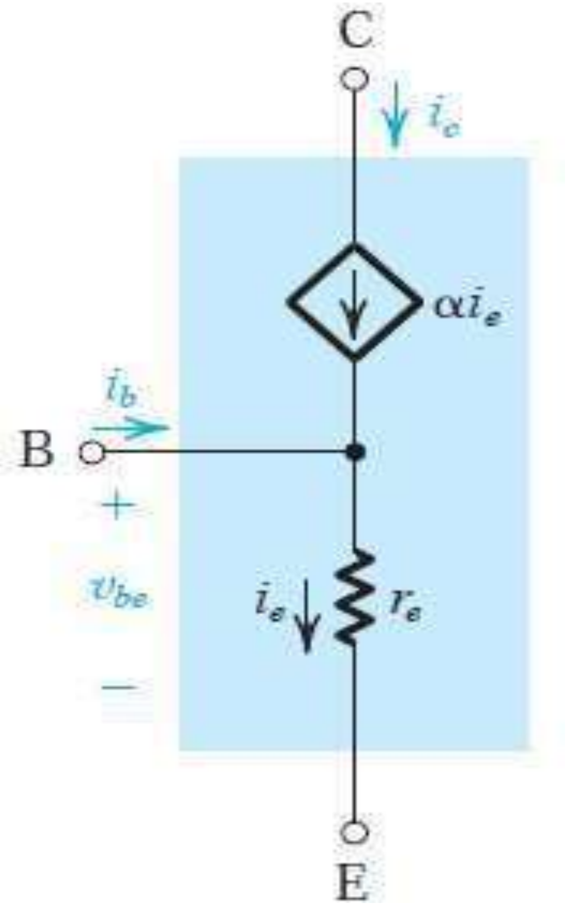
The current of the controlled source can be expressed in terms of the emitter current.

$$g_m v_{be} = g_m(i_e r_e)$$

$$= (g_m r_e) i_e = \alpha i_e$$

T Model

npn T-Model



$$i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e)$$

$$= \frac{v_{be}}{r_e}(1-\alpha) = \frac{v_{be}}{r_e}(1 - \frac{\beta}{\beta+1})$$

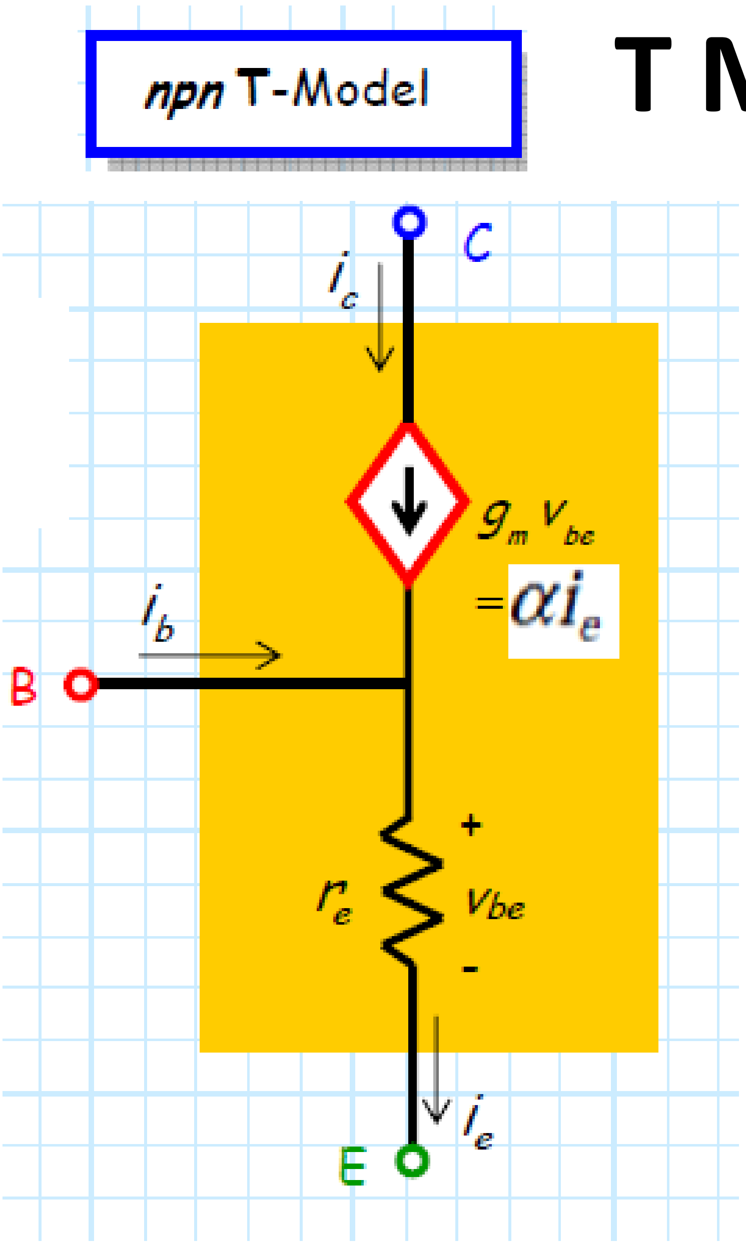
$$= \frac{v_{be}}{(\beta+1)r_e} = \frac{v_{be}}{r_\pi}$$

The current of the controlled source can be expressed in terms of the emitter current.

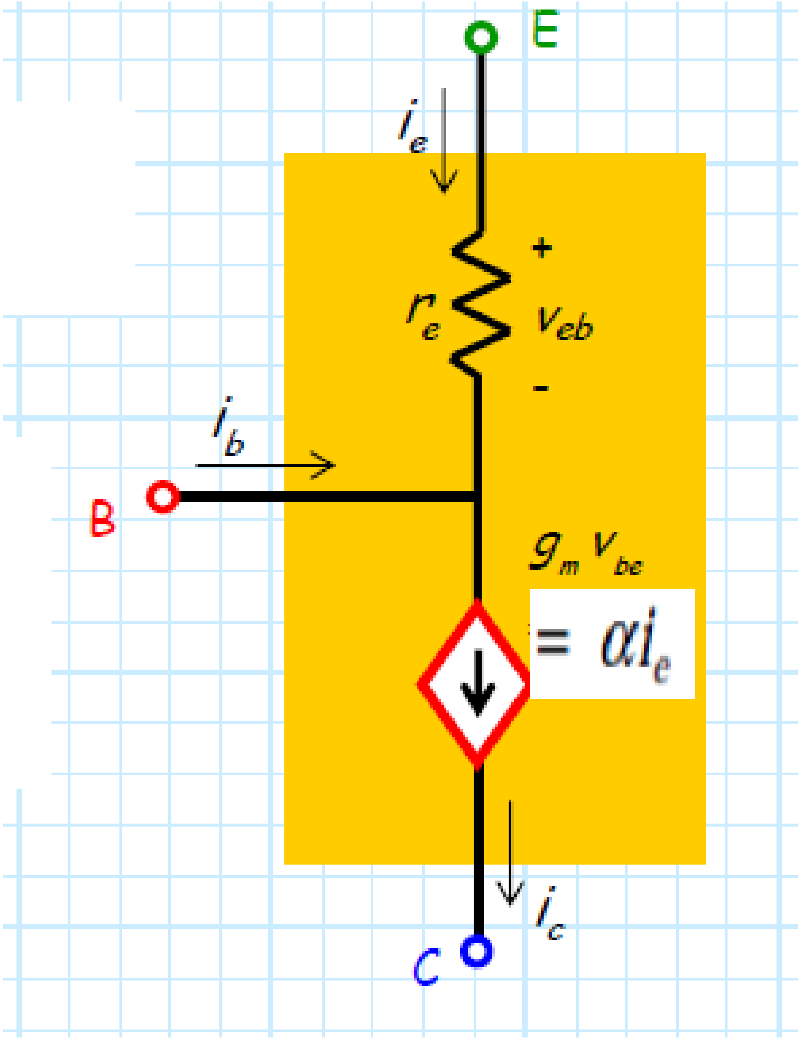
$$g_m v_{be} = g_m(i_e r_e)$$

$$= (g_m r_e) i_e = \alpha i_e$$

T Model



npn T-Model
C
i_c
$g_m v_{be}$
$=\alpha i_e$
i_b
B
r_e
+
v_{be}
-
i_e
E



E
i_e
+
r_e
v_{eb}
-
i_b
B
$g_m v_{be}$
$= \alpha i_e$
i_c
C

APPLICATION OF THE SMALL SIGNAL EQUIVALENT CIRCUIT

1. **Eliminate the signal source and determine the dc operating point of the BJT and in particular** the dc collector current *IC.*

$$\boxed{I_C, I_B, I_E \; and \; V_C}$$

2. **Calculate the values of the small-signal model parameters:**

$$\boxed{g_m = \frac{I_C}{V_T}} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_\pi = \frac{\beta}{g_m} \qquad r_e = \frac{\alpha}{g_m} \simeq \frac{1}{g_m} \qquad r_e = \frac{V_T}{I_E}$$

3. **Eliminate the dc sources by replacing each dc voltage source with a short circuit and each** dc current source with an open circuit.

APPLICATION OF THE SMALL SIGNAL EQUIVALENT CIRCUIT

4. **Replace the BJT with one of its small-signal equivalent circuit models. Although any** one of the models can be used, one might be more convenient than the others for the particular circuit being analyzed.
   - **Hybrid-π Model**
   - **T Model**
   - **Hybrid Model**

5. **Analyze the resulting circuit to determine the required quantities (e.g., voltage gain, input and output** resistance).

$$\boxed{A_v = \frac{v_0}{v_i}} \qquad \boxed{R_o \, and \, R_i}$$