CO-1

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20/01/24
Monday

Matrix

Introduction:

Matrix: The arrangement of elements in rows wise (or) column wise (or) in both and placed in a rectangular array boxes is called Matrics.

→ Always the Matrics are represented by capital Alphabets.

Ex: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$

Order: The No. of rows and columns in a matrix is called order of the matrise.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & -1 \\ 3 & 0 & -4 \\ 5 & 1 & 2 \end{bmatrix}_{4\times 3}$$

Row matrix: A Matrics which contains only a row is called row matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$$

Column matrix: A Matrix which contains only one column is called column matrix

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$$

Square Matrix: A Matrix which has equal No. of rows and column is called square matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Rectangular Matrix: A Matrix which has unequal No. of rows and columns is called Rectangular Matrix.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

Diagnal Matrix: In a Matrix all the elements equal to zero except principal Diagnal Matrix.

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Triangular Matrix:

upper triangular matrix: In a triangular Matrixs all the elements below the principal diagnal are ~~(or)~~ Equal to be Zero is called upper triangular matrix.

$$A = \begin{bmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{bmatrix}$$

Lower triangular matrix: In a triangular Matrix all the elements above the principal diagnal are ~~(or)~~ Equal to be Zero is called lower triangular matrix.

$$A = \begin{bmatrix} a & 0 & 0 \\ b & e & 0 \\ c & d & f \end{bmatrix}$$

Addition of Matrix:

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1 & 1 \\ 5 & 7 \end{bmatrix}$$

Multiplication:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+0 & -1+2 \\ 6+0 & -3+4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2+(-3) & 4+(-4) \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 3 & 4 \end{bmatrix}$$

Determinant:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a\left(\begin{vmatrix} e & f \\ h & i \end{vmatrix}\right) - b\left(\begin{vmatrix} d & f \\ g & i \end{vmatrix}\right) + c\left(\begin{vmatrix} d & e \\ g & h \end{vmatrix}\right)$$

$$= a(ei - hf) - b(di - gf) + c(dh - ge)$$

Inverse: (A^{-1})

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

echelon form: A Matrics is said to be in echelon form if.

1, 0 rows if any Exist there should be below the Non-zero rows

2, the first Non-zero Entry in Each Non-zero row is one. (optional)

3, The No. of zero's before the first Non-zero element in a row is the less than the No. of such zero's in the next row.

Rank: The No. of Non-zero rows in a echelon form is called Rank of the Matrix.
And it is denoted by row (Rho) ρ.

Session-2

1. optain the Rank of the co-efficient Matrix for the following system of Equations.

$$2x + 3y + z = 6$$
$$4x + 5y + z = 10$$
$$x + y + 3z = 5$$

sol: Here the co-efficent matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \\ 1 & 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 10 \\ 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix}$$

$$R_1 \longleftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 3 \\ 4 & 5 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 4R_1$

$R_3 \longrightarrow R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -11 \\ 0 & 1 & -5 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -11 \\ 0 & 0 & 6 \end{bmatrix}$$

$\rho(A) = 3$

2. Find the Rank of the Matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

Sol:

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 3R_1$

$R_4 \longrightarrow R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$R_4 \longrightarrow R_4 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

NOTE: If the Rank of the Matrix is 1 it is called collinear.

→ If the Rank of the Matrix is 2 it is called coplanar.

→ If the Rank of the Matrix is 3 it is called Non-coplanar.

3. check whether the vectors (should be consider in column.) $[1\ -2\ 3]$ $[-2\ 4\ 6]$ $[3\ -6\ 9]$ coplanar (or) Not

Sol:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 3 & 6 & 9 \end{bmatrix}$$

$R_2 \longrightarrow R_2 + 2R_1$

$R_3 \longrightarrow R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \\ 0 & 1012 & 0 \end{bmatrix}$$

$\rho(A) = 2$. (It is a coplanar)

4. Mr. James invest a total of Dollar 12,000 in 2 municipal Bonds one paying 10.5% intrest and other paying 12% intrest. The annual intrest earned on the two investiments last year was \$1,335. How much was invested at each rate. Model this phenomenon by the system of linear equation and Hence obtained Rank of the Matrix.

Sol: Let x be the 1st invester and y be the 2nd invester

$$x + y = 12000 \longrightarrow (1)$$

~~10.5%~~

so the intrest 1st got = 10.5%

and 2nd got = 12%

$$10.5\% \, x + 12\% \, y = 1335.$$

$$\frac{105}{1000} x + \frac{12}{100} y = 1335$$

$$0.105 x + 0.12 y = 1335 \longrightarrow (2)$$

$$A = \begin{bmatrix} 1 & 1 \\ 0.105 & 0.12 \end{bmatrix} \quad B = \begin{bmatrix} 1200 \\ 1335 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0.105 & 0.12 \end{bmatrix}$$

$$R_2 \longrightarrow R_2 - (0.105) R_1$$

$$\sim \begin{bmatrix} 1 & 1 \\ 0 & 0.015 \end{bmatrix}$$

$$\rho(A) = 2$$

5. Ava invest of \$10,000 in 3 accounts, 1 paying 5% intrest, another paying 8% intrest and third paying 9% intrest. The annual intrest Earned on the three investment last year was Dollar 770. The amount invested at 9% was twice the amount invested at 5%. determine the co-efficient Matrix and find it's Rank.

Sol: Let x be the amount invested at 5% intrest.

Let y be the amount invested at 8%.
Let z be the amount invested at 9%.

$$x + y + z = 10,000 \longrightarrow ①$$
$$5x + 8y + 9z = 770 \longrightarrow ②$$

$$z = 2(x)$$
$$2x - z = 0 \longrightarrow ③$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 9 \\ 2 & 0 & -1 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 5(R_1)$

$R_3 \longrightarrow R_3 - 2(R_1)$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 4 \\ 0 & -2 & -3 \end{bmatrix}$$

$R_3 \longrightarrow 3R_3 + 2R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{bmatrix}$$

$\rho(A) = 3.$

6. check whether it is co-planar are not
$[1 \ 1 \ 2]$ $[3 \ -1 \ 2]$ $[4 \ 0 \ 4]$ Vectors.

Sol:

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & -1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - R_1$

$R_3 \longrightarrow R_3 - 2(R_1)$

$$\sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & -4 & -4 \\ 0 & -4 & -4 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & -4 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

Session - 3

Gauss - elimination:

1. Determine the solutions of the equations

$$x + y + z = 9$$

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

sol:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Argumented Matrix $[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{array}\right]$

$$R_2 \longrightarrow R_2 - 2R_1$$

$$R_3 \longrightarrow R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{array}\right]$$

$$R_3 \longrightarrow 3R_3 + R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$$x+y+z=9 \longrightarrow (1)$$

$$3y+5z=34 \longrightarrow (2)$$

$$-4z=-20 \longrightarrow (3)$$

from Eq^n (3). $z=5$

substitute $z=5$ in eq(2).

$$3y+5(5)=34$$

$$3y+25=34$$

$$3y=34-25$$

$$3y=9$$

$$y=\frac{\cancel{9}\,3}{\cancel{3}}$$

$$y=3.$$

substitute $y=3$ and $z=5$ in eq(1).

$$x+3+5=9$$

$$x=9-8$$

$$x=1$$

2. $2x+y+z=10$

$3x+2y+3z=18$

$x+4y+9z=16$

Sol:

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \quad B=\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix} \quad X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Augmented Matrix $\cdot \left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$

$$R_3 \longleftrightarrow R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$$

$$R_2 \longrightarrow R_2 - 3R_1$$

$$R_3 \longrightarrow R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -11 & -17 & -22 \end{array}\right]$$

$$R_3 \longrightarrow 10R_3 - 7R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & 0 & -2 & -10 \end{array}\right]$$

$$x + 4y + 9z = 16 \longrightarrow (1)$$

$$-10y - 24z = -30 \longrightarrow (2)$$

$$-2z = -10 \longrightarrow (3)$$

from eq(3) $z = 5$

substitute $z=5$ in eq(2)

$$-10y - 24(5) = -30$$

$$-10y - 120 = -30$$

$$-10y = -30 \cancel{+70} + 120$$

$$-10y = 90$$

$$y = \frac{90}{-10}$$

$$y = -9$$

substitute $z = 5$, $y = -9$ in eqn (1)

$$x + 4(-9) + 9(5) = 16$$

$$x - 36 + 45 = 16$$

$$x - 9 = 16$$

$$x = 16 - 9$$

$$x = 7$$

6. $T_1 + 2T_2 + 4T_3 = 3$

$5T_1 + 5T_2 + 5T_3 = 7$

$4T_1 + 2T_3 = 4$

Sol:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 5 & 5 \\ 4 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\text{Augmented Matrix} = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 5 & 5 & 5 & 7 \\ 4 & 0 & 2 & 4 \end{array}\right]$$

$$R_2 \longrightarrow R_2 - 5R_1$$

$$R_3 \longrightarrow R_3 - 4R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & -5 & -15 & -8 \\ 0 & -8 & -14 & -8 \end{array}\right]$$

$R_3 \longrightarrow 5R_3 - 8R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & -5 & -15 & -8 \\ 0 & 0 & 50 & 24 \end{array}\right]$$

$x + 2y + 4z = 3$ — (1)

$-5y - 15z = -8$ — (2)

$50z = 24$ — (3)

from eq^n 3 $\boxed{z = \frac{12}{25}}$

$50z = 24$

$z = \frac{\cancel{24}^{12}}{\cancel{50}_{25}}$

substitute $z = \frac{12}{25}$ in eq^n (2)

$-5y - \cancel{15}^{3} \frac{12}{\cancel{25}_{5}} = -8$

$-5y - \frac{36}{5} = -8$

$\frac{-(5\times5)y - 1\times36}{5} = -8$

$-25y - 36 = -8\times5$

$-25y - 36 = -40$

$-25y - 36 = -40$

$-25y = -40 + 36$

$-25y = -4$

$y = \frac{+4}{+25}$

$\boxed{y = \frac{4}{25}}$

substitute $y = \frac{4}{25}$ and $z = \frac{12}{25}$ in eq^n (1)

$x + 2\frac{4}{25} + 4\times\frac{12}{25} = 3$

$x + \frac{8}{25} + \frac{48}{25} = 3$

$\frac{25x + 8 + 48}{25} = 3$

$25x + 8 + 48 = 3\times25$

$25x + 56 = 75$

$25x = 75 - 56$

$25x = 19$

$\boxed{x = \frac{19}{25}}$

sol!

$$\begin{array}{c|ccc|c} & A & B & C & \\ \hline R & 2 & 1 & 6 & 9 \\ G & 8 & 3 & 2 & 13 \\ B & 1 & 5 & 1 & 7 \end{array}$$

$2x + y + 6z = 9$

$8x + 3y + 2z = 13$

$x + 5y + z = 7$

$$A = \begin{bmatrix} 2 & 1 & 6 \\ 8 & 3 & 2 \\ 1 & 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 13 \\ 7 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Argumented matrix: $\left[\begin{array}{ccc|c} 2 & 1 & 6 & 9 \\ 8 & 3 & 2 & 13 \\ 1 & 5 & 1 & 7 \end{array}\right]$

~~RANK of A = R~~

$R_3 \longleftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 7 \\ 8 & 3 & 2 & 13 \\ 2 & 1 & 6 & 9 \end{array}\right]$$

$R_2 \longrightarrow R_2 - 8R_1$

$R_3 \longrightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 7 \\ 0 & -37 & -6 & -43 \\ 0 & -9 & 4 & -5 \end{array}\right]$$

$R_3 \longrightarrow 37R_3 - 9R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 7 \\ 0 & -37 & -6 & -43 \\ 0 & 0 & 202 & 202 \end{array}\right]$$

$x + 5y + z = 7 \longrightarrow (1)$

$-37y - 6z = -43 \longrightarrow (2)$

$202z = 202 \longrightarrow (3)$

from eq^n 3

$z = 1$

$202z = 202$

$z = \frac{202}{202}$, $\boxed{z=1}$

substitute $z=1$ in eq^n 2

$-37y - 6(1) = -43$

$-37y - 6 = -43$

$-37y = -43 + 6$

$-37y = -37$

$y = \frac{-37}{-37}$, $\boxed{y=1}$

substitute $z=1$ and $y=1$ in eq^n 1

$x + 5(1) + 1 = 7$

$x + 5 + 1 = 7$

$x + 6 = 7$

$x = 7 - 6$

$\boxed{x = 1}$

$2x + y + 6z = 9$

$2 + 1 + 6 = 9$

$9 = 9$

$x + 5y + z = 7$

$1 + 5 + 1 = 7$

$7 = 7$

$8x + 3y + 2z = 13$

$8 + 3 + 2 = 13$

$13 = 13$

LU-Decomposition (or) factorization (or) Do-little.

Consider the co-efficient Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The system of Equations in Matrix form is $AX = B$

According to LU-Decomposition method we have $A = LU$

where,

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

According to Do-little method we have

$$l_{11} = l_{22} = l_{33} = 1$$

Now,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = LU$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

By the simplification we can get the 9 unknown values

l_{21}, l_{31}, l_{32} U_{11} U_{12} U_{13} U_{22} U_{23} U_{33}

We have, $AX = B \longrightarrow (1)$

where $A = LU \longrightarrow (2)$

substitute eq'n (2) in (1)

$$LUX = B \longrightarrow (3)$$

put $UX = V \longrightarrow (4)$

substitute eq'n (4) in (3)

$$LV = B \longrightarrow (5)$$

from this we can get V matrix

from eq'n (4)

$$UX = V$$

we can get the unknown Matrix X.

1. Apply LU-Decomposition method find lower and upper triangular matrixs and, Hence solve the Equations

$$3x + 2y + 7z = 4$$

$$2x + 3y + z = 5$$

$$3x + 4y + z = 7$$

Sol:

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = B$$

$$A = LU$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{21} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} U_{11}+0+0 & U_{12}+0+0 & U_{13}+0+0 \\ l_{21}U_{11}+0+0 & l_{21}U_{12}+U_{22}+0 & l_{21}U_{13}+U_{23}+0 \\ l_{31}U_{11}+0+0 & l_{31}U_{12}+l_{32}U_{22}+0 & l_{31}U_{13}+l_{32}U_{23}+U_{33} \end{bmatrix}$$

$U_{11} = 3$

$U_{12} = 2$

$U_{13} = 7$

$l_{21}U_{11} = 2 \Rightarrow l_{21}(3) \Rightarrow 2/3$

$l_{21}U_{12} + U_{22} = 3 \Rightarrow 2/3(2) + U_{22} = 3 \Rightarrow U_{22} = \frac{5}{3}$

$$l_{21}U_{13}+U_{23}=1 \Rightarrow \frac{2}{3}(7)+U_{23}=1 \Rightarrow U_{23}=\frac{-11}{3}$$

$$l_{31}U_{11}=3 \Rightarrow l_{31}(3) \Rightarrow l_{31}=1$$

$$l_{31}U_{12}+l_{32}U_{22}=4 \Rightarrow 1(2)+l_{32}\left(\frac{5}{3}\right)=4$$

$$2+l_{32}\left(\frac{5}{3}\right)=4$$

$$l_{32}=\frac{6}{5}$$

$$l_{31}U_{13}+l_{32}U_{23}+U_{33}=1 \Rightarrow (1)(7)+\frac{6}{5}\left(\frac{-11}{3}\right)+U_{33}=1$$

$$7+\frac{6}{5}\left(\frac{-11}{3}\right)+U_{33}=1$$

$$U_{33}=\frac{-18}{5}$$

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix}$$

$$U=\begin{bmatrix} 3 & 2 & 7 \\ 0 & \frac{5}{3} & -\frac{11}{3} \\ 0 & 0 & \frac{-8}{5} \end{bmatrix}$$

$$Ax = B \longrightarrow (1)$$

$$A = LU \longrightarrow (2)$$

$$LUx = B \longrightarrow (3)$$

$$Ux = V \longrightarrow (4)$$

$$LV = B \longrightarrow (5)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ 1 & \frac{6}{5} & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} V_1 + 0 + 0 \\ \frac{2}{3}V_1 + V_2 + 0 \\ V_1 + \frac{6}{5}V_2 + V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$V_1 = 4$$

$$\frac{2}{3} \times 4 + V_2 + 0 = 5 \Rightarrow \frac{2}{3}(4) + V_2 = 5 \Rightarrow V_2 = \frac{7}{3}$$

$$V_1 + \frac{6}{5}V_2 + V_3 = 7 \Rightarrow 4 + \frac{6}{5}\left(\frac{7}{3}\right) + V_3 = 7 \Rightarrow V_3 = \frac{1}{5}$$

$$V = \begin{bmatrix} 4 \\ \frac{7}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$Ux = V$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & \frac{5}{3} & -11/3 \\ 0 & 0 & \frac{-8}{5} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{7}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$3x + 2y + 7z = 4$$

$$\frac{5}{3} y - \frac{11}{3} z = \frac{7}{3}$$

$$\frac{-8}{5} z = \frac{1}{5}$$

$$z = \frac{\frac{1}{5}}{\frac{-8}{5}}$$

$$z = \frac{1}{5} \times \frac{-5}{8}$$

$$z = \frac{-1}{8}$$

substitute in eqn (2) $\boxed{z = -\frac{1}{8}}$

$$\longrightarrow \frac{5}{3} y - \frac{11}{3}(z) = \frac{7}{3}$$

$$\frac{5}{3} y - \frac{11}{3}\left(\frac{-1}{8}\right) = \frac{7}{3}$$

$$= \frac{5}{3} y + \frac{11}{24} = \frac{7}{3}$$

$$= \frac{(8 \times 5) y + 1 \times 11}{24} = \frac{7}{3}$$

$$= \frac{40y + 11}{24} = \frac{7}{3}$$

$$= 40y + 11 = \frac{7}{\cancel{3}} \times \cancel{24}^{8}$$

$$- 40y + 11 = 56$$

$\cancel{y = \frac{9}{8}}$

$$= 40y = 56 - 11$$

$$- 40y = 45$$

$$y = \frac{\cancel{45}^{9}}{\cancel{40}_{8}}$$

$$\boxed{y = \frac{9}{8}}$$

$3x+2y+7z=4$

$x=\frac{21}{24}=\frac{7}{8}$

$=3x+\cancel{2}^{1}\left(\frac{9}{\cancel{8}_4}\right)+7\left(\frac{-1}{8}\right)=4$

$3x+\frac{9}{4}-\frac{7}{8}=4$

$\frac{(8\times3)x+2\times9-1\times7}{8}=4$

$=\frac{24x+18-7}{8}=4$

$=24x+11=4\times8$

$=24x+11=32$

$=24x=32-11$

$24x=21$

$x=\frac{\cancel{21}\,7}{\cancel{24}\,8}$

$\boxed{x=\frac{7}{8}}$

2.

	A	B	C	Time
MA-I	2	1	1	180
	1	3	2	300
	2	1	2	240

$2x+y+z=180$

$x+3y+2z=300$

$2x+y+2z=240$

Let x is time available of a type A

y be the time available of type B

z be the time available on type C

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 180 \\ 300 \\ 240 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = B$$

$$A = LU.$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{23} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} U_{11}+0+0 & U_{12}+0+0 & U_{13}+0+0 \\ l_{21}U_{11}+0+0 & l_{21}U_{12}+U_{22}+0 & l_{21}U_{13}+U_{23}+0 \\ l_{31}U_{11}+\overset{0}{l_{32}}U_{22}+0 & l_{31}U_{12}+l_{32}U_{22}+0 & l_{31}U_{13}+l_{32}U_{23}+U_{33} \end{bmatrix}$$

$U_{11} = 2$

$U_{12} = 1$

$U_{13} = 1$

$l_{21}U_{11} = 1 \rightarrow l_{21}(2) \Rightarrow \frac{1}{2}$

$l_{21}U_{12} + U_{22} = 3 \Longrightarrow \frac{1}{2}(1) + U_{22} = 3 \Longrightarrow \frac{3}{1} - \frac{1}{2}$

$\frac{(2 \times 3) - 1}{2} = \frac{5}{2}$

$l_{21} u_{13} + u_{23} = 2 \Rightarrow \frac{1}{2}(1) + u_{23} = 2 \Rightarrow u_{23} = 2 - \frac{1}{2}$

$= \frac{(2\times 2) - 1\times 1)}{2}$

$= \frac{3}{2}$

$l_{31} u_{11} = 2 \rightarrow l_{31}(2) = 1$

$l_{31} u_{12} + l_{32} u_{22} = 1 \Rightarrow \cancel{1(1) + 1 \times \frac{5}{2}} = 1$ $\quad l_{32} = 0$

$\cancel{1 + \frac{5}{2}} = 1$

$\cancel{\frac{2+5}{2}} = 1$

$\cancel{\frac{7}{2}} = 1$

$= \cancel{\frac{1}{1} = \frac{7}{2}}$

$= \cancel{\frac{2 - 1\times 7}{2}}$

$l_{31} u_{13} + l_{32} u_{23} + u_{33} = 2$ $\quad u_{33} = 1$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$Ax = B \longrightarrow (1)$

$A = LU \longrightarrow (2)$

$LUx = B \longrightarrow (3)$

$Ux = V \longrightarrow (4)$

$LV = B \longrightarrow (5)$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 180 \\ 300 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} v_1 + 0 + 0 \\ \frac{1}{2}v_1 + v_2 + 0 \\ v_1 + 0 + v_3 \end{bmatrix} = \begin{bmatrix} 180 \\ 300 \\ 240 \end{bmatrix}$$

$\rightarrow \quad v_1 = 180$

$\rightarrow \quad \frac{1}{\cancel{2}} \times \cancelto{90}{180} + v_2 + 0 = 300$

$90 + v_2 + 0 = 300$

$v_2 = 300 - 90$

$v_2 = 210$

$\rightarrow \quad 180 + 0 + v_3 = 240$

$v_3 = 240 - 180$

$= 60$

$$v = \begin{bmatrix} 180 \\ 210 \\ 60 \end{bmatrix}$$

$$Ux = v$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & 3/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 180 \\ 210 \\ 60 \end{bmatrix}$$

$2x + y + z = 180$ $\boxed{y=48}$

$\frac{5}{2}y + \frac{3}{2}z = 210$

$z = 60$

$2x + 48 + 60 = 180$

$2x + 108 = 180$

$2x = 180 - 108$

$2x = 72$

$\rightarrow \frac{5}{2}y + \frac{3}{\cancel{2}_1} \times \cancel{60}^{30} = 210$

$x = \frac{72}{2}$

$\frac{5}{2}y + 90 = 210$

$\boxed{x = 36}$

$\frac{5}{2}y = 210 - 90$

$\frac{5}{2}y = 120$

$y = \cancel{120}^{24} \times \frac{2}{\cancel{5}}$

Eigen values and Eigen vectors:

Let x be any non-zero vector, of a matrix A, is said to be Eigen vector if there Exist a scalar λ such that $Ax = \lambda x$ where λ is called Eigen value.

properties:

1. Eigen values are also called as characteristic roots are roots. characteristic roots are Latent roots.
2. The Eigen vector are also called as characteristic vectors are Latent vectors.
3. the characteristic matrix is $[A - \lambda I]$
4. the characteristic equation is $|A - \lambda I| = 0$
5. For 2×2 matrix the characteristic equation is $\lambda^2 - \lambda(Tr(A)) + \det A = 0$
6. For 3×3 matrix the characteristic equation is $\lambda^3 - \lambda^2(Tr(A)) + \lambda$(sum of minors of the principal diagonal elements) $- \det A = 0$

Properties of Eigen Values:

① If the given matrix is triangular Matrix that is Lower triangular Matrix (or) upper triangular Matrix then the diagonal elements of the Eigen Matrix Values

Ex: find the Eigen values for the Matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 7 & 5 \end{bmatrix}$$

sol: the given Matrix is lower triangular Matrix so the diagnal elements of the Eigen values.

$$\lambda = 2, 3, 5$$

② find the Eigen values for the $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 5 \end{bmatrix}$

sol: The given Matrix is upper triangular Matrix. so the diagnal elements of the Eigen values.

$$\lambda = 1, 0, 5.$$

③ If $\lambda_1, \lambda_2, \lambda_3$ are the Eigen values of the Matrix A then the Eigen values of the A^{-1} Matrix are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$

Ex: If 1, 3 and 5 are the Eigen values of the Matrix A. then the Eigen values of the A^{-1}

sol: The Eigen values of the A^{-1} Matrix is

$\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$

(3) If λ_1, λ_2 and λ_3 are the Eigen values of the Matrix A then the Eigen values of the A^n Matrix is $\lambda_1^n, \lambda_2^n, \lambda_3^n$

Ex: if 2, -3, 5 are the eigen values of the Matrix A then the Eigen values of the A^3 Matrix

sol: $2^3, (-3)^3, 5^3$

(4) Always sum of the Eigen values must be equal to the trace of the Matrix and the product of the Eigen values is equal's to the determinent of the Matrix

session - 5

Q. Determine Eigen Values and Eigen Vectors of the Matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

Sol: The Given Matrix is $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

The characteristic Matrix $[A - \lambda I]$ =

$$[A - \lambda I] = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

The characteristic equation = $|A - \lambda I| = 0$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

(Q3)

$$\lambda^2 - \lambda(\operatorname{Tr}(A)) + \det A = 0$$

$$\lambda^2 - \lambda(7) + 6 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$$

$$\boxed{\lambda = 1, 6}$$

<u>Eigen-Vectors</u>

$$(A - \lambda I) = \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

Now $[A - \lambda I]\, X = 0$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Now $\lambda = 1$

$$\begin{bmatrix} 5-1 & 4 \\ 1 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$4x + 4y = 0 \Rightarrow x + y = 0$$

$$x + y = 0$$

$$x = -y$$

$$\frac{x}{1} = \frac{y}{-1}$$

$$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ (or) } \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Now $\lambda = 6$

$$\begin{bmatrix} 5-6 & 4 \\ 1 & 5-6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x + 4y = 0 \Rightarrow x - 4y = 0$$

$$x - 4y = 0$$

$$x - 4y = 0$$

$$x = 4y.$$

$$\frac{x}{4} = \frac{y}{1}$$

$$x_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

2. $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

Sol: The given matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

Matrix

the characteristic equation $\begin{bmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{bmatrix}$

the characteristic equation

$$\begin{bmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(4-\lambda) \mp 10 = 0$$

$$4 - \lambda - 4\lambda + \lambda^2 - 10 = 0$$

$$\lambda^2 - 5\lambda - 6 = 0$$

$$\lambda = 6, -1$$

$$[A - \lambda I] = \begin{bmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Now $\lambda = 6$

$$\begin{bmatrix} -5 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-5x - 2y = 0 \Rightarrow x + y = 0$$

$$-5x + 2y = 0$$

$$5x = -2y$$

$$\Rightarrow \frac{x}{-2} = \frac{y}{5}$$

$$x_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$\lambda = -1$

$$\begin{bmatrix} 2 & -2 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x - 2y = 0 \quad x - y = 0$$
$$-5x + 5y = 0 \Rightarrow x - y = 0$$

$$\cancel{2x - 2y}$$
$$x - y = 0$$
$$x = y$$
$$\frac{x}{1} = \frac{y}{1}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3. $$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 0\,4 \\ 0\,3 & 2-\lambda \end{bmatrix}$$

$$= \begin{vmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda(Tra(A)) + \det A = 0$$
$$\lambda^2 - 3\lambda - 10 = 0$$
$$\lambda^2 + 2\lambda - 5\lambda - 10 = 0$$
$$\lambda(\lambda+2) - 5(\lambda+2) = 0$$
$$(\lambda-5)(\lambda+2) = 0$$
$$\boxed{\lambda = 5, -2}$$

<u>Eigen vectors</u>

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Now $\lambda = 5$

$$\begin{bmatrix} 1-5 & 4 \\ 3 & 2-5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-4x+4y=0 \Rightarrow x-y=0$

$3x-3y=0 \Rightarrow x-y=0$

$x=y$

$\frac{x}{1}=\frac{y}{1}$

$x_1=\begin{bmatrix}1\\1\end{bmatrix}$

Now. $\lambda=2$

$\begin{bmatrix}1+2 & 4\\ 3 & 2+2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}3 & 4\\ 3 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$3x+4y=0$ $\quad \frac{x}{-4}=\frac{y}{3}$

$3x+4y=0$

$3x=-4y$ $\quad x_2=\begin{bmatrix}-4\\3\end{bmatrix}$

4. find the Eigen Values and Eigen vectors for the Matrix $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$

Sol: The charateristic matrix $\begin{bmatrix}8-\lambda & -6 & 2\\ -6 & 7-\lambda & -4\\ 2 & -4 & 3-\lambda\end{bmatrix}$

The chalateristic matrix $\begin{vmatrix}8-\lambda & -6 & 2\\ -6 & 7-\lambda & -4\\ 2 & -4 & 3-\lambda\end{vmatrix}=0$

$\lambda^3-\lambda^2(Tr(A))+\lambda$(sum of minors of Pd element) $-$ det $A=0$

$Tr(A)=8+7+3=18$

sum of minor $=\begin{vmatrix}7 & -4\\ -4 & 3\end{vmatrix}+\begin{vmatrix}8 & 2\\ 2 & 3\end{vmatrix}+\begin{vmatrix}8 & -6\\ -6 & 7\end{vmatrix}$

$=(21-16)+(24-4)+(56-36)$

$=5+20+20$

$=45.$

$\det A = 8(21-16) + 6(-18+8) + 2(24-14)$

$= 8(5) + 6(-10) + 2(10)$

$= 40 - 60 + 20$

$\det A = 0$

Characteristic eqn: $\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\lambda(\lambda^2 - 18\lambda + 45) = 0$

$\lambda = 0$ and $\lambda^2 - 18\lambda + 45 = 0$

$\lambda^2 - 3\lambda - 15\lambda + 45 = 0$

$\lambda(\lambda - 3) - 15(\lambda - 3) = 0$

Eigen value $0, 3, 15, 0$.

Eigen vectors:

characteristic matrix $\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$

$\lambda = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$8x - 6y + 2z = 0$

$-6x + 7y - 4z = 0$

$2x - 4y + 3z = 0$

$$\begin{array}{cccc} & x & y & z \\ -6 & 2 & 8 & -6 \\ 7 & -4 & -6 & 7 \end{array}$$

$$\frac{x}{24-14} = \frac{y}{-12+32} = \frac{z}{56-36}$$

$$\frac{x}{10} = \frac{y}{20} = \frac{z}{20}$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$\lambda = 3$

sub $\lambda = 3$ in eq (1)

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x - 6y + 2z = 0$$
$$-6x + 4y - 4z = 0$$
$$2x - 4y + 0 = 0$$

$$\begin{array}{cccc} & x & y & z \\ -6 & 2 & 5 & -6 \\ 4 & -4 & -6 & 4 \end{array}$$

$$\frac{x}{24-8} = \frac{y}{-12+20} = \frac{z}{20-36}$$

$$\frac{x}{16} = \frac{y}{8} = \frac{z}{-16}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$$

$$x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

If $\lambda = 15$

sub in eqn (1)

$$\begin{bmatrix} -4 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -3-\lambda 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow (9) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4x - 6y + 2z = 0$$
$$-6x - 8y - 4z = 0$$
$$2x - 4y - 12z = 0$$

x		y		z
-6	2	-4		-6
-8	-4	-6		-8

$$\frac{x}{-24+16} = \frac{y}{-12-28} = \frac{z}{-56-36}$$

$$\frac{x}{\cancel{-8}\ 40} = \frac{y}{-40} = \frac{z}{20} \qquad x_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

5. find Eigen Values and Eigen Vectors for the Matrix $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Sol: the characteristic matrix $\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

The characteristic eq'n $\begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$

$\lambda^3 - \lambda^2 (Tr(A)) + \lambda(\text{sum of Minors of p.d}) - \det A = 0$

$Tr(A) = 2-5+3 = 0$

$\text{minors} = \begin{vmatrix} -5 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 2 & -5 \end{vmatrix}$

$= -15 + 6 + (-10+6)$

$= -15+6-4$

$= -13$

$\det A = 2(-15-0) + 3(6-0) + 0(\quad)$

$= -12$

The characteristic eq'n

$$\lambda^3 - \lambda^2(0) + \lambda(-13) - (-12) = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

$1 \quad 0 \quad -13 \quad 12$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & 0 & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$\lambda = 1$

$$\lambda^2 + \lambda - 12 = 0$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$

$$\lambda(\lambda+4) - 3(\lambda+4) = 0$$

$$(\lambda-3)(\lambda+4) = 0$$

$$\boxed{\lambda = 1, 3, -4}$$

Eigen vectors

The charac'l eq'n $\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

If $\lambda = 1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x - 3y + 0 = 0$$
$$2x - 6y + 0 = 0 \Rightarrow x - 3y = 0 \Rightarrow x = 3y$$
$$0 + 0 + 2z = 0.$$
$$\frac{x}{3} = \frac{y}{1}.$$

If $\lambda = 3$

$$= \begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= -1x - 3y + 0 = 0$$
$$2x - 8y = 0$$

$$-2\cancel{x} - 6y = 0 \longrightarrow 1 \times 2$$
$$2\cancel{x} - 8y = 0 \longrightarrow 2$$
$$-14y = 0$$
$$y = 0$$
$$x = 0$$

$$x_2 = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}$$

$$\begin{array}{cccc} x & y & z & \\ -3 & 0 & -1 & -3 \\ -8 & 0 & -2 & -8 \end{array}$$

$$\frac{x}{0-0} = \frac{y}{0-0} = \frac{z}{8+6}$$

$$\frac{x}{0} = \frac{y}{0} = \frac{z}{14} = \begin{bmatrix} 0 \\ 0 \\ 14 \end{bmatrix}$$

$\lambda = -4$

$$\begin{bmatrix} -2 & -3 & 0 \\ 2 & -7 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-2x - 3y + 0 = 0$$
$$2x - 7y = 0$$
$$-1z = 0$$

x		y	z
-3	0	-2	-3
-7	0	2	-7

Session - 6

Applications of Eigen Values. $\dfrac{dx}{dt} = Ax$.
(stability Analysis)

The system $\dfrac{dx}{dt} = Ax$ is stable only when the Eigen values are NEGATIVE other than other values of λ the system is unstable.

Ex: $\lambda = 1, 2, 3$ then the system is unstable

If $\lambda = -1, 2, 0$ then the system is unstable

If $\lambda = -1, -2, -3$ then the system is stable.

1. Verify the system $\frac{dx}{dt} = Ax$ is stable (or) Not $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

<u>sol:</u> $\lambda = 6, -1$

The system is unstable

2. $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

<u>sol:</u> $\lambda = 5, -2$

The system is unstable.

3. $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

<u>sol:</u> The characteristic Matrix $= \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1-\lambda \end{bmatrix}$

The characteristic eq'n $\begin{vmatrix} 1-\lambda & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$

$\lambda^3 - \lambda^2 (Tr(A)) + \lambda$ (sum of minors of p.d elements) $- \det A = 0$

$Tr(A) = 1+2+1 = 4.$

sum of minors = $\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$

$= (2-1) + (1-0) + (2-1)$

$= (1) + (1) + (1)$

$= 3$

$\det A = 1(2-1) + 1(-1-0) + 0$

$= 1(1) + 1(-1)$

$= 1-1$

$= 0$

$\lambda^3 - \lambda^2(4) + \lambda(3) = 0$

$\lambda^3 - 4\lambda^2 + 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda + 3) = 0$

$\lambda = 0$, and $\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda-3) - 1(\lambda-3) = 0$

$(\lambda-3)(\lambda-1) = 0$

$= 3, 1, 0$

It is unstable

4. $\begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix}$

sol: the charatristic matrix. $\begin{bmatrix} 1-\lambda & -2 & 1 \\ -2 & 4-\lambda & -2 \\ 1 & -2 & 1-\lambda \end{bmatrix}$

charateristic eq'n. $\begin{vmatrix} 1-\lambda & -2 & 1 \\ -2 & 4-\lambda & -2 \\ 1 & -2 & 1-\lambda \end{vmatrix} = 0$

$\lambda^3 - \lambda^2 (Tr(A)) + \lambda$ (sum of minors of p.d elements)

$- \det A = 0.$

$Tr(A) = 1 + 4 + 1 = 6$

sum of minors $= \begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix}$

$= (4-4) + (1-1) + (4-4)$

$= 0 + 0 + 0$

$= 0$

$\det A = 1(4-4) + 2(-2+2) + 1(4-4)$

$= 1(0) + 2(0) + 1(0)$

$= 0$

det A = 1 $\quad \lambda^3 - 6\lambda^2 = 0$

$\lambda^2(\lambda-6) = 0$

$\lambda - 6 = 0$

$\lambda = 6$

$\lambda = 0, 0, 6$

Session-7

Diagonalization:

1. Consider the given matrix
2. find it's eigen values
3. find it's eigen vectors
4. Construct the modal (P)
5. Find P^{-1}
6. Find $P^{-1} A P = D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

1. Check whether the Matrix is diagonalized or not. $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

sol: $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

The characteristic Matrix $\begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$

The characteristic eq'n $\begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0$

$(1-\lambda)(3-\lambda)-8=0$

$\lambda^2-4\lambda-5=0$

$\lambda=-1,5$

Eigen vectors:

cha-tic matrix $\begin{bmatrix}1-\lambda & 2\\ 4 & 3-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$\lambda=-1$

$$\begin{bmatrix}2 & 2\\ 4 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$2x+2y=0$

$4x+4y=0$

$x+y=0$

$x=-y$

$$\frac{x}{-1}=\frac{y}{1}$$

$\lambda=5$

$$\begin{bmatrix}-4 & 2\\ 4 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$-4x+2y=0$

$4x-2y=0 \Rightarrow 2x-y=0$

$2x-y \Rightarrow \frac{x}{1}=\frac{y}{2}$

$$x_2=\begin{bmatrix}1\\ 2\end{bmatrix}$$

The model Matrix $\cdot$ $P = [x_1 \quad x_2]$

$$\text{Model } P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$P^{-1} \rightarrow \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$P^{-1} = \frac{1}{-2-1}\begin{bmatrix} 2 & -1 \\ -1 & -1 \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} 2 & -1 \\ -1 & -1 \end{bmatrix}$$

$P^{-1}AP$

$$P^{-1}A = \frac{-1}{3}\begin{bmatrix} 2 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} 2-4 & 4-3 \\ -1-4 & -2-3 \end{bmatrix}$$

$$P^{-1}A = -\frac{1}{3}\begin{bmatrix} -2 & 1 \\ -5 & -5 \end{bmatrix}$$

$$P^{-1}AP = \frac{-1}{3}\begin{bmatrix} -2 & 1 \\ -5 & -5 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} 2+1 & -2+2 \\ 5-5 & -5-10 \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} 3 & 0 \\ 0 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} = 0$$

2. $\begin{bmatrix} -2 & 5 \\ -1 & 4 \end{bmatrix}$

Sol: $A = \begin{bmatrix} -2 & 5 \\ -1 & 4 \end{bmatrix}$

The characteristic Matrix $= \begin{bmatrix} -2-\lambda & 5 \\ -1 & 4-\lambda \end{bmatrix}$

The characteristic eq'n $= \begin{vmatrix} -2-\lambda & 5 \\ -1 & 4-\lambda \end{vmatrix} = 0$

$= (-2-\lambda)(4-\lambda)+5=0$

$= (-2-\lambda)(4-\lambda)+5=0$

$-8+2\lambda-4\lambda+\lambda^2+5=0$

$-8-2\lambda+\lambda^2+5=0$

$\lambda^2-2\lambda-8+5=0$

$\lambda^2-2\lambda-3=0$

-3

$3 \quad 1$

$\cancel{\lambda^2}$