ALM-4 solutions

1) $i_1 = 100 \sin(100t)$

$i_1 = 100 \cos(100t - 90^\circ)$

$i_2 = 200 \cos(100t + 30^\circ)$

Phasor form $\Rightarrow i_1 = 100 \angle -90^\circ$

$i_2 = 200 \angle 30^\circ$

(a) $i_1 + i_2 = 100 \angle -90^\circ + 200 \angle 30^\circ$

$= 100 [\cos(-90^\circ) + \sin(-90^\circ)]$

$+ 200 [\cos 30^\circ + j \sin 30^\circ]$

$= -j100 + 173.2 + j100$

$= 173.2$

$i_1 + i_2 = 173.2 \angle 0^\circ$

$$\boxed{i_1 + i_2 = 173.2 \cos(100t)}$$

(b) $i_1 - i_2 = -173.2 - j200$

$|i_1 - i_2| = \sqrt{(-173.2)^2 + (-200)^2} = 264.57$

$\theta = \tan^{-1}\left(\frac{-200}{-173.2}\right) = 229.1$

$$\boxed{i_1 - i_2 = 264.57 \cos(100t + 229.1)}$$

c) $i_1 \cdot i_2 = 100\angle 90^\circ \cdot 200\angle 30^\circ$

$= 20000\angle -60^\circ$

$\boxed{i_1 \cdot i_2 = 20000 \cos(100t - 60^\circ)}$

d) $\frac{i_1}{i_2} = \frac{100\angle -90^\circ}{200\angle 30^\circ} = \frac{1}{2}\angle -90^\circ - 30^\circ$

$= \frac{1}{2}\angle -120^\circ$

$\boxed{\frac{i_1}{i_2} = \frac{1}{2}\cos(100t - 120^\circ)}$

e) $\left(\frac{i_1}{i_2}\right)^2 = \left[\frac{100\angle -90^\circ}{200\angle 30^\circ}\right]^2$

$= \left[\frac{1}{2}\angle(-120)\right]^2$

$= \frac{1}{4}\angle -240^\circ$

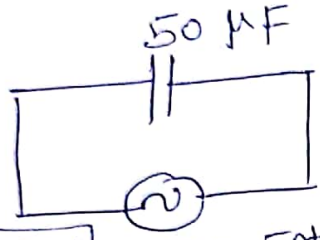
$\boxed{\left(\frac{i_1}{i_2}\right)^2 = \frac{1}{4}\cos(100t - 240^\circ)}$

f) $(i_1 \cdot i_2)^3 = \left[100\angle -90^\circ \times 200\angle 30^\circ\right]^3$

$= \left[20000\angle -60^\circ\right]^3 = 20000^3\angle -180^\circ$

$= 200\,00^5 \cos(100t - 180^\circ)$

(2)



$V_{rms} = 230V$

$\omega = 2\pi f$

a) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$= \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$

$X_C = 63.66\,\Omega$

b) $I_{max} = \frac{V_{max}}{X_C}$

$V_{max} = V_{rms} \times \sqrt{2}$

$= 230 \times 1.414$

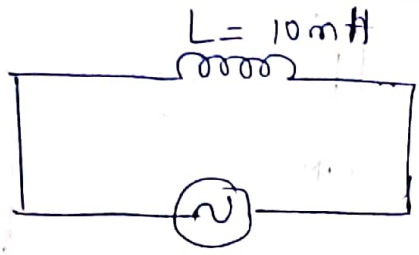
$V_{max} = 325.22V$

$I_{max} = \frac{325.22}{63.66} = 5.11A \Rightarrow \frac{V_{max}}{X_C}$

$I_{max} = 5.11A$

c) $I_{rms} = \frac{V_{rms}}{X_C} = \frac{230}{63.66} = 3.61A$

(3)



$V_{rms} = 230V$

$f = 50\,Hz$

$L = 10mH$

a) $X_L = \omega_L = 2\pi f L$

$= 2\pi \times 50 \times 10 \times 10^{-3}$

$$\boxed{X_L = 3.142\,\Omega}$$

b) $I_{max} = \dfrac{V_{max}}{X_L}$

$V_{max} = V_{rms} \times \sqrt{2} = 230 \times 1.414 = 325.22$

$$\boxed{V_{max} = 325.22\,V}$$

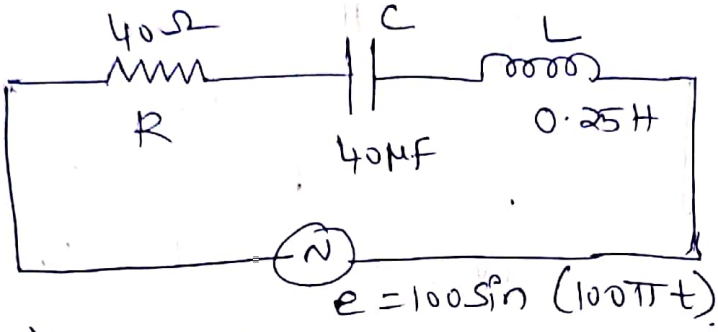
$I_{max} = \dfrac{325.22}{3.142} = 103.5\,A$

$$\boxed{I_{max} = 103.5A}$$

c) $I_{rms} = \dfrac{V_{rms}}{X_L} = \dfrac{230}{3.142} = 73.2A$

$$\boxed{I_{rms} = 73.2A}$$

(4)



$V_{max} = 100$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7\ V.$$

$$\boxed{V_{rms} = 70.7\ V}$$

(i) $X_L = \omega_L = 2\pi f L = 100\pi \times 0.25$

$= 78.54\Omega$

$$\boxed{X_L = 78.54\Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 40 \times 10^{-6}} = 79.58\Omega$$

$$\boxed{X_C = 79.58\Omega}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{40^2 + (78.54^2 - 79.58^2)}$$

$$\boxed{Z = 40.01\Omega}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{70.7}{40.01} = 1.77\ A$$

$$\boxed{I_{rms} = 1.77\ A}$$

(ii) power supplied

(or)

$$I_{rms}^2 R = V_{rms} I_{rms} \cos\phi$$

$$= 70.7 \times 1.77 \times 0.998$$

$$= 125 W$$

$$\cos\phi = \frac{R}{Z}$$

$$= \frac{40}{40.01}$$

$$\boxed{\cos\phi = 0.998}$$

(iii) power factor $= \cos\phi = 0.998$

$$\Rightarrow \frac{R}{Z} = 0.998.$$

(5) RLC Series Circuit

$$v(t) = 283 \sin(100\pi t)$$

$V_m = 283$ | $I_{max} = 0.314$

$\omega = 100\pi$ | $V_c = 300V$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{283}{\sqrt{2}} = 200V$$

$$I_{max} = \frac{V_{rms}}{R} \Rightarrow R = \frac{V_{rms}}{I_{max}} = \frac{200}{0.314}$$

at resonance.

$$= 637\Omega$$

$$V_c = I_{max} X_C$$

$$\Rightarrow X_C = \frac{V_c}{I_{max}} = \frac{300}{0.314}$$

$$= 955\Omega$$

$$C = \frac{1}{\omega_0 X_C} *$$

$$= \frac{1}{100\pi \times 955}$$

$$\boxed{C = 3.34\,\mu F}$$

At resonance $X_L = X_C$; So. $X_L = 955\Omega$

$$L = \frac{X_L}{\omega_0} = \frac{955}{100\pi} = 3.04H$$

Q-factor $= \frac{\omega_0 L}{R}$

$$Q. = \frac{100\pi \times 3.04}{637} = 1.5$$

Bandwidth $= \frac{\omega_0}{Q.}$

$$= \frac{100\pi}{1.5}$$

B.W = 209.44 rad/sec.