# Problems and Solutions on Classification of Systems

1. Validate that the system with excitation $x(t)$ and response $y(t)$ described by the following equations are linear, time variant, static, causal and stable.

   (i) $y(t) = x(t)\sin 3t$ (ii) $y(t) = x(\sin(t))$ (iii) $y(t) = \frac{d}{dt}x(t)$ (iv) $y(t) = t\, x(t)$ (v) $y(t) = x(t)r(t)$

   where $r(t) = t\, u(t)$ (vi) $y(t) = \begin{cases} x(t), & \text{if } x(t) > 0 \\ 0, & \text{if } x(t) \le 0 \end{cases}$ (vii) $y(t) = x(t-5) - x(3-t)$

**Solution:**

i). $y(t) = x(t)\sin 3t$

a) Linear

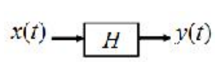
Satisfies the superposition principal

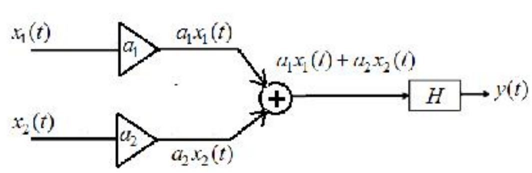
It states that the response of system to a weighted sum of the signals is equal to the corresponding weighed sum of the responses to each of the individual input signals.

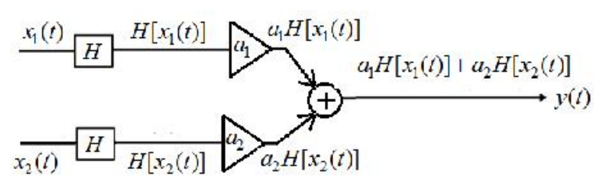
$$T\left[a_1x_1(t) + a_2x_2(t)\right] = a_1T[x_1(t)] + a_2T[x_2(t)]$$

$$= a_1y_1(t) + a_2y_2(t)$$

Where $a_1$ and $a_2$ are arbitrary constants




Procedure

i) Let $x_1(t)$ and $x_2(t)$ be two inputs to the system $H$ and $y_1(t)$ and $y_2(t)$ be corresponding responses.

ii) Consider a signal, $x_3(t) = a_1 x(t) + a_2 x_2(t)$ is weighted sum of $x_1(t)$ and $x_2(t)$.

iii) Let $y_3(t)$ be the response of $x_3(t)$, i.e $y_3(t) = H\left[x_3(t)\right]$

iv) Check whether $y_3'(t) = a_1 y_1(t) + a_2 y_2(t)$

v) Comparing the both equations $y_3(t)$ and $y_3'(t)$ which are equal, then the system is linear or otherwise Non-linear.

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1 x(t) + a_2 x_2(t)$

Therefore, the response of the output system

$$y_3(t) = H\left[x_3(t)\right] = x_3(t)\sin 3t = \left[a_1 x(t) + a_2 x_2(t)\right]\sin 3t$$

Let $y_1(t) = H[x_1(t)] = x_1(t)\sin 3t$

$y_2(t) = H[x_2(t)] = x_2(t)\sin 3t$ ,

Then,

$$y_3'(t) = a_1 y_1(t) + a_2 y_2(t) = a_1 x_1(t)\sin 3t + a_2 x_2(t)\sin 3t = [a_1 x_1(t) + a_2 x_2(t)]\sin 3t$$

Comparing the both equations $y_3(t)$ and $y_3'(t)$, both equations are equal, then the system is linear.

b) Time invariant:

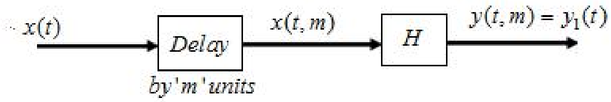
If it's input varies at any instant of time. Then output is also varied at specified instant of time of input. For example if the input gets delayed by m units of times then output is also get delayed by m units of times only. This is known as time invariant system or otherwise time variant system.

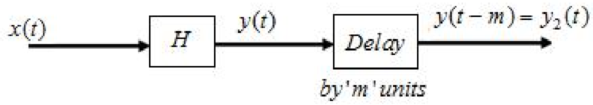
Procedure:

i) Delay the input signal m units of time and determine the response of the system for this delayed input signal. Let this response $y_1(t)$.

ii) Delay the response of the system for unshifted by m units of time .Let this delayed response $y_2(t)$.

iii) Check the equations $y_1(t)$ and $y_2(t)$ are equal,then the system is time invariant system or otherwise time variant system




Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t,m) = x(t,m)\sin 3t = x(t-m)\sin 3t$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t-m$)

$$y_2(t) = y(t-m) = x(t-m)\sin 3(t-m).$$

Comparing both equations $y_1(t)$ and $y_2(t)$ both are not equal. Hence the system is time variant system.

c) Static

If it’s output at any instant of time depends at most on the input signal at same time. But not on past and future inputs.

$$y(t) = x(t)\sin 3t$$

The output depends only at present time of the input.

If

$$t = 0, y(0) = x(0).\sin 3(0)$$

$$t = 1, y(1) = x(1).\sin 3(1)$$

Above equations intend that the output at any instant of time t depends on the input signal at the same time. Hence system is static or memoryless system.

d) Causal:

The response of the output system at any instant ‘t’ depends only on the present and past inputs. But it does not depend on the future inputs ,Or otherwise the system is Non-causal system.

$$y(t) = x(t)\sin 3t$$

If

$$t = 0, y(0) = x(0).\sin 3(0)$$

$$t = 1, y(1) = x(1).\sin 3(1)$$

Above equations shows that the output at any instant of time ‘t’ depends only on present inputs .Hence the system is causal system.

e) Stable:

An arbitrary relaxed system is said to be BIBO stable(Bounded Input Bounded Output).If and only if every bounded input produces bounded output.

$$y(t) = x(t)\sin 3t$$

Except $t = \infty$,The output is bounded as long as input the input is bounded. Hence this is BIBO stable system.

ii) $y(t) = x(\sin(t))$

a) Linear

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1x(t) + a_2x_2(t)$

Therefore, the response of the output system

$$y_3(t) = H[x_3(t)] = x_3(\sin(t))$$

$$y_3(t) = a_1x_1(\sin(t)) + a_2x_2(\sin(t))$$

Let

$$y_1(t) = H[x_1(t)] = x_1(\sin(t))$$

$$y_2(t) = H[x_2(t)] = x_2(\sin(t))$$

$$y_3^{'}(t) = a_1y_1(t) + a_2y_2(t) = a_1x_1(\sin(t)) + a_2x_2(\sin(t))$$

Comparing the both equations $y_3(t)$ and $y_3^{'}(t)$ ,both equations are equal. Hence the system is linear.

b) Time invariant

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t, m) = x(\sin(t, m)) = x(\sin(t - m))$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t - m$)

$$y_2(t) = y(t-m) = x(\sin(t-m))$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are equal.Hence the system is time invariant system.

c) Static

$$y(t) = x(\sin(t))$$

The output depends only at present time of the input. Hence the system is static or memory less system.

d) Causal:

$$y(t) = x(\sin(t))$$

The output at any instant of time ‘t’ depends only on present inputs .Hence the system is causal system.

e) Stable

$$y(t) = x(\sin(t))$$

The output is bounded as long as input the input is bounded. Hence this is BIBO stable system.

iii) $y(t) = \frac{d}{dt}x(t)$

a) Linear

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1x(t) + a_2x_2(t)$

Therefore, the response of the output system

$$y_3(t) = H[x_3(t)] = \frac{d}{dt}x_3(t)$$

$$y_3(t) = \frac{d}{dt}[a_1x_1(t) + a_2x_2(t)]$$

Let

$$y_1(t) = H[x_1(t)] = \frac{d}{dt}x_1(t)$$

$$y_2(t) = H[x_2(t)] = \frac{d}{dt}x_2(t)$$

$$y_3'(t) = a_1 y_1(t) + a_2 y_2(t) = a_1\frac{d}{dt}x_1(t) + a_2\frac{d}{dt}x_2(t)$$

$$y_3'(t) = \frac{d}{dt}[a_1 x_1(t) + a_2 x_2(t)]$$

Comparing the both equations $y_3(t)$ and $y_3'(t)$ ,both are equal, then the system is linear.

b) Time invariant

Response of the delayed inputs by 'm' units of time is

$$y_1(t) = y(t,m) = \frac{d}{dt}x(t,m) = \frac{d}{dt}x(t-m)$$

Delay by 'm' units of time for response of the output (i.e substitute $t \to t-m$)

$$y_2(t) = y(t-m) = \frac{d}{dt}x(t-m)$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are equal.Hence the system is time invariant system.

c) Static

$$y(t) = \frac{d}{dt}x(t)$$

Using definition of differentiation

$$y(t) = \lim_{\triangle t \to 0} \frac{x(t)-x(t-\triangle t)}{\triangle t}$$

At any value of t. the $x(t)$ is present input and $x(t-\triangle t)$ is past input.Hence the system is dynamic.

d) Causal

$$y(t) = \frac{d}{dt}x(t)$$

Using definition of differentiation

$$y(t) = \lim_{\triangle t \to 0} \frac{x(t) - x(t - \triangle t)}{\triangle t}$$

At any value of t, the output depends on present and past inputs.Hence the system is causal system.

e) Stable

$$y(t) = \frac{d}{dt}x(t)$$

The output is bounded as long as input the input is bounded. Hence this is BIBO stable system.

iv) $y(t) = tx(t)$

a) Linear

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1x(t) + a_2x_2(t)$ Therefore, the response of the output system

$$y_3(t) = H[x_3(t)] = tx_3(t)$$

$$y_3(t) = t[a_1x_1(t) + a_2x_2(t)]$$

Let

$$y_1(t) = H[x_1(t)] = tx_1(t)$$

$$y_2(t) = H[x_2(t)] = tx_2(t)$$

$$y_3'(t) = a_1y_1(t) + a_2y_2(t) = a_1tx_1(t) + a_2tx_2(t)$$

$$y_3'(t) = t[a_1x_1(t) + a_2x_2(t)]$$

Comparing the both equations $y_3(t)$ and $y_3'(t)$ ,both are equal, then the system is linear.

b) Time invariant

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t,m) = tx(t,m) = tx(t-m)$$

Delay by 'm' units of time for response of the output (i.e substitute $t \to t-m$)

$$y_2(t) = y(t-m) = (t-m)x(t-m)$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are not equal. Hence the system is time variant system.

c) Static

$$y(t) = tx(t)$$

The output depends only at present time of the input. Hence the system is static or memory less system

d) Causal

$$y(t) = tx(t)$$

The output at any instant of time 't' depends only on present inputs. Hence the system is causal system.

e) Stable

$$y(t) = tx(t)$$

Except $t = \infty$,The output is bounded as long as input the input is bounded. Hence this is BIBO stable system

v) $y(t) = x(t)r(t);$ where $r(t) = t.u(t)$

a) Linear

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1x(t) + a_2x_2(t)$ Therefore, the response of the output system

$$y_3(t) = H[x_3(t)] = x_3(t)r(t)$$

$$y_3(t) = r(t)[a_1x_1(t) + a_2x_2(t)]$$

Let

$$y_1(t) = H[x_1(t)] = x_1(t)r(t)$$

$$y_2(t) = H[x_2(t)] = x_2(t)r(t)$$

$$y_3^{'}(t) = a_1 y_1(t) + a_2 y_2(t) = a_1 x_1(t) r(t) + a_2 x_2(t) r(t)$$

$$y_3^{'}(t) = r(t)[a_1 x_1(t) + a_2 x_2(t)]$$

Comparing the both equations $y_3(t)$ and $y_3^{'}(t)$ ,both are equal, then the system is linear.

b) Time invariant

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t, m) = x(t, m) r(t) = x(t-m) r(t)$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t - m$)

$$y_2(t) = y(t-m) = x(t-m) r(t-m)$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are not equal. Hence the system is time variant system.

c). Static

$y(t) = x(t) r(t);$ where $r(t) = t.u(t)$

The output depends only at present time of the input. Hence the system is static or memory less system

d) Causal

$y(t) = x(t) r(t);$ where $r(t) = t.u(t)$

The output at any instant of time‘t’ depends only on present inputs. Hence the system is causal system.

e) Stable

$y(t) = x(t) r(t);$ where $r(t) = t.u(t)$

The output is bounded as long as input the input is bounded. Hence this is BIBO stable system

vi) $y(t) = \begin{cases} x(t), if\ t>0 \\ 0 \quad , if\ t\le 0 \end{cases}$

a) Linear

When $t > 0$, the output $y(t) = x(t)$ ,it is linear.

When $t \leq 0$.the output $y(t) = 0$.it is Non-linear

b) Time invariant

$$y(t) = \begin{cases} x(t), if\ t>0 \\ 0 \quad , if\ t\leq 0 \end{cases}$$

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t, m) = x(t, m) = x(t - m)$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t - m$)

$$y_2(t) = y(t - m) = x(t - m)$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are equal. Hence the system is time invariant system.

c) Static

$$y(t) = \begin{cases} x(t), if\ t>0 \\ 0 \quad , if\ t\leq 0 \end{cases}$$

The output depends only at present time of the input. Hence the system is static or memory less system.

d) Causal

$$y(t) = \begin{cases} x(t), if\ t>0 \\ 0 \quad , if\ t\leq 0 \end{cases}$$

The output at any instant of time‘t’ depends only on present inputs. Hence the system is causal system.

e) Stable

$$y(t) = \begin{cases} x(t), if\ t>0 \\ 0 \quad , if\ t\leq 0 \end{cases}$$

The output is bounded as long as input the input is bounded. Hence this is BIBO stable system

vii) $y(t) = x(t - 5) - x(3 - t)$

a) Linearity

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t)=a_1x(t)+a_2x_2(t)$
Therefore, the response of the output system is

$$y_3(t) = H[x_3(t)] = x_3(t-5) - x_3(3-t)$$

$$y_3(t) = a_1[x_1(t-5) - x_1(3-t)] + a_2[x_2(t-5) + x_2(3-t)]$$

Let

$$y_1(t) = H[x_1(t)] = x_1(t-5) - x_1(3-t)$$

$$y_2(t) = H[x_2(t)] = x_2(t-5) + x_2(3-t)$$

$$y_3^{'}(t) = a_1y_1(t) + a_2y_2(t) = a_1\left[x_1(t-5) - x_1(3-t)\right] + a_2\left[x_2(t-5) + x_2(3-t)\right]$$

Comparing the both equations $y_3(t)$ and $y_3^{'}(t)$ ,both are equal, then the system is linear.

b) Time invariant

$$y(t) = x(t-5) - x(3-t)$$

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t,m) = x(t-5,m) - x(3-t,m) = x(t-5-m) - x(3-t-m)$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t-m$)

$$y_2(t) = y(t-m) = x(t-m-5) - x(3-(t-m) = x(t-5-m) - x(3-t+m)$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are not equal.
Hence the system is time variant system.

c) Static

$$y(t) = x(t-5) - x(3-t)$$

$$when$$

$$t = 0, y(0) = x(0-5) - x(3-0) = x(-5) - x(3)$$

$$t = 1, y(1) = x(1-5) - x(3-1) = x(-4) - x(2)$$

$$t = -1, y(-1) = x(-1-5) - x(3-(-1)) = x(-6) - x(4)$$

The output at instant of time‘t’ depends on past and future time of the input. Hence the System is Dynamic.

d) Causal

$$y(t) = x(t-5) - x(3-t)$$

*when*

$$t = 0, y(0) = x(0-5) - x(3-0) = x(-5) - x(3)$$

$$t = 1, y(1) = x(1-5) - x(3-1) = x(-4) - x(2)$$

$$t = -1, y(-1) = x(-1-5) - x(3-(-1)) = x(-6) - x(4)$$

The output at instant of time'$t$' depends on not only past input and but also future input. Hence the System is Non-causal.

e) Stable

$$y(t) = x(t-5) - x(3-t)$$

The output is bounded as long as input the input is bounded. Hence this is BIBO stable system.

2. Consider two systems with input output relations are described by

(i) $y(t) = x(t/2)$ and (ii) $y(t) = x(2t)$

Test whether these systems is linear, time-invariant and causal.

Solution:

i) $y(t) = x(t/2)$

a) Linear

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1 x(t) + a_2 x_2(t)$

Therefore, the response of the output system

$$y_3(t) = H[x_3(t)] = x_3(t/2)$$
$$y_3(t) = a_1 x_1(t/2) + a_2 x_2(t/2)$$

Let

$$y_1(t) = H[x_1(t)] = x_1(t/2)$$
$$y_2(t) = H[x_2(t)] = x_2(t/2)$$
$$y_3'(t) = a_1 x_1(t/2) + a_2 x_2(t/2)$$

Comparing the both equations $y_3(t)$ and $y_3'(t)$ ,both are equal, then the system is linear.

b) Time invariant

$$y(t) = x(t/2)$$

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t,m) = x(\frac{t}{2}, m) = x(\frac{t}{2} - m)$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t - m$)

$$y_2(t) = y(t-m) = x(\frac{t-m}{2})$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are not equal. Hence the system is time variant system.

c) Causal

$$y(t) = x(t/2)$$

*when*

$$t = 0, y(0) = x(0/2) = x(0)$$
$$t = 1, y(1) = x(1/2) = x(0.5)$$
$$t = -1, y(-1) = x(-1/2) = x(-0.5)$$

The output at any instant of time‘t’ depends on present and past inputs. Hence the system is causal.

ii) $y(t) = x(2t)$

a) Linear

Consider the two signals $x_1(t)$ and $x_2(t)$ then $x_3(t) = a_1 x(t) + a_2 x_2(t)$
Therefore, the response of the output system

$$y_3(t) = H[x_3(t)] = x_3(2t)$$
$$y_3(t) = a_1 x_1(2t) + a_2 x_2(2t)$$

Let

$$y_1(t) = H[x_1(t)] = x_1(2t)$$
$$y_2(t) = H[x_2(t)] = x_2(2t)$$
$$y_3^{'}(t) = a_1 x_1(2t) + a_2 x_2(2t)$$

Comparing the both equations $y_3(t)$ and $y_3^{'}(t)$ ,both are equal, then the system is linear.

b) Time invariant

$$y(t) = x(2t)$$

Response of the delayed inputs by ‘m’ units of time is

$$y_1(t) = y(t,m) = x(2t,m) = x(2t-m)$$

Delay by ‘m’ units of time for response of the output (i.e substitute $t \to t-m$)

$$y_2(t) = y(t-m) = x(2(t-m))$$

Comparing both equations $y_1(t)$ and $y_2(t)$, both are not equal. Hence the system is time variant system.

c) Causal

$$y(t) = x(2t)$$

*when*

$$t = 0, y(0) = x(2.0) = x(0)$$
$$t = 1, y(1) = x(2.1) = x(2)$$
$$t = -1, y(-1) = x(2.-1) = x(-2)$$

The output at any instant of time‘t’ depends on present and future inputs. Hence the system is Non-causal.

3. For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, explain with reasons which of the systems are time-invariant parameter systems and which are time-varying-parameter systems.

(i) $y(t) = x(t-2)$

Step1: Find the delayed ($^2$ units) output $y(t-{}^2) = x(t-{}^2-2)$

Step2: Delay the input by same units of delay ($^2$ units), i. e., , then apply to the system and find the output. According to the input-output relation, the output of the system is determined as

$$y(t,{}^2) = x(t-2-{}^2)$$

Since $y(t,{}^2) = y(t-{}^2)$

the system is Time Invariant

(ii) $y(t) = x(-t)$

Step1: Find the delayed ($^2$ units) output $y(t-{}^2) = x(-(t-{}^2)) = x(-t+{}^2)$

Step2: Delay the input by same units of delay ($^2$ units), i. e., , then apply to the system and find the output. According to the input-output relation, the output of the system is determined as

$$y(t,{}^{2}) = x(-t-{}^{2})$$

Here $y(t,{}^{2}) \neq y(t-{}^{2})$

So the system is Time Variant

(iii) $y(t) = x(at)$

Step1: Find the delayed (${}^{2}$ units) output $y(t-{}^{2}) = x(a(t-{}^{2})) = x(at-a{}^{2})$

Step2: Delay the input by same units of delay (${}^{2}$ units), i. e., , then apply to the system and find the output. According to the input-output relation, the output of the system is determined as

$$y(t,{}^{2}) = x(at-{}^{2})$$

Here $y(t,{}^{2}) \neq y(t-{}^{2})$

So the system is Time Variant

(iv) $y(t) = t\ x(t-2)$

Step1: Find the delayed (${}^{2}$ units) output $y(t-{}^{2}) = (t-{}^{2})x(t-{}^{2}-2)$

Step2: Delay the input by same units of delay (${}^{2}$ units), i. e., , then apply to the system and find the output. According to the input-output relation, the output of the system is determined as

$$y(t,{}^{2}) = t \times x(t-2-{}^{2})$$

Here $y(t,{}^{2}) \neq y(t-{}^{2})$

So the system is Time Variant

**4. For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, determine which are causal and which are non-causal.**

(i) $y(t) = x(t-2)$ (ii) $y(t) = x(-t)$ (iii) $y(t) = x(at), a > 0$ (iv) $y(t) = x(at), a < 0$

(v) $y(t) = t\ x(t-2)$

**Ans:** (i) $y(t) = x(t-2)$

| | | |
|---|---|---|
| For | $t = 0$ | $y(0) = x(-2)$ |
| | $t = 1$ | $y(1) = x(-1)$ |
| | $t = 2$ | $y(2) = x(0)$ |

$t=-1$ $\quad y(-1)=x(-3)$

$t=-2$ $\quad y(-2)=x(-4)$

Here it is observed that for zero positive and negative values of $t$, the present output $y(t)$ depends on past values of input $x(t)$. So the systems is Causal.

(ii) $y(t)=x(-t)$

For $\quad t=0 \quad y(0)=x(0)$

$t=1 \quad y(1)=x(-1)$

$t=2 \quad y(2)=x(-2)$

$t=-1 \quad y(-1)=x(1)$ future value

$t=-2 \quad y(-2)=x(2)$ future value

Here it is observed that for zero and positive values of $t$, the present output $y(t)$ depends on past values of input $x(t)$ .But for negative values of $t$, the present output $y(t)$ depends on future values of input $x(t)$. So the systems is Non-Causal.

(iii) $y(t)=x(at), a>0$ (iv) $y(t)=x(at), a<0$

For $a>0$

For all values t, $y(t)$ depends on present and past values of input $x(t)$. So the systems is Causal.

For $a<0$

For all zero and positive values t, $y(t)$ depends on present and past values of input $x(t)$. But for Negative values of $t$, the present output $y(t)$ depends on future values of input $x(t)$. So the systems is Non-Causal.

(v) $y(t)=t\,x(t-2)$

As the causality of a system depends on time variable only, changing amplitude will not affect the causality. So the case is similar to (i). So the systems is Causal.