I/IV B. Tech. (CSE,ECE,CSIT,AI&DS and IOT) EVEN Semester, 2024-25

Subject Code: 23MT1001 TITLE: Linear Algebra & Calculus for Engineers

CO-4
Classroom delivery problems

SESSION-23 & 24: Scalar and vector point functions, Concept of the gradient, directional derivative

1. Determine $grad\ f$ where $f = x^3 - y^3 + 3xyz$ at the point (1,2,3).
2. The temperature at the point (x, y, z) in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1, -1, 2) desires to fly in such a direction that it gets cooled faster. Compute the direction in which it should fly.
3. Determine a unit normal vector to the surface $x^2 y - 2xz = 4$ at the point (1,-1,0)
4. Compute the directional derivative of $f = xy^2 - y^3 z - z^2 x$ in the direction of vector $\bar{i} + \bar{j} + 2\bar{k}$ at the point (2, 1,3).
5. Find the directional derivative of $\varphi = xy - yz - zx$ at A in the direction of AB where A= (1, 2,0), B = (1, 0, 3).
6. Find the angle between the surfaces $x^2 - y^2 - z^2 = 4$ and $x^2 - y^2 - z = 2$ at the point (0, –1, 2).
7. Identify the angle between the normal to the surface $xy^2 z = 3x - z^2$ at the points (2, 0, 2) and (1 ,-1, 1).

SESSION-25: Determine divergence, and curl of a vector and scalar point function.

1. If $\bar{f} = x^2 y\,\bar{i} + y^2 z\,\bar{j} + z^2 x\,\bar{k}$, then illustrate $div\ \bar{f}$ at (1, 1, 2)
2. Determine p, if $\vec{F} = (x+3y)i + (y-2z)j + (x+pz)k$ is solenoidal vector.
3. If $\bar{f} = -x\,\bar{i} - y\,\bar{j} - z\,\bar{k}$, then calculate $curl\ \bar{f}$
4. Compute the divergence and curl of $\vec{v} = (xyz)i - (2x^2 y)j - 3y^2 z\,k$ at (2, –1, 3).
5. Show that the fluid motion $\bar{f} = (y+z)\bar{i} + (x+z)\bar{j} + (x+y)\bar{k}$ is irrotational.
6. Determine a, b, c if $\bar{f} = (2x + 3y + az)\,\bar{i} + (bx + 2y + 3z)\,\bar{j} + (2x + cy + 3z)\bar{k}$ is irrotational.

SESSION-26: Compute the work done by a Force field of a vector, Line Integral

1. Evaluate the line integral $\int_C \left(x^2 - xy\right)dx + \left(x^2 - y^2\right)dy$ where C is the square formed by the lines $y = \pm 1$ *and* $x = \pm 1$.

2. Compute the work done by the force $\vec{F} = 2xy\,i - 3z\,j + 5x\,k$ when it moves a particle along the curve $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.

3. If $\vec{F} = (2xy)\vec{i} - 3yz\vec{j} + 4xz\vec{k}$, evaluate $\int_C \vec{F}.d\vec{r}$ from (0,0,0) to (1,1,1) along the path C : $x = t, y = t^2, z = t^3$.
4. Compute the work done by the force $\vec{F} = \left(2x^2\right)i + \left(4xz - y\right)j + 2z\,k$ when it moves a particle along the straight line from the point (0,0,0) to (2,1,3).

5. Evaluate $\iint_S \vec{\mathbf{F}}\,.\,\hat{\mathbf{n}}\,dS$, where $\vec{\mathbf{F}} = 18z\hat{\imath} - 12y\hat{\jmath} + 3y\hat{k}$ and S is the surface of the plane $2x + 3y + 6z = 12$ in the first octant.

6. If $\vec{\mathbf{F}} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4z\hat{k}$, then evaluate $\iiint_V \nabla \cdot \vec{\mathbf{F}}\,dV$, where V is bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + 2y + z = 4$.

SESSION-27 & 28: Apply Green's Theorem to transform line integrals to surface integrals & Apply Stoke's Theorem to transform surface integrals to line integral.

1. Apply Green's theorem to evaluate the integral $\int_C \left[\left(xy - y^2\right)dx - x^2dy\right]$ where C is bounded by $y = x$ and $y = x^2$.
2. Apply Green's theorem, evaluate $\int_C \left[\left(3x - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$ where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$.
3. Apply Green's theorem, evaluate $\int_C (3x^2 - 2y^2)\,dx + (4y - 3xy)\,dy$ where C is the region bounded by $y = \sqrt{x}$ *and* $y = x^2$.
4. Apply Stoke's theorem to evaluate $\int_c \mathrm{F}.\overline{\mathrm{dr}}$ where $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ where C is the boundary of the rectangle bounded by the lines x = ±a, y = 0, y = b.