Signals and Communication Systems

Properties of FT

Properties of FT

Linearity Property

$$\text{If} \quad x_1(t) \overset{\text{F.T}}{\longleftarrow - \longrightarrow} X_1(j\Omega),$$

$$\text{and} \quad x_2(t) \overset{\text{F.T}}{\longleftarrow - \longrightarrow} X_2(j\Omega)$$

$$\text{Then} \quad x(t) = x_1(t) + x_2(t) \overset{\text{F.T}}{\longleftarrow - \longrightarrow} X(j\Omega) = X_1(j\Omega) + X_2(j\Omega)$$

Proof:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t} dt = \int_{-\infty}^{\infty} \{x_1(t) + x_2(t)\}\, e^{-j\Omega t} dt$$

$$= \int_{-\infty}^{\infty} x_1(t)\, e^{-j\Omega t} dt + \int_{-\infty}^{\infty} x_2(t)\, e^{-j\Omega t} dt = X_1(j\Omega) + X_2(j\Omega)$$

Ex1: Find the FT of a signal

$$x(t) = 3e^{-3t}u(t) - 2e^{-2t}u(t)$$

$$3e^{-3t}u(t) \xleftrightarrow{\text{F.T}} \frac{3}{3 + j\Omega}$$

$$2e^{-2t}u(t) \xleftrightarrow{\text{F.T}} \frac{2}{2 + j\Omega}$$

$$x(t) = 3e^{-3t}u(t) - 2e^{-2t}u(t) \xleftrightarrow{\text{F.T}}$$

$$X(j\Omega) = \frac{3}{3 + j\Omega} - \frac{2}{2 + j\Omega} = \frac{j\Omega}{6 + j5\Omega - \Omega^2}$$

Find Inverse FT of a signal: $X(j\Omega) = \dfrac{1}{2 + j3\Omega - \Omega^2}$

Ans: $X(j\Omega) = \dfrac{1}{2 + j3\Omega - \Omega^2} = \dfrac{1}{(1 + j\Omega)(2 + j\Omega)}$

$$= \frac{1}{(1 + j\Omega)} + \frac{1}{(2 + j\Omega)}$$

Therefore $x(t) = e^{-t}u(t) + e^{-2t}u(t)$

Properties of FT

Time Shifting Property

$$\text{If} \quad x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$$

$$\text{Then} \quad x(t-t_0) \xleftrightarrow{\text{F.T}} e^{-j\Omega t_0} X(j\Omega)$$

Proof: $FT\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0)\, e^{-j\Omega t}\, dt$

Let $t - t_0 = p \Rightarrow t = p + t_0$ and $dt = dp$

$$= \int_{-\infty}^{\infty} x(p)\, e^{-j\Omega(p+t_0)} dp = \int_{-\infty}^{\infty} x(p)\, e^{-j\Omega t_0} e^{-j\Omega p} dp$$

$$= e^{-j\Omega t_0} \int_{-\infty}^{\infty} x(p)\, e^{-j\Omega p} dp = e^{-j\Omega t_0}\, X(j\Omega)$$

(DEEMED TO BE UNIVERSITY)

Find the F.T. of a signal $x(t) = \delta(t-1)$

Ans: We know that $\delta(t) \xleftrightarrow{F.T} 1$

By Time Shifting property,

$$\delta(t-1) \xleftrightarrow{F.T} 1.e^{-j\Omega}$$

Similarly $\delta(t+1) \xleftrightarrow{F.T} 1.e^{+j\Omega}$

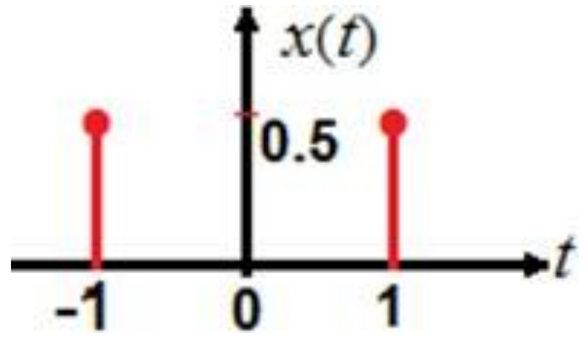
Find the signal $x(t)$, if $X(j\Omega) = e^{+j\Omega}$

$$x(t) = \delta(t+1)$$

Find the signal $x(t)$, if $X(j\Omega) = e^{-j\Omega}$

$$x(t) = \delta(t-1)$$

Find the F.T. of a signal shown in figure



$\delta(t+1)$ $\delta(t-1)$

$$x(t)=\frac{1}{2}\{\delta(t+1)+\delta(t-1)\}$$

Ans: We know that

$$\delta(t) \xleftrightarrow{F.T} 1$$

By Time Shifting property,

$$\delta(t-1) \xleftrightarrow{F.T} e^{-j\Omega}$$

$$\delta(t+1) \xleftrightarrow{F.T} e^{+j\Omega}$$

$$X(j\Omega) = \frac{1}{2}\left\{e^{+j\Omega}+e^{-j\Omega}\right\} = \cos\Omega$$

$$\frac{1}{2}\{\delta(t+1)+\delta(t-1)\} \xleftrightarrow{F.T} \cos\Omega$$

Find Time domain signal $x(t)$, *if* $X(j\Omega) = \cos\Omega$

Ans: The given $X(j\Omega)$ *can be written as*

$$X(j\Omega) = \cos\Omega = \frac{1}{2}\left\{e^{+j\Omega} + e^{-j\Omega}\right\}$$

We know that $x(t-t_0) \xleftrightarrow{\text{F.T}} e^{-j\Omega t_0}\, X(j\Omega)$

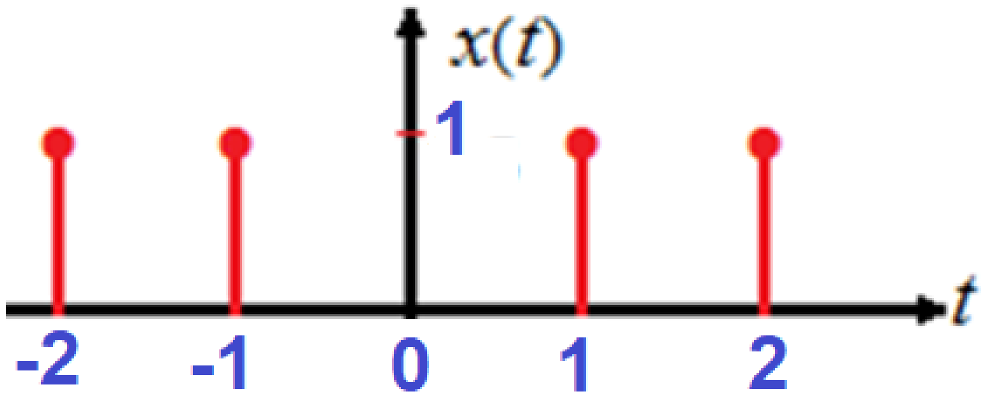
$$x(t) = \delta(t) \xleftrightarrow{\text{FT}} X(j\Omega) = 1$$

$$\delta(t-1) \xleftrightarrow{\text{F.T}} e^{-j\Omega} = e^{-j\Omega 1} \times 1$$

$$\delta(t+1) \xleftrightarrow{\text{F.T}} e^{+j\Omega}$$

$$\frac{1}{2}\left\{\delta(t-1) + \delta(t+1)\right\} \xleftrightarrow{\text{F.T}} \frac{1}{2}\left\{e^{+j\Omega} + e^{-j\Omega}\right\}$$

Find the F.T. of a signal shown in figure



Ans: $X(j\Omega) = 2\{\cos\Omega + \cos 2\Omega\}$

Find the FT of a signal $x(t) = \delta(t) + \frac{1}{2}\{\delta(t+1) + \delta(t-1)\}$

Find the signal $x(t)$ if $X(j\Omega) = 2\cos\Omega$

(DEEMED TO BE UNIVERSITY)

Find the FT of a signal $x(t) = e^{-2(t-2)}u(t)$

Ans: We know that

$$e^{-2t}u(t) \xleftrightarrow{\text{F.T}} \frac{1}{2 + j\Omega}$$

Then $$e^{4}e^{-2t}u(t) \xleftrightarrow{\text{F.T}} \frac{e^{4}}{2 + j\Omega}$$

Find the F.T. of $x(t) = e^{-2t}u(t-2)$

$$If\ \ y(t) = e^{-2t}u(t),\ \ Then\ \ y(t-2) = \ \ e^{-2(t-2)}u(t-2)$$

Ans: The given signal is represented by

$$x(t) = e^{-2t}u(t-2) = e^{-2(t-2+2)}u(t-2) \ = e^{-4}e^{-2(t-2)}u(t-2)$$

We know that $\quad e^{-2t}u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{2+j\Omega}$

Then by Time shifting property

$$x(t) = e^{-4}e^{-2(t-2)}u(t-2) \xleftrightarrow{\text{F.T}} \quad X(j\Omega) = \ e^{-4}\ \frac{e^{-j2\Omega}}{2+j\Omega} = \frac{e^{-2(2+j\Omega)}}{2+j\Omega}$$

Frequency Shifting Property

If $\quad x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$

Then $\quad y(t) = e^{j\Omega_0 t} x(t) \xleftrightarrow{\text{F.T}} X(j(\Omega - \Omega_0))$

Proof: The FT of given signal $y(t) = e^{j\Omega_0 t} x(t)$

$$Y(j\Omega) = \int_{-\infty}^{\infty} e^{j\Omega_0 t} x(t)\, e^{-j\Omega t} dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j(\Omega - \Omega_0)t} dt$$

$$= X(j(\Omega - \Omega_0))$$

Hence proved.

$x(t - t_0) \xleftrightarrow{\text{F.T}} e^{-j\Omega t_0} X(j\Omega)$ Time Shifting roperty

Find the FT of a signal $x(t) = e^{-j\Omega_0 t}$

Ans: We know that $1 \xleftrightarrow{\text{F.T}} 2\pi\,\delta(\Omega)$

Then by Frequency Shifting theorem

$$1.\, e^{-j\Omega_0 t} \xleftrightarrow{\text{F.T}} 2\pi\,\delta(\Omega + \Omega_0)$$

Find the FT of a signal $x(t)$ *If* $X(j\Omega) = \delta(\Omega + \Omega_0)$

Find the FT of a signal $x(t) = A\cos\Omega_0 t$

Ans: The given signal is represented by

$$x(t) = A\cos\Omega_0 t = \frac{1}{2}\left\{e^{j\Omega_0 t} + e^{-j\Omega_0 t}\right\}$$

We know that $1 \xleftrightarrow{\text{F.T}} 2\pi\,\delta(\Omega)$

The by Frequency Shifting theorem

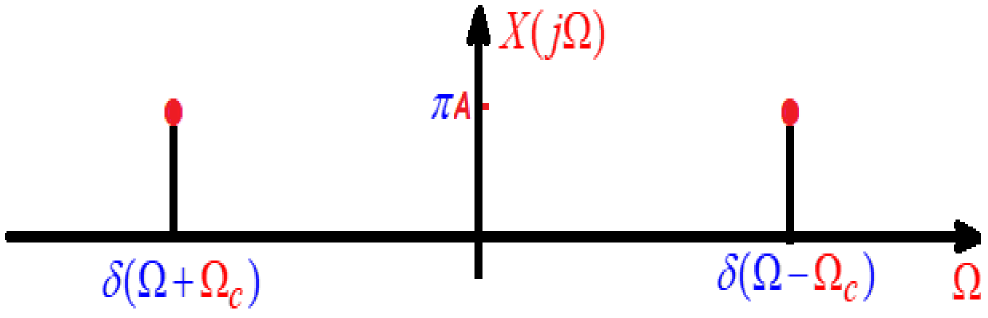
$$1.\,e^{j\Omega_0 t} \xleftrightarrow{\text{F.T}} 2\pi\,\delta(\Omega-\Omega_0)$$

and $1.\,e^{-j\Omega_0 t} \xleftrightarrow{\text{F.T}} 2\pi\,\delta(\Omega+\Omega_0)$

Therefore $x(t) = A\cos\Omega_0 t \xleftrightarrow{\text{F.T}}$

$$X(j\Omega) = \frac{1}{2}2\,\pi A\left\{\delta(\Omega+\Omega_0) + \delta(\Omega-\Omega_0)\right\}$$

$$= \pi A\left\{\delta(\Omega+\Omega_0) + \delta(\Omega-\Omega_0)\right\}$$



Find the FT of a signal $x(t) = 4\sin^2 t$

We know that $\sin^2\theta = \dfrac{1+\cos 2\theta}{2}$

Then we can write $x(t) = 4\sin^2 t = 4\dfrac{1+\cos 2t}{2} = 2 + 2\cos 2t$

$$A \xleftrightarrow{\text{FT}} A\,2\pi\,\delta(-\Omega) = A\,2\pi\,\delta(\Omega)$$

$$2 \xleftrightarrow{\text{FT}} 2\,2\pi\,\delta(\Omega) = 4\pi\,\delta(\Omega)$$

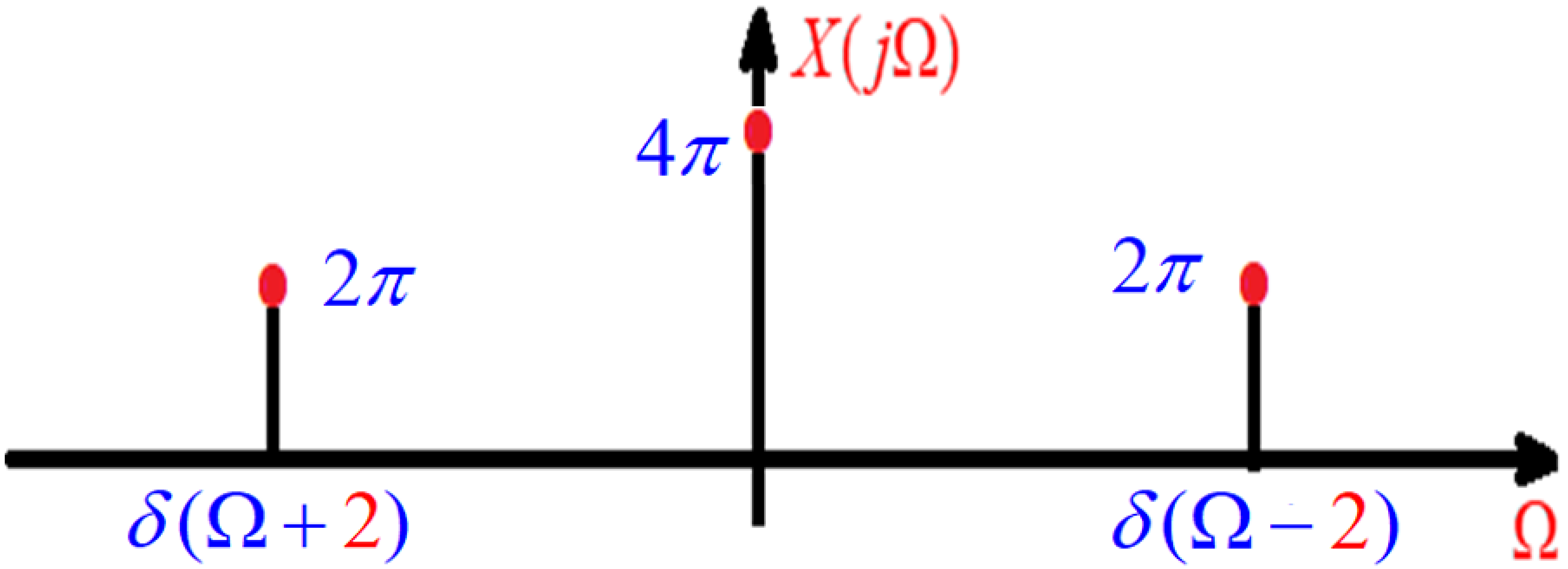
$$A\cos(\Omega_0 t) \xleftrightarrow{\text{FT}} A\pi\left\{\delta(\Omega+\Omega_0) + \delta(\Omega-\Omega_0)\right\}$$

$$2\cos 2t \xleftrightarrow{\text{FT}} 2\pi\left\{\delta(\Omega+2) + \delta(\Omega-2)\right\}$$

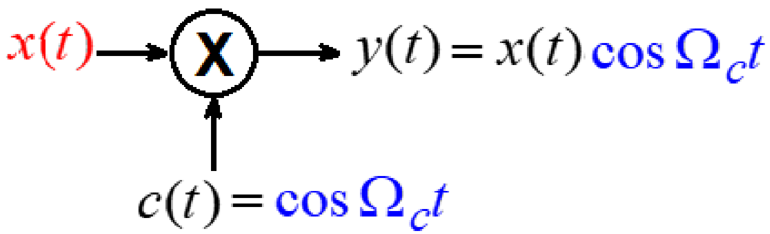
$$x(t) = 4\sin^2 t \xleftrightarrow{\text{FT}} 4\pi\,\delta(\Omega) + 2\pi\left\{\delta(\Omega+2) + \delta(\Omega-2)\right\}$$

$$= 2\pi\left\{2\delta(\Omega) + \delta(\Omega+2) + \delta(\Omega-2)\right\}$$

$$x(t) = 4\sin^2 t \overset{\text{FT}}{\longleftrightarrow} 4\pi\,\delta(\Omega) + 2\pi\,\{\delta(\Omega+2) + \delta(\Omega-2)\}$$



Modulation Theorem



$$\text{If} \quad x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$$

$$\text{Then } \cos\Omega_c t\, x(t) \xleftrightarrow{\text{F.T}} \frac{1}{2}\{X(j(\Omega+\Omega_c)) + X(j(\Omega-\Omega_c))\}$$

Proof: The given signal is represented by

$$\cos\Omega_c t\, x(t) = \frac{1}{2}\left\{e^{j\Omega_c t} - e^{-j\Omega_c t}\right\} x(t) = \frac{1}{2}\left\{e^{j\Omega_c t}x(t) - e^{-j\Omega_c t}x(t)\right\}$$

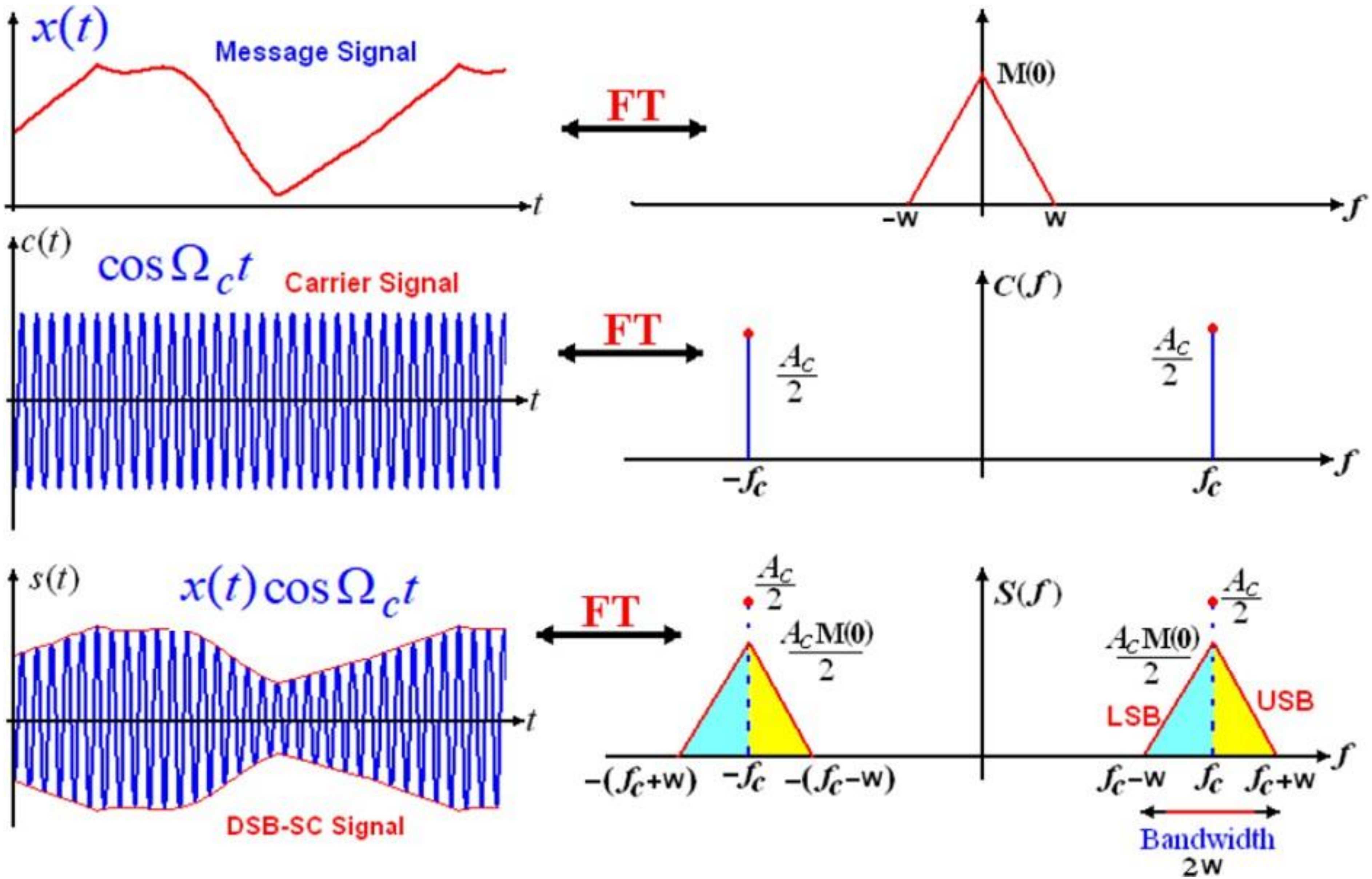
We know that $x(t) \xleftrightarrow{\text{F.T}} X(j\Omega)$

Then by Frequency Shifting theorem

$$e^{j\Omega_c t}x(t) \xleftrightarrow{\text{F.T}} X(j(\Omega-\Omega_c)),$$

$$\text{and} \quad e^{-j\Omega_c t}x(t) \xleftrightarrow{\text{F.T}} X(j(\Omega+\Omega_c)),$$

Therefore $\cos\Omega_c t\, x(t) \xleftrightarrow{\text{F.T}} \frac{1}{2}\{X(j(\Omega+\Omega_c)) + X(j(\Omega-\Omega_c))\}$



x(t)
Message Signal
t
FT
M(0)
-W
W
f
c(t)
cos Ωct
Carrier Signal
FT
C(f)
Ac/2
Ac/2
-fc
fc
s(t)
x(t) cos Ωct
DSB-SC Signal
FT
S(f)
Ac/2
AcM(0)/2
Ac/2
AcM(0)/2
LSB
USB
-(fc+W)
-fc
-(fc-W)
fc-W
fc
fc+W
Bandwidth
2W

Duality Property

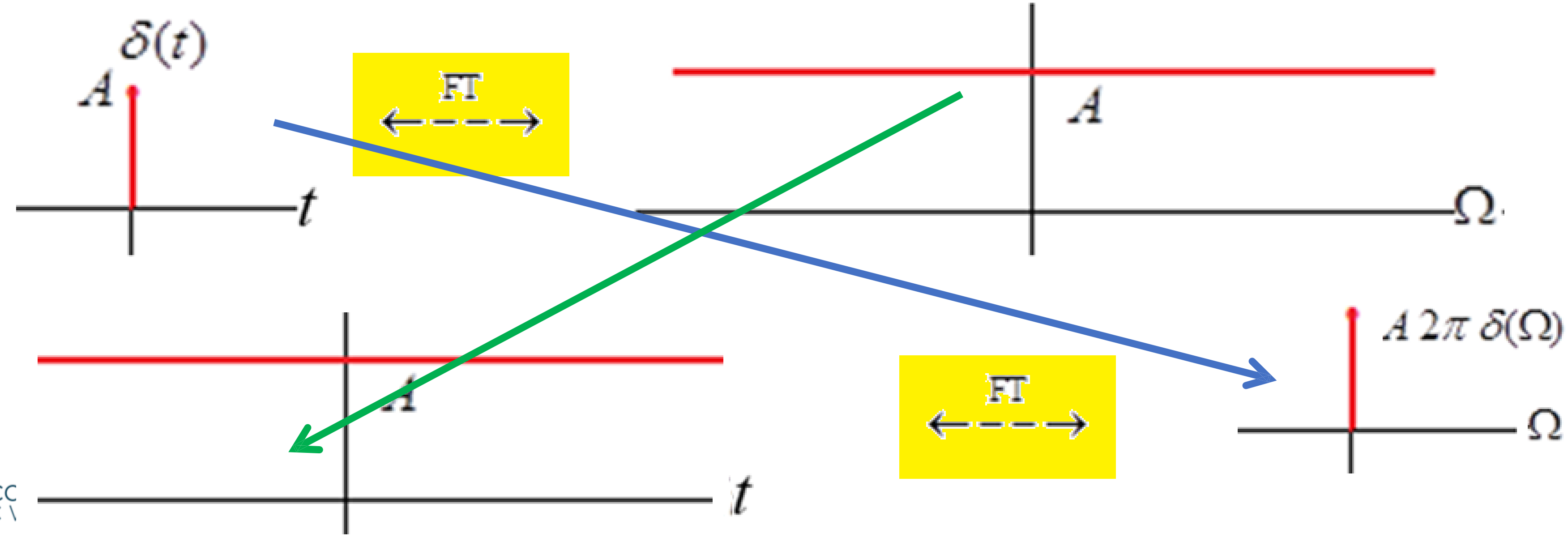
If $x(t) \overset{F.T}{\longleftrightarrow} X(j\Omega)$

Then $X(t) \overset{F.T}{\longleftrightarrow} 2\pi\, x(-j\Omega)$

$= 2\pi\ x(t)\big|_{t=-\Omega}$

$$\delta(t) \overset{F.T}{\longleftrightarrow} 1,\ \textit{for all}\ \Omega$$

$$1,\ \textit{for all}\ t \overset{F.T}{\longleftrightarrow} 2\pi\,\delta(-\Omega) = 2\pi\delta(\Omega)$$



Find the signal $x(t)$, if its FT is $X(j\Omega) = \delta(\Omega)$

Ans: $x(t) = \dfrac{1}{2\pi}$

Find the FT of a signal $x(t) = \dfrac{1}{a + jt}$

Ans: We Know that $e^{-at}u(t) \xleftrightarrow{\text{F.T}} \dfrac{1}{a + j\Omega}$

Then by duality, $\dfrac{1}{a + jt} \xleftrightarrow{\text{F.T}} 2\pi\, x(t)\Big|_{t=-\Omega}$

$$= 2\pi\, e^{a\Omega} u(-j\Omega)$$

Find the signal $x(t)$, if its FT is $X(j\Omega) = e^{2\Omega} u(-\Omega)$

Ans: $x(t) = \dfrac{1}{2\pi} \dfrac{1}{2 + jt}$

Find the FT of a signal $x(t)=\dfrac{1}{9+t^2}$ using duality property

Ans: The given signal $x(t)$ is represented in the form $\left(\dfrac{2a}{a^2+\Omega^2}\right)$

as $x(t)=\dfrac{1}{9+t^2}=\dfrac{1}{6}\left(\dfrac{6}{9+t^2}\right)$

We know that $e^{-a|t|} \xleftrightarrow{\text{F.T}} \dfrac{2a}{a^2+\Omega^2}$ or

$$e^{-3|t|} \xleftrightarrow{\text{F.T}} \frac{6}{9+\Omega^2}$$

By duality property, $x(t)=\dfrac{1}{9+t^2}=\dfrac{1}{6}\left(\dfrac{6}{9+t^2}\right) \xleftrightarrow{\text{F.T}}$

$$X(j\Omega)=\frac{1}{6}2\pi e^{-3|\Omega|}=\frac{\pi}{3}e^{-3|\Omega|}$$

$$\frac{1}{2}\{\delta(t+1)+\delta(t-1)\} \xleftrightarrow{\text{FT}} \frac{1}{2}\{e^{j\Omega}+e^{-j\Omega}\} = \cos\Omega$$

By duality $\cos t \xleftrightarrow{\text{FT}} \pi\{\delta(\Omega+1)+\delta(\Omega-1)\}$

End