Problems and Solutions on Even and Odd Signals

1. Determine and sketch the even and odd parts of the signals defined below:

(i) $x(t) = u(t)$ (ii) $x(t) = \sin \Omega_0 t$ (iii) $x(t) = e^{-2t} u(t)$ (iv) $x(t) = \begin{cases} 2\cos(4t), & t > 0 \\ 0, & Otherwise \end{cases}$
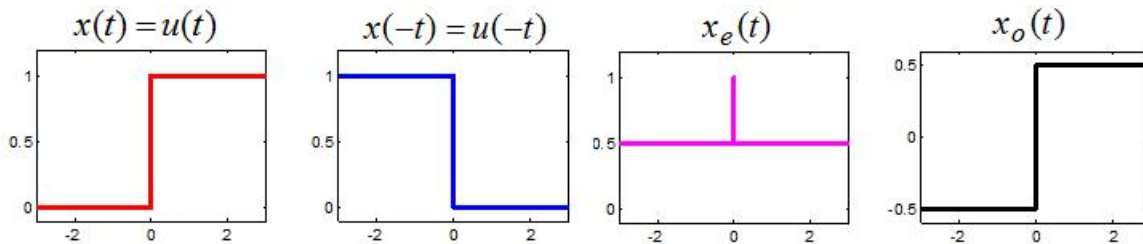
Solutions:

1. (i) Given that $x(t) = u(t)$. By time reversal $x(-t) = u(-t)$.

Then $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} = \frac{1}{2}\{u(t) + u(-t)\}$, and

$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} = \frac{1}{2}\{u(t) - u(-t)\}$

The plots of these signals are shown below
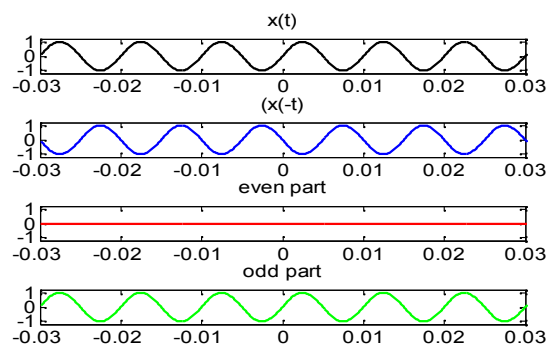

(ii) Given that $x(t) = \sin \Omega_0 t$. By time reversal $x(-t) = \sin \Omega_0(-t) = -\sin \Omega_0 t$.

Then $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} = \frac{1}{2}\{\sin \Omega_0 t - \sin \Omega_0 t\} = 0$, and

$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} = \frac{1}{2}\{\sin \Omega_0 t + \sin \Omega_0 t\} = \sin \Omega_0 t$

The plots of these signals are shown below
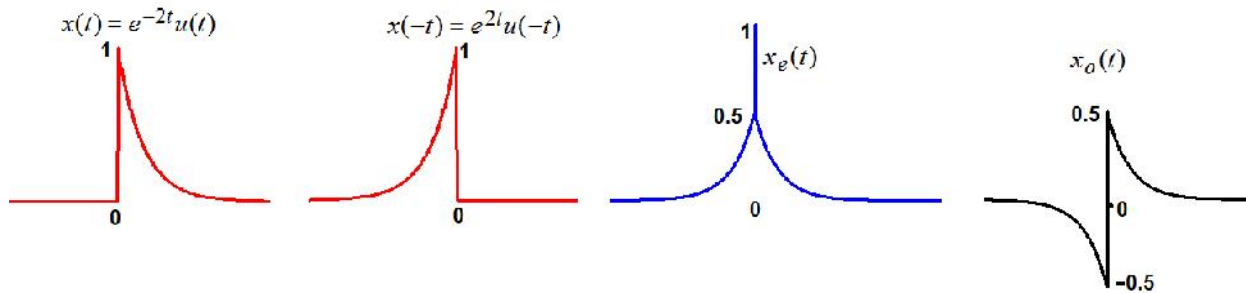

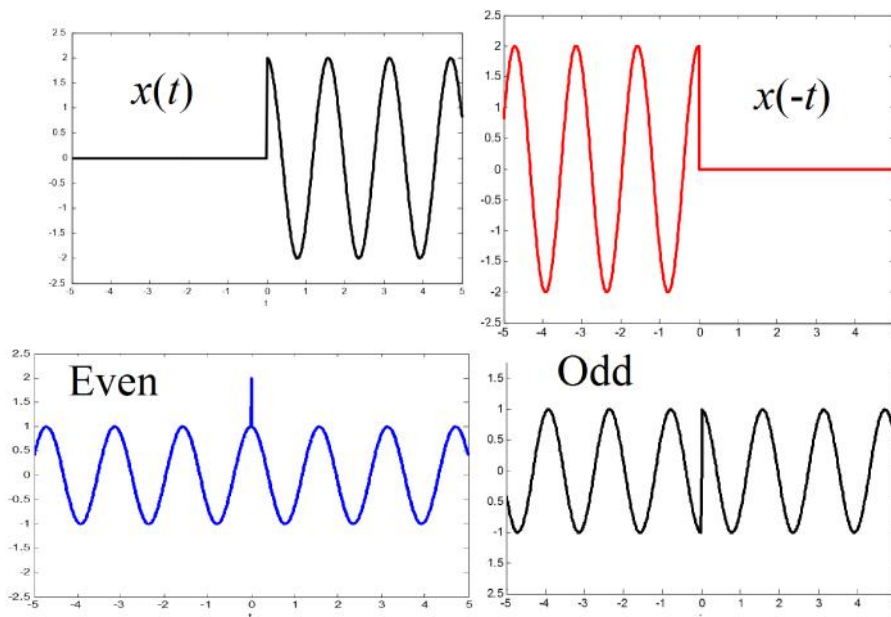(iii) Given that $x(t) = e^{-2t} u(t)$. By time

reversal $x(-t) = e^{2t}u(-t)$.

Then $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} = \frac{1}{2}\{e^{-2t}u(t) + e^{2t}u(-t)\}$, and

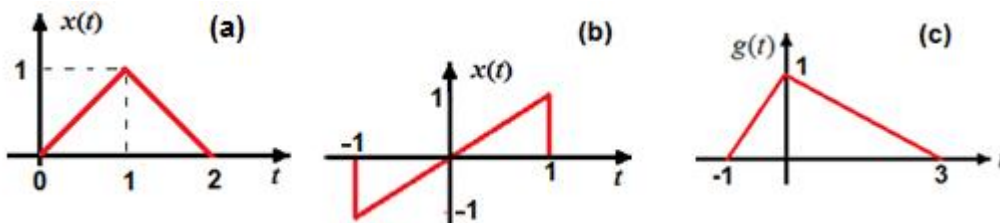$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} = \frac{1}{2}\{e^{-2t}u(t) - e^{2t}u(-t)\}$$

The plots of these signals are shown below

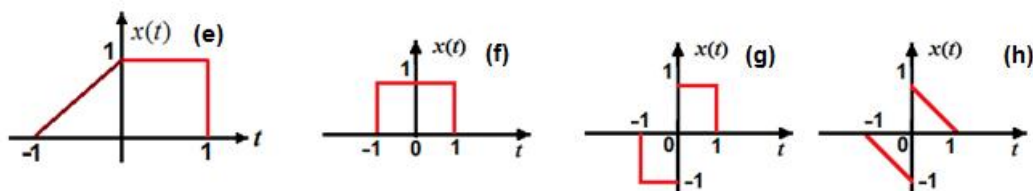(iv) $x(t) = \begin{cases} 2\cos(4t), & t > 0 \\ 0, & Otherwise \end{cases}$

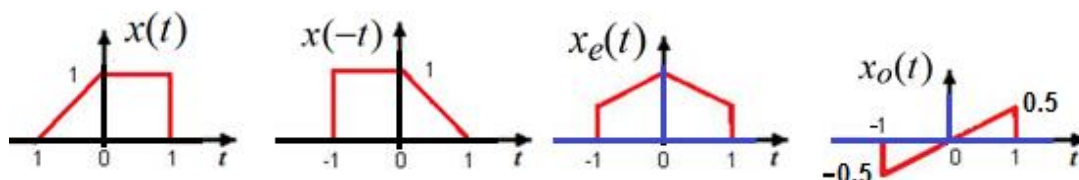2. Determine and sketch the even and odd parts of the signals shown below:

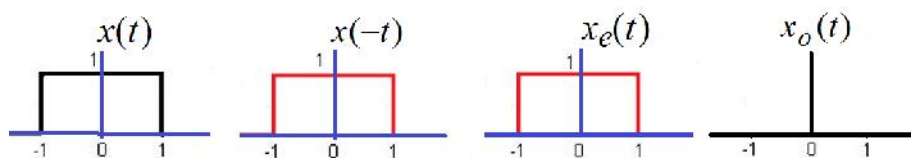3. Determine and sketch the even and odd parts of the signals shown below:

Solutions:

(e)

(f)

5. Find the even and odd components of the signal $x(t) = e^{jt}$

Ans: Given that $x(t) = e^{jt}$. By time reversal $x(-t) = e^{-jt}$

Then $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} = \frac{1}{2}\{e^{jt} + e^{-jt}\} = \cos t$, and

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} = \frac{1}{2}\{e^{jt} - e^{-jt}\} = \sin t$$

(6) Show that the product of two even signals or of two odd signals is an even signal and that the product of an even and an odd signal is an odd signal.

Let $x(t) = x_1(t)x_2(t)$. If $x_1(t)$ and $x_2(t)$ are both even, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ and $x_2(t)$ are both odd, then

$$x(-t) = x_1(-t)x_2(-t) = -x_1(t)[-x_2(t)] = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ is even and $x_2(t)$ is odd, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)[-x_2(t)] = -x_1(t)x_2(t) = -x(t)$$

and $x(t)$ is odd. Note that in the above proof, variable t represents either a continuous or a discrete variable.