Problems and Solutions on Signal Operations/Transformations

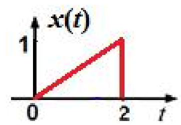
1. Justify the following

   (a) $t\text{¢}(t) = 0$ (b) $\sin t\, \text{¢}(t) = 0$

   Ans: (a) At time $t = 0, \quad 0\text{¢}(0) = 0$

   (b) At time $t = 0, \quad \sin 0\, \text{¢}(0) = 0$

2. Consider a signal shown in figure



(a) Express this signal in functional form and in terms of step functions.

(b) Find and sketch $y(t) = \frac{d}{dt} x(t)$. Express this signal in terms of step functions.

(c) Find and sketch $z(t) = \frac{d}{dt} y(t)$. Express this signal in terms of singularity functions.

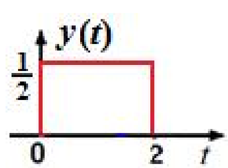
Ans: (a) The given signal in functional form $x(t) = \begin{cases} \frac{1}{2}t, & 0 < t < 2 \\ 0, & Elsewhere \end{cases}$

The given signal is represented in terms of step functions $x(t) = \frac{1}{2} t \left\{ u(t) - u(t-2) \right\}$

(b) We know that $\frac{d}{dt} r(t) = \frac{d}{dt} t\, u(t) = u(t)$,

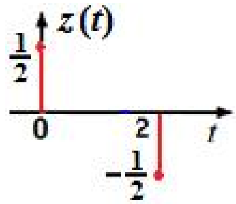
$$y(t) = \frac{d}{dt} x(t) = \begin{cases} \frac{d}{dt}\left\{\frac{1}{2}t\right\}, & 0 < t < 2 \\ 0, \ Elsewhere \end{cases} = \begin{cases} \frac{1}{2}, & 0 < t < 2 \\ 0, & Elsewhere \end{cases} = \frac{1}{2}\left\{ u(t) - u(t-2) \right\}$$

The plot of the signal $y(t)$ is shown in figure.



(c) We know that $\frac{d}{dt} u(t) = \text{¢}(t)$, Then $z(t)=\frac{d}{dt}y(t)=\frac{d}{dt}\left(\frac{1}{2}\{u(t)-u(t-2)\}\right)=\frac{1}{2}\{\delta(t)-\delta(t-2)\}$

The plot of the signal $z(t)$ is shown in figure.



3. Evaluate the following integrals and sketch them.

(a) $x(t)=\int_{-\infty}^{t}\{\delta(\tau)-\delta(\tau-2)\}\,d\tau$ (b) $x(t)=\int_{-\infty}^{t}\{u(\tau)-u(\tau-2)\}\,d\tau$

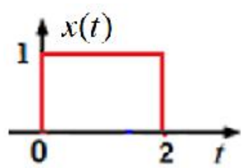
(c) $x(t)=\int_{-\infty}^{t}\{u(t-\tau)u(\tau-4)\}\,d\tau$

Ans: (a) Given that $x(t)=\int_{-\infty}^{t}\{\delta(\tau)-\delta(\tau-2)\}\,d\tau$.

We know that $\int_{-\infty}^{t}\delta(\tau)d\tau=u(t)$. Then the given equation can be written as

$$x(t)=\int_{-\infty}^{t}\{\delta(\tau)-\delta(\tau-2)\}\,d\tau=\int_{-\infty}^{t}\delta(\tau)d\tau-\int_{-\infty}^{t}\delta(\tau-2)\,d\tau$$
$$=u(t)-u(t-2)$$

The plot of the signal $x(t)$ is shown in the following figure

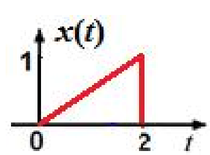


(b) Given that $x(t)=\int_{-\infty}^{t}\{u(\tau)-u(\tau-2)\}\,d\tau$

We Know that $\int_{-\infty}^{t}u(\tau)\,d\tau=r(t)=t\,u(t)$. Then the given equation can be written as

$$x(t)=\int_{-\infty}^{t}\{u(\tau)-u(\tau-2)\}\,d\tau=\int_{-\infty}^{t}u(\tau)d\tau-\int_{-\infty}^{t}u(\tau-2)\,d\tau$$
$$=\int_{0}^{t}1\,d\tau-\int_{2}^{t}1\,d\tau=tu(t)-(t-2)u(t-2)=r(t)-r(t-2)$$

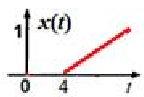
The plot of the signal $x(t)$ is shown in the following figure



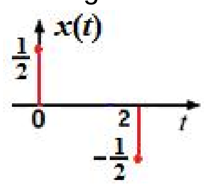
(c) Given that $x(t)=\int_{-\infty}^{t}\{u(t-\tau)u(\tau-4)\}\,d\tau$. Since the variable is τ , this signal can be simplified as below.

$$x(t)=\int_{-\infty}^{t}\{u(t-\tau)u(\tau-4)\}\,d\tau=[\tau]_{4}^{t}=(t-4)u(t-4)$$

The plot of the signal $x(t)$ is shown in the following figure



4. Consider a signal shown in figure



(a) Represent the signal in mathematical equation.

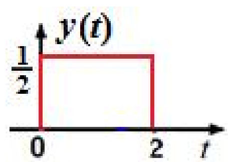
(b) Find and sketch the signal $y(t)=\int_{-\infty}^{t}x(\tau)\,d\tau$.

(c) Find and sketch the signal $z(t)=\int_{-\infty}^{t}y(\tau)\,d\tau$.

Ans: (a) The given signal is represented by $x(t)=\frac{1}{2}\{\delta(t)-\delta(t-2)\}$

(b) $y(t)=\int_{-\infty}^{t}x(\tau)\,d\tau=\int_{-\infty}^{t}\frac{1}{2}\{\delta(t)-\delta(t-2)\}\,d\tau=\frac{1}{2}\{u(t)-u(t-2)\}$

The plot of the signal $y(t)$ is shown in figure.



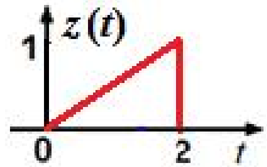
(c) $z(t)=\int_{-\infty}^{t}y(\tau)\,d\tau=\int_{-\infty}^{t}\frac{1}{2}\{u(\tau)-u(\tau-2)\}\,d\tau$

$$=\frac{1}{2}\int_{-\infty}^{t}u(\tau)\,d\tau-\frac{1}{2}\int_{-\infty}^{t}u(\tau-2)\,d\tau=\frac{1}{2}\int_{0}^{t}1\,d\tau-\frac{1}{2}\int_{2}^{t}u(\tau-2)\,d\tau$$

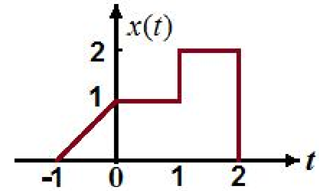
$$=\frac{1}{2}[\tau]_{0}^{t}-\frac{1}{2}[\tau]_{2}^{t}=\frac{1}{2}tu(t)-\frac{1}{2}(t-2)u(t-2)$$

$$=\frac{1}{2}\{u(t)-u(t-2)\}$$

The plot of the signal $z(t)$ is shown in figure.

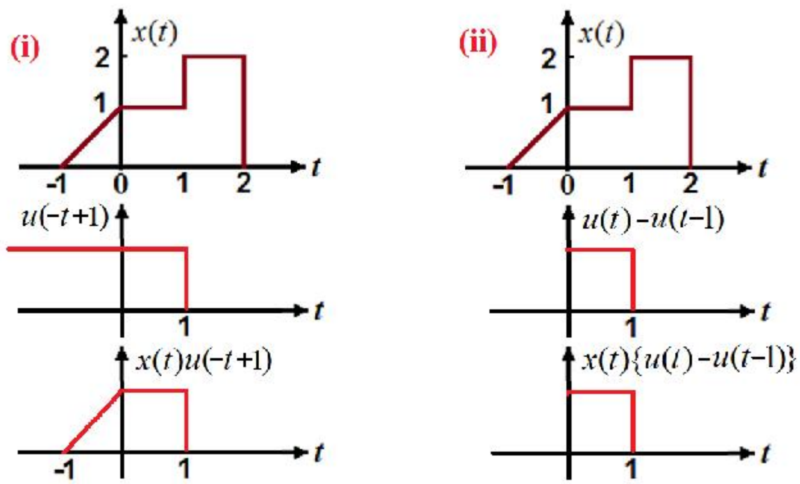


5. A continuous-time signal is shown in Fig. Sketch and label each of the following signals.



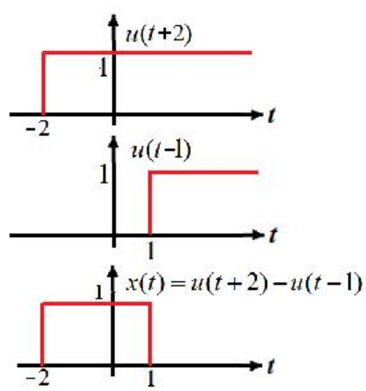
(i) $y_1(t) = x(t)u(1-t)$ (ii) $y_2(t) = x(t)\{u(t) - u(t-1)\}$

Ans:

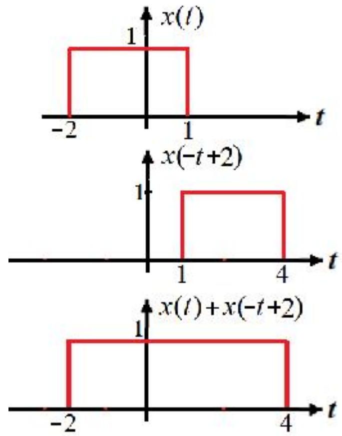


6. Determine and sketch $y(t) = x(t) + x(2-t)$ where $x(t) = u(t+2) - u(t-1)$. Express $y(t)$ in terms of step functions.

Ans: The given signal is shown below



The corresponding $\{x(t) + x(2-t)\}$ are illustrated below



Given that $x(t) = u(t+2) - u(t-1)$. From the figure, we write $x(2-t) = u(t-1) - u(t-4)$

Therefore $y(t) = x(t) + x(2-t) = u(t+2) - u(t-1) + u(t-1) - u(t-4) = u(t+2) - u(t-4)$.

7 (a) Sketch and label the signal $x(t) = r(t-2)$, where $r(t)$ is a unit ramp function. Express the signal in terms of step functions.

(b) Find and sketch $y(t) = \dfrac{d}{dt}x(t)$. Express this signal in terms of step functions.

(c) Find and sketch $z(t) = \dfrac{d}{dt}y(t)$. Express this signal in terms of singularity functions.