

The current in a circuit is given by  $I = 42.42 \sin(628t)$ . Calculate

- i) Frequency
- ii) RMS Value
- iii) Average Value
- iv) Form Factor

$$I = I_m \sin \omega t$$

$$I = 42.42 \sin(628t)$$

$$I_m = 42.42 \text{ A}$$

$$\omega = 628$$

$$2\pi f = 628$$

$$f = \frac{628}{2\pi} = 99.95 \text{ Hz}$$

$$\approx 100 \text{ Hz}$$

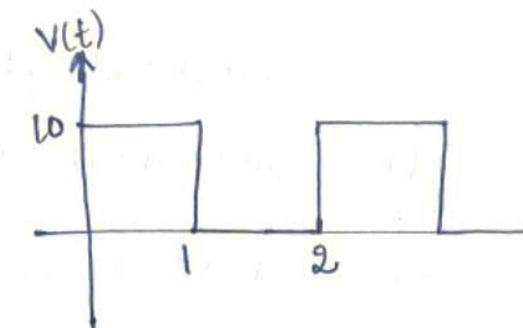
$$\text{RMS} \Rightarrow I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{42.42}{\sqrt{2}} = 30 \text{ A}$$

$$\text{Average} \Rightarrow I_{\text{ave}} = \frac{2I_m}{\pi} = \frac{2 \times 42.42}{\pi} = 27 \text{ A}$$

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{ave}}} = \frac{30/\sqrt{2}}{27} = \frac{30}{27} = 1.11$$

$$\text{Peak factor} = \frac{I_m}{I_{\text{rms}}} = \frac{42.42}{30/\sqrt{2}} = \sqrt{2} = 1.414$$

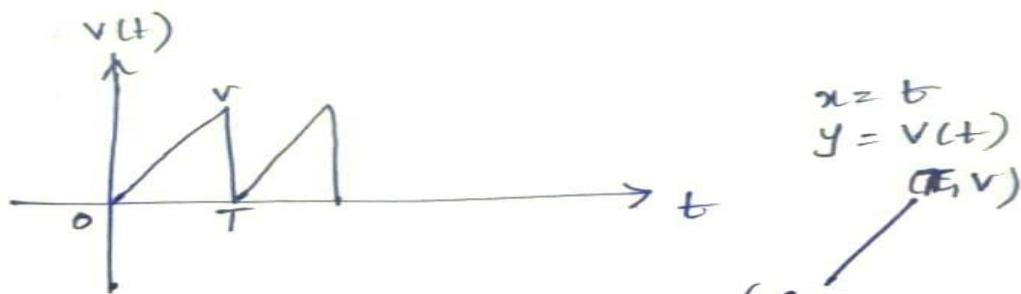
2. Calculate the average and RMS value of the square wave.



$$V_{rms} = \sqrt{\frac{\int_0^2 V^2(t) dt}{2}} = \sqrt{\frac{\int_0^1 10^2 \cdot dt + \int_1^2 0^2 \cdot dt}{2}}$$
$$= \sqrt{\frac{100(t)|_0^1 + 0}{2}} = \sqrt{\frac{50}{2}} = 7.071V.$$

$$V_{ave} = \frac{\int_0^2 V(t) \cdot dt}{2} = \frac{\int_0^1 10 \cdot dt + \int_1^2 0 \cdot dt}{2}$$
$$= \frac{10(t)|_0^1}{2} = \frac{10[1]}{2} = 5V.$$

3. Calculate the form factor and peak factor of a triangular wave in which the voltage rises uniformly from 0 to V volts in time T seconds and completes the cycle by instant fall back to zero?



$$\text{book } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \Rightarrow y = \frac{V - 0}{T - 0} (t - 0) = \frac{V}{T} t$$

$$\text{book } y = 0 \Rightarrow \frac{V - 0}{T - 0} (t - 0) = 0 \Rightarrow t = 0$$

$$\text{Ansatz } y = \frac{V}{T} t \Rightarrow V(t) = \frac{V}{T} \cdot t = \frac{V}{T} t$$

$$V_{\text{rms}} = \sqrt{\frac{\int_0^T \left(\frac{V}{T} t\right)^2 dt}{T}} = \sqrt{\frac{\frac{V^2}{T^2} \left[\frac{t^3}{3}\right]_0^T}{T}} = \sqrt{\frac{V^2}{T^2} \cdot \frac{T^3}{3}} = \frac{V}{\sqrt{3}}$$

$$V_{\text{ave}} = \frac{1}{T} \int_0^T V(t) dt = \frac{1}{T} \int_0^T \frac{V}{T} t dt = \frac{V}{T^2} \int_0^T t dt$$

$$V_{\text{ave}} = \frac{V}{T^2} \left[ \frac{t^2}{2} \right]_0^T = \frac{V}{T^2} \cdot \frac{T^2}{2} = \frac{V}{2}$$

$$\text{Form factor} = \frac{V_{\text{max}}}{V_{\text{ave}}} = \frac{V}{\frac{V}{2}} = 2$$

$$\text{Peak factor} = \frac{V_{\text{max}}}{V_{\text{rms}}} = \frac{V}{\frac{V}{\sqrt{3}}} = \sqrt{3}$$

$$\text{Peak factor} = \frac{V_{\text{max}}}{V_{\text{ave}}} = \frac{V}{\frac{V}{2}} = 2$$

The current drawn by a pure capacitance of  $20\mu F$  is 1.382 A from 220V ac supply, find the supply frequency?

$$C = 20\mu F$$

$$I = 1.382 A$$

$$V = 220 V$$

$$Z = X_C = \frac{V}{I} = \frac{220}{1.382} = 159 \Omega$$

$$X_C = \frac{1}{2\pi f L}$$

$$f = \frac{1}{2\pi X_C L} = 49.99 \text{ Hz}$$

A  $300 \mu\text{F}$  capacitor is connected across a  $240\text{V}, 50\text{Hz}$  system. Determine i) the capacitance reactance ii) R.M.S value of current iii) Equations for voltages and currents.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(300 \times 10^{-6})} = 10 \Omega$$

$$Z = R + j(X_L - X_C) = 0 + j(0 - 10) = -10j$$

$$= 10 \angle -90^\circ$$

$$I = \frac{V}{Z} = \frac{240}{10} = 24 \text{ A} \cdot (24 \angle 90^\circ)$$

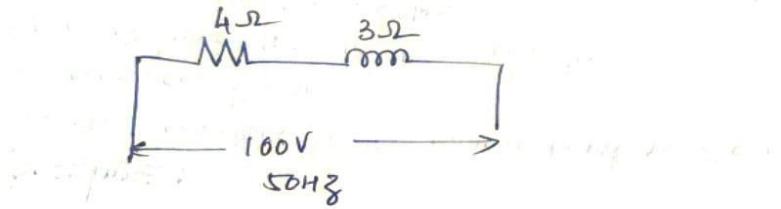
$$V_m = \sqrt{2}V = \sqrt{2} \times 240 = 325 \text{ V}$$

$$I_m = \sqrt{2}I = \sqrt{2} \times 24 = 32.5 \text{ A}$$

$$V = V_m \sin \omega t = 325 \sin(100t \pm \theta) \text{ V.}$$

$$I = I_m \sin \omega t = 32 \sin(100\pi t + 90^\circ) \text{ A.}$$

A Series RL circuit connected with  $R=4 \Omega$  and inductive reactance  $X_L = 3\Omega$  is connected to  $100\text{V}, 50\text{Hz}$ . Find the amount of current, power drawn by the circuit and power factor?



$$V = 100 \angle 0^\circ$$

$$Z = R + jX_L$$

$$Z = 4 + j3 = 5 \angle 36.86^\circ \Omega$$

$$I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{5 \angle 36.86^\circ} = 20 \angle -36.86^\circ A$$

$$P = I^2 R = (20)^2 \times 4 = 1600W$$

$$\cos\phi = \cos(-36.86^\circ) = 0.8 \text{ Lagging}$$

$$\text{power} \Rightarrow P = V \cdot I \cdot \cos\phi$$

$$= 100 \times 20 \times 0.8 \text{ Watts}$$

$$= 1600 \text{ Watts}$$

- ③ Write the polar form of the voltage  
 $v(t) = \sin(10\pi t + \frac{\pi}{3})$ . obtain its rectangular form.

Sol:  $v(t) = \sin(10\pi t + \frac{\pi}{3})$

$$v(t) = V_m \sin(\omega t + \theta)$$

By comparison,

$$V_m = 1 \quad \omega = 10\pi \quad \theta = \frac{\pi}{3} = 60^\circ$$

$$\text{voltage in polar form} = V_{ms} \angle \theta = \frac{V_m}{\sqrt{2}} \angle \theta$$

$$\boxed{V = \frac{1}{\sqrt{2}} \angle 60^\circ \text{ v.} \\ = 0.707 \angle 60^\circ \text{ v.}} \quad \text{polar form}$$

$$\text{voltage in rectangular form} = \frac{1}{\sqrt{2}} \cos 60^\circ + j \frac{1}{\sqrt{2}} \sin 60^\circ$$

$$= 0.707 \cos 60^\circ + j 0.707 \sin 60^\circ$$

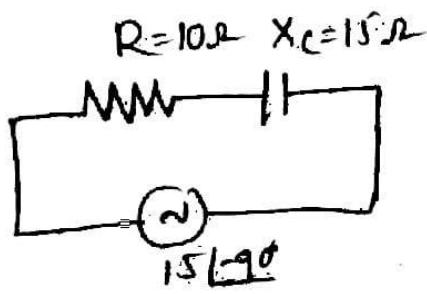
$$\boxed{V = 0.354 + j 0.612 \text{ v.}} \quad \text{rectangular form.}$$

A series circuit of  $R=10\Omega$ ,  $X_C=15\Omega$  has an applied phase voltage  $V=50 \angle -90^\circ$  Vrms. Find the real power, reactive power, and power factor?

$$\text{Resistance (R)} = 10 \Omega$$

$$\text{Capacitive reactance (Xc)} = 15 \Omega$$

$$\text{Applied phase voltage (V)} = 50 \angle -90^\circ = \text{Vrms}$$



$$\begin{aligned} Z &= R + jX_C \\ &= 10 + j(-15) \\ Z &= \sqrt{10^2 + 15^2} \tan^{-1}\left(\frac{-15}{10}\right) \\ &= 18.02 \tan^{-1}(15/10) \\ Z &= 18.02 \angle -56.3^\circ \end{aligned}$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{50}{18} = 2.7 \text{ A}$$

$$\begin{aligned} \text{Real Power} &= V_{\text{rms}} \times I_{\text{rms}} \times \cos \phi = 50 \times 2.7 \times \cos(56.3^\circ) \\ &= 99 \text{ W} \end{aligned}$$

$$\text{Reactive power} = V_{\text{rms}} \times I_{\text{rms}} \times \sin \phi = 109 \text{ VAR}$$

$$\text{Power factor} = \cos(\phi) = \cos(56.3^\circ) = 0.547$$