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Group 915

① $g: \mathbb{R}^n \to \mathbb{R}$

$$g(v) = \frac{1}{2} v^T A v + b^T \cdot v \to \min$$

To prove that the minimum of g is given by $v = -A^{-1} b$, we can take its derivative and set it to 0.

$$\frac{d}{dv}\left(\frac{1}{2} \cdot v^T \cdot A \cdot v\right) = Av \text{ if } A \text{ is symmetric}$$

because the derivative of $v^T \cdot A \cdot v$ with respect v is $(A + A^t) \cdot v$ and since we have the $\frac{1}{2}$ factor it simplifies with 2, hence the result.

$$\frac{d}{dv} g(v) = A \cdot v + b$$

$$A \cdot v + b = 0$$

$$v = -A^{-1} \cdot b \qquad (q.e.d)$$

② $f: \mathbb{R}^n \to \mathbb{R}$, $H(x)$ is positive definite.

$$f(x+h) \approx f(x) + \nabla f(x)^T h + \frac{1}{2} h^T H(x) \cdot h \to \min$$

To find the direction of h that minimizes $f(x+h)$, we can take the derivative of the Taylor approximation and set it equal to 0.

$$\frac{d}{dh} f(x+h) = \nabla f(x) + H(x) \cdot h$$

$$\nabla f(x) + H(x) \cdot h = 0$$

$$H(x) \cdot h = -\nabla f(x)$$

$$h = -H^{-1}(x) \cdot \nabla f(x)$$

This is the direction in which $f(x+h)$ decreases the fastest ($H(x)$ is positive definite). This is because $f(x+h)$ is convex and $\min(f(x+h))$ is reached when its derivative is 0.