Optional Homework

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Compose and solve 5 exercises of reasoning modelling using propositional logic.

1. Proof method used - truth table

Premises:

H_1: If it is sunny, then John goes ~~stop~~ jogging.
H_2: If John goes jogging, then he does not watch TV.
H_3: If it is not sunny, then Mary goes shopping.
H_4: If Mary goes shopping, then ~~it~~ John does not go jogging

Conclusion:

C: Either John watches TV or goes jogging

We transform the premises and conclusions into propositional logic form

S - it is sunny
J - John goes jogging

$T \rightarrow$ John watches TV

$M \rightarrow$ Mary goes shopping

Propositional formulas:

H_1: $S \rightarrow J$

H_2: $J \rightarrow \neg T$

H_3: $\neg S \rightarrow M$

H_4: $M \rightarrow \neg J$

C: $T \vee J$

$T \equiv 1$
$F \equiv 0$ in the Truth table below

					H_1	H_2	H_3	H_4	C
	S	J	T	M	$S \rightarrow J$	$J \rightarrow T$	$\neg S \rightarrow M$	$M \rightarrow \neg J$	$T \vee J$
1	0	0	0	0	1	1	0	1	0
2	0	0	0	1	1	1	1	1	0
3	0	0	1	0	1	1	0	1	1
4	0	0	1	1	1	1	1	1	1
5	0	1	0	0	1	0	0	1	1
6	0	1	0	1	1	0	1	0	1
7	0	1	1	0	1	1	0	1	1
8	0	1	1	1	1	1	1	0	1
9	1	0	0	0	0	1	1	1	0
10	1	0	0	1	0	1	1	1	0
11	1	0	1	0	0	1	1	1	1
12	1	0	1	1	0	1	1	1	1
13	1	1	0	0	1	0	1	1	1
14	1	1	0	1	1	0	1	0	1
15	1	1	1	1	1	0	1	0	1
16	1	1	1	0	1	1	1	1	1

If we look on row 2, all premises are T and the conclusion is false, therefore the conclusion does not hold.

2.- Proof method definition of deduction

H_1: Mihnea will go to the lecture if Luca will go and Luscan will not go.

H_2: If Bogdan will go to the lecture then Luca will go too.

H_3: If Bogdan is in Cluj he will go to the lecture.

H_4: Lușcan is sick and will not go to the lecture.

H_5: Last week Bogdan has returned in Cluj from Bistrița.

We have to check wheter the fallowing deduction holds.

$H_1, H_2, H_3, H_4, H_5 \vdash C$

C: Will Mihnea go to the lecture?

Notations for propositional variables:

B - Bogdan will go to the lecture.

M - Mihnea will go to the lecture.

L - Lucian will go to the lecture.

Lu - Luca will go to the lecture.

C - Bogdan is in Cluj.

Propositional formulas:

$H_1: \neg L \wedge Lu \to M = f_1$

$H_2: B \to Lu = f_2$

$H_3: C \to B = f_3$

$H_4: \neg L = f_4$

$H5: C = f_5$

$C: M$

$f_5, f_3 \vdash_{mp} B : f_6$

$f_6, f_2 \vdash_{mp} Lu : f_7$

$f_7, f_4 \vdash Lu \wedge \neg L$ (conjunction of conclusions) $: f_8$

$f_8, f_1 \vdash_{mp} \cancel{Lu \wedge \neg L \to} M : f_9 = C$

The sequence of formulas is the deduction of

C => Mihnea will go to the lecture

3. Proof method: semantic tableaux

H_1: If Bogdan will go to the party, then Ana and Lucreția will ~~be sad~~ also go.

H_2: If Lucreția washes the dishes then she will go to the party

H_3: Lucreția has a broken hand, so she did not wash the dishes.

H_4: Ana is in love with Bogdan, so she will go to the party

C: Will Lucreția go to the party?

Notations for the propositional variables:

B - Bogdan will go to the party

A - Ana will go to the party

L - Lucreția will go to the party

D - Lucreția washes the dishes

$H_1: B \to (A \land L)$

$H_2: D \to L$

$H_3: \neg D$

$H_4: A$

$C: L$

We have to negate the conclusion and use the theorem of soundness and completion.

$H_1, H_2, H_3, H_4 \models C$ iff

$H_1 \land H_2 \land H_3 \land H_4 \land \neg C$ has a closed semantic tableaux

$B \to (A \land L) \land (D \to L) \land \neg D \land A \land \neg L$ (1)

| α rule for (1)

$B \to (A \land L)$ (2)

$D \to L$ (3)

$\neg D$

A

$\neg L$

β rule for (3)

- D $\otimes$
- L (β rule for (2))
  - $\neg B$ $\circ$
  - $A \land L$ (4)
    - α rule for (4): A, L $\otimes$

We have optained a open and complete tableau with one open branch and two closed branches containing the following pairs of opposite literals $(D, \neg D)$, $(L, \neg L)$

Therefore $H_1, H_2, H_3, H_4 \nvDash C$ based on hypothesis we can't conclude if Lucreția will go to the party

4. Proof method used - resolution (general resolution)

H_1: If it is sunny then, Andrei and Bogdan will go to play football and Mihnea will play Basketball

H_2: Andrei goes to football on Monday.

H_3: It was sunny last monday.

H_4: Mihnea goes to basketball everyday of the week.

C: Did Mihnea, Bogdan and Andrei play a sport on Monday?

Notations for the propositional variables

$S \to$ it is sunny

$B \to$ Bogdan plays football

$A \to$ Andrei plays football

$M \to$ Mihnea plays basketball

$Mon \to$ It is monday

$H_1: S \to (B \wedge A \wedge M) \equiv (\neg S \vee B) \wedge (\neg S \vee A) \wedge (\neg S \vee M)$

$\equiv C_1 \wedge C_2 \wedge C_3$

$H_2: Mon \to A : C_4 \equiv \neg Mon \vee A$

$H_3: Mon \wedge S : C_5 \wedge C_6$

$H_4: M : C_7$

$C: M \wedge B \wedge A :$

$\neg C : \neg M \vee \neg B \vee \neg A : C_8$

$Cnf(H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C) \vdash_{Res} \square ?$

$X = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$

$C_1 = \neg S \vee B$

$C_2 = \neg S \vee A$

$C_3 = \neg S \vee M$

$C_4 : \neg Mor \vee A$

$C_5 : Mor$

$C_6 : S$

$C_7 : M$

$C_8 : \neg M \vee \neg B \vee \neg A$

$C_9 = Res_S(C_6, C_3) = M = C_7$

$C_{10} = Res_M(C_7, C_8) = \neg B \vee \neg A$

$C_{11} = Res_S(C_6, C_1) = B$

$C_{12} = Res_S(C_6, C_2) = A$

$C_{13} = Res_B(C_{11}, C_{10}) = \neg A$

$C_{14} = Res_A(C_{13}, C_{12}) = \square$

$CNF(H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C) \vdash_{Res} \square$, so C is deductible from the hypothesis

~~5. Proof method used - lock Resolution~~

5. Proof method used - resolution (level saturation strategy)

H_1: If it is raining, the ground is wet.

H_2: If the ground is wet then, then wear boots.

H_3: If John wear boots, he will not ride his bike

H_4: It is raining.

Conclusion: John will not ride his bike.

The sequence $S^0, S^1, S^2 \dots$ represent the level of resolvents

$$S^k := \{ Res(C_i, C_j) \mid C_i \in S^{k-1}, C_j \in S^0 \cup S^1 \cup \dots \cup S^{k-1} \}, k = 1, 2, \dots$$

Notations for the propositional variables:

R - It is raining

W → The ground is wet

B - John wear boots

Bi - John rides his bike

H_1: $R \to W \equiv \neg R \lor W : C_1$

H_2: $W \to B \equiv \neg W \lor B : C_2$

H_3: $B \to \neg Bi \equiv \neg B \lor \neg Bi : C_3$

H_4: $R : C_4$

C: $\neg Bi$:

$\neg C = Bi \quad C_5$

Initial level $S^0 = S = \{\overbrace{\neg R \vee W}^{C_1}, \overbrace{\neg W \vee B}^{C_2}, \overbrace{\neg B \vee \neg Bi}^{C_3},$

$\overbrace{R}^{C_4}, \overbrace{Bi}^{\neg C = C_5}\}$

First Level

$C_6 = Res_W(C_1, C_2) = \neg R \vee B$

$C_7 = Res_R(C_1, C_4) = W$

$C_8 = Res_B(C_2, C_3) = \neg W \vee \neg Bi$

$C_9 = Res_{Bi}(C_3, C_5) = \neg B$

$S^1 = \{\underbrace{\neg R \vee B}_{C_6}, \underbrace{W}_{C_7}, \underbrace{\neg W \vee \neg Bi}_{C_8}, \underbrace{\neg B}_{C_9}\}$

Second level

$C_8 = Res_B(C_6, C_2) = \neg W \vee \neg R$

$C_9 = Res_B(C_6, C_3) = \neg R \vee \neg Bi$

$C_{10} = Res_R(C_6, C_4) = B$

$C_{11} = Res_W(C_8, C_1) = \neg R \vee \neg Bi$

$C_{12} = Res_{Bi}(C_8, C_5) = \neg W$

$S^2 = \{C_8, C_9, C_{10}, C_{11}, C_{12}\}$

Third level

$C_{13} = Res_B(C_{10}, C_9) = \square$

$\Rightarrow Cnf(C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5) \vdash_{Res} \square$

$\Rightarrow$ The conclusion is valid, therefore John will not ride his bike

Solve 5 exercises from the list in the attached file using predicate logic

1. Proof method used – Predicate resolution

H_1: All hounds howl at night.

H_2: Anyone who has any cats will not have any mice.

H_3: Light sleepers do not have anything which howls at night

H_4: John has either a cat or a hound.

C: If John is a light sleeper, then John does not have any mice.

Now we tranform the hypothesis and the conclusion into predicate logic formulas.

predicate symbols used:

$H: D \to \{T, F\}$ $H(x)$ is T if x is a hound

$N: D \to \{T, F\}$ $N(x)$ is T if x howls at night

$M: D \to \{T, F\}$ $M(x)$ is T if x is a mice

$C: D \to \{T, F\}$ $C(x)$ is T if x is a cat

$Owns: D \times D \to \{T, F\}$ $Owns(x, y)$ is T if x owns y

$H_1: \forall(x) \left(H(x) \to N(x) \right)$

$H_2: \forall(x)(y) \left((C(x) \wedge Owns(y, x)) \to \forall(z) (M(z) \wedge \neg Owns(y, z)) \right)$

$H_3: \forall(x)(y) \left(L(x) \to \neg(N(y) \wedge \neg Owns(x, y)) \right)$

~~$H_4: L(John) \to \forall(x) (M(x) \wedge \neg Owns(John, x))$~~

$H_4: \forall(x) \left((C(x) \wedge Owns(John, x)) \vee (H(x) \wedge Owns(John, x)) \right)$

$C = L(John) \to \forall(x) \left(M(x) \wedge \neg Owns(John, x) \right)$

$\neg C = \neg \left(L(John) \to \forall(x) (M(x) \wedge \neg Owns(John, x)) \right)$

$H_1^c = H(x) \to N(x) \equiv \neg H(x) \vee N(x) : C_1$

$H_2^c = \neg(C(x) \wedge Owns(y,x)) \vee (M(z) \wedge \neg Owns(y,z)) \equiv$

$\neg C(x) \vee Owns(y,x) \vee (M(z) \wedge \neg Owns(y,z)) \equiv$

$(\neg C(x) \vee Owns(John,x) \vee M(z)) \wedge (\neg C(x) \wedge Owns(John,x) \wedge \neg Owns(John, z)) : C_2 \wedge C_3$

$H_4^c : \neg C(x) \vee \neg Owns(John,x) \vee \neg H(x)$ ~~not~~ $: C_6$

$H_3^c : (\neg L(x) \vee N(y)) \wedge (\neg L(x) \vee \neg Owns(x,y)) \equiv$

$(\neg L(John) \vee N(y)) \wedge (\neg L(John) \vee Owns(John,y))$

$: \quad C_4 \quad \wedge C_5$

$S = \{C_1, C_2, C_3, C_4, C_5\}$

We can eliminate the clauses with pure literals $\neg H(x), \neg C(x)$

and $\neg L(x)$ and obtain S'

but $S' = \emptyset$, all clauses from S containing pure literals $\Rightarrow$ The conclusion holds

$H_1, H_2, H_3, H_4 \vdash V$

13. Proof method used - semantic tableaux

H_1: Every boy or girl is a child.

H_2: Every child gets a doll or a train or a lump of coal.

H_3: No boy gets any doll.

H_4: No child who is good gets any lump of coal

C: If no child gets a train, then no boy is good.

predicate logic symbols used:

$B: D \to \{T, F\}$ T if Bx is a boy

$G: D \to \{T, F\}$ $G(x)$ T if x is a girl

$C: D \to \{T, F\}$ $C(x)$ is T if x is a child

$T: D \to \{T, F\}$ $T(x)$ is T if x gets a train

$L: D \to \{T, F\}$ $L(x)$ is T if x gets a lump of coal

$D: D \to \{T, F\}$ $D(x)$ is T if x gets a doll.

$Good: D \to \{T, F\}$ $Good(x)$ is T if x is good

H_1: $\forall(x)\left((B(x) \vee G(x)) \to C(x)\right)$

H_2: $\forall(x)\left(C(x) \to (L(x) \vee T(x) \vee D(x))\right)$

H_3: $\forall(x)\left(B(x) \to \neg D(x)\right)$

H_4: $\forall(x)\left((C(x) \wedge G(x)) \to \neg L(x)\right)$

C: $\forall(x)\left(((C(x) \to \neg T(x)) \to \forall(y)(B(y) \to \neg Good(y))\right)$

$H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C$ (1) ✓

| α rule (1)

$H_1: \forall(x)\big((B(x) \vee G(x)) \to C(x)\big)$ (2)

|

$H_2: \forall(x)\big(C(x) \to (L(x) \vee T(x) \vee D(x))\big)$ (3) ✓

|

$H_3: \forall(x)\big(B(x) \to \neg D(x)\big)$ (4) ✓

|

$H_4: \forall(x)\big((C(x) \wedge \underset{\text{and}}{G(x)}) \to \neg L(x)\big)$ (5) ✓

|

✓ $\neg C: \forall(x)(C(x) \to \neg T(x)) \wedge \exists y (B(y) \wedge Good(y))$ (6)

| δ rule for 6
| a-new const

$(C(a) \to \neg T(a)) \wedge B(a) \wedge Good(a))$ (7) ✓

| α rule for (7)

$C(a) \to \neg T(a)$ (8) ✓

|

$B(a)$

|

$Good(a)$

| β rules for (8)

$\neg C(a)$ — $\neg T(a)$ β rules for 5

β rules (2)

$\neg B(a) \wedge \neg G(a)$ (9) — $C(a)$ ✗

α rules for (9)

|

$\neg G(a)$

✗ $\neg B(a)$

β rules

$\neg(C(a) \wedge G(a))$ (9) — $\neg L(a)$

✗ $\neg C(a)$ — $\neg Good(a)$ (✗)

$T(a)$ ✗ — $L(a)$ (7) — $\neg C(a)$ ✗

$H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C$ has a closed semantic tablea

$\rightarrow C$ is a logical consequence of H_1, H_2, H_3, H_4

6. Proof method used -

H_1: Every coyote chases some roadrunner.

H_2: Every roadrunner who says „beep-beep" is smart.

H_3: No coyote catches any smart roadrunner.

H_4: Any coyote who chases some roadrunner but does not catch it is frustrated

C: If all roadrunners say „beep-beep", then all coyotes are frustrated

predicate symbols used:

$C: D \rightarrow \{T, F\}$ $C(x)$ is T if x is a coyote

$R: D \rightarrow \{T, F\}$ $R(x)$ is T if x is a roadrunner.

$B: D \rightarrow \{T, F\}$ $B(x)$ is T if it says „beep-beep"

$S: D \rightarrow \{T, F\}$ $S(x)$ is T if x is a smart roadrunner

$F: D \rightarrow \{T, F\}$ $F(x)$ is T if x is frustrated

$Chase: D \times D \rightarrow \{T, F\}$ $Chase(x, y)$ is T if x chases y

$Catch: D \times D \rightarrow \{T, F\}$ $Catch(x, y)$ is T if x catches y

f_1: $H_1: \forall x(C(x) \to \exists y(R(y) \wedge Chare(x,y)))$

f_2: $H_2: \forall x((R(x) \wedge B(x)) \to S(x)) \equiv \forall(x)(\neg R(x) \vee \neg B(x) \vee S(x))$

f_3: $H_3: \forall(x)\forall(y)((C(x) \wedge R(y) \wedge S(y)) \to \neg catch(x,y))$

f_4: $H_4: \forall(x)\forall(y)((C(x) \wedge R(y) \wedge Chare(x,y) \wedge \neg Catch(x,y)) \to F(x))$

$C: \forall(x)\forall(y)((R(y) \wedge B(y)) \to (C(x) \to F(x)))$

$\equiv \forall(x)(y)(R(y) \wedge B(y) \vee \neg C(x) \vee F(x))$

a → new const

f_1: $H_1: \neg C(x) \vee (R(a) \wedge Chare(x,a))$

f_2: $H_2: \neg R(a) \vee \neg B(a) \vee S(a)$

f_3: $H_3: \neg C(x) \vee \neg R(a) \vee \neg S(a) \vee \neg Catch(x,a)$

f_4: $H_4: \neg C(x) \vee \neg R(a) \vee \neg Chare(x,a) \vee Catch(x,a) \vee F(x)$

$\neg C$: $\cancel{(R(a) \to B(a)) \wedge (C(x) \wedge \neg F(x))}$

: $\cancel{\neg R(a) \vee B(a) \wedge C(x) \wedge \neg F(x)}$

f_5: $\vdash_{H_3 \vee H_4}$ $\neg C(x) \vee \neg R(a) \vee \neg S(a) \vee \neg Chare(x,a) \vee F(x)$

f_6: $\vdash_{H_1 \vee H_4}$ $\neg C(x) \vee Catch(x,a) \vee F(x)$

f_7: $\vdash_{f_5 \vee H_4}$ $\neg C(x) \vee F(x)$

f_8: $\vdash_{f_2 / mk}$ $R(x) \wedge B(x)$

f_9: $\vdash_{f_7 \vee f_8}$ $R(x) \wedge B(x) \vee \neg C(x) \vee F(x) \equiv C$

5.

H_1: Anyone whom Mary loves is a football star.

H_2: Any student who does not pass, does not play.

H_3: John is a student

H_4: Any student who does not ~~play~~ study, does not pass.

H_5: Anyone who does not play is not a football star.

6 (Conclusion): If John does not study, then Mary does not love John.

We transform the hypothesis and conclusion into predicate logic form using:

variables: x

constants: Mary, John

domain: D (all people)

predicate symbols:

loves : $D \times D \to \{T, F\}$ T if x loves y, "loves (x, y)"

student : $D \to \{T, F\}$ student (x) is T if x is a student

pass : $D \to \{T, F\}$ pass (x) is T if x passed

play : $D \to \{T, F\}$ play (x) is T if x plays

fs : $D \to \{T, F\}$ fs (x) is T if x is a football star

study : $D \to \{T, F\}$ study (x) is T if x studies

H_1: $\forall(x)\big(loves(Mary, x) \rightarrow fs(x)\big)$

H_2: $\forall(x)\big((student(x) \land \neg pass(x)) \rightarrow \neg play(x)\big)$

H_3: $Student(John)$

H_4: $\forall(x)\big((student(x) \land \neg study(x)) \rightarrow \neg pass(x)\big)$

H_5: $\forall x\big(\neg play(x) \rightarrow \neg fs(x)\big)$

$f_6 \vdash_{H_4 \lor H_2} \forall(x)\big(\neg student(x) \lor \cancel{pass(x)} \lor study(x) \lor \neg play(x)$

$f_7 \vdash_{H_5 \lor H_1} \forall(x)\big(play(x) \lor \neg loves(Mary, x)\big)$

$f_8 \vdash_{f_6 \lor f_7} \forall(x)\big(\neg student(x) \lor study(x) \lor \neg loves(Mary, x)\big)$

$f_9 \equiv f_8 \quad \forall(x)\big(student(x) \rightarrow \neg study(x) \rightarrow \neg loves(Mary, x)\big)$

$f_{10} \vdash_{\text{John}}^{\text{Univ-inst}} student(John) \rightarrow \neg study(John) \rightarrow \neg loves(Mary, John)$

$f_{11} \vdash_{\text{modus ponens}}^{f_{10},\ H_3} \neg study(John) \rightarrow \neg loves(Mary, John)$

$f_{11} \equiv C \Rightarrow$ The conclusion holds

17.

1. Everyone who is an ace at any final exam studies or is brilliant or lucky.

2. Everyone who makes an A is an ace at some final exam

3. No CS major is lucky

4. Anyone who drinks beer does not study

C: If every CS major makes an A, then every CS major who drinks beer is brilliant.

We transform the hypothesis and conclusions into predicate form using

variables: x, y domain: D

predicate symbols:

Ace: $D \times D \to \{T, F\}$ T if x aced on exam y

S: $D \to \{T, F\}$ T if x studies

B: $D \to \{T, F\}$ T if x is brilliant

L: $D \to \{T, F\}$ T if x is lucky

Makes: $D \to \{T, F\}$ T if x makes an A at some f. exam

Beer: $D \to \{T, F\}$ T if x drinks beer

$H_1: \forall(x)\forall(y)\big(Ace(x,y) \rightarrow (S(x) \vee B(x) \vee L(x))\big)$

$H_2: \forall x \exists y \big(Makes(x) \rightarrow Ace(x,y)\big)$

$\equiv \forall(x)\exists(y)\big(\neg Makes(x) \vee Ace(x,y)\big)$

$H_3: \forall(x)\big(Cs(x) \rightarrow \neg L(x)\big)$

$\equiv \forall(x)\big(\neg Cs(x) \vee \neg L(x)\big)$

$H_4: \forall(x)\big(Beer(x) \rightarrow \neg S(x)\big)$

$\equiv \forall(x)\big(\neg Beer(x) \vee \neg S(x)\big)$

$C: \forall(x)\forall(y)\big((Cs(x) \wedge Makes(x)) \rightarrow ((Cs(y) \wedge Beer(y)) \rightarrow B(y)\big)$

$\equiv \forall(x)\forall(y)\big(\neg Cs(x) \vee \neg Makes(x) \vee \neg Cs(y) \vee \neg Beer(y) \vee B(y)\big)$

$f_5 \xrightarrow{H_1 \vee H_3} \forall(x)\forall(y)\big(\neg Ace(x,y) \vee \neg Cs(x) \vee S(x) \vee B(x)\big)$

$f_6 \xrightarrow{f_5 \vee H_2} \forall(x)\forall(y)\big(\neg Makes(x) \vee \neg Cs(x) \vee S(x) \vee B(x)\big)$

$f_7 \xrightarrow{f_6 \vee H_4} \forall(x)\left(\neg Makes(x) \vee \neg Cs(x) \vee \neg Beer(x) \vee B(x)\right)$

$f_8 \equiv f_7 \forall(x)(Cs(x) \wedge Beer(x) \wedge Makes(x)) \to B(x)$

f_8: Every Cs major who drinks beer and makes an a is brilliant

=> counter example

a Cs major who drinks beer is brilliant just if he makes a ten, but the conclusion states that every Cs major who drinks beer is brilliant

=> conclusion does not hold !!!