



Circles

1. Find the equation of the circle:

- diameter $[A, B]$, with $A(1, 2)$ and $B(-3, -1)$,
- with center $(2, -3)$ and radius $R = 7$,
- with center $(-1, 2)$ and passing through $A(2, 6)$,
- centered at the origin and tangent to $\ell: 3x - 4y + 20 = 0$,
- passing through $A(3, 1)$ and $B(-1, 3)$ and having the center on the line $\ell: x - 3y - 2 = 0$,
- passing through $A(1, 1)$, $B(-1, -1)$ and $C(0, 2)$,
- tangent to both $\ell_1: x + y - 5 = 0$ and $\ell_2: 2x + y + 15 = 0$ if one tangency point is $M(3, -1)$.

$$\text{a) } [A, B] \text{ diameter} \Rightarrow \begin{cases} x_0 = \frac{x_A + x_B}{2} = -1 \\ y_0 = \frac{y_A + y_B}{2} = \frac{1}{2} \end{cases}$$

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{4^2 + 3^2} = 5 \Rightarrow R = \frac{AB}{2} = \frac{5}{2}$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$(x - 2)^2 + (y - \frac{1}{2})^2 = \frac{25}{4} \Leftrightarrow x^2 - 4x + 4 + y^2 - y + \frac{1}{4} = \frac{25}{4} \Leftrightarrow x^2 + 2x + y^2 - y = 11$$

$$\text{b) } (x - 2)^2 + (y - 3)^2 = r^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = r^2$$

$$x^2 - 4x + y^2 - 6y = r^2 - 14$$

$$\text{c) } R = 1 \text{ i.e. } \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$(x+1)^2 + (y-2)^2 = 25$$

$$x^2 + 2x + y^2 - 4y + 1 + 4 = 25 \Rightarrow x^2 + 2x + y^2 - 4y = 20$$

$$\text{d) } l: x + y - 5 = 0$$

$$d(0, l) = R = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$d(0, l) = \frac{5}{\sqrt{2}} = 5$$

$$x^2 + y^2 = 25$$

$$\text{e) } l: 2x + y - 5 = 0$$

$$(x - 2)^2 + (y - 1)^2 = r^2$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = r^2$$

$$x^2 - 4x + y^2 - 2y = r^2 - 5$$

$$x^2 - 4x + 4 + y^2 - 2y = 10 \Rightarrow x^2 - 4x + y^2 - 2y = 10$$

$$x^2 - 4x + 4 + y^2 - 2y = -10$$

$$\text{f) } l: x + 2y - 5 = 0$$

$$(x - 5)^2 + (y - 2.5)^2 = r^2$$

$$x^2 - 10x + 25 + y^2 - 5y + 6.25 = r^2$$

$$x^2 - 10x + y^2 - 5y = r^2 - 31.25$$

$$x^2 - 10x + 25 + y^2 - 5y = 31.25 \Rightarrow x^2 - 10x + y^2 - 5y = 16.25$$

$$x^2 - 10x + 25 + y^2 - 5y = 16.25 \Rightarrow x^2 - 10x + y^2 - 5y = -8.75$$

$$x^2 - 10x + 25 + y^2 - 5y = -8.75 \Rightarrow x^2 - 10x + y^2 - 5y = -34.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -34.25 \Rightarrow x^2 - 10x + y^2 - 5y = -69.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -69.25 \Rightarrow x^2 - 10x + y^2 - 5y = -94.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -94.25 \Rightarrow x^2 - 10x + y^2 - 5y = -119.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -119.25 \Rightarrow x^2 - 10x + y^2 - 5y = -144.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -144.25 \Rightarrow x^2 - 10x + y^2 - 5y = -169.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -169.25 \Rightarrow x^2 - 10x + y^2 - 5y = -184.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -184.25 \Rightarrow x^2 - 10x + y^2 - 5y = -209.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -209.25 \Rightarrow x^2 - 10x + y^2 - 5y = -234.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -234.25 \Rightarrow x^2 - 10x + y^2 - 5y = -264.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -264.25 \Rightarrow x^2 - 10x + y^2 - 5y = -294.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -294.25 \Rightarrow x^2 - 10x + y^2 - 5y = -324.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -324.25 \Rightarrow x^2 - 10x + y^2 - 5y = -354.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -354.25 \Rightarrow x^2 - 10x + y^2 - 5y = -384.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -384.25 \Rightarrow x^2 - 10x + y^2 - 5y = -414.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -414.25 \Rightarrow x^2 - 10x + y^2 - 5y = -444.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -444.25 \Rightarrow x^2 - 10x + y^2 - 5y = -474.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -474.25 \Rightarrow x^2 - 10x + y^2 - 5y = -504.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -504.25 \Rightarrow x^2 - 10x + y^2 - 5y = -534.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -534.25 \Rightarrow x^2 - 10x + y^2 - 5y = -564.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -564.25 \Rightarrow x^2 - 10x + y^2 - 5y = -604.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -604.25 \Rightarrow x^2 - 10x + y^2 - 5y = -634.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -634.25 \Rightarrow x^2 - 10x + y^2 - 5y = -664.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -664.25 \Rightarrow x^2 - 10x + y^2 - 5y = -704.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -704.25 \Rightarrow x^2 - 10x + y^2 - 5y = -734.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -734.25 \Rightarrow x^2 - 10x + y^2 - 5y = -764.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -764.25 \Rightarrow x^2 - 10x + y^2 - 5y = -794.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -794.25 \Rightarrow x^2 - 10x + y^2 - 5y = -824.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -824.25 \Rightarrow x^2 - 10x + y^2 - 5y = -854.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -854.25 \Rightarrow x^2 - 10x + y^2 - 5y = -884.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -884.25 \Rightarrow x^2 - 10x + y^2 - 5y = -914.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -914.25 \Rightarrow x^2 - 10x + y^2 - 5y = -944.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -944.25 \Rightarrow x^2 - 10x + y^2 - 5y = -974.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -974.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1004.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1004.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1034.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1034.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1064.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1064.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1104.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1104.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1134.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1134.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1164.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1164.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1194.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1194.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1224.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1224.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1254.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1254.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1284.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1284.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1314.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1314.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1344.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1344.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1374.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1374.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1404.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1404.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1434.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1434.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1464.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1464.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1494.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1494.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1524.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1524.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1554.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1554.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1584.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1584.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1614.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1614.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1644.25$$

$$x^2 - 10x + 25 + y^2 - 5y = -1644.25 \Rightarrow x^2 - 10x + y^2 - 5y = -1674.25$$