

Seminar 12

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message $m \in \mathbb{Z}_2^k$ encode vector $v \in \mathbb{Z}_2^n$ channel output vector $v' \in \mathbb{Z}_2^{n+k}$ error correction \downarrow decode v' correct \downarrow compare v' to the accepted output and choose the most likely candidate

(n, k) -code $\rightarrow (e_1, e_2, \dots, e_k) \in \mathbb{Z}_2^{n+k}$

If we have a linear code, i.e. if $\gamma: \mathbb{Z}_2^k \rightarrow \mathbb{Z}_2^n$ the encoder function, where

$$G = [e_1 | e_2 | \dots | e_k] = [\gamma(e_1) | \gamma(e_2) | \dots | \gamma(e_k)] = \begin{pmatrix} P \\ E \end{pmatrix}$$

generator matrix

$$H = \begin{pmatrix} I_{n-k} & P \end{pmatrix}$$

$v \in \mathbb{Z}_2^n$ is a codeword iff $H \cdot [v]_{E'} = 0$

Polynomial code

$$m = a_0 a_1 \dots a_{k-1} \rightarrow p = a_0 + a_1 x + \dots + a_{k-1} x^{k-1}$$

$$1) Q = p_m \cdot x^k$$

$$2) \text{Divide } Q \text{ by } p$$

$$Q = p \cdot q + r$$

$$3) \text{The encoded polynomial is } Q - R = Q + R = m$$

Ex 16.3 code generator

$$\text{by } P = 1 + x^2 + x^3$$

$$\text{Let us encode } m = 101$$

$$1) m = 101 \rightsquigarrow p = 1 + x^2$$

$$2) Q = p \cdot x^3 = x^3 + x^5$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^3 + x^5 & x^3 + x^2 + 1 \\ \hline x^4 + x^3 + x^2 & x^2 + x \\ x^4 + x^3 + x^2 & x^2 + x \\ x^4 + x^3 + x^2 & x^2 + x \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^2 + x$$

$$\text{The encoded polynomial is } Q + R = x^3 + x^5 + x^2 + x$$

$$\text{The encoded vector is } 011101$$

Determine the generator matrix and the parity check matrix for:

$$7. \text{ The } (4,1)\text{-code generated by } p = 1 + X + X^2 + X^3 \in \mathbb{Z}_2[X].$$

$$8. \text{ The } (7,3)\text{-code generated by } p = 1 + X^2 + X^3 + X^4 \in \mathbb{Z}_2[X].$$

$$e_1 = (1, 0, 0)$$

$$m = 100$$

$$1) m = 100 \rightsquigarrow p = 1$$

$$2) Q = p \cdot x^4 = x^4$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^4 & x^4 + x^3 + x^2 + 1 \\ \hline x^4 + x^3 + x^2 + 1 & x+1 \\ x^4 + x^3 + x^2 + 1 & x+1 \\ x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^3 + x^2 + 1$$

$$\text{The encoded polynomial is } Q + R = x^4 + x^3 + x^2 + x + 1$$

$$\text{The encoded message is } 110101$$

$$e_3 = (0, 0, 1)$$

$$1) m = 001 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_5 = (0, 1, 0)$$

$$1) m = 010 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_7 = (0, 0, 0)$$

$$1) m = 000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_9 = (0, 0, 0, 0)$$

$$1) m = 0000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_{11} = (0, 0, 0, 0, 0)$$

$$1) m = 00000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_{13} = (0, 0, 0, 0, 0, 0)$$

$$1) m = 000000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_{15} = (0, 0, 0, 0, 0, 0, 0)$$

$$1) m = 0000000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_{17} = (0, 0, 0, 0, 0, 0, 0, 0)$$

$$1) m = 00000000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x$$

$$4) \text{The encoded polynomial is } Q + R = x^6 + x^5 + x^4 + x^3 + x^2 + x$$

$$\text{The encoded message is } 011110$$

$$e_{19} = (0, 0, 0, 0, 0, 0, 0, 0, 0)$$

$$1) m = 000000000 \rightsquigarrow p = x^2$$

$$2) Q = p \cdot x^4 = x^6$$

$$3) \text{Divide } Q \text{ by } p$$

$$\begin{array}{r|rr} x^6 & x^6 + x^5 + x^4 + x^3 + x^2 + 1 \\ \hline x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ x^6 + x^5 + x^4 + x^3 + x^2 + 1 & x+1 \\ \hline x^2 + x & \end{array}$$

$$\Rightarrow R = x^5 + x^4 + x^3 + x^2 + x</math$$