Seminar 13

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$$Q: \underbrace{a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_{10}x + a_{01}y + a_{00}}_{= f(x,y)} = 0$$

$$Q: (x \;\; y)\underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}}_{M(Q)} \cdot \begin{pmatrix} x \\ y\end{pmatrix} + (a_{10} \;\; a_{01}) \cdot \begin{pmatrix} x \\ y\end{pmatrix} + a_{00} = 0$$

If we build an orthonormal basis B of eigenvectors, then if $B = (v_1, v_2)$

$$M_{E,B} = [id]_{B,E} = ([v_1]_E, [v_2]_E)$$

$$\underbrace{\begin{pmatrix} x \\ y\end{pmatrix}}_{\text{coordinates in initial basis } E} = M_{E,B} \cdot \underbrace{\begin{pmatrix} x_1 \\ y_1\end{pmatrix}}_{\text{coordinates in the basis } B \text{ of eigenvectors}}$$

$$Q: (x_1 \;\; y_1) \cdot \overbrace{M_{E,B}^t \cdot M(Q) \cdot M_{E,B}}^{\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix}} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + (a_{10} \;\; b_{01}) \cdot M_{E,B} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + a_{00} = 0$$

$$Q: \lambda_1 x_1^2 + \lambda_2 x_2^2 + \alpha x_1 + \beta y_1 + a_{00} = 0$$

We choose v_1, v_2 so that $M_{E,B} \in SO_2(\mathbb{R})$

The next step is a translation.

We force squares: $\lambda_1\left(x_1^2 + 2\frac{\alpha}{2\lambda_1}x_1 + \frac{\alpha^2}{4\lambda_1^2}\right) + \lambda_2\left(y_1^2 + 2\frac{\beta}{2\lambda_2}y_1 + \frac{\beta^2}{4\lambda_2^2}\right) + a_{00} - \frac{\alpha^2}{4\lambda_1^2} - \frac{\beta^2}{4\lambda_2^2} =$

$$\Leftrightarrow \lambda_1\left(x_1 + \frac{\alpha}{2\lambda_1}\right)^2 + \lambda_2\left(y_1 + \frac{\beta}{2\lambda_2}\right)^2 + h = 0$$

$$\begin{cases} x_2 = x + \frac{\alpha}{2\lambda_1} \\ y_2 = y_1 + \frac{\beta}{2\lambda_2}\end{cases}$$

$$\Rightarrow Q: \lambda_1 x_2^2 + \lambda_2 y_2^2 = -h$$

$$Q: \frac{x_2^2}{\frac{-h}{\lambda_1}} + \frac{y_2^2}{\frac{-h}{\lambda_2}} = 1$$

$$\begin{cases} x_3 = \frac{x_2}{\sqrt{\left|\frac{h}{\lambda_1}\right|}} \\ y_3 = \frac{y_2}{\sqrt{\left|\frac{h}{\lambda_2}\right|}}\end{cases}$$

4. For each of the following equations write down the associated matrix and bring the equation in canonical form.

a) $-x^2 + xy - y^2 = 0$,

b) $6xy + x - y = 0$.

b) $Q: 6xy + x - y = 0$

$$M(Q) = \begin{pmatrix} 0 & 3 \\ 3 & 0\end{pmatrix}$$

$\lambda_1 = 3, \; \lambda_2 = -3$

$$\begin{pmatrix} -3 & 3 \\ 3 & -3\end{pmatrix} \cdot \begin{pmatrix} x \\ y\end{pmatrix} = \begin{pmatrix} 0 \\ 0\end{pmatrix} \Rightarrow \{-3x + 3y = 0 \Leftrightarrow y = x \Rightarrow \text{So } S(\lambda_1) = \{(x, x) \mid x \in \mathbb{R}\} = \langle(1,1)\rangle$$

We chose $v_1 = \frac{1}{\sqrt{2}}(1,1)$

$S(\lambda_2) = \{(x, -x) \mid x \in \mathbb{R}\} = \langle(1,-1)\rangle$

We chose $v_2 = \frac{1}{\sqrt{2}}(1,-1)$

$B = (v_1, v_2) = \left(\frac{1}{\sqrt{2}}(1,1), \frac{1}{\sqrt{2}}(1,-1)\right)$ orthonormal basis of $\mathbb{R}^2$

$$M_{E,B} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1\end{pmatrix}$$

But $\det M_{E,B} = -1$. We want the transformation to be rotation

We chose $v_2 = \frac{1}{\sqrt{2}}(-1, 1)$

$$M_{E,B} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1\end{pmatrix} \in SO_2(\mathbb{R})$$

$Q: 6xy + x - y = 0$

$$(x \;\; y) \cdot \begin{pmatrix} 0 & 3 \\ 3 & 0\end{pmatrix}\begin{pmatrix} x \\ y\end{pmatrix} + (1 \;\; -1)\begin{pmatrix} x \\ y\end{pmatrix} = 0$$

$$Q: (x \;\; y) \cdot \overbrace{\frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1\end{pmatrix} \cdot \begin{pmatrix} 0 & 3 \\ 3 & 0\end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1\end{pmatrix}}^{A} \cdot \begin{pmatrix} x \\ y\end{pmatrix} + (1 \;\; -1) \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1\end{pmatrix}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} = 0$$

$$A = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1\end{pmatrix} \cdot \begin{pmatrix} 0 & 3 \\ 3 & 0\end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1\end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & -3\end{pmatrix}$$

$$(1 \;\; -1) \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1\end{pmatrix} = (0 \;\; -2)$$

$$Q: (x_1 \;\; y_1) \cdot \begin{pmatrix} 3 & 0 \\ 0 & -3\end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + \frac{1}{\sqrt{2}}(0 \;\; -2)\begin{pmatrix} x_1 \\ y_1\end{pmatrix} = 0 \Rightarrow$$

$$\Rightarrow Q: 3x_1^2 - 3y_1^2 - \sqrt{2}y_1 = 0$$

$$Q: 3x_1^2 - 3\left(y_1^2 + 2\frac{\sqrt{2}}{6}y_1 + \frac{1}{18}\right) = -\frac{1}{6}$$

$$3x_1^2 - 3\left(y_1 + \frac{\sqrt{2}}{6}\right)^2 = -\frac{1}{6}$$

$$\begin{cases} x_2 = x_1 \\ y_2 = y_1 + \frac{\sqrt{2}}{6}\end{cases}$$

$$Q: 3x_2^2 - 3y_2^2 = -\frac{1}{6}$$

$$-\frac{x_2^2}{\frac{1}{18}} + \frac{y_2^2}{\frac{1}{18}} = 1 \rightarrow \text{hyperbola that is } y \text{ oriented}$$

$$\begin{cases} x_3 = y_2 \\ y_3 = x_2\end{cases}$$

$$\frac{x_3^2}{-\frac{1}{18}} + \frac{y_3^2}{-\frac{1}{18}} = 1$$

Not necessarily req

$$x_4 = \frac{x_3}{\frac{1}{3\sqrt{2}}}, \; y_4 = \frac{y_3}{\frac{1}{3\sqrt{2}}}$$

$$\Rightarrow x_4^2 - y_4^2 = 1$$

5. In each of the following cases, decide the type of the quadratic curve based on the parameter $a \in \mathbb{R}$.

a) $x^2 - 4xy + y^2 = a$,

b) $x^2 + 4xy + y^2 = a$.

a) $x^2 - 4xy + y^2 - 4y - 2x + 1 = a$

$(x^2 - 4xy + 4y^2) - 3y^2 - 4y - 2x + 1 = a$

$(x - 2y)^2 - 3y^2 + 4y - 2x + 1 = a$

$x_1 = x - 2y \Leftrightarrow x = x_1 + 2y$

$Q: x_1^2 - 3y^2 - 2x_1 + 1 = a$

$x_1^2 - 2x_1 + 1 - 3y^2 = a$

$(x_1 - 1)^2 - 3y^2 = a$

$$\begin{cases} x_2 = x_1 - 1 \\ y_2 = y\end{cases}$$

$x_2^2 - 3y_2^2 = a$

- $a > 0$: hyperbola (x - aligned)
- $a = 0$: 2 lines, $x_2 = \pm\sqrt{3}y_2$
- $a < 0$: hyperbola (y - aligned)

Lagrange Method

7. Find the canonical equation for each of the following cases

a) $5x^2 + 4xy + 8y^2 - 32x - 56y + 80 = 0$,

b) $8y^2 + 6xy - 12x - 26y + 11 = 0$,

c) $x^2 - 4xy + y^2 - 6x + 2y + 1 = 0$.

a) $5x^2 + 2xy + 2yx + 8y^2 - 32x - 56y + 80 = 0$

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 8\end{pmatrix}$$

$a_{10} = -32$

$a_{01} = -56$

$$\begin{vmatrix} 5-\lambda & 2 \\ 2 & 8-\lambda\end{vmatrix} = 0 \Leftrightarrow (5-\lambda)(8-\lambda) - 4 = 0$$

$40 - 8\lambda - 5\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 13\lambda + 36 = 0$

$\Delta = 169 - 4 \cdot 36 = 25$

$\lambda_1 = \frac{13+5}{2} = 9$

$\lambda_2 = \frac{13-5}{2} = 4$

$$\lambda_1 = 9: \quad A - \lambda i = \begin{pmatrix} -4 & 2 \\ 2 & -1\end{pmatrix} \Rightarrow \begin{cases} -4x + 2y = 0 \\ 2x - y = 0\end{cases} \Rightarrow V(\lambda_1) = \langle(1,2)\rangle$$

We choose $V_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2\end{pmatrix}$

$$\lambda_2 = 4: \quad A - \lambda i = \begin{pmatrix} 1 & 2 \\ 2 & 4\end{pmatrix} \Rightarrow \begin{cases} x + 2y = 0 \\ 2x + 4y = 0\end{cases} \Rightarrow V(\lambda_2) = \langle(-2,1)\rangle$$

We choose $V_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 1\end{pmatrix}$

$$M_{E,B} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1\end{pmatrix}$$

$$Q: (x_1 \;\; y_1) \cdot \underbrace{M_{E,B}^t \cdot M(Q) \cdot M_{E,B}}_{\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix}}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + (a_{10} \;\; a_{01}) \cdot M_{E,B}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + a_{00} = 0$$

$$(x_1 \;\; y_1) \cdot \begin{pmatrix} 9 & 0 \\ 0 & 4\end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} - \frac{8}{\sqrt{5}}(4 \;\; 7) \cdot \begin{pmatrix} 1 & -2 \\ 2 & 1\end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + 80 = 0$$

$$9x_1^2 + 4y_1^2 - \frac{8}{\sqrt{5}}(18 \;\; -1)\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + 80 = 0$$

$$9x_1^2 + 4y_1^2 - \frac{144\sqrt{5}}{5}x_1 + \frac{8\sqrt{5}}{5}y_1 + 80 = 0$$

$$(3x_1)^2 - 2 \cdot \frac{\sqrt{5}}{5} \cdot 3x_1 + \frac{24^2}{5} + (2y_1)^2 + 2 \cdot \frac{2\sqrt{5}}{5} \cdot 2y_1 + \frac{4}{5} + 27 = 0$$

$$\left(3x_1 - \frac{24\sqrt{5}}{5}\right)^2 + \left(2y_1 - \frac{2\sqrt{5}}{5}\right)^2 = 36$$

$$\begin{cases} x_2 = 3x_1 - \frac{16\sqrt{5}}{5} \\ y_2 = 2y_1 - \frac{2\sqrt{5}}{5}\end{cases}$$

$x_2^2 + y_2^2 = 36 \quad | : 36$

$$\frac{x_2^2}{6^2} + \frac{y_2^2}{6^2} = 1$$

b) $8y^2 + 3xy + 3yx + 0 \cdot y^2 - 12x - 26y + 11 = 0$

$$A = \begin{pmatrix} 8 & 3 \\ 3 & 0\end{pmatrix}$$

$$\begin{vmatrix} 8-\lambda & 3 \\ 3 & -\lambda\end{vmatrix} = 0 \Leftrightarrow \lambda(\lambda - 8) - 9 = 0$$

$(\lambda - 4 + 4)(\lambda - 4 - 4) - 9 = 0$

$(\lambda - 4)^2 - 16 - 9 = 0$

$|\lambda - 4| = 5 \Rightarrow \lambda_1 = -1 \quad \nearrow$ We have a hyperbola

$\lambda_2 = 9$

$$\lambda_1 = -1: \quad A - 9\lambda = \begin{pmatrix} 9 & 3 \\ 3 & 1\end{pmatrix} \Rightarrow \begin{cases} 9x + 3y = 0 \\ 3x + y = 0\end{cases} \Rightarrow V(\lambda_1) = \langle(-1, 3)\rangle$$

We choose $V_1 = \frac{1}{\sqrt{10}}\begin{pmatrix} -1 \\ 3\end{pmatrix}$

$$\lambda_2 = 9: \quad A - 9\lambda = \begin{pmatrix} -1 & 3 \\ 3 & -9\end{pmatrix} \Rightarrow \begin{cases} -x + 3y = 0 \\ 3x - 9y = 0\end{cases} \Rightarrow V(\lambda_2) = \langle(3, 1)\rangle$$

We choose $V_2 = \frac{1}{\sqrt{10}}\begin{pmatrix} -3 \\ -1\end{pmatrix}$

$$M_{E,B} = \frac{1}{\sqrt{10}}\begin{pmatrix} -1 & -3 \\ 3 & -1\end{pmatrix}$$

$$Q: (x_1 \;\; y_1) \cdot M_{E,B}^t \cdot M(Q) \cdot M_{E,B} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + (a_{10} \;\; a_{01}) \cdot M_{E,B} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + a_{00} = 0$$

2. For each of the following matrices A, write down a quadratic equation with associated matrix A and find the matrix $M \in SO(2)$ which diagonalizes A.

a) $\begin{bmatrix} 6 & 2 \\ 2 & 9\end{bmatrix}$

b) $\begin{bmatrix} 5 & -13 \\ -15 & 5\end{bmatrix}$

c) $\begin{bmatrix} 7 & -2 \\ -2 & 5/3\end{bmatrix}$

a) $6x^2 + 4xy + 9y^2 = 1$

$$\begin{vmatrix} 6 & 2 \\ 2 & 9\end{vmatrix} = 16 \neq 0$$

$$\begin{vmatrix} 6-\lambda & 2 \\ 2 & 9-\lambda\end{vmatrix} = 0 \Leftrightarrow (6-\lambda)(9-\lambda) - 4 = 0$$

$54 - 9\lambda - 6\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 15\lambda + 50 = 0$

$\Delta = 225 - 200 = 25$

$\lambda_1 = \frac{15+5}{2} = 10$

$\lambda_2 = \frac{15-5}{2} = 5$

$\lambda = 5$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4\end{pmatrix} \cdot \begin{pmatrix} x \\ y\end{pmatrix} = \begin{pmatrix} 0 \\ 0\end{pmatrix} \Rightarrow \begin{cases} x + 2y = 0 \\ 2x + 4y = 0\end{cases} \Rightarrow S(\lambda) = \langle(-2, 1)\rangle$$

We choose $v_1 = \frac{1}{\sqrt{5}}(-2, 1)$

$\lambda = 10$

$$\begin{pmatrix} -4 & 2 \\ 2 & -1\end{pmatrix} \cdot \begin{pmatrix} x \\ y\end{pmatrix} = \begin{pmatrix} 0 \\ 0\end{pmatrix} \Rightarrow \begin{cases} -4x + 2y = 0 \\ 2x - y = 0\end{cases} \Rightarrow V(\lambda_2) = \langle(1, 2)\rangle$$

We choose $V_2 = \frac{1}{\sqrt{5}}(-1, -2)$

$$M_{E,B} = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 & -1 \\ 1 & -2\end{pmatrix}$$

$$Q: (x_1 \;\; y_1) \cdot \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix} + (a_{10} \;\; b_{01}) \cdot M_{E,B}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + a_{00} = 0$$

b) $$\begin{vmatrix} 5 & -13 \\ -15 & 5\end{vmatrix} = 25 + 13 \cdot 15 \neq 0$$

$$\begin{vmatrix} 5-\lambda & -13 \\ -15 & 5-\lambda\end{vmatrix} = (5-\lambda)^2 - 13 \cdot 15 = 0$$

$\lambda^2 - 10\lambda + 25 - 13 \cdot 15 = 0$

$\lambda^2 - 10\lambda - 170 = 0$

$\Delta = 100 + 4 \cdot 170 = 780$

$\lambda_1 = \frac{10 + \sqrt{780}}{2} = \frac{10 + 2\sqrt{195}}{2} = 5 + \sqrt{195}$

$\lambda_2 = 5 - \sqrt{195}$

$\lambda_1 = 5 + \sqrt{195}$

$$\begin{pmatrix} \sqrt{195} & -13 \\ -15 & \sqrt{195}\end{pmatrix} \cdot \begin{pmatrix} x \\ y\end{pmatrix} = \begin{pmatrix} 0 \\ 0\end{pmatrix}$$

$$\begin{cases} \sqrt{155}x - 13y = 0 \\ -15x + \sqrt{155}y = 0\end{cases} \Leftrightarrow \begin{cases} \sqrt{195}x - 13y = 0 \\ x = \sqrt{\frac{13}{15}}y\end{cases} \Leftrightarrow \begin{cases} 0 = 0 \end{cases} \Rightarrow V(\lambda_1) = \left\langle\left(\sqrt{\frac{13}{15}}, 1\right)\right\rangle$$

$x = \sqrt{\frac{13}{15}}a$

$y = a$

We choose $V_1 = \frac{\sqrt{35}}{5}\begin{pmatrix} \sqrt{\frac{13}{15}} \\ 1\end{pmatrix}$

$$Q: a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_{10}x + a_{01}y + a_{00} = 0$$

Step 1: $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22}\end{pmatrix}$

Step 2: Find eigenvalues

Step 3: Find eigenvectors (normalized)

$M_{E,B} = (v_1 \;\; v_2)$

Step 4: $$Q: (x_1 \;\; y_1) \cdot \underbrace{M_{E,B}^t \cdot A \cdot M_{E,B}}_{\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix}}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + (a_{10} \;\; a_{01}) M_{E,B}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + a_{0,0} = 0$$

$$(x_1 \;\; y_1)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1\end{pmatrix} + (a_{10} \;\; a_{01}) M_{E,B}\begin{pmatrix} x_1 \\ y_1\end{pmatrix} + a_{00} = 0$$

$$\lambda_1 x^2 + \lambda_2 y_1^2 + \alpha x_1 + \beta y_1 + a_{00} = 0$$

Step 5: Force squares

$$\lambda_1\left(x_1^2 + 2 \cdot \frac{\alpha}{2\lambda_1}x_1 + \left(\frac{\alpha}{2\lambda_1}\right)^2\right) + \lambda_2\left(y_1^2 + 2 \cdot \frac{\beta}{2\lambda_2}y_1 + \left(\frac{\beta}{2\lambda_2}\right)^2\right) + h$$

$$h = a_{00} - \frac{\alpha^2}{4\lambda_1} - \frac{\beta^2}{4\lambda_2}$$

Step 5 $$\begin{cases} x_2 = x_1 + \frac{\alpha}{2\lambda_1} \\ y_2 = y_1 + \frac{\alpha}{2\lambda_2}\end{cases}$$

$$\lambda_1 x_2^2 + \lambda_2 y_2^2 = -h$$

$$\frac{x_2^2}{\frac{1}{\lambda_1}} + \frac{y_2^2}{\frac{1}{\lambda_2}} = -h \quad \Big| \cdot \frac{1}{-h}$$

$$\frac{x_2^2}{-\frac{h}{\lambda_1}} + \frac{y_2^2}{\frac{-h}{\lambda_2}} = 1$$

6. Consider the rotation $R_{90°}$ of $\mathbb{E}^2$ around the origin and the translation $T_{\mathbf{v}}$ of $\mathbb{E}^2$ with vector $\mathbf{v}(1,0)$.

a) Give the algebraic form of the isometries $R_{90°}$, $T_{\mathbf{v}}$ and $T_{\mathbf{v}} \circ R_{90°}$.

b) Determine the equations of the hyperbola $\mathcal{H}: \frac{x^2}{4} - \frac{y^2}{9} - 1 = 0$ and the parabola $\mathcal{P}: y^2 - 8x = 0$ after transforming them with $R_{90°}$ and with $T_{\mathbf{v}} \circ R_{90°}$ respectively.

$$R_{90}(P) = \begin{pmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90°\end{pmatrix} \cdot [P] = \begin{pmatrix} 0 & -1 \\ 1 & 0\end{pmatrix}[P] = \begin{pmatrix} 0 & -1 \\ 1 & 0\end{pmatrix}\begin{pmatrix} x_P \\ y_P\end{pmatrix} = \begin{pmatrix} -y_P \\ x_P\end{pmatrix}$$

$$T_v(P) = \begin{pmatrix} x_P \\ y_P\end{pmatrix} + \begin{pmatrix} 1 \\ 0\end{pmatrix} = \begin{pmatrix} x_P + 1 \\ y_P\end{pmatrix}$$

$$T_v \circ R_{90°} = T_v\begin{pmatrix} -y_P \\ x_P\end{pmatrix} = \begin{pmatrix} -y_P + 1 \\ x_P\end{pmatrix}$$

b) $$\left.\begin{aligned} &\frac{x^2}{4} - \frac{y^2}{9} - 1 = 0 \\ &R_{90°} = \begin{pmatrix} -y_P \\ x_P\end{pmatrix}\end{aligned}\right\} \Rightarrow \frac{y'^2}{4} - \frac{x'^2}{9} - 1 = 0$$

$$-\frac{x'^2}{9} + \frac{y'^2}{4}$$

$$T_v \circ R_{90°}(P) = \begin{pmatrix} -y_P + 1 \\ x_P\end{pmatrix}$$

$$\begin{cases} x' = -y + 1 \\ y' = x\end{cases} \Leftrightarrow \begin{cases} x = y' \\ y = 1 - x'\end{cases}$$

$$\frac{y'^2}{4} - \frac{(1 - x')^2}{9} - 1 = 0 \quad | \cdot 36$$

$$9y'^2 - 4(x'^2 - 2x' + 1) - 36 = 0$$

$$-4x'^2 + 9y'^2 + 8x' - 40 = 0$$

10. Using the classification of quadrics, decide what surfaces are described by the following equations.

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$,

b) $xy + yz + zx = 1$,

c) $x^2 + xy + yz + zx = 1$,

d) $xy + yz + zx = 0$.

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$

①

$$\begin{array}{c|ccc} & x & y & z \\ \hline x & 1 & \frac{1}{2} & \frac{1}{2} \\ y & \frac{1}{2} & 2 & \frac{1}{2} \\ z & \frac{1}{2} & \frac{1}{2} & 1 \end{array}$$

② Find eigenValues

$$Q: (x \;\; y \;\; z) \cdot M_{E,B}^t \cdot M(Q) \cdot M_{E,B}\begin{pmatrix} x \\ y \\ z\end{pmatrix} + (0 \;\; 0 \;\; 0) \cdot M_{E,B}\begin{pmatrix} x \\ y \\ z\end{pmatrix} - 1 = 0$$

b) $$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}$$