Modelling of opinion formation process on different social networks

Mateusz Borowiec[1], Tomasz Gwizdałła[1[0000−0002−3981−6037]], and Paweł Maślanka[1[0000−0002−6456−7423]]

Faculty of Physics and Applied Informatics, University of Łódź, Pomorska 149/153, 90-236 Łódź, Poland norbert.borowiec@edu.nui.lodz.pl, tomasz.gwizdalla, pawel.maslanka@uni.lodz.pl

Abstract. Different processes related to the phenomena of interaction across communities are of particular interest, especially since the beginning of the current century. It concerns such processes as disease spreading, opinion distribution, and formation or recommendation systems. The last two are of special interest in the rapid development of social networks. In our paper we will present the proposition of the update scheme intended for the opinion in continuous space. We show it for the 2D opinion space following Nolan's idea of multidimensional opinion presentation. We implement our mechanism for different types of networks and different parameters. The paper shows that similarities exist between the networks with longer tails of node degree distribution, and we can observe the emergence of two groups around two different points in the space.

Keywords: opinion formation · scale-free networks · social networks

1 Introduction

The history of studying the process of opinion formation using numeric models consists of history related to the two different problems. The first is the description of the community, while the second is the definition of opinion and the rule that makes it possible to change it.

Regarding the first topic, we must emphasize that the image of community is completely different today compared to the pioneering times of computing. While today, online social networks create a magnificent area to study human behavior, when coming to the 50s or 60s, we have to mention the lack of information from sociological sources. The first, revolutionary in some sense approach to the process of society modeling came from the late 50's [9] and one of the most famous experiments showing the range and possibilities of interpersonal contacts (I consider here the famous Milgram "small world" experiment was described almost ten years later. Even when considering that the original Erdos-Renyi model seems unrealistic compared to some contemporary models, which commonly with the Erdos-Renyi one, will be studied in this paper, the original one is still of great importance.

When considering the attempts to describe the interaction between the individuals, we have to go back to the 70s and the Clifford model of spatial conflict with Glauber update [8]. Since then, a lot of different approaches have been studied: the analytical model of Galam [11], the Voters model, where the opinion is spread from the influencer and not imposed on individual [20]. The group of papers dealing with outward opinion dissemination also consists of papers related to the Sznajd-Weron model and its extensions [?][15][27]. Many works also use Cellular Automata as the playground for opinion formation [19].

Some different types of approaches, mainly taking into account swarm-based updates also related to modified logistic functions, were presented in our earlier papers [16] [17].

2 Model

2.1 Basics

In this subsection, we are going to show the crucial assumptions of our calculations. It contains two parts: information about the most popular graphs used in the simulation of communities and information on how the opinions of individuals are determined.

Three social network models will be used in our paper:

- Erdös-Rényi graph
- Watts-Strogatz graph
- Barabasi-Albert graph

The Erdös-Rényi is the most classical, known for more than 60 years model, called usually "Random graph" and introduced by authors in 1959 [9]. Since the two approaches to this graph exist, we must emphasize using the more frequent $G(N, p)$ approach. According to this model, the edge between nodes (connection between individuals) exists for every pair of nodes independently and with the probability p. N certainly means the number of nodes.

It is interesting that a great comeback of social networks took place at the turn of the 20th and 21st centuries. This resulted in the creation of two of the most famous models. There are Watts-Strogatz and Barabasi-Albert ones. Both these models have different backgrounds and different final properties but both are often used in the study of different complex systems related to the analysis of communities behavior.

The basic assumption underlying the Watts-Strogatz model [28][25] was maintaining each node's average number of neighbors. When defining the WS model $G(N, K, \beta)$, we initially connect every node to the even number of K of its neighbors. For simplicity of visualization, this phase is usually presented as a connection of nodes distributed along the ring with $K/2$ to its left nearest neighbors and to $K/2$ of right nearest neighbors. Then, the procedure of rewiring starts. For every node, $K/2$ of its left or right edges is rewired with probability β to an arbitrarily selected node. The only restriction is that we cannot duplicate connections, and we cannot create self-loops.

The Barabasi-Albert approach [3][1] is usually called the preferential one. This term comes from the fact that, for the basic model, we do not make any assumptions about its parameters. We start from the three nodes, creating a complete graph. Then, successive nodes are added to the structure, and edges between the added node and the existing one are created with the probability:

$$P(k_i) = \frac{k_i}{\sum_j k_j}. \tag{1}$$

In the formula above, k_i is the number of neighbors of $i-th$ node, and the sum is taken over all nodes.

From the point of view of simulations performed on all of the above-presented graphs, especially two values or functions are crucial:

- the distribution of node's degrees
- the clustering coefficient

The distribution of degrees is self-explanatory, however we would like to emphasize the role of this parameter in the process of information passing. It is obvious that by increasing the number of neighbors we increase the influence of particular node as well as its susceptibility to the opinions imposed by them. The closer the node degree values are to each other, the more similar the behavior of the individuals. On the other hand, the strong asymmetry of distribution leads to the existence of individuals which are better "interfaces" in the providing feedback.

type of graph	node's degree distribution	clustering coefficient
Erdös-Rényi $G(N,p)$	binomial distribution $p(k) = (\frac{N-1}{k})p^k(1-p)^{N-1-k}$	$C = p$
Watts-Strogatz $G(N,K,\beta)$ [5]	$p(k) = \sum_{n=0}^{min(k-K/2,K/2)}(\frac{K/2}{n})$ $\times(1-\beta)^n \beta^{K/2-n} e^{\beta K/2^n}$ $\times \frac{\beta K/2^{k=K/2-n}}{(k-K/2-n)!}$	$C = \frac{3(K-2)}{4(K-1)}(1-\beta)^2$
Barabasi-Albert [4]	$p(k) \propto k^{-3}$	$\frac{(lnN)^2}{N}$

Table 1: The properties of three types of graphs corresponding to the three models used across the paper.

The formulas describing the values of parameters for all three types of plots presented above are summarized in table ??. The comparison shows visible similarities between the first two models and the differences when considering the third one. For the ER and WS models, the distribution of node's degrees has a visible maximum, whose position is defined by the parameters p or K, respectively. The width of the distribution for the ER model is given by the properties of the binomial distribution, while the WS model increases with increasing β. On the contrary, the BA model expresses the exponential decrease of the plot with the maximum, only slightly higher than for smaller values, existing for the very low degree (usually less than 10). Interestingly, WS, as well as BA models, lead to the emergence of so-called hubs - the nodes/individuals with an extremely high number of neighbors. Because this fact is connected to the distances between nodes (usually expressed as graph's radii and diameters) it has been considered, with variable success, to model the famous Milgram experiment, [24], or the similar problems. This concerns Watts-Strogatz [22], [18], [23] as well as Barabasi-Albert model [14][12].

The individual's opinion is described within the frame of the "Nolan chart" concept. The scientific community does not often use the name, but some submissions use it in the mentioned form [16] [2] [10] [13]. Quite often the similar model is called simply "twodimensional" or even "multidimensional" [7] [26] [6] [21]. The Nolan chart's crucial property is to present individuals' opinions on two different issues on perpendicular axes and limit the possible value on every issue to a scaled interval $[-1, 1]$. According to the original Nolan's model, two axes correspond to "personal freedom" and "economic freedom". This approach allows an attempt to define several typical political attitudes, such as liberal or conservative, as certain areas on the diagram. The idea can certainly be easily adapted to an arbitrary number of opinions concerning arbitrary topics. This leads to a simple generalization of Nolan's chart to an arbitrary number of dimensions, forming the hypercube as a space available for opinions.

2.2 The details of approach

When introducing the model used throughout the paper, it's crucial to focus on two key aspects: the features of individuals and the update scheme. Each individual is characterized by a specific set of data, which we will delve into.

- The twodimensional vector corresponding to the node's position on Nolan's diagram. It is clear that both numbers should be float and taken from the $[0, 1]$ interval. In fact, the original values are drawn from this range. The presence of safeguards preventing crossing the limits of this range depends on the method of the update. It means that, although it is not correct and even not sensible, we enable greater values and this, as it will be seen in the section devoted to the presentation of results, can lead to quite interesting effects.
- Influence. This parameter, constant through the whole simulation and sampled uniformly from the interval $[0, 1]$ corresponds to the power to influence the opinion of other individuals.
- Susceptibilty. The parameter, also constant through the simulation and sampled uniformly from the interval $[0, 1]$, describes the sensitivity of individuals to the social pressure exerted by neighbors.

In the simplest case, we can assume that both factors, influence and susceptibility, correlate or even equal one to another. In the presented study, we distinguish them for every individual, so there is no connection between them.

We use two update schemes in the proposed paper, which are different from the most popular ones. For both of them, the crucial information is the direction of motion of the individual in the space given by Nolan's diagram. The direction is given as the vector between the individual's current position and the average of positions of all nearest neighbors of this individual. The position of the point selected as a potential new opinion is chosen according to the exponential distribution given by the distance between the current position and the average of neighbors multiplied by the sum of neighbors' influences, its own susceptibility, and some general parameter defined for a particular type of update and constant through the whole simulation. On the line which direction is described earlier, the new opinion is determined according to one of the following schemes:

1. The point is chosen from the exponential distribution between the minimal coordinate and the maximal coordinate. The old point will be conserved if the new value exceeds the interval limits.
2. The new point is chosen with the exponential distribution starting from the old position.

As can be observed, the first model preserves the correctness of the new choice, while the second one does not. This leads to some different behaviors and makes interesting intervals of parameters for both updates significantly different.

3 Results

The main goal of the simulation was to analyze the dynamics of the opinion-forming process. We can either study the behavior of single individuals or try to determine the characteristics of an entire population. In most models that are used to study the opinion formation process, the final result is the unanimity of the entire population. The question is usually about the time and characteristics of reaching this state. Therefore, to show the model's general properties, we concentrate on the global characteristics, particularly the speed of going to unanimity, by presenting the dependence of the sum of squared distances between all nodes and their center of mass.

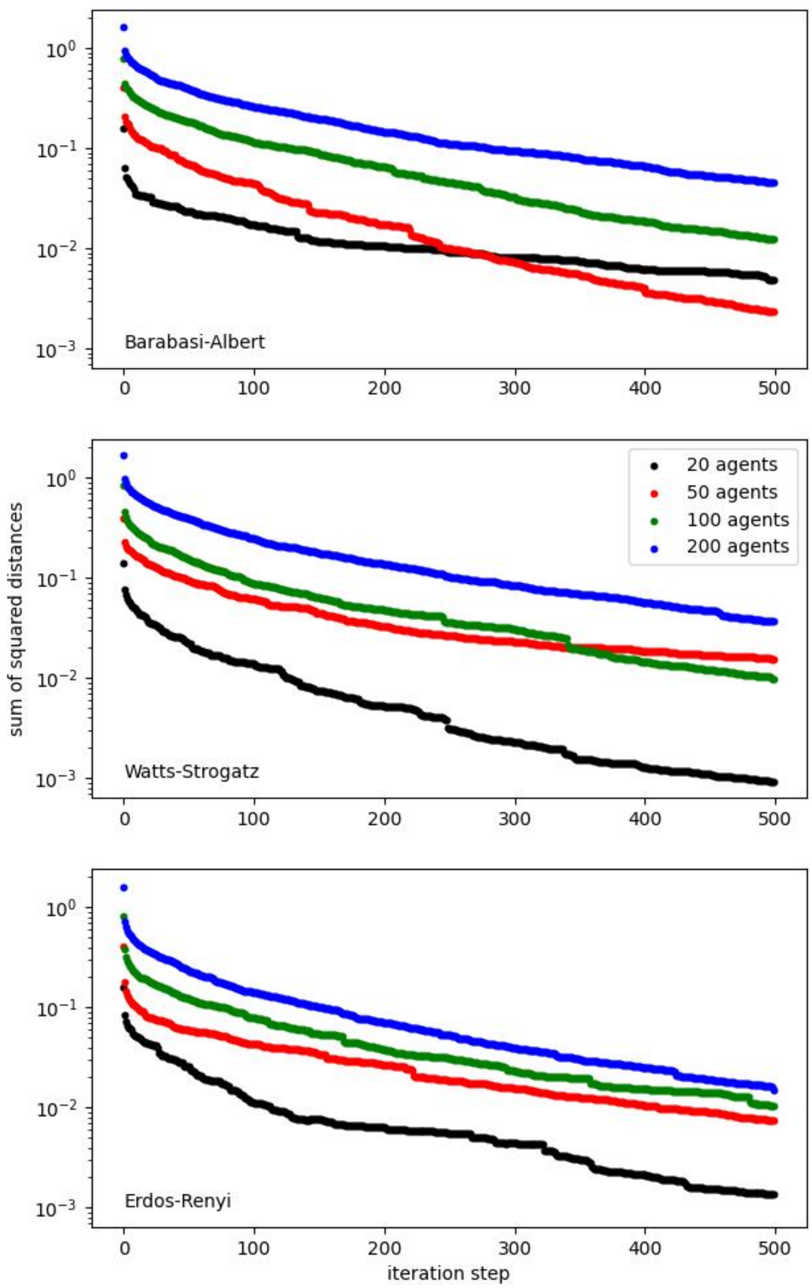

Fig. 1: The decrease of the average distance from the mean position in the function of time of simulation for the first type of update.

The simulation was made for three networks, two methods of update and the sets of parameters characteristic for these updates. As was written earlier, the three network models are the Barabasi-Albert (BA), Watts-Strogatz (WS), and Erdos-Renyi (ER) ones. The only additional information we have to provide is that the probability of rewiring is for the Watts-Strogatz network assumed to be equal $\beta = 0.3$. The updates are described simply as the first and the second one, but it must be remembered that the first protects the system against going beyond the limits of the Nolan diagram space, and the second does not. An interesting observation was made for the update parameters. From the preliminary tests, we obtained that the value area for the first update is two or even three orders of magnitude larger than that for the second one. We certainly do not need to analyze the absolute values of this parameter because it describes the speed of movement in the solution space and, therefore, the tendency to change opinions, and we can take any values. Finally, we adopted the set $\{200, 500, 1000\}$ for the first update as the set of parameter values. For the second update, it is $\{5, 10\}$. All calculations are made for different graph sizes: $\{20, 50, 100, 200\}$. These values are indeed relatively small, but they allow for the most essential characteristics of the process to be shown.

For every set of parameters we perform 5 simulations, 500 simulation steps each. The simulation step means the synchronized update of the positions of all individuals according to the assumed rule.

In Fig.1, the results for the update #1 are presented. All plots are obtained after averaging over five runs with the model parameter equal 500. Successive plots correspond to different network models, while colors are related to the size of the graph. The image for this update does not show any anomalies or deviations from the general image. For every set of parameters, the sum of square distance decreases. The decline is monotonic without any abrupt changes. The only debatable issue is the difference in slope value for some particular cases (20 agents for the BA model, 50 agents for the WS model), which, following the tendency, decrease significantly slower. We will discuss this problem later.

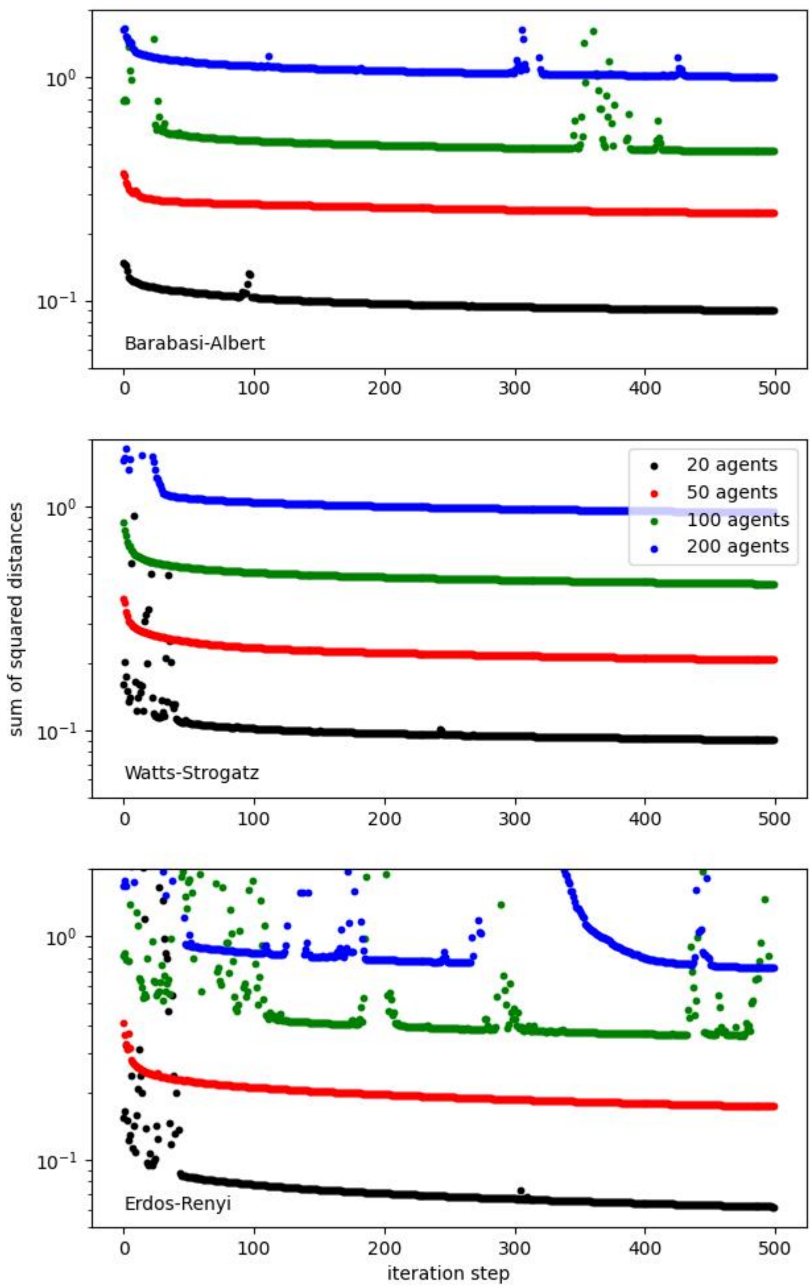

Fig. 2: The decrease of average distance from the mean position in the function of time of simulation for the second type of update..

The situation for the second update is much more interesting, shown in Fig.2. The plots were obtained for model parameters equal to 5 and, as earlier, averaged over five runs. We must mention that for parameter equal 10 the majority of runs lead to the dispersal of individuals beyond the acceptable area. Only one run led to such a situation for the presented case, but we must mention this problem. Generally, for this update, we can see several previously unobservable effects. The decrease of the squared distance value sum is much slower than for the first model. It certainly exists but seems to reach in 500 steps the value obtained for update#1 in the first ten steps. The more interesting is that sometimes the system is brought out of a state of stable development. This process, observed as a sudden increase in distance, can be observed for every graph size and different network models, so we can say that this is the effect of the update scheme. The important fact is that the system returns to the previously observed relationship, which is best visible in the graph for the ER model and 200 agents in the fourth century of simulation steps.

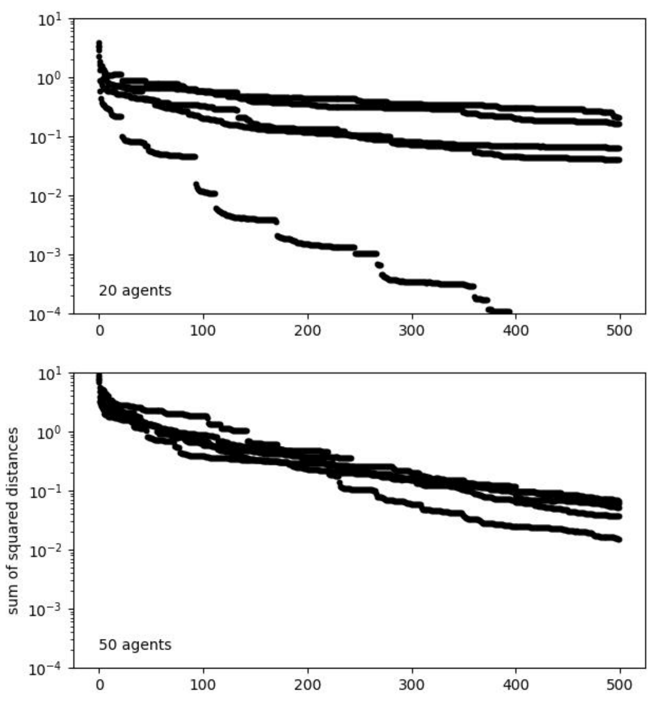

Fig. 3: The comparison of individual runs averaged in Fig.1.

In Fig.3, we can observe the source of differences shown earlier in Fig.1. Both plots are prepared for the Barabasi-Albert model. One can confirm the expected effect that the results are more stable for a higher number of nodes, and all runs lead to more similar results. On the other hand, the plots for 20 nodes lead to two conclusions, one of which is quite surprising. It can be expected that the spread of curves will be higher, and the single result can impact the average more strongly than for larger networks. The unexpected effect is that most runs lead to results going down much slower than for graphs with more nodes. Such expectations come clearly from the Fig.1. The only rapid descent comes from the single run, and the decrease here is extremely strong compared to other cases. This can be easily compared when considering that horizontal axes have the same range on both plots.

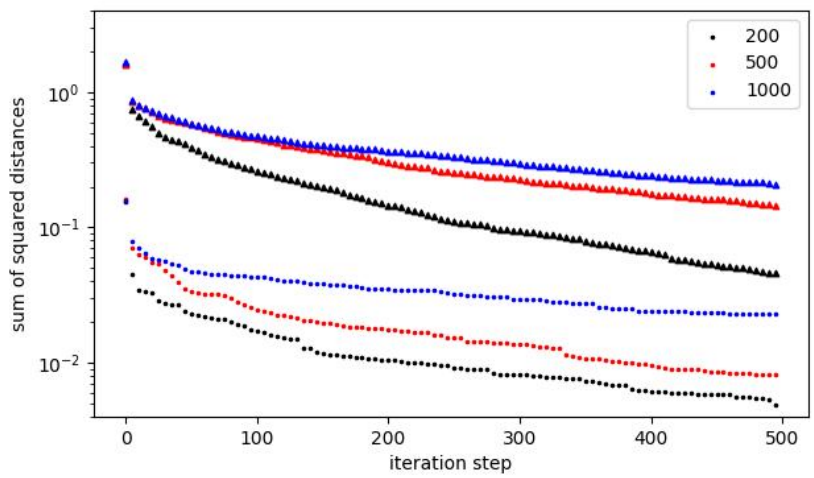

Fig. 4: The comparison of parameter dependence for the Barabasi-Albert model and two network sizes (20 nodes - lower bunch of curves, 200 nodes - the upper one). The values of the parameter are shown in the legend.

We must answer the question about the dependence of results on the update parameter. Since more successfully finished calculations we made for the first update technique, the results shown in Fig.4 are also presented for this update. We can notice the flattening of the decrease process with increasing value. The effect is monotonic and reproducible for all sets of simulation parameters applied.

Finally, we can compare curves for different network models in Fig.5. As it was done earlier, two sizes are considered: 20 nodes and 200 nodes. For every graph, the bunch for lower size is visible below the one for greater size. Since all curves are visible in earlier plots, we have to pay attention to some common behavior observed for different models. The analysis of the figure shows that, independent of the type of update, value of update parameter and graph size, the Barabasi-Albert model behaves similarly to the Watts-Strogatz one and Erdos-Renyi model presents different behavior. It is a little surprising since the WS model's origin is closer to the ER one than to BA, but it seems that the observed effect is the result of the presence of hubs. These nodes with a high number of neighbors appear in the WS model.

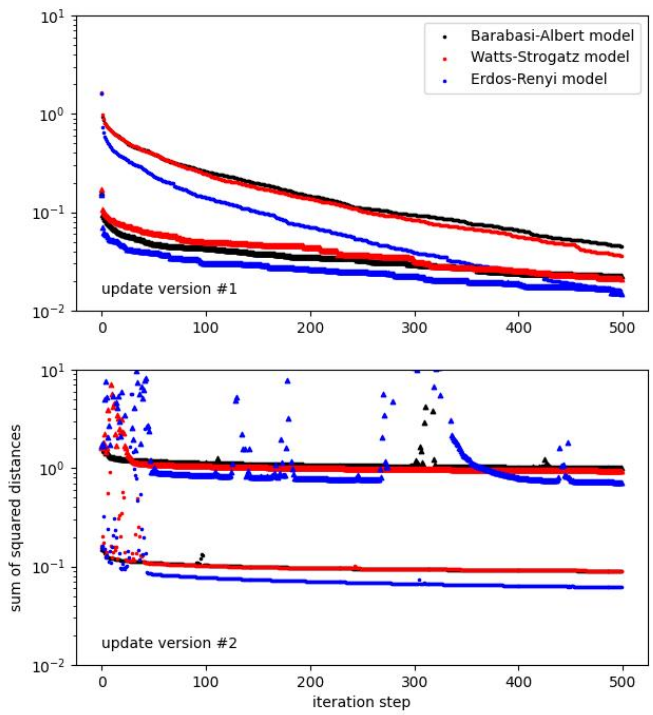

Fig. 5: The averaged runs for different models and parameters.

The difference between the two models, similar in behavior, becomes visible when we plot the simulation snapshot. In Fig.6 we can see the comparison of two selected runs for Barabasi-Albert and Watts-Strogatz networks. Although the right image shows the typical slow convergence of individuals to a common point, in the left plot, some different pattern is created. In the figure, we can see that there are two centers, instead of one, that are common to all points.

Indeed, they are close one to another but the possibility to distinguish both of then is obvious. It shows that by manipulating individual and global parameters of simulation, we can reproduce the situation with more than one opinion existing in the community.

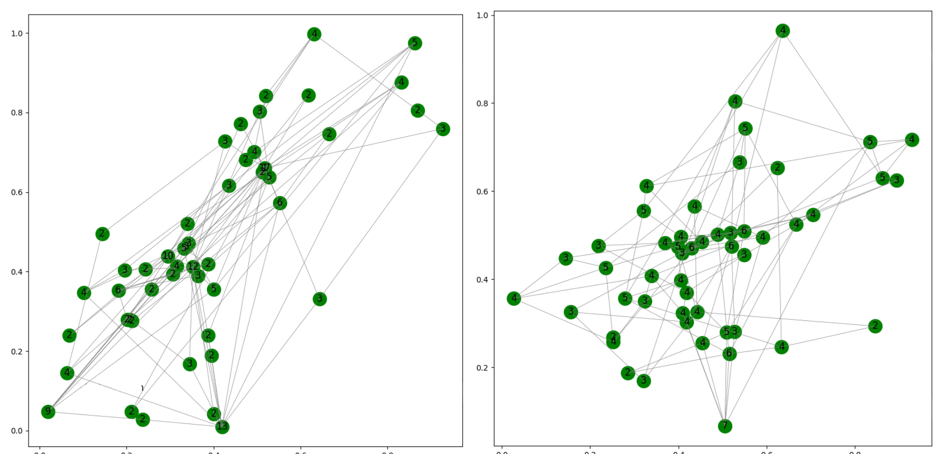

Fig. 6: Creation of two clusters for Barabasi-Albert model (left) as compared to the Watts-Strogatz one (right).

4 Conclusions

In the paper, we present the first test of some new class of opinion update scheme, applied to different network models corresponding usually to the social networks of various sizes and local and global parameters. From the results obtained, we can deduce the main directions in which further research should be conducted. The calculations are performed for relatively low graph sizes. But even such small sizes enable us to formulate some conclusions.

The presence of hubs is probably the reason for the more similar behavior of Barabasi-Albert and Watts-Strogatz models despite the similarity of node degree distributions for WS and ER models. We can also observe the emergence of clusters concentrated around some common opinion.

When discussing future, we can indicate some interesting problems which presentation goes beyond the scope of this short, introductory paper. First, more attention should be paid to the individuals' features. Here, some issues need discussion. Are the features related to someone's susceptibility constant in time? We know from different sociological news that people are usually conservative in their opinions and are rarely ready to change them. How should this effect influence the value of individual parameters? Another question concerns the connection between the number of neighbors and individual parameters. Usually, hubs are more convinced of their opinions and have a greater influence on their surroundings. Nowadays, influencers are an excellent example of such a phenomenon. Maybe some personal factors, like relationships between people, also have to be included.

We can also take into account some properties related to the geometry of the Nolan's diagram as the opinion space. I mean mainly the determination of the motion direction from the weighted average or the increase of dimensionality.

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