Name
Date
Course

Polynomials (Part III): Long Division and Synthetic Division

Review

Perform the following division.

① $(5x^3+5-20x)\div(-2+3x+x^2)$

$x^2+3x-2 \;\overline{)\; 5x^3 \quad -20x+5}$ $\quad 5x$

$5x^3+15x^2-10x$

$-15x^2-10x+5$

$-15x^2-45x+30$

$35x-25$

$5x-15+\dfrac{35x-25}{x^2+3x-2}$

② $(6x^5-9x^6+8)\div(3x^3+3)$

$3x^3+3 \;\overline{)\; -9x^6+6x^5 \quad +8}$ $\quad -3x^2$

$-9x^6 \quad -9x^3$

$6x^5+9x^3 \quad +8$

$6x^5 \quad +6x^2$

$9x^3-6x^2+8$

$9x^3 \quad +9$

$-6x^2-1$

$-3x^3+2x^2+3+\dfrac{-6x^2-1}{3x^3+3}$

$-3x^3+2x^2+3-\dfrac{6x^2+1}{3x^3+3}$

③ $(15+10x+3x^2-2x^4)\div(x+1)$

$x+1 \;\overline{)\; -2x^4+3x^2+10x+15}$ $\quad -2x^3+2x^2+x+9$

$-2x^4-2x^3$

$2x^3+3x^2+10x+15$

$2x^3+2x^2$

$x^2+10x+15$

x^2+x

$9x+15$

$9x+9$

6

$-2x^3+2x^2+x+9+\dfrac{6}{x+1}$

④ $(10-x^5-8x^2)\div(-5+x^2)$

$x^2-5 \;\overline{)\; -x^5-8x^2+10}$ $\quad -x^3$

$-x^5+5x^3$

$-5x^3-8x^2 \quad +10$

$-5x^3 \quad +25x$

$-8x^2-25x+10$

$-8x^2 \quad +40$

$-25x-30$

$-x^3-5x-8+\dfrac{-25x-30}{x^2-5}$

$-x^3-5x-8-\dfrac{25x+30}{x^2-5}$

More Long Division Problems

⑤ $(-4x^3+x^2+4x)\div(x-2)$

$x-2$ ⟌ $-4x^2-7x-10$

$-4x^3+x^2+4x$

$-4x^3+8x^2$

$-7x^2+4x$

$-7x^2+14x$

$-10x$

$-10x+20$

-20

$-4x^2-7x-10-\frac{20}{x-2}$

⑥ $(x^3-4x^2+5x)\div(x^2+4)$

x^2+4 ⟌ $x-4$

x^3-4x^2+5x

x^3+4x

$-4x^2+x$

$-4x^2-16$

$x+16$

$x-4+\frac{x+16}{x^2+4}$

⑦ $(x^3+6)\div(x-4)$

$x-4$ ⟌ $x^2+4x+16$

x^3+6

x^3-4x^2

$4x^2+6$

$4x^2-16x$

$16x+6$

$16x-64$

70

$x^2+4x+16+\frac{70}{x-4}$

⑧ $(4x^4-35)\div(x+1)$

$x+1$ ⟌ $4x^3-4x^2+4x-4$

$4x^4-35$

$4x^4+4x^3$

$-4x^3-35$

$-4x^3-4x^2$

$4x^2-35$

$4x^2+4x$

$-4x-35$

$-4x-4$

-31

$4x^3-4x^2+4x-4-\frac{31}{x+1}$

⑨ $(-2x^3) \div (x+5)$

$x+5 \;\big|\; -2x^3$

$-2x^3 - 10x^2$

$10x^2$

$10x^2 + 50x$

$-50x$

$-50x - 250$

250

$-2x^2 + 10x - 50$

$-2x^2 + 10x - 50 + \frac{250}{x+5}$

⑩ $(x^4) \div (x-3)$

$x-3 \;\big|\; x^4$

$x^4 - 3x^3$

$3x^3$

$3x^3 - 9x^2$

$9x^2$

$9x^2 - 27x$

$27x$

$27x - 81$

81

$x^3 + 3x^2 + 9x + 27$

$x^3 + 3x^2 + 9x + 27 + \frac{81}{x-3}$

Synthetic Division

Synthetic Division: An algorithm used to divide polynomials that eliminates the need to write variables.

* This algorithm can only be used when dividing by expressions of the form $x+c$ or $x-c$, where "c" is a constant.

Example

$(2x^4 + x^3 - 23x^2 - 46x - 24) \div (x-4)$

Change sign

4	2	1	−23	−46	−24
		8			
	2	9			

Step I: Write the numbers as seen above, remembering to change the sign of the constant in the divisor.

Step II: Bring the first number from the dividend down.

Step III: The remaining part of the algorithm is 2-step cycles. 1) Multiply these two numbers and write the product. Add these two numbers and write the sum. Repeat this process.

Divide the following polynomials using synthetic division.

(11) $(x^4-7x^3+x^2+27x+18)\div(x+1)$

-1	1	-7	1	27	18
		-1	8	-9	-18
	1	-8	9	18	0

$x^3-8x^2+9x+18$

(12) $(3x^6-14x^5-16x^4+46x^3+49x^2-16x-20)\div(x-5)$

5	3	-14	-16	46	49	-16	-20
		15	5	-55	-45	20	20
	3	1	-11	-9	4	4	0

$3x^5+x^4-11x^3-9x^2+4x+4$

(13) $(-6x^4+2x^3-7x^2+4x-8)\div(x^2+4x-9)$

Cannot use synthetic division

(14) $(-5x^3-32x^2-52x-16)\div(x+2)$

-2	-5	-32	-52	-16
		10	44	16
	-5	-22	-8	0

$-5x^2-22x-8$

(15) $(6x^4-13x^3-28x^2+41x-6)\div(x-3)$

3	6	-13	-28	41	-6
		18	15	-39	6
	6	5	-13	2	0

$6x^3+5x^2-13x+2$

(16) $(-x^4-3x^3+15x^2-17x+6)\div(x+6)$

-6	-1	-3	15	-17	6
		6	-18	18	-6
	-1	3	-3	1	0

$-x^3+3x^2-3x+1$

(17) $(4x^5+19x^4-45x^3-70x^2+116x-24)\div(x-2)$

2	4	19	−45	−70	116	−24
		8	54	18	−104	24
	4	27	9	−52	12	0

$$4x^4+27x^3+9x^2-52x+12$$

(18) $(9x^5+x^4+10x^3+4x^2-3x+1)\div(x^2+4)$

Cannot use synthetic division

(19) $(-12x^3-47x^2+10x+24)\div(x+4)$

−4	−12	−47	10	24
		48	−4	−24
	−12	1	6	0

$$-12x^2+x+6$$

(20) $(8x^3-30x^2+16x+6)\div(x-1)$

1	8	−30	16	6
		8	−22	−6
	8	−22	−6	0

$$8x^2-22x-6$$

(21) $(-25+13x^4+48x^2-2x^6)\div(x-3)$

$(-2x^6+13x^4+48x^2-25)\div(x-3)$

3	−2	0	13	0	48	0	−25
		−6	−18	−15	−45	9	27
	−2	−6	−5	−15	3	9	2

$$-2x^5-6x^4-5x^3-15x^2+3x+9+\frac{2}{x-3}$$

(22) $(4x-15+x^5)\div(x+2)$

$(x^5+4x-15)\div(x+2)$

−2	1	0	0	0	4	−15
		−2	4	−8	16	−40
	1	−2	4	−8	20	−55

$$x^4-2x^3+4x^2-8x+20-\frac{55}{x+2}$$

(23) $(x^6+10-3x^3-5x)\div(x-2)$

$(x^6-3x^3-5x+10)\div(x-2)$

2	1	0	0	−3	0	−5	10
		2	4	8	10	20	30
	1	2	4	5	10	15	40

$$x^5+2x^4+4x^3+5x^2+10x+15+\frac{40}{x-2}$$

(24) $(-10x^3-4x^4+27)\div(x+3)$

$(-4x^4-10x^3+27)\div(x+3)$

−3	−4	−10	0	0	27
		12	−6	18	−54
	−4	2	−6	18	−27

$$-4x^3+2x^2-6x+18-\frac{27}{x+3}$$

(25) $(x^3+x^2+x+5)\div(3x-4)$

Cannot use synthetic division

(26) $(2x^4-8x^3+3x^2-2x)\div(x-4)$

4	2	−8	3	−2	0
		8	0	12	40
	2	0	3	10	40

$$2x^3+3x+10+\frac{40}{x-4}$$

(27) $(-8x^5+3x^4+11x^3+9x^2)\div(x+1)$

−1	−8	3	11	9	0	0
		8	−11	0	−9	9
	−8	11	0	9	−9	9

$$-8x^4+11x^3+9x-9+\frac{9}{x+1}$$

(28) $(17x^5-20x+100)\div(x^5-x-1)$

Cannot use synthetic division

(29) $(-x^4+1-3x)\div(2+x)$

$(-x^4-3x+1)\div(x+2)$

$$\begin{array}{r|ccccc} -2 & -1 & 0 & 0 & -3 & 1 \\ & & 2 & -4 & 8 & -10 \\ \hline & -1 & 2 & -4 & 5 & -9 \end{array}$$

$$-x^3+2x^2-4x+5-\frac{9}{x+2}$$

* You may use a calculator.

(30) $(3+x-x^2+6x^5)\div(-3+x)$

$(6x^5-x^2+x+3)\div(x-3)$

$$\begin{array}{r|cccccc} 3 & 6 & 0 & 0 & -1 & 1 & 3 \\ & & 18 & 54 & 162 & 483 & 1452 \\ \hline & 6 & 18 & 54 & 161 & 484 & 1455 \end{array}$$

$$6x^4+18x^3+54x^2+161x+484+\frac{1455}{x-3}$$

(31) $(7+4x^2-10x^5)\div(8-x)$

Cannot use synthetic division.

(32) $(x^3+10x-1)\div(-x+4)$

Cannot use synthetic division