Polynomials (Part I): A Review of Polynomials

Polynomial: A sum of two or more expressions of the form bx^n, where n is a whole number (i.e. 0, 1, 2, 3, ...), b is a constant (called a coefficient), and x is a variable.

Example: $3x^5 + 8x^3 - x^2 + 4x - 7$

Warning: The definition above excludes polynomials with different variables (e.g. $5x^2y^4 + 2xy$). We will learn these types of polynomials later in the course.

Determine if the expressions below are polynomials. If not, circle the term(s) that disqualify each expression.

① $x^6 - 8x^4 + 5^x + 2$
No

② $10x^3 + 4x$
Yes

③ $4x^{-2} + 9x^5 - x^{3/4}$
No

④ $-108x^2 + 5x + 40$
Yes

⑤ $3x^2 + 4\sqrt{x} + 6$
No

⑥ $2 + \frac{4}{5}x^{16} + x$
Yes

⑦ $-3x^3 + 1$
Yes

⑧ $8x^5 + 2x^4 - \frac{5}{x^2} + 8x^2$
No

⑨ $42x + x^0 + \pi$
Yes

⑩ $0.002x^6 - 23^{-2/3}$
No

⑪ $-2 + 90x^4 + x$
Yes

⑫ $\frac{3}{x^4} + 5x - x^{-1}$
No

⑬ $6\sqrt[3]{x} + 10^{x-7} + 26$
No

⑭ $\log_6 x - 4x^2 + 3.872$
No

⑮ $47x^{101} + 83x^{100} - 3\frac{1}{9}$
Yes

⑯ $\frac{1}{2}x^2 - 70x + \frac{1}{8}$
Yes

⑰ $\frac{x^3}{7} - x^2 + x + 2,000$
Yes

⑱ $500\pi + \frac{3}{8x^4} + 0.44x^2$
No

⑲ $75x^9 + 15x^7 + 3x^0$
Yes

⑳ $36x^{10} - 7(2^{4x})$
No

Simplify each polynomial, write in descending order of powers, and identify the degree.

㉑ $8x^7 - 2x^5 + 3x^7 + 6x - 9x^5 + 4$
$11x^7 - 11x^5 + 6x + 4$, $D = 7$

㉒ $-5x^3 + 3x^4 + 10 + 3x^2 + 6x - 9 + 8x^4 - 2x + 5x - 4x^2$
$11x^8 - 6x^2 + 9x + 1$, $D = 8$

㉓ $2 - x^3 - 12x^2 + 3x^2 + 6x + 1 - 8x + x^2$
$-x^3 - 8x^2 - 2x + 3$, $D = 3$

(24) $8x + 3x^4 - 5x^2 + x^2 - 3x + 2x^4 + 1$

$\boxed{5x^4 - 4x^2 + 5x + 1}$, $\boxed{D = 4}$

Add the following polynomials.

(25) $(5x^3 - 2x^2 + 6x - 8) + (3x^2 + 6x - 4)$

$5x^3 - 2x^2 + 6x - 8 + 3x^2 + 6x - 4$

$\boxed{5x^3 + x^2 + 12x - 12}$

(26) $(x^3 - 2x^2 + 3) + (4x^3 - 5)$

$\boxed{5x^3 - 2x^2 - 2}$

(27) $(-9x^5 + 2x^3 - 6) + (4x^4 - 3x^3 + 8x^2 - 1)$

$\boxed{-9x^5 + 4x^4 - x^3 + 8x^2 - 7}$

(28) $(5x + 7) + (2x^3 + 6x + 7)$

$\boxed{2x^3 + 11x + 14}$

Subtract the following polynomials.

(29) $(4x^3 - 9x^2 - 2x + 3) - (6x^3 - 7x^2 + 2x - 10)$

$4x^3 - 9x^2 - 2x + 3 - 6x^3 + 7x^2 - 2x + 10$

$\boxed{-2x^3 - 2x^2 - 4x + 13}$

(30) $(10x^4 + 3x^3 - 7) - (8x^3 + 2x^2 - 3x + 2)$

$10x^4 + 3x^3 - 7 - 8x^3 - 2x^2 + 3x - 12$

$\boxed{10x^4 - 5x^3 - 2x^2 + 3x - 19}$

(31) $(-5x^3 - 2x^2 + 6x + 5) - (3x^4 - 6x^3 + 2x^2 - 3x - 3)$

$-5x^3 - 2x^2 + 6x + 5 - 3x^4 + 6x^3 - 2x^2 + 3x + 3$

$\boxed{-3x^4 + x^3 - 4x^2 + 9x + 8}$

(32) $(-4x^3 + x) - (-2x^3 + 6x - 9)$

$-4x^3 + x + 2x^3 - 6x + 9$

$\boxed{-2x^3 - 5x + 9}$

Multiply the following polynomials.

(33) $8x^2(3x^4 - 3x + 5) = \boxed{24x^6 - 24x^3 + 40x^2}$

(34) $-2z^3(2z^4 - 8z^3 + 3z^2 - 4z - 9) = \boxed{-4z^7 + 16z^6 - 6z^5 + 8z^4 + 18z^3}$

(35) $3y^4(y^3 - 5y^2 - y + 7) = \boxed{3y^7 - 15y^6 - 3y^5 + 21y^4}$

(36) $3x^8(3x^4 - 2x^3 - 7) = \boxed{9x^{12} - 6x^{11} - 21x^8}$

(37) $(x - 2)(x + 6) = x^2 + 6x - 2x - 12 = \boxed{x^2 + 4x - 12}$

(38) $(4x + 3)(3x + 7) = 12x^2 + 28x + 9x + 21 = \boxed{12x^2 + 37x + 21}$

(39) $(x - 1)(x - 5) = x^2 - 5x - x + 5 = \boxed{x^2 - 6x + 5}$

㊵ $(x+2)(x-8) = x^2-8x+2x-16 = \boxed{x^2-6x-16}$

㊶ $(x+3)(x^2+4x-1) = x^3+4x^2-x+3x^2+12x-3$

$= \boxed{x^3+7x^2+11x-3}$

㊷ $(x-5)(2x^2-x+6) = 2x^3-x^2+6x-10x^2+5x-30$

$= \boxed{2x^3-11x^2+11x-30}$

㊸ $(x+10)(x^2-7x+5) = x^3-7x^2+5x+10x^2-70x+50$

$\boxed{x^3+3x^2-65x+50}$

㊹ $(2x-1)(3x^2+8x+11) = 6x^3+16x^2+22x-3x^2-8x-11$

$= \boxed{6x^3+13x^2+14x-11}$

Divide the following polynomials.

㊺ $\dfrac{10x^6+20x^5-16x^4+4x^3}{2x^2}$ ㊻ $\dfrac{21z^{10}+9z^5}{3z^5}$

$\boxed{5x^4+10x^3-8x^2+2x}$ $\boxed{7z^5+3}$

㊼ $\dfrac{-36y^7-27y^6+45y^5-81y^4}{-9y^3}$ ㊽ $\dfrac{25x^5+60x^4-35x^3-70x^2+40x}{-5x}$

$\boxed{4y^4+3y^3-5y^2+9y}$ $\boxed{-5x^4-12x^3+7x^2+14x-8}$

A Review of Arithmetic Long Division

Example I) $7,491 \div 3$ Example II) $2,135 \div 8$

```
    2497        2,497
3 | 7,491
    6↓
    14
    12 ↓
    29
    27 ↓
    21
    21
    0
```

```
      266           266 + 7/8
  8 | 2135
     16↓
     53
     48 ↓
     55
     48
     7
```

Example III) $5,576 \div 4$ Example IV) $1,287 \div 5$

```
    1394        1,394
4 | 5576
    4↓
    15
    12↓
    37
    36 ↓
    16
    16
    0
```

```
      257
  5 | 1287
     10↓
     28
     25 ↓
     37
     35
     2

     257 + 2/5
```