Name
Date
Course

Polynomials (Part VI): Graphs of Polynomial Functions and Finding Polynomials Given Their Zeros

Review on Complex Numbers

Simplify.

① $(5+2i)+(-1+3i)$

$\boxed{4+5i}$

② $(-3-5i)+(-2+4i)$

$\boxed{-5-i}$

③ $(-6+3i)-(4-9i)$

$-6+3i-4+9i$

$\boxed{-10+12i}$

④ $(11+8i)-(-7+i)$

$11+8i+7-i$

$\boxed{18+7i}$

⑤ $(8-i)+(-6-7i)$

$\boxed{2-8i}$

⑥ $(1+i)+(3-10i)$

$\boxed{4-9i}$

⑦ $(-1-4i)-(2+5i)$

$-1-4i-2-5i$

$\boxed{-3-9i}$

⑧ $(12-6i)-(-1-6i)$

$12-6i+1+6i$

$\boxed{13}$

⑨ $3i(4i) = 12i^2 = 12(-1) = \boxed{-12}$

⑩ $-8(-6i) = \boxed{48i}$

⑪ $-7i(4i) = -28i^2 = -28(-1) = \boxed{28}$

⑫ $5i(7) = \boxed{35i}$

⑬ $3i(-2i) = -6i^2 = -6(-1) = \boxed{6}$

⑭ $(-5i)(-5i) = 25i^2 = 25(-1) = \boxed{-25}$

FOIL

⑮ $(4-2i)(5+3i)$

$20+12i-10i-6i^2$

$20+2i-6(-1)$

$20+2i+6$

$\boxed{26+2i}$

⑯ $(8-5i)(6-7i)$

$48-56i-30i+35i^2$

$48-86i+35(-1)$

$48-86i-35$

$\boxed{13-86i}$

⑰ $(-2+3i)(-1+10i)$

$2-20i-3i+30i^2$

$2-23i+30(-1)$

$2-23i-30$

$\boxed{-28-23i}$

⑱ $(-1+i)(-3-2i)$

$3+2i-3i-2i^2$

$3-i-2(-1)$

$3-i+2$

$\boxed{5-i}$

⑲ $(7-6i)(7-8i)$

$49-56i-42i+48i^2$

$49-98i+48(-1)$

$49-98i-48$

$\boxed{1-98i}$

⑳ $(-4+i)(-3+2i)$

$12-8i-3i+2i^2$

$12-11i+2(-1)$

$12-11i-2$

$\boxed{10-11i}$

Finding a Polynomial Given Its Zeros

Leading Coefficient of a Polynomial: The coefficient of the term with the highest power (i.e. greatest degree).

Examples: $2x^4+10x^3-4x^2+9$, $-x^3+x-8$, $0.9x^5+2x+4$

$LC=2$ $\quad LC=-1$ $\quad LC=0.9$

Write a polynomial function in standard form that has the following zeros, leading coefficient (LC), and degree (D).

(21) $-1, 2, 4, 1$ $\quad LC=-3$ $\quad D=4$

$$f(x) = -3(x+1)(x-2)(x-4)(x-1)$$
$$= -3(x^2-2x+x-2)(x^2-x-4x+4)$$
$$= -3(x^2-x-2)(x^2-5x+4)$$
$$= -3(x^4-5x^3+4x^2-x^3+5x^2-4x-2x^2+10x-8)$$
$$= -3(x^4-6x^3+7x^2+6x-8)$$
$$= \boxed{-3x^4+18x^3-21x^2-18x+24}$$

(22) $3, -3, -5$ $\quad LC=1$ $\quad D=3$

$$f(x) = 1(x-3)(x+3)(x+5)$$
$$= (x^2+3x-3x-9)(x+5)$$
$$= (x^2-9)(x+5)$$
$$= \boxed{x^3+5x^2-9x-45}$$

The Complex Conjugate Theorem

If a polynomial has real coefficients and a complex zero $a+bi$, then $a-bi$ must be a zero of the same polynomial.

Write a polynomial function in standard form with real coefficients that has the following zeros, leading coefficient (LC), and degree (D).

(23) $2i, 6$ $\quad LC=2$ $\quad D=3$

$$f(x) = 2(x-2i)(x+2i)(x-6)$$
$$= 2(x^2+2ix-2ix-4i^2)(x-6)$$
$$= 2(x^2-4(-1))(x-6)$$
$$= 2(x^2+4)(x-6)$$
$$= 2(x^3-6x^2+4x-24)$$
$$= \boxed{2x^3-12x^2+8x-48}$$

(24) $3-5i, 1, -2$ $\quad LC = -2 \quad D = 4$

$f(x) = -2(x-(3-5i))(x-(3+5i))(x-1)(x+2)$

$= -2(x-3+5i)(x-3-5i)(x^2+2x-x-2)$

$= -2(x^2-3x-5ix-3x+9+15i+5ix-15i-25i^2)(x^2+x-2)$

$= -2(x^2-6x+9-25(-1))(x^2+x-2)$

$= -2(x^2-6x+34)(x^2+x-2)$

$= -2(x^4+x^3-2x^2-6x^3-6x^2+12x+34x^2+34x-68)$

$= -2(x^4-5x^3+26x^2+46x-68)$

$= \boxed{-2x^4+10x^3-52x^2-92x+136}$

(25) $1+4i, -4$ $\quad LC = -1 \quad D = 3$

$f(x) = -1(x-(1+4i))(x-(1-4i))(x+4)$

$= -1(x-1-4i)(x-1+4i)(x+4)$

$= -1(x^2-x+\cancel{4ix}-x+1-\cancel{4i}-\cancel{4ix}+\cancel{4i}-16i^2)(x+4)$

$= -1(x^2-2x+1-16(-1))(x+4)$

$= -1(x^2-2x+17)(x+4)$

$= -1(x^3+4x^2-2x^2-8x+17x+68)$

$= -1(x^3+2x^2+9x+68)$

$= \boxed{-x^3-2x^2-9x-68}$

(26) $-i, -1, 2$ $\quad LC = 3 \quad D = 4$

$f(x) = 3(x+i)(x-i)(x+1)(x-2)$

$= 3(x^2-ix+ix-i^2)(x^2-2x+x-2)$

$= 3(x^2-(-1))(x^2-x-2)$

$= 3(x^2+1)(x^2-x-2)$

$= 3(x^4-x^3-2x^2+x^2-x-2)$

$= 3(x^4-x^3-x^2-x-2)$

$= \boxed{3x^4-3x^3-3x^2-3x-6}$

Graphs of Polynomial Functions

The graphs of polynomial functions move in one direction vertically (either rise or fall) as x-values become very large or very small.

The End Behavior of the Graph of a Polynomial Function: The direction of a graph of a polynomial function as the x-values increase or decrease indefinitely.

The term in a polynomial function with highest power determines the end behavior because it grows or decreases faster than all other terms combined.

Example: $2x^5+3x^4-8x^3-6x^2-100x-37$

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Notation for Describing End behavior

Notation: As $x \longrightarrow \infty$

Translation: As x approaches infinity.

Notation: $f(x) \longrightarrow \infty$

Translation: f(x) approaches infinity.

Raising a Negative Number to an Even or Odd Power

Even Powers Produce Positive Numbers

I) $(-2)^2 = (-2)(-2) = 4$ II) $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

III) $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$

Odd Powers Produce Negative Numbers, Assuming that the Base is Negative

IV) $(-2)^3 = (-2)(-2)(-2) = -8$ V) $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$

VI) $(-2)^7 = (-2)(-2)(-2)(-2)(-2)(-2)(-2) = -128$

Indicate the end behavior for each function.

(27) $f(x) = -2x^6 + 4x^5 - x^4 + x^3 - 20x^2 - 47x + 25$

End Behavior: As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to -\infty$

(28) $f(x) = x^3 - 10x^2 - 2$

End Behavior: As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

(29) $f(x) = -3x^5 + 2x^2 - 4x + 6$

End Behavior: As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to \infty$

(30) $f(x) = 9x^4 - 8x^3 - 4x^2 - 71x - 1$

End Behavior: As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$

(31) $f(x) = -7x^7 + 2x^5 - 3x^3 + 2x$

End Behavior: As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to \infty$

(32) $f(x) = 25x^8 + 92x^7 - 22x^6 + 15x^5 - 4x^3 - 2x + 1$

End Behavior: As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$

(33) $f(x) = -4x^4 + 27x - 4$

End Behavior: As $x \to \infty, f(x) \to -\infty$
As $x \to -\infty, f(x) \to -\infty$

(34) $f(x) = 17x^3 + 2x^2 + x - 10$

End Behavior: As $x \to \infty, f(x) \to \infty$
As $x \to -\infty, f(x) \to -\infty$

Describing Where a Function Is Increasing or Decreasing

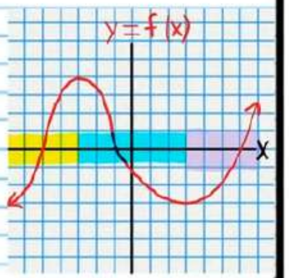
A function is increasing if the y-values increase as the x-values increase.

A function is decreasing if the y-values decrease as the x-values increase.

The graphs of various functions are written below. Write the intervals where each function is increasing or decreasing.

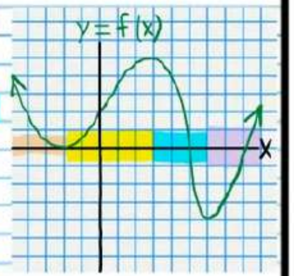
(35) Increasing: $(-7,-3)$ and $(3,7)$

Decreasing: $(-3,3)$



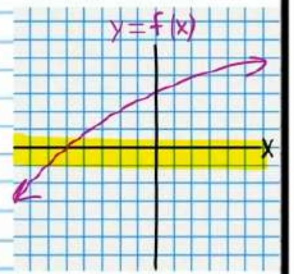
(36) Increasing: $(-2,3)$ and $(6,9)$

Decreasing: $(-5,-2)$ and $(3,6)$



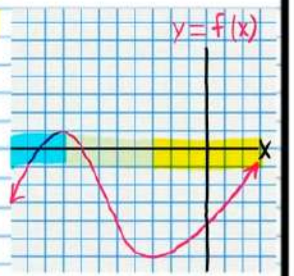
(37) Increasing: $(-8,6)$

Decreasing: X



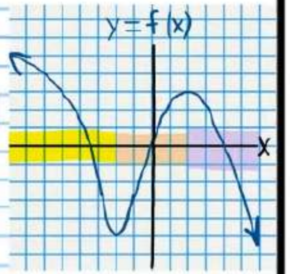
(38) Increasing: $(-11,-8)$ and $(-3,3)$

Decreasing: $(-8,-3)$



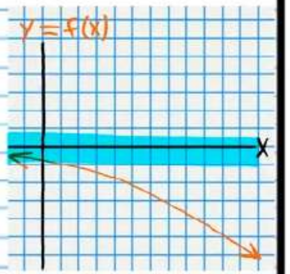
(39) Increasing: $(-2,2)$

Decreasing: $(-8,-2)$ and $(2,6)$



(40) Increasing: X

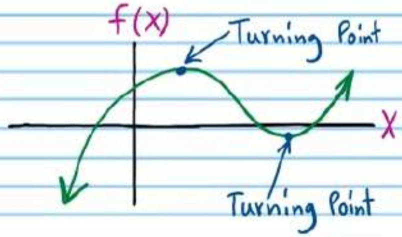
Decreasing: $(-2,12)$



Turning Point: A point where a function changes from increasing to decreasing or decreasing to increasing.

Example:

$f(x) = x^3 - 8x^2 + 11x + 20$



The Turning Points Theorem: The graph of a polynomial function with degree n has at most n-1 turning points.

For each polynomial function, state the maximum number of turning points of the corresponding graph.

(41) $f(x) = -3x^5 + 2x^2 - 4x + 6$

Max # of Turning Points: $n-1 = 5-1 = \boxed{4}$

(42) $f(x) = 9x^4 - 8x^3 - 4x^2 - 71x - 1$

Max # of Turning Points: $n-1 = 4-1 = \boxed{3}$

(43) $f(x) = -7x^7 + 2x^2 - 3x^3 + 2x$

Max # of Turning Points: $n-1 = 7-1 = \boxed{6}$

(44) $f(x) = 25x^8 + 92x^7 - 22x^6 + 15x^5 - 4x^3 - 2x + 1$

Max # of Turning Points: $n-1 = 8-1 = \boxed{7}$

(45) $f(x) = -4x^4 + 27x - 4$

Max # of Turning Points: $n-1 = 4-1 = \boxed{3}$

(46) $f(x) = 17x^3 + 2x^2 + x - 10$

Max # of Turning Points: $n-1 = 3-1 = \boxed{2}$

The Multiplicity of a Zero: If a zero is repeated m times, then the multiplicity of the zero is defined to be m.

Example: $f(x) = (x-7)^2(x+2)^3(x-4)$

7 is a zero of multiplicity 2.

-2 is a zero of multiplicity 3.

4 is a zero of multiplicity 1.

Using Multiplicity When Graphing

If a polynomial function has a real zero that has an odd multiplicity, then the graph of the polynomial crosses the x-axis at that number. If a polynomial function has a real zero that has an even mutiplicity, then the graph of the polynomial touches the x-axis at that number, but does not pass through it.

Polynomial Functions are written below in factored form. Write the x-intercepts of each corresponding graph and determine if the function crosses or touches the x-axis at each intercept.

(47) $f(x) = 2(x-6)^3(x+4)^2(x-2)$

$6 \Rightarrow$ Crosses $2 \Rightarrow$ Crosses

$-4 \Rightarrow$ Touches

By Definition, the Converse of the Factor Theorem is Part of the Factor Theorem: Let f be a polynomial function. If $x-c$ is a factor of $f(x)$, then $f(c)=0$.

(48) $f(x) = -9(x+2)^5(x+1)^4$

$-2 \Rightarrow$ Crosses

$-1 \Rightarrow$ Touches

(49) $f(x) = 7(x-8)(x+\sqrt{3})(x+\pi)^6$

$8 \Rightarrow$ Crosses

$-\sqrt{3} \Rightarrow$ Crosses

$-\pi \Rightarrow$ Touches

(50) $f(x) = -3(x+5)^7(x-10)^{12}$

$-5 \Rightarrow$ Crosses

$10 \Rightarrow$ Touches

+

m
aph.

(51) $f(x) = 53(x-9)^{10}(x+2)(x+5)^{5}$

$9 \Rightarrow$ Touches

$-2 \Rightarrow$ Crosses

$-5 \Rightarrow$ Crosses

(52) $f(x) = -(x-2.4)(x-4)^{3}(x-e)^{11}$

$2.4 \Rightarrow$ Crosses

$4 \Rightarrow$ Crosses

$e \Rightarrow$ Crosses