Quantum Bogosort

Chicago STEM Fair, Design

George Huebner (Grade 12), Michael Caines (Teacher Sponsor)

March 18, 2022

Walter Payton College Prep

Attachments

(ALL ITEMS LISTED MUST BE TYPED)

ABSTRACT

The Illinois Junior Academy of Science

CATEGORY Computer Science STATE REGION # 3

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CHECK ONE: ☐ EXPERIMENTAL INVESTIGATION ■ DESIGN INVESTIGATION
(Choice will determine rubric used for assessment)

NAME OF EXHIBITOR * George Huebner GRADE 12

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* If this project is awarded a monetary prize, the check will be written in this scientist's name and it will be his/her responsibility to distribute the prize money equally among all participating exhibitors.

PROJECT TITLE Quantum Bogosort

Purpose	Procedure	Conclusion
To explore quantum computing and development of NISQ devices by implementing the theoretical "quantum bogosort" gag algorithm.	Implemented with Qiskit and tested on Aer simulator and IBMQ Lima (5 qubit machine).	Quantum bogosort is a funky way to introduce quantum computing fundamentals to beginners. Novel QRNG algorithm provides insight into crafting arbitrary qubit states but is not practically useful.

1. Limit Abstract to 3 paragraphs (250 words or less). Include: a) Purpose– what you set out to investigate; b) Procedure – how you did it; c) Conclusion – based on your results. Label each paragraph.
2. Must be typed, single-spaced, on the front side of this form. DO NOT write on back side of this form.
3. Three (3) copies of your COMPLETE paper are required at the State Science Project Exposition. Four (4) copies of your COMPLETE paper are required for the State Paper Session Competition.

The above form must be duplicated. (Student-generated forms must be in essentially the same format.)
*This form **MUST** be displayed on the front of the exhibitor's display board. It may be reduced to fit on a half sheet of paper. (Print at 75% reduction) Displayed abstract cannot be smaller than 8.5 inches (vertical) x 5.5 inches (horizontal).*

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SAFETY SHEET

The Illinois Junior Academy of Science

Directions: The student is asked to read this introduction carefully, fill out the bottom of this sheet. The science teacher and/or advisor must sign in the indicated space. By signing this sheet, the sponsor assumes all responsibilities related to this project.

Safety and the Student: Experimentation or design may involve an element of risk or injury to the student, test subjects and to others. Recognition of such hazards and provision for adequate control measures are joint responsibilities of the student and the sponsor. Some of the more common risks encountered in research are those of electrical shock, infection from pathogenic organisms, uncontrolled reactions of incompatible chemicals, eye injury from materials or procedures, and fire in apparatus or work area. Countering these hazards and others with suitable controls is an integral part of good scientific research. In the **chart** below, list the principal hazards associated with your project, if any, and what specific precautions you have used as safeguards. Be sure to read the entire section in the *Policy and Procedure Manual of the Illinois Junior Academy of Science* entitled "Safety Guidelines for Experimentation" before completing this form.

Possible hazards	Precautions taken to deal with each hazard
N/A	

Please check off any other possible endorsements needed. Include these documents in your paper and on your board.
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_____Microorganism-for any projects involving bacteria, viruses, yeasts, fungi or protozoa
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_____Recombinant DNA-must be conducted in a registered research laboratory under professional supervision
_____Use of Firearms – including all required documents
_____Letter from institution where research was done or IJAS SRC-if an exception to the IJAS rules has been granted.

SIGNED _____
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Introduction

Abstract

Gag algorithms are a staple of computer science and provide many a good chuckle. Take, for instance, MiracleSort[1], an algorithm that relies on alpha particle emission to cause erroneous bit flips to sort the list:

```
while isSorted(unsorted_list) is False:
    time.sleep(1000)
```

Despite their apparent uselessness, joke algorithms provide valuable insight into algorithm design and complexity[2]. In this paper we propose an implementation to Quantum Bogosort, one such joke algorithm.

What is Quantum Bogosort?

Classical Bogosort is a sorting algorithm.

1. Randomly permute the list.
2. If the list is sorted, done! Otherwise go to step 1.

This has unbounded worst-case time complexity and an expected time complexity of $O(n \cdot n!)$.

Quantum Bogosort is very similar to classical Bogosort, except it uses a quantum computer to permute the list.

Attachments

Code + Paper: github.com/Borris-the-real-OG/Quantum_Bogosort

Acknowledgements

- Qiskit
- *We acknowledge the use of IBM Quantum services for this work. The views expressed are those of the authors, and do not reflect the official policy or position of IBM or the IBM Quantum team.*[3]
- All code was written and tested with Qiskit 0.32.0 & Python 3.8 on WSL2 Ubuntu 20.04.1.
- Special thanks to Joe Clapis

Purpose

Just like its classical counterpart, Quantum Bogosort is pretty useless. In fact, classical sorting algorithms have been proven to have a lower bound time complexity of $\Omega(N \log_2 N)$: there's nothing to optimize!

However, QBS presents a good case study for quantum algorithms; we present an analysis of implementation and complexity of QBS in a similar vein to Gruber et al.'s[2] analysis of classic bogosort.

Algorithm Design

QRNG

Desired State:

$$\sum_{x=0}^{N-1} \frac{1}{\sqrt{N}} |x_{BE}\rangle$$

This is quite easy to do with powers of 2; we just need to use Hadamard gates with control qubits to chunk a big number into smaller subproblems.

$$|\psi\rangle = \frac{1}{\sqrt{2}} |0000\rangle + \frac{1}{\sqrt{2}} |1000\rangle$$

$$\mathbf{CH}(\psi_{\mathbf{0}}, \psi_{[\mathbf{1},\mathbf{3}]}),\ \mathbf{X}(\psi_{\mathbf{0}})$$

$$|\psi\rangle = \frac{1}{\sqrt{16}} (|0000\rangle + |0001\rangle + \ldots + |0111\rangle) + \frac{1}{\sqrt{2}} |1000\rangle$$

We mostly care about amplitude as opposed to register state

QRNG

Limitation: Hadamard gates create the state $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.
What about states that aren't powers of 2?

Solution: Arbitrary rotation gates allow for qubit superposition with arbitrary amplitudes.

Hadamard is just a rotation of $\frac{\pi}{2}$ around the Pauli Y-axis!

$$|\psi\rangle = |0\rangle$$

$$\mathrm{R_y}(\theta)\,|\psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} |\psi\rangle = -\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle$$

Everything in this project is measured in Pauli-Z basis, so we don't worry about phase or imaginary numbers.

Algorithm

1. If 'parts' $= 2^{\texttt{len(register)}}$, use the control register to apply multi-controlled Hadamard gates to the rest of the register. Done.
2. If 'parts' $\leq 2^{\texttt{len(register)}-1}$, add `register[0]` to the control register and skip to step 5.
3. Calculate θ, $\mathrm{R_y}(\theta)$ the first register qubit, and add it to the control register.
4. Using the control register, apply multi-controlled Hadamard gates to the rest of the register.
5. Recurse on the remaining qubits.

Worked Example

$$i = 77,\ |\psi\rangle = |0000000\rangle$$

$$\theta_0 = 2 \arccos \sqrt{\frac{77 - 64}{77}}$$

Most Significant Bit in register

$$\mathsf{R}_y(\theta_0, \psi_0),\ \mathsf{X}(\psi_0),\ \mathsf{CH}(\psi_0, \psi_{[1,6]}),\ \mathsf{X}(\psi_0) =$$

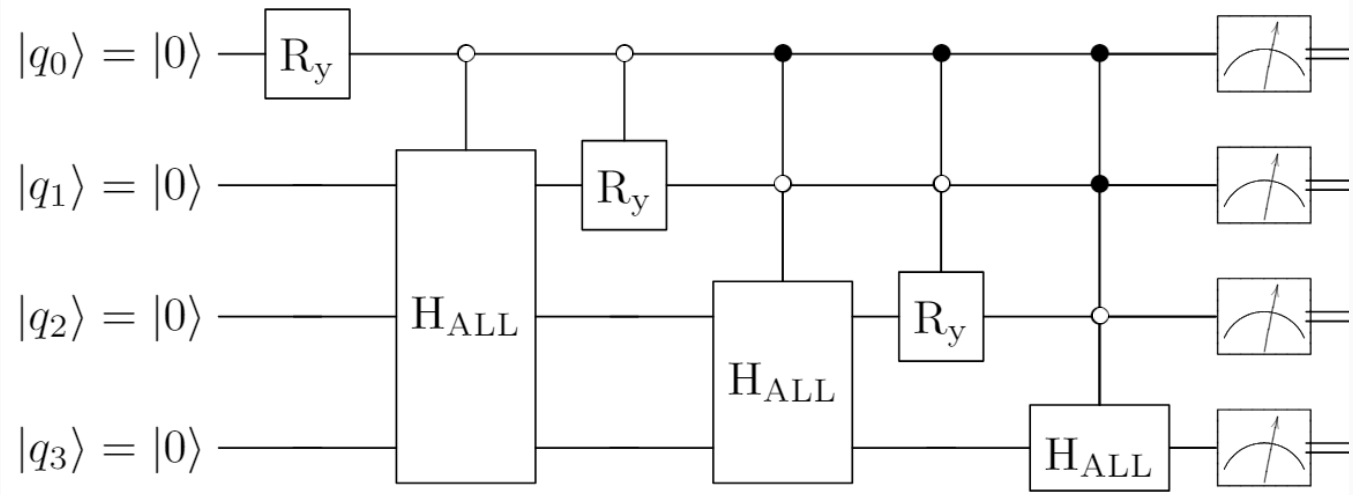
$$\frac{1}{\sqrt{77}}(|0000000\rangle + |0000001\rangle + ... + |0111111\rangle) + \sqrt{\frac{13}{77}}\,|1000000\rangle$$

Done with this part

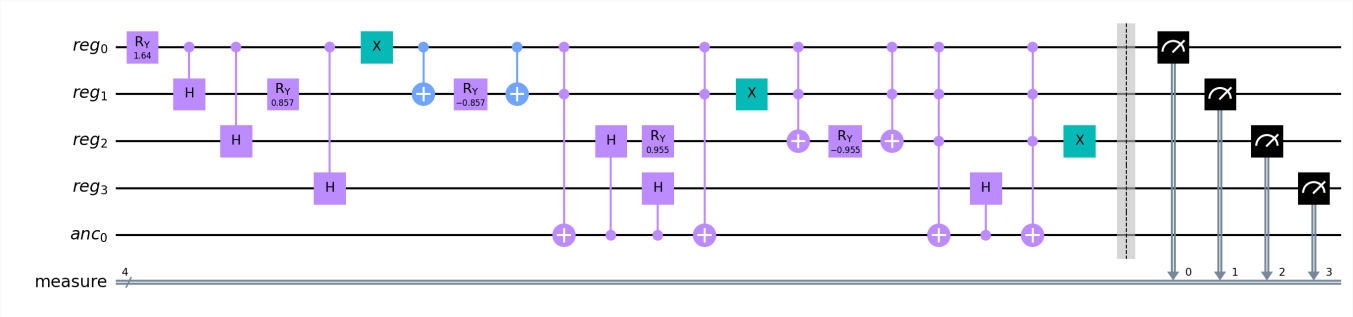
New subproblem

Practicality Assessment/Analysis

Circuit

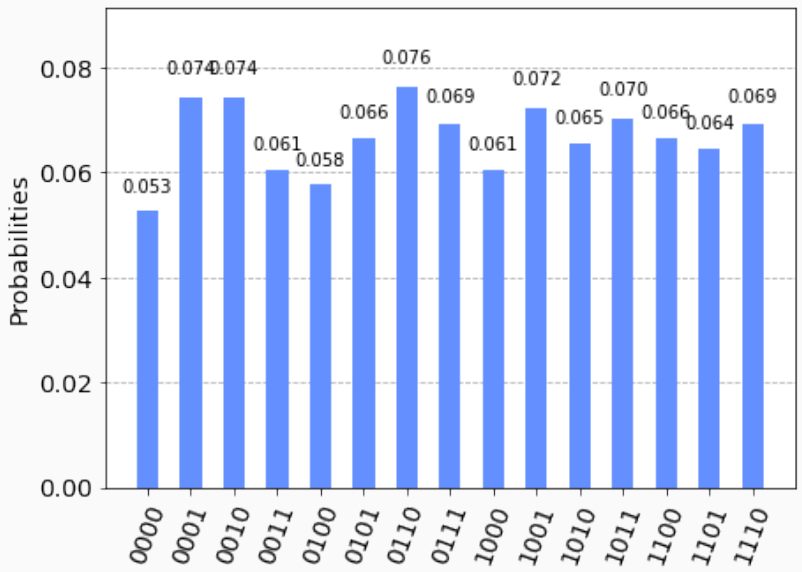


1 Conceptual Circuit[4]



2 Compiled to Aer Simulator

Results



Example measurement with `parts = 15`.

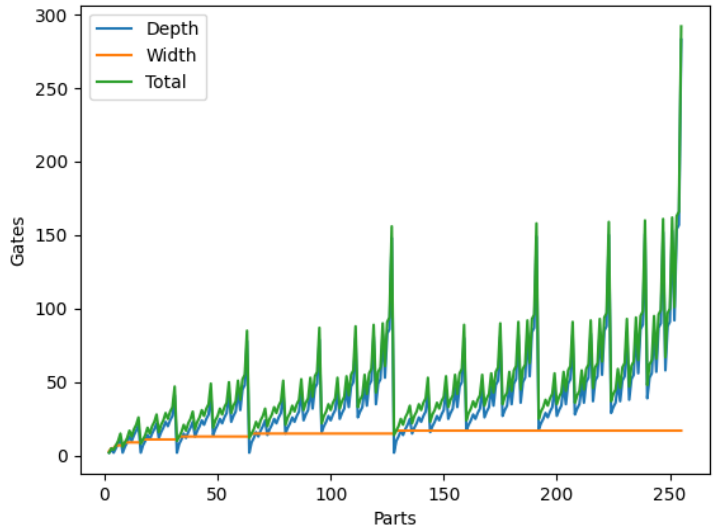
Complexity

$n = \#$ elements, $o =$ `bin(n).count('1')`

Many-worlds: $O(o)$

Copenhagen: $O(o \cdot n!)$

Gate count scales with the number of '1's in the binary representation, **not** the size of the number.



Circuit depth, width, and total gate count in relation to i.

Conclusion

Quantum Bogosort is not a useful algorithm, but it's helpful in teaching quantum computing fundamentals in a fun way.
Although infeasible due to its ballooning gate count, the QRNG algorithm could be repurposed for quickly preparing evenly balanced states among qubits.
By trying to optimize around the abysmal $O(\log_2 n!)$ space complexity via 'dividing and conquering', we end up implementing quicksort.

Bibliography

[1] K. Thompson, "Are there any worse sorting algorithms than bogosort (a.k.a monkey sort)?," Feb 2013.

[2] H. Gruber, M. Holzer, and O. Ruepp, "Sorting the slow way: An analysis of perversely awful randomized sorting algorithms," *Lecture Notes in Computer Science Fun with Algorithms*, p. 183–197, 2007.

[3] IBM Quantum, 2021.

[4] I. Chuang, "qasm2circ."