Automated Reasoning
Practical Assignment – Part 2

Jelmer Firet (s1023433, jelmer.firet@ru.nl)
Bram Pulles (s1015194, bram.pulles@ru.nl)

November 21, 2022

1. Groups

a) We proved that $I * x = x$, $inv(inv(x)) = x$ and $inv(x) * x = I$ by putting all of the given formulas *literally* in prover9, see `groups_a1.in`.

Using mace4, we show that, in general, $inv(x * y) = inv(x) * inv(y)$ does not hold. The smallest group for which this does not hold has size 6, see `groups_a2.in` and appendix A.

b) Using mace4, we show that the smallest non-Abelian group has size 6, again by *literally* putting in the formulas provided, see `groups_b.in` and appendix A.

c) We use the fact that $x^2 = x * x$, $x^3 = x * (x * x)$ and $x^4 = x * (x * (x * x))$ this makes encoding $x^n = I$ straightforward for $n = 2, 3, 4$.

Using prover9, we show that for $n = 2$ all such groups are Abelian, see `groups_c1.in`.

Using mace4, we show that for $n = 3$ the group is not Abelian in general and the smallest counterexample has size 27, see `groups_c2.in`.

Using mace4, we show that for $n = 4$ the group is not Abelian and the smallest counterexample has size 8, see `groups_c3.in` and appendix A.

A. Groups

a) Smallest finite group for which $inv(x * y) = inv(x) * inv(y)$ does not hold.

```
interpretation( 6, [number=1, seconds=0], [

    function(I, [ 0 ]),

    function(c1, [ 1 ]),

    function(c2, [ 2 ]),

    function(inv(_), [ 0, 1, 2, 4, 3, 5 ]),

    function(*(_,_), [
        0, 1, 2, 3, 4, 5,
        1, 0, 3, 2, 5, 4,
        2, 4, 0, 5, 1, 3,
        3, 5, 1, 4, 0, 2,
        4, 2, 5, 0, 3, 1,
        5, 3, 4, 1, 2, 0 ])
]).
```

b) Smallest non-abelian group.

```
interpretation( 6, [number=1, seconds=0], [

    function(I, [ 0 ]),

    function(c1, [ 1 ]),

    function(c2, [ 2 ]),

    function(inv(_), [ 0, 1, 2, 4, 3, 5 ]),

    function(*(_,_), [
        0, 1, 2, 3, 4, 5,
        1, 0, 3, 2, 5, 4,
        2, 4, 0, 5, 1, 3,
        3, 5, 1, 4, 0, 2,
        4, 2, 5, 0, 3, 1,
        5, 3, 4, 1, 2, 0 ])
]).
```

c) Smallest non-abelian group with $x^4 = I$.

```
interpretation( 6, [number=1, seconds=0], [

    function(I, [ 0 ]),

    function(c1, [ 1 ]),

    function(c2, [ 2 ]),

    function(inv(_), [ 0, 1, 2, 4, 3, 5, 6, 7 ]),

    function(*(_,_), [
        0, 1, 2, 3, 4, 5, 6, 7,
        1, 0, 3, 2, 5, 4, 7, 6,
        2, 4, 0, 6, 1, 7, 3, 5,
        3, 5, 1, 7, 0, 6, 2, 4,
        4, 2, 6, 0, 7, 1, 5, 3,
        5, 3, 7, 1, 6, 0, 4, 2,
        6, 7, 4, 5, 2, 3, 0, 1,
        7, 6, 5, 4, 3, 2, 1, 0 ])
]).
```