Automated Reasoning IMC009
Practical Assignment – Part 1

The assignment

Each part of the practical assignment has to be executed by one or two persons. (It is not mandatory to use the same group for part 2.) The result of each part should be described in a report that should be submitted in PDF via Brightspace, preferably less than 20 pages.

For all used formulas an extensive documentation is required, explaining the approach and the overall structure. A generic approach is preferred, since this may result in clearer descriptions, increasing the confidence in the correctness of the results. Formulas of more than half a page should not be contained in the report; instead the structure of the formula should be explained. From the output of the programs relevant parts should be contained in the report, and observations on computation time should be reported. The answers on the problems should be motivated, where relevant including pictures are appreciated. Every report should contain name, student number and email address of each of the authors. In case of two authors each of them is considered to be responsible for the full text and all results.

Grading

- Clear and generic descriptions are appreciated, both of the formulas themselves and the way they were designed. An example of appreciated style is given in the assignment on Brightspace.
- To obtain a 7, at least 3 out of the four solutions should be correct.
- Not giving a solution at all for one problem is preferred over giving a wrong solution.
- Reasons for obtaining higher than a 7 may be:
  - all problems correctly solved,
  - remarkably clear and structured descriptions,
  - approaches allowing generalizations,
  - original approaches and solutions.

The programs to be used

- `Z3`: `https://github.com/Z3Prover/z3` .
- `Yices`: `http://yices.csl.sri.com/`.
- `cvc5`: `https://cvc5.github.io/`.

  All of `Z3`, `Yices` and `cvc5` are programs for satisfiability modulo theories (SMT). They accept standard SMT format, in particular boolean SAT format. They also have their own input language.

Each of the problems should be solved using one of these tools. The tools should do the job: manual modifications of the problems should be avoided.

1 Pallets

Eight trucks have to deliver pallets of obscure building blocks to a magic factory. Every truck has a capacity of 8000 kg and can carry at most eight pallets. In total, the following has to be delivered:

- Four pallets of nuzzles, each of weight 700 kg.
- A number of pallets of prittles, each of weight 400 kg.
- Eight pallets of skipples, each of weight 1000 kg.
- Ten pallets of crottles, each of weight 2500 kg.
- Twenty pallets of dupples, each of weight 200 kg.

Skipples need to be cooled; only three of the eight trucks have facility for cooling skipples.

Nuzzles are very valuable: to distribute the risk of loss no two pallets of nuzzles may be in the same truck.

1. Investigate what is the maximum number of pallets of prittles that can be delivered, and show how for that number all pallets may be divided over the eight trucks.
2. Do the same, with the extra information that prittles and crottles are an explosive combination: they are not allowed to be put in the same truck.

2 Chip design

Give a chip design containing two power components and ten regular components satisfying the following constraints:

- Both the width and the height of the chip is 30.
- The power components have width 4 and height 3.
- The sizes of the ten regular components are 4×5, 4×6, 5×20, 6×9, 6×10, 6×11, 7×8, 7×12, 10×10, 10×20, respectively.
- All components may be turned $90°$, but may not overlap.
- In order to get power, all regular components should directly be connected to a power component, that is, an edge of the component should have a part of length > 0 in common with an edge of the power component.
- Due to limits on heat production the power components should be not too close: their centres should differ at least 16 in either the x direction or the y direction (or both).

What if this last distance requirement of 16 is increased to 17? And what if it is increased to 18?

3 Dinner

Five couples each living in a separate house want to organize a dinner.

Since all restaurants are closed due to some lock-down, they will do it in their own houses. The dinner will consist of 5 rounds. Due to the 1.5 meter restriction in every house presence of at most 5 people is allowed, by which every round has to be prepared and served in two houses simultaneously, each with the corresponding couple and three guests. Every couple will serve two rounds in their house, and in between the rounds participants may move from one house to another.

Every two people among the 10 participants meet each other at most 4 times during these 5 rounds. Further there are four desired properties:

(A) Every two people among the 10 participants meet each other at least once.
(B) Every two people among the 10 participants meet each other at most 3 times.
(C) Couples never meet outside their own houses.
(D) For every house the six guests (three for each of the two rounds) are distinct.

1. Show that (A) is possible, both in combination with (C) and (D), but not with both (C) and (D).

2. Show that (B) is possible in combination with both (C) and (D).

4 Program safety

Consider the following program:

$a := 1;\ b := 1;$

for $i := 1$ to 10 do

    if ? then $\{a := a + 2b;\ b := b + i\}$ else $\{b := a + b;\ a := a + i\}$;

if $b = 700 + n$ then crash

Here '?' is an unknown test that may yield false or true in any situation. Note that the test on crash is outside the loop, so is only tested at the end.

Establish for which values of $n = 1, 2, \ldots, 10$ it is safe, that is, will not reach 'crash'. Show for one of the non-safe values of n how $b = 700 + n$ can be reached.