Automated Reasoning
Practical Assignment – Part 1

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1 Introduction

We have chosen to use Z3 for solving all of the assignments. In particular we use the pythonic Z3 library.

2 Pallets

In this task we need to distribute pallets of various goods over a limited number of trucks. To this end we have formulated the problem as a system of linear equations. We created an integer variable $v_{g,t}$ for every *good* and *truck* combination such that $v_{g,t} \in good \times truck$. This variable describes how many pallets of a certain good are in the specific truck. Using just these variables we can describe all of the constraints.

- We can only have a positive number of pallets in a truck.

$$\forall_{v_{g,t} \in good \times truck} \; v_{g,t} \geq 0$$

- Every truck has at most eight pallets.

$$\forall_{t \in truck} \left(\sum_{g \in good} v_{g,t} \right) \leq 8$$

- Every truck can carry at most 8000 kg. Let w be a function giving the weight of a given good $w : good \to \mathbb{N}$.

$$\forall_{t \in truck} \left(\sum_{g \in good} w(g) \right) \leq 8000$$

- For every good all the pallets are distributed. Except for prittles, as no number of pallets is specified, we need to maximize this. Let p be a function giving the number of pallets of a given good $p : good \to \mathbb{N}$.

$$\forall_{g \in good \setminus prittles} \left(\sum_{t \in truck} v_{g,t} \right) = p(g)$$

- Only three trucks can contain skipples. Let t_i be the $i-$th truck, starting at $i = 1$.

$$\forall_{t_i \in truck, i>3}\ v_{skipples,t} = 0$$

- No two pallets of nuzzles may be in the same truck.

$$\forall_{t \in truck}\ v_{nuzzles,t} \leq 1$$

- Prittles and crottles are not allowed to be put in the same truck.

$$\forall_{t \in truck}\ v_{prittles,t} = 0 \vee v_{crottles,t} = 0$$

All of the constraints described above can be easily converted to Z3. In order to let Z3 automatically maximize the number of pallets with prittles we set the maximisation function to maximize the formula shown below.

$$\sum_{t \in truck} v_{prittles,t}$$

Running our program *without* the last constraint (question 1) gives us the result shown below. Every row is a truck, every column is the good starting with that letter. As can be seen from the table, all the pallets can be distributed and there are a total of $8 + 5 + 5 + 4 = 22$ prittles pallets.

```
   n s c d p
0 : 0 0 0 0 8
1 : 0 8 0 0 0
2 : 1 0 2 0 5
3 : 1 0 2 0 5
4 : 0 0 2 6 0
5 : 0 0 0 4 4
6 : 1 0 2 5 0
7 : 1 0 2 5 0
```

Running our program *with* the last constraint (question 2) gives us the result shown below. Again, all the pallets can be distributed and there are a total of $4 + 8 + 8 = 20$ prittles pallets.

```
   n s c d p
0 : 0 4 0 0 4
1 : 1 2 2 1 0
2 : 0 2 2 4 0
3 : 0 0 0 0 8
4 : 1 0 2 5 0
5 : 1 0 2 5 0
6 : 1 0 2 5 0
7 : 0 0 0 0 8
```

Our program and formalisation are generalised under the number of trucks, the truck maximum weight and maximum number of pallets. It is also very easy to add more goods, change their weight or change the number of pallets.

3 Chip design

4 Dinner

5 Program safety