APM466 Assignment 1

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Fundamental Questions-25 Points

1.

(a) If government prints money instead, the action devalues the money in the market, causing inflation rate to rise, whereas issuing bonds is a debt which does not cause inflation itself.

(b) A high risk premium is already applied to a long term bond, so increasing bond term contributes less to YTM (YTM increase becomes slower), which makes the long term part flatten.

(c) Quantitative easing basically means increasing money supply. U.S. Fed employed this method by buying massive amount of financial assets (bonds, stocks, etc.) since the beginning of the COVID-19 pandemic.

2.

The 10 bonds I selected are **CAN 0.25 Jul 31 22, CAN 0.25 Jan 31 23, CAN 0.25 Jul 31 23, CAN 0.75 Jan 31 24, CAN 1.5 Aug 31 24, CAN 1.25 Feb 28 25, CAN 0.5 Aug 31 25, CAN 0.25 Feb 28 26, CAN 1 Aug 31 26, CAN 1.25 Feb 28 27.**

I selected these bonds since ①All of them have been issued if we use Jan 31st as today. ② These bonds have similar coupon rate. None of them have insane coupon rates like 6% or 8%. ③Suitable for semi-annul coupons. Except for CAN 0.75 Jan 31 24 and CAN 1.5 Aug 31 24 whose terms are 7 months apart, all other bonds' terms are 6 months apart from each other. I will ignore this single 1 month difference in later calculations so everything can be in the 5-year range.

3.

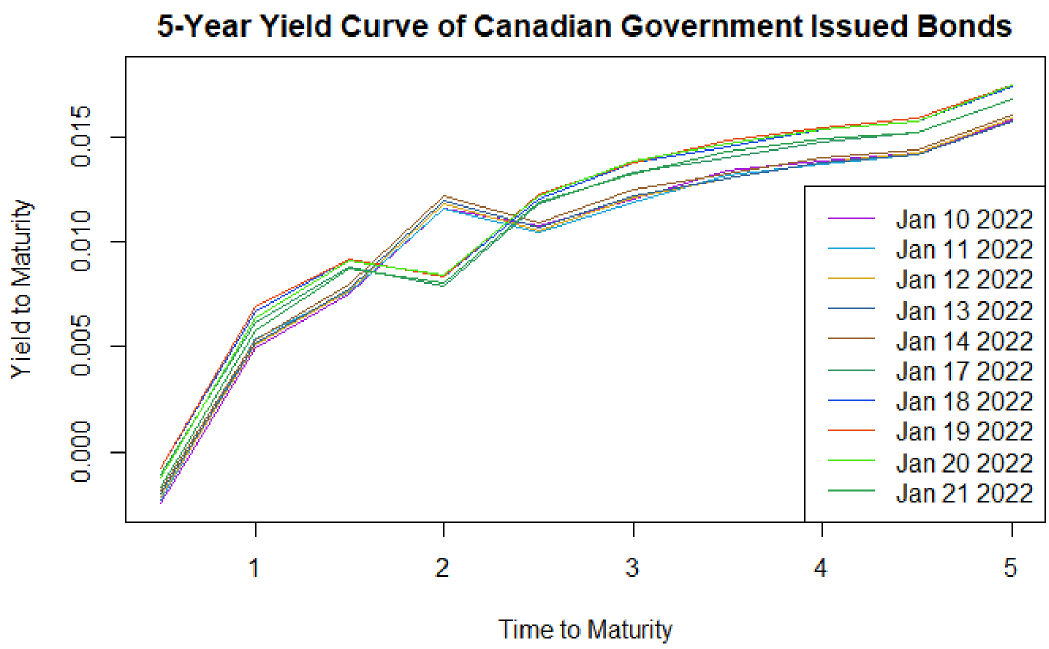
By finding the eigenvectors and eigenvalues of the covariance matrix, we can summarize the information in a large dataset by means of a much smaller set of summary parameters which can be more easily analyzed and visualized. Intuitively, they give us a lower dimensional picture of the object. (1: Aug 18 2020)

Empirical Questions-75 Points

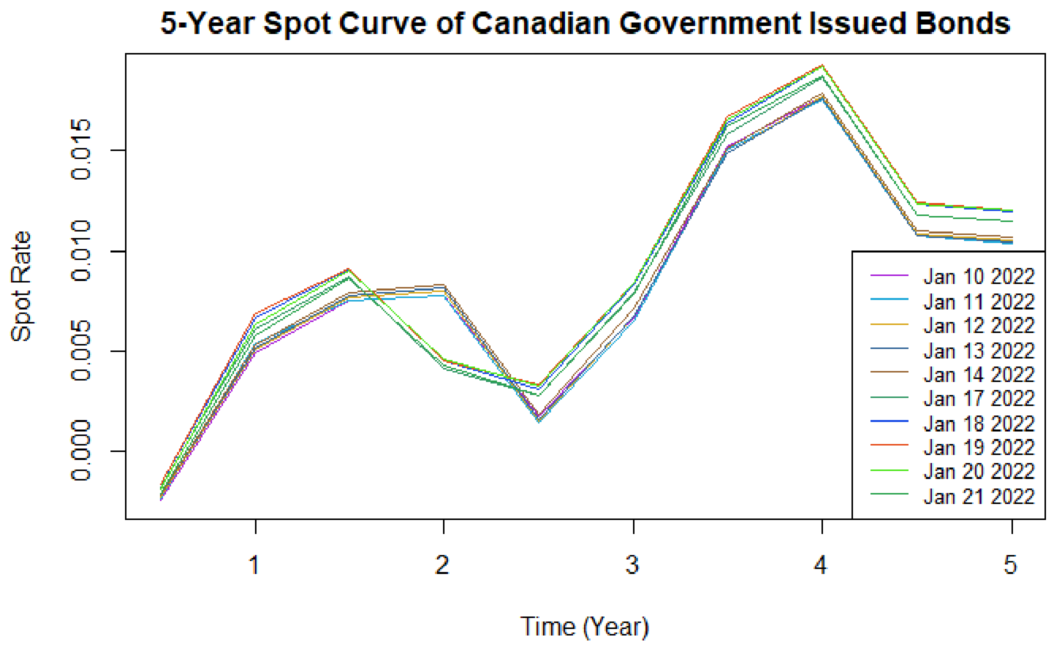
4.

It quite surprises me that the bond CAN 0.25 Jul 31 22 that matures in 6 months from Today (Jan 31) has negative yield when I use dirty price to calculate. Since there is only 1 coupon payment left (the one right before date of maturity), and the coupon payment is too low 0.25% annually, 0.125% semi-annually. Further it makes sense that current interest rate is quite low, thus the return of this bond is really low, resulting in the bond price to be high to have a negative yield.

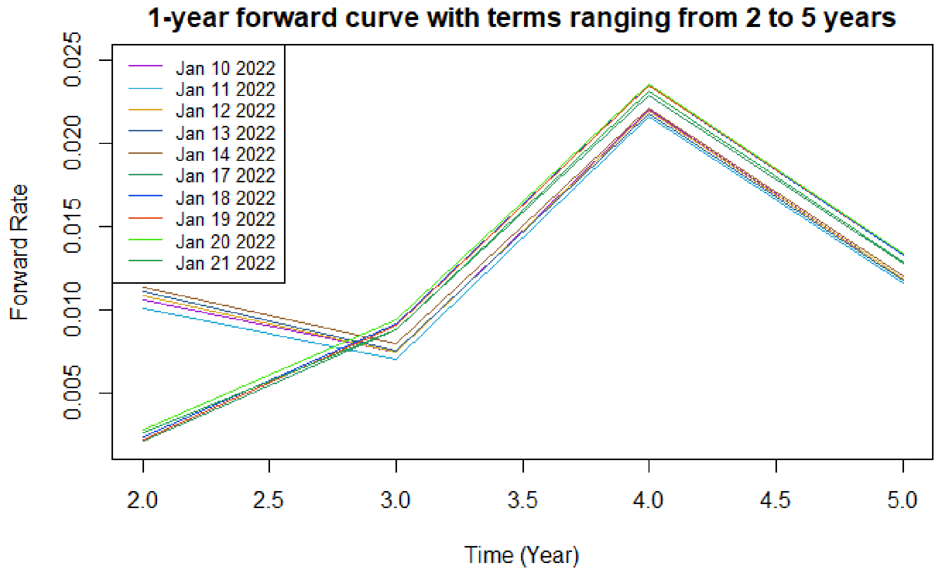
(a) According to my answer of Q2, I use semi-annual compounding to calculate YTM. In this case, multiplying the result by two gives us the actual YTM.



(b) Since spot rate is a function of time t, so I will use instantaneous compounding to calculate spot rate. We know that the spot rate of bonds which mature within a year, the spot rate is equal to its ytm, so we can easily calculate this spot rate. Then, we use bootstrapping. Since we know the dirty price, cash flow of bonds, the 0.5-year spot curve and the face value. The only unknown in the bootstrapping equation the next spot rate and it is calculatable in this case. We repeat this process to get all spot rates.



(c) According to the formula on note of week 2 slide 9, we can see that forward yield can be calculated from 2 spot rates. We simplify the equation and get -(-r2 * (T2 - t) - (-r1(T1-t))) / (T2- T1), then we can acquire forward yield by spot rates. (2)



5. Covariance matrix of daily log-returns of yield:

```
             x1           x2            x3            x4            x5
x1  0.006542549 -0.005549820  0.0015510714  0.0014436647  0.0013593731
x2 -0.005549820  0.021796329 -0.0023914879 -0.0020152751 -0.0016793773
x3  0.001551071 -0.002391488  0.0008603828  0.0007402593  0.0006930997
x4  0.001443665 -0.002015275  0.0007402593  0.0006736245  0.0006266966
x5  0.001359373 -0.001679377  0.0006930997  0.0006266966  0.0005893211
```

Covariance matrix of daily log-returns of forward rate:

```
             fx1           fx2           fx3           fx4
fx1  0.325955021 -0.0140232557 -0.0053166171 -0.0092937183
fx2 -0.014023256  0.0028252133  0.0009006576  0.0015577977
fx3 -0.005316617  0.0009006576  0.0003277149  0.0005632480
fx4 -0.009293718  0.0015577977  0.0005632480  0.0009794311
```

6. Eigenvalues and Eigenvectors of Covariance matrix of daily log-returns of yield:

```
[1] 2.437819e-02 5.161785e-03 8.978251e-04 2.292514e-05 1.477730e-06
            [,1]      [,2]        [,3]        [,4]         [,5]
[1,]  0.31555826 0.8806771  0.35254773  0.02172736 -0.008298923
[2,] -0.93093734 0.3590263 -0.06461572  0.01208563 -0.011598138
[3,]  0.12142163 0.1775345 -0.60160176  0.76216082 -0.104520263
[4,]  0.10453548 0.1766355 -0.51557917 -0.55031827 -0.623859083
[5,]  0.09004318 0.1810783 -0.49374927 -0.34006257  0.774384898
[1] 3.269190e-01 3.059872e-03 1.056510e-04 2.841237e-06
```

Eigenvalues and Eigenvectors of daily log-returns of forward rate:

```
[1] 3.269190e-01 3.059872e-03 1.056510e-04 2.841237e-06
            [,1]        [,2]         [,3]          [,4]
[1,]  0.99851070 -0.05434779 -0.004762211  0.0001413602
[2,] -0.04338831 -0.84398523  0.534582093  0.0053266713
[3,] -0.01642403 -0.26763154 -0.415201183 -0.8693167347
[4,] -0.02870691 -0.46163688 -0.736075597  0.4942266903
```

The largest eigenvalue and its correspondence eigenvector give the direction of the maximum variability in the data, which in this case, means a change in this factor should be in the same direction for all rates.

However, we can see a number (in red bracket) in the two largest vector has different sign. These two values correspond to ytm of bonds matures in 1 year and 2 years and 1yr-1yr forward rate. Both rates are calculated from bond CAN 0.25 Jan 31 23 & CAN

0.75 Jan 31 24 (1yr-1yr forward rate are calculated from the corresponding spot rate at t=1 and t=2) And since the bond matures in 2 years was issued on 10/21/2022, so there is no previous coupon payment, thus there is no dirty price for this bond. Hence the ytm and spot rate at t = 2, differ from the trend significantly (Can see this in my plots of 4(a) and 4(b).

Further, the coupon payment for the 1-year and 2- year bonds differ significantly as well. Although there is no insane coupon rate like 6% or something, the latter rate is still 3 times the front one. (0.75% to 0.25%) The two reasons combine result in a value of different sign in the largest eigenvectors.

References

(1) What Is Principal Component Analysis (PCA) and How It Is Used? Aug 18 2020, https://www.sartorius.com/en/knowledge/science-snippets/what-is-principal-component-analysis-pca-and-how-it-is-used-507186

(2) https://seco.risklab.ca/apm466-mat1856-library/#

Code Link: https://github.com/BoruiTian/APM466-Assignmeent1.git