Digital Image Processing

What Is Digital Image Processing?

DIP Definition:

A Discipline in which Both Input and Output of a Process are Images.

What is Image ?

- An image is a spatial representation of a two-dimensional or three-dimensional scene.
- An image is an array, or a matrix pixels (picture elements) arranged in columns and rows.

WHY…..digital image processing…???

- Interest in digital image processing methods stems from two principal application areas:

1. **Improvement of pictorial information** for human interpretation
2. Processing of image data for **storage, transmission, and representation** for autonomous machine perception

What Is Digital Image ?

- An image may be defined as a two-dimensional function, **f(x, y)**, where **x** and **y** are spatial (plane) coordinates, and the amplitude of **f** at any pair of coordinates (x,y) is calledthe **intensity** or **gray level** of the image at that point.

- **Digital Image:**
  When x, y and the intensity values of f are all finite, discrete quantities, then the image is a digital image.

- **Color Image:**

  f(x,y)= [r(x,y), g(x,y), b(x,y)]

What Is Digital Image ?

An Image:

$$g(x , y)$$

Discretization

$$g(i , j)$$

Quantization

$$f(i , j)$$ Digital Image

$f(i_0 , j_0)$ **:** Picture Element, Image Element, Pixel

What Is Digital Image Processing?

Image Processing

Low-Level Process

- Reduce Noise
- Contrast Enhancement
- Image Sharpening

Image Analysis

Mid-Level Process

- Segmentation
- Classification

Vision

High-Level Process

Making Sense of an Ensemble of Recognized Objects

Origins of Digital Image Processing

- One of the first applications of digital images was in the newspaper industry, when pictures were first sent by submarine cable between London and New York.
- Introduction of the Bartlane cable picture transmission system in the early 1920s reduced the time required to transport a picture across the Atlantic from more than a week to less than three hours.

A digital picture produced in 1921 from a coded tape by a telegraph printer with special type faces.

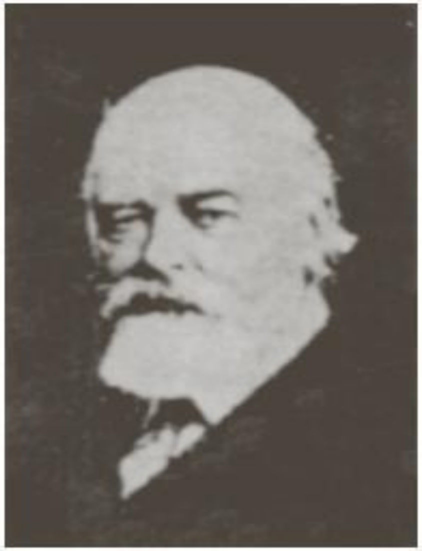


FIGURE 1.2 A digital picture made in 1922 from a tape punched after the signals had crossed the Atlantic twice. (McFarlane.)

Fields that Use Digital Image Processing

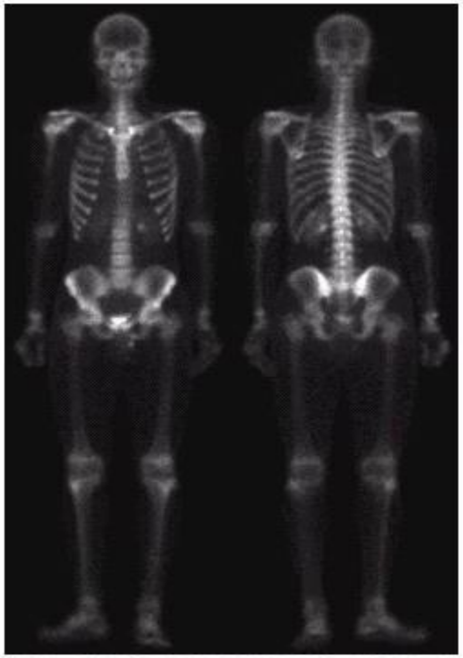
- Today, there is almost no area of technical endeavor that is not impacted in some way by digital image processing.

- Gamma-Ray Imaging
- X-Ray Imaging
- Imaging in the Ultraviolet Band
- Imaging in the Visible and Infrared Bands
- Imaging in the Microwave Band
- Imaging in the Radio Band

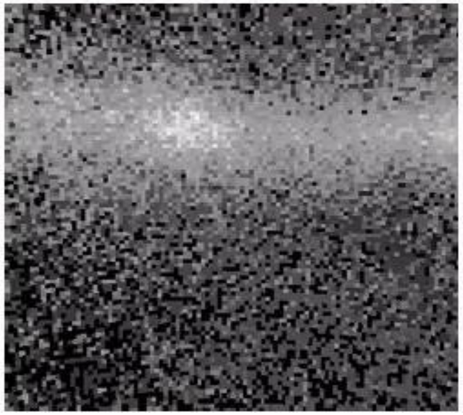
Gamma-Ray Imaging

- Major uses of imaging based on gamma rays include nuclear medicine.
- In nuclear medicine, the approach is to inject a patient with a radioactive isotope that emits gamma rays as it decays.
- Images are produced from the emissions collected by gamma ray detectors.

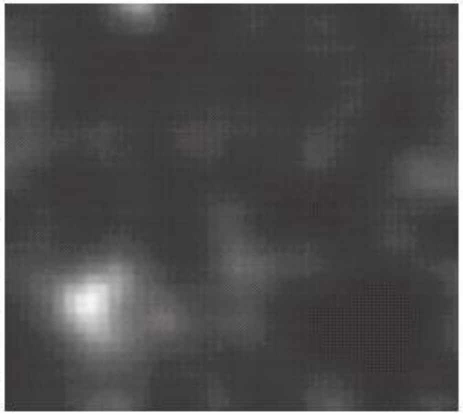
Bone scan



PET



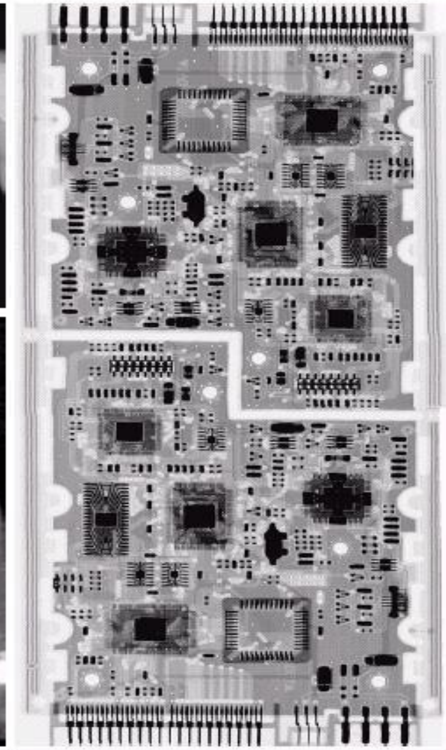
Cygnus loop



Reactor valve

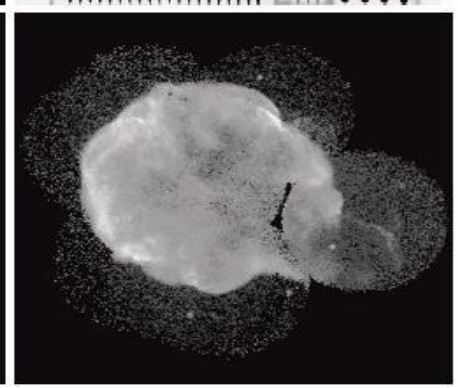
X-Ray Imaging

Chest X-Ray



PCB

Angiogram



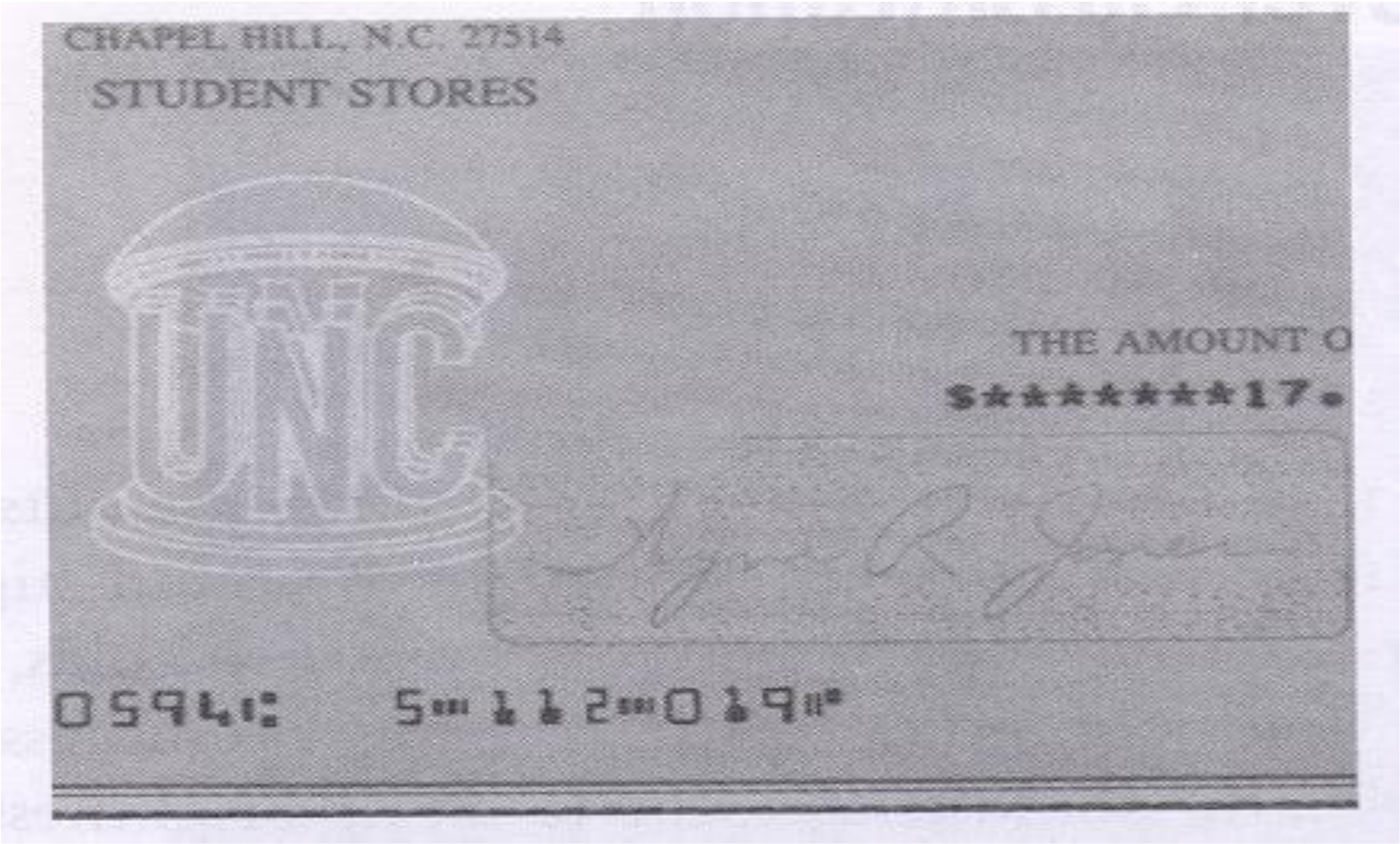
Head CT

Cygnus loop

Applications and Research Topics

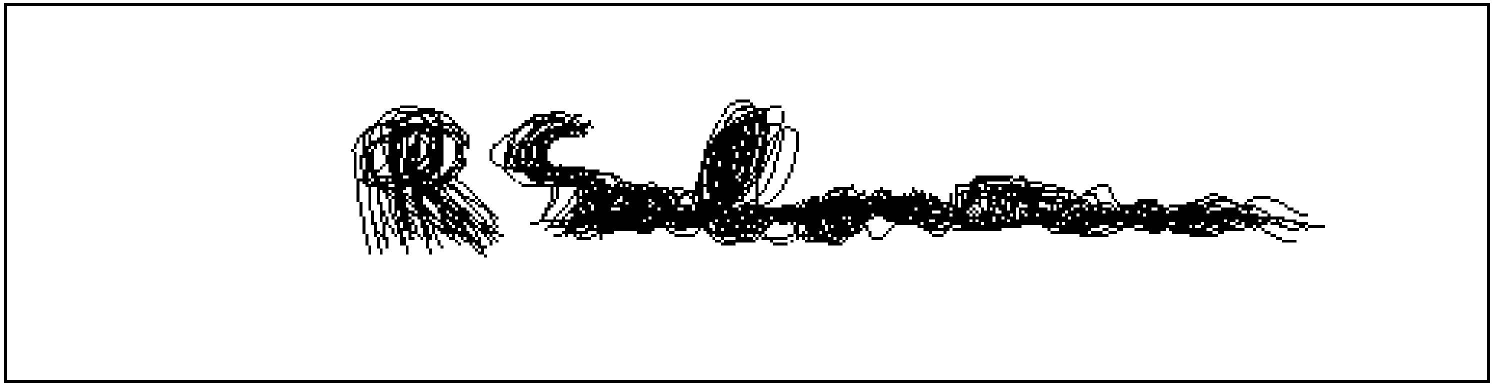
Applications and Research Topics

- Document Handling



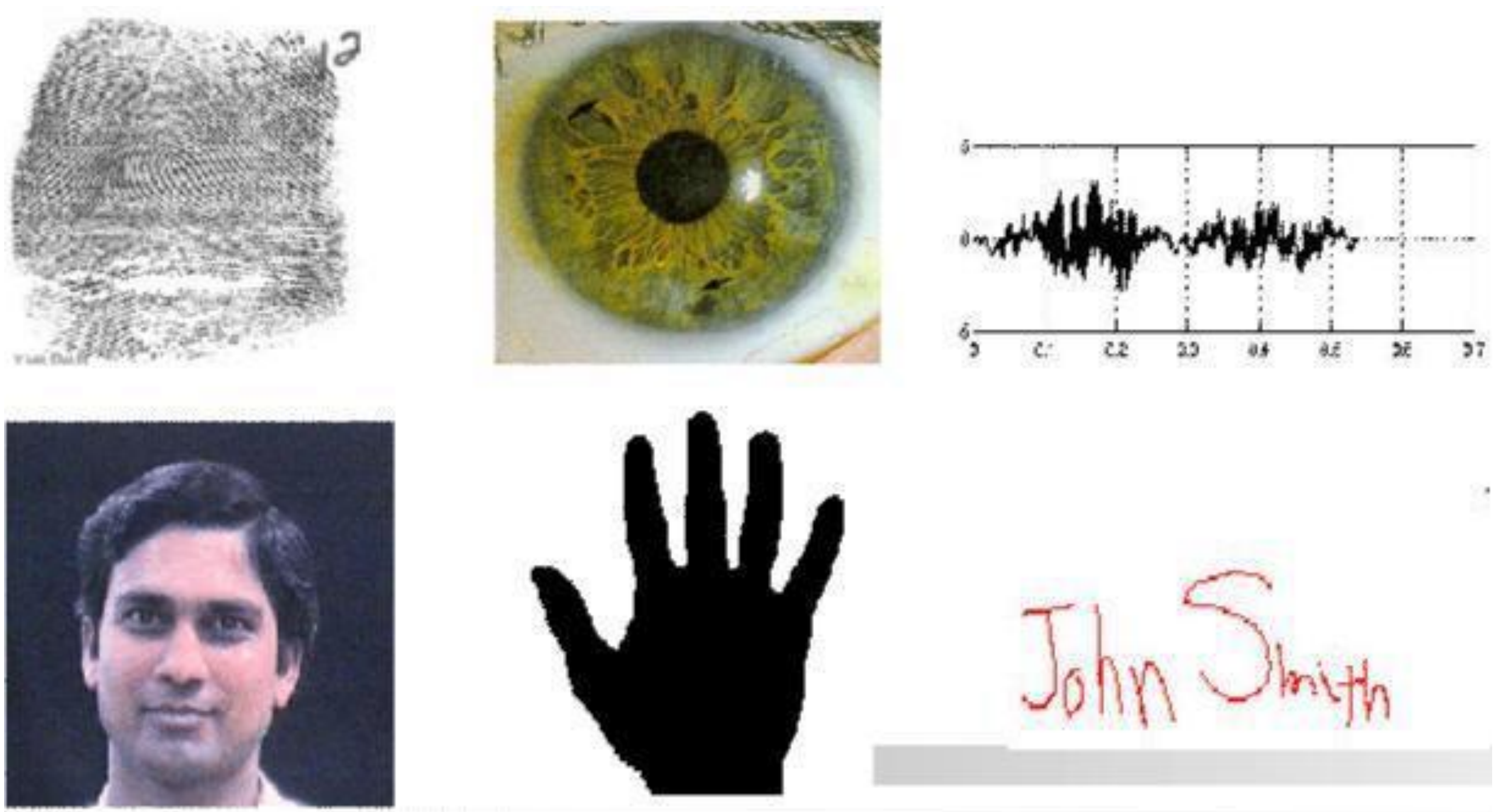
Applications and Research Topics

- Signature Verification



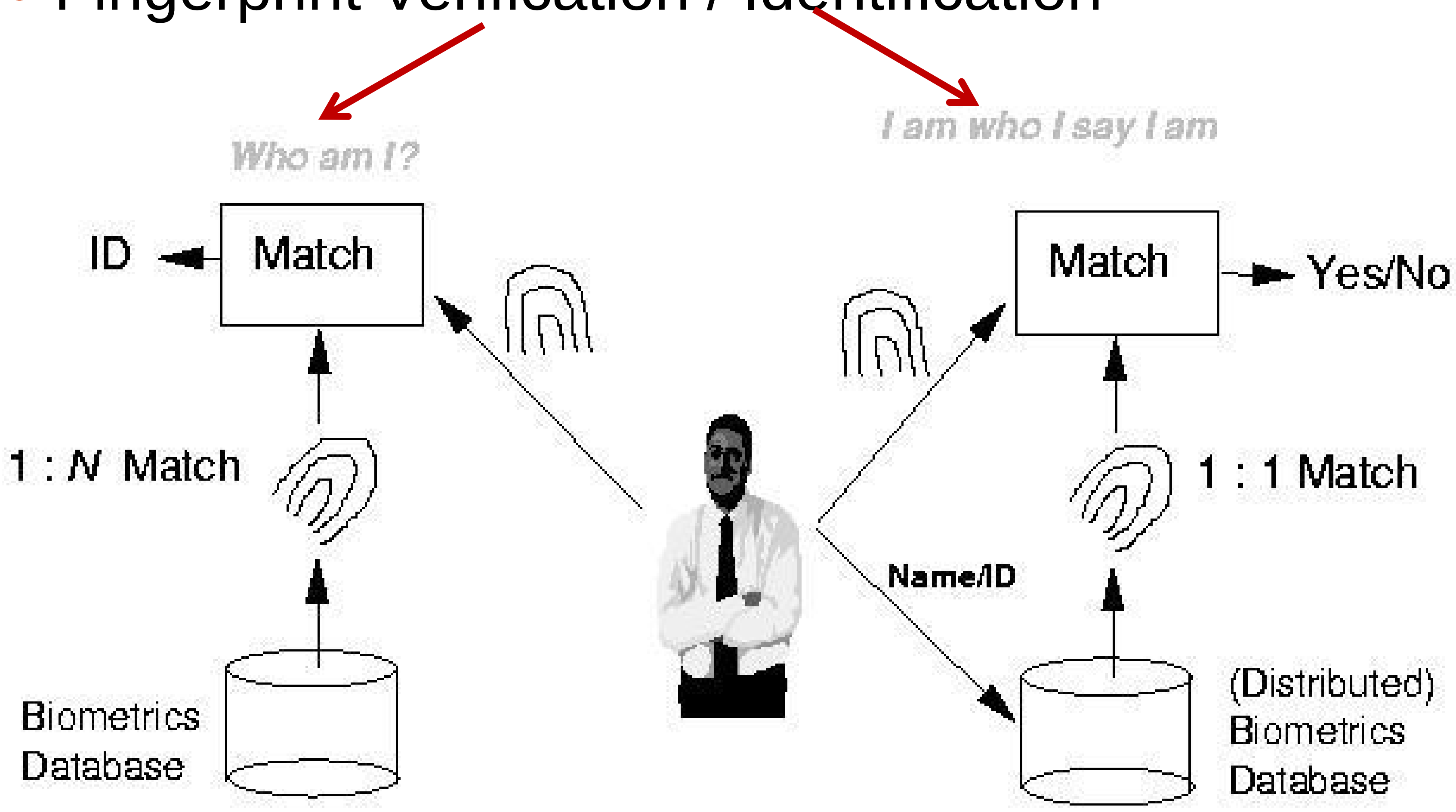
Applications and Research Topics

- Biometrics



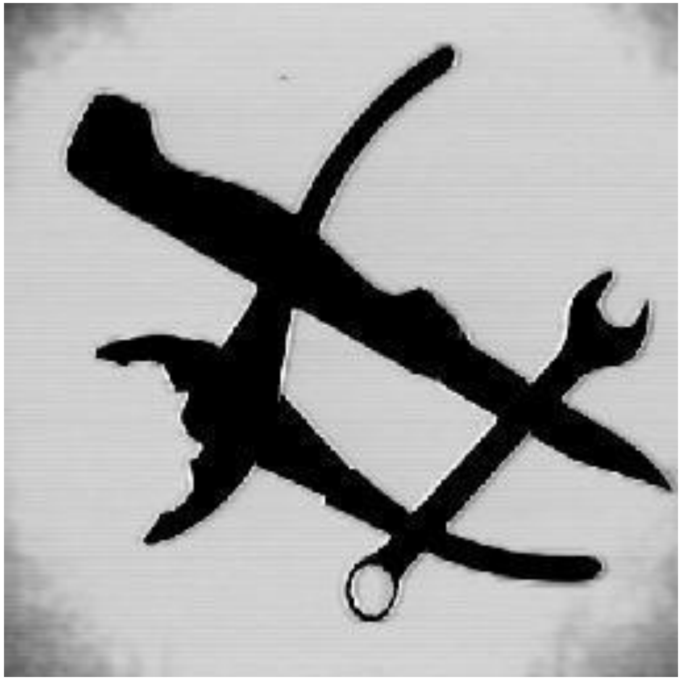
Applications and Research Topics

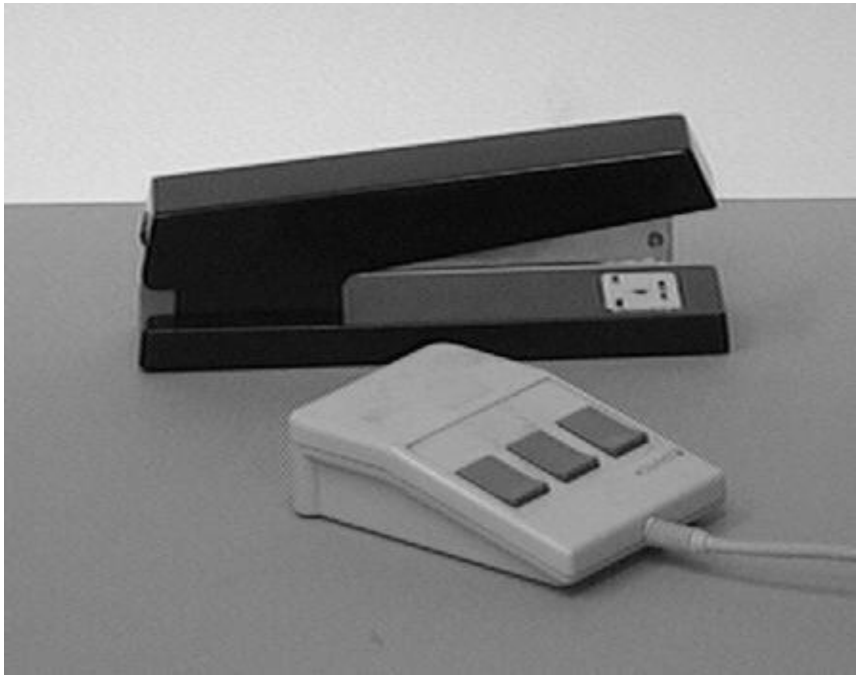
- Fingerprint Verification / Identification



Applications and Research Topics

- Object Recognition

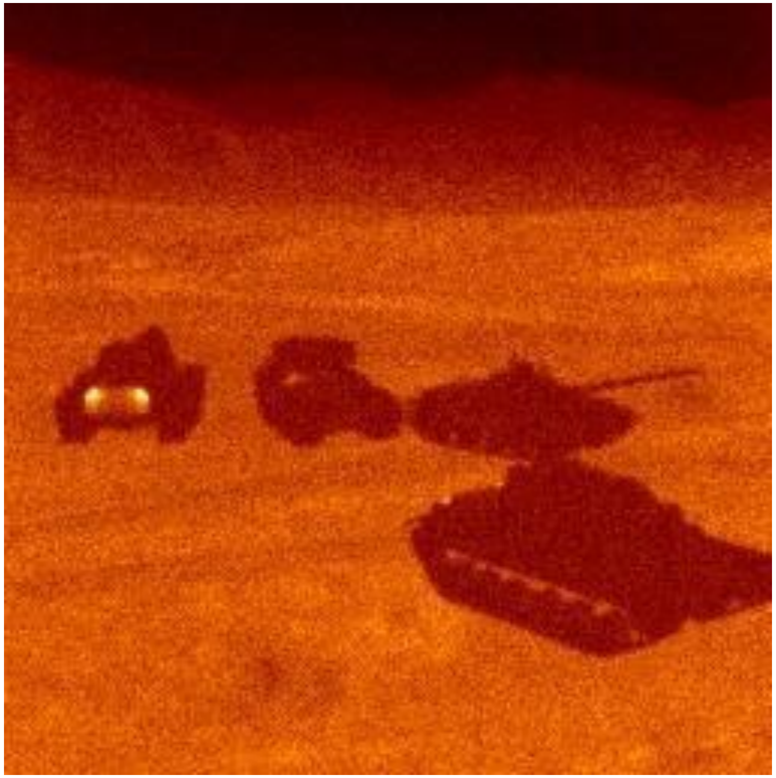




Applications and Research Topics

- Target Recognition

  Department of Defense (Army, Air force, Navy)



Applications and Research Topics

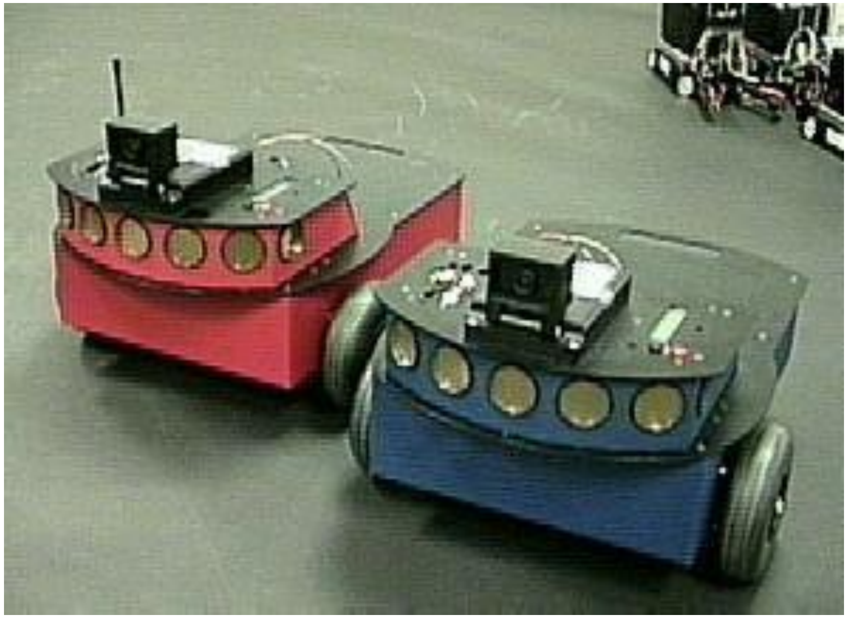
- Interpretation of Aerial Photography

Interpretation of aerial photography is a problem domain in both computer vision and registration.

Applications and Research Topics

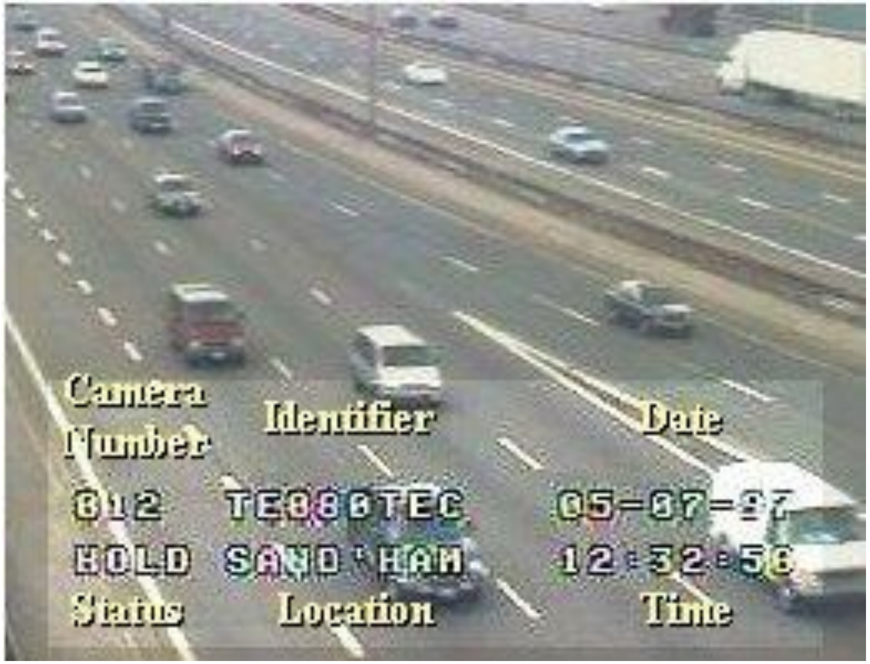
- Autonomous Vehicles

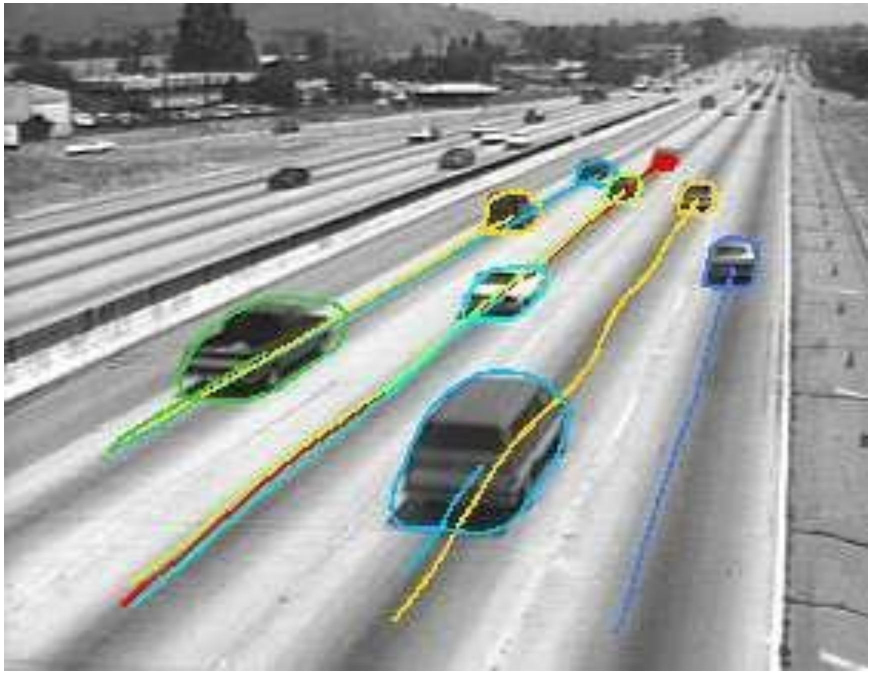
  Land, Underwater, Space



Applications and Research Topics

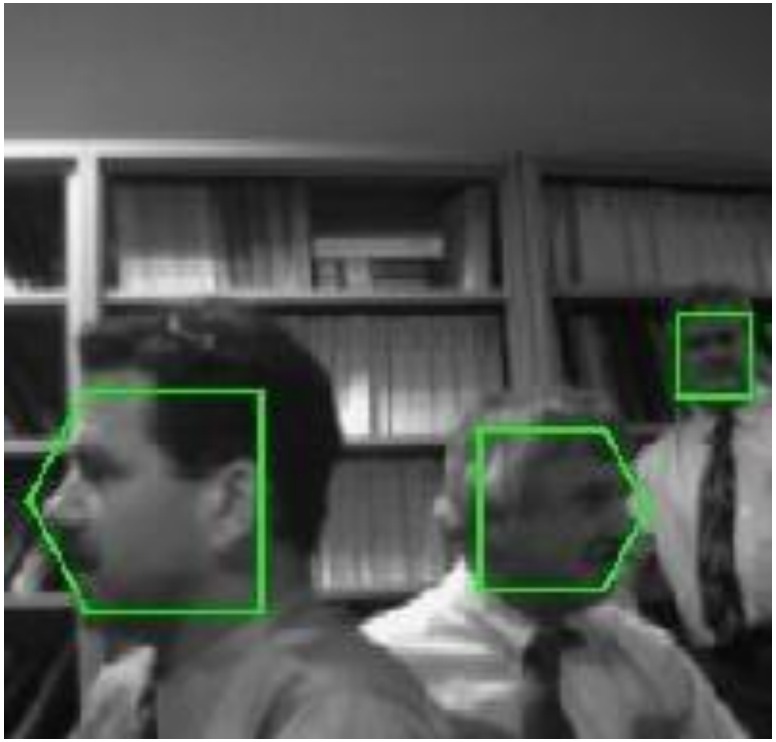
- Traffic Monitoring





Applications and Research Topics

- Face Detection

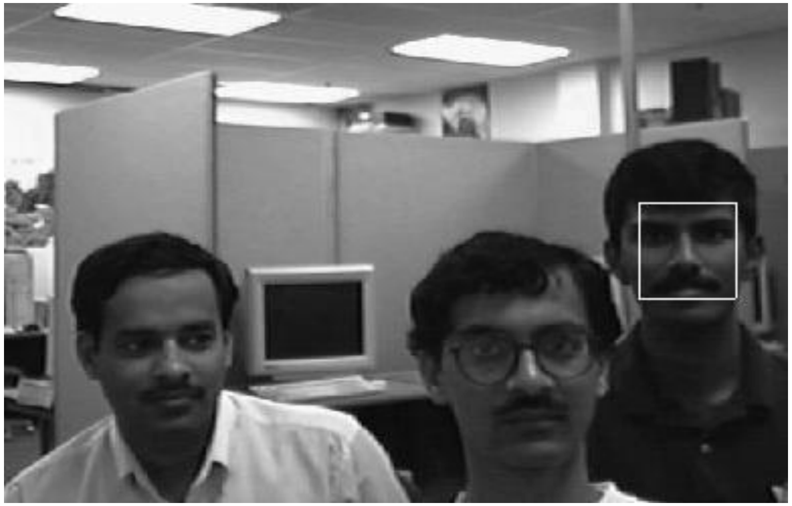


Applications and Research Topics

- Face Recognition

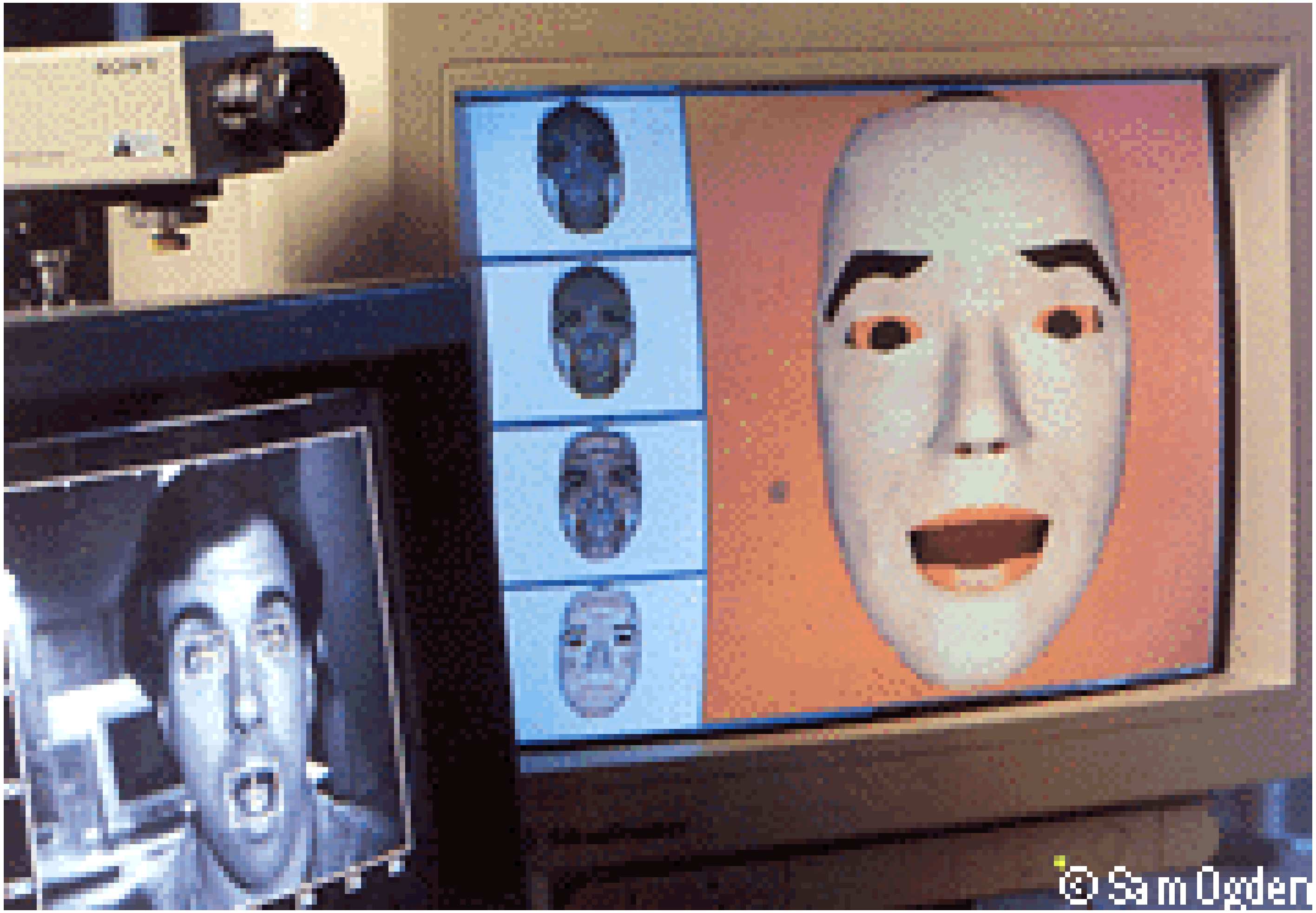
Applications and Research Topics

- Face Detection/Recognition Research



Applications and Research Topics

- Facial Expression Recognition



Applications and Research Topics

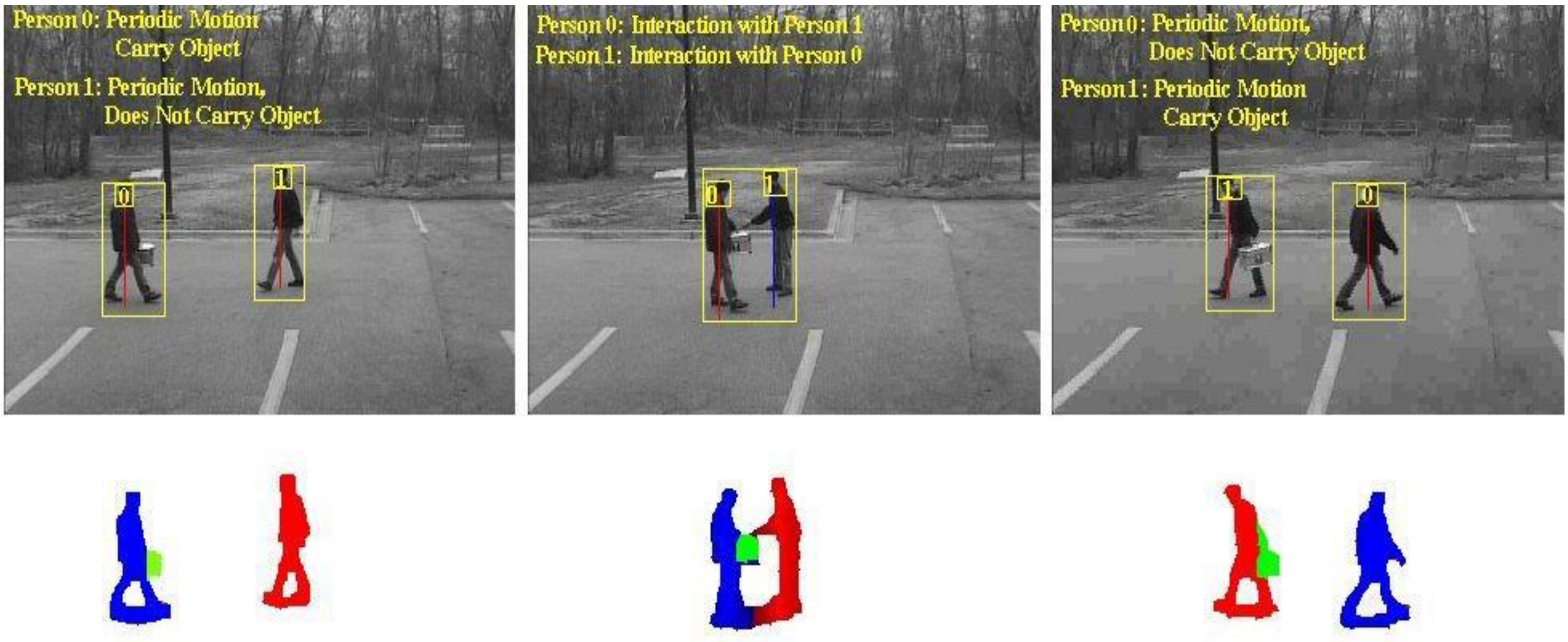
- Hand Gesture Recognition

  Smart Human-Computer User Interfaces

  Sign Language Recognition

Applications and Research Topics

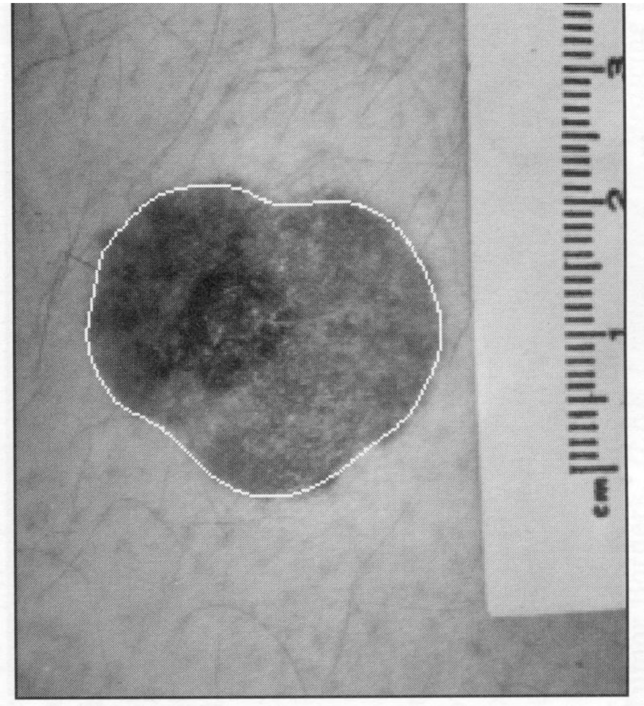
- Human Activity Recognition



Applications and Research Topics

- Medical Applications

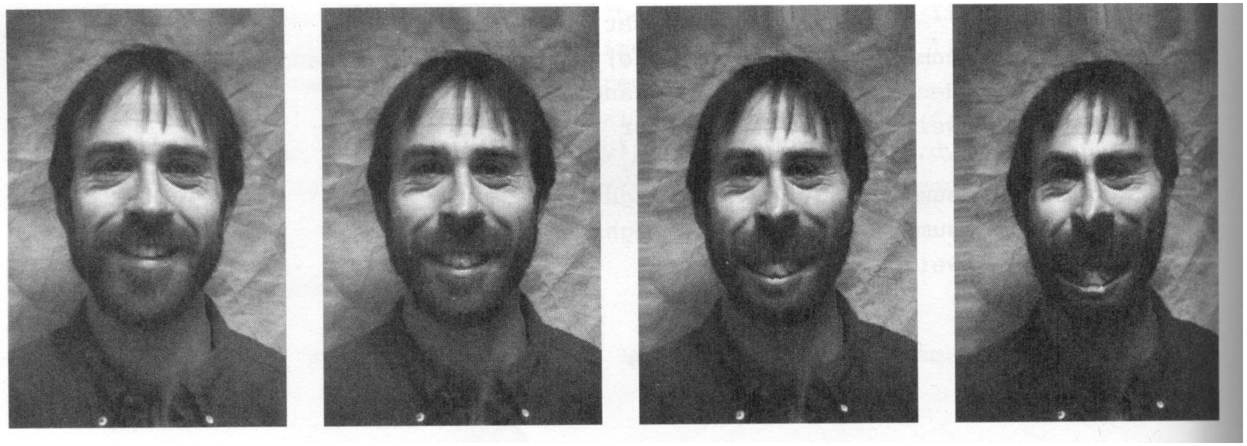
skin cancer

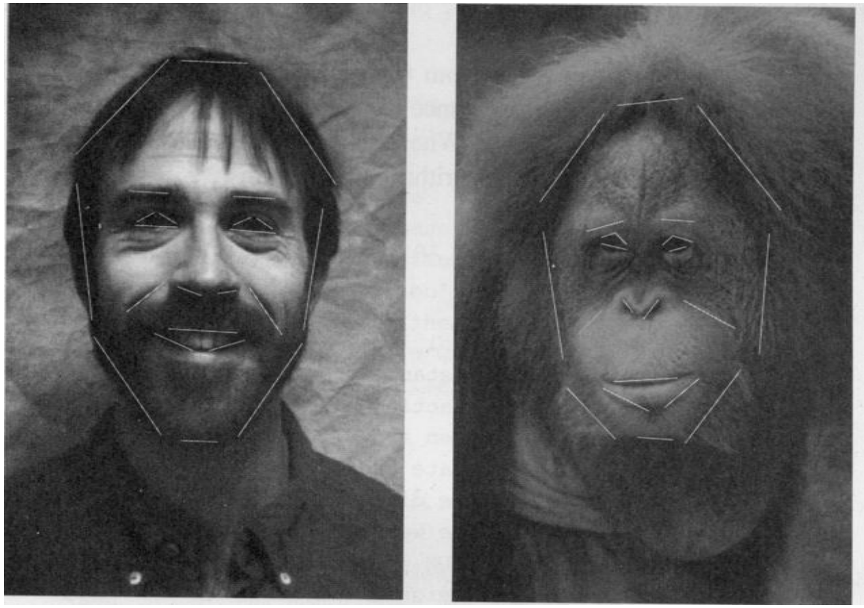


breast cancer

Applications and Research Topics

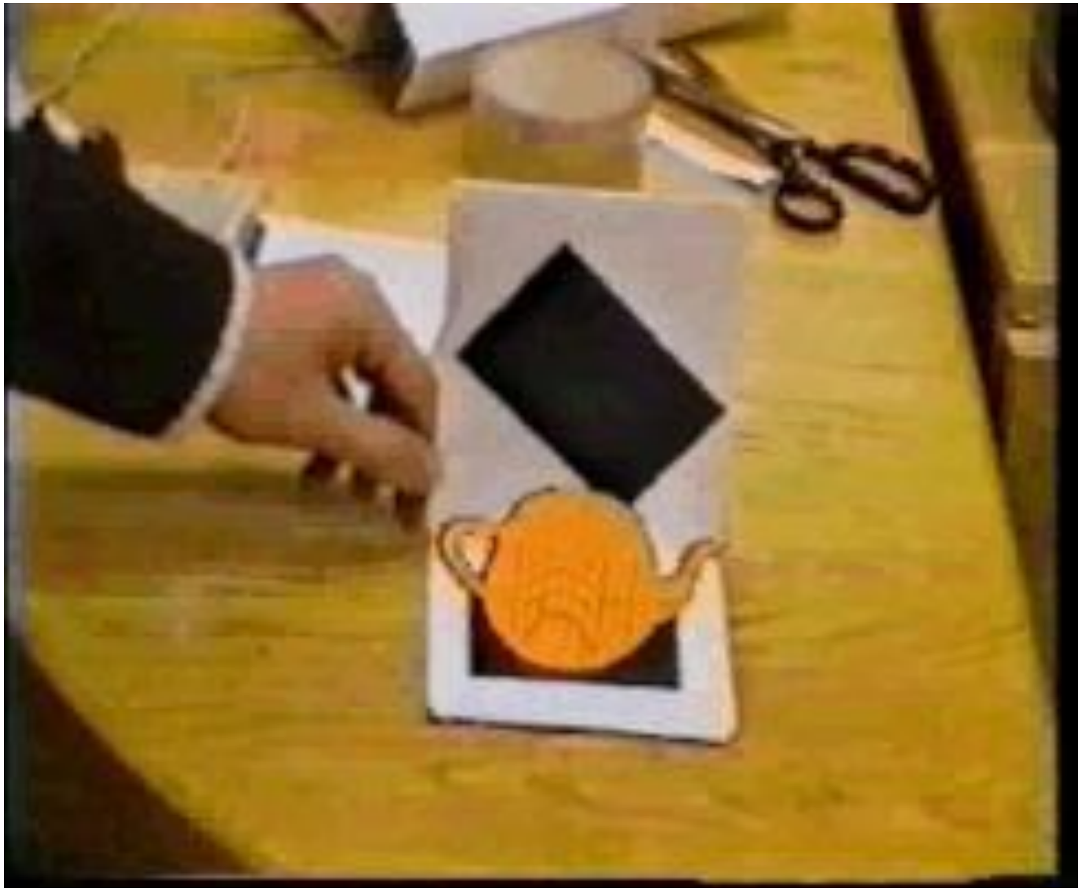
- Morphing



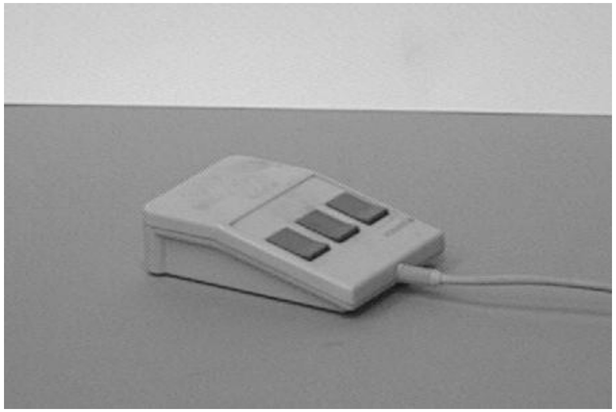


Applications and Research Topics

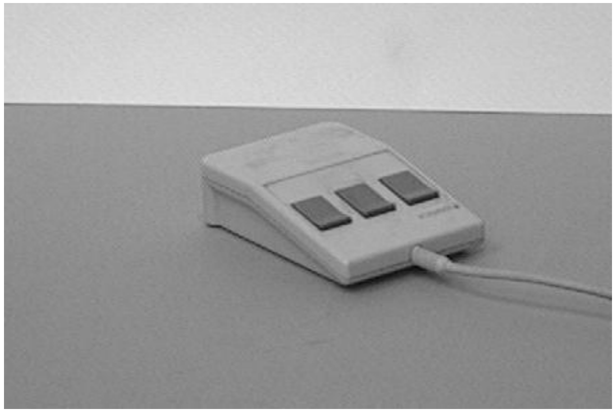
- Inserting Artificial Objects into a Scene



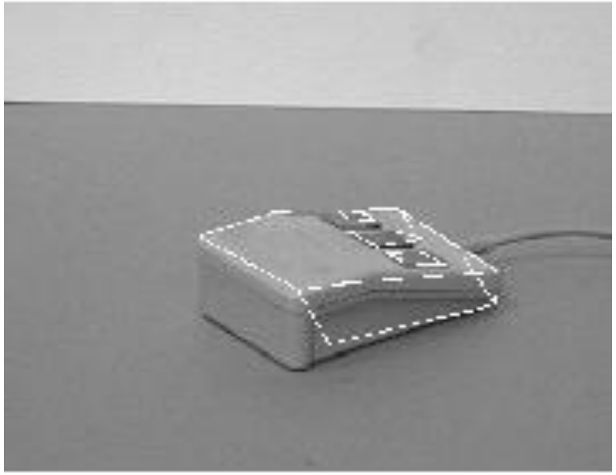
reference view 1



reference view 2

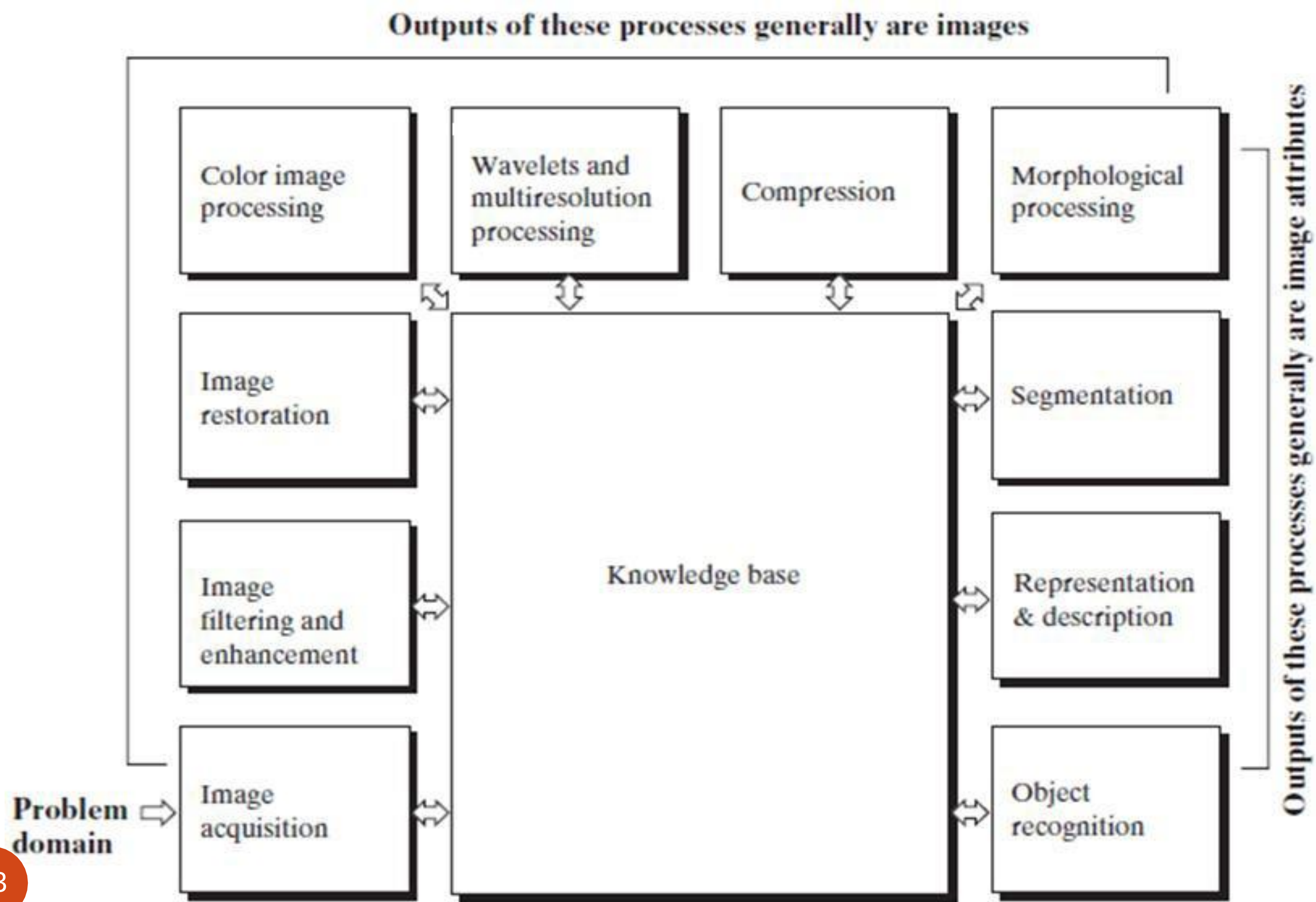


novel view recognized



Fundamental Steps in Digital Image Processing

Fundamental Steps in Digital Image Processing



Fundamental Steps in Digital Image Processing

Essential steps when processing digital images:

Acquisition

Enhancement

Restoration

Color image restoration

Wavelets

Morphological processing

Segmentation

Representation

Recognition

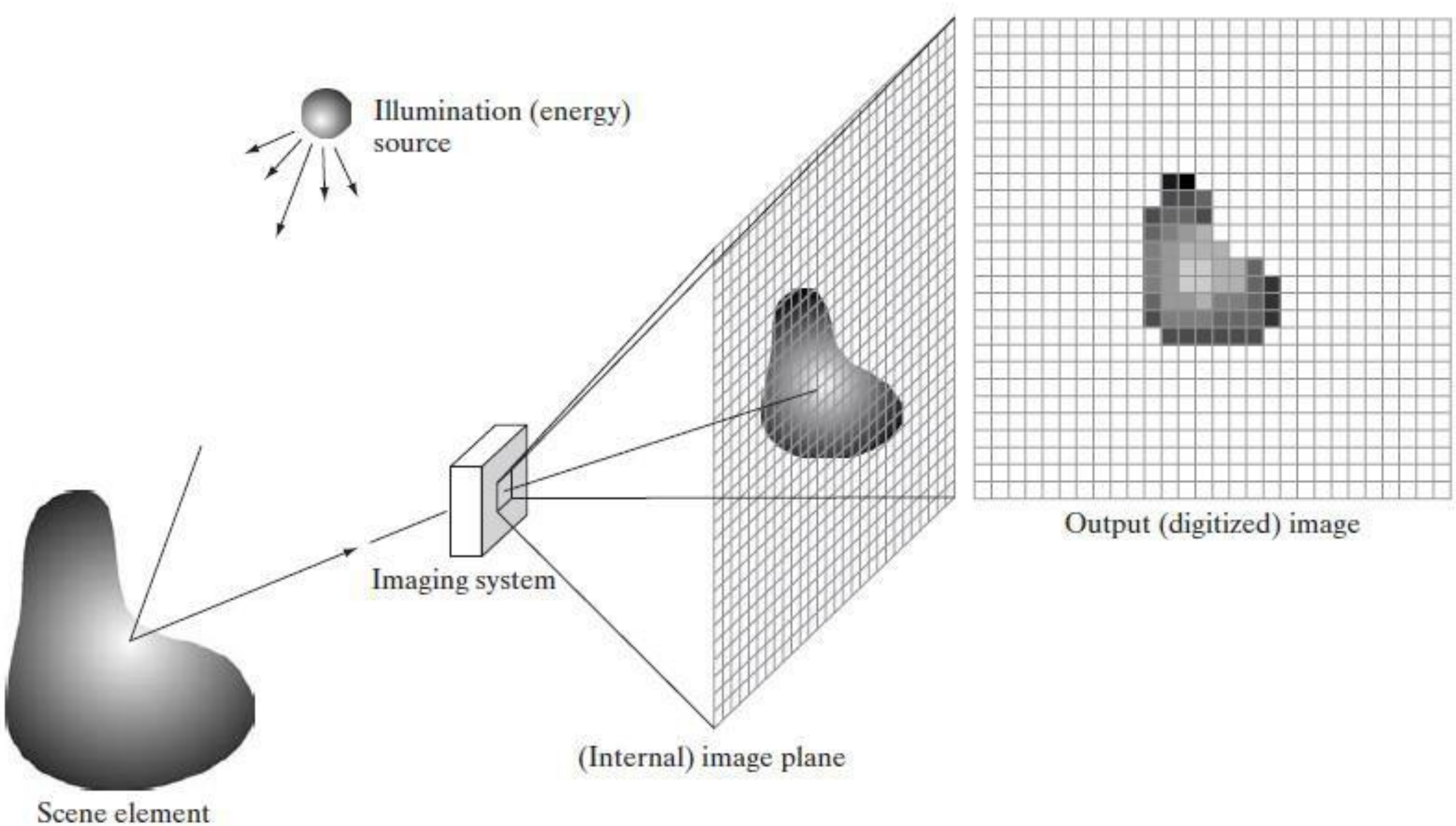
Outputs are digital images

Outputs are attributes of the image

Fundamental Steps in Digital Image Processing

- ***Image acquisition*** is the first process.

  Generally, the image acquisition stage involves preprocessing, such as scaling.

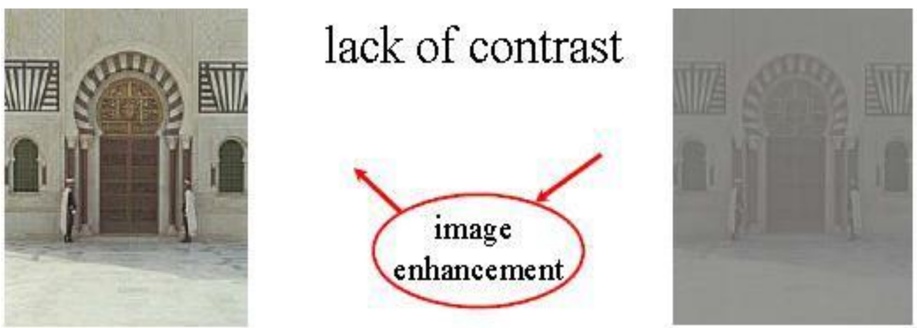


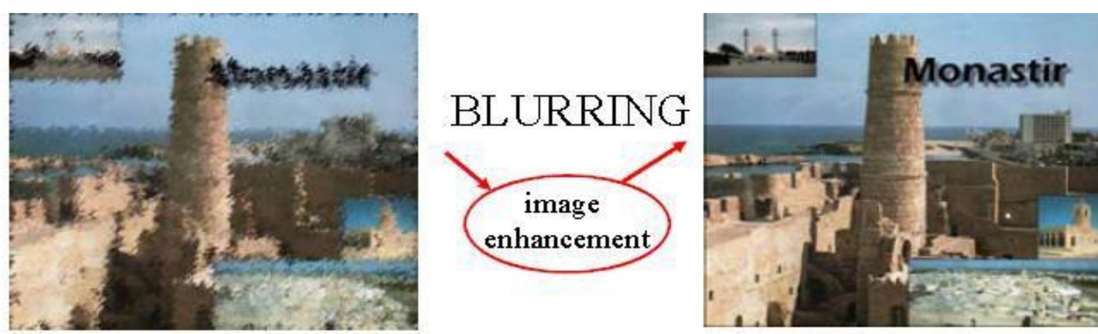
Fundamental Steps in Digital Image Processing

- ***Image enhancement*** is the process of manipulating an image so that the result is more suitable than the original for a specific application.

  There is no general "theory" of image enhancement.

  When an image is processed for visual interpretation, the viewer is the ultimate judge of how well a particular method works.

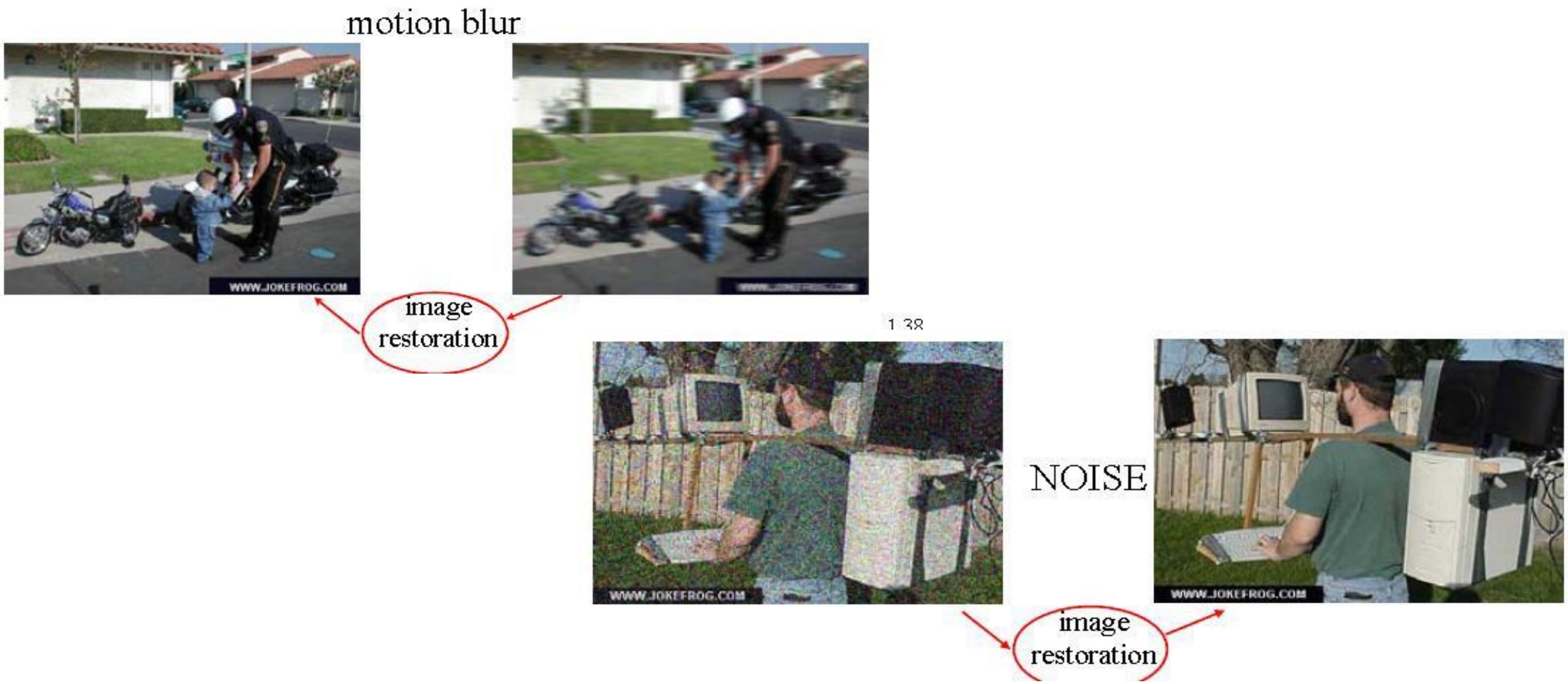




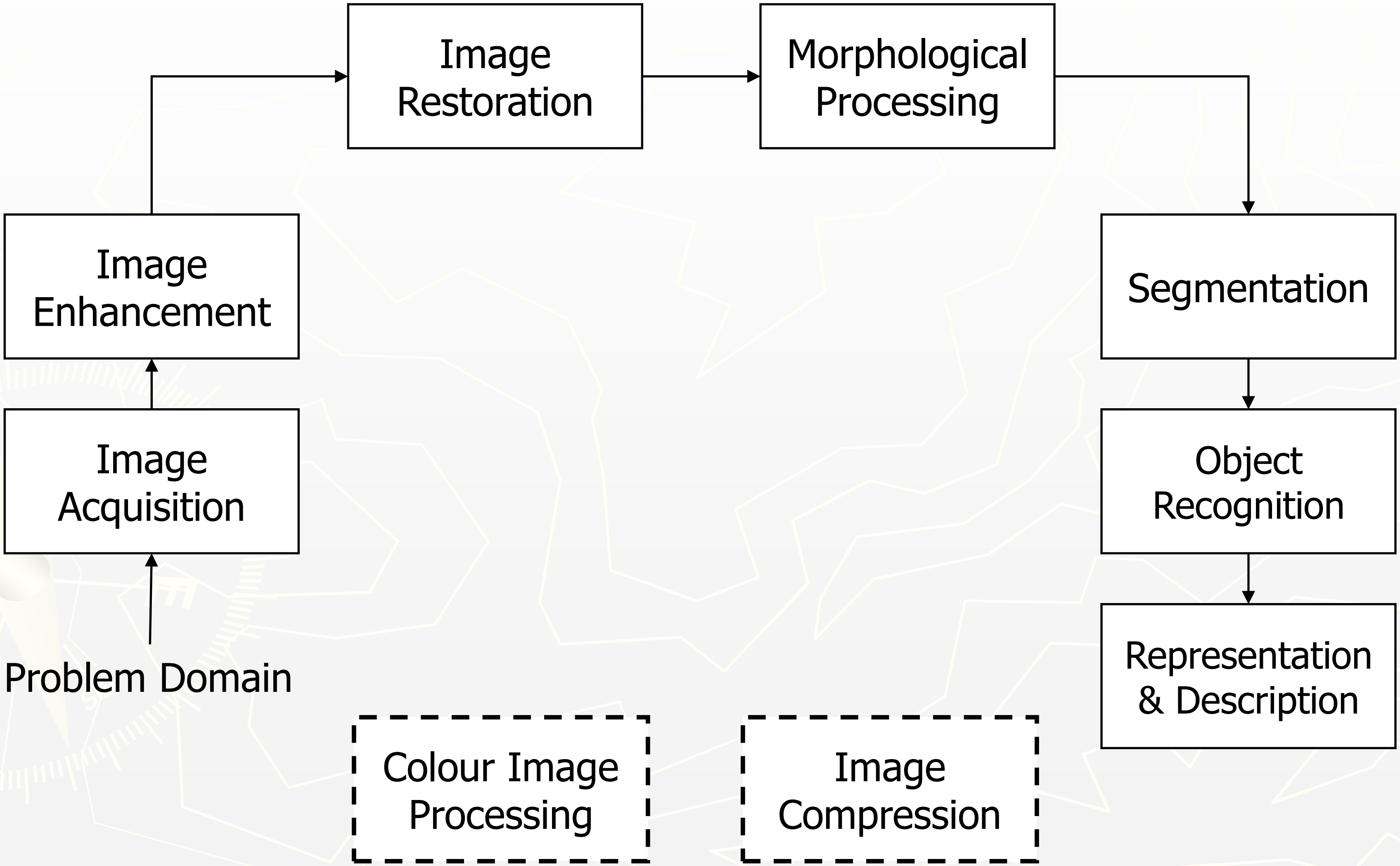
Fundamental Steps in Digital Image Processing

- ***Image Restoration*** is an area that also deals with improving the appearance of an image.

  However, unlike enhancement, which is subjective, image restoration is objective, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image degradation.



Key Stages in Digital Image Processing



Key Stages in Digital Image Processing: Image Aquisition

Image Restoration

Morphological Processing

Image Enhancement

Segmentation

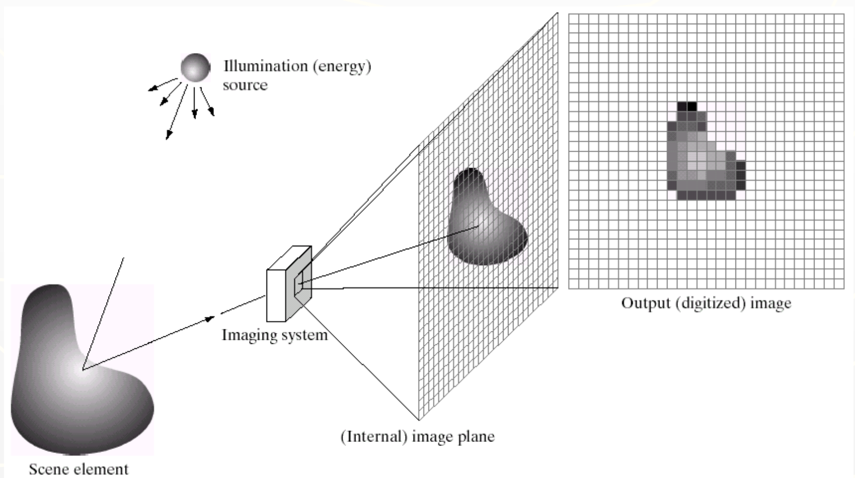


Image Acquisition

Object Recognition

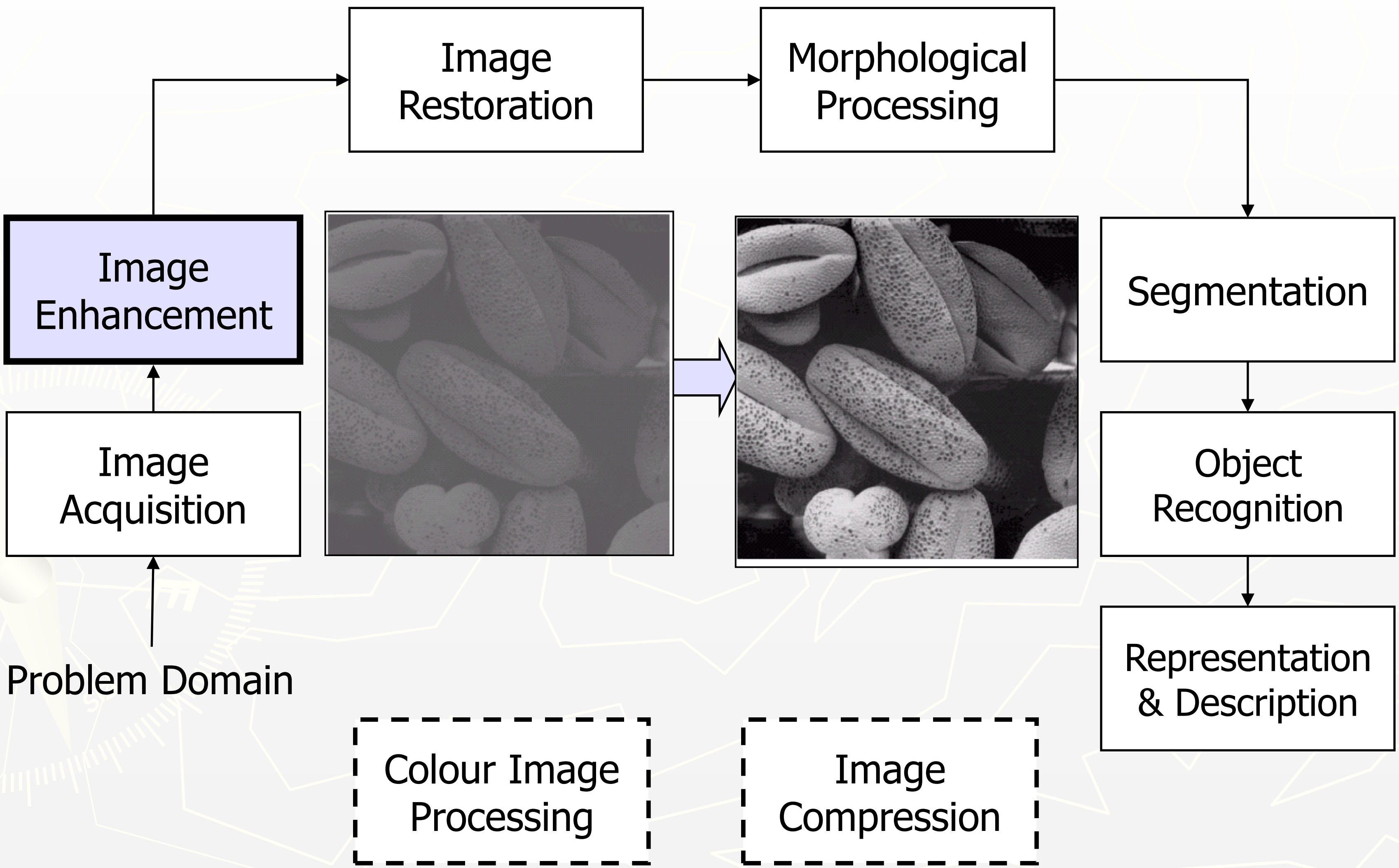
Representation & Description

Problem Domain

Colour Image Processing

Image Compression

Key Stages in Digital Image Processing: Image Enhancement



Key Stages in Digital Image Processing: Image Restoration

Image Acquisition

Image Enhancement

Image Restoration

Morphological Processing

Segmentation

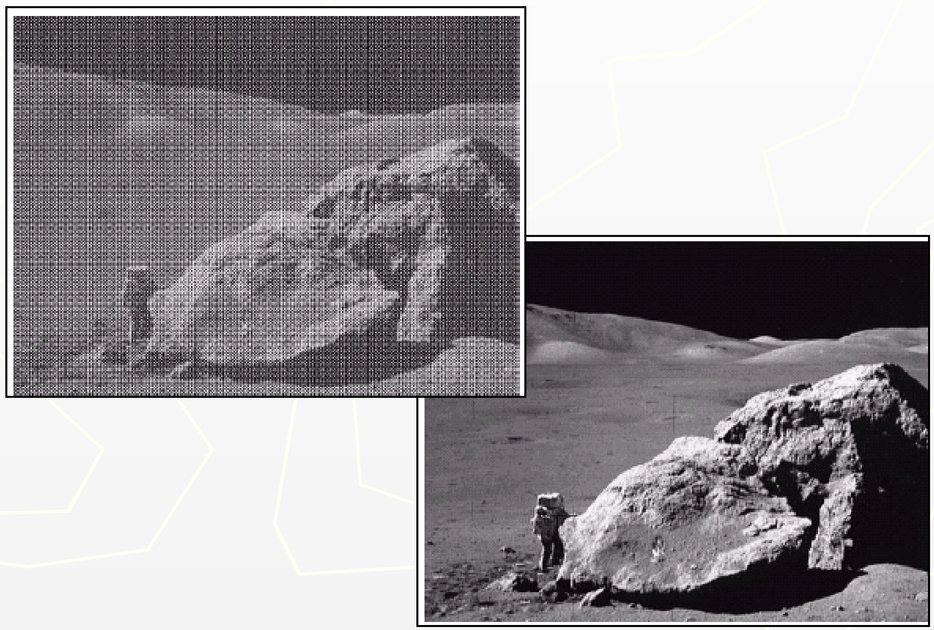
Object Recognition

Representation & Description

Problem Domain

Colour Image Processing

Image Compression



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Morphological Processing

Image Acquisition

Image Enhancement

Image Restoration

Morphological Processing

Segmentation

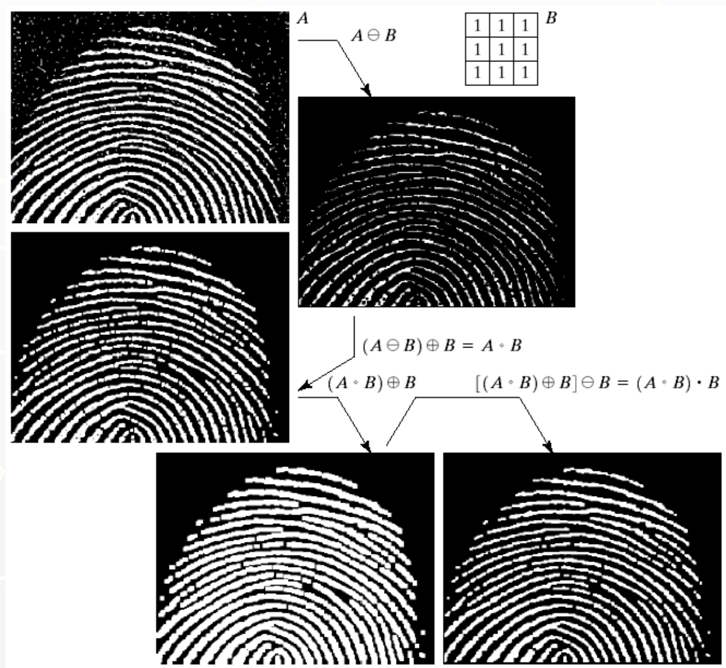
Object Recognition

Representation & Description

Colour Image Processing

Image Compression

Problem Domain



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Segmentation

Image Acquisition

Problem Domain

Image Enhancement

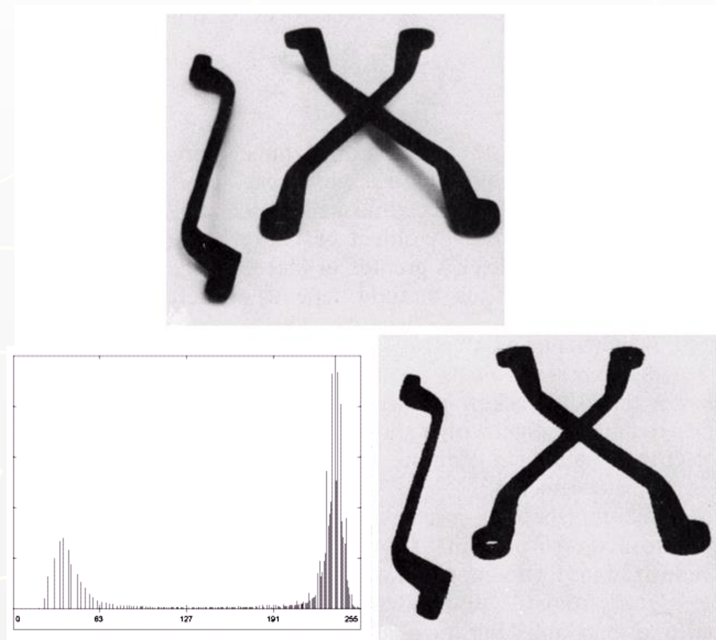
Image Restoration

Morphological Processing

Segmentation

Object Recognition

Representation & Description



Colour Image Processing

Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Object Recognition

Image Restoration

Morphological Processing

Image Enhancement

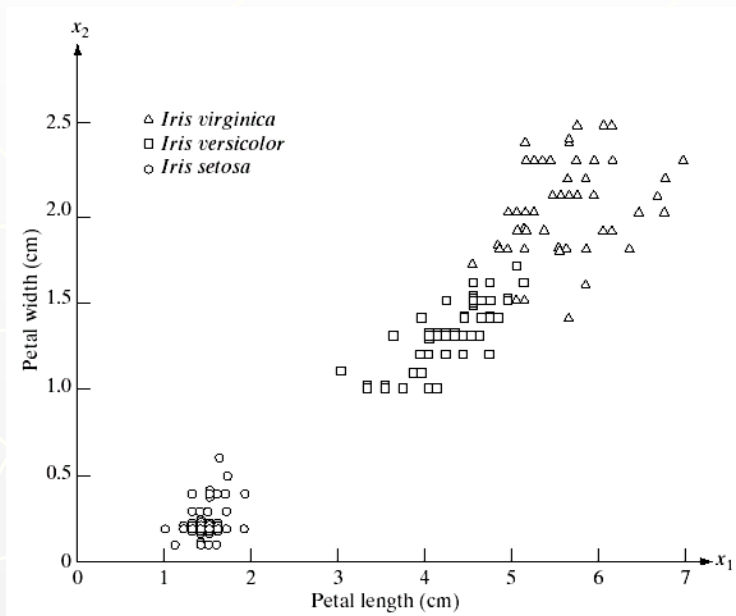
Segmentation

Image Acquisition

Object Recognition

Problem Domain

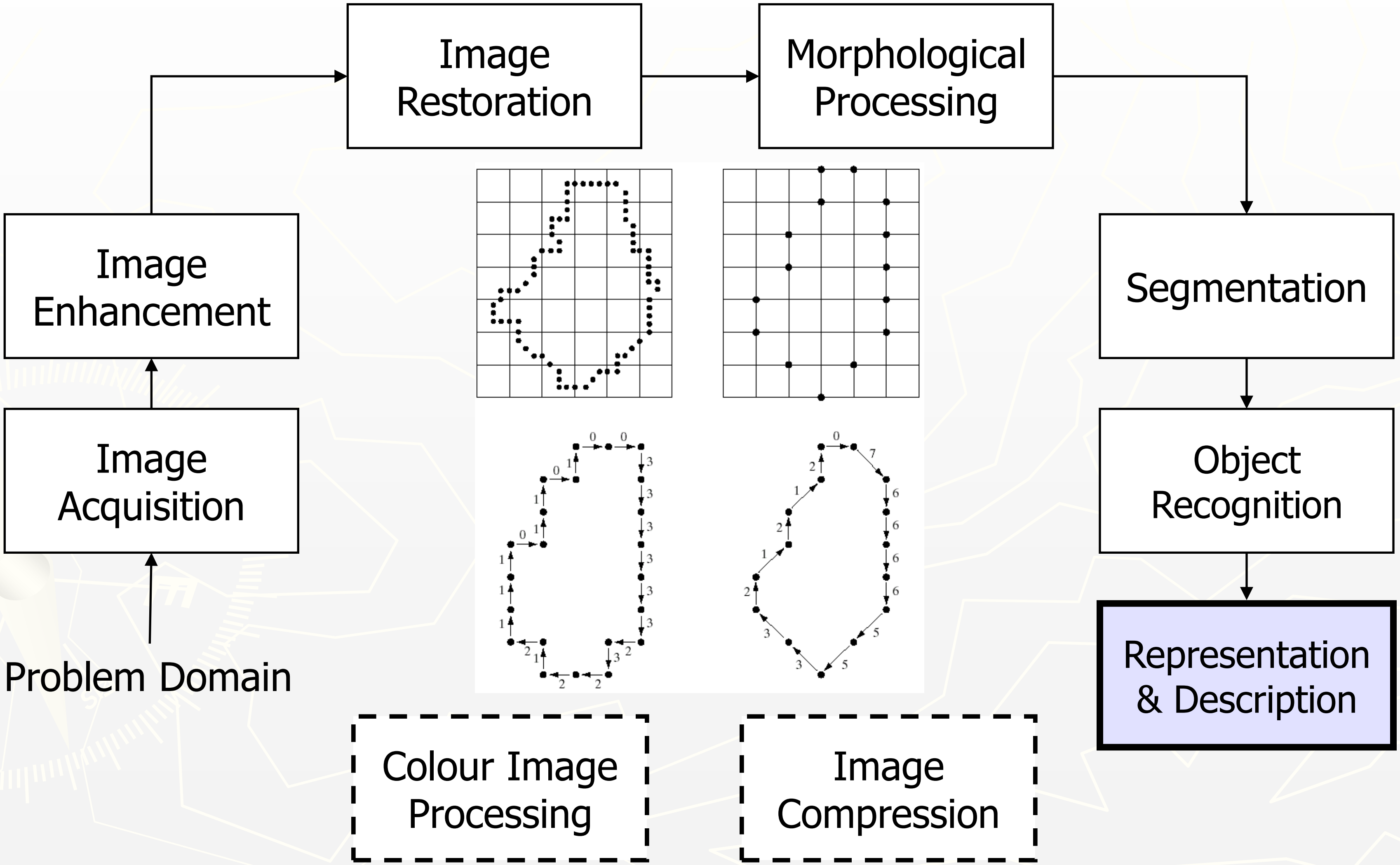
Representation & Description



Colour Image Processing

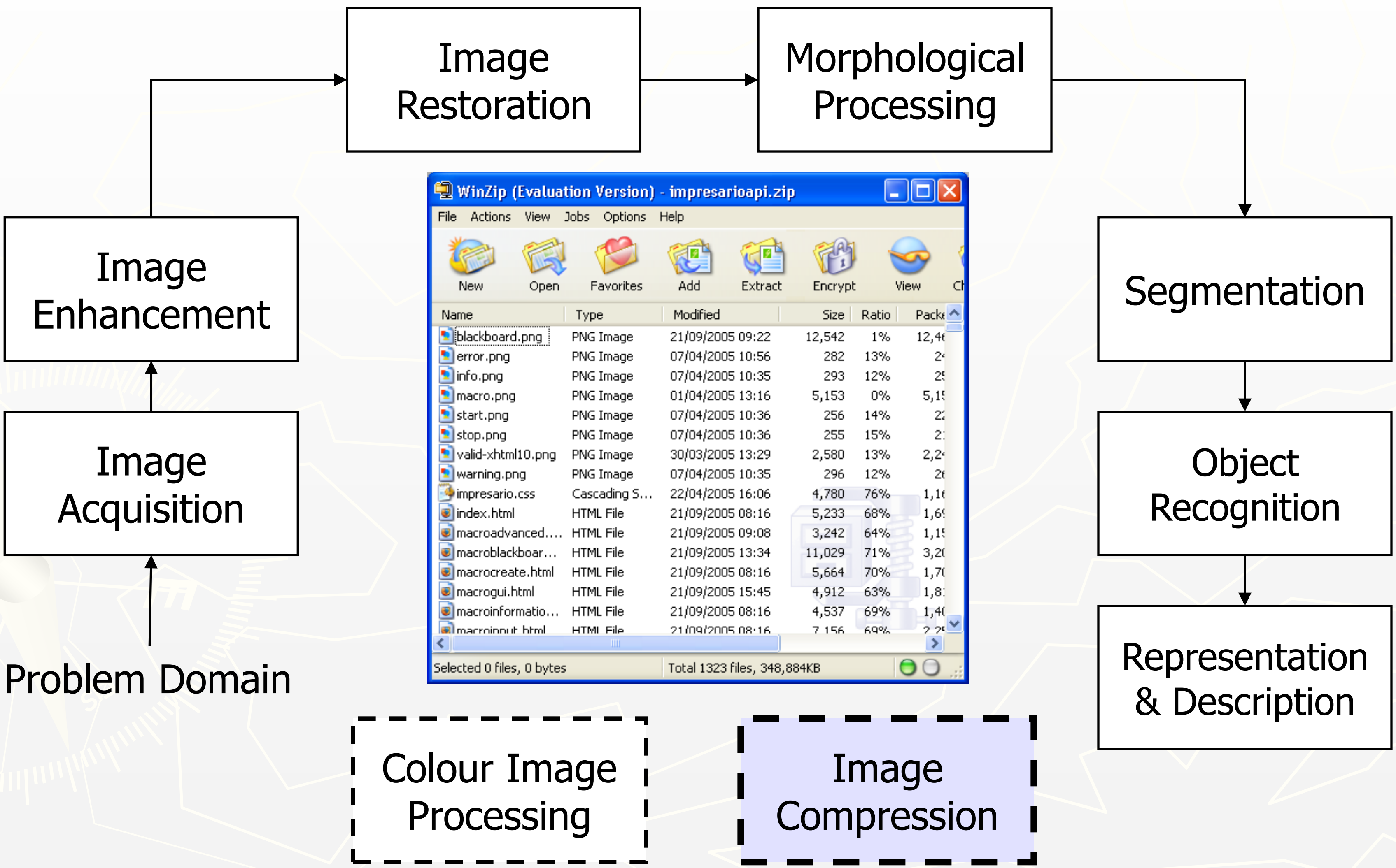
Image Compression

Key Stages in Digital Image Processing: Representation & Description



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Image Compression

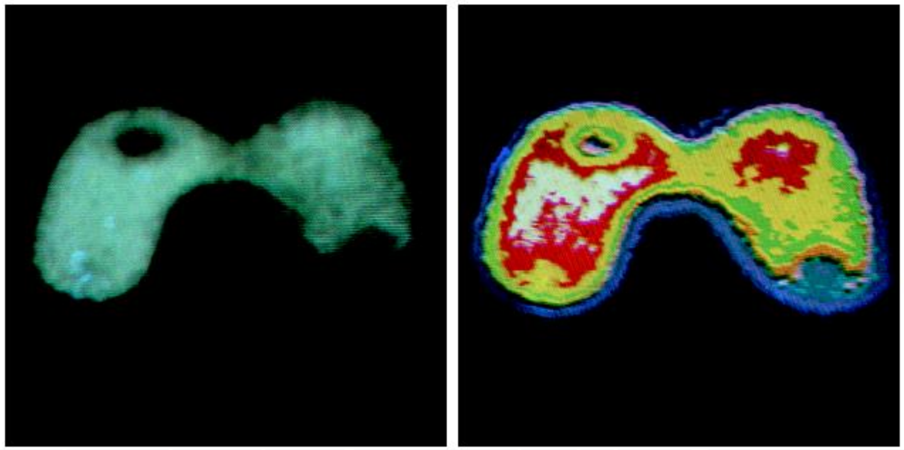


Key Stages in Digital Image Processing: Colour Image Processing

Image Restoration

Morphological Processing

Image Enhancement



Segmentation

Image Acquisition

Object Recognition

Representation & Description

Problem Domain

Colour Image Processing

Image Compression

Fundamental Steps in Digital Image Processing

- ***Color Image Processing*** is an area that has been gaining in importance because of the significant increase in the use of digital images over the Internet.

- ***Wavelets*** are the foundation for representing images in various degrees of resolution.

Fundamental Steps in Digital Image Processing

- ***Compression***, as the name implies, deals with techniques for reducing the storage required to save an image, or the bandwidth required to transmit it. This is true particularly in uses of the Internet.

- ***Morphological processing*** deals with tools for extracting image components that are useful in the representation and description of shape.

- ***Segmentation*** procedures partition an image into its constituent parts or objects.

  A segmentation procedure brings the process a long way towards successful solution of imaging problems that require objects to be identified individually.

  In general, the more accurate the segmentation, the more likely recognition is to succeed.

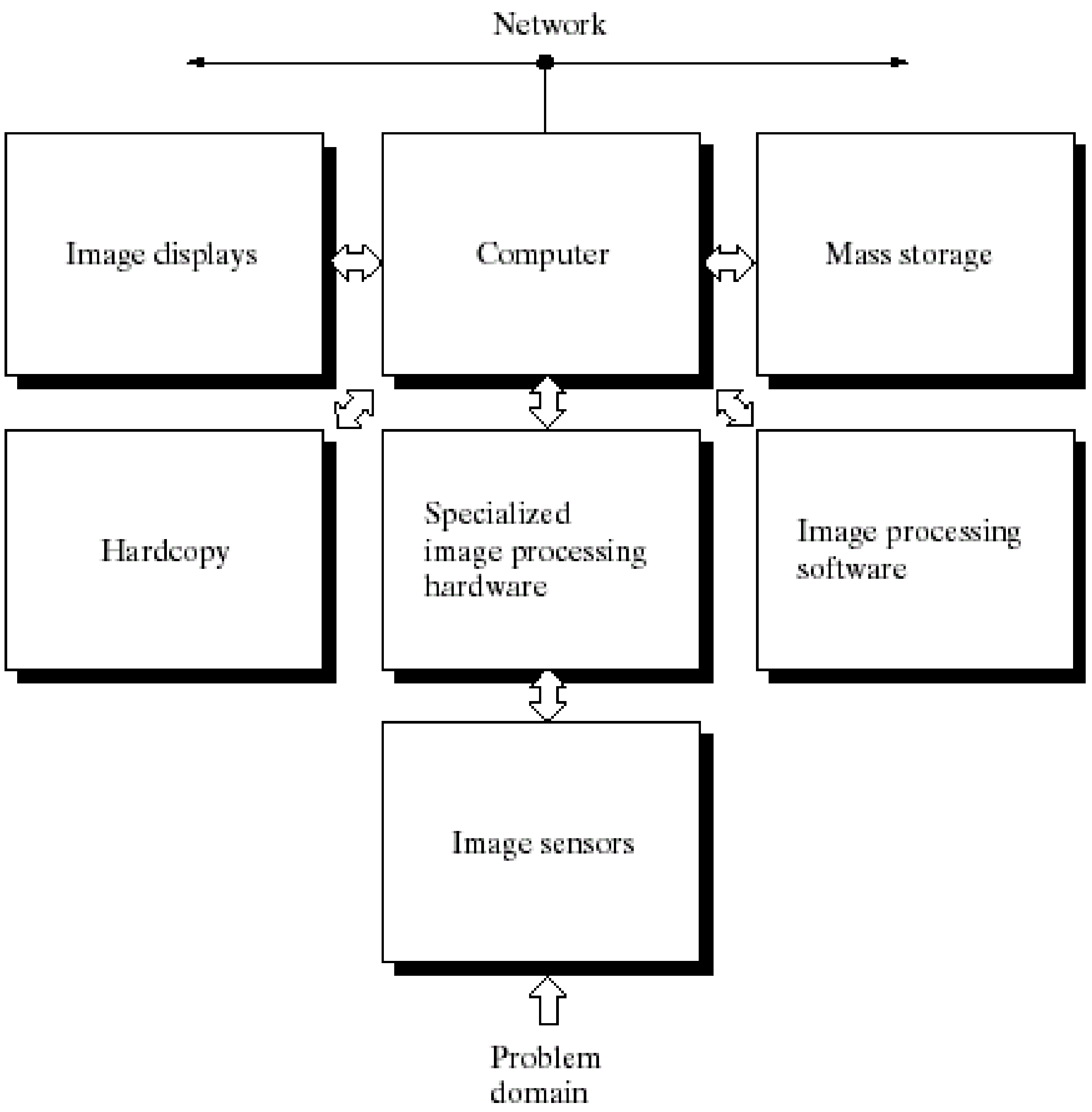
Fundamental Steps in Digital Image Processing

- **Representation and description** almost always follow the output of a segmentation stage, which usually is raw pixel data.
  - Boundary representation is appropriate when the focus is on external shape characteristics, such as corners and inflections.
  - Regional representation is appropriate when the focus is on internal properties, such as texture or skeletal shape.

  Description, also called feature selection; deals with extracting attributes that result in some quantitative information of interest or are basic for differentiating one class of objects from another.

- ***Recognition*** is the process that assigns a label (e.g., "vehicle") to an object based on its descriptors. Digital image processing with the development of methods for recognition of individual objects.

General Purpose Image Processing System



General Purpose Image Processing System

- ***Specialized image processing hardware*** usually consists of the digitizer, plus hardware that performs other primitive operations, such as (ALU), that performs arithmetic and logical operations in parallel on entire images.

- This type of hardware sometimes is called a *front-end subsystem,* and its most distinguishing characteristic is speed.

Image Processing Basics

Image Representation

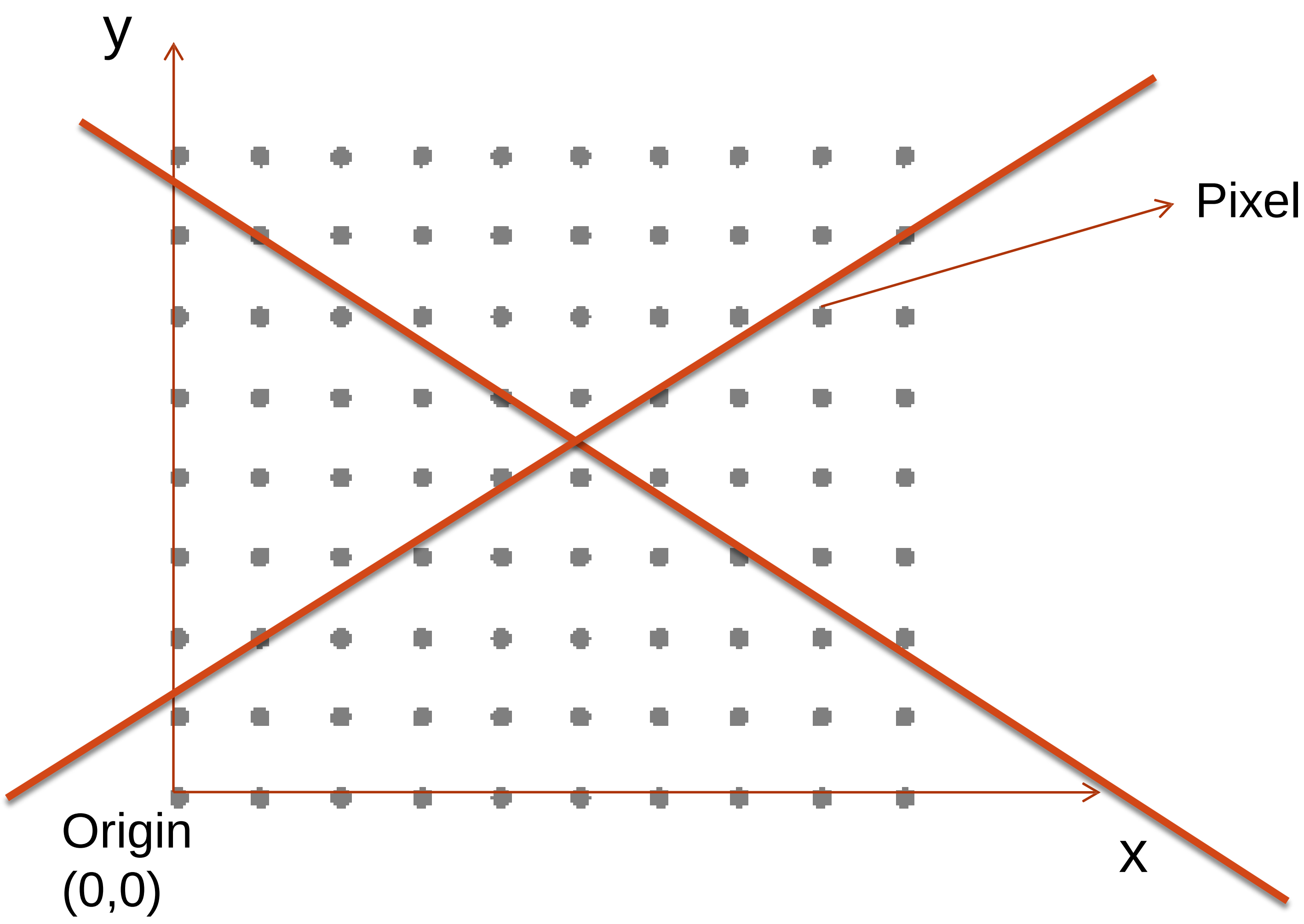
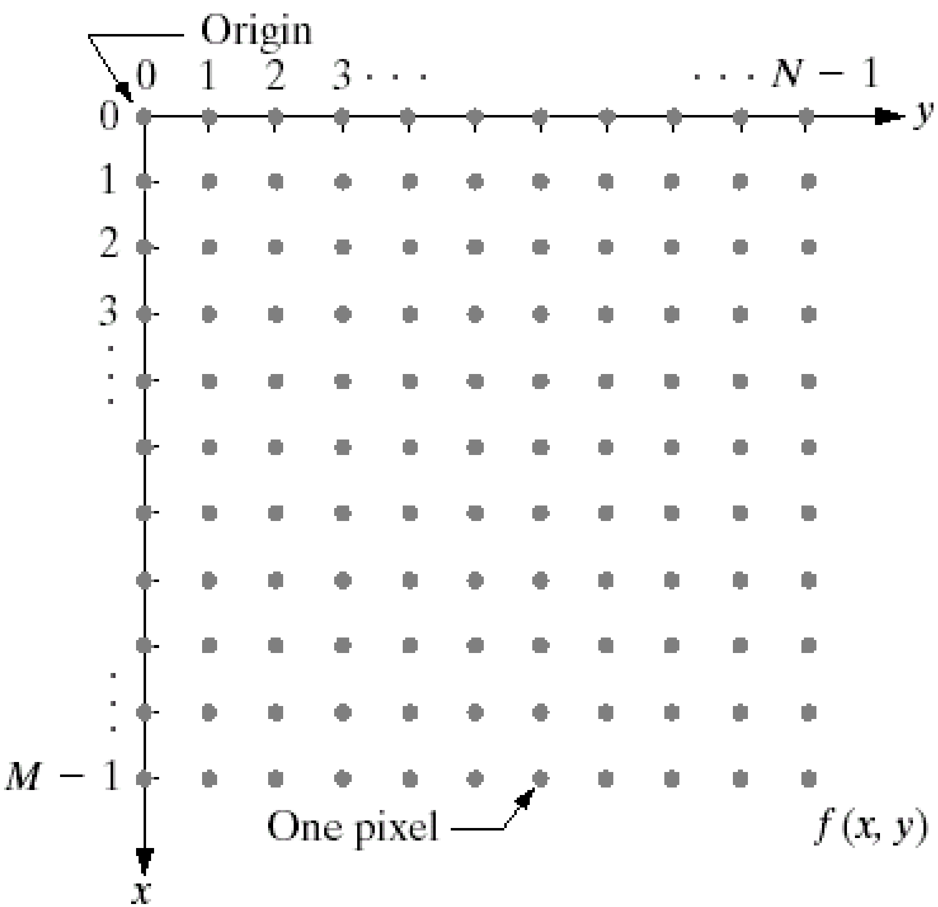
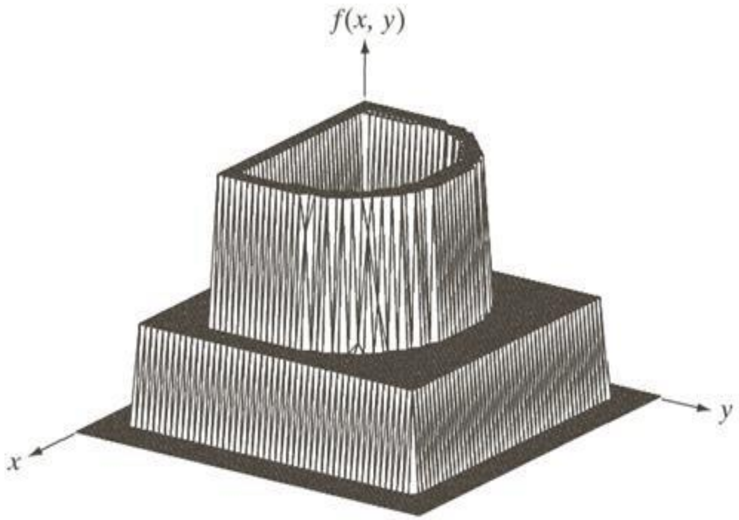


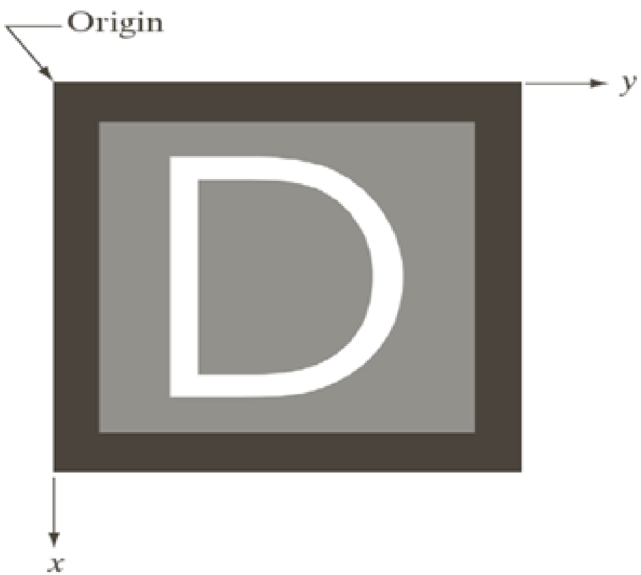
Image Representation



- A digital image is composed of M rows and N columns of pixels each storing a value
- Pixel values are most often grey levels in the range 0-255(black-white)
- Images can easily be represented as matrices.

Image Representation





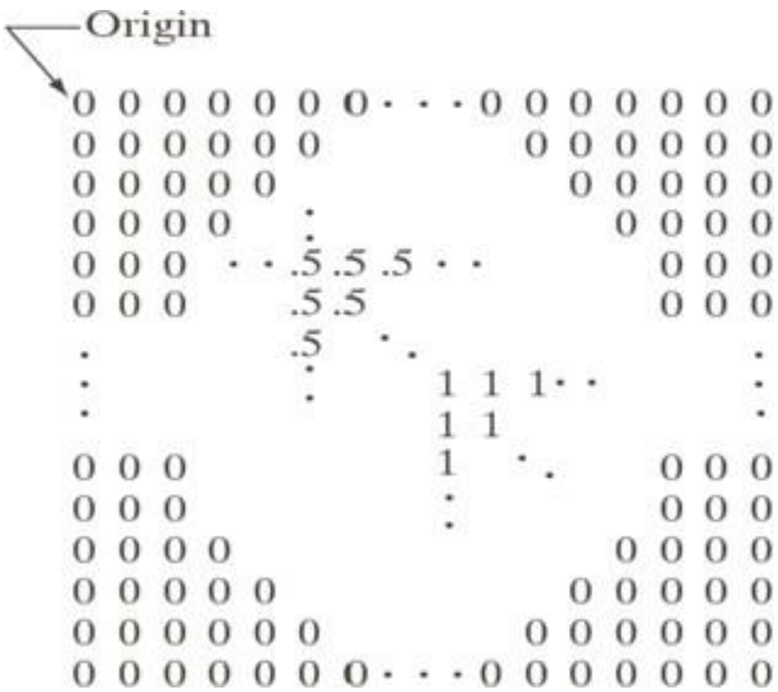
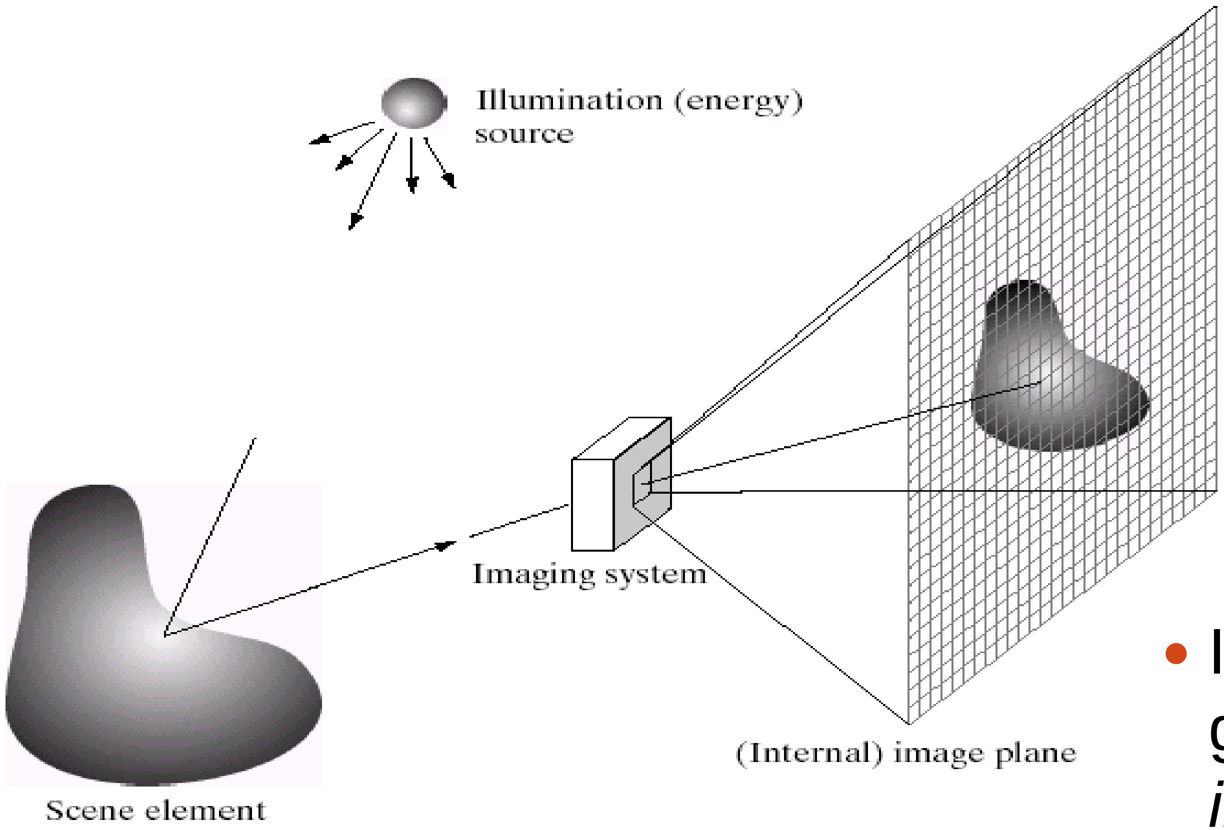


Image Acquisition



- Images are typically generated by *illuminating* a *scene* and absorbing the energy reflected by the objects in that scene

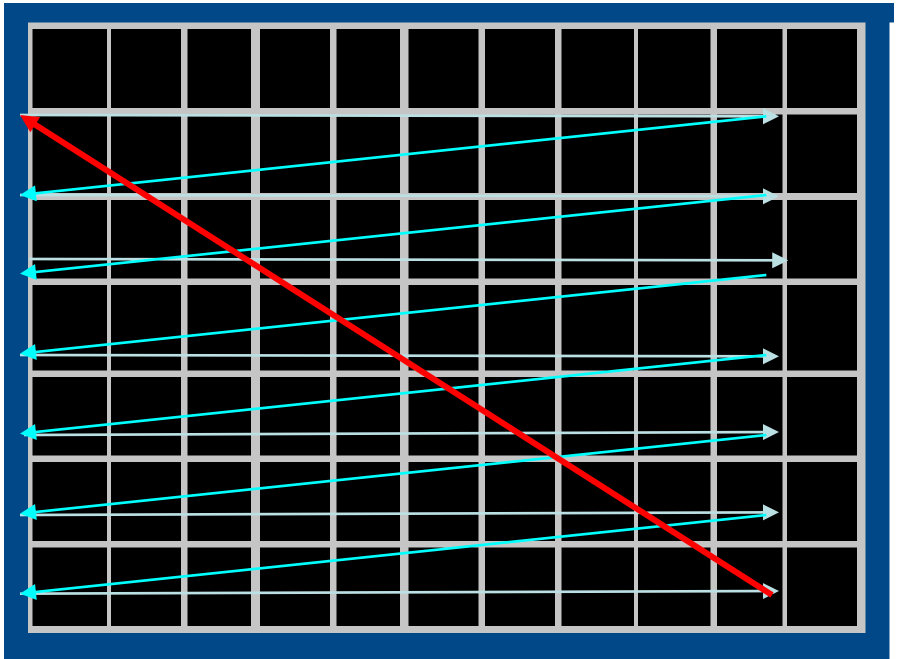
Raster-Scan Display

- Frame must be “refreshed” to draw new images
- As new pixels are struck by electron beam, others are decaying
- Electron beam must hit all pixels frequently to eliminate flicker
- Critical fusion frequency
  - Typically 60 times/sec
  - Varies with intensity, individuals, phosphor persistence, lighting...

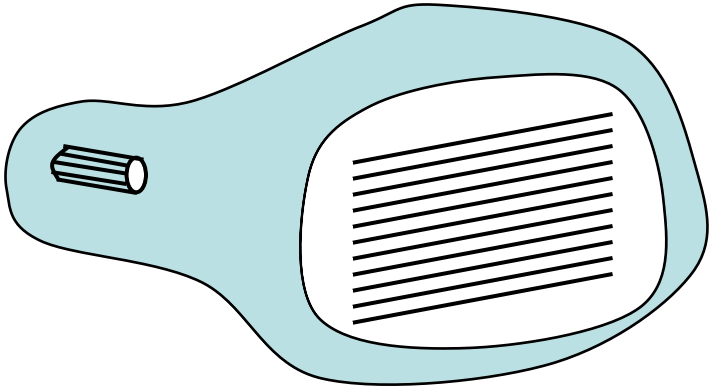
Scanning *(left to right, top to bottom)*

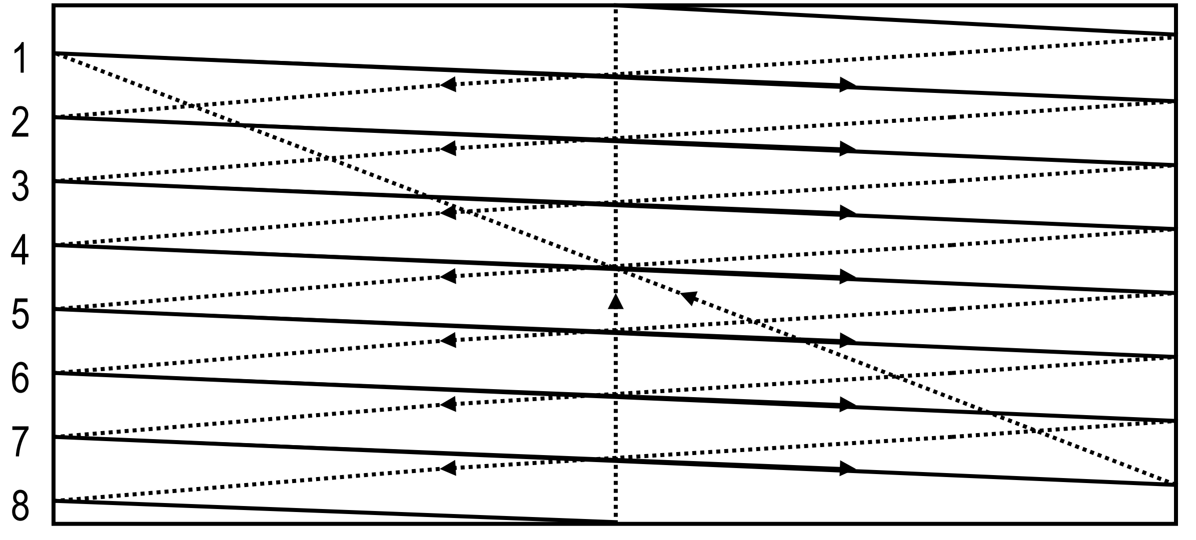
- **HORIZONTAL SYNC PULSE** — Signals the start of the new scan line.
- **HORIZONTAL RETRACE** — Time needed to get from the end of the current scan line to the start of the next scan line.
- **VERTICAL SYNC PULSE** — Signals the start of the next field.
- **VERTICAL RETRACE** — Time needed to get from the bottom of the current field to the top of the next field.

During Horizontal and Vertical retrace electron gun is turn off and so no phosphor is illuminated



Interlacing





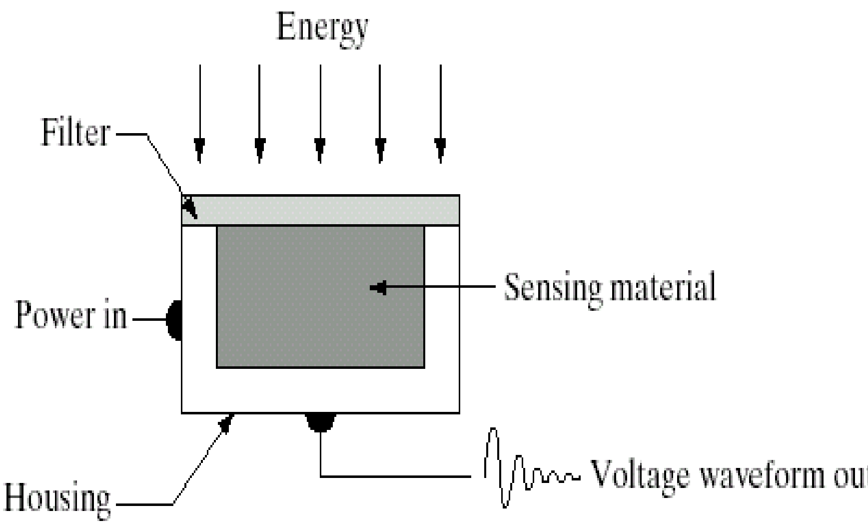
Interlacing

- Assume can only scan 30 frames/second
- To reduce flicker, divide frame into two "fields" of odd and even lines

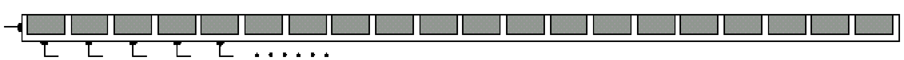
1/30 Sec		1/30 Sec	
1/60	1/60 Sec	1/60 Sec	1/60 Sec
Field 1	Field 2	Field 1	Field 2
Frame		Frame	

Image Sensing

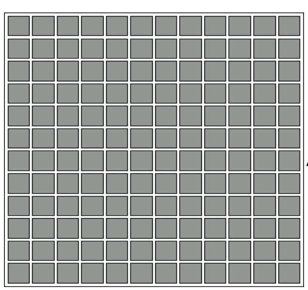
- Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage
- Collections of sensors are arranged to capture images



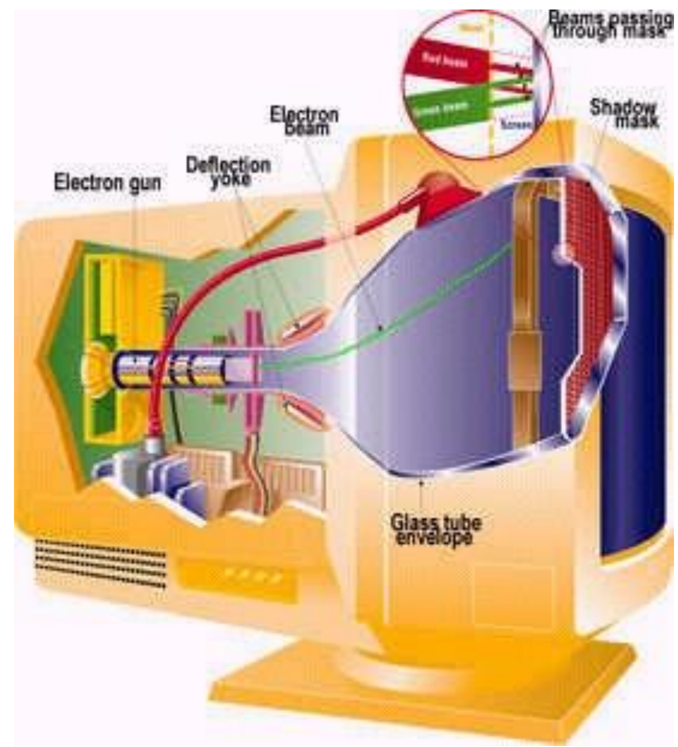
Imaging Sensor



Line of Image Sensors



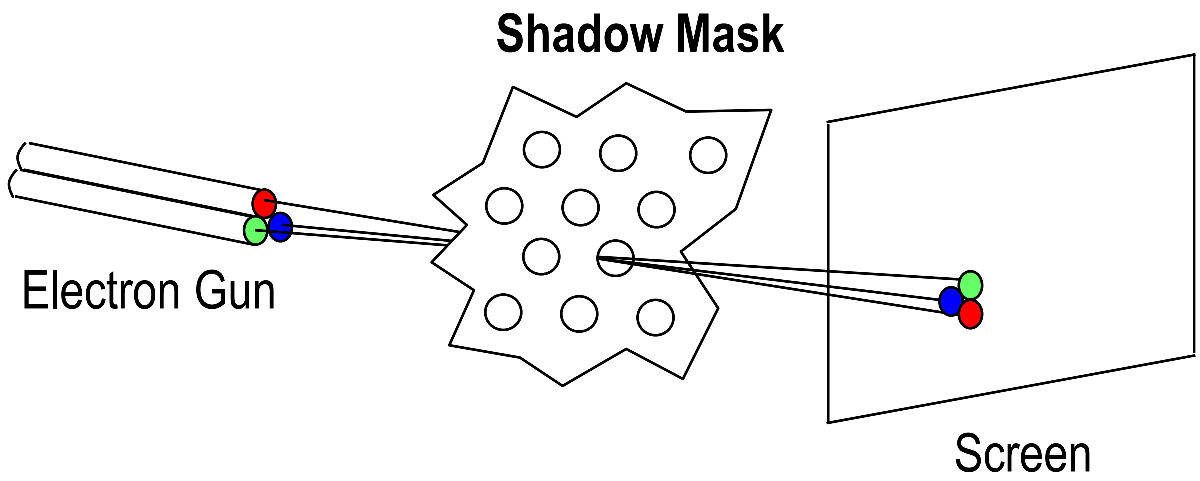
Array of Image Sensors

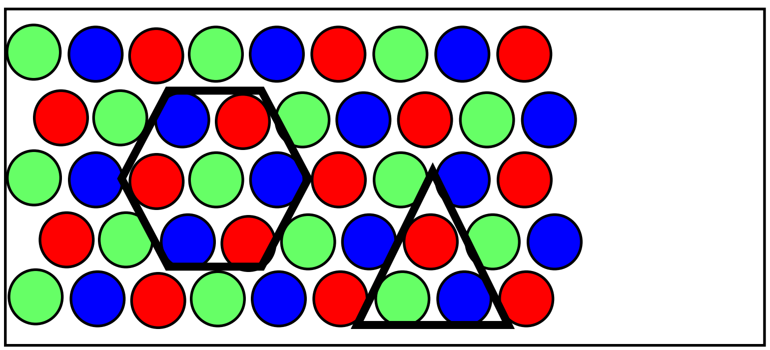


Phosphors

- Fluorescence: Light emitted while the phosphor is being struck by electrons
- Phosphorescence: Light emitted once the electron beam is removed
- Persistence: The time from the removal of the excitation to the moment when phosphorescence has decayed to 10% of the initial light output
- CRT screen resolution depends on *phosphor spot intensity distribution* (Gaussian)

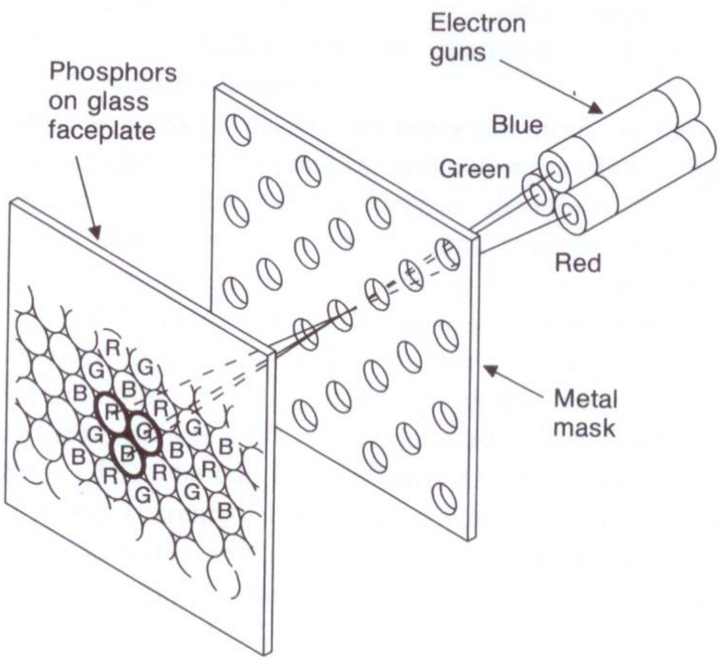
Color CRT (Shadow Mask)



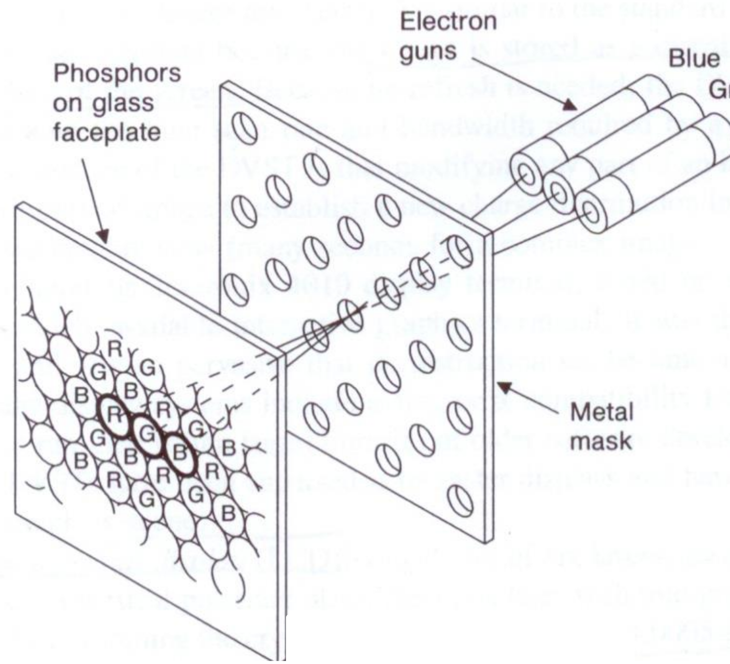


Phosphor dot pattern

Different phosphor for each color !!!

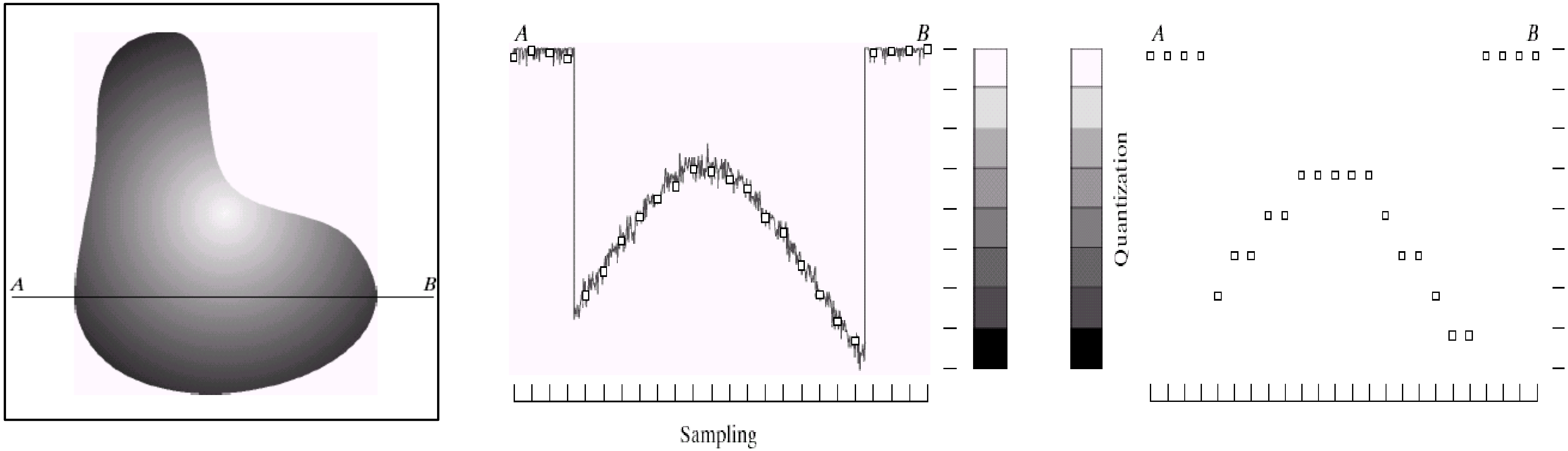


Delta-delta shadow-mask CRT



Precision in-line delta CRT

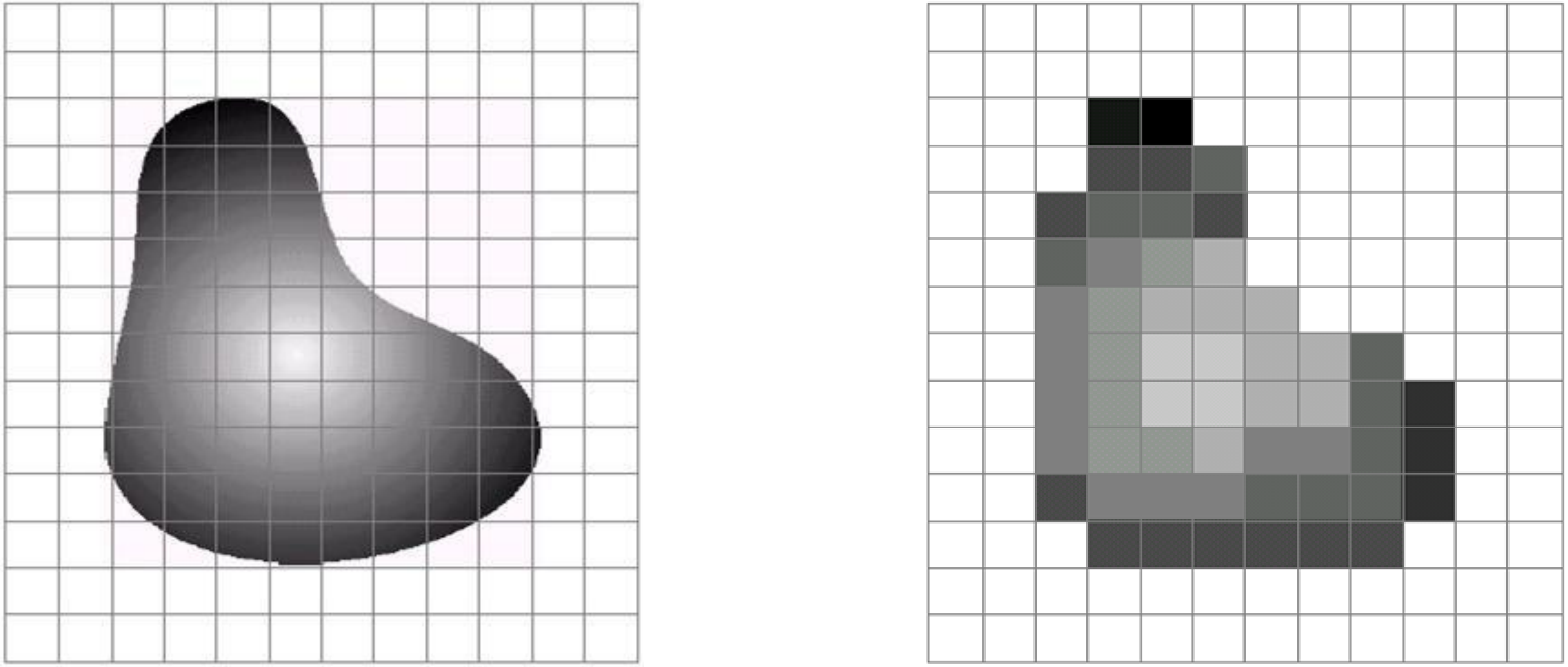
Image Sampling And Quantization

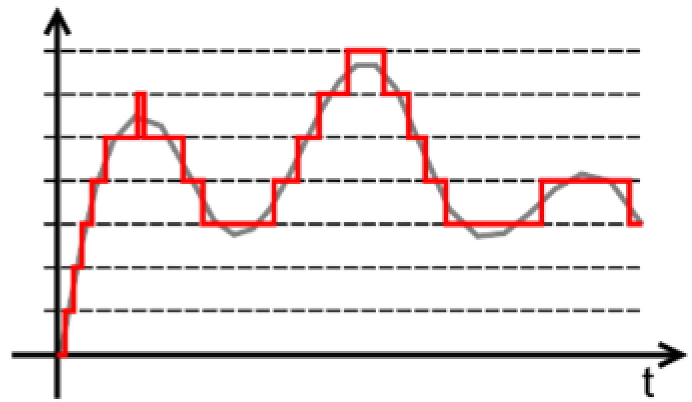


- A digital sensor can only measure a limited number of **samples** at a **discrete** set of energy levels
- *Quantisation* is the process of converting a continuous **analogue** signal into a digital representation of this signal

Image Sampling And Quantization

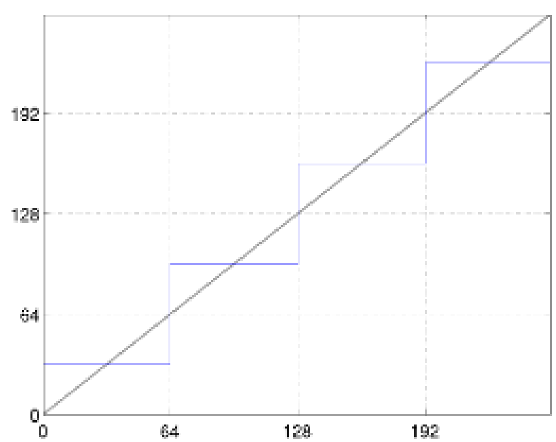
- Remember that a digital image is always only an **approximation** of a real world scene.



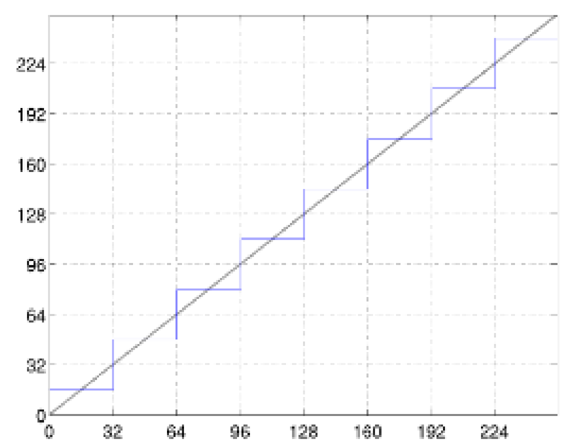


- After sampling and quantization, we get $f : [1, \ldots, N] \times [1, \ldots, M] \longrightarrow [0, \ldots, L]$.

4 levels

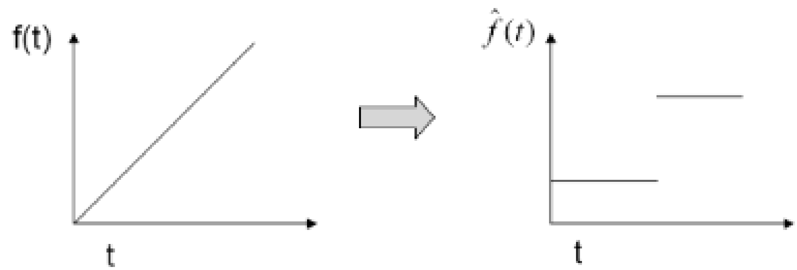


8 levels

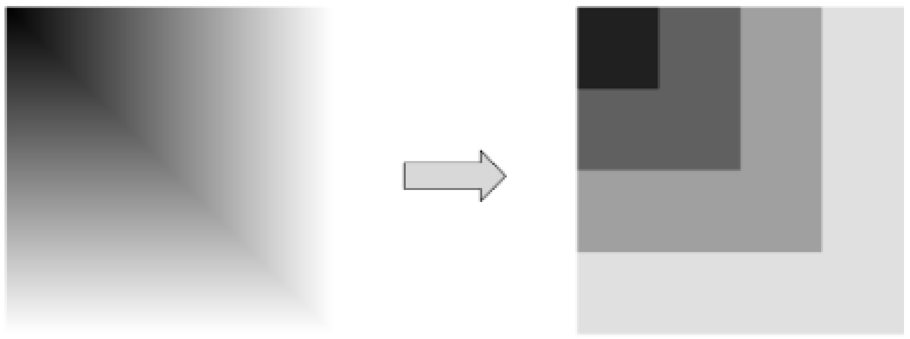


The “false contour” effect I

- By quantizing the images we introduce discontinuities in the image intensities which look like contours.
  - in 1-D,



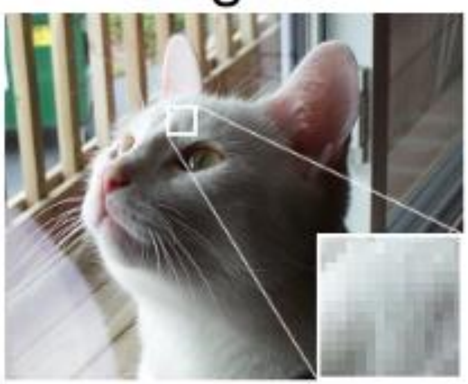
  - in 2-D,



The “false contour” effect II

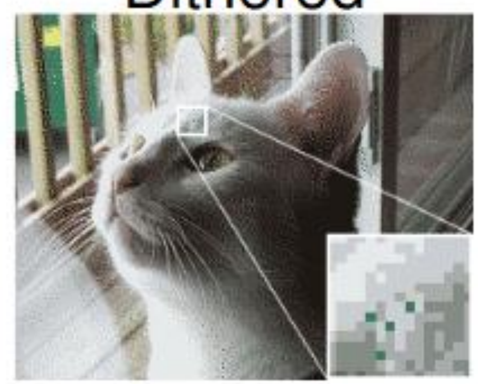
- To mitigate the “false contour” effect we can use dither.
  - Basically, we add noise before quantization to create a more natural distribution of the new intensity values.

Original



Undithered

Dithered

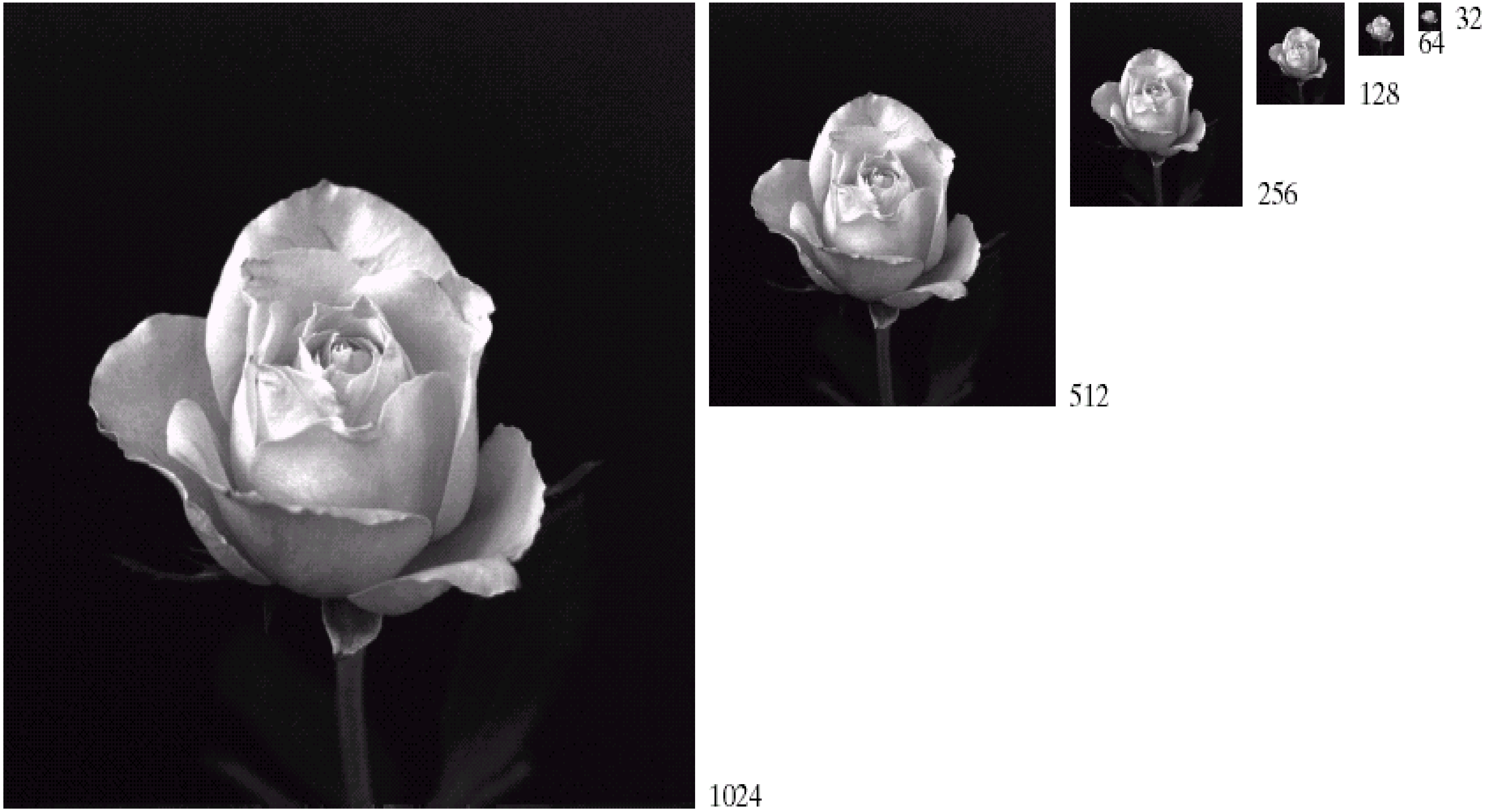


(Images from Wikipedia.)

Spatial Resolution

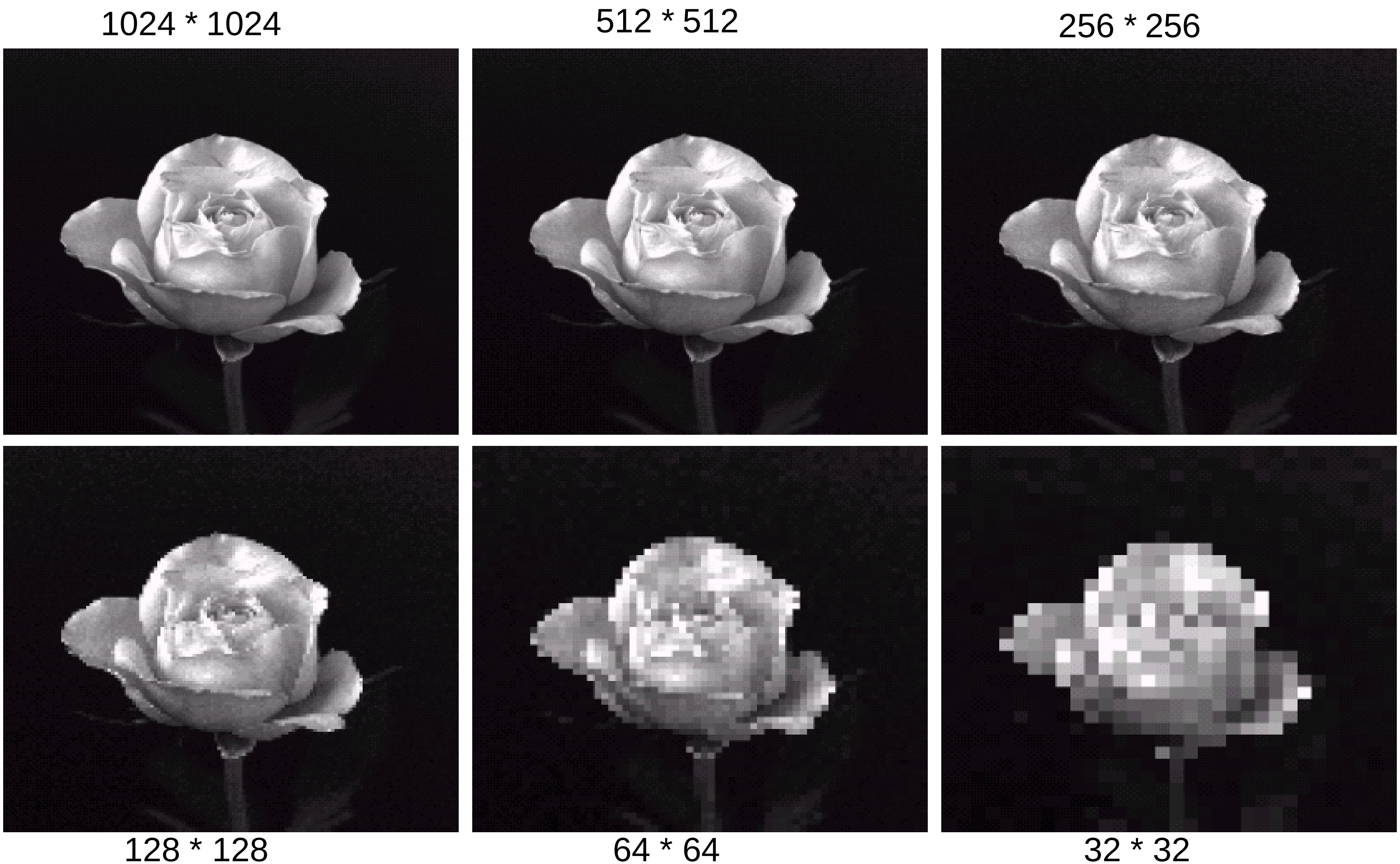
- *The spatial resolution* of an image is determined by how sampling was carried out
- Spatial resolution simply refers to the smallest discernable detail in an image

Spatial Resolution



Vision specialists often talk about pixel size

Spatial Resolution



Graphic designers talk about *dots per inch*

Intensity Level Resolution

- *Intensity level resolution* refers to the number of intensity levels used to represent the image
  - The more intensity levels used, the finer the level of detail discernable in an image
  - Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

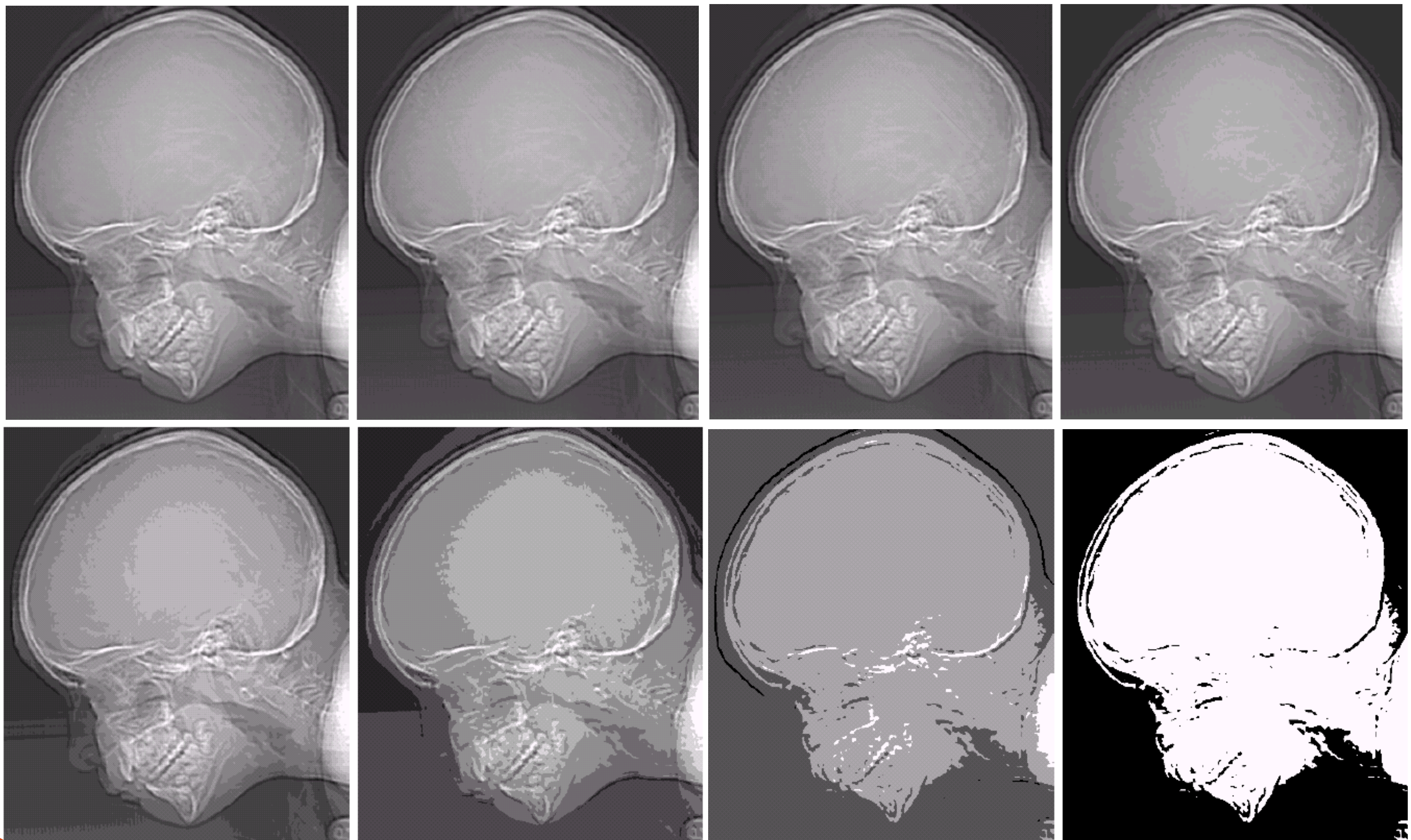
Number of Bits	Number of Intensity Levels	Examples
1	$2^1=2$	0, 1
2	$2^{2=}4$	00, 01, 10, 11
4	$2^{4=}16$	0000, 0101, 1111
8	256	00110011,
16	65,536	01010101 1010101010101010

The representation of an M×N numerical array in MATLAB

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Intensity Level Resolution

256 grey levels (8 bpp) | 128 grey levels (7 bpp) | 64 grey levels (6 bpp) | 32 grey levels (5 bpp)



16 grey levels (4 bpp) | 8 grey levels (3 bpp) | 4 grey levels (2 bpp) | 2 grey levels (1bpp)

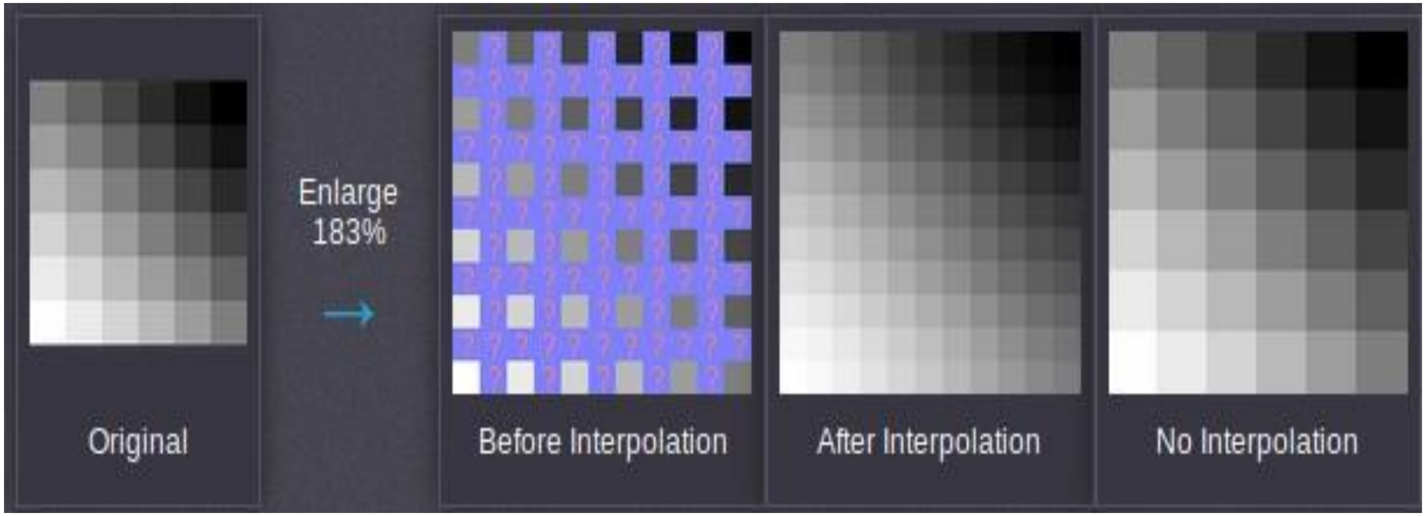
Resolution: How Much Is Enough?

- The big question with resolution is always *how much is enough*?
  - This all depends on what is in the image and what you would like to do with it
  - Key questions include
    - Does the image look aesthetically pleasing?
    - Can you see what you need to see within the image?

- The picture on the right is fine for counting the number of cars, but not for reading the number plate

Image Interpolation

- Interpolation is a process of using known data to estimate values at unknown locations.
- Extensively used for zooming and shrinking the image.



Basic relationships between pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Basic Relationships Between Pixels

- **Neighbors** of a pixel p at coordinates (x,y)

- **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
  (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

- **4 diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
  (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

- **8 neighbors of p**, denoted $\mathbf{N_8(p)}$
  $N_8(p) = N_4(p) \cup N_D(p)$

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **m-adjacency**: Two pixels p and q with values from V are m-adjacent (mixed adjacency) if

(i) q is in the set $N_4(p)$, or

(ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Basic Relationships Between Pixels

- **Path**
- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

- Here n is the *length* of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

V = {1, 2}

0	1	1	0	1	1	0	1	1
0	2	0	0	2	0	0	2	0
0	0	1	0	0	1	0	0	1

Examples: Adjacency and Path

V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

8-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent **m-adjacent**

Examples: Adjacency and Path

V = {1, 2}

$$\begin{array}{ccc} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent **m-adjacent**

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

- **Connected in S**

  Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

  $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$

  Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.
- If S has only one connected component, then S is called ***Connected Set*.**
- We call R a **region** of the image if R is a connected set
- Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not to be adjacent are said to be ***disjoint.***

Connectivity :

To determine whether the pixels are adjacent in some sense.

- Let V be the set of gray-level values used to define connectivity; then Two pixels p, q that have values from the set V are:

a. 4-connected, **if q is in the set $N_4(p)$**

b. 8-connected, **if q is in the set $N_8(p)$**

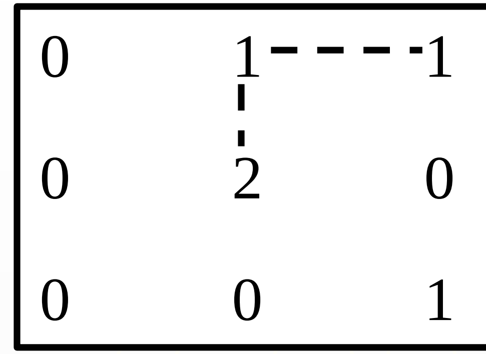
c. m-connected, **iff**

i. q is in $N_4(p)$ or

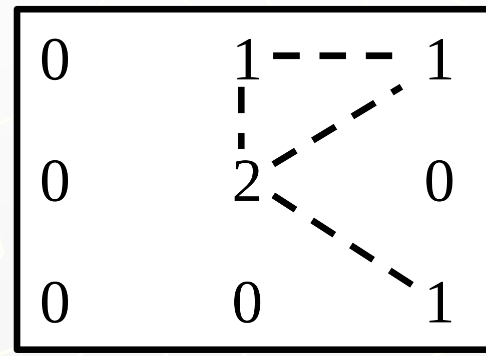
ii. q is in $N_D(p)$ and the set

$N_4(p) \cap N_4(q)$ **is empty**

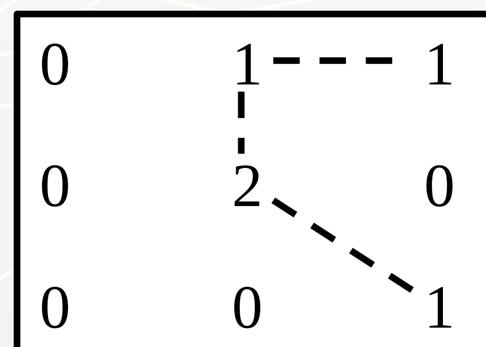
V = {1, 2}



a.



b.



c.

Adjacency, Connectivity

Adjacency: Two pixels are adjacent if they are neighbors and their intensity level 'V' satisfy some specific criteria of similarity.

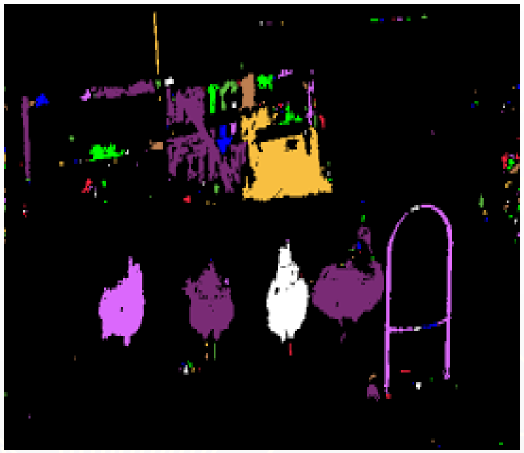
e.g. Binary image = { 0, 1} ➔ V = {1}, V = {0}, V = {0, 1}

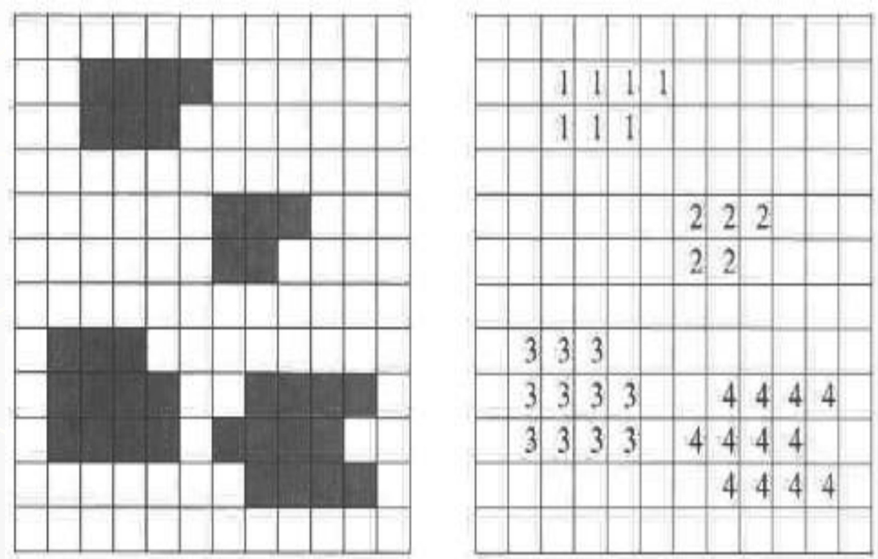
Gray scale image = { 0, 1, 2, ------, 255} ➔ V = {1}, V = {0}, V = {0, 1}, V = {0, 2}, V = {1, 2},……

In binary images, 2 pixels are adjacent if they are neighbors & have some intensity values either 0 or 1.

In gray scale, image contains more gray level values in range 0 to 255.

Regions and Boundaries





Connected Component examples

Basic Relationships Between Pixels

- **Boundary (or border)**

- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

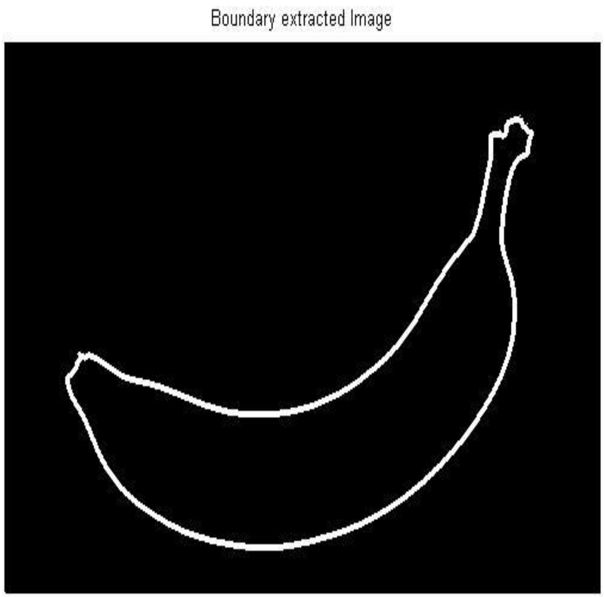
- **Foreground and background**

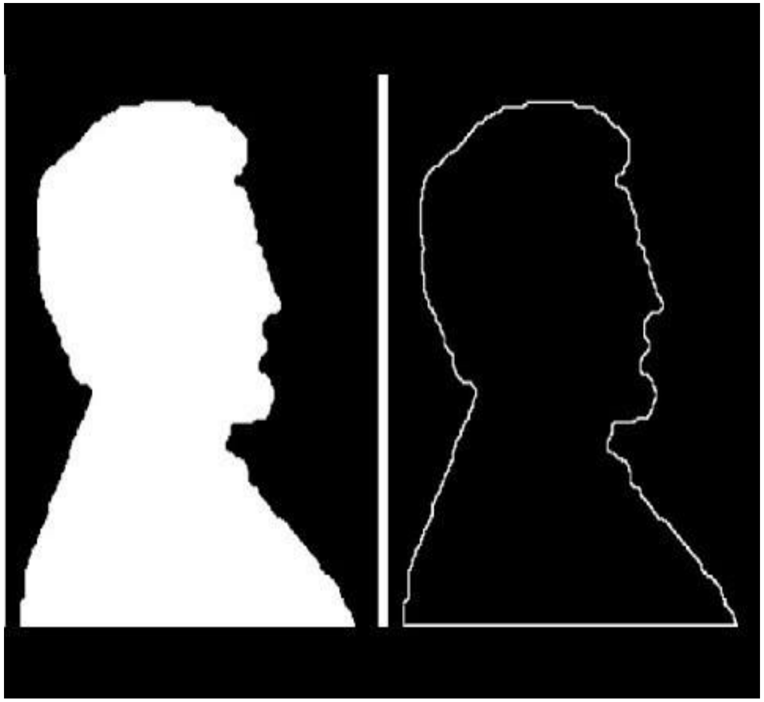
- An image contains K disjoint regions, R_k, $k = 1, 2, \ldots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

All the points in R_u is called **foreground**;

All the points in $(R_u)^c$ is called **background**.

Adjacency, Connectivity, Regions and Boundaries





Distance Measures

Distance Measures: Distance between pixels p, q & z with co-ordinates (x, y), (s, t) & (v, w) resp. is given by:

a) $D(p, q) \geq 0$ [$D(p, q) = 0$ if $p = q$] ....called reflexivity
b) $D(p, q) = D(q, p)$ ....called symmetry
c) $D(p, z) \leq D(p, q) + D(q, z)$ ....called transmitivity

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$

b. City Block Distance:

$D_4(p, q) = |x-s| + |y-t|$

c. Chess Board Distance:

$D_8(p, q) = \max(|x-s|, |y-t|)$

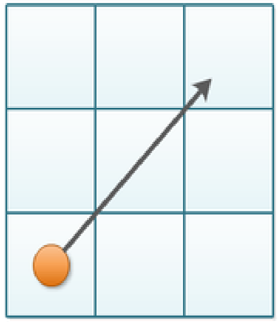
Distance measures

- Euclidean distance between p and q is defined as

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

- For this distance measure, the pixels having a distance less than or equal to some value r from (x,y) i.e points contained in a disk of radius r.

Euclidean Distance



$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance Measures

- City-block distance D_4 between p and q is defined as

$$D_4(p,q) = |x - s| + |y - t|$$

- In this case, pixels having D_4 distance from (x, y) less than or equal to some value r form a diamond centered at (x, y).

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with D_4 distance ≤ 2 forms the following contour of constant distance.

Distance Measures

- D_8-distance called as chessboard distance and defined as

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

In this case, pixels having D_8 distance from (x, y) less than or equal to some value r form a square centered at (x, y).

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with D_8 distance ≤ 2 forms the following contour of constant distance.

Introduction to Mathematical Operations in DIP

- **Array vs. Matrix Operation**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \,.\!*\, B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

- **Linear vs. Nonlinear Operation**

$$H\left[f(x,y)\right]=g(x,y)$$

$$H\left[a_i f_i(x,y)+a_j f_j(x,y)\right]$$

$$=H\left[a_i f_i(x,y)\right]+H\left[a_j f_j(x,y)\right]$$

Additivity

$$=a_i H\left[f_i(x,y)\right]+a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$=a_i g_i(x,y)+a_j g_j(x,y)$$

H is said to be a **linear operator;**

H is said to be a **nonlinear operator** if it does not meet the above qualification.

Mathematical tools used in DIP

- Linear versus non linear operations
- eg.1: Array summation on images is a linear operation

  eg.2: Max operation is not linear.

Arithmetic Operations

- Arithmetic operations between images are array operations:
- $s(x,y) = f(x,y) + g(x,y)$

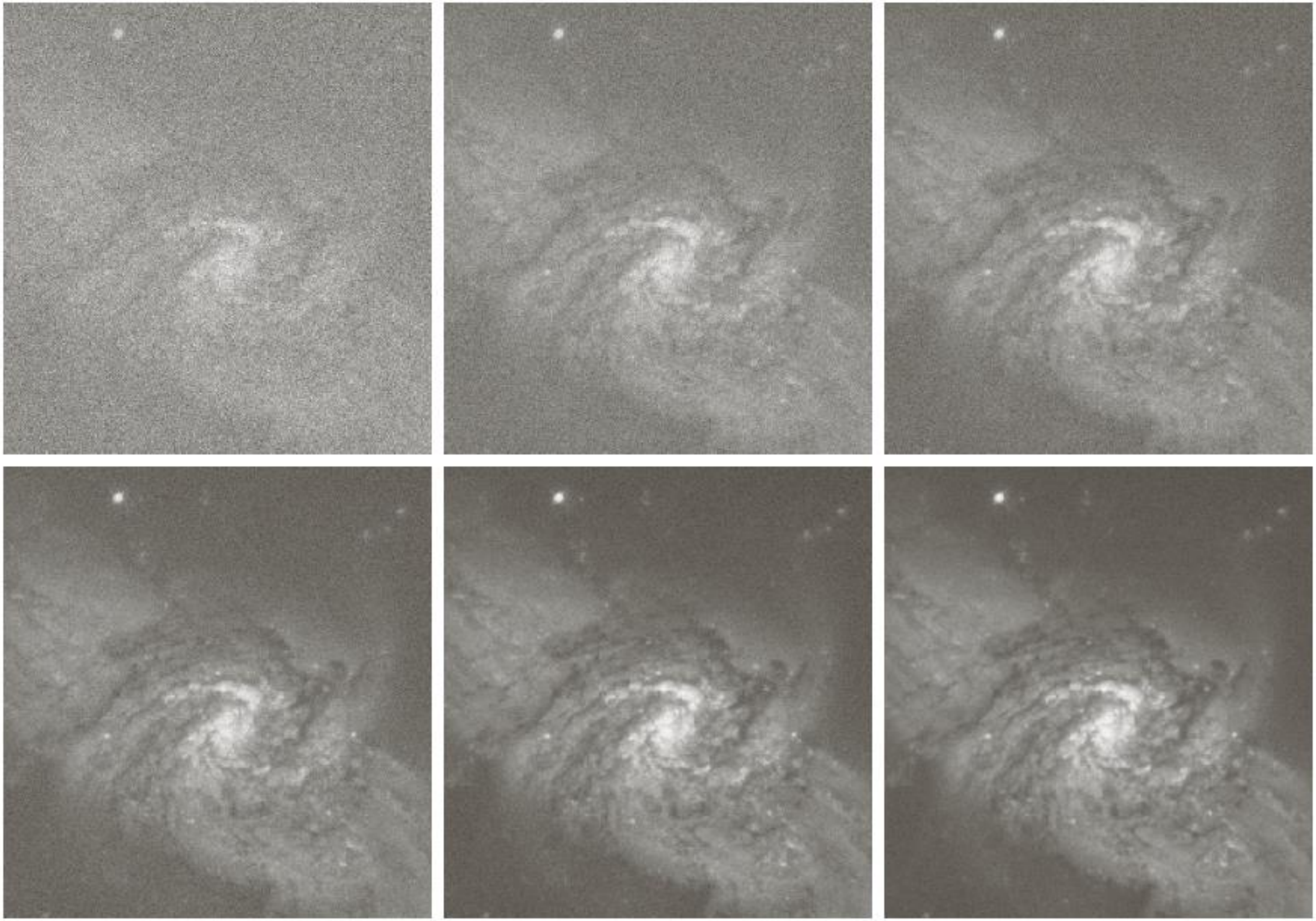
  $d(x,y) = f(x,y) - g(x,y)$

  $p(x,y) = f(x,y) \times g(x,y)$

  $v(x,y) = f(x,y) \div g(x,y)$

- Set and Logical Operations

- Vector and matrix operations



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

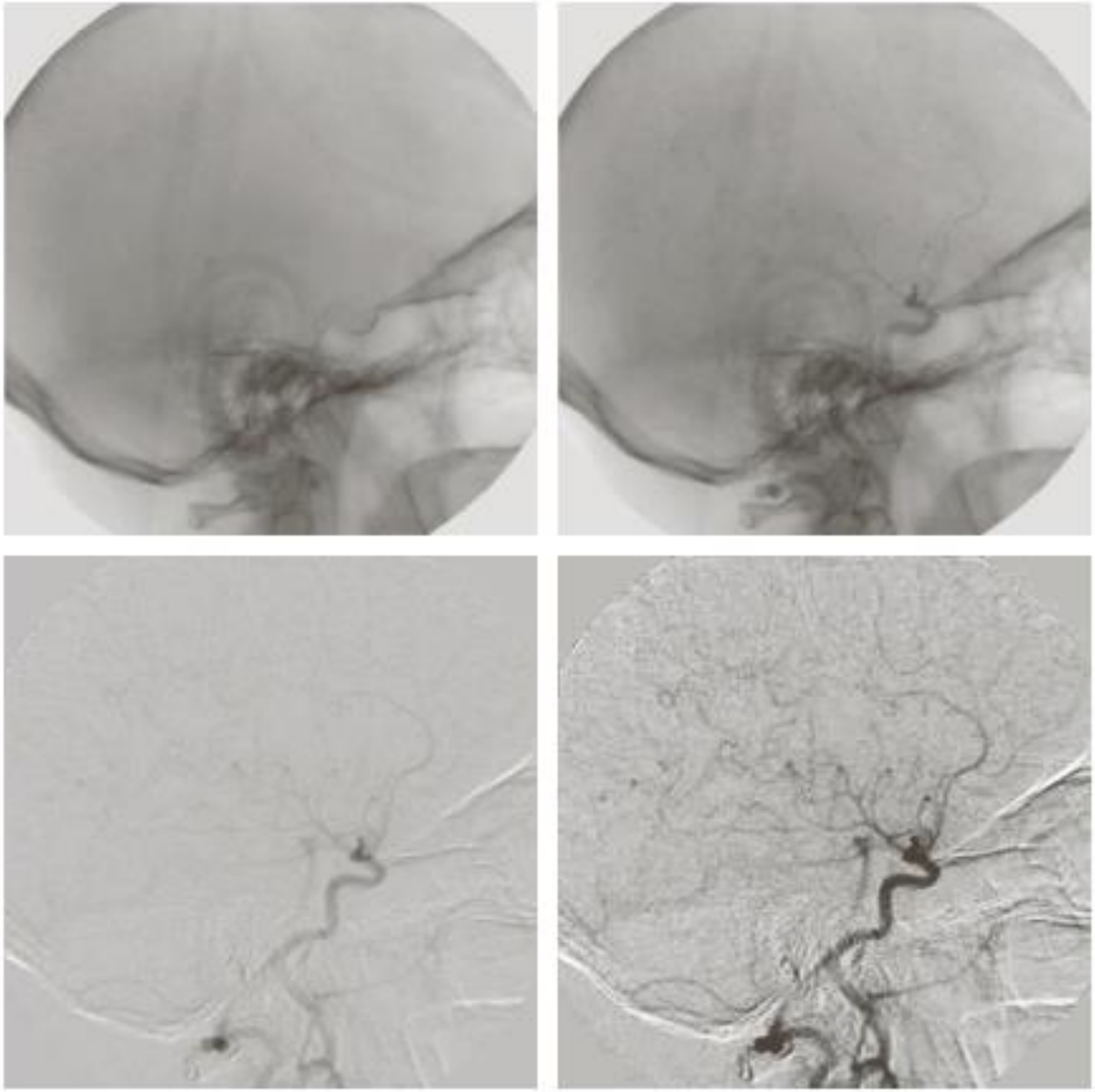
Mask h(x,y): an X-ray image of a region of a patient's body

Live images f(x,y): X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail g(x,y)

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.



a	b
c	d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

An Example of Image Multiplication

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Set and Logical Operations

- Let A be the elements of a gray-scale image

  The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

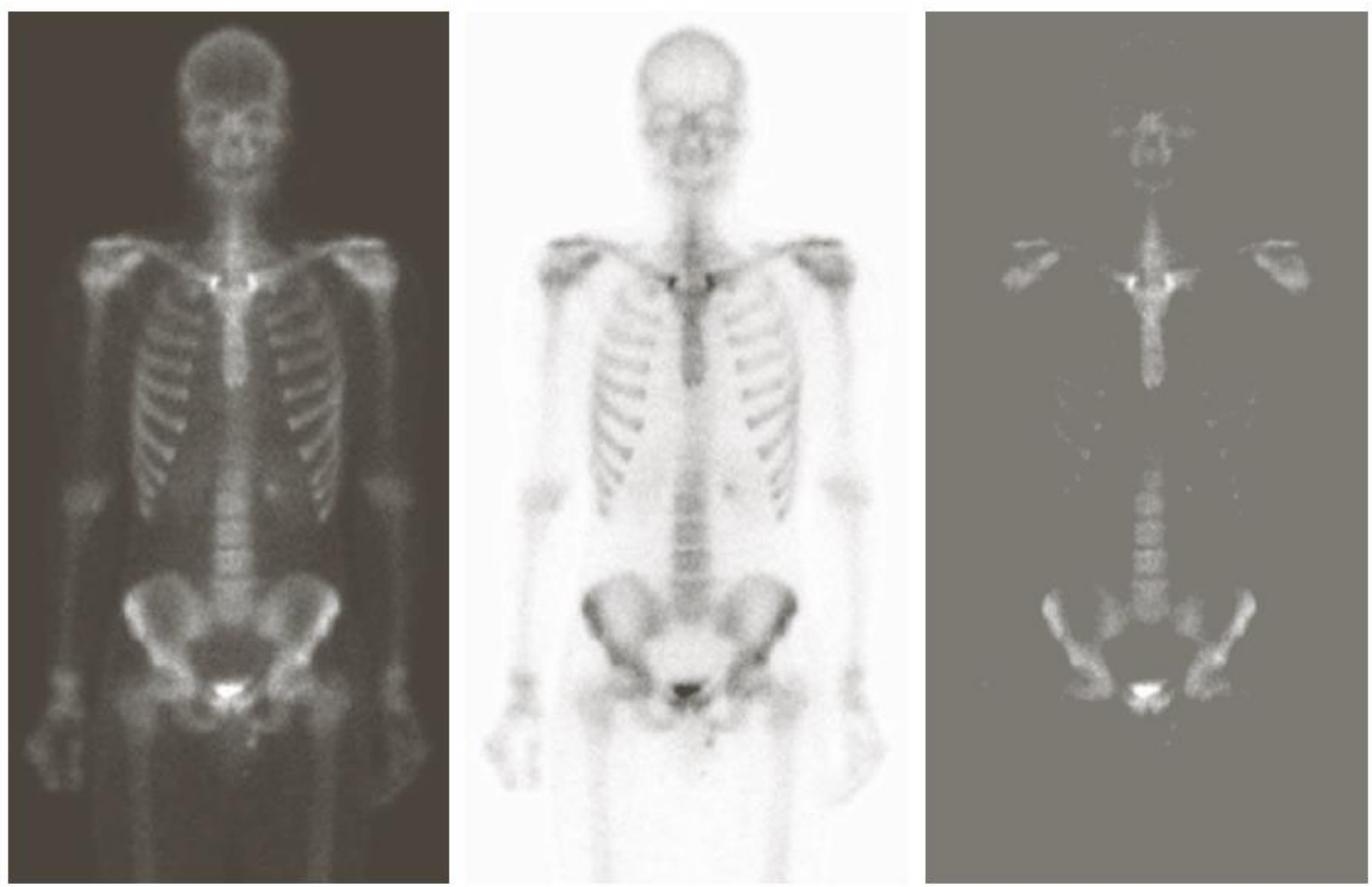
$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$$K = 2^k - 1; k \text{ is the number of intensity bits used to represent } z$$

Set and Logical Operations



Set and Logical Operations

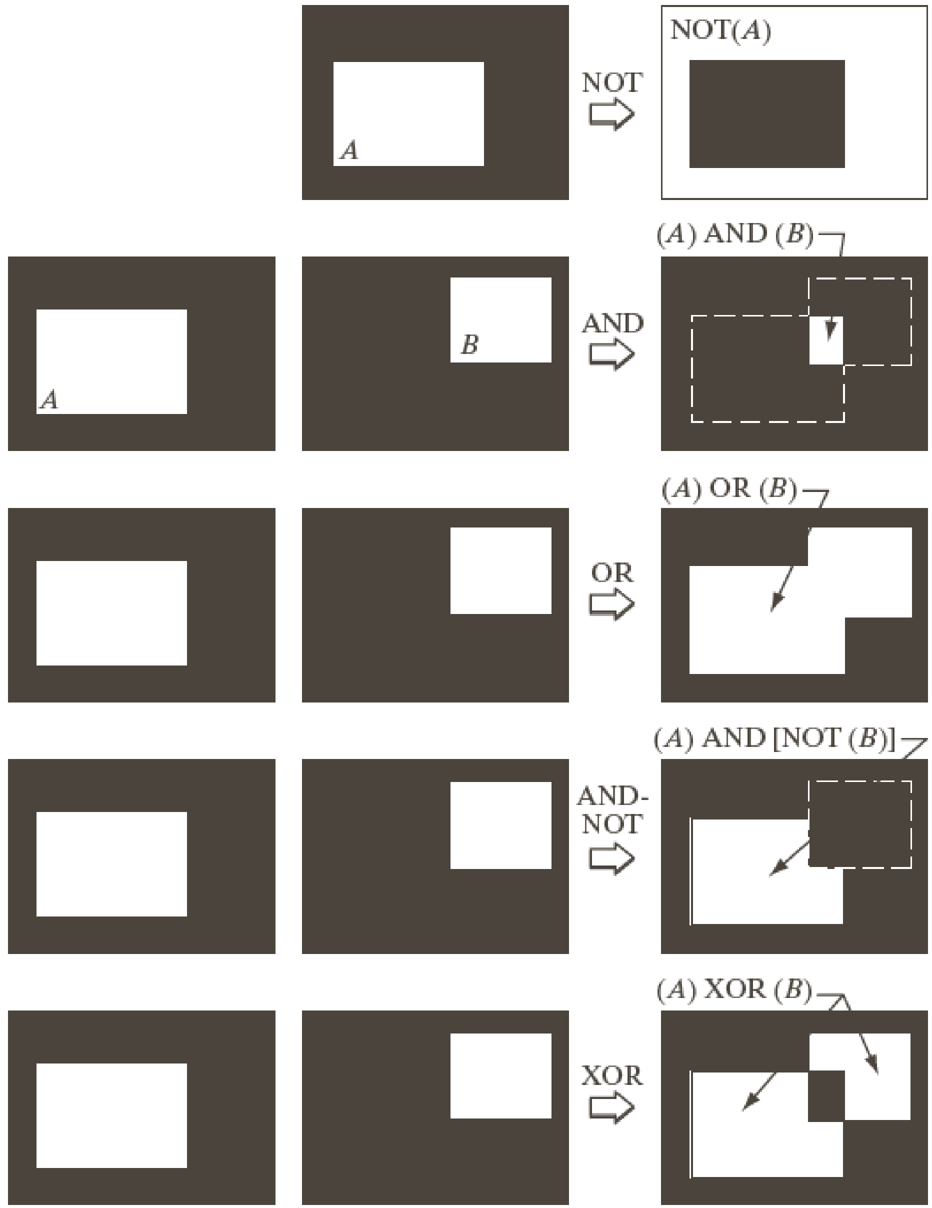


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Spatial Operations

- Single-pixel operations

  Alter the values of an image's pixels based on the intensity.

e.g.

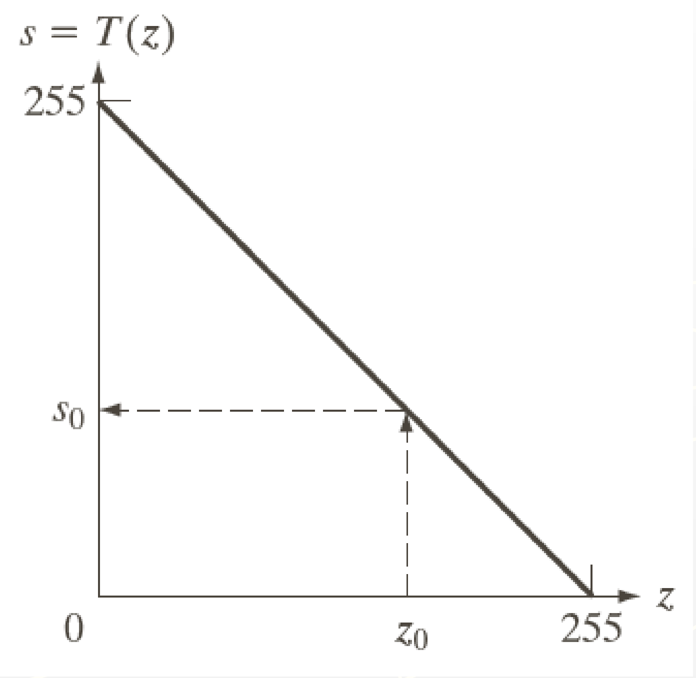
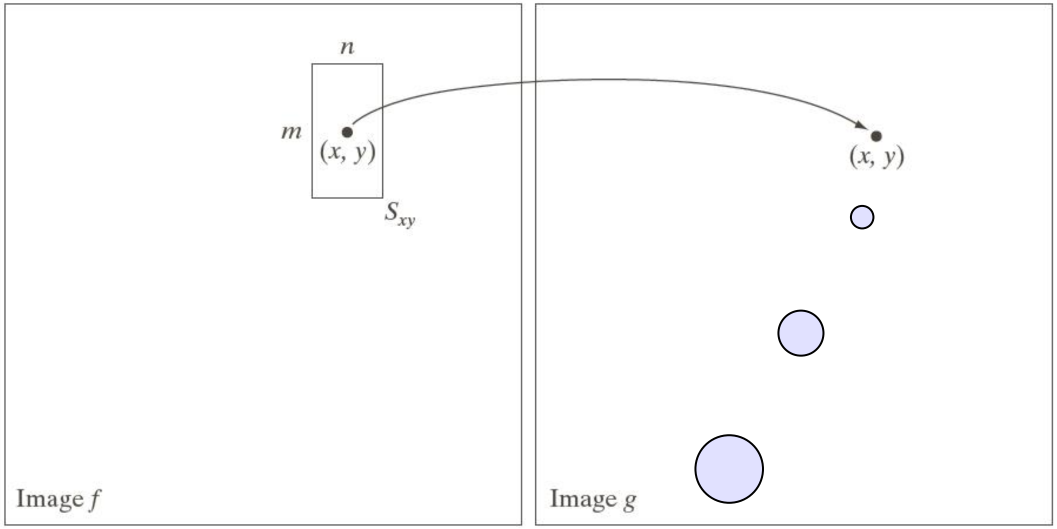


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Spatial Operations

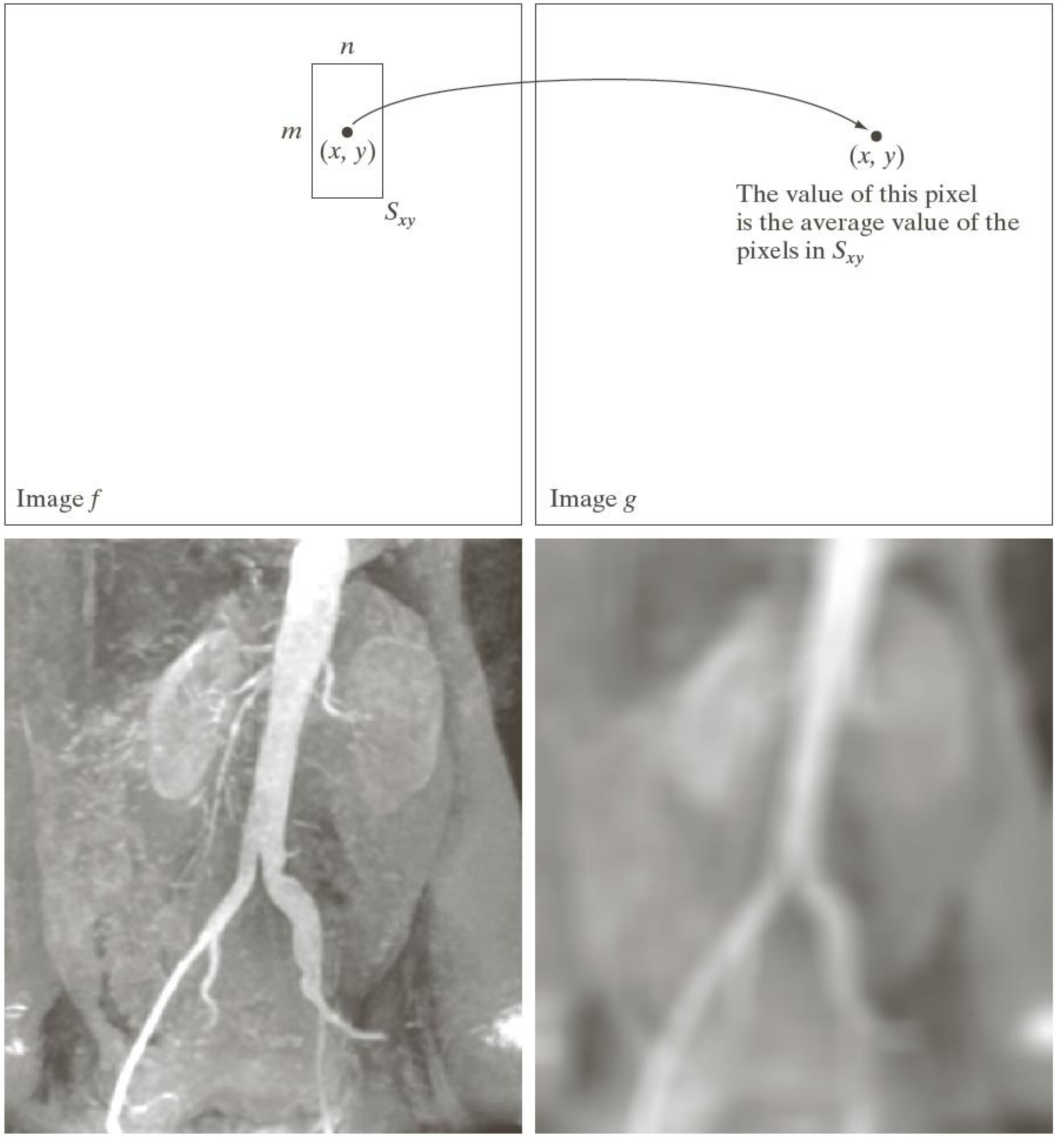
- Neighborhood operations



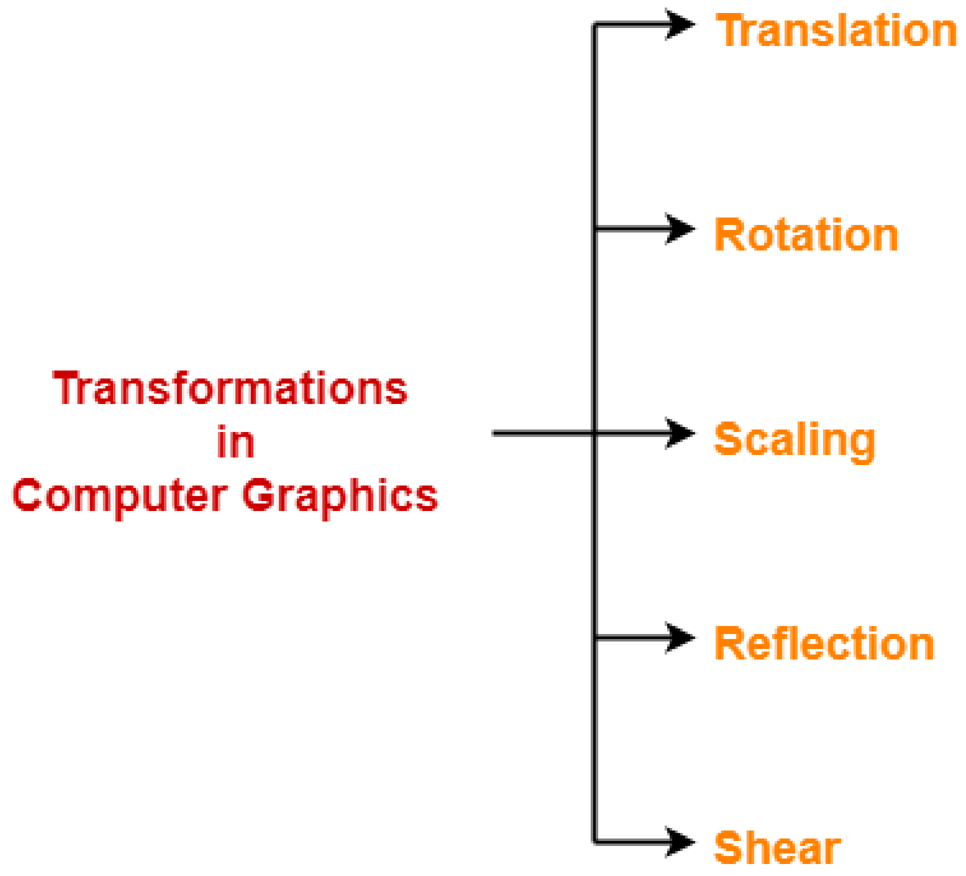
The value of this pixel is determined by a specified operation involving the pixels in the input image with coordinates in S_{xy}

Spatial Operations

- Neighborhood operations



Affine Transformation



Geometric Spatial Transformations

- Geometric transformation (rubber-sheet transformation)

  — A spatial transformation of coordinates:

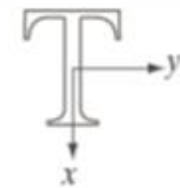
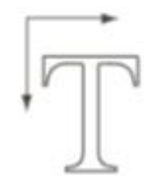
(Where Original point = (v,w) and transformed point = (x,y))

$$(x, y) = T\{(v, w)\}$$

— intensity interpolation that assigns intensity values to the spatially transformed pixels.

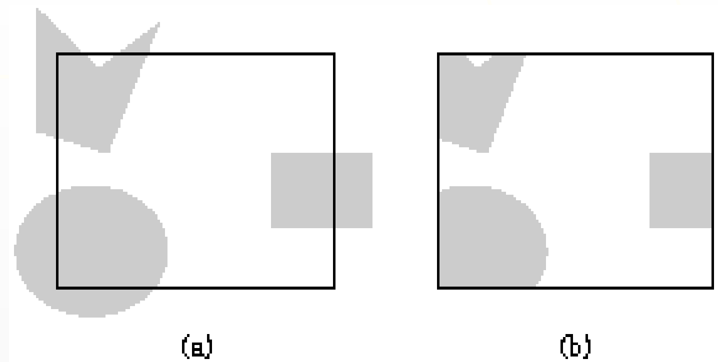
- Affine transform

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

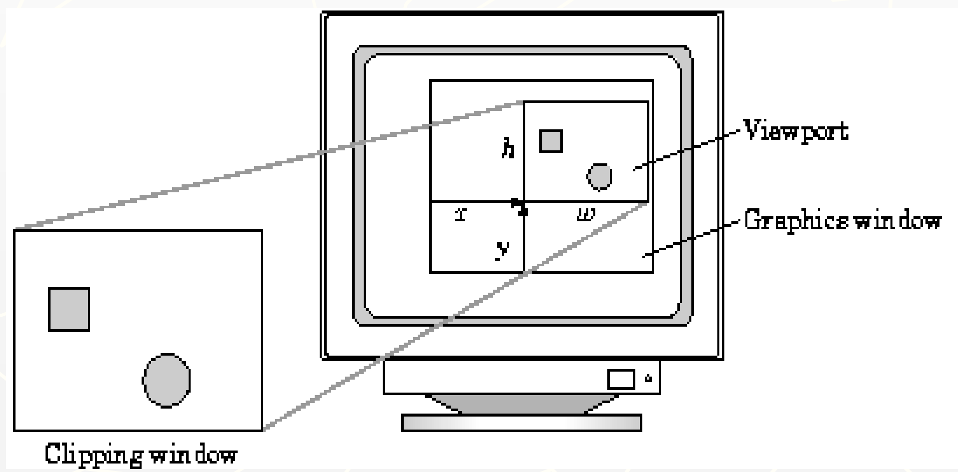
Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cancel{\cos\theta} + w\cancel{\sin\theta}$ $\sin\theta$ $\cos\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

2D Viewing: Window to Viewport Transformation

- **Viewing/clipping window**: part of projected image that is being viewed. In view/projection coordinates



- **Viewport**: part of X-window (output window) where the viewing window is mapped. In normalized device coordinates [0,1]
  Device independent.
- **Screen (device specific) coordinates, integers.**



Window-to-Viewport Transformation

- Range map: given values in range A, map them linearly in range B

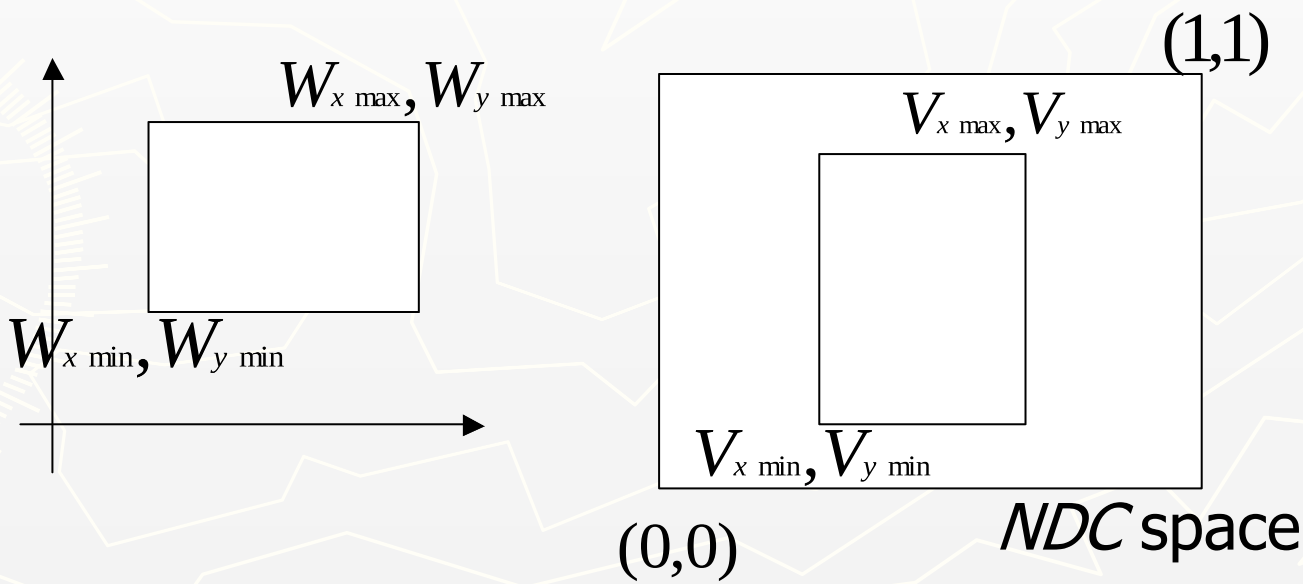
$$A = [A_{min}, A_{max}] \xrightarrow{r} B = [B_{min}, B_{max}]$$

$$A_{min} <= a <= A_{max}$$

$$b = r(a) = (B_{max} - B_{min})(a - A_{min}) / (A_{max} - A_{min}) + B_{min}$$

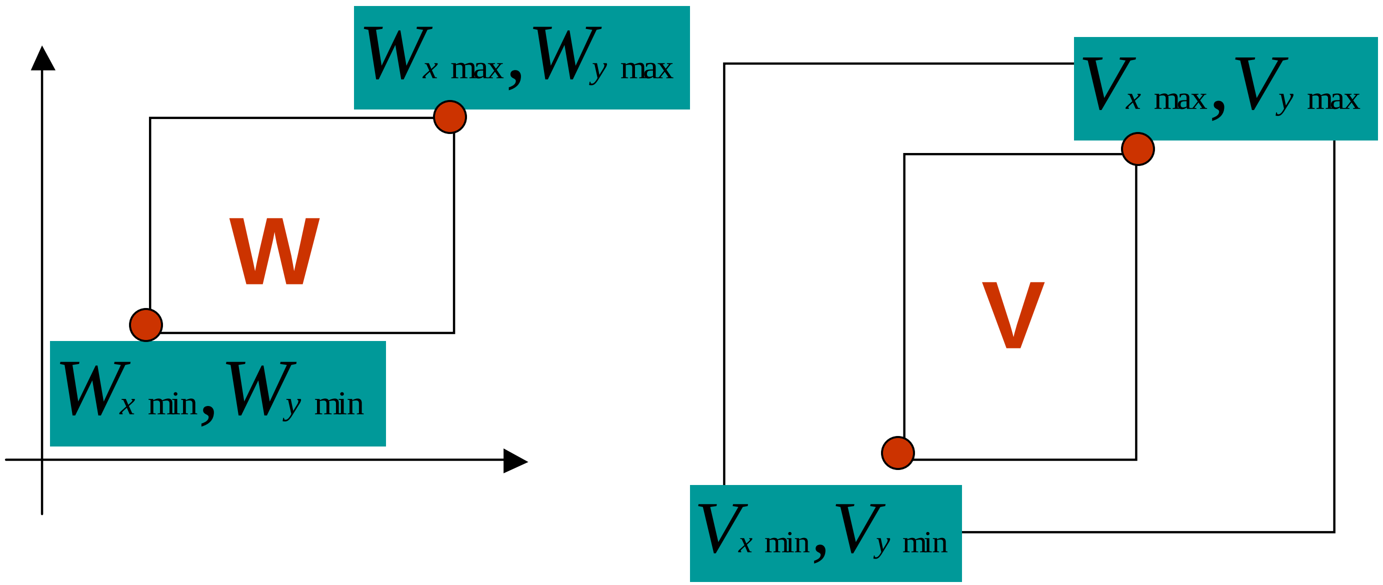
Window-to-Viewport Transformation

- From world/view coordinates to NDC space
- Simply 2D case of range map
- Apply range map in x and y independently



Window-to-Viewport Transformation

- $Sx = (V_{xmax} - V_{xmin}) / (W_{xmax} - W_{xmin})$
- $Sy = (V_{ymax} - V_{ymin}) / (W_{ymax} - W_{ymin})$



Window-to-Viewport Transformation

$$T =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -W_{x\min} & -W_{y\min} & 1 \end{bmatrix} \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ +V_{x\min} & +V_{y\min} & 1 \end{bmatrix}$$

Image Storage Types (Tutorial)

- JPEG (Joint Photographic Expert Group)
- BITMAP
- PNG
- TIFF (Tagged Image File Format)
- GIF (Graphics Interchange Format)