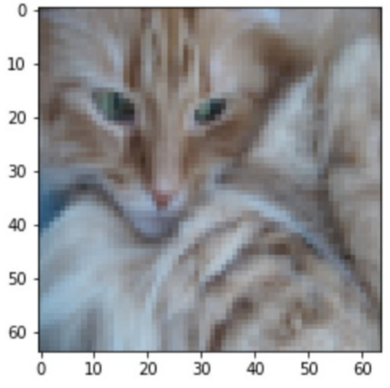
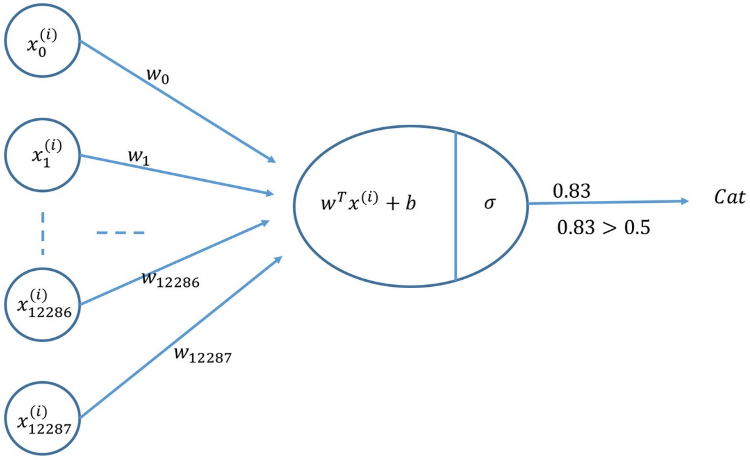
Exercise 12: Deep learning

In this exercise, you will build and train a 1 neuron network for classification of images of cats.

- Follow the tutorial: https://towardsdatascience.com/build-a-simple-neural-network-using-numpy-2add9aad6fc8 (The article is also uploaded as pdf on itsLearning)
- Hint: You can use https://colab.research.google.com/ if you do not have a pc with an nvidia gpu
- Hint: Observe carefully the shape of the numpy arrays (aka. tensors). Make sure you understand which dimensions holds which information, otherwise it will be difficult to debug
- Hint: The loss function is “cross-entropy”. It is optional to read more about this specific loss
- Hint: The derivatives have been done for you. If you are interested see the derivation on the following slides



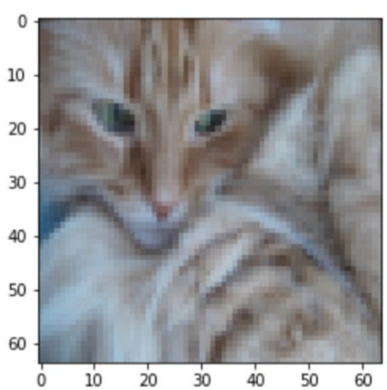


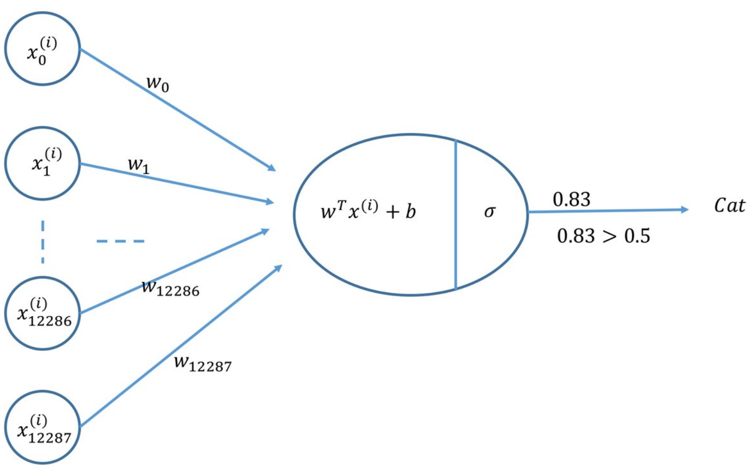
Shape of train_x and train_y

Train x : 209 images, 12288 $\times$ [...]

$(64 \times 64 \times 3 = 12288)$

Train y : 1 [y_1 y_2 .....] 209 labels





Derivative of sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\sigma'(x) = \frac{d}{dx}\sigma(x) = \sigma(x)(1 - \sigma(x))$$

Notation used for A

$$A = \sigma(w^T X + b) = \sigma\left(\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}^T \begin{bmatrix} | & | & | & | & | & | \end{bmatrix} + b\right)$$

$$= \sigma\left(\begin{bmatrix} w^T x_1 + b & w^T x_2 + b & \cdots \end{bmatrix}\right)$$

$$= \begin{bmatrix} \sigma(w^T x_1 + b) & \sigma(w^T x_2 + b) & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & a_2 & \cdots & a_{209} \end{bmatrix}$$

Computing the partial derivatives

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial a^{(1)}}\frac{\partial a^{(1)}}{\partial w} + \frac{\partial L}{\partial a^{(2)}}\frac{\partial a^{(2)}}{\partial w} + \cdots$$

$$\frac{\partial L}{\partial a^{(i)}} = -\frac{1}{m}\left(Y^{(i)}\frac{1}{a^{(i)}} - \left(1-Y^{(i)}\right)\frac{1}{1-a^{(i)}} \right)$$

$$= -\frac{1}{m}\left(Y^{(i)}\frac{(1-a^{(i)})}{a^{(i)}(1-a^{(i)})} - \left(1-Y^{(i)}\right)\frac{a^{(i)}}{a^{(i)}(1-a^{(i)})} \right)$$

$$= -\frac{1}{m}\left(\frac{-Y^{(i)}a^{(i)} + Y^{(i)} + Y^{(i)}a^{(i)} - a^{(i)}}{a^{(i)}(1-a^{(i)})} \right)$$

$$= -\frac{1}{m}\left(\frac{Y^{(i)} - a^{(i)}}{a^{(i)}(1-a^{(i)})} \right)$$

$$\frac{\partial a^{(i)}}{\partial w} = X^{(i)}a^{(i)}(1-a^{(i)})$$

$$= \Sigma \quad \frac{\partial L}{\partial a^{(i)}}\frac{\partial a^{(i)}}{\partial w} = \frac{1}{m} X^{(i)}\left(a^{(i)} - Y^{(i)}\right)$$

$$\frac{\partial L}{\partial w} = \sum \frac{\partial L}{\partial a^{(i)}}\frac{\partial a^{(i)}}{\partial w} = \frac{1}{m}\left[X^{(1)}\ X^{(2)}\ \cdots\right]\begin{bmatrix} a^{(1)} - Y^{(1)} \\ a^{(2)} - Y^{(2)} \\ \vdots \end{bmatrix} = \frac{1}{m}X(A-Y)^{T}$$