CASE A

1)

Decision Variable

Let T be the number of tickets to sell for each route

H stands for Houston, C stands for Chicago, M stands for Miami, P stands for Phoenix

The decision variables are:

3 Inbound flights: T_{CH}, T_{MH}, T_{PH}

3 outbound flights: T_{HC}, T_{HM}, T_{HP}

6 two-flights route: T_{CM}, T_{CP}, T_{MC}, T_{MP}, T_{PC}, T_{PM}

Objective Function

The objective function is to maximize the revenue over a single bank

R, which stands for the revenue over a single bank, can be expressed as:

$$
\begin{aligned}
R = {} & 190 \times T_{CH} + 108 \times T_{MH} + 110 \times T_{PH} + \\
& 197 \times T_{HC} + 110 \times T_{HM} + 125 \times T_{HP} + \\
& 282 \times T_{CM} + 195 \times T_{CP} + 292 \times T_{MC} + \\
& 238 \times T_{MP} + 192 \times T_{PC} + 230 \times T_{PM}
\end{aligned}
$$

Constraints

Capacity:

$T_{CH} + T_{CM} + T_{CP} \leq 240$ $\quad$ $T_{HC} + T_{MC} + T_{PC} \leq 240$

$T_{MH} + T_{MC} + T_{MP} \leq 240$ $\quad$ $T_{HM} + T_{CM} + T_{PM} \leq 240$

$T_{PH} + T_{PC} + T_{PM} \leq 240$ $\quad$ $T_{HP} + T_{CP} + T_{MP} \leq 240$

Non-negativity.

Demand

Demand [redacted]

$0 \le T_{CH} \le 130$

$0 \le T_{HC} \le 123$

$0 \le T_{HM} \le 80$

$0 \le T_{MH} \le 72$

$0 \le T_{HP} \le 110$

$0 \le T_{PH} \le 115$

$0 \le T_{CM} \le 98$

$0 \le T_{MC} \le 105$

$\cancel{0 \le T_{C}}$

$0 \le T_{CP} \le 88$

$0 \le T_{PC} \le 90$

$0 \le T_{MP} \le 68$

$0 \le T_{PM} \le 66$

2)

Decision Variables					
		Destination			
		Houston	Chicago	Miami	Phoenix
Origin	Houston	-	123	80	110
	Chicago	84	-	94	62
	Miami	72	100	-	68
	Phoenix	115	17	66	-

Objective Function	
Inbound	36386
Outbound	46781
Two-flights	102426
Total	185593

Constraints			
	LHS	Type	RHS
Capacity	240	<=	240
	240	<=	240
	198	<=	240
	240	<=	240
	240	<=	240
	240	<=	240
Non-negativity			

Demand				
0	<=	123	<=	123
0	<=	80	<=	80
0	<=	110	<=	110
0	<=	84	<=	130
0	<=	94	<=	98
0	<=	62	<=	88
0	<=	72	<=	72
0	<=	100	<=	105
0	<=	68	<=	68
0	<=	115	<=	115
0	<=	17	<=	90
0	<=	66	<=	66

3)

According to the output, the optimal solution would be selling tickets for the following routes:

Houston-Chicago: 123

Houston-Miami: 80

Houston-Phoenix:110

Chicago-Houston: 84

Chicago-Miami:94

Chicago-Phoenix: 62

Miami-Houston: 72

Miami-Chicago: 100
Miami-Phoenix: 68
Phoenix-Houston: 115
Phoenix-Chicago: 90
Phoenix-Miami: 66
And the corresponding optimal revenue is $185593

CASE B

1)

There are some changes to the calculation of revenue and the constraints:

- For revenue, the revenue of Miami-Houston shall change from (Tmh*108) to [(Tmh-5)*108+5*104]
- Also, we need to change the demand constraint: 0<=Tmh<=72 to 0<=Tmh<=77 since we have another 5 fixed reservation

Decision Variables					
		Destination			
		Houston	Chicago	Miami	Phoenix
Origin	Houston	-	123	80	110
	Chicago	84	-	94	62
	Miami	77	95	-	68
	Phoenix	115	22	66	-

Objective Function	
Inbound	36906
Outbound	46781
Two-flights	101926
Total	185613

Constraints			
	LHS	Type	RHS
Capacity	240	<=	240
	240	<=	240
	203	<=	240
	240	<=	240
	240	<=	240
	240	<=	240
Non-negativity			

Demand				
0	<=	123	<=	123
0	<=	80	<=	80
0	<=	110	<=	110
0	<=	84	<=	130
0	<=	94	<=	98
0	<=	62	<=	88
0	<=	77	<=	77
0	<=	95	<=	105
0	<=	68	<=	68
0	<=	115	<=	115
0	<=	22	<=	90
0	<=	66	<=	66

The total revenue has increased from 185593 to 185613. So, maybe BlueSky should accept the offer.

2)

We need to change the calculation of revenue and the constraints:

- ❖ The revenue of Miami-Chicago shall change from (Tmc*292) to [(Tmc-10)*292+10*285]
- ❖ Also, we need to add another constraint: 0<=Tmc<=95 to 0<=Tmc<=105 since we have another 10 fixed reservation

Decision Variables					
		Destination			
		Houston	Chicago	Miami	Phoenix
Origin	Houston	-	123	80	110
	Chicago	84	-	94	62
	Miami	77	95	-	68
	Phoenix	115	22	66	-

Objective Function	
Inbound	36906
Outbound	46781
Two-flights	101856
Total	185543

Constraints			
	LHS	Type	RHS
Capacity	240	<=	240
	240	<=	240
	203	<=	240
	240	<=	240
	240	<=	240
	240	<=	240
Non-negativity			

Demand				
0	<=	123	<=	123
0	<=	80	<=	80
0	<=	110	<=	110
0	<=	84	<=	130
0	<=	94	<=	98
0	<=	62	<=	88
0	<=	77	<=	77
0	<=	95	<=	105
0	<=	68	<=	68
0	<=	115	<=	115
0	<=	22	<=	90
0	<=	66	<=	66

The total revenue has decreased from 185593 to 185543. So, maybe BlueSky shouldn't accept the offer.

CASE C

1)

We need to change the decision variable, objective function, and constraints:

- For decision variable, we set S as the number of seats for each flight
- For the objective function, we focus on maximizing the profit, which is (Revenue-Cost). The cost is calculated as C=6*12000+6*S*40
- For the constraints, we need to:
  - Change all the previous capacity constraints to total tickets<= S.
  - Since the maximum number of seats, 380, is larger than any demanded number of tickets of any flight, we can add demand constraints that can fit the exact demand for customers of each flight.
  - Also, since we need to satisfy the demand, we need to set a constraint that the number of seats is at least the maximum demand for tickets for a flight, which is 318. We can set 318<=S<=380

Decision Variables					
		Destination			
		Houston	Chicago	Miami	Phoenix
Origin	Houston	-	123	80	110
	Chicago	130	-	98	88
	Miami	72	105	-	68
	Phoenix	115	88	66	-
Seat Capacity	For each flight	316			

Objective Function		
Cost		147840
Revenue	Inbound	45126
	Outbound	46781
	Two-flight	123716
	Total	215623
Profit	Total	67783

Constraints			
	LHS	Type	RHS
Capacity	316	<=	316
	245	<=	316
	269	<=	316
	316	<=	316
	244	<=	316
	266	<=	316
Flight Seat	316	<=	380
Non-negativity			

Demand				
0	<=	123	<=	123
0	<=	80	<=	80
0	<=	110	<=	110
0	<=	130	<=	130
0	<=	98	<=	98
0	<=	88	<=	88
0	<=	72	<=	72
0	<=	105	<=	105
0	<=	68	<=	68
0	<=	115	<=	115
0	<=	88	<=	90
0	<=	66	<=	66

Here we have the new ticket selling strategy and we can assume that if BlueSky purchases three identical aircraft, BlueSky should order 316 coach seats for the three new aircraft.

2)

a. Since the the cost parameters for the six flights in each bank in and out of Houston are nearly identical and each aircraft needs to have an inbound and outbound flight in a bank, the total cost of three aircrafts in a bank is, assuming we have three aircrafts with S1, S2, S3 seats, is always 6*12000+40*2*S1+40*2*S2+40*2*S3 no matter which route the aircraft is flying. We only need to focus on how to have all the demands satisfied while each flight has a comparably lower empty-seat rate so that the profit (Revenue-cost) will be maximized. According to the demand chart, we can see that the demand of Chicago-Houston and Houston-Chicago are close, the demand of Miami-Houston and Houston-Miami are close, and the demand of Phoenix-Houston and Houston-Phoenix are close. We can assume that having the same aircraft satisfying the demand between Houston and the same city with a minimum number of seats will have a lower cost, thus maximizing the profit.

b. We need to change the decision variable, objective function, and constraints:
   - We need to add another 2 decision variables, having S1, S2, S3 for the number of Seats for each aircraft.
   - For the objective function, we need to change the calculation of cost from (C=6*12000+6*S*40) to (C=6*12000+40*2*S1+40*2*S2+40*2*S3)
   - For the constraints:
     - We assign flight1 with S1 seats to fly the route between Chicago and Houston
     - We assign flight2 with S2 seats to fly the route between Miami and Houston
     - We assign flight3 with S3 seats to fly the route between Phoenix and Houston
     - So, we need S1 to satisfy the demand of the route between Chicago and Houston. We need S2 to satisfy the demand of the route between Miami and Houston. We need S3 to satisfy the demand of the route between Phoenix and Houston. Also, we need to reset the capacity constraint so that the total ticket sold for each inbound or outbound flight are less or equal to the capacity of the aircraft.
     - The new constraints will be added as follow:

Capacity

$T_{CH} + T_{CM} + T_{CP} \leq S_1 \quad T_{MH} + T_{MC} + T_{MP} \leq S_2$

$T_{HC} + T_{MC} + T_{PC} \leq S_1 \quad T_{HM} + T_{CM} + T_{PM} \leq S_2$

$T_{PH} + T_{PC} + T_{PM} \leq S_3 \quad T_{HP} + T_{CP} + T_{MP} \leq S_3$

Demand

$T_{CH} + T_{CM} + T_{CP} = 316 \quad T_{MH} + T_{MC} + T_{MP} = 245$

$T_{HC} + T_{MC} + T_{PC} = 318 \quad T_{HM} + T_{CM} + T_{PM} = 244$

$T_{PH} + T_{PC} + T_{PM} = 271 \quad T_{HP} + T_{CP} + T_{MP} = 266$

Flight Seat: $318 \leq S1 \leq 380$

$245 \leq S2 \leq 380$

$271 \leq S_3 \leq 380$

Non-negativity.

Decision Variables					
		Destination			
		Houston	Chicago	Miami	Phoenix
Origin	Houston	-	123	80	110
	Chicago	130	-	98	88
	Miami	72	105	-	68
	Phoenix	115	90	66	-
Seat Capacity	Flight1	318			
	Flight2	245			
	Flight3	271			

Objective Function		
Cost	Fixed	138720
Revenue	Inbound	45126
	Outbound	46781
	Two-flights	124100
	Total	216007
Profit	Total	77287

Regarding the output, we have a new selling strategy.
The aircraft flying between Chicago and Houston should have 318 seats.
The aircraft flying between Miami and Houston should have 245 seats.
The aircraft flying between Phoenix and Houston should have 271 seats.

3)
From CASE C (1), we know that the profit of each bank with three identical aircraft is $67783.
From CASE C (2), we know that the profit of each bank with three aircraft with different number of seats is $77287.

If Bluesky runs three banks each weekday, the daily profit with three identical aircrafts is $67783*3=$203349
If Bluesky runs three banks each weekday, the daily profit with three aircrafts with different number of seats is $77287*3=$231861

We add the 5 million discounts to the profit with three identical aircraft and assume that BlueSky runs this bank for x days.

If Bluesky runs x days with three identical aircrafts, the total profit would be 203349x+5000000
If Bluesky runs x days with three aircrafts with different number of seats, the total profit would be 231861x

We would like to know how many days the two options would reach the same profit. So we solve the equation: 203349x+5000000=231861x and get x is about 175.4

This indicates that if BlueSky plans to run the banks for more than 176 weekdays, then BlueSky should order three different aircraft since they will bring more profit in the future. If BlueSky plans to run the banks for less than 176 weekdays, then BlueSky should order three identical aircraft since they will bring more profit during the total number of weekdays less than 176.