

Decision Variable

We have 8 locations that are considered to be potential warehouse locations

1. Atlanta
2. Buffalo
3. Chicago
4. Cincinnati
5. Detroit
6. Pittsburgh
7. Richmond
8. St. Louis

We have 22 cities that need the distribution centers to send products

1. Atlanta
2. Birmingham
3. Buffalo
4. Charleston
5. Charlotte
6. Chattanooga
7. Chicago
8. Cincinnati
9. Cleveland
10. Columbus
11. Detroit
12. Evansville
13. Ft Wayne
14. Indianapolis
15. Knoxville
16. Louisville
17. Memphis
18. Nashville
19. Peoria
20. Pittsburgh
21. Richmond
22. St. Louis

We assign X_{ij} ($i=1,\dots,8, j=1,\dots,22$) to represent how much cwt of product should each of the 8 potential warehouse locations send to the 22 cities

We assign Z_i ($i=1,\dots,8$) to represent how much cwt of product should each of the 8 potential warehouse locations hold

$$Z_i = \sum (X_{ij}), i=1,\dots,8$$

We assign binary decision variable Y_i ($i=1,\dots,8$) to represent if the potential warehouse location should have a warehouse

$$Y_i = 0 \text{ if } Z_i = 0$$

$$Y_i = 1 \text{ if } Z_i > 0$$

Objective Function

We would like to minimize the leasing cost

Let FC_i ($i=1, \dots, 8$) represent the fixed cost of each of the 8 potential warehouse locations

Let VC_i ($i=1, \dots, 8$) represent the variable cost of each of the 8 potential warehouse locations

Let Li ($i=1, \dots, 8$) represent the minimum capacity of each of the 8 potential warehouse locations

Let U_i ($i=1, \dots, 8$) represent the maximum capacity of each of the 8 potential warehouse locations

Let N_j ($j=1, \dots, 22$) represent the need of each 22 cities

Let D_{ij} ($i=1, \dots, 8, j=1, \dots, 22$) represent the distribution cost of each of the 8 potential warehouse locations sending products to the 22 cities

Total cost = Leasing cost + Distribution cost

Leasing cost = fixed cost + variable cost = $\sum (FC_i * Y_i) + \sum [(Z_i - Li) * VC_i * Y_i]$

Distribution cost = $\sum (X_{ij} * D_{ij})$

Total cost = $\sum (FC_i * Y_i) + \sum [(Z_i - Li) * VC_i * Y_i] + \sum (X_{ij} * D_{ij})$

Constraints

1. The total volume of a single warehouse is equal to the sum of all the amount of product should the potential warehouse send to the 22 cities

$$Z_i = \sum (X_{ij}), i=1, \dots, 8$$

2. The binary decision variable Y_i ($i=1, \dots, 8$) should be 0 if the total volume of a single warehouse is 0

$$Y_i = 0 \text{ if } Z_i = 0$$

$$Y_i = 1 \text{ if } Z_i > 0$$

3. The total volume of a single warehouse should be less or equal to the maximum capacity of each of the potential warehouse location

$$Z_i \leq U_i$$

4. Meet the demand

$$\sum (X_{ij}) \geq N_j \quad (j=1, \dots, 22)$$

5. Non-negativity

Optimal Solution

According to the output, the optimal solution in chart is shown below:

| Open or not | Yes | Yes | No | Yes | No | No | Yes | Yes |
|---------------------------|-------------|-------------|----------|------------|----------|------------|------------|-------------|
| | Atlanta | Buffalo | Chicago | Cincinnati | Detroit | Pittsburgh | Richmond | St. Louis |
| Atlanta | 275 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Birmingham | 160 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Buffalo | 0 | 240 | 0 | 0 | 0 | 0 | 0 | 0 |
| Charleston | 135 | 0 | 0 | 0 | 0 | 0 | 260 | 0 |
| Charlotte | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Chattanooga | 160 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Chicago | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 400 |
| Cincinnati | 0 | 0 | 0 | 200 | 0 | 0 | 0 | 0 |
| Cleveland | 0 | 320 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columbus | 0 | 0 | 0 | 220 | 0 | 0 | 0 | 0 |
| Detroit | 0 | 190 | 0 | 0 | 0 | 0 | 0 | 0 |
| Evansville | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 |
| Ft Wayne | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 40 |
| Indianapolis | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 310 |
| Knoxville | 125 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Louisville | 0 | 0 | 0 | 340 | 0 | 0 | 0 | 0 |
| Memphis | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 240 |
| Nashville | 210 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Peoria | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 150 |
| Pittsburgh | 0 | 340 | 0 | 0 | 0 | 0 | 0 | 0 |
| Richmond | 0 | 0 | 0 | 0 | 0 | 0 | 300 | 0 |
| St. Louis | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 260 |
| Total product held | 1065 | 1090 | 0 | 860 | 0 | 0 | 560 | 1500 |

Hornby can lease warehouses in Atlanta, Buffalo, Cincinnati, Richmond, and St. Louis

The chart shows how to distribute the products from the warehouses to the 22 cities as well as the total capacity each warehouse should have

Optimal Cost

The optimal cost for is 56075.0