NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

CE1012/CZ1012 – ENGINEERING MATHEMATICS II

Nov/Dec 2015 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer **ALL** questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. Useful information is provided in the APPENDIX on Pages 4 and 5.

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1. (a) Use the definition of derivative to calculate $\dfrac{dy}{dx}$ at $x = 4$, where $y = \dfrac{\sqrt{x}}{2+\sqrt{x}}$.

(6 marks)

(b) Given $\dfrac{d}{dx}(e^x) = e^x$, prove that $\dfrac{d}{dx}(a^x) = a^x \ln a$.

(5 marks)

(c) Evaluate $\lim\limits_{x\to 0^-}(2-\cos x)^{\frac{1}{|x|}}$.

(7 marks)

Note: Question No. 1 continues on Page 2

(d) Find $\frac{dy}{dx}$ if $\sqrt{4-\sqrt{x}} - x^{\frac{1}{4}} = y\frac{\cos x}{1+\sin x}$.

(7 marks)

2. (a) Evaluate $\int \frac{\cos x}{\sin x + 2} dx$.

(6 marks)

(b) Evaluate $\int_{-1}^{1} f(x)dx$, where

$$f(x) = \begin{cases} e^x & if \quad x < 0 \\ x+1 & if \quad x \geq 0 \end{cases}.$$

(6 marks)

(c) Evaluate $\int x(x^2+1)\sin(x^2+1)dx$.

(6 marks)

(d) Solve the differential equation $y''+4y'+3y=0$ with the initial conditions $y(0)=0$ and $y'(0)=1$.

(7 marks)

3. (a) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$-7, +3, -\frac{9}{7}, +\frac{27}{49}, -\frac{81}{343}, -\frac{243}{2401}, \ldots$$

(3 marks)

(b) Determine and show whether the sequence is convergent or divergent. If it is convergent, find the limit.

$$a_n = \sqrt{\frac{2n+2}{7n+3}}$$

(4 marks)

Note: Question No. 3 continues on Page 3

(c) Determine and show whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=0}^{\infty} \frac{1}{\left(\sqrt{3}\right)^n}$$

(4 marks)

(d) Determine and show whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin 3n}{1+3^n}$$

(4 marks)

(e) Singapore median monthly household incomes are S\$7,037, S\$7,566, S\$7,872, and S\$8,292 in years 2011, 2012, 2013, and 2014, respectively.

(i) Derive the linear regression expression in the form of $y = ax + b$, where y is median monthly household income, x is year, a is a slope, and b is an offset.

(8 marks)

(ii) Use the linear regression expression obtained from Q3(e)(i) to estimate the median monthly household income in year 2015.

(2 marks)

4. (a) Find the Taylor series of $f(x) = 3x^3 + 2x^2 + x + 1$ centered at the given value of $a = 2$.

(10 marks)

(b) Use the trapezoidal rule to compute

$$\int_0^2 \frac{x^2}{1+x^3}$$

for the number of subintervals $n = 5$.

(5 marks)

Note: Question No. 4 continues on Page 4

(c) Obtain Fourier coefficients of the signal shown in Figure Q4.

(10 marks)

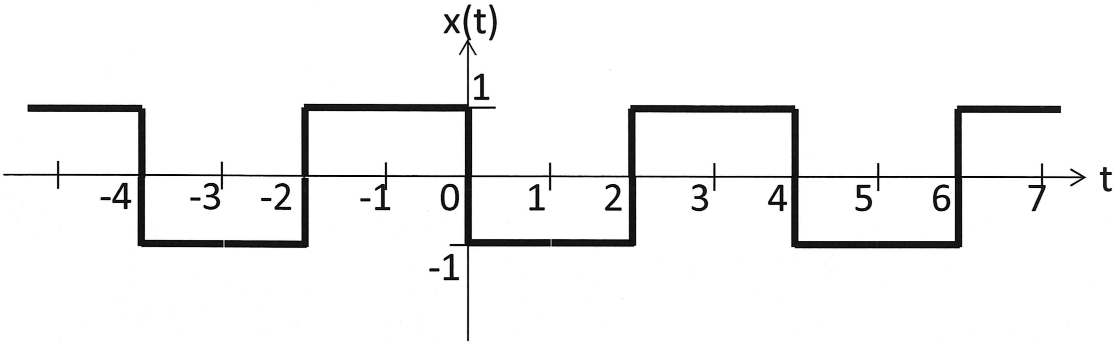

Figure Q4

Appendix

If $\{a_n\}$ and $\{b_n\}$ are convergent sequences and c is a constant, then

$$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} (a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n \qquad \lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty} (a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad \text{if } \lim_{n\to\infty} b_n \neq 0$$

$$\lim_{n\to\infty} a_n^p = \left[\lim_{n\to\infty} a_n\right]^p \quad \text{if } p > 0 \text{ and } a_n > 0$$

Note: Appendix continues on Page 5

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$

$$\cos y = \frac{e^{jy} + e^{-jy}}{2} \qquad \sin y = \frac{e^{jy} - e^{-jy}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Name	Equation
Exponential	$\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}; \quad C_k = \lvert C_k \rvert e^{j\theta_k}, C_{-k} = C_k^*$
Combined trigonometric	$C_0 + \sum_{k=1}^{\infty} 2\lvert C_k \rvert \cos(k\omega_0 t + \theta_k)$
Trigonometric	$A_0 + \sum_{k=1}^{\infty} (A_k \cos k\omega_0 t + B_k \sin k\omega_0 t)$
	$2C_k = A_k - jB_k, C_0 = A_0$
Coefficients	$C_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.