NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

CE1012/CZ1012 – ENGINEERING MATHEMATICS II

Nov/Dec 2015

Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer **ALL** questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. Useful information is provided in the APPENDIX on Pages 4 and 5.

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1. (a) Use the definition of derivative to calculate $\frac{dy}{dx}$ at $x = 4$, where $y = \frac{\sqrt{x}}{2+\sqrt{x}}$.

(6 marks)

(b) Given $\frac{d}{dx}(e^x) = e^x$, prove that $\frac{d}{dx}(a^x) = a^x \ln a$.

(5 marks)

(c) Evaluate $\lim_{x \to 0^-} (2 - \cos x)^{\frac{1}{|x|}}$.

(7 marks)

Note: Question No. 1 continues on Page 2

(d) Find $\frac{dy}{dx}$ if $\sqrt{4-\sqrt{x}} - x^{\frac{1}{4}} = y\frac{\cos x}{1+\sin x}$.

(7 marks)

2. (a) Evaluate $\int \frac{\cos x}{\sin x + 2} dx$.

(6 marks)

(b) Evaluate $\int_{-1}^{1} f(x)dx$, where

$$f(x) = \begin{cases} e^x & if \quad x < 0 \\ x+1 & if \quad x \geq 0 \end{cases}.$$

(6 marks)

(c) Evaluate $\int x(x^2+1)\sin(x^2+1)dx$.

(6 marks)

(d) Solve the differential equation $y''+4y'+3y=0$ with the initial conditions $y(0)=0$ and $y'(0)=1$.

(7 marks)

3. (a) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$-7, +3, -\frac{9}{7}, +\frac{27}{49}, -\frac{81}{343}, -\frac{243}{2401}, \ldots$$

(3 marks)

(b) Determine and show whether the sequence is convergent or divergent. If it is convergent, find the limit.

$$a_n = \sqrt{\frac{2n+2}{7n+3}}$$

(4 marks)

Note: Question No. 3 continues on Page 3

(c) Determine and show whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=0}^{\infty} \frac{1}{\left(\sqrt{3}\right)^n}$$

(4 marks)

(d) Determine and show whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin 3n}{1+3^n}$$

(4 marks)

(e) Singapore median monthly household incomes are S$7,037, S$7,566, S$7,872, and S$8,292 in years 2011, 2012, 2013, and 2014, respectively.

(i) Derive the linear regression expression in the form of $y = ax + b$, where y is median monthly household income, x is year, a is a slope, and b is an offset.

(8 marks)

(ii) Use the linear regression expression obtained from Q3(e)(i) to estimate the median monthly household income in year 2015.

(2 marks)

4. (a) Find the Taylor series of $f(x) = 3x^3 + 2x^2 + x + 1$ centered at the given value of $a = 2$.

(10 marks)

(b) Use the trapezoidal rule to compute

$$\int_0^2 \frac{x^2}{1+x^3}$$

for the number of subintervals $n = 5$.

(5 marks)

Note: Question No. 4 continues on Page 4

(c) Obtain Fourier coefficients of the signal shown in Figure Q4.

(10 marks)

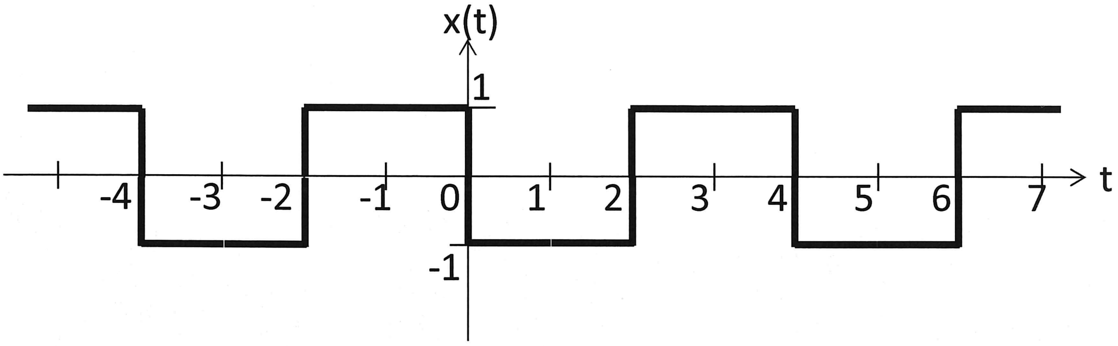

Figure Q4

Appendix

If $\{a_n\}$ and $\{b_n\}$ are convergent sequences and c is a constant, then

$$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} (a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n \qquad \lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty} (a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad \text{if } \lim_{n\to\infty} b_n \neq 0$$

$$\lim_{n\to\infty} a_n^p = \left[\lim_{n\to\infty} a_n\right]^p \quad \text{if } p > 0 \text{ and } a_n > 0$$

Note: Appendix continues on Page 5

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \dbinom{k}{n} x^n = 1 + kx + \dfrac{k(k-1)}{2!} x^2 + \dfrac{k(k-1)(k-2)}{3!} x^3 + \cdots$	$R = 1$

$$\cos y = \frac{e^{jy} + e^{-jy}}{2} \qquad \sin y = \frac{e^{jy} - e^{-jy}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Name	Equation
Exponential	$\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t};\quad C_k = \lvert C_k\rvert e^{j\theta_k}, C_{-k} = C_k^*$
Combined trigonometric	$C_0 + \sum_{k=1}^{\infty} 2\lvert C_k\rvert \cos(k\omega_0 t + \theta_k)$
Trigonometric	$A_0 + \sum_{k=1}^{\infty} (A_k \cos k\omega_0 t + B_k \sin k\omega_0 t)$ $2C_k = A_k - jB_k, C_0 = A_0$
Coefficients	$C_k = \dfrac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

CE1012/CZ1012 – ENGINEERING MATHEMATICS II

Apr/May 2016 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 6 pages.
2. Answer **ALL** questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. Appendix is on Page 5 and Page 6.

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1. (a) Evaluate $\lim_{x\to\infty}\frac{x}{e^x}$.

(6 marks)

(b) Evaluate $\lim_{x\to 0} xe^{-x}$.

(4 marks)

(c) Evaluate $\lim_{h\to 0}\frac{\sqrt{\sin(x+h)-\cos(x+h)+5}-\sqrt{\sin(x)-\cos(x)+5}}{h}$.

(8 marks)

(d) Find $\frac{dy}{dx}$ if $y=\cos x\sin x+\sin(\sqrt{x^2+1})$.

(7 marks)

2. (a) Find $\frac{dy}{dx}$ if $xy^2 + 3y^2 = e^{xy}$.

(7 marks)

(b) Evaluate $\int (x^2 + \sin x - \cos x + e^x - \frac{1}{x}) dx$.

(4 marks)

(c) Evaluate $\int_0^5 f(x)\, dx$, where

$$f(x) = \begin{cases} \lfloor x \rfloor & \text{if} \quad x \le 3 \\ x \ln(x^2) & \text{if} \quad x > 3 \end{cases}.$$

(7 marks)

(d) Solve the differential equation $y'' + 2y' + 1 = 0$ with the initial conditions $y(0) = 0$ and $y'(0) = 1$.

(7 marks)

3. (a) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues as

-1, +3, -5/3, +5, -27/5, +70/6, -121/7, +33, …

(3 marks)

(b) Determine and show whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{2\ln n}{\ln 3n}$$

(2 marks)

Note: Question No. 3 continues on Page 3

(c) Determine and show whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{\cos n}{1+\sqrt{2n}}$$

(2 marks)

(d) Find the range of values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=1}^{\infty} (2x-1)^n$$

(4 marks)

(e) Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{5n^3+5}}$$

(4 marks)

(f) Singapore GDPs are \$38,576, \$46,569, \$52,870, and \$54,007 in years 2009, 2010, 2011 and 2012, respectively. Derive the linear regression expression in the form of $y = ax + b$, where y is a GDP, x is a year, a is a slope, and b is an offset by using

(i) The first and last point method.

(5 marks)

(ii) The optimal linear regression method.

(5 marks)

4. (a) Find the Maclaurin series for $f(x) = 5^{2x}$ by using the definition of a Maclaurin series. Also find the associated radius of convergence.

(5 marks)

(b) Use the trapezoidal rule to compute

$$\int_0^2 \sin(2x)\cos(x)dx$$

for n = 5, where n is the number of intervals. Compare the result with that of an actual integration.

(5 marks)

(c) Find the fundamental frequency of the following signal

$x(t) = \cos(5t\pi/3) + \sin(10t\pi/4) + \cos(3t\pi/2)$.

(5 marks)

(d) Find the Fourier transform for the signal

$$f(t) = 2\cos(t\pi)[u(t+2) - u(t-2)]$$

where $u(t)$ is the unit step function.

(10 marks)

Appendix

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$$\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad \text{if } \lim_{n\to\infty} b_n \neq 0$$

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$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty}\binom{k}{n}x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$

Note: Appendix continues on Page 6

$$\cos y = \frac{e^{jy} + e^{-jy}}{2} \qquad \sin y = \frac{e^{jy} - e^{-jy}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

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Coefficients	$C_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$

END OF PAPER