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Question 1

(a) Prove:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (by definition)

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (by definition)

$\equiv [(\neg p \vee q) \wedge \neg q] \vee [(\neg p \vee q) \wedge p]$ (DeMorgan's)

$\equiv [(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \vee [(\neg p \wedge p) \vee (p \wedge q)]$ (DeMorgan's)

$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$

$\equiv p \wedge q \vee \neg p \wedge \neg q$ (operator precedence)

(b) $\neg(\exists x\in X, \forall y\in Y, P(x, y)) \equiv (\forall x\in X, \exists y\in Y, P(x, y))$

X can only be 2 or 3:

1 When x = 2, y = 7:
$x \equiv y \bmod 5 = 2$
That is, P (2, 7) is true.

2 When x = 3, y = 8:
$x \equiv y \bmod 5 = 3$
That is, P (3, 8) is true.

So $\forall x\in X, \exists y\in Y, P (x, y)$ is true, which means $\neg(\exists x\in X, \forall y\in Y, P (x, y))$ is true.

(c) A valid argument satisfies: "If the premises are true, then the conclusion is true".

∵$p \wedge q$ (premise 1)

∴p is true, and q is true. (i)

∵$p \rightarrow \neg r$ and p is true (premise 2 and i)

∴$\neg r$ is true (ii)

∵$q \rightarrow \neg s$ and q is true (premise 3 and i)
∴$\neg s$ is true (iii)

So from (ii) and (iii), $\neg r \wedge \neg s$ is true.
According to the definition, this argument is valid.

Question 2

(a) (1) Let y ∈ f (S ∪ T) arbitrary.
Then there exists x ∈ (S ∪ T) such that f(x) = y.
Since x ∈ (S ∪ T), x∈S or x ∈T, then f(x) ∈f(S) or f(x) ∈ f(T).
That is, f(x) ∈ f(S) ∪ f(T)
Thus, f(S ∪ T) ⊆ f(S) ∪ f(T) (i)

Let y∈ f(S) ∪ f(T)arbitrary.
Then there exists x∈S or x ∈T such that f(x) = y.
Since x∈S or x ∈T, x ∈ (S ∪ T), then f(x) ∈f (S ∪ T)
Thus, f(S) ∪ f(T) ⊆ f(S ∪ T) (ii)

From (i) and (ii), f(S ∪ T) = f(S) ∪ f(T).

(2) Disprove:

Suppose set A = {x1, x2, x3}, set Y = {y1, y2}, f(x1) = f(x3) = y1, f(x2) = y2.
If S = {x1, x2}, T = {x2, x3}, then S∩T = x2, f(x2) = y2.
However, f(S)={ y1, y2} , f(T)= { y1, y2} , f(S)∩f(T) = { y1, y2}, which is not equal to y2.

(b) Yes, we can use membership table to solve this question. We should notice that premise is A⊕B = B⊕C, and we should get the conclusion that A = C.

A⊕B	B⊕C	B	A	C
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

And we can get the conclusion that A = C.

Question 3

We can use matrix representation to solve this question.

Since the relation should be reflexive, which means every element of A is related to itself, the matrix's diagonal entries should all be true while the others can be true or false.

$$\begin{bmatrix} T & T/F & T/F & T/F \\ T/F & T & T/F & T/F \\ T/F & T/F & \ddots & T/F \\ T/F & T/F & T/F & T \end{bmatrix}$$

So we can construct 2^{n*n-n} distinct matrixes. That is, there are 2^{n*n-n} distinct reflexive relations on a set with n elements.

Question 4

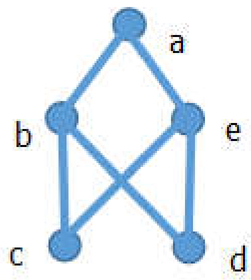

Figure 1: Graph

(a) Yes, this graph is bipartite. Bipartite graph is a graph whose vertices can be partitioned into 2 (disjoint) subsets V and W such that each edge only connects a v $\in$ V and a w $\in$ W.

Suppose V = {a,c,d} , W = {e,b}, then each edge only connects a node in V and a node in W. Thus, this graph is bipartite.

(b) An Euler path (Eulerian trail) is a walk on the edges of a graph which uses each edge in the original graph exactly once. (The beginning and end of the walk may or not be the same vertex.)

If we follow this order: e→c→b→d→e→a→b, we can walk on the edges of the graph which use each edge exactly once.