NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

CE1012/CZ1012 – ENGINEERING MATHEMATICS II

Apr/May 2016 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 6 pages.
2. Answer **ALL** questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. Appendix is on Page 5 and Page 6.

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1. (a) Evaluate $\lim_{x \to \infty} \frac{x}{e^x}$.

(6 marks)

(b) Evaluate $\lim_{x \to 0} x e^{-x}$.

(4 marks)

(c) Evaluate $\lim_{h \to 0} \frac{\sqrt{\sin(x+h) - \cos(x+h) + 5} - \sqrt{\sin(x) - \cos(x) + 5}}{h}$.

(8 marks)

(d) Find $\frac{dy}{dx}$ if $y = \cos x \sin x + \sin(\sqrt{x^2+1})$.

(7 marks)

2. (a) Find $\frac{dy}{dx}$ if $xy^2 + 3y^2 = e^{xy}$.

(7 marks)

(b) Evaluate $\int (x^2 + \sin x - \cos x + e^x - \frac{1}{x}) dx$.

(4 marks)

(c) Evaluate $\int_0^5 f(x)\, dx$, where

$$f(x) = \begin{cases} \lfloor x \rfloor & if \quad x \le 3 \\ x \ln(x^2) & if \quad x > 3 \end{cases}.$$

(7 marks)

(d) Solve the differential equation $y'' + 2y' + 1 = 0$ with the initial conditions $y(0) = 0$ and $y'(0) = 1$.

(7 marks)

3. (a) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues as

-1, +3, -5/3, +5, -27/5, +70/6, -121/7, +33, …

(3 marks)

(b) Determine and show whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{2 \ln n}{\ln 3n}$$

(2 marks)

Note: Question No. 3 continues on Page 3

(c) Determine and show whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{\cos n}{1+\sqrt{2n}}$$

(2 marks)

(d) Find the range of values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=1}^{\infty} (2x-1)^n$$

(4 marks)

(e) Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{5n^3+5}}$$

(4 marks)

(f) Singapore GDPs are \$38,576, \$46,569, \$52,870, and \$54,007 in years 2009, 2010, 2011 and 2012, respectively. Derive the linear regression expression in the form of $y = ax + b$, where y is a GDP, x is a year, a is a slope, and b is an offset by using

(i) The first and last point method.

(5 marks)

(ii) The optimal linear regression method.

(5 marks)

4. (a) Find the Maclaurin series for $f(x)=5^{2x}$ by using the definition of a Maclaurin series. Also find the associated radius of convergence.

(5 marks)

(b) Use the trapezoidal rule to compute

$$\int_0^2 \sin(2x)\cos(x)dx$$

for n = 5, where n is the number of intervals. Compare the result with that of an actual integration.

(5 marks)

(c) Find the fundamental frequency of the following signal

$$x(t)=\cos(5t\pi/3)+\sin(10t\pi/4)+\cos(3t\pi/2).$$

(5 marks)

(d) Find the Fourier transform for the signal

$$f(t)=2\cos(t\pi)[u(t+2)-u(t-2)]$$

where $u(t)$ is the unit step function.

(10 marks)

Appendix

If $\{a_n\}$ and $\{b_n\}$ are convergent sequences and c is a constant, then

$$\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty}(a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} ca_n = c\lim_{n\to\infty} a_n \qquad\qquad \lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty}(a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad \text{if } \lim_{n\to\infty} b_n \neq 0$$

$$\lim_{n\to\infty} a_n^p = \left[\lim_{n\to\infty} a_n\right]^p \quad \text{if } p > 0 \text{ and } a_n > 0$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty}\binom{k}{n}x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$

Note: Appendix continues on Page 6

$$\cos y = \frac{e^{jy} + e^{-jy}}{2} \qquad \sin y = \frac{e^{jy} - e^{-jy}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Name	Equation
Exponential	$\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}; \quad C_k = \lvert C_k \rvert e^{j\theta_k}, C_{-k} = C_k^*$
Combined trigonometric	$C_0 + \sum_{k=1}^{\infty} 2\lvert C_k \rvert \cos(k\omega_0 t + \theta_k)$
Trigonometric	$A_0 + \sum_{k=1}^{\infty} (A_k \cos k\omega_0 t + B_k \sin k\omega_0 t)$
	$2C_k = A_k - jB_k, C_0 = A_0$
Coefficients	$C_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.