NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

MH1812 - Discrete Mathematics

November 2014 | TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(20 marks)**

(a) Let A and B be two sets. Write the definition of $A \neq B$ (in terms of predicate logic).

(b) Show that

$$(A \neq B) \to \neg((A \cup B) \subseteq (A \cap B)).$$

(c) Deduce that

$$((A \cup B) \subseteq (A \cap B)) \to (A = B).$$

QUESTION 2. **(25 marks)**

(a) Consider the set S of 3×3 matrices with binary coefficients, that is the coefficients are integers modulo 2.

(i) Compute $|S|$.

(ii) Consider the subset R of S formed by 3×3 matrices with binary coefficients such that they are equal to their transposes:

$$R = \left\{M \in S,\ M = M^T\right\}.$$

Compute $|R|$.

(b) Consider the 3×3 real matrix

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ x & 0 & 2 \end{pmatrix}.$$

Compute the value(s) of x for which M is invertible, in which case(s), compute M^{-1}.

QUESTION 3. **(25 marks)**

(a) Compute the real part and the imaginary part of the following complex number:

$$\frac{5-3i}{1+i}.$$

(b) Let z be a complex number. Prove that the real part of z is given by $\frac{z+\bar{z}}{2}$. Give a formula to express the imaginary part of z in terms of z and $\bar{z}$.

(c) Define a relation R on the set of complex numbers by

$$zRw \iff |z| \leq |w|.$$

Is this relation R a partial order? Justify your answer.

QUESTION 4. **(20 marks)**

(a) Let A_5 denote the set of integers modulo 5, and A_8 denote the set of integers modulo 8. Consider the maps $f_1 : A_8 \to A_8$, $f_2 : A_5 \to A_5$ given by

$$\begin{aligned} f_1(x) &= 2x \mod 8, \\ f_2(x) &= 4x+1 \mod 5. \end{aligned}$$

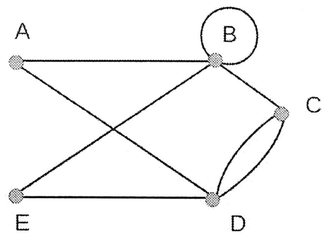
(i) Is the map f_1 injective (one-to-one)? Justify your answer.

(ii) Is the map f_2 invertible? If so, give its inverse. Justify your answer.

(b) Let B be a finite set. Let $f : B \to B$ be an injective map. Show that f is surjective.

QUESTION 5. **(10 marks)**

Does the following graph contain an Euler circuit? Justify your answer.



END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2014-2015

MH1812 - DISCRETE MATHEMATICS

April 2015 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1.

(a) Compute 3^{2015} mod 7. **(7 marks)**

(b) Let i denote the imaginary unit of complex numbers, i.e., $i = \sqrt{-1}$. Find all roots of the following equation and express them with i: **(8 marks)**

$$x^2 + 2x + 5 = 0$$

(c) Write the following system of linear equations in a matrix form and solve it using Gaussian elimination. **(15 marks)**

$$\begin{cases} x_1 - x_2 + 2x_3 = 11 \\ x_1 + x_2 + x_3 = 8 \\ 2x_1 - 3x_2 = 1 \end{cases}$$

QUESTION 2.

(a) Find the solution of the recurrence relation, $a_n = 5a_{n-1} - 6a_{n-2}$, with $a_1 = 8$ and $a_2 = 20$. **(10 marks)**

(b) A deck of 52 different cards consists of 4 suits (spade, heart, diamond, club) with 13 distinguished cards each. A "Flush" refers to 5 cards with the same suit, what is the probability for a randomly chosen 5 cards from one deck to be a Flush? **(10 marks)**

(c) Prove by mathematical induction that

$$\sum_{k=1}^{2^n} \frac{1}{k} \geq 1 + \frac{n}{2}$$

for all integer $n \geq 1$. (note: $\sum_{k=1}^{2^n} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{2^n - 1} + \frac{1}{2^n}$) **(10 marks)**

QUESTION 3.

(a) Prove, for any two sets A and B, **(10 marks)**

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

(b) In a party of 6 people, any two people can be either strangers to (they meet for the first time) or friends with (they met before) the other. Prove, among these 6 people, there exist at least 3 people such that they are all friends, or they are all strangers. **(10 marks)**

QUESTION 4.

(a) Let $\mathbb{Z}^+$ be the set of positive integers, i.e. $\{1, 2, 3, \ldots\}$, and let R be the relation defined by "$x, y \in \mathbb{Z}^+$ and $(x, y) \in R \Leftrightarrow x^2 | y$". Is R a partial order? Is R an equivalence relation? Justify your answer. **(10 marks)**

(b) Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y. Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$. **(10 marks)**

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

MH1812 - Discrete Mathematics

November 2015 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(40 marks)**

(a) Prove or disprove the following statement using logical equivalences:

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q).$$

(b) Consider the domains $X = \{2, 3\}$ and $Y = \{6, 7, 8\}$, and the predicate

$$P(x, y) = \text{“}x \equiv y \mod 5\text{”}.$$

Decide what is the truth value of the statement

$$\neg(\exists x \in X,\ \forall y \in Y,\ P(x, y)).$$

(c) Decide whether the following argument is valid:

$p \wedge q$;
$p \rightarrow \neg r$;
$q \rightarrow \neg s$;
$\therefore \neg r \wedge \neg s$.

QUESTION 2. **(30 marks)**

(a) Let f be a function from the set A to the set B. Let S and T be subsets of A.

(i) Show that

$$f(S \cup T) = f(S) \cup f(T).$$

(ii) Prove or disprove that

$$f(S \cap T) = f(S) \cap f(T).$$

(b) Given two sets A and B, their symmetric difference $A \oplus B$ is the set containing those elements in either A or B, but not in both A and B. Suppose that $A \oplus B = C \oplus B$ for three sets A, B, C. Does it imply that $A = C$? Justify your answer.

QUESTION 3. **(15 marks)**
How many distinct reflexive relations are there on a set with n elements? Justify your answer.

QUESTION 4. **(15 marks)**

(a) Is the following graph shown on Figure 1 bipartite? Justify your answer.

(b) Does the following graph shown on Figure 1 contain an Euler path? Justify your answer.

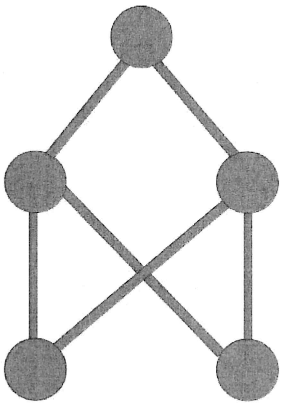


Figure 1: Graph

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

MH1812 - DISCRETE MATHEMATICS

May 2016 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1.

(a) Compute $5^{2016} \mod 7$. **(8 marks)**

(b) Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology. (list all intermediate propositions if you choose to prove by truth table) **(10 marks)**

(c) How many solutions are there for the following equation

$$x_1 + x_2 + \cdots + x_r = n$$

with r, n, x_i positive integers for $i = 1, 2, \ldots, r$ and $n \geq r$. **(10 marks)**

(d) Let sets $A = \{1, 3\}$, $B = \{0, 2, 4\}$, and $P(x, y)$ denote "$5 \mid (x + y)$", i.e., $(x + y)$ is multiple of 5. Determine the truth value of the following and justify your answer: **(12 marks)**

- $\forall x \in A, \exists y \in B, P(x, y)$.
- $\exists y \in B, \forall x \in A, P(x, y)$.

QUESTION 2.

(a) Find the solution of the recurrence relation, $a_n = 4a_{n-1} - 4a_{n-2}$, with $a_1 = 2$ and $a_2 = 8$. **(10 marks)**

(b) Prove

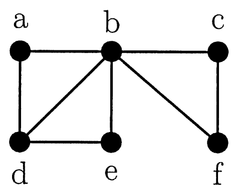
$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n + 1) = n(n + 1)(n + 2)/3$$

for every positive integer n. **(10 marks)**

QUESTION 3.

(a) Let A, B, and C be sets, show $(B - A) \cup (C - A) = (B \cup C) - A$. **(10 marks)**

(b) Refer to the graph below, find Euler Path, Euler Circuit and Hamilton Circuit if any, justify your answer if it does not exist. **(8 marks)**



QUESTION 4.

(a) Let set $A = \{a, b, c, d\}$ and relation $R = \{(a, a), (a, b), (b, c), (c, d), (d, c)\}$. **(12 marks)**

- Is R reflexive, symmetric, transitive ?
- Find R^t, i.e., the transitive closure of R.

(b) Let function $f(x) = x^2 + 2x + 3$ with x being real numbers and $x \leq -1$, find **(10 marks)**

- the range of f.
- the inverse function f^{-1}.

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SPECIAL TERM I EXAMINATION 2015–2016

MH1812 – Discrete Mathematics

June 2016 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(30 Marks)**

(a) Prove or disprove the following statement:

$$p \wedge (\neg(q \rightarrow r)) \equiv (p \rightarrow r)$$

(b) Decide whether the following argument is valid. Justify your answer:

$(p \vee q) \rightarrow \neg r;$
$\neg r \rightarrow s;$
$p;$
$\therefore s$

QUESTION 2. **(30 Marks)**

(a) Let R be a relation from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $xRy \Leftrightarrow 2|(x-y)$. Show that if n is odd, then n is related to 1. Here $\mathbb{Z}$ is the set of integers.

(b) For all sets A and B, prove

$$(A \cup B) \cap \overline{(A \cap B)} = (A - B) \cup (B - A)$$

by showing that each side of the equation is a subset of the other.

QUESTION 3. (20 Marks)

Solve the following linear recurrence relation by using the characteristic equation.

$$b_n = 4b_{n-1} - b_{n-2}, \quad b_0 = 2, b_1 = 4.$$

QUESTION 4. (20 Marks)

(a) How many edges are needed to build a complete bipartite graph with 10 and 20 vertices, respectively, on each side ?

(b) Does the graph shown on Figure 1 contain an Euler path? Justify your answer.

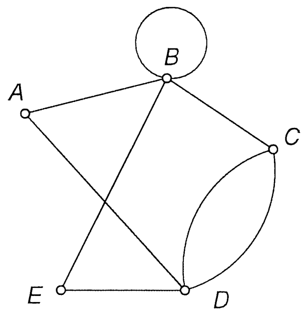


Figure 1: Graph

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