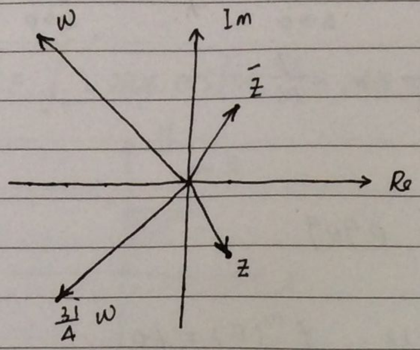
Q1.

(a)




(b) $z = 2e^{-\frac{\pi}{3}i}$ , $w = 4\sqrt{2}\,e^{\frac{3\pi}{4}i}$

(c) $|z^{26} \cdot w^{-10}| = |z|^{26}|w|^{-10} = 2^{26} \cdot (4\sqrt{2})^{-10} = 2^{26} \cdot 2^{-25} = 2$

(d) $z^{26} \cdot w^{-10} = 2e^{-\frac{\pi}{3}i \times 26 - \frac{3\pi}{4}i \times 10} = 2e^{-\frac{97}{6}\pi i} = 2e^{-\frac{\pi}{6}i}$

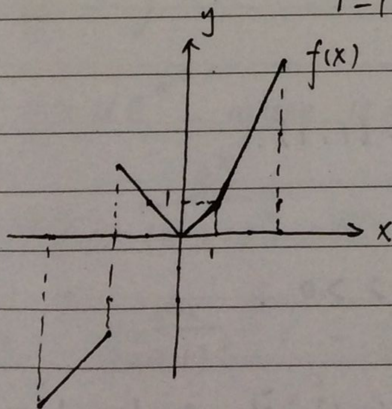
Q2.

(a)(i) $\vec{AB} = (-1, 2, 2)$ . $\vec{AC} = (-1, -1, 4)$.

$\cos\langle \vec{AB}, \vec{AC} \rangle = \dfrac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \dfrac{7\sqrt{2}}{18}$ . $\langle \vec{AB}, \vec{AC} \rangle = 0.315$

(ii) $S = |\vec{AB} \times \vec{AC}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 2 \\ -1 & -1 & 4 \end{vmatrix} = |10\vec{i} + 2\vec{j} + 3\vec{k}| = \sqrt{113}$

(b)




$$\int_{-4}^{3} f(x)\,dx = -\frac{3+5}{2} \times 2 + \frac{1}{2} \times 2^2 + \frac{1}{2} \times 1^2 + \frac{1+5}{2} \times 2$$

$$= \frac{1}{2}$$

Q3.

(a) $f'(1) = \lim_{h\to 0} \dfrac{\dfrac{1}{\sqrt{5(1+h)-1}} - \dfrac{1}{\sqrt{5-1}}}{h} = \lim_{h\to 0} \dfrac{2-\sqrt{4+5h}}{h} \lim_{h\to 0} \dfrac{1}{2\sqrt{4+5h}}$

$= \lim_{h\to 0} -\dfrac{5}{2+\sqrt{4+5h}} \lim_{h\to 0} \dfrac{1}{2\sqrt{4+5h}} = -\dfrac{5}{16}$.

(b)(i) $\frac{1}{1.1} \doteq 1 - 0.1 + 0.01 - 0.001 = 0.909$.

(ii) $f(0)=1$. $f'(0)=3$. $f''(0)=12$. $f'''(0)=60$.

$f(x) = 1+3x+6x^2+10x^3$. $\quad C_n = \dfrac{(n+2)(n+1)}{2}$

Q4.

(a)(i) $\vec{\nabla} T = \dfrac{2y}{1+2x}\vec{i} + \ln(1+2x)\vec{j}$.

$\vec{\nabla} T(0,3) = 6\vec{i} \quad \hat{v} = \dfrac{1}{\sqrt{5}}\vec{i} - \dfrac{2}{\sqrt{5}}\vec{j}$

$T'_{\vec{v}}(0,3) = \vec{\nabla} T(0,3)\cdot \hat{v} = \dfrac{6\sqrt{5}}{5}$

(ii) $\dfrac{\partial T}{\partial t} = \dfrac{\partial T}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial T}{\partial y}\cdot\dfrac{dy}{dt} = \dfrac{8t\,e^{2t-t^2}}{1+4t^2} + \ln(1+4t^2)(2-2t)e^{2t-t^2}$

$\left.\dfrac{\partial T}{\partial t}\right|_{t=2} = -4.725$

(b) $\begin{cases} f_x = -\dfrac{1}{x^2}+y=0 \\ f_y = -\dfrac{1}{y^2}+x=0 \end{cases} \Rightarrow (x,y)=(1,1)$.

$f_{xx} = \dfrac{2}{x^3} = 2>0$. $\quad f_{yy} = \dfrac{2}{y^3} = 2>0$.

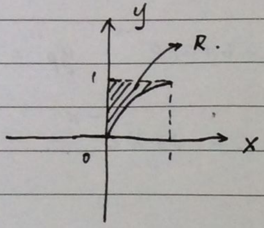
$f_{xx}f_{yy} - f_{xy}^2 = 3>0$. So, $(1,1)$ is a local minima.

Q5.

(a) $dV = 2\pi x\,(y+1)\,dx$.

$$V = \int_0^1 2\pi x \cos(\pi x^2) + 2\pi x \, dx = \sin(\pi x^2) + \pi x^2 \Big|_0^1 = \pi.$$

(b)




Change the region $\{(x,y) \mid \sqrt{x} \le y \le 1,\ 0 \le x \le 1\}$ to $\{(x,y) \mid 0 \le x \le y^2,\ 0 \le y \le 1\}$

$$\int_0^1 \int_{\sqrt{x}}^1 \sin(\pi y^3)\,dy\,dx = \int_0^1 \int_0^{y^2} \sin(\pi y^3)\,dx\,dy = \int_0^1 y^2 \sin(\pi y^3)\,dy$$

$$= -\frac{1}{3\pi}\cos(\pi y^3)\Big|_0^1 = \frac{2}{3\pi}.$$

Q6.

(a). $y' - 2xy = \frac{e^{x^2}}{(x+1)^2}$

Consider $y' - 2xy = 0$ i.e. $\frac{dy}{dx} - 2xy = 0$

$\int \frac{dy}{y} = \int 2x\,dx$. $\ln y = x^2 + C_1$. $\Rightarrow y = C_2 e^{x^2}$.

Let $y = ue^{x^2}$, where $u = u(x)$. $\frac{dy}{dx} = \frac{du}{dx} e^{x^2} + 2xue^{x^2} = \frac{du}{dx} e^{x^2} + 2xy$.

$y' - 2xy = \frac{e^{x^2}}{(x+1)^2} \Leftrightarrow \frac{du}{dx} e^{x^2} = \frac{e^{x^2}}{(x+1)^2} \Leftrightarrow \frac{dy}{dx} = \frac{1}{(x+1)^2}$

$u = \int \frac{dx}{(x+1)^2} = -\frac{1}{x+1} + C$.

$y = ue^{x^2} = -\frac{e^{x^2}}{x+1} + Ce^{x^2}$.

let $(x,y) = (0,5)$, we have $C = 6$.

So, the solution is $y = -\frac{e^{x^2}}{x+1} + 6e^{x^2}$.