

Lecture 4: Convolutional Neural Networks: Basic operations

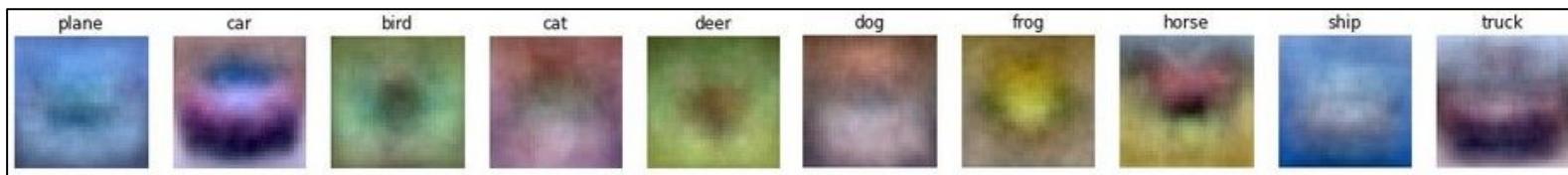
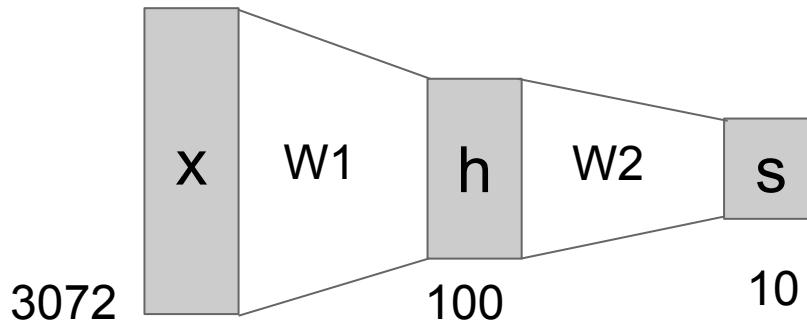
Last time: Neural Networks

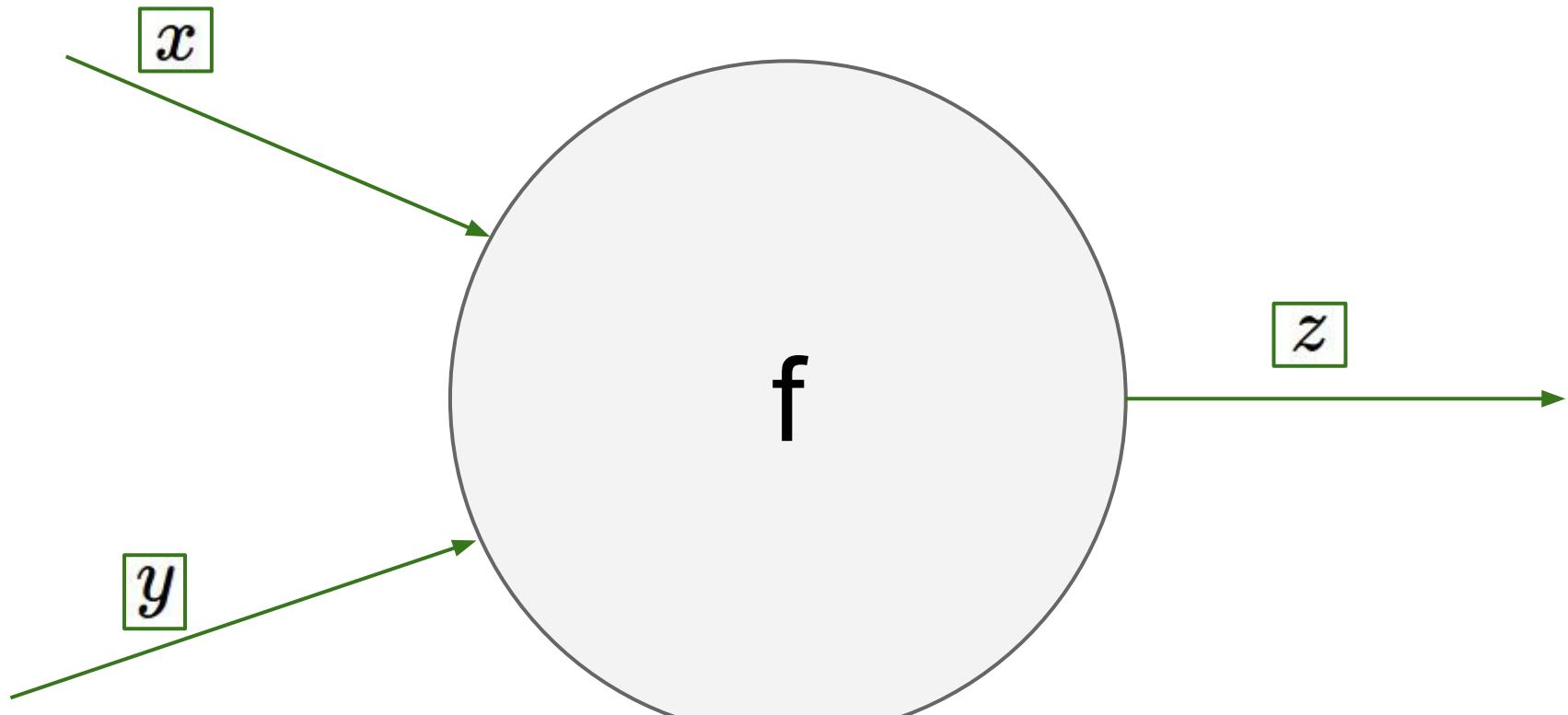
Linear score function:

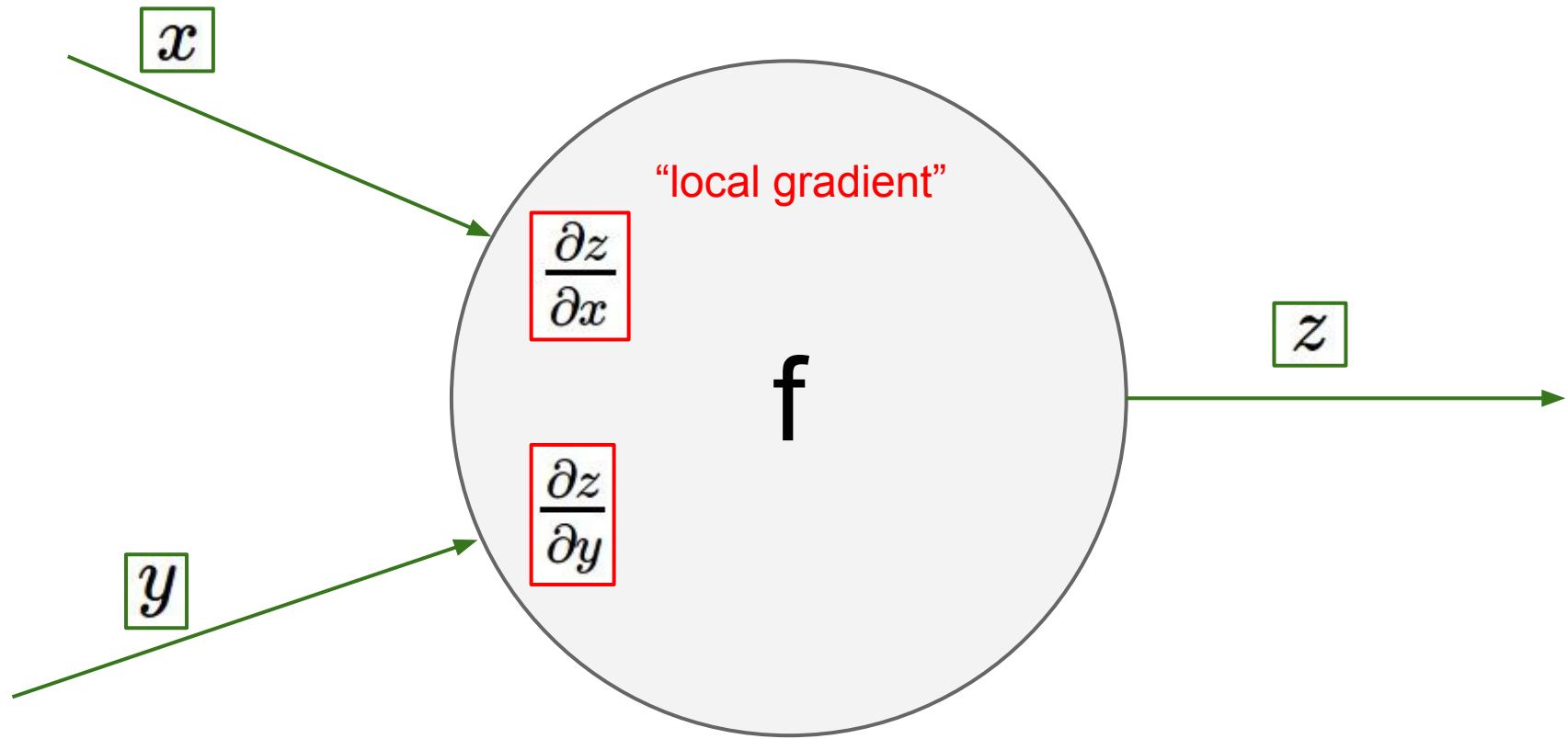
$$f = Wx$$

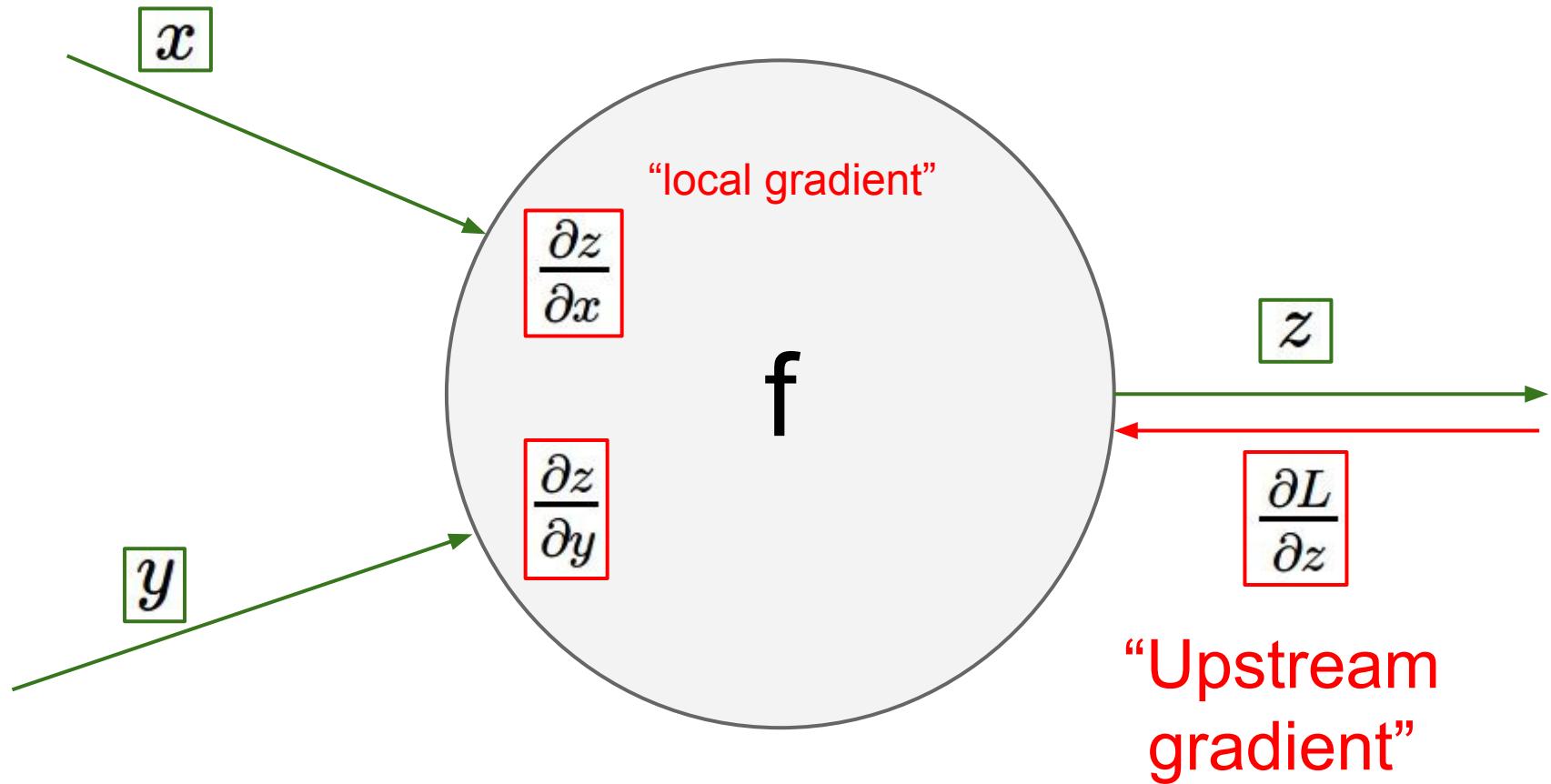
2-layer Neural Network

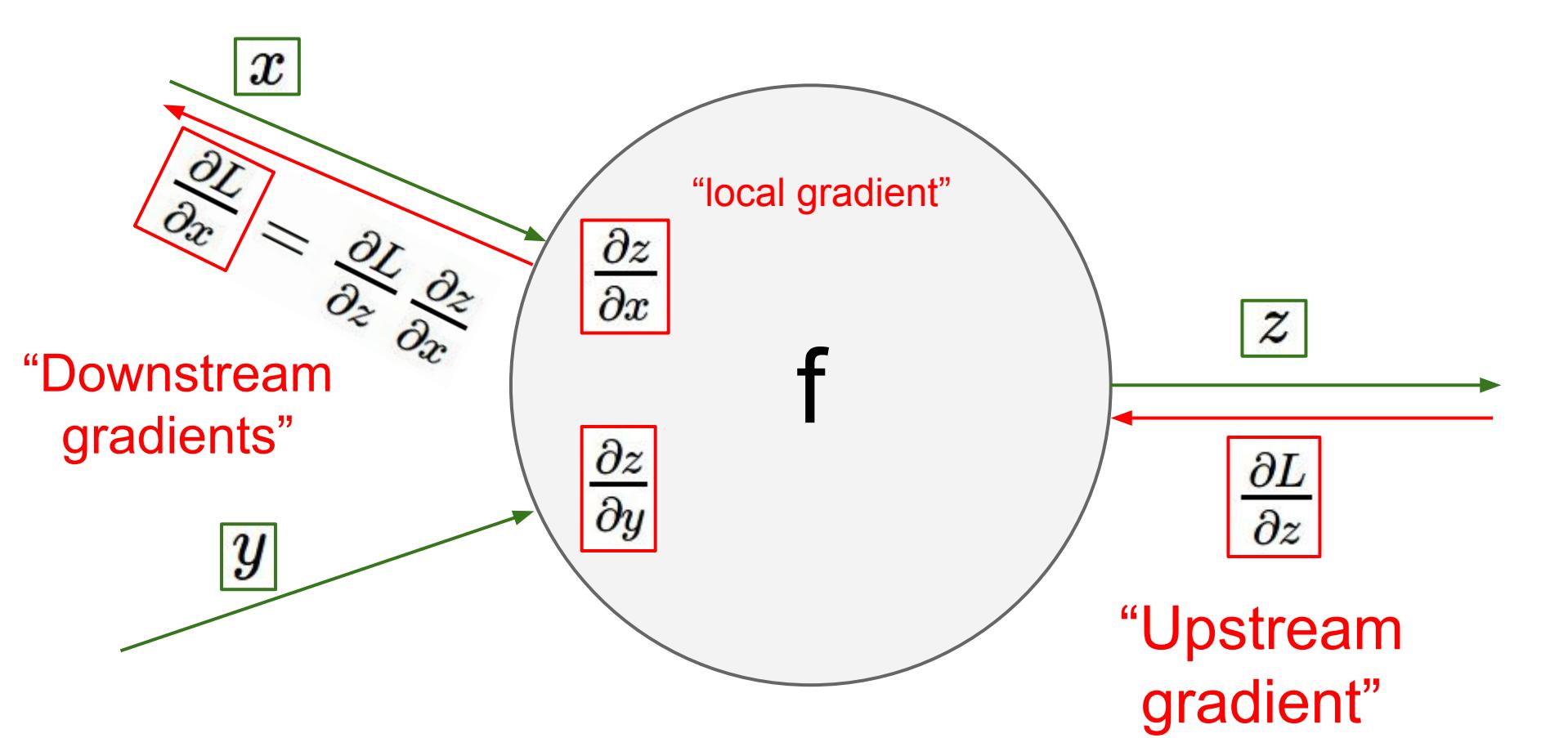
$$f = W_2 \max(0, W_1 x)$$

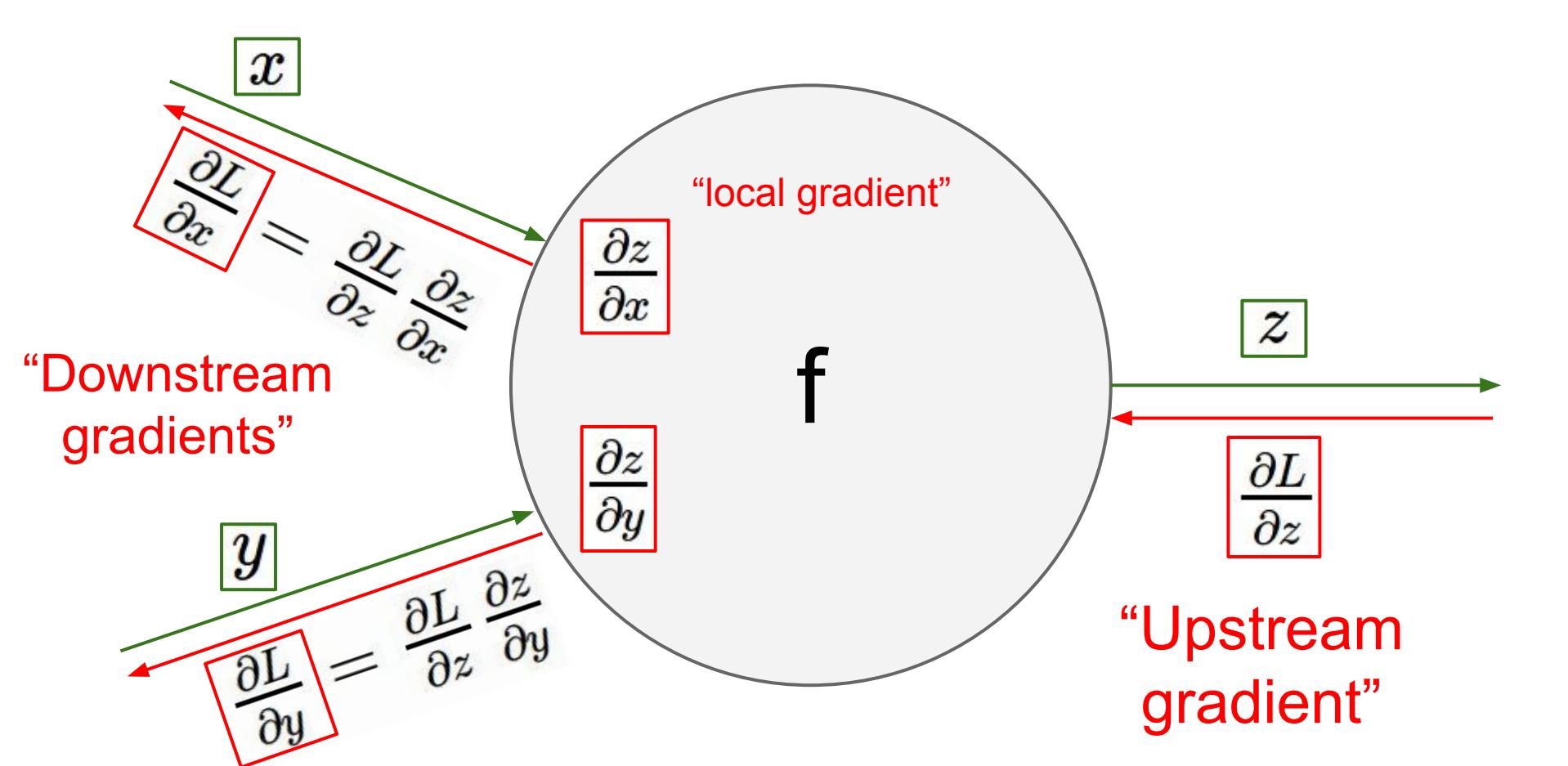


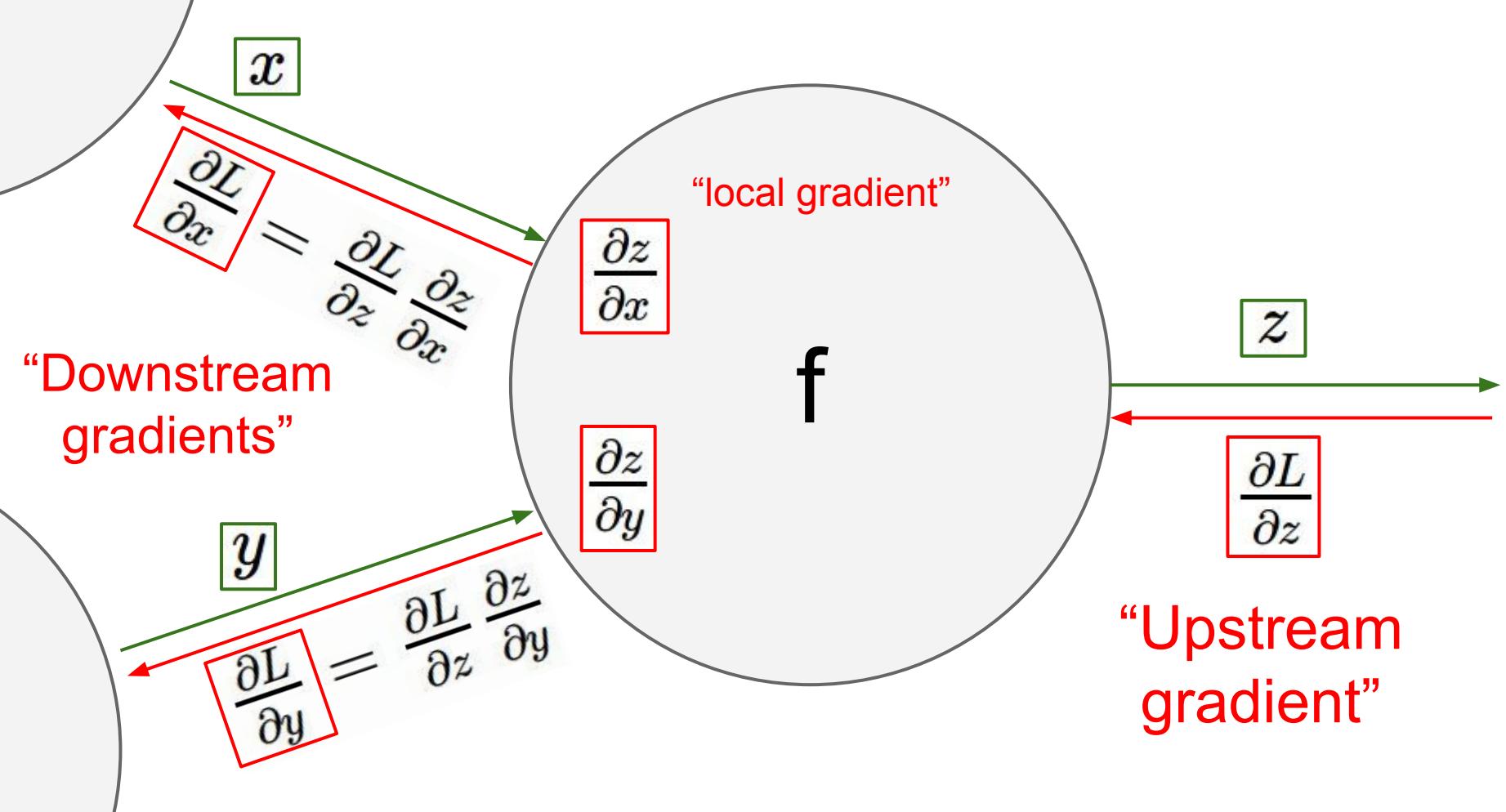












So far: backprop with scalars

What about vector-valued functions?

Recap: Vector derivatives

Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If x changes by a small amount, how much will y change?

Recap: Vector derivatives

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Vector to Scalar

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N \quad \left(\frac{\partial y}{\partial x} \right)_n = \frac{\partial y}{\partial x_n}$$

For each element of x , if it changes by a small amount then how much will y change?

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Vector to Vector

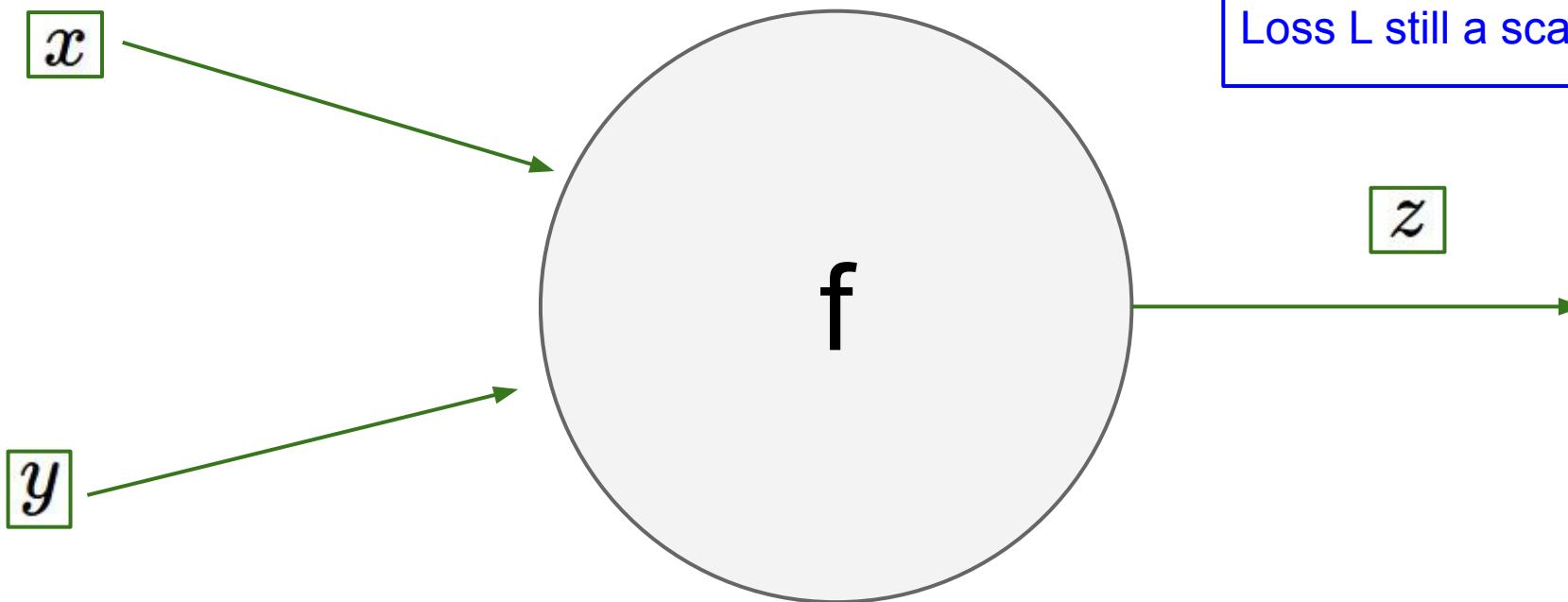
$$x \in \mathbb{R}^N, y \in \mathbb{R}^M$$

Derivative is **Jacobian**:

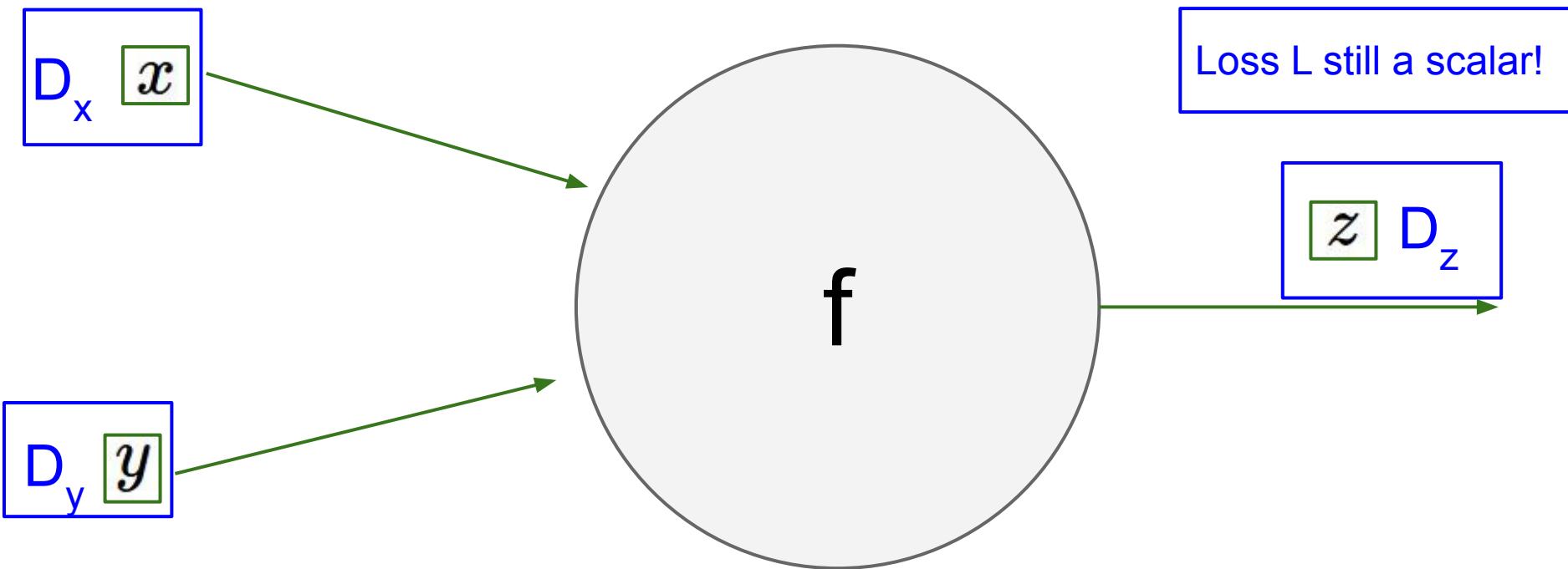
$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M} \quad \left(\frac{\partial y}{\partial x} \right)_{n,m} = \frac{\partial y_m}{\partial x_n}$$

For each element of x , if it changes by a small amount then how much will each element of y change?

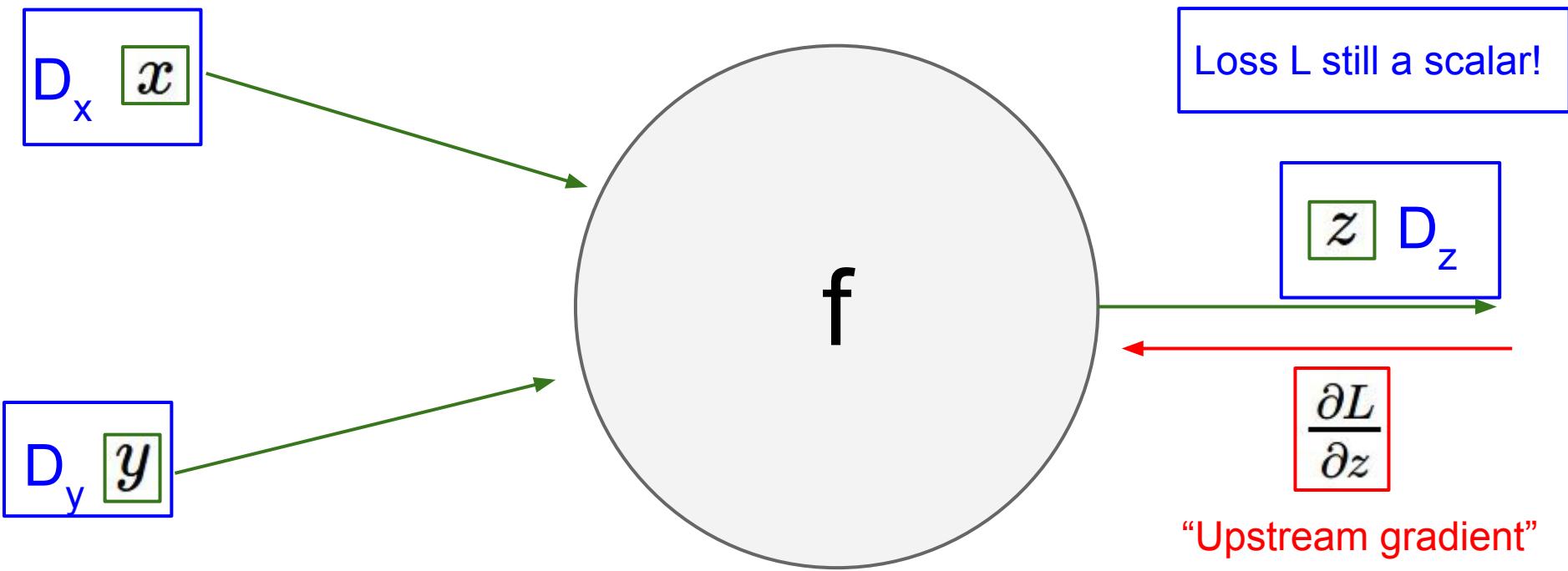
Backprop with Vectors



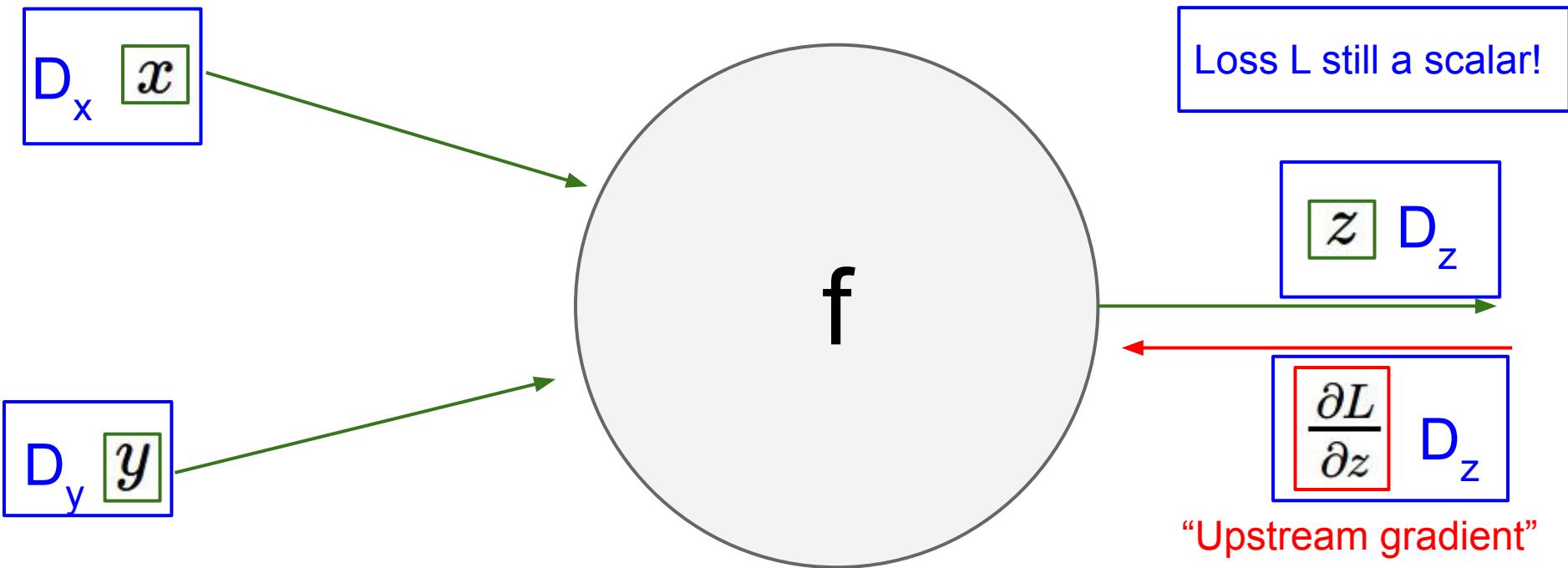
Backprop with Vectors



Backprop with Vectors



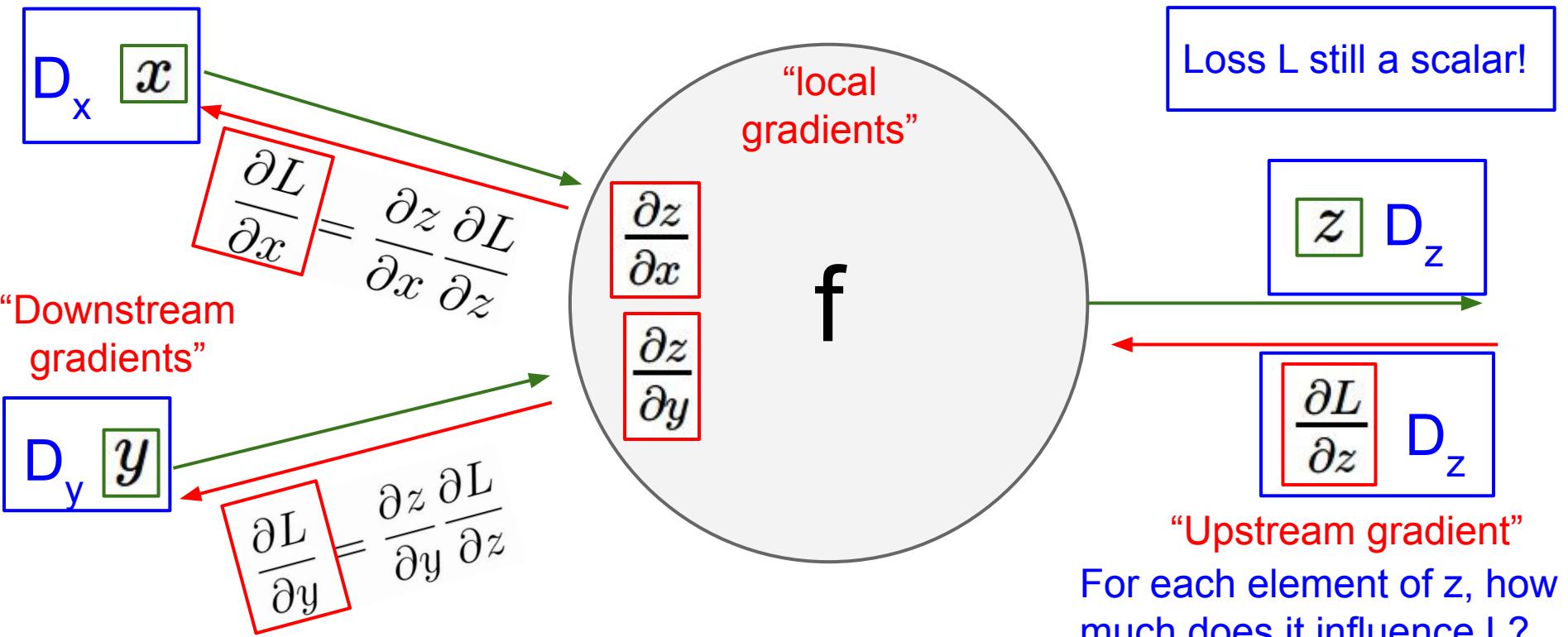
Backprop with Vectors



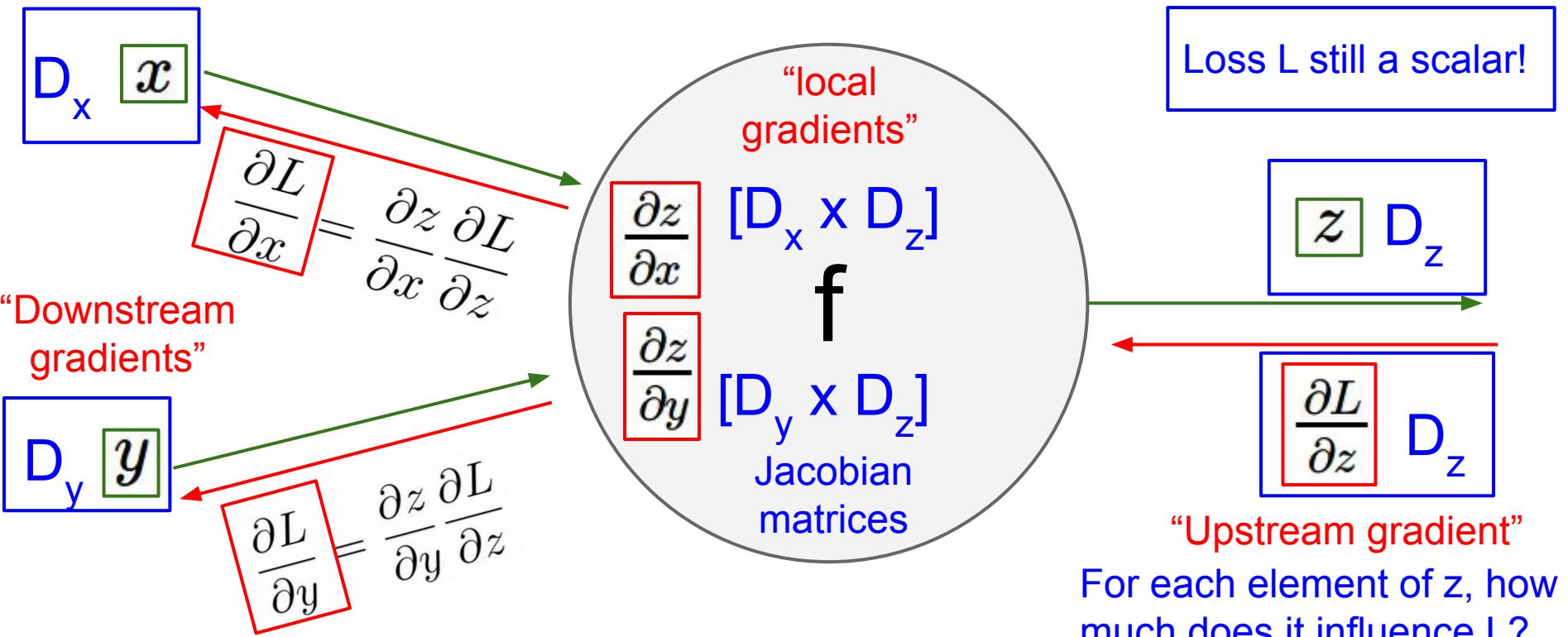
“Upstream gradient”

For each element of z , how
much does it influence L ?

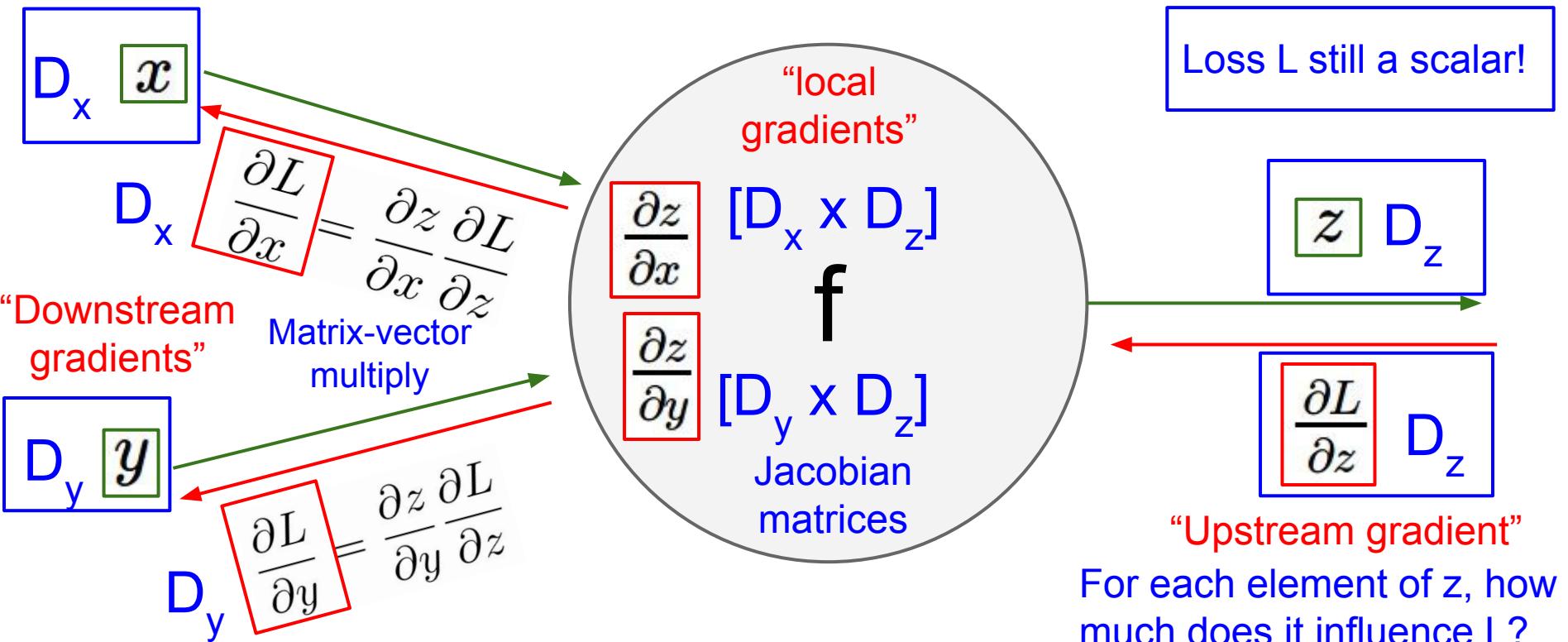
Backprop with Vectors



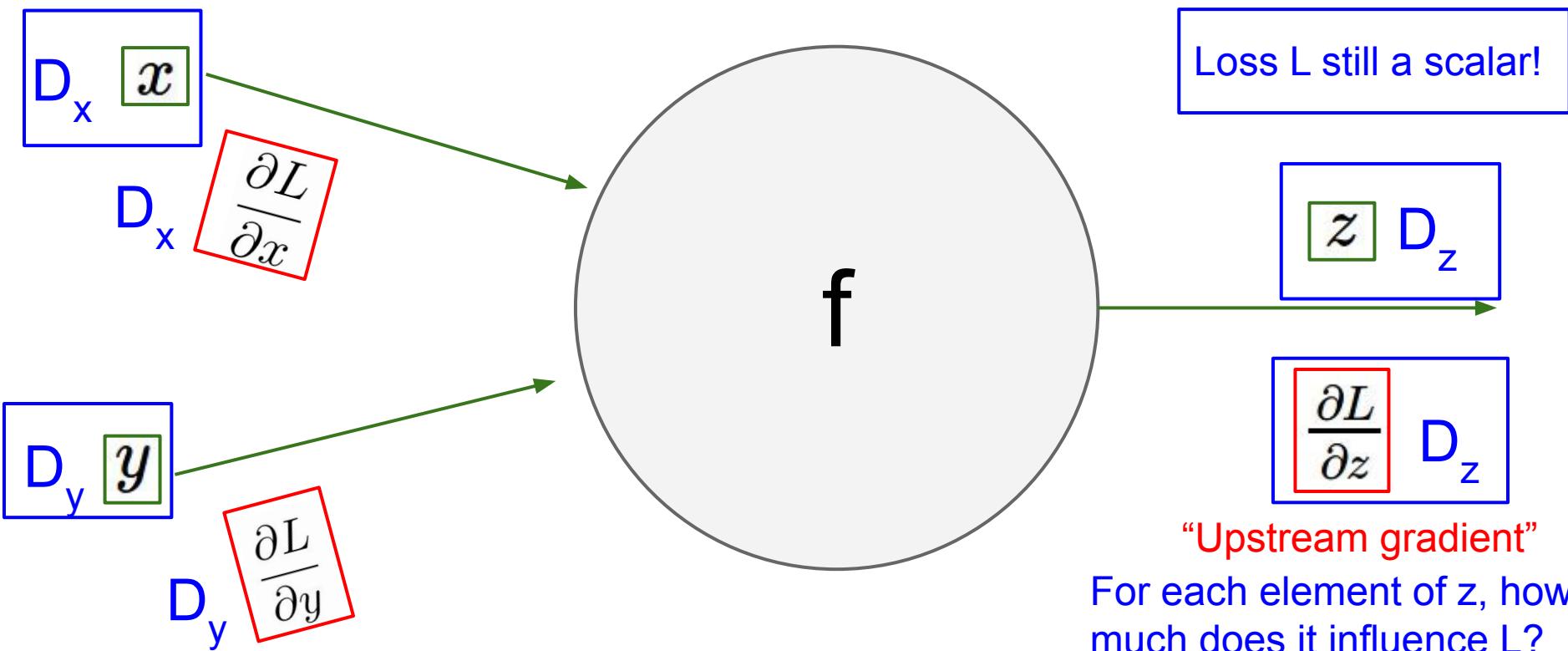
Backprop with Vectors



Backprop with Vectors



Gradients of variables wrt loss have same dims as the original variable



Backprop with Vectors

4D input x :

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \xrightarrow{\hspace{1cm}} \begin{array}{c} \text{f}(x) = \max(0, x) \\ (\text{elementwise}) \end{array}$$

4D output z :

$$\begin{array}{l} \xrightarrow{\hspace{1cm}} \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} \\ \xrightarrow{\hspace{1cm}} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} \\ \xrightarrow{\hspace{1cm}} \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \xrightarrow{\hspace{1cm}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array}$$

Backprop with Vectors

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4D dL/dz :

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix} \leftarrow$$

Upstream
gradient

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Jacobian $\frac{dz}{dx}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4D $\frac{dL}{dz}$:

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Upstream
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Backprop with Vectors

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$[dz/dx] [dL/dz]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

4D dL/dz :

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Jacobian is **sparse**:
off-diagonal entries
always zero! Never
explicitly form
Jacobian -- instead
use **implicit**
multiplication

4D dL/dx :

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4D dL/dx :

$$\begin{bmatrix} 4 \\ 0 \\ 5 \\ 0 \end{bmatrix} \leftarrow$$

$$\left(\frac{\partial L}{\partial x} \right)_i = \begin{cases} \left(\frac{\partial L}{\partial z} \right)_i & \text{if } x_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

$[dz/dx]$ $[dL/dz]$

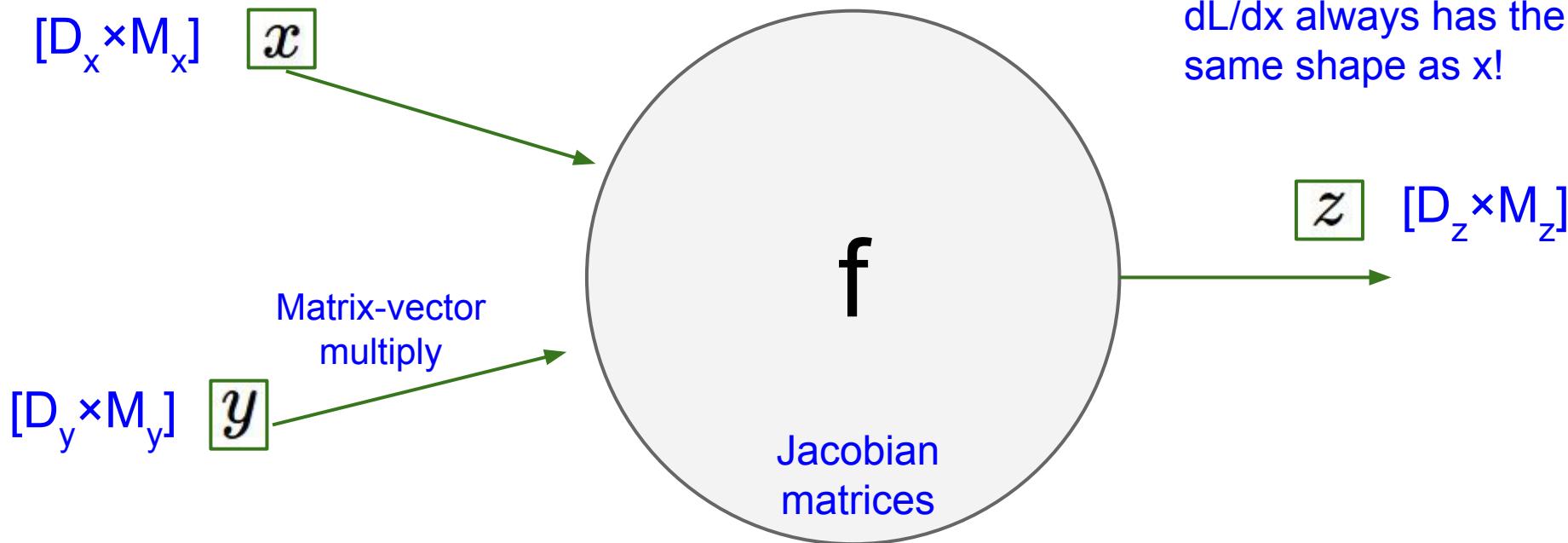
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Upstream
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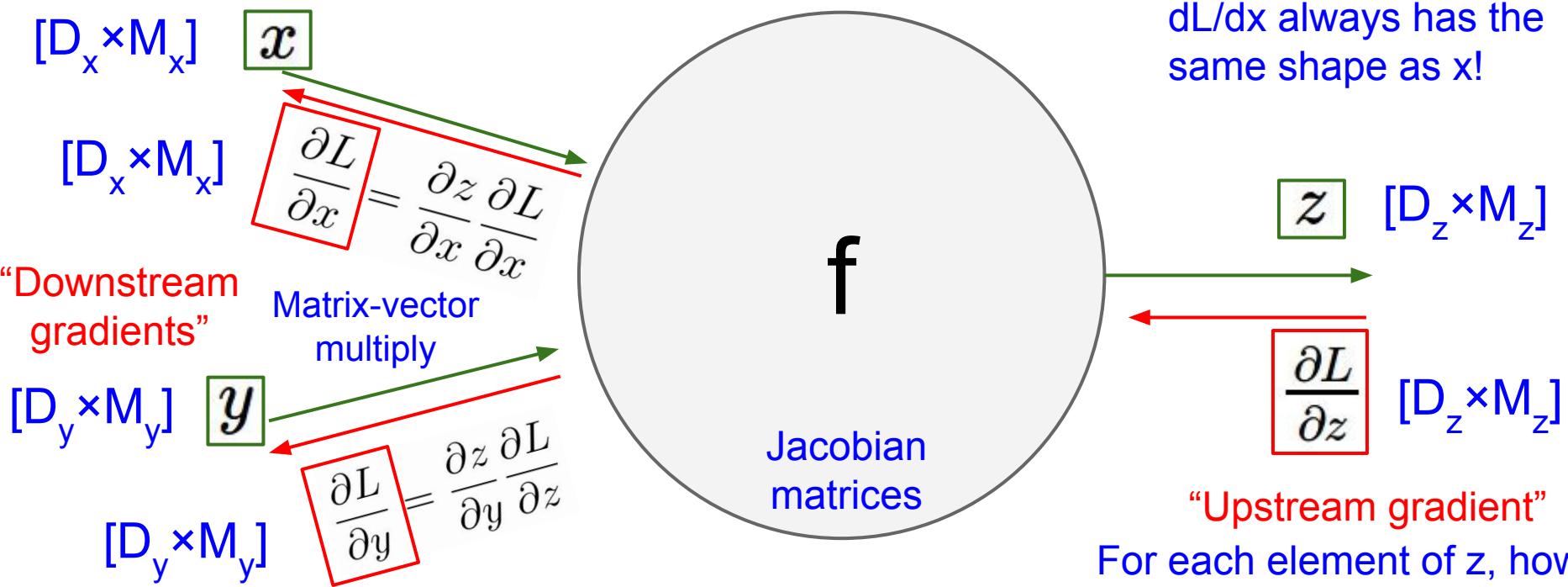
Backprop with Matrices (or Tensors)

Loss L still a scalar!



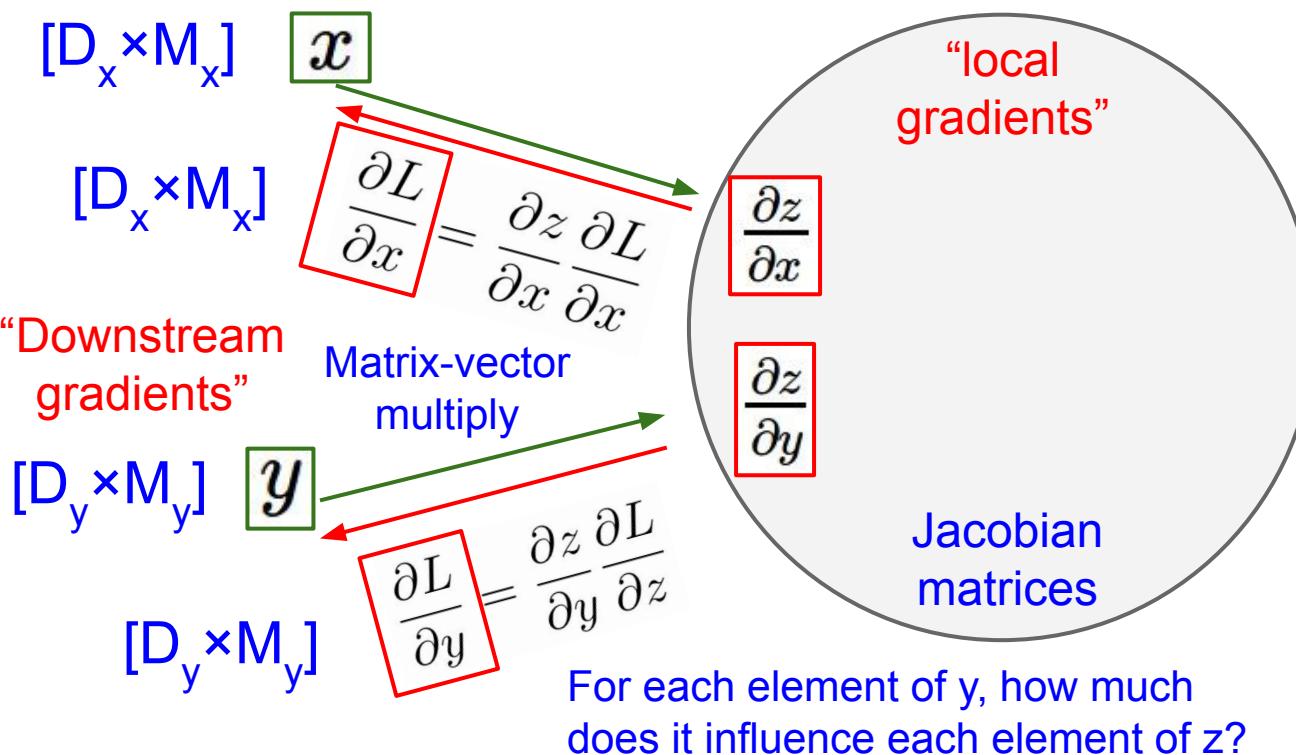
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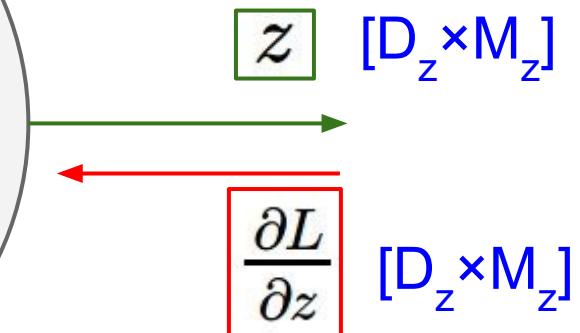


Backprop with Matrices (or Tensors)

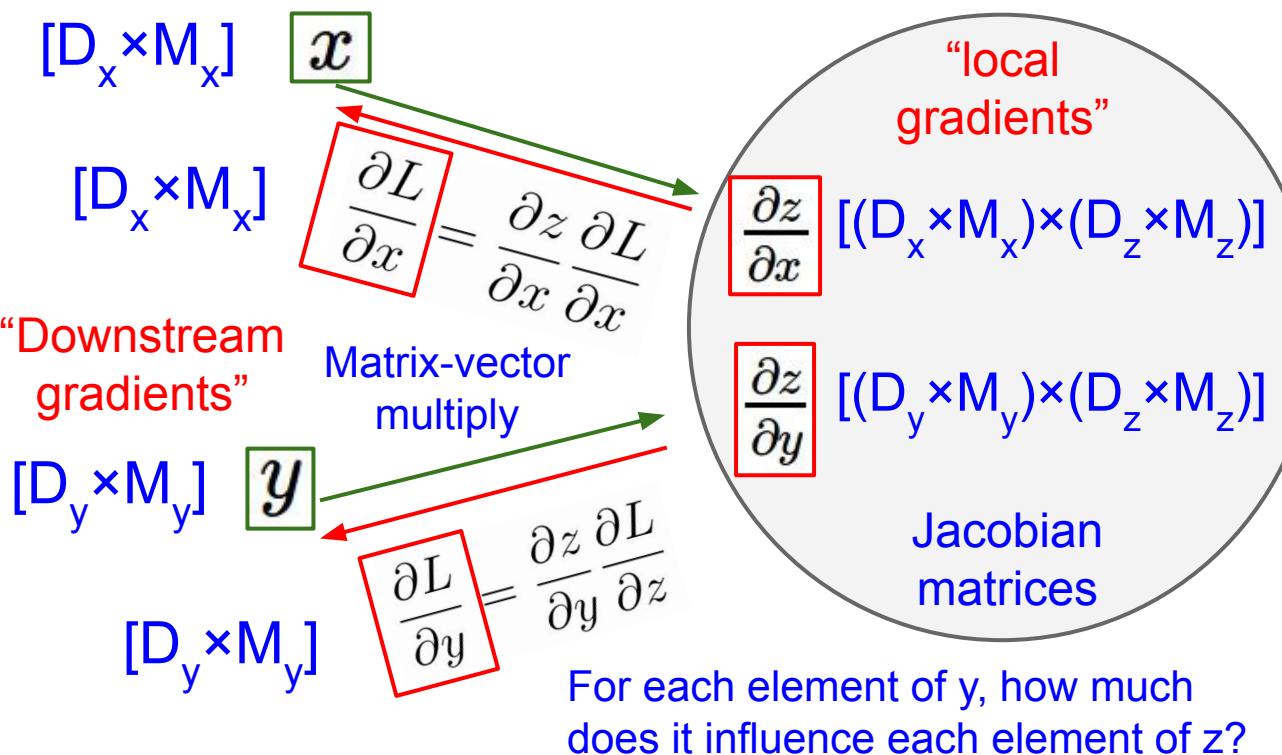
Loss L still a scalar!



dL/dx always has the same shape as x !

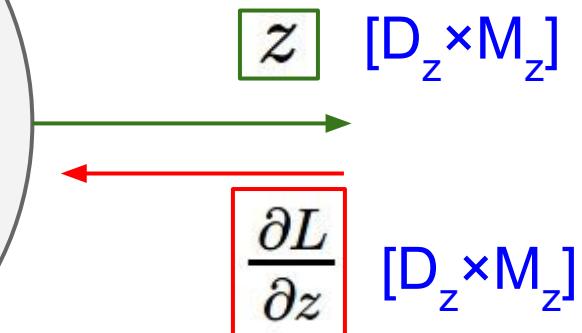


Backprop with Matrices (or Tensors)



Loss L still a scalar!

dL/dx always has the same shape as x !



"Upstream gradient"

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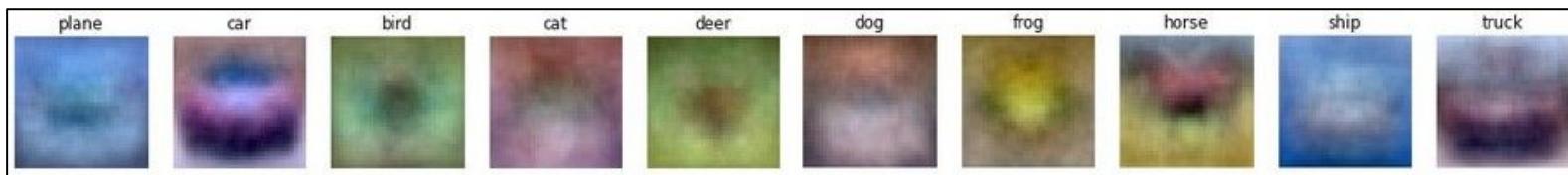
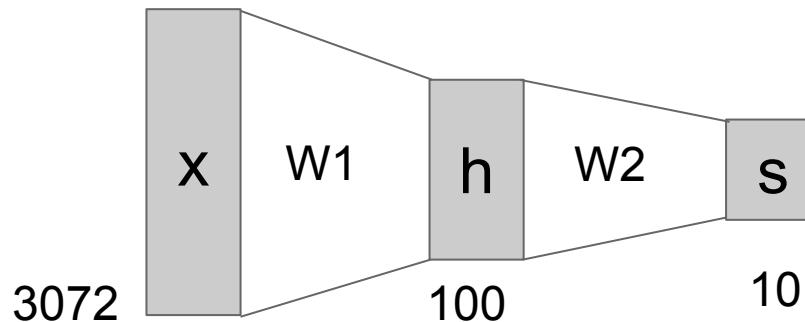
Wrapping up: Neural Networks

Linear score function:

$$f = Wx$$

2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$



Next: Convolutional Neural Networks

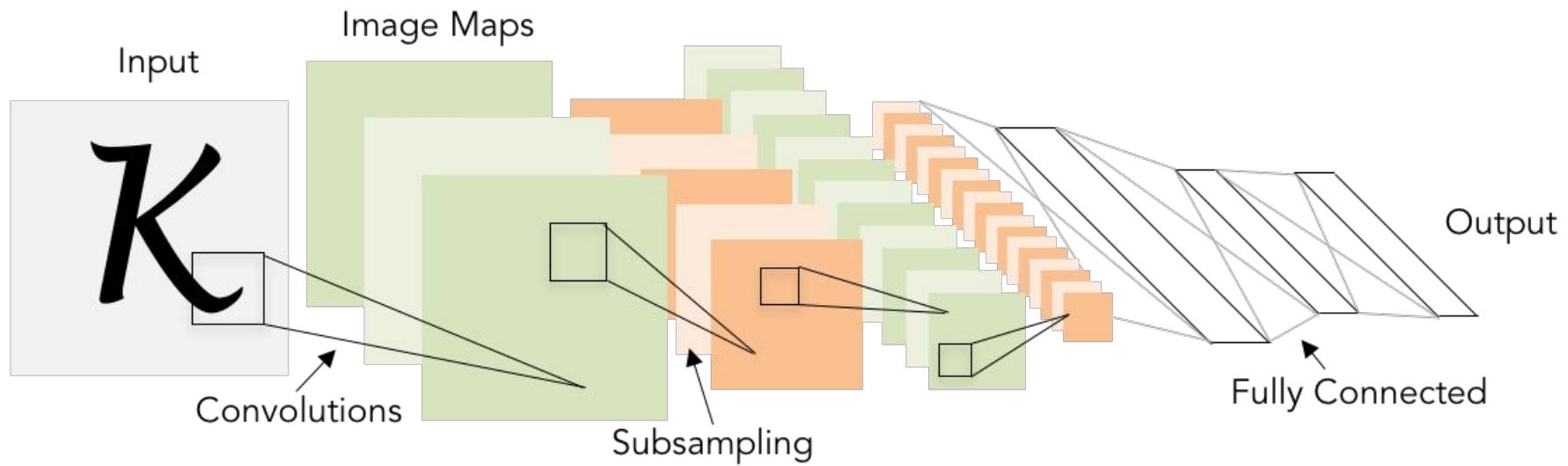


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

A bit of history...

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

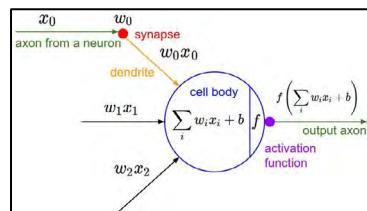
The machine was connected to a camera that used 20×20 cadmium sulfide photocells to produce a 400-pixel image.

recognized
letters of the alphabet

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

update rule:

$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i}$$

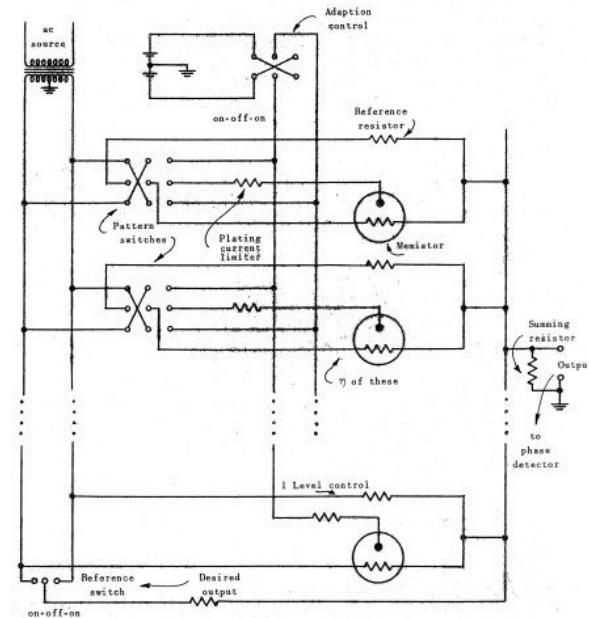
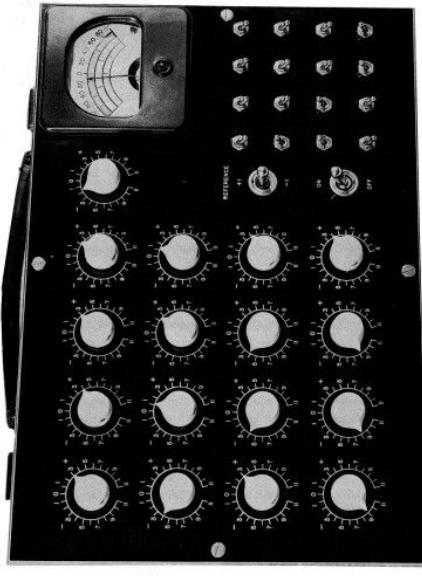
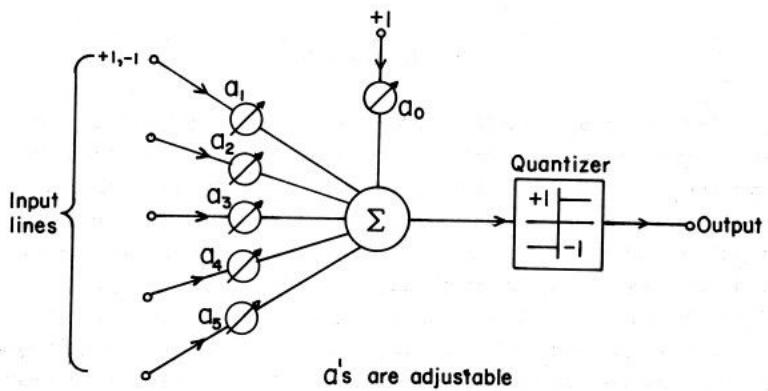


Frank Rosenblatt, ~1957: Perceptron



[This image](#) by Rocky Acosta is licensed under [CC-BY 3.0](#)

A bit of history...

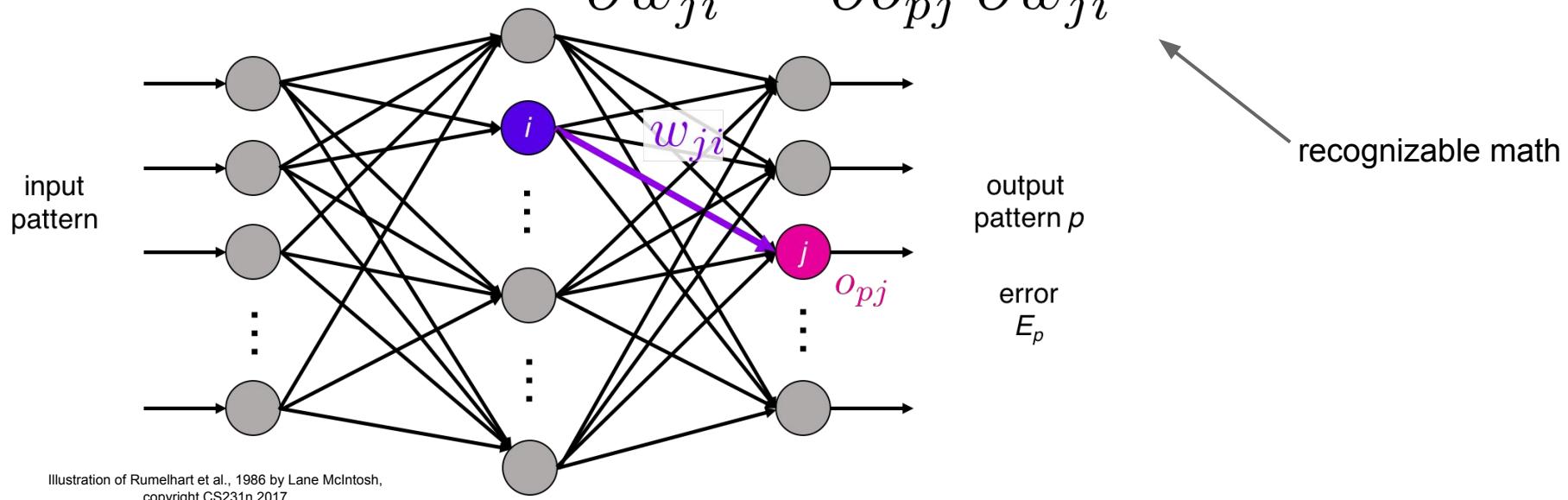


Widrow and Hoff, ~1960: Adaline/Madaline

These figures are reproduced from [Widrow 1960, Stanford Electronics Laboratories Technical Report](#) with permission from [Stanford University Special Collections](#).

A bit of history...

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}$$



Rumelhart et al., 1986: First time back-propagation became popular

A bit of history...

[Hinton and Salakhutdinov 2006]

Reinvigorated research in
Deep Learning

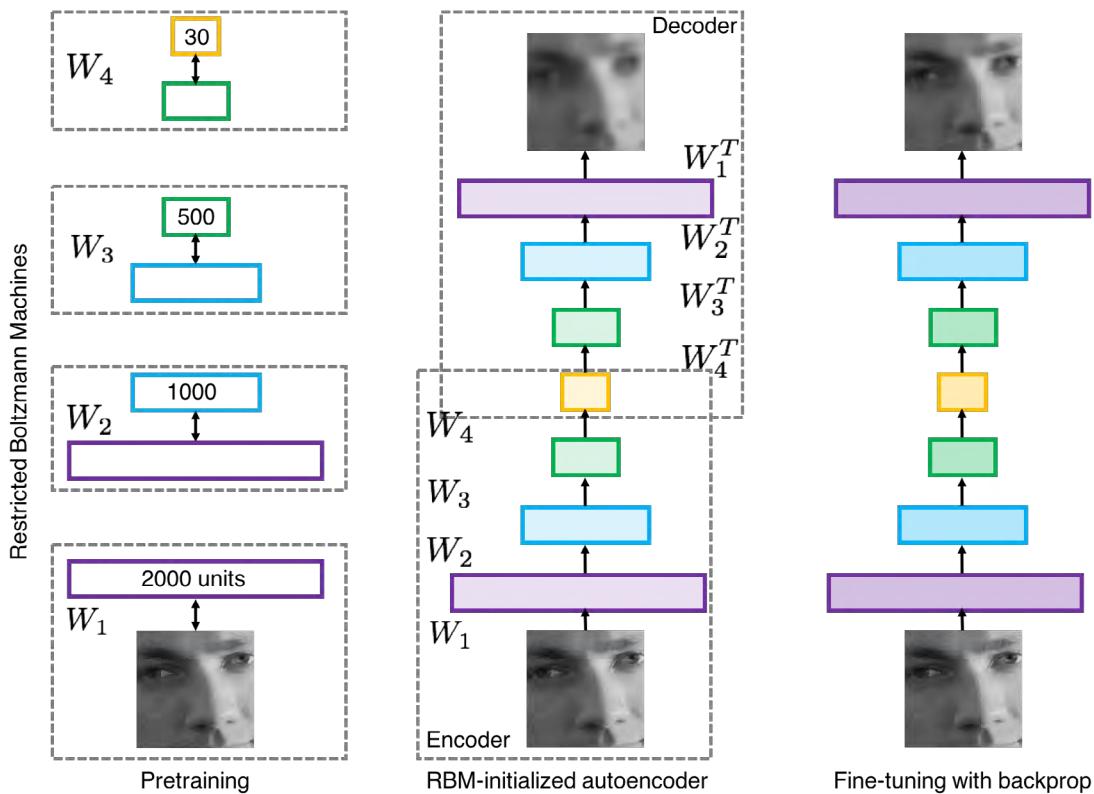


Illustration of Hinton and Salakhutdinov 2006 by Lane McIntosh, copyright CS231n 2017

First strong results

Acoustic Modeling using Deep Belief Networks

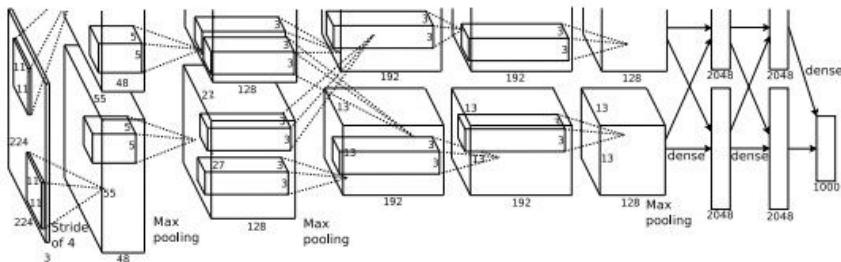
Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010

Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition

George Dahl, Dong Yu, Li Deng, Alex Acero, 2012

Imagenet classification with deep convolutional neural networks

Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012



Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

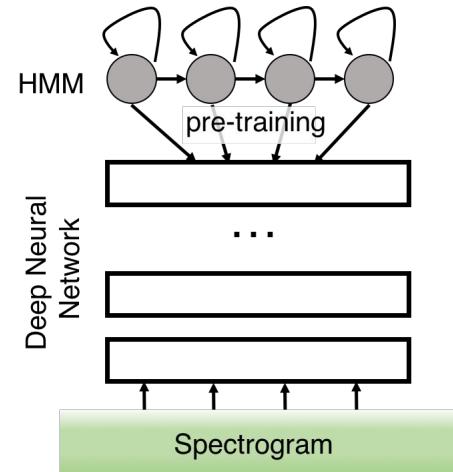


Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017

A bit of history:

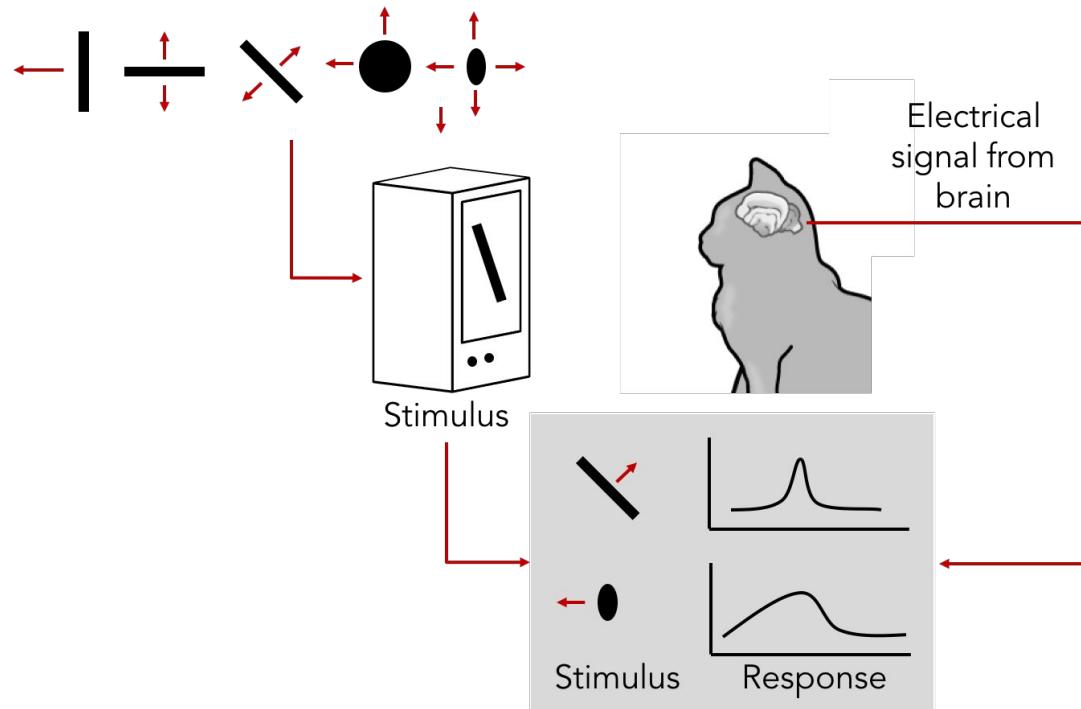
**Hubel & Wiesel,
1959**

RECEPTIVE FIELDS OF SINGLE
NEURONES IN
THE CAT'S STRIATE CORTEX

1962

RECEPTIVE FIELDS, BINOCULAR
INTERACTION
AND FUNCTIONAL ARCHITECTURE IN
THE CAT'S VISUAL CORTEX

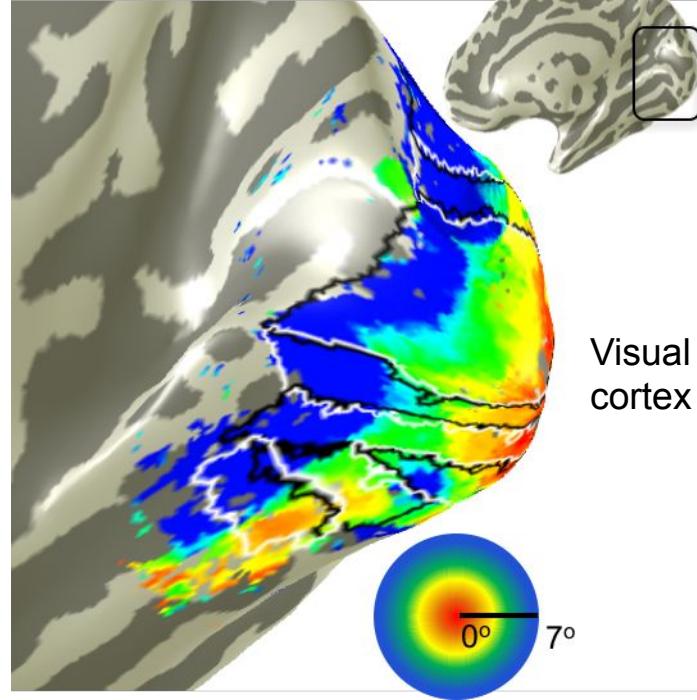
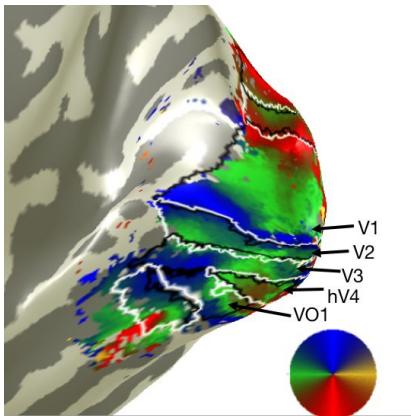
1968...



[Cat image](#) by CNX OpenStax is licensed
under CC BY 4.0; changes made

A bit of history

Topographical mapping in the cortex:
nearby cells in cortex represent
nearby regions in the visual field



Retinotopy images courtesy of Jesse Gomez in the Stanford Vision & Perception Neuroscience Lab.

Hierarchical organization

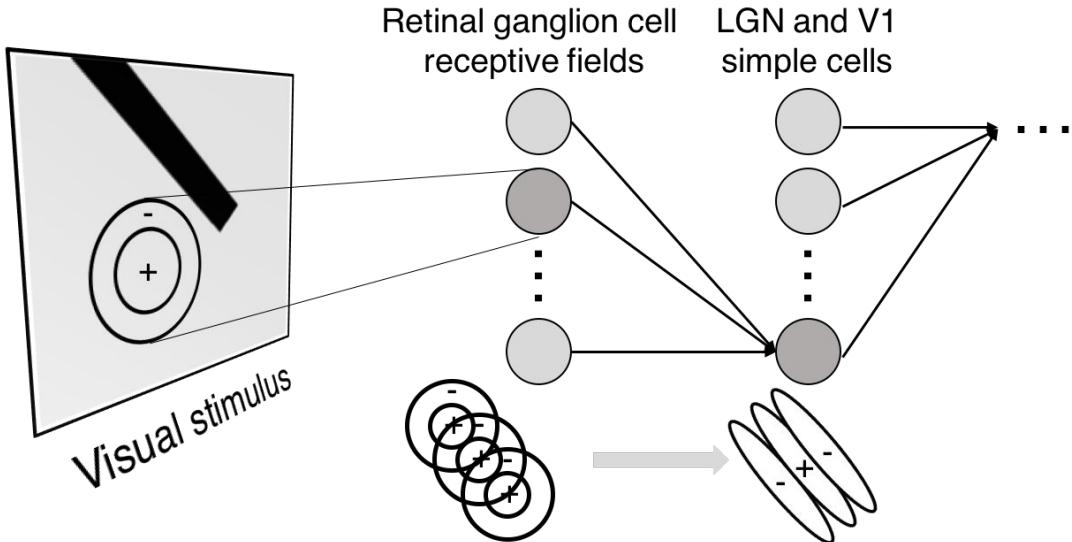
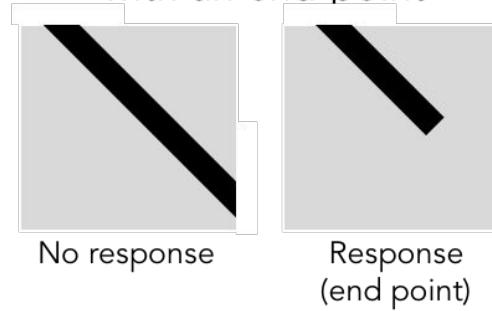


Illustration of hierarchical organization in early visual pathways by Lane McIntosh, copyright CS231n 2017

Simple cells:
Response to light orientation

Complex cells:
Response to light orientation and movement

Hypercomplex cells:
response to movement with an end point



A bit of history



Rainer Goebel

Professor of Cognitive Neuroscience, [Maastricht University](#)
Verified email at maastrichtuniversity.nl - [Homepage](#)

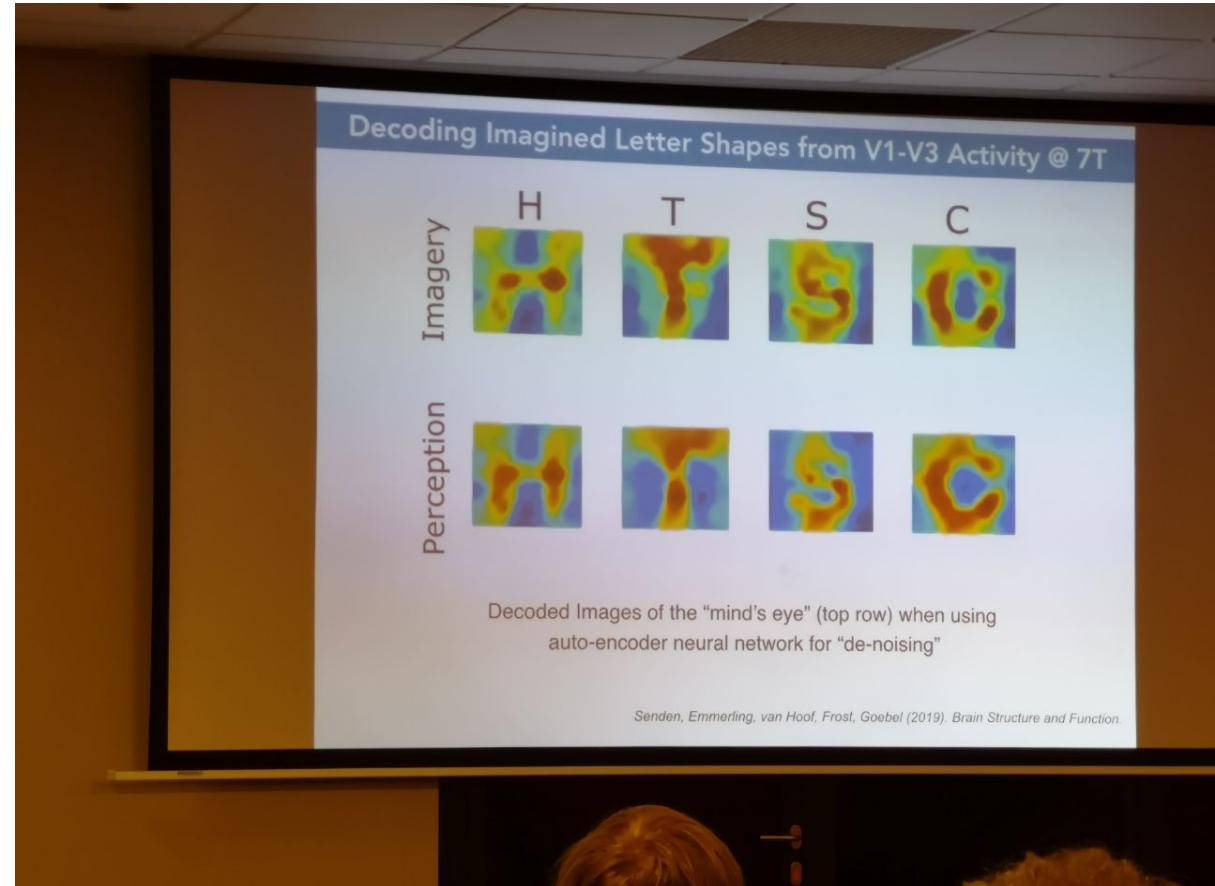
Ultra-High Field fMRI Human Brain Research Cognitive Neuroscience
Neural Network Modelling Brain-Computer Interfaces

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| Citations | 34458 | 15380 |
| h-index | 101 | 67 |
| i10-index | 274 | 241 |

Создатель Turbo BrainVoyager
Наша альтернатива для TBV:
<http://opennft.org/>



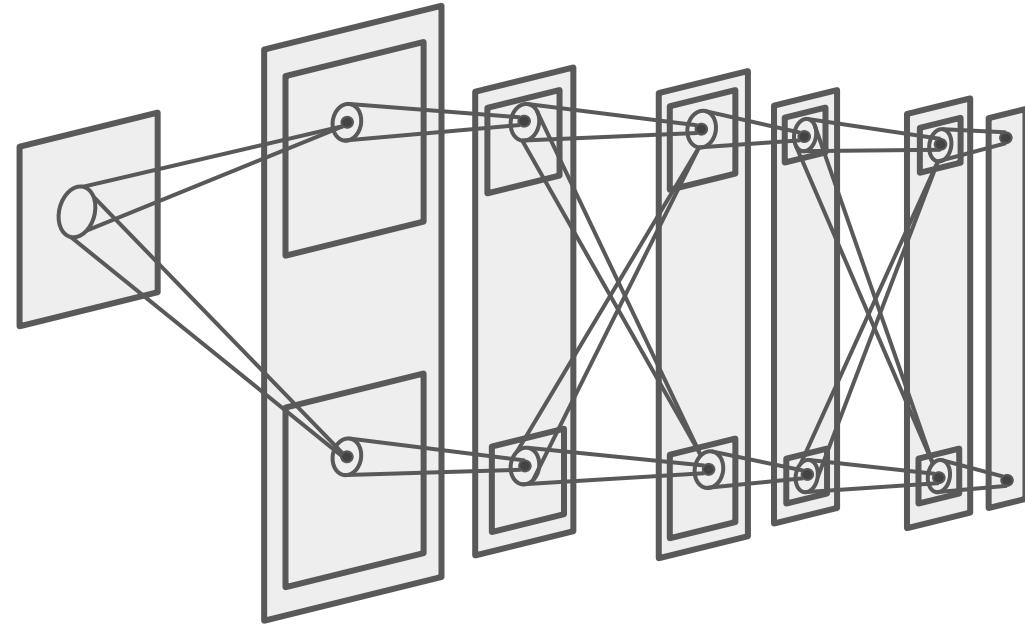
A bit of history:

Neocognitron [Fukushima 1980]

“sandwich” architecture (SCSCSC...)

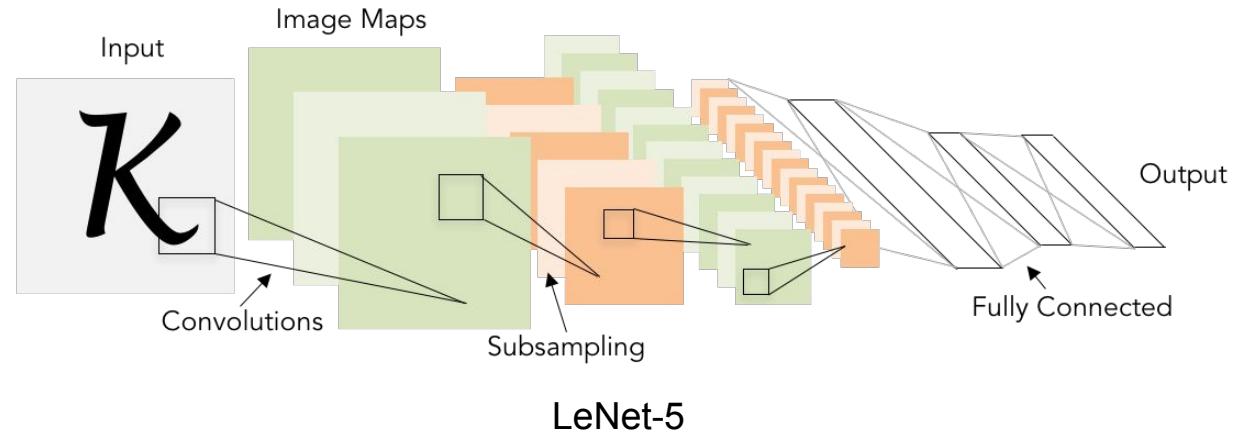
simple cells: modifiable parameters

complex cells: perform pooling



A bit of history: Gradient-based learning applied to document recognition

[LeCun, Bottou, Bengio, Haffner 1998]



A bit of history: ImageNet Classification with Deep Convolutional Neural Networks *[Krizhevsky, Sutskever, Hinton, 2012]*

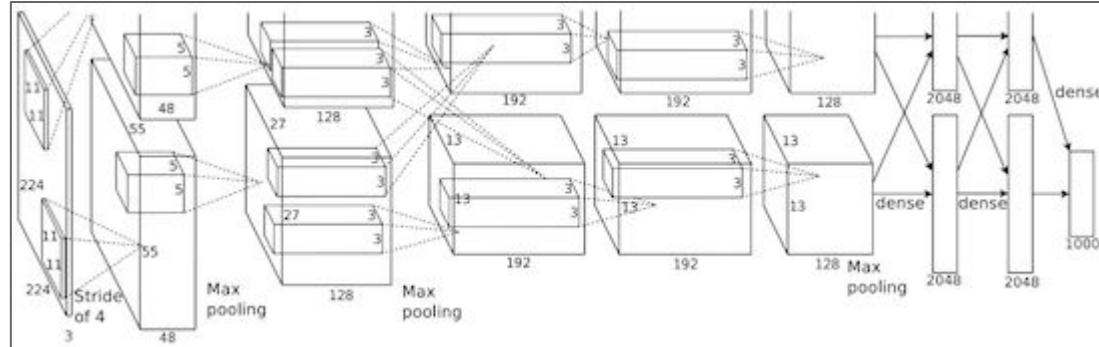


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

“AlexNet”

A bit of history

1989 G Cybenko

Теорема об универсальной аппроксимации

1998 Yann LeCun
сверточные сети

2007 – Выход NVIDIA CUDA,

2009 – Google отказывается от нейронных сетей

2012 – AlexNet

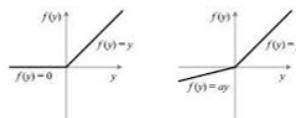


Figure 1. ReLU vs. PReLU. For PReLU, the coefficient of the negative part is not constant and is adaptively learned.

[Approximation by superpositions of a sigmoidal function - Springer Link](#)

<https://link.springer.com/article/10.1007/BF02551274> - Перевести эту страницу

автор: G Cybenko - 1989 - Цитируется: 10688 - Похожие статьи

[ieeexplore.ieee.org › document](http://ieeexplore.ieee.org/document/) - Перевести эту страницу

[Gradient-based learning applied to document recognition ...](#)

Gradient-based learning applied to document recognition. ... A new learning paradigm, called graph transformer networks (GTN), allows such multimodule systems to be trained globally using gradient-based methods so as to minimize an overall performance measure. Two systems for online handwriting recognition are described.

автор: Y Lecun - 1998 - Цитируется: 28105 - Похожие статьи



[\[PDF\] ImageNet Classification with Deep Convolutional Neural Networks](#)

<https://papers.nips.cc/.../4824-imagenet-classification-with-de...> ▾ Перевести эту страницу

автор: A Krizhevsky - 2012 - Цитируется: 34232 - Похожие статьи

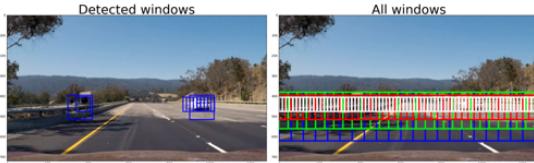
[Delving Deep into Rectifiers: Surpassing Human-Level Performance .](#)

<https://arxiv.org/.../4824-imagenet-classification-with-de...> ▾ Перевести эту страницу

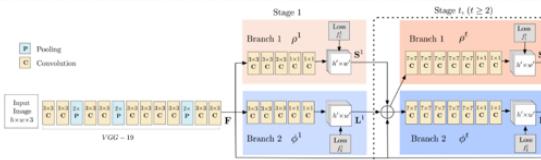
автор: K He - 2015 - Цитируется: 3856 - Похожие статьи

Fast-forward to today: ConvNets are everywhere

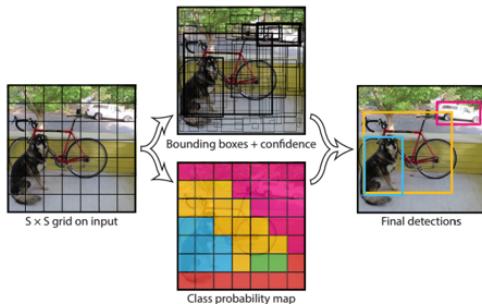
Fast Detection



Tracking



Sliding window detection -NO



YOLO - You Only Look Once – Yes!

Real time pose tracking, CVPR 17



Openpose

Вычислительная фотография –

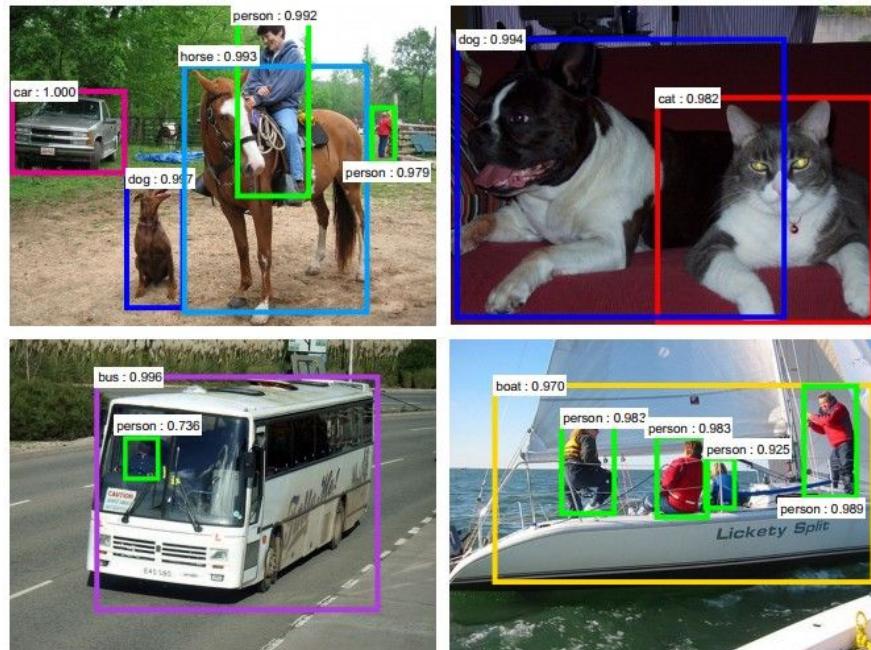
- Стекинг фото
- Сверхразрешение
- Съемка в темноте



Light.co

Fast-forward to today: ConvNets are everywhere

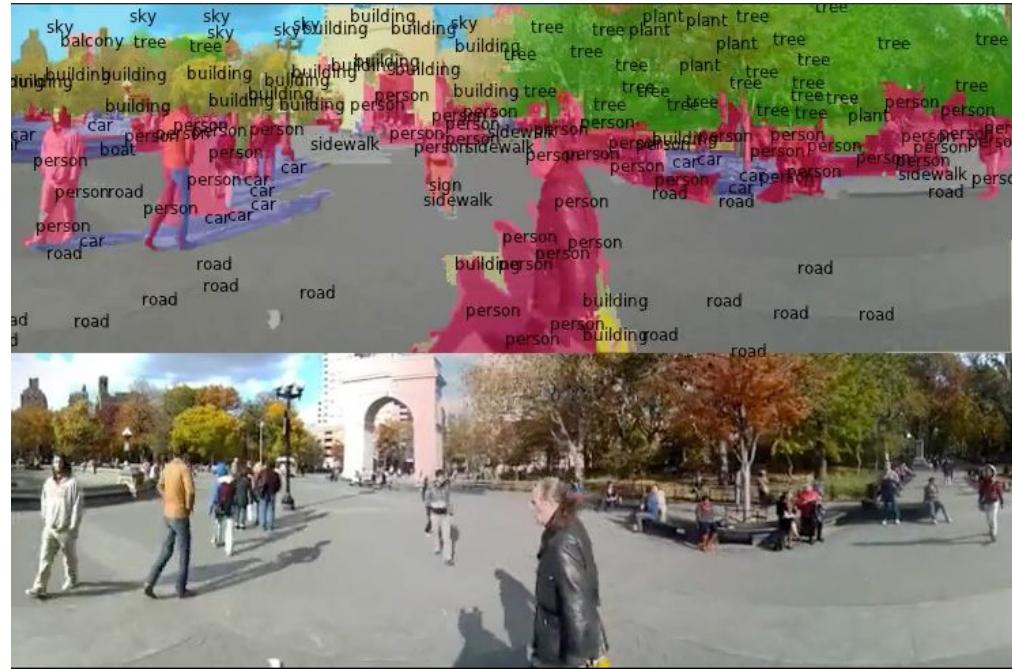
Detection



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[Faster R-CNN: Ren, He, Girshick, Sun 2015]

Segmentation



Figures copyright Clement Farabet, 2012.
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[Farabet et al., 2012]

Fast-forward to today: ConvNets are everywhere



self-driving cars

Photo by Lane McIntosh. Copyright CS231n 2017.



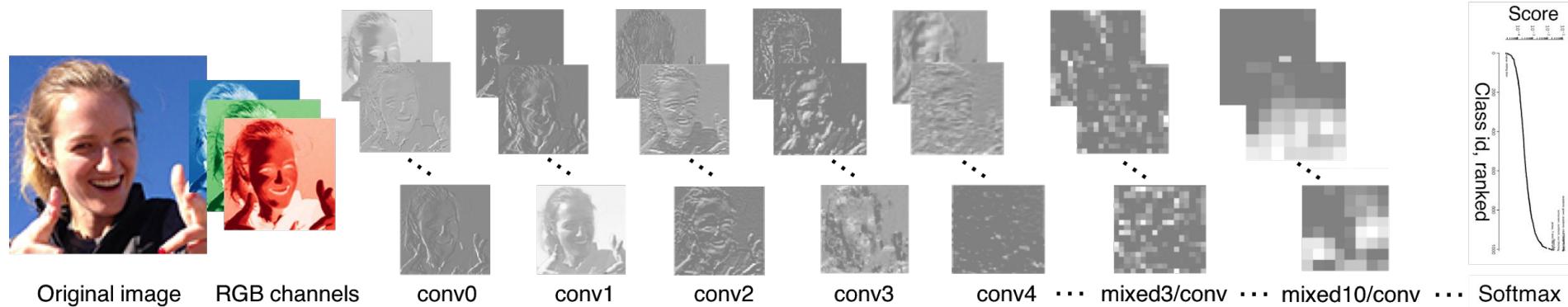
[This image](#) by GBPublic_PR is licensed under [CC-BY 2.0](#)

NVIDIA Tesla line

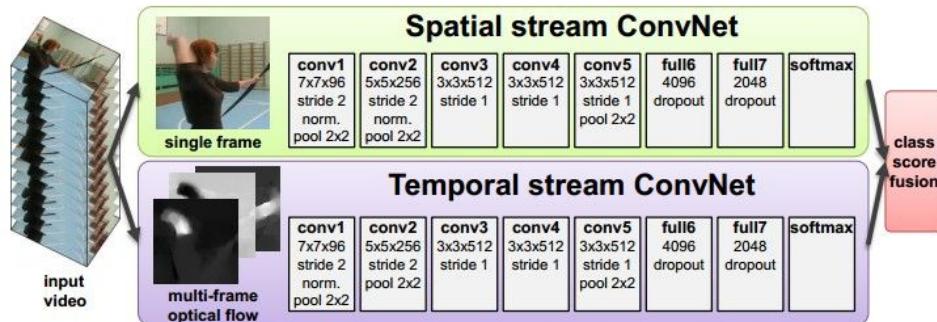
(these are the GPUs on rye01.stanford.edu)

Note that for embedded systems a typical setup would involve NVIDIA Tegras, with integrated GPU and ARM-based CPU cores.

Fast-forward to today: ConvNets are everywhere



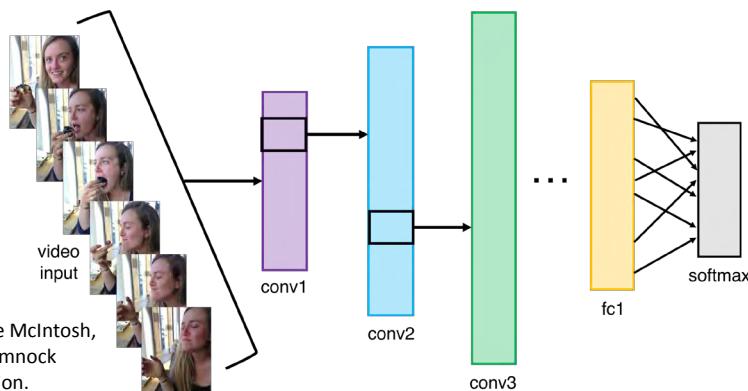
Activations of [inception-v3 architecture](#) [Szegedy et al. 2015] to image of Emma McIntosh, used with permission. Figure and architecture not from Taigman et al. 2014.



[Simonyan et al. 2014]

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Fast-forward to today: ConvNets are everywhere

Classification



Retrieval



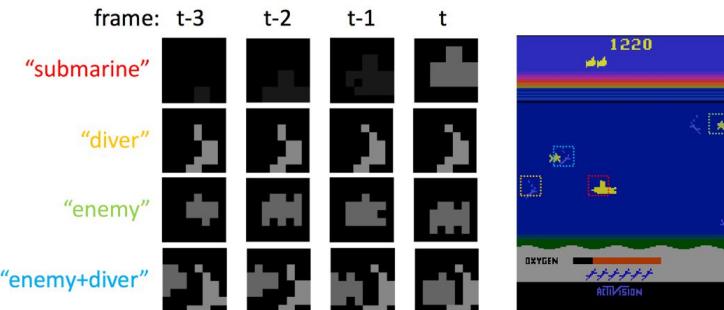
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Fast-forward to today: ConvNets are everywhere



Images are examples of pose estimation, not actually from Toshev & Szegedy 2014. Copyright Lane McIntosh.

[Toshev, Szegedy 2014]

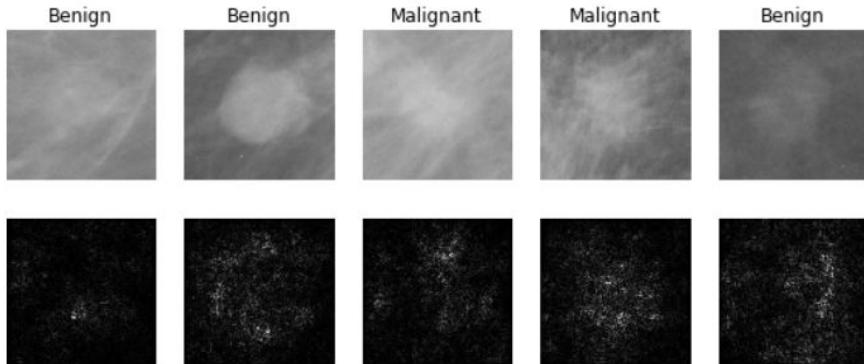


[Guo et al. 2014]



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Fast-forward to today: ConvNets are everywhere



[Levy et al. 2016]

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[Dieleman et al. 2014]

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Photos by Lane McIntosh.
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[Sermanet et al. 2011]
[Ciresan et al.]

[This image](#) by Christin Khan is in the public domain and originally came from the U.S. NOAA.



Whale recognition, Kaggle Challenge

Photo and figure by Lane McIntosh; not actual example from Mnih and Hinton, 2010 paper.



Mnih and Hinton, 2010

No errors



A white teddy bear sitting in the grass



A man riding a wave on top of a surfboard

Minor errors



A man in a baseball uniform throwing a ball



A cat sitting on a suitcase on the floor

Somewhat related



A woman is holding a cat in her hand



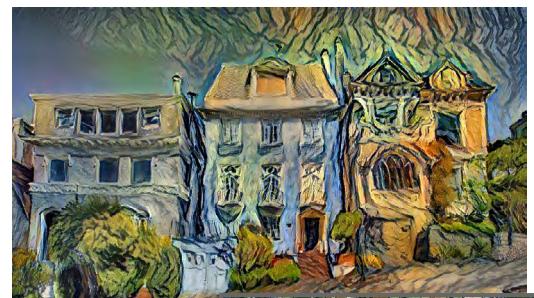
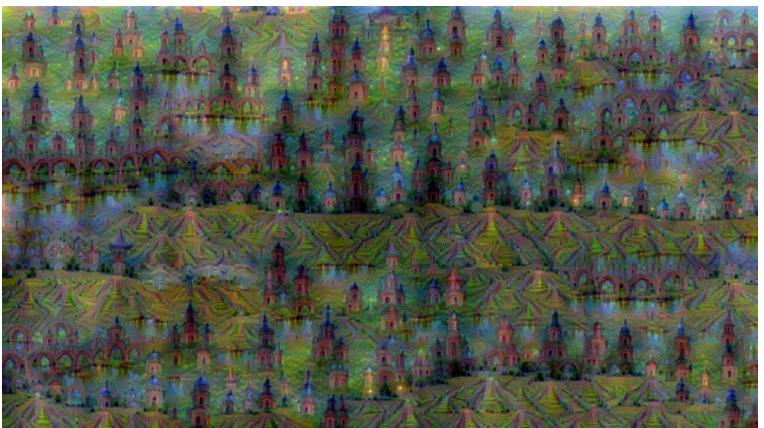
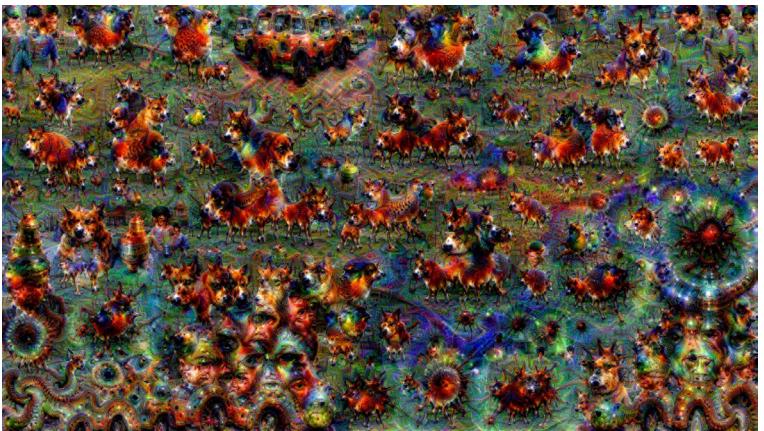
A woman standing on a beach holding a surfboard

Image Captioning

[Vinyals et al., 2015]
[Karpathy and Fei-Fei, 2015]

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Captions generated by Justin Johnson using [Neuraltalk2](#)



Figures copyright Justin Johnson, 2015. Reproduced with permission. Generated using the Inceptionism approach from a [blog post](#) by Google Research.

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[Starry Night](#) and [Tree Roots](#) by Van Gogh are in the public domain

[Bokeh image](#) is in the public domain

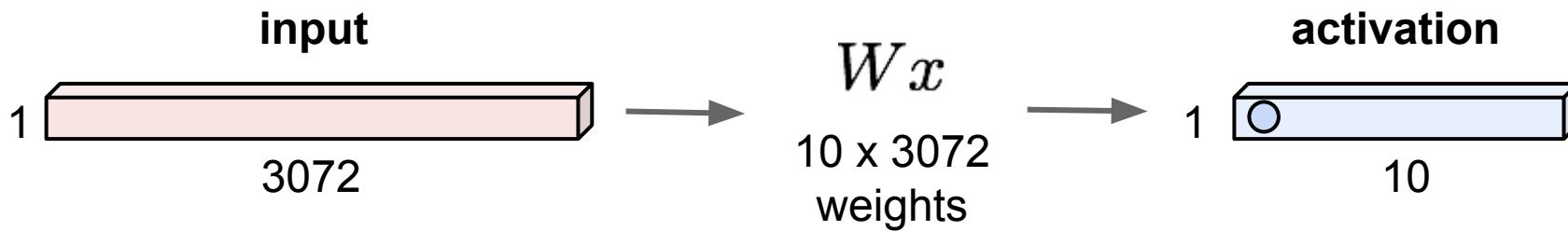
Stylized images copyright Justin Johnson, 2017;
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Gatys et al, "Image Style Transfer using Convolutional Neural Networks", CVPR 2016
Gatys et al, "Controlling Perceptual Factors in Neural Style Transfer", CVPR 2017

Convolutional Neural Networks

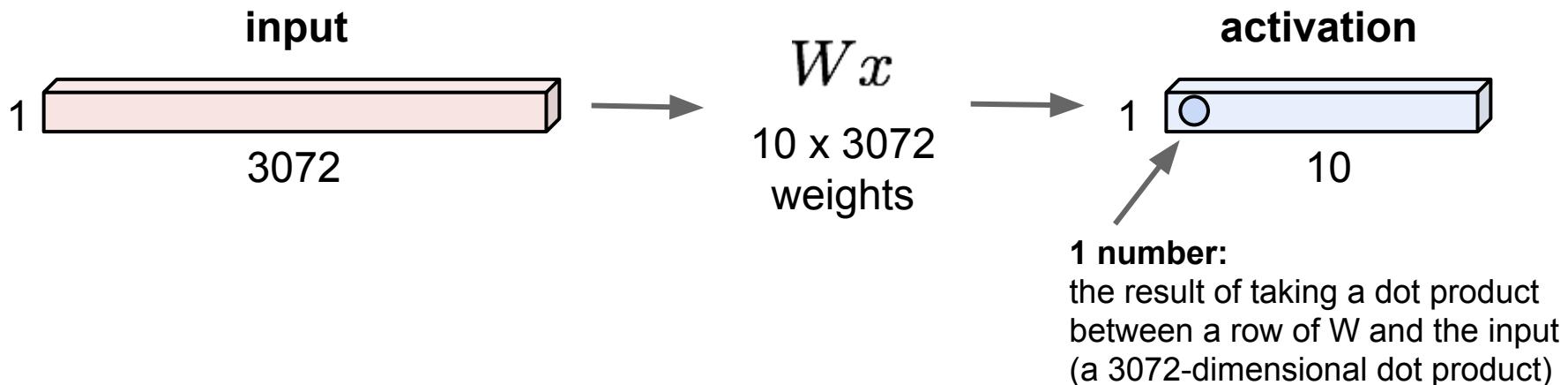
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



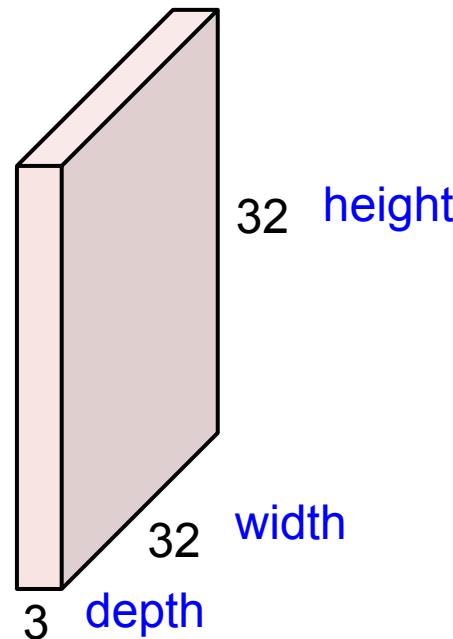
Fully Connected Layer

32x32x3 image -> stretch to 3072×1



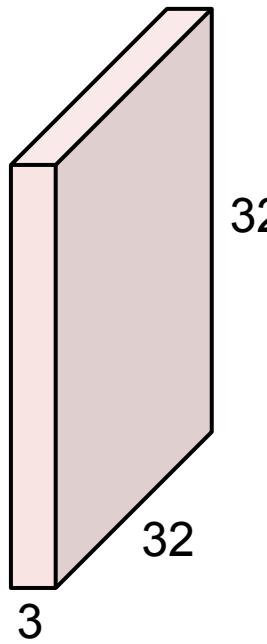
Convolution Layer

32x32x3 image -> preserve spatial structure

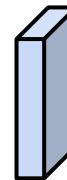


Convolution Layer

32x32x3 image



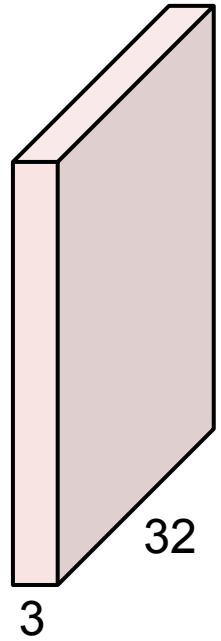
5x5x3 filter



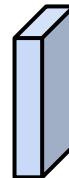
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolution Layer

32x32x3 image



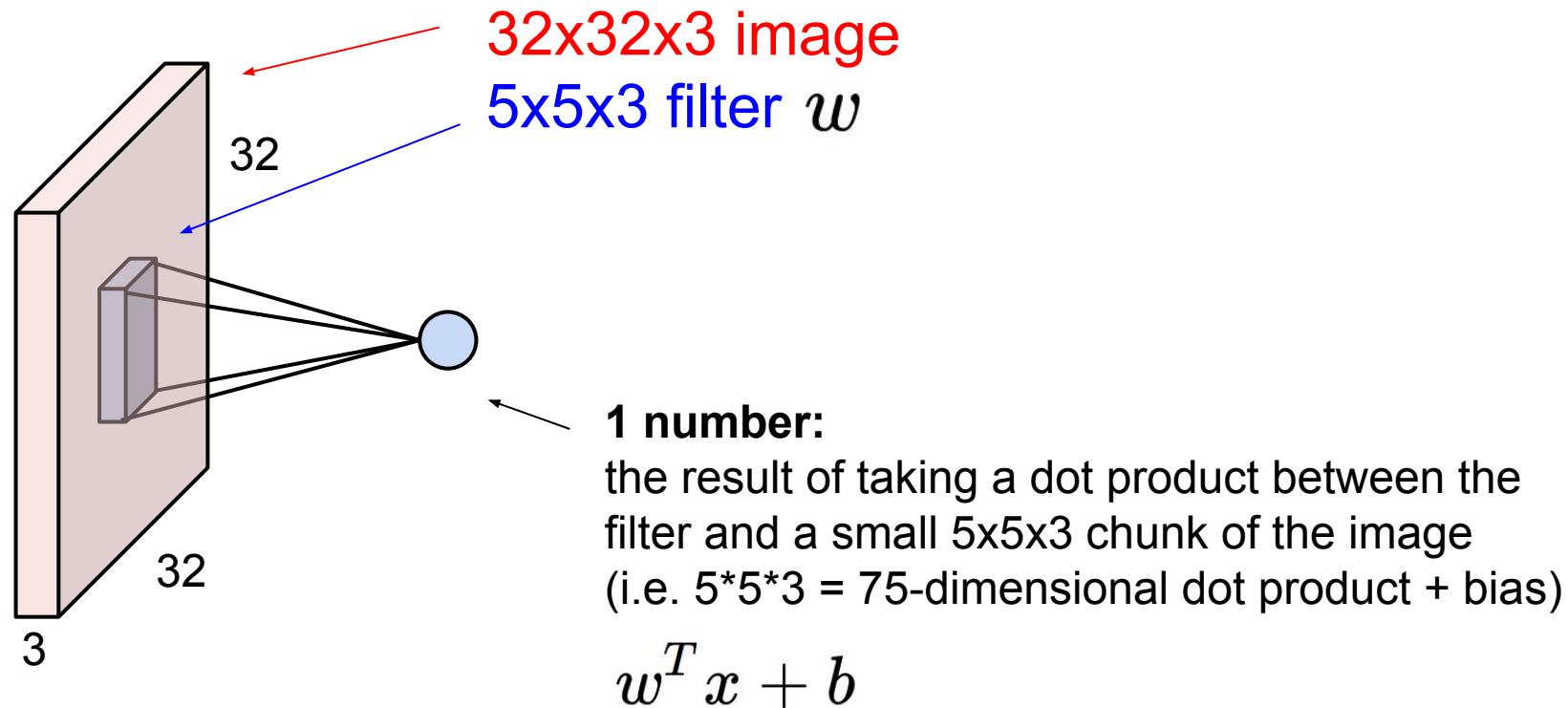
5x5x3 filter



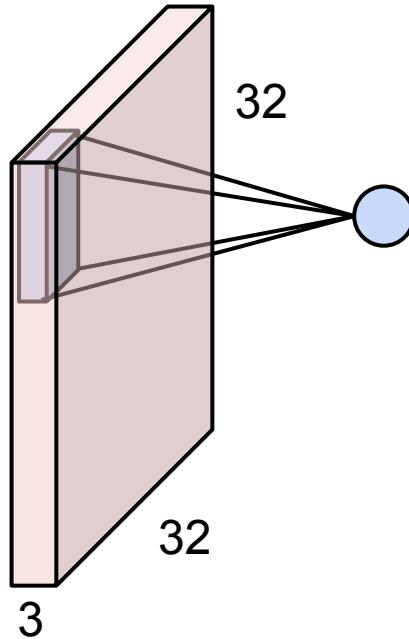
Filters always extend the full depth of the input volume

Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

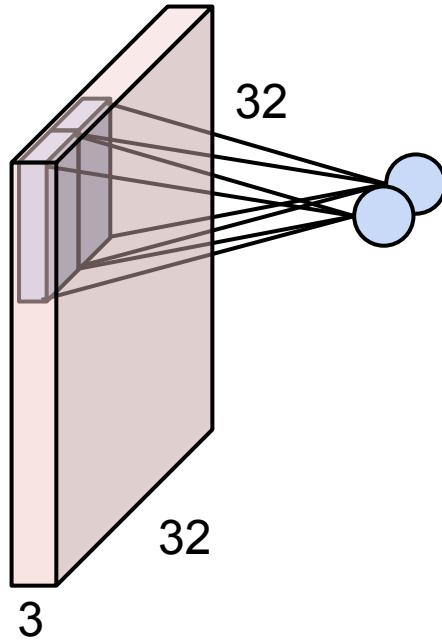
Convolution Layer



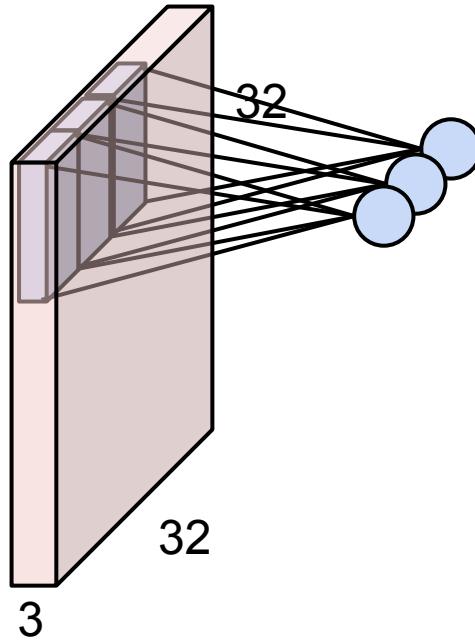
Convolution Layer



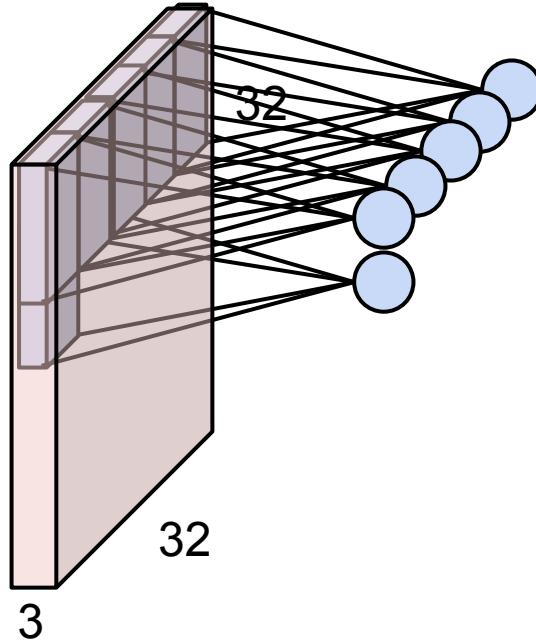
Convolution Layer



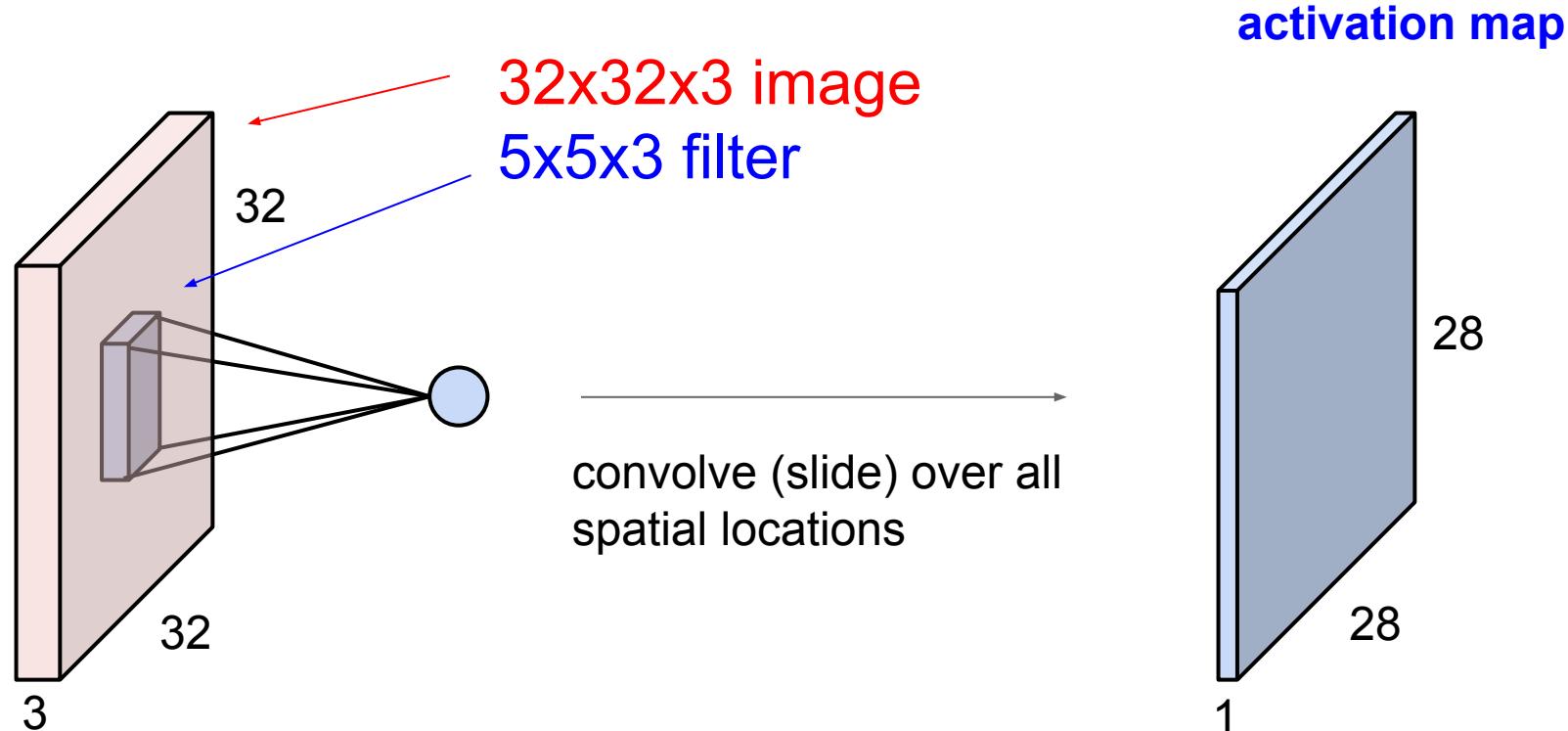
Convolution Layer



Convolution Layer

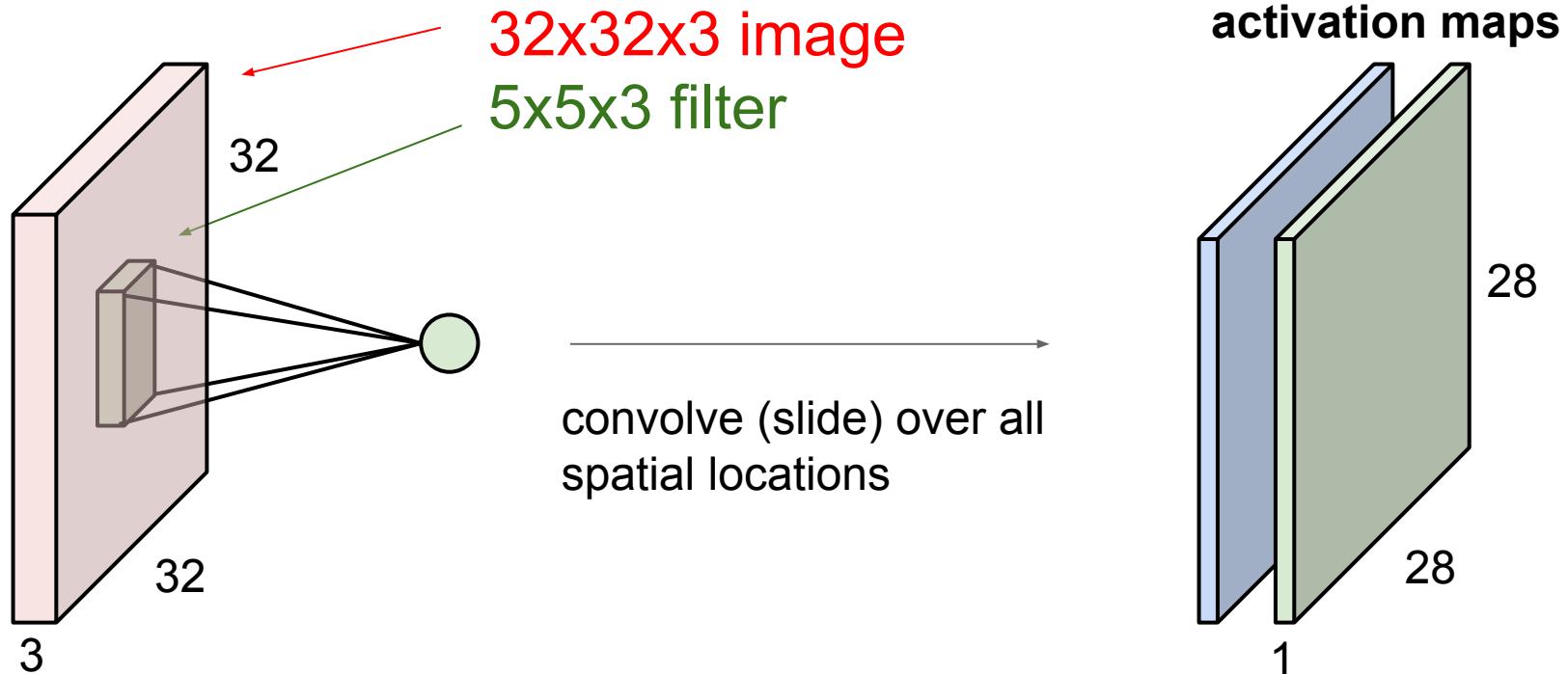


Convolution Layer

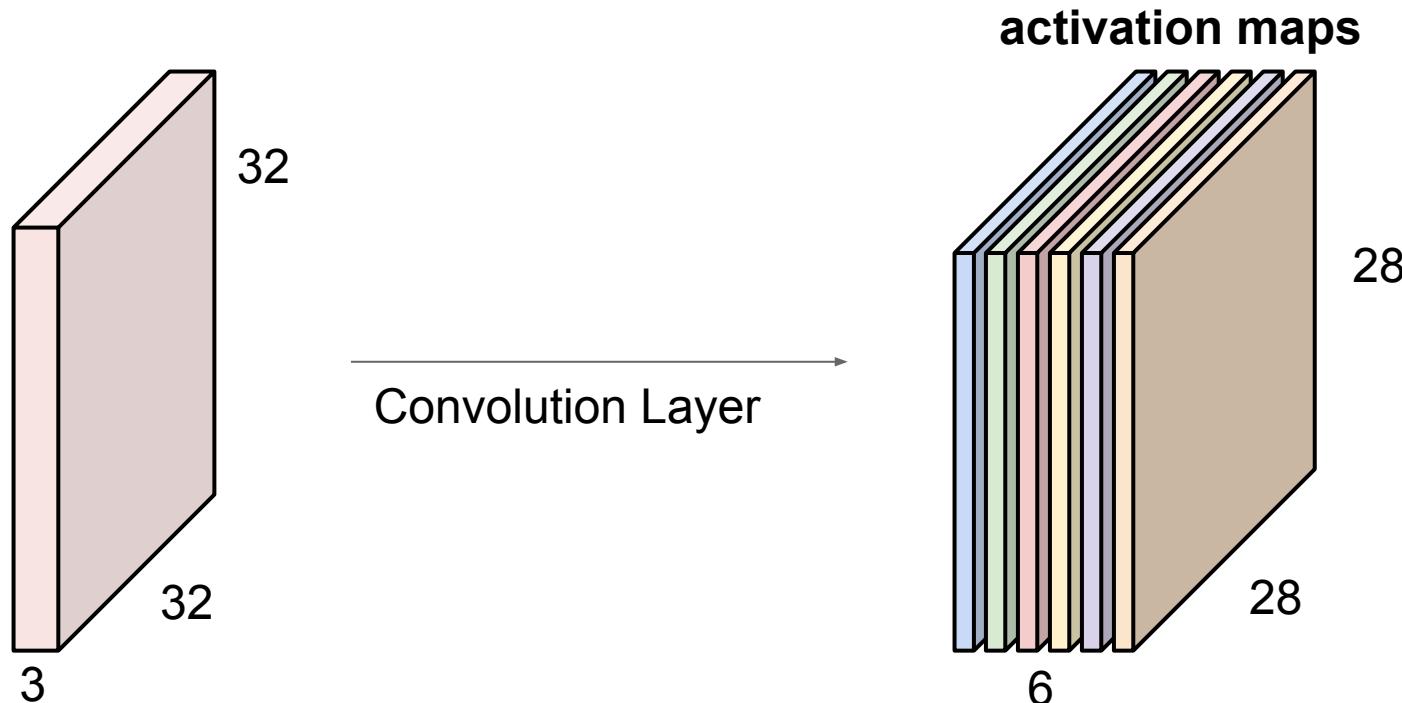


Convolution Layer

consider a second, green filter

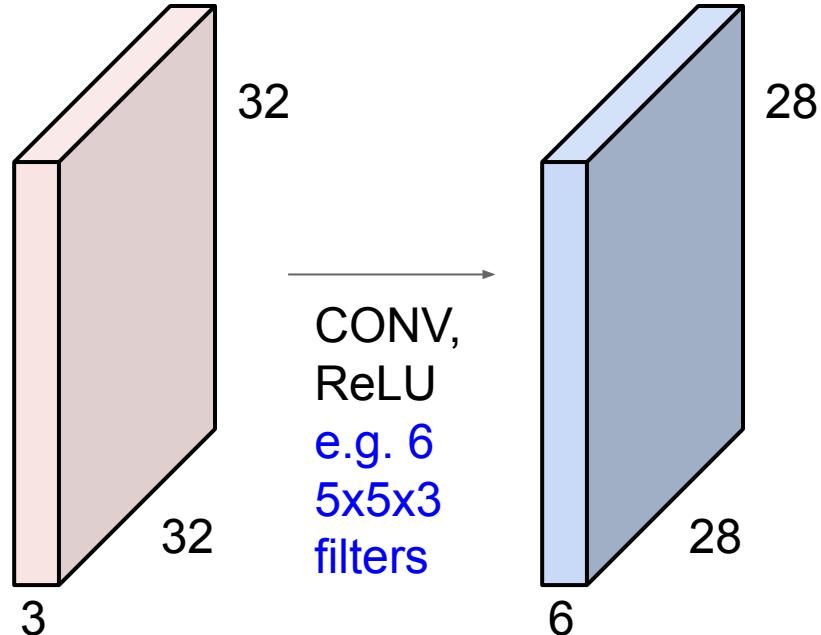


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

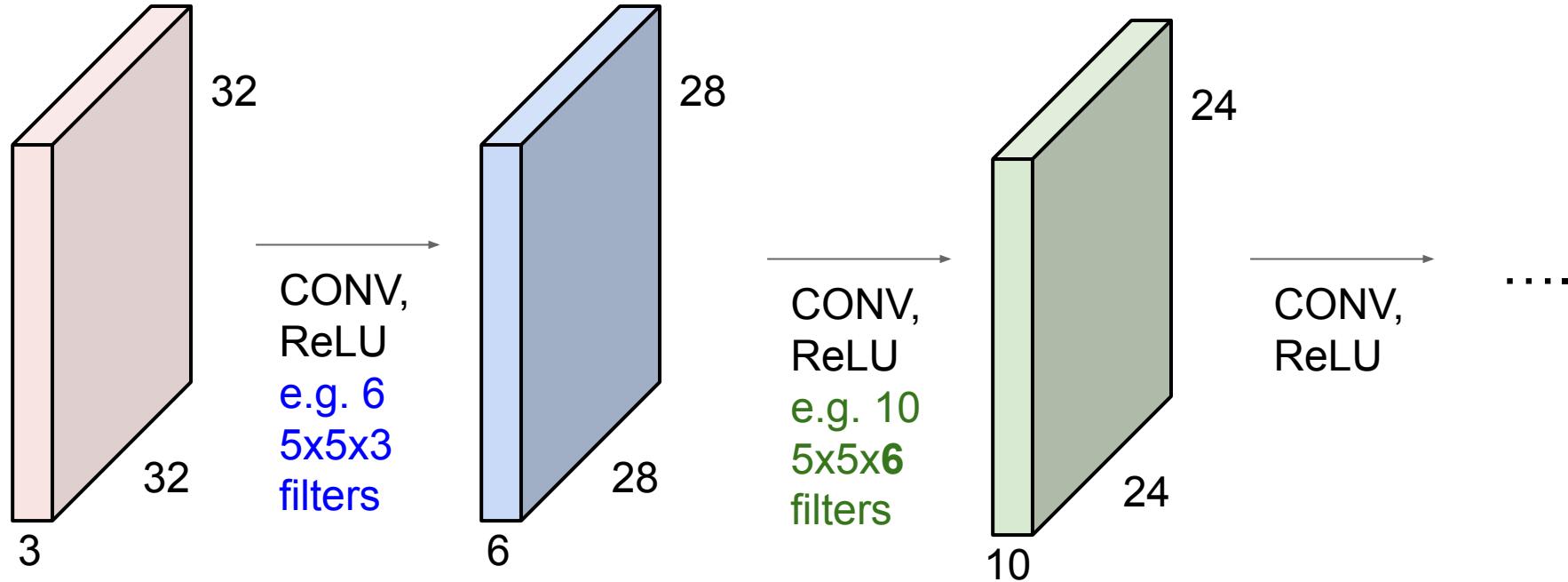


We stack these up to get a “new image” of size $28 \times 28 \times 6$!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



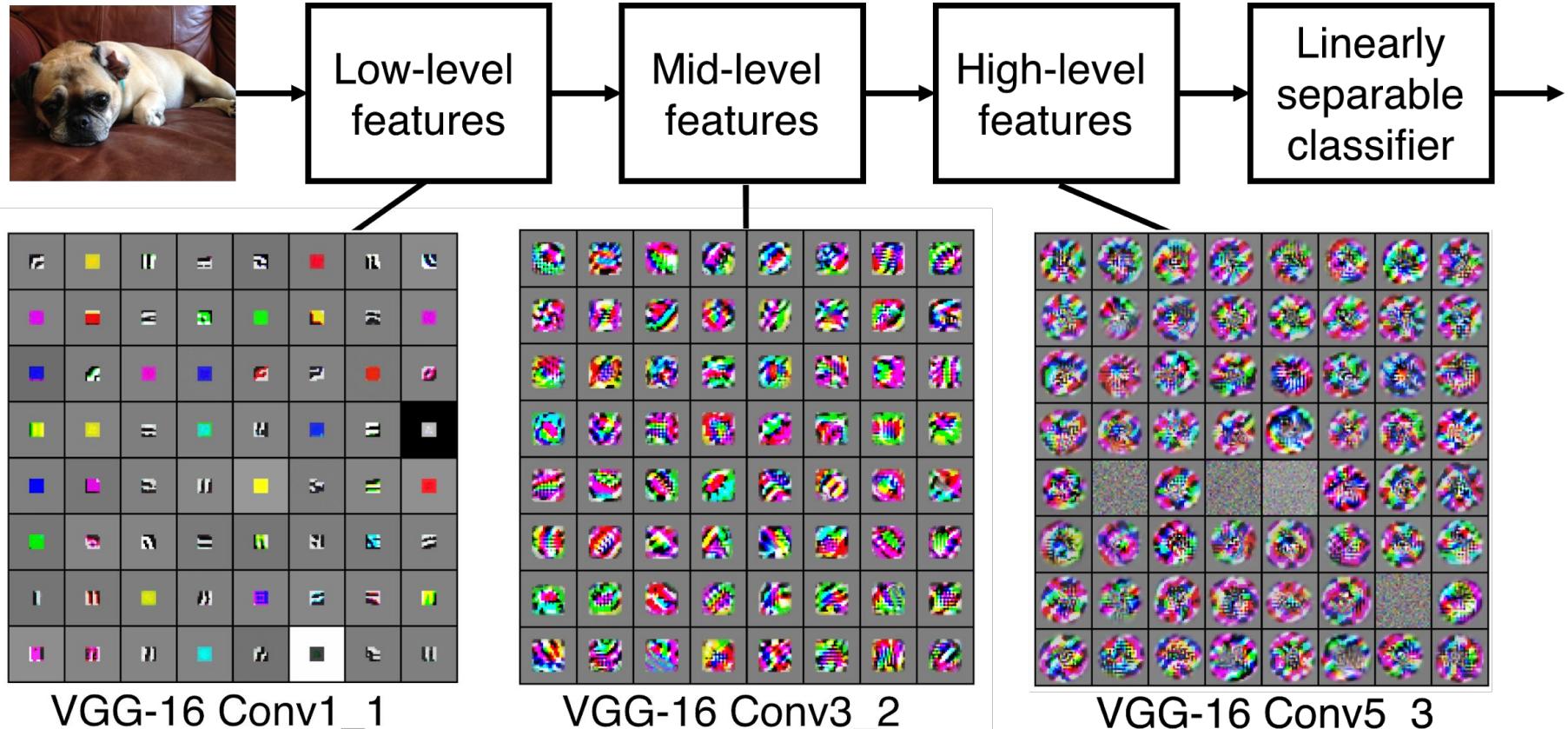
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



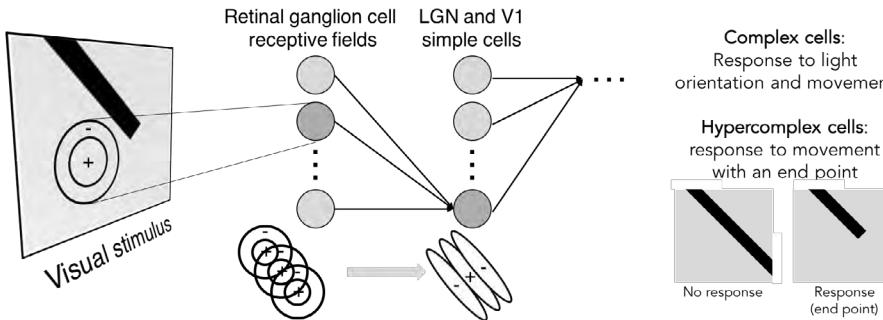
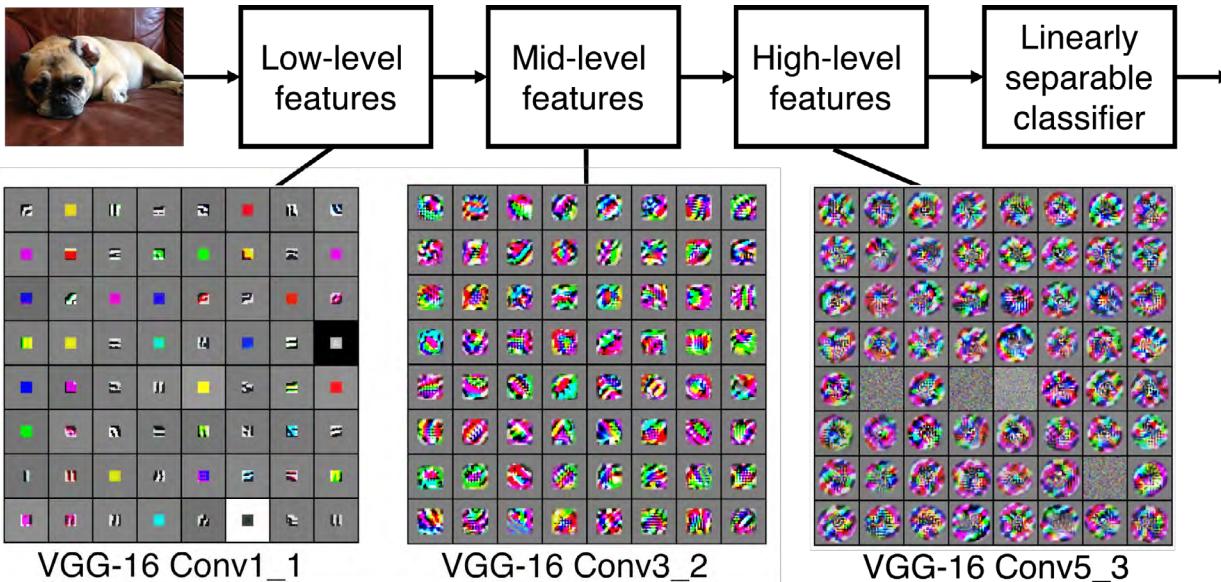
Preview

[Zeiler and Fergus 2013]

Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].

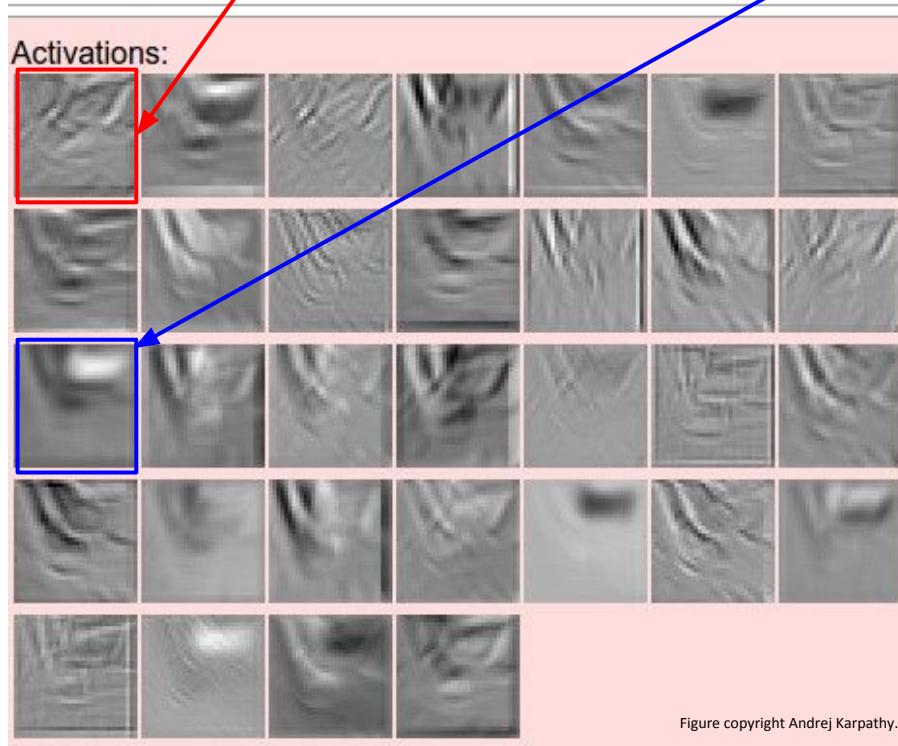


Preview





one filter =>
one activation map



example 5x5 filters
(32 total)

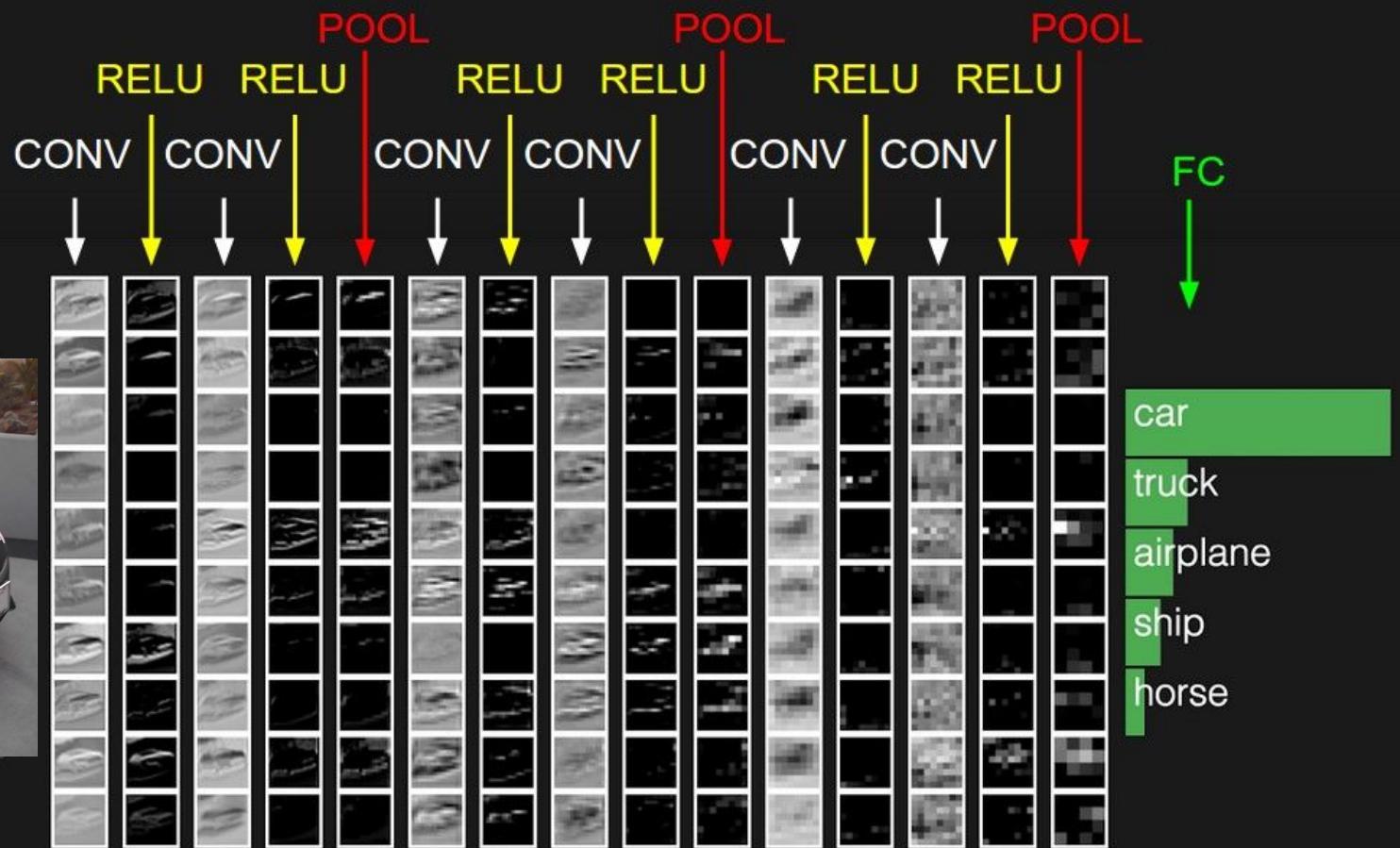
We call the layer convolutional
because it is related to convolution
of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1, n_2] \cdot g[x - n_1, y - n_2]$$

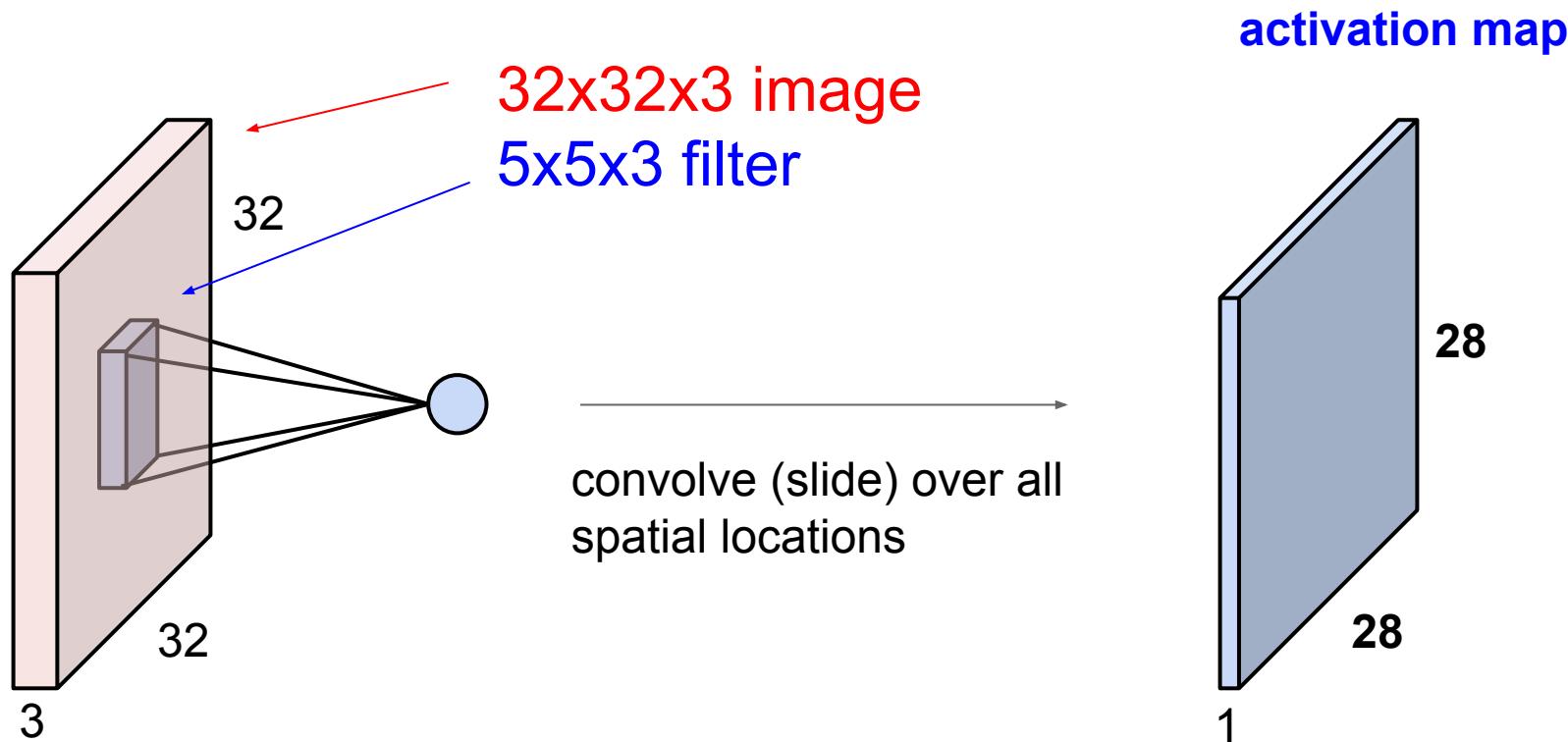


elementwise multiplication and sum of
a filter and the signal (image)

preview:

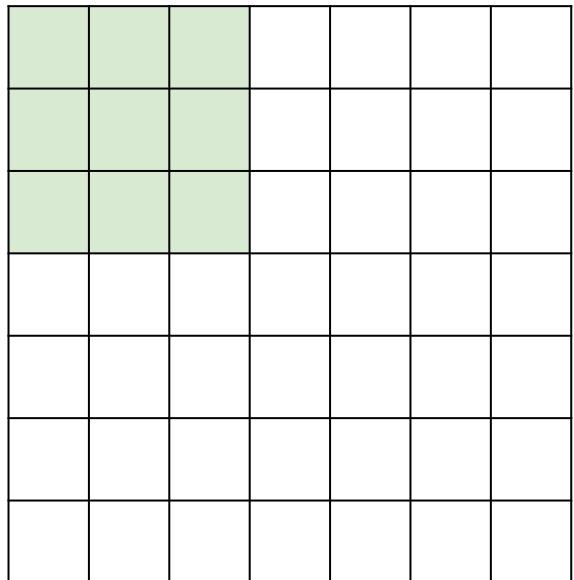


A closer look at spatial dimensions:



A closer look at spatial dimensions:

7

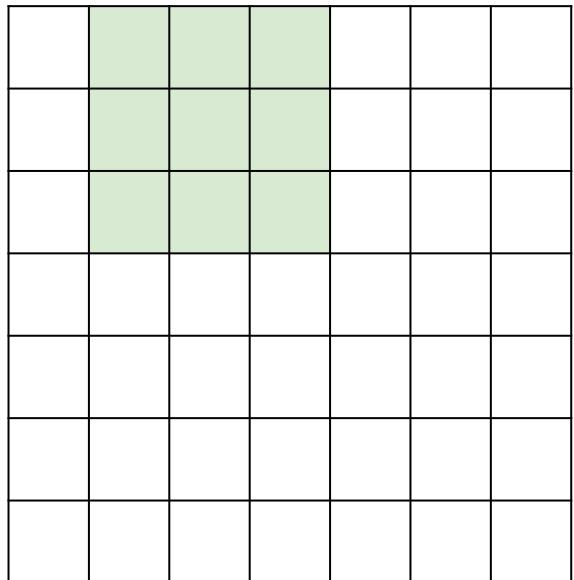


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

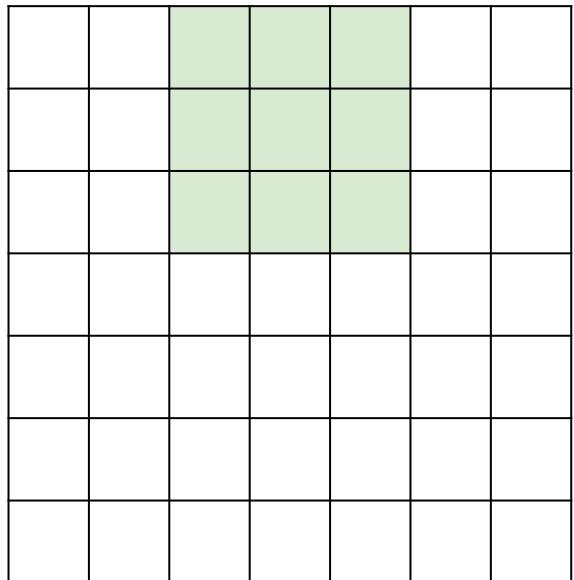


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

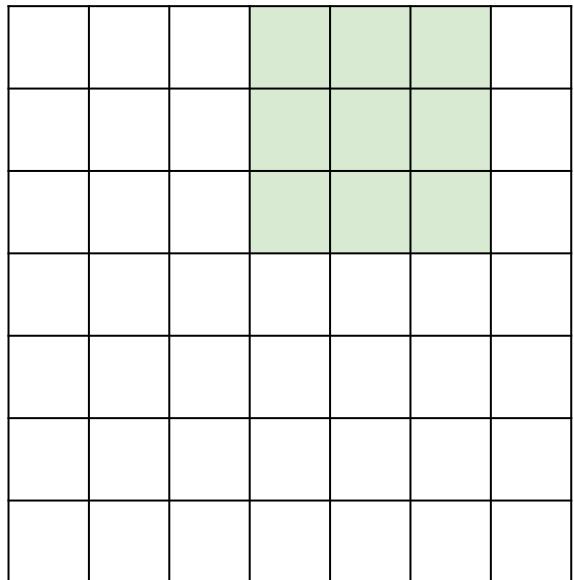


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

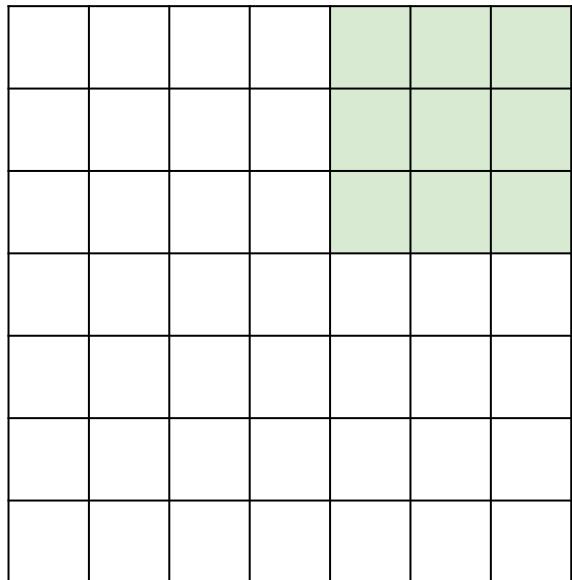


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

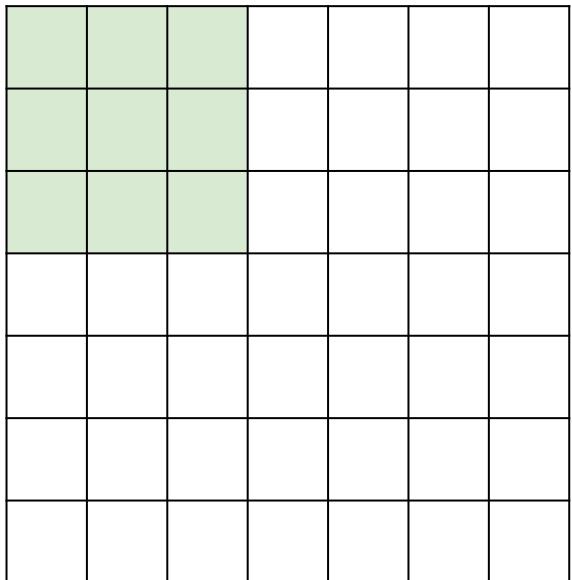


7x7 input (spatially)
assume 3x3 filter

=> 5x5 output

A closer look at spatial dimensions:

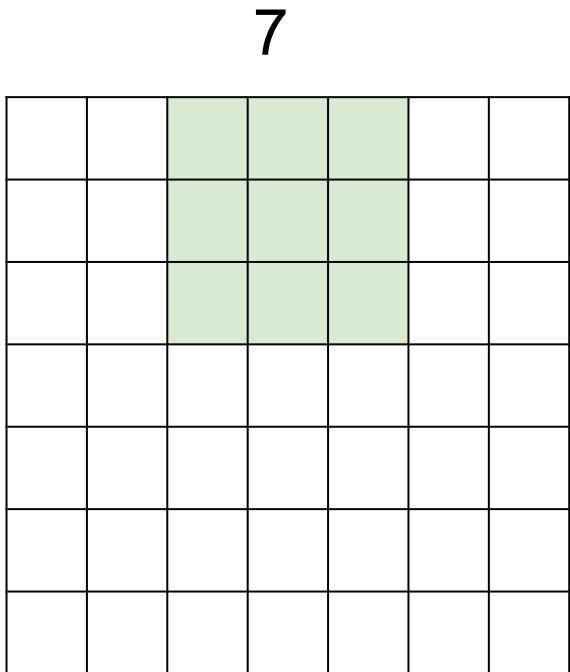
7



7

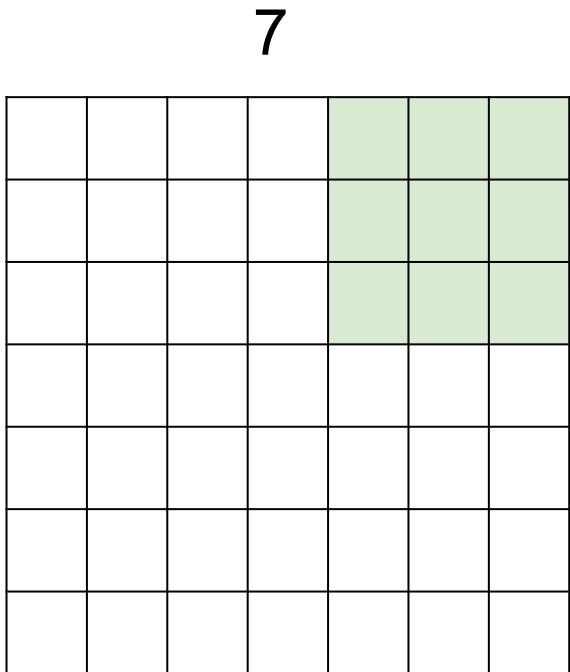
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

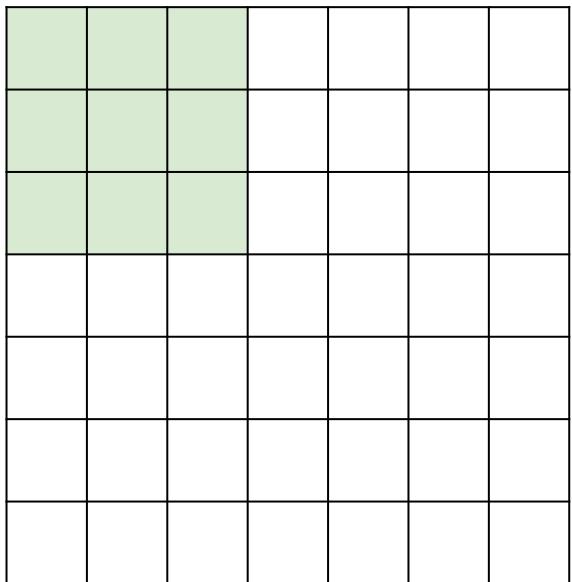
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:

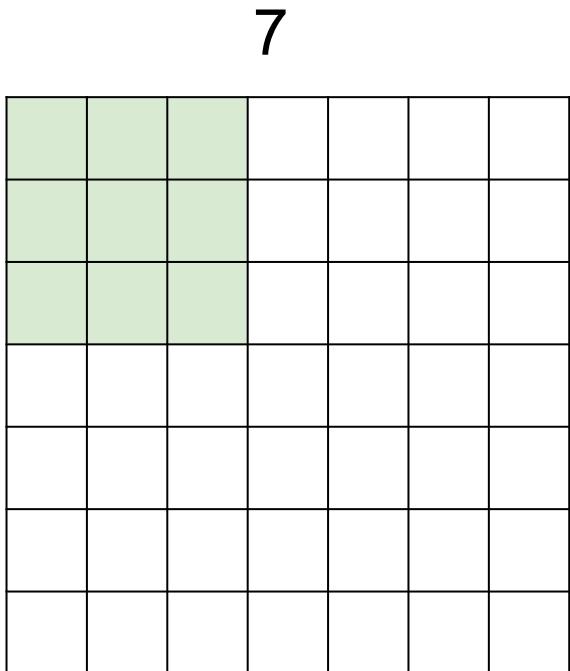
7



7

7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

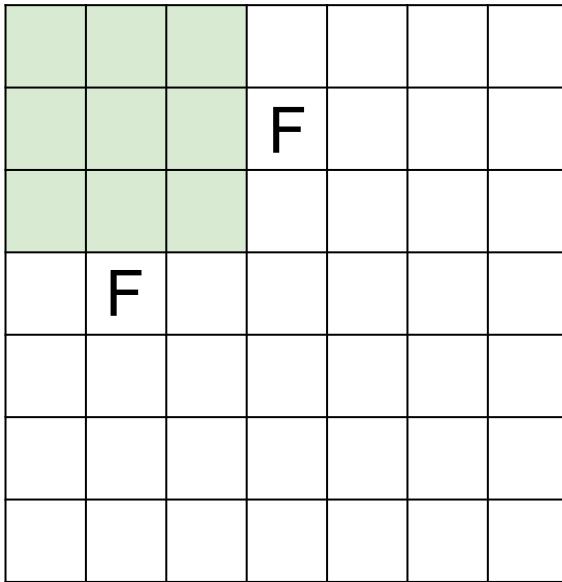
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.

N



N

Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:

$$\text{stride 1} \Rightarrow (7 - 3)/1 + 1 = 5$$

$$\text{stride 2} \Rightarrow (7 - 3)/2 + 1 = 3$$

$$\text{stride 3} \Rightarrow (7 - 3)/3 + 1 = 2.33 : \backslash$$

In practice: Common to zero pad the border

| | | | | | | | |
|---|---|---|---|---|---|--|--|
| 0 | 0 | 0 | 0 | 0 | 0 | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

e.g. input 7x7

3x3 filter, applied with stride 1

pad with 1 pixel border => what is the output?

(recall:)

$$(N - F) / \text{stride} + 1$$

In practice: Common to zero pad the border

| | | | | | | | |
|---|---|---|---|---|---|--|--|
| 0 | 0 | 0 | 0 | 0 | 0 | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

e.g. input 7x7

3x3 filter, applied with **stride 1**

pad with 1 pixel border => what is the output?

7x7 output!

(recall:)

$$(N + 2P - F) / \text{stride} + 1$$

In practice: Common to zero pad the border

| | | | | | | | |
|---|---|---|---|---|---|--|--|
| 0 | 0 | 0 | 0 | 0 | 0 | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| 0 | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

e.g. input 7x7

3x3 filter, applied with stride 1

pad with 1 pixel border => what is the output?

7x7 output!

in general, common to see CONV layers with stride 1, filters of size $F \times F$, and zero-padding with $(F-1)/2$. (will preserve size spatially)

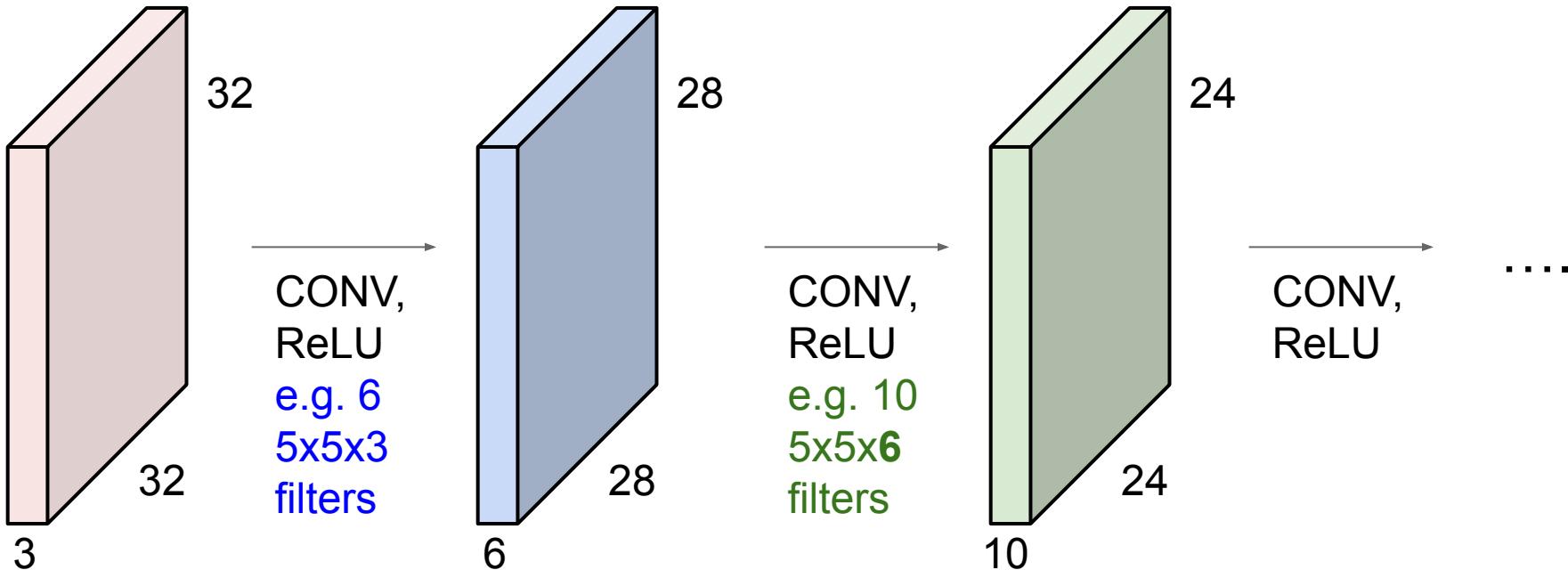
e.g. $F = 3 \Rightarrow$ zero pad with 1

$F = 5 \Rightarrow$ zero pad with 2

$F = 7 \Rightarrow$ zero pad with 3

Remember back to...

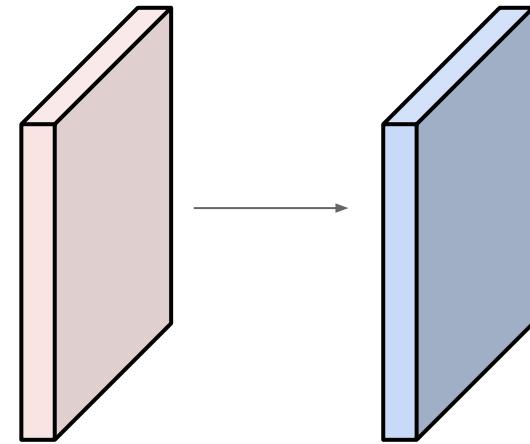
E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially!
(32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.



Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

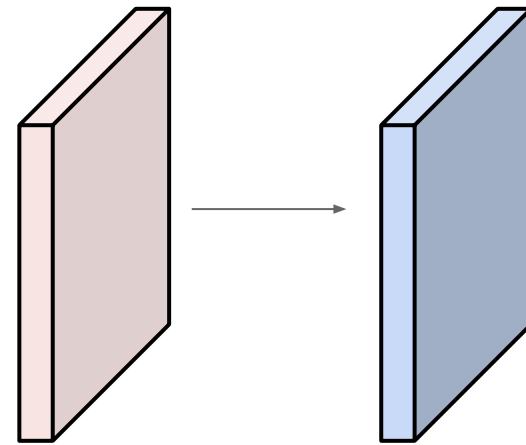


Output volume size: ?

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad **2**



Output volume size:

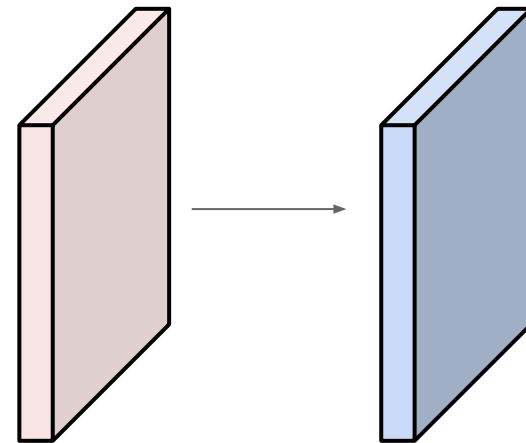
$(32+2*2-5)/1+1 = 32$ spatially, so

32x32x10

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

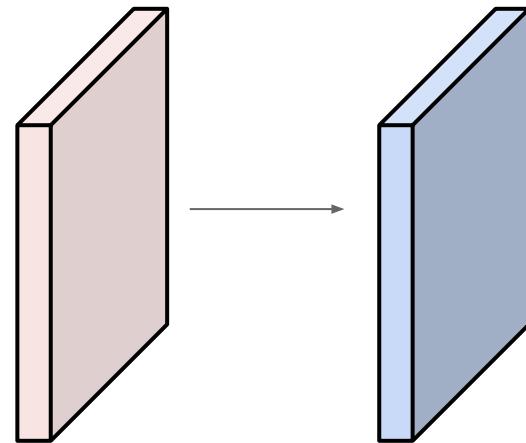


Number of parameters in this layer?

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

each filter has $5*5*3 + 1 = 76$ params (+1 for bias)
 $\Rightarrow 76*10 = 760$

Convolution layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 4 hyperparameters:

- Number of filters K
- The filter size F
- The stride S
- The zero padding P

This will produce an output of $W_2 \times H_2 \times K$

where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

Number of parameters: F^2CK and K biases

Convolution layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of $W_2 \times H_2 \times K$

where:

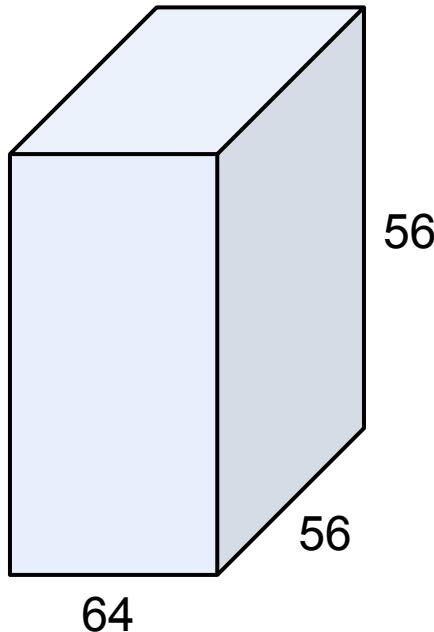
- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

Number of parameters: F^2CK and K biases

Common settings:

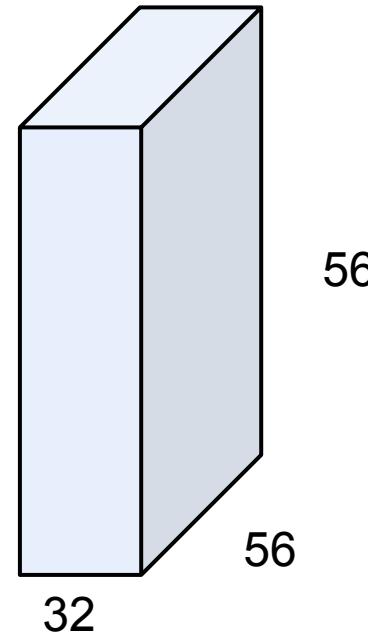
- K** = (powers of 2, e.g. 32, 64, 128, 512)
- $F = 3, S = 1, P = 1$
 - $F = 5, S = 1, P = 2$
 - $F = 5, S = 2, P = ?$ (whatever fits)
 - $F = 1, S = 1, P = 0$

(btw, 1x1 convolution layers make perfect sense)

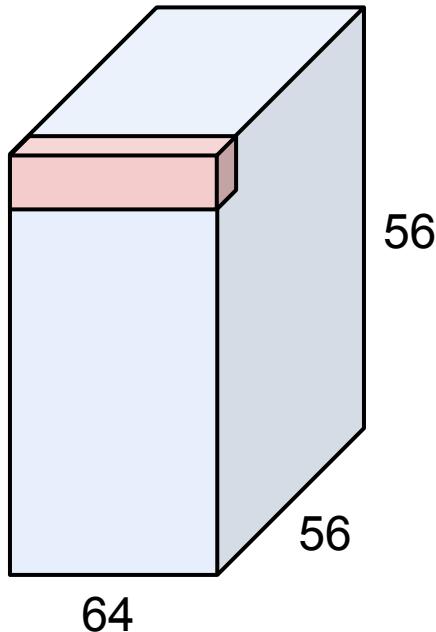


1x1 CONV
with 32 filters

(each filter has size
1x1x64, and performs a
64-dimensional dot
product)

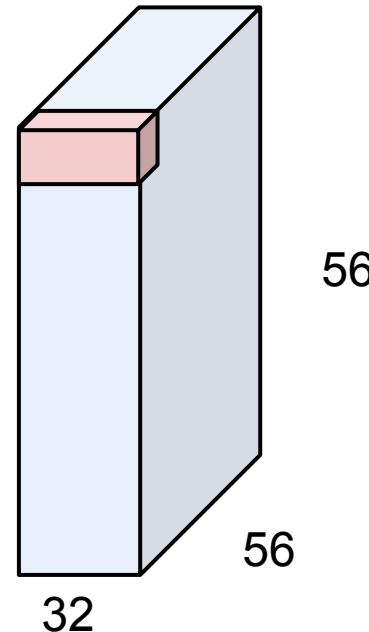


(btw, 1x1 convolution layers make perfect sense)



1x1 CONV
with 32 filters

(each filter has size
1x1x64, and performs a
64-dimensional dot
product)



Example: CONV layer in TF

TensorFlow > API > TensorFlow Core v2.3.0 > Python ☆☆☆☆☆

tf.keras.layers.Conv2D

 TensorFlow 1 version  View source on GitHub

2D convolution layer (e.g. spatial convolution over images).

 View aliases

```
tf.keras.layers.Conv2D(  
    filters, kernel_size, strides=(1, 1), padding='valid', data_format=None,  
    dilation_rate=(1, 1), groups=1, activation=None, use_bias=True,  
    kernel_initializer='glorot_uniform', bias_initializer='zeros',  
    kernel_regularizer=None, bias_regularizer=None, activity_regularizer=None,  
    kernel_constraint=None, bias_constraint=None, **kwargs  
)
```

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

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Example: CONV layer in Keras

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

Conv2D

[source]

```
keras.layers.Conv2D(filters, kernel_size, strides=(1, 1), padding='valid', data_format=None, d:
```

2D convolution layer (e.g. spatial convolution over images).

This layer creates a convolution kernel that is convolved with the layer input to produce a tensor of outputs. If `use_bias` is True, a bias vector is created and added to the outputs. Finally, if `activation` is not `None`, it is applied to the outputs as well.

When using this layer as the first layer in a model, provide the keyword argument `input_shape` (tuple of integers, does not include the batch axis), e.g. `input_shape=(128, 128, 3)` for 128x128 RGB pictures in `data_format="channels_last"`.

Arguments

- **filters**: Integer, the dimensionality of the output space (i.e. the number of output filters in the convolution).
- **kernel_size**: An integer or tuple/list of 2 integers, specifying the height and width of the 2D convolution window. Can be a single integer to specify the same value for all spatial dimensions.
- **strides**: An integer or tuple/list of 2 integers, specifying the strides of the convolution along the height and width. Can be a single integer to specify the same value for all spatial dimensions. Specifying any stride value != 1 is incompatible with specifying any `dilation_rate` value != 1.
- **padding**: one of `"valid"` or `"same"` (case-insensitive). Note that `"same"` is slightly inconsistent across backends with `strides` != 1, as described here
- **data_format**: A string, one of `"channels_last"` or `"channels_first"`. The ordering of the dimensions in the inputs. `"channels_last"` corresponds to inputs with shape `(batch, height, width, channels)` while `"channels_first"` corresponds to inputs with shape `(batch, channels, height, width)`. It defaults to the `image_data_format` value found in your Keras config file at `~/.keras/keras.json`. If you never set it, then it will be `"channels_last"`.

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Example: CONV layer in PyTorch

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True)`

[SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) * \text{input}(N_i, k)$$

where $*$ is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

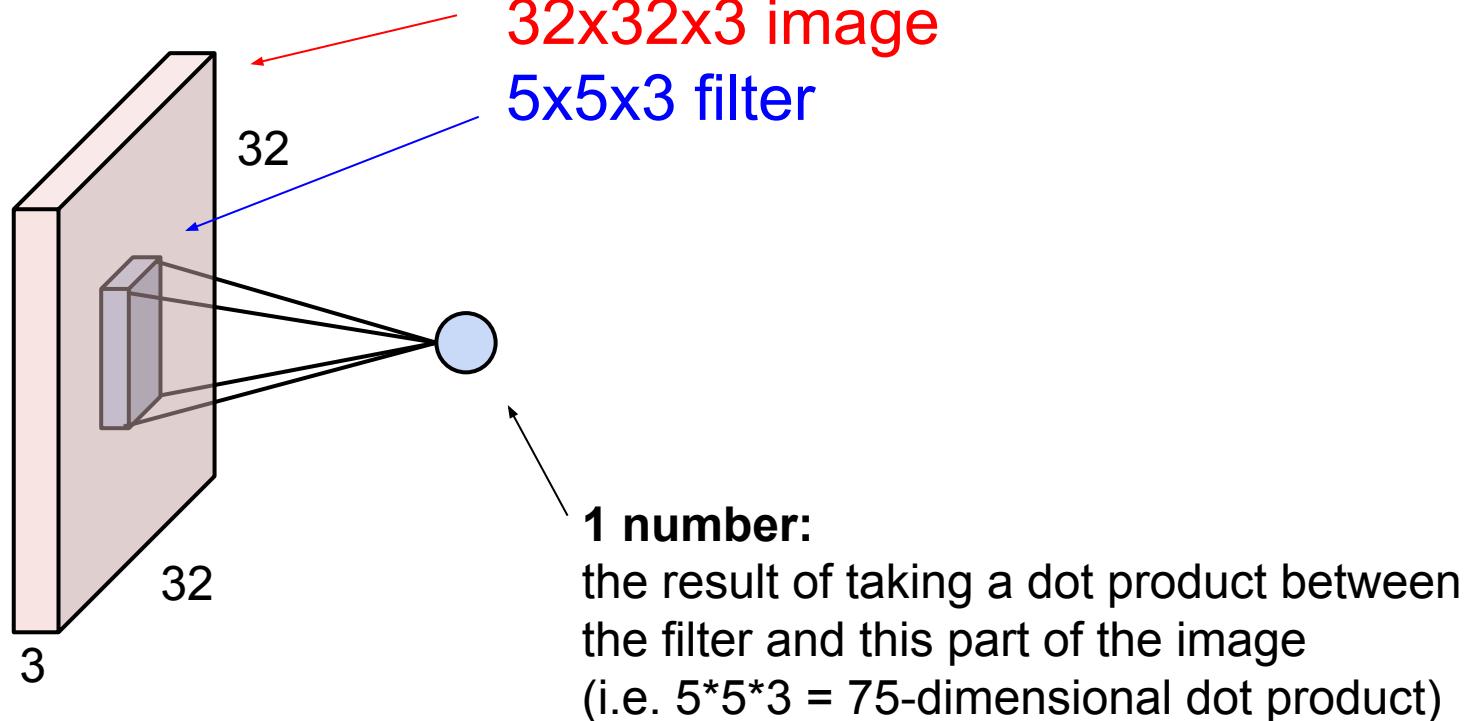
- `stride` controls the stride for the cross-correlation, a single number or a tuple.
- `padding` controls the amount of implicit zero-paddings on both sides for `padding` number of points for each dimension.
- `dilation` controls the spacing between the kernel points; also known as the à trous algorithm. It is harder to describe, but this [link](#) has a nice visualization of what `dilation` does.
- `groups` controls the connections between inputs and outputs. `in_channels` and `out_channels` must both be divisible by `groups`. For example,
 - At `groups=1`, all inputs are convolved to all outputs.
 - At `groups=2`, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
 - At `groups= in_channels`, each input channel is convolved with its own set of filters, of size: $\left\lfloor \frac{C_{\text{out}}}{C_{\text{in}}} \right\rfloor$.

The parameters `kernel_size`, `stride`, `padding`, `dilation` can either be:

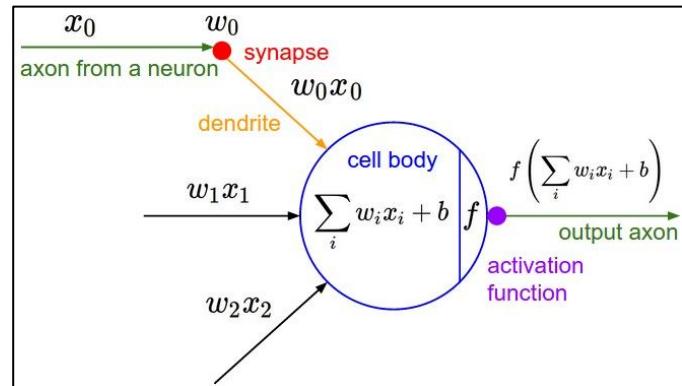
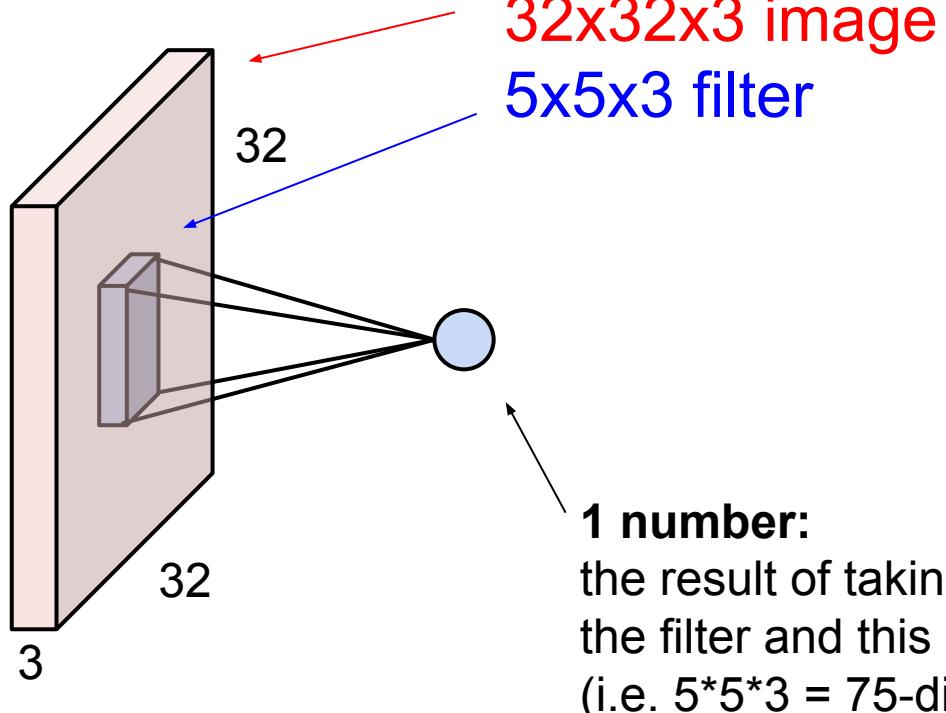
- a single `int` - in which case the same value is used for the height and width dimension
- a `tuple` of two `ints` - in which case, the first `int` is used for the height dimension, and the second `int` for the width dimension

[PyTorch](#) is licensed under [BSD 3-clause](#).

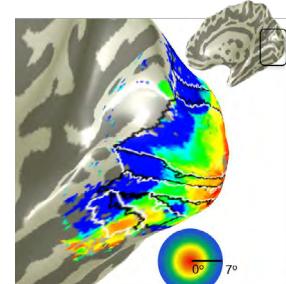
The brain/neuron view of CONV Layer



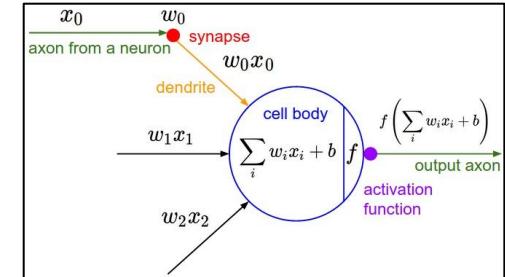
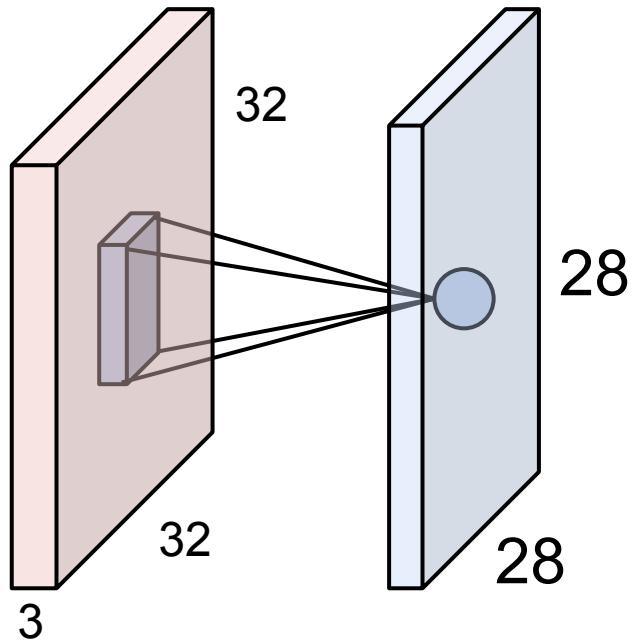
The brain/neuron view of CONV Layer



It's just a neuron with local connectivity...



The brain/neuron view of CONV Layer

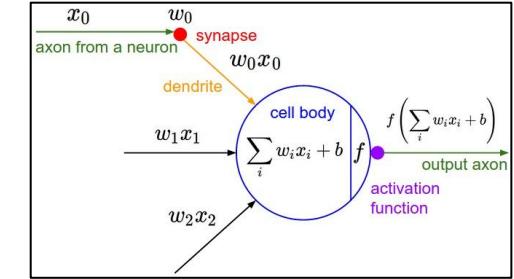
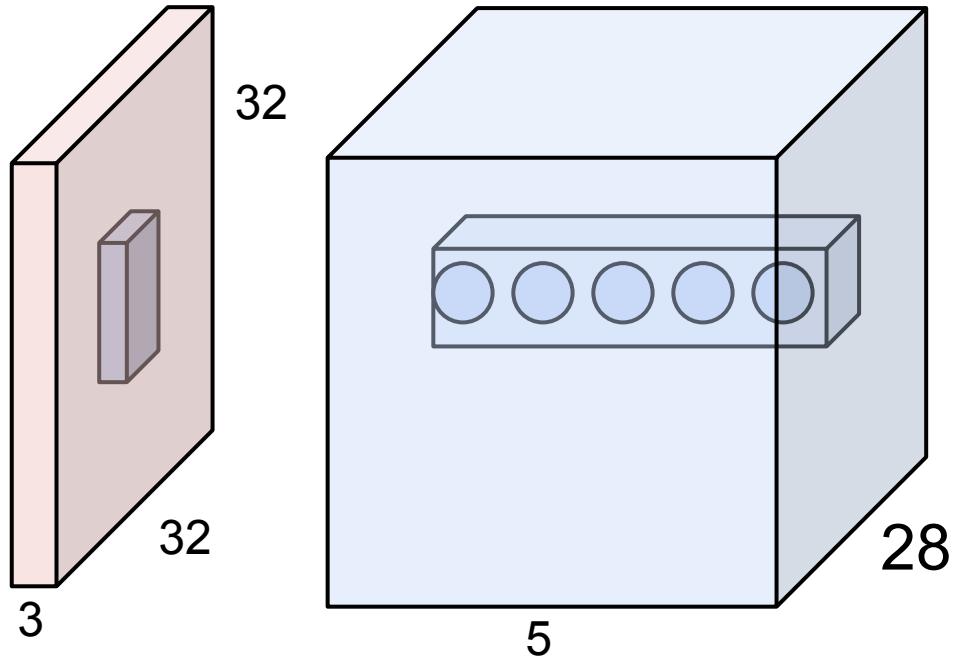


An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

“5x5 filter” -> “5x5 receptive field for each neuron”

The brain/neuron view of CONV Layer



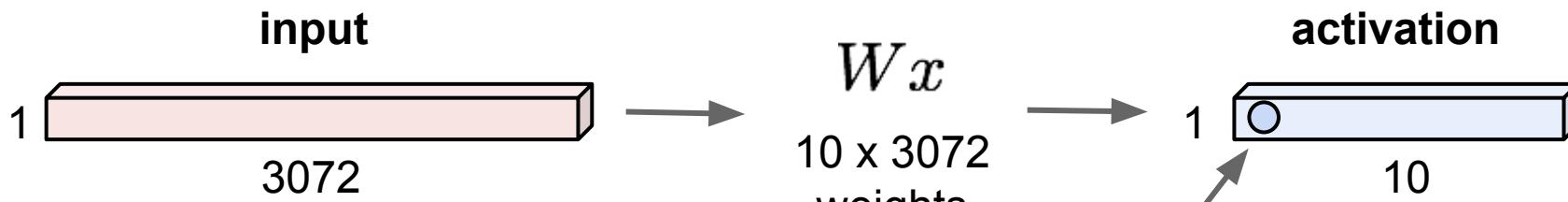
E.g. with 5 filters,
CONV layer consists of
neurons arranged in a 3D grid
(28x28x5)

There will be 5 different
neurons all looking at the same
region in the input volume

Reminder: Fully Connected Layer

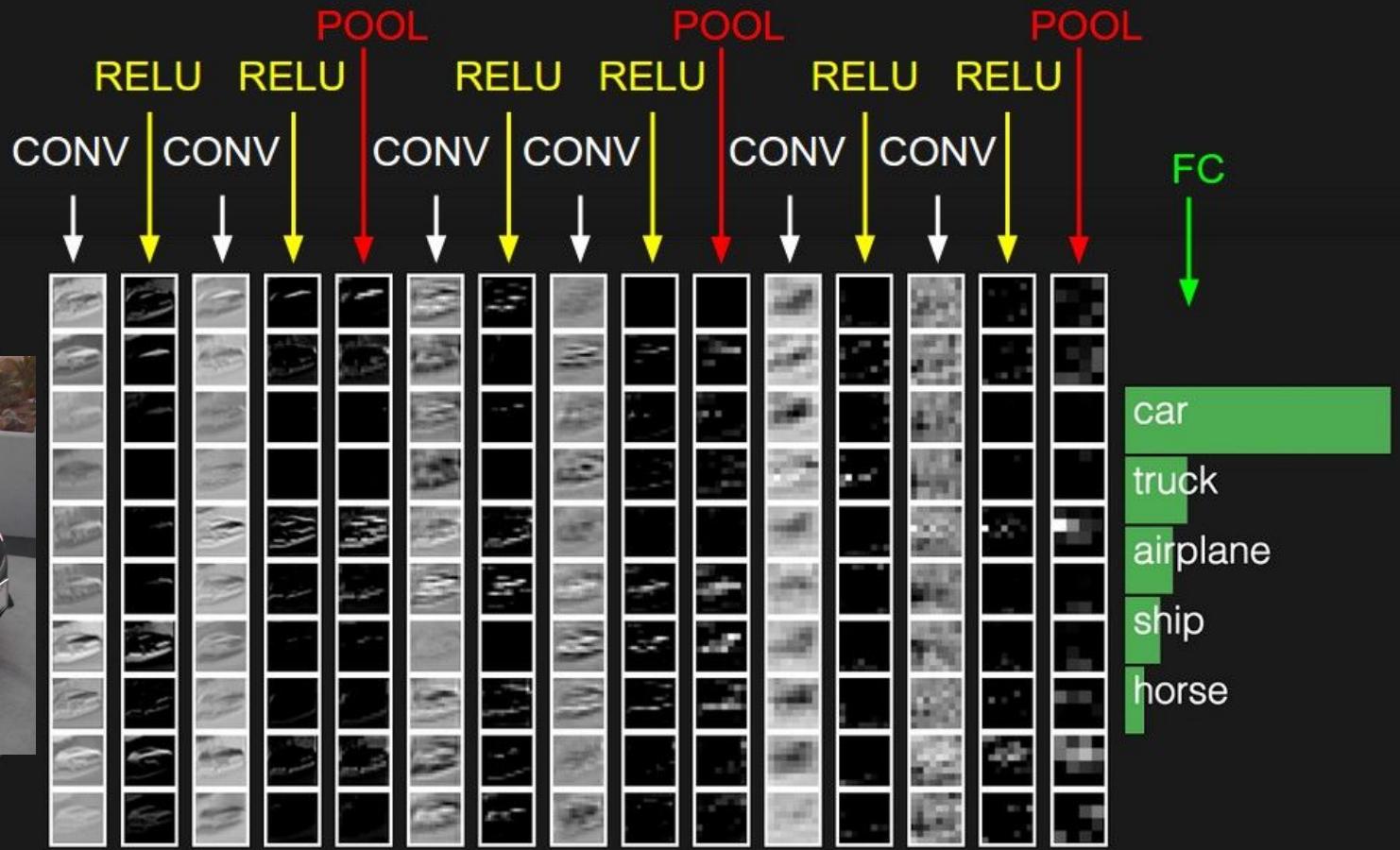
32x32x3 image -> stretch to 3072×1

Each neuron
looks at the full
input volume



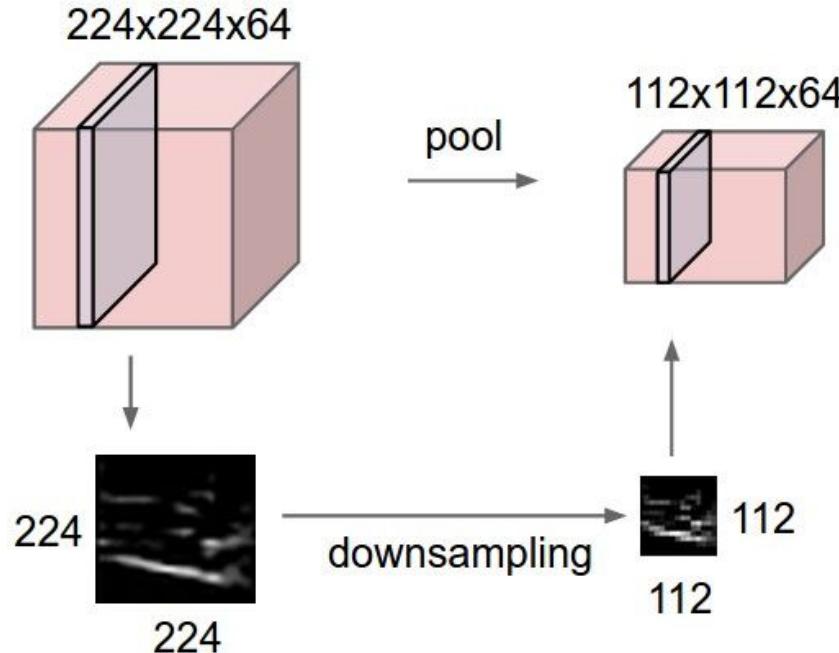
1 number:
the result of taking a dot product
between a row of W and the input
(a 3072-dimensional dot product)

two more layers to go: POOL/FC

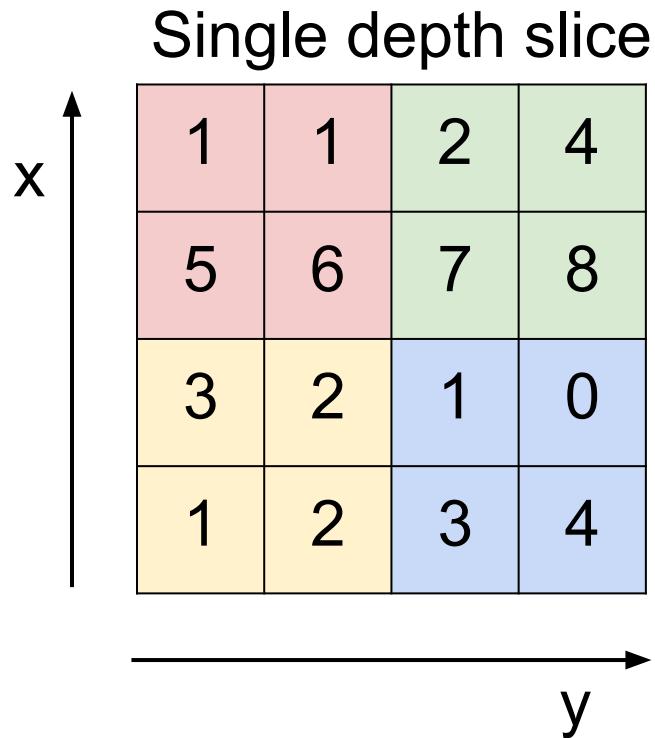


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX POOLING



max pool with 2x2 filters
and stride 2

| | |
|---|---|
| 6 | 8 |
| 3 | 4 |

Pooling layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 2 hyperparameters:

- The spatial extent **F**
- The stride **S**

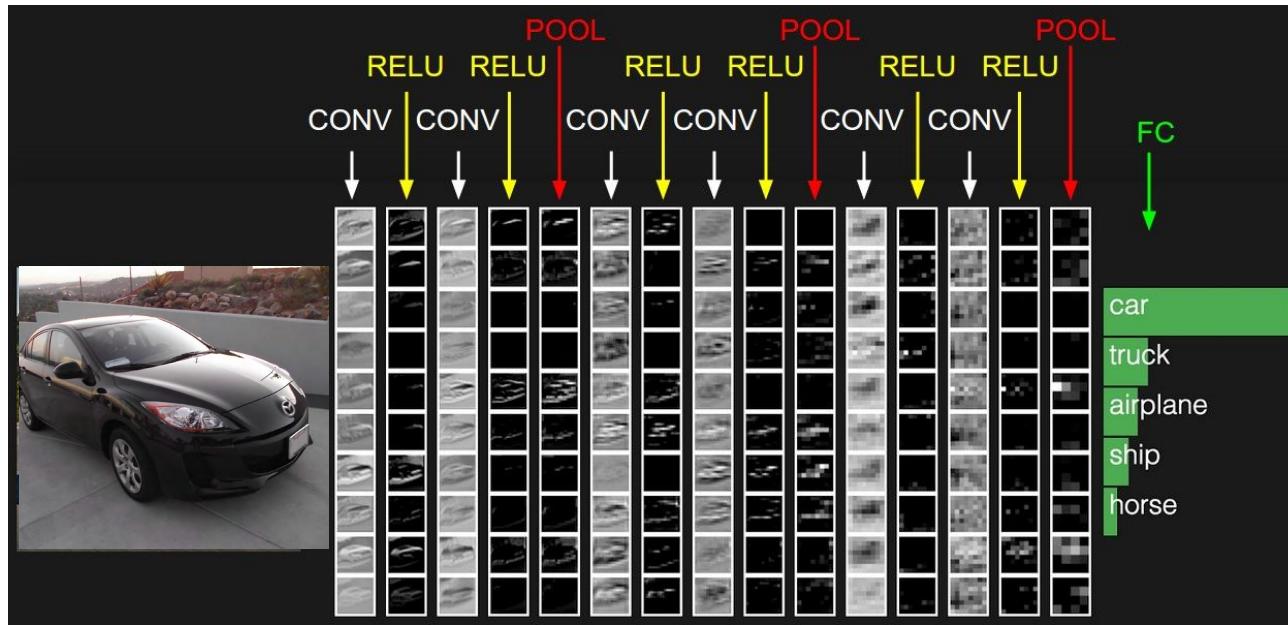
This will produce an output of $W_2 \times H_2 \times C$ where:

- $W_2 = (W_1 - F)/S + 1$
- $H_2 = (H_1 - F)/S + 1$

Number of parameters: 0

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



[ConvNetJS demo: training on CIFAR-10]

ConvNetJS CIFAR-10 demo

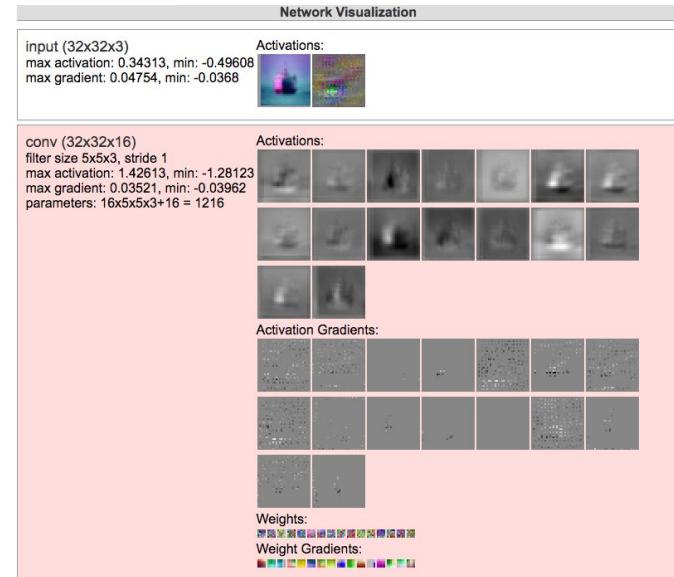
Description

This demo trains a Convolutional Neural Network on the [CIFAR-10 dataset](#) in your browser, with nothing but Javascript. The state of the art on this dataset is about 90% accuracy and human performance is at about 94% (not perfect as the dataset can be a bit ambiguous). I used [this python script](#) to parse the [original files](#) (python version) into batches of images that can be easily loaded into page DOM with img tags.

This dataset is more difficult and it takes longer to train a network. Data augmentation includes random flipping and random image shifts by up to 2px horizontally and vertically.

By default, in this demo we're using Adadelta which is one of per-parameter adaptive step size methods, so we don't have to worry about changing learning rates or momentum over time. However, I still included the text fields for changing these if you'd like to play around with SGD+Momentum trainer.

Report questions/bugs/suggestions to [@karpathy](#).



<http://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html>

Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Historically architectures looked like
 $[(CONV-RELU)^*N-POOL?]^*M-(FC-RELU)^*K, SOFTMAX$
where N is usually up to ~5, M is large, $0 \leq K \leq 2$.
 - but recent advances such as ResNet/GoogLeNet have challenged this paradigm

Задача на дом:

Входное изображение: CONV фильтр:

$$\begin{array}{c} [1 \ 2 \ 3 \ 4 \ 5] \\ [2 \ 2 \ 1 \ 1 \ 1] \\ [3 \ 2 \ 1 \ 1 \ 1] \\ [4 \ 1 \ 1 \ 1 \ 1] \\ [5 \ 1 \ 1 \ 1 \ 1] \end{array} \quad \begin{array}{c} [0 \ -1 \ 0] \\ [1 \ 1 \ 1] \\ [0 \ -1 \ 0] \end{array}$$

Посчитать выход сети: conv(depth=1, stride=2) -> ReLU -> MaxPool