Calculus Homework – Real sequences

Exercise 1. *Investigate the following sequences for monotonicity, boundedness, and convergence! Calculate the upper limit and the lower limit of the sequences as well!*

(1) $a_n := (-2)^n + n$, (2) $a_n := \dfrac{2n-3}{5n+1}$, (3) $a_n := \dfrac{3n+1}{3n+2}$, (4) $a_n := \sqrt{n+2}$

(5) $a_n := \dfrac{7n+1}{2n+3}$, (6) $a_n := (-n)^{n+1}$, (7) $a_n := \dfrac{n+1}{n}$, (8) $a_n := \dfrac{5n+4}{6n-3}$

Exercise 2. *Calculate the limit of the following real sequences!*

(1) $a_n := \dfrac{n5^n + (-6)^{n+1}}{(-6)^n + 2^{n-1}}$, (2) $a_n := \dfrac{n^3 + 2n + 1}{(3n+1)^3}$, (3) $a_n := \dfrac{(n+1)3^n + n^2}{5^n + 3^{n+1}}$

(4) $a_n := \left(\dfrac{2n+1}{2n+10}\right)^{3n-1}$, (5) $a_n := \dfrac{n^2+4}{n^4-16}$, (6) $a_n := \sqrt{2n^2+2} - \sqrt{2n^2+1}$

(7) $a_n := \sqrt[3]{n^3+1} - n$, (8) $a_n := \left(\dfrac{n+1}{n-3}\right)^{2n+1}$, (9) $a_n := \dfrac{(-1)^n + 5^{n+2}}{5^n + (-2)^{n-1}}$

(10) $a_n := \dfrac{n^3 + 2n^2 - 4}{n^2 + 2n^3 + 4}$, (11) $a_n := n - \sqrt{n^2-1}$, (12) $a_n := \left(\dfrac{3n+4}{3n+3}\right)^{n+1}$

(13) $a_n := \dfrac{n^4 + 2n + 3}{5^{n+1} + 2^n}$, (14) $a_n := \dfrac{(n+1)^8 7^n + n^2 + 1}{(-8)^{n+1} + n^3(-7)^{n+1}}$, (15) $a_n := \sqrt[n]{2^n + 3^n}$

(16) $a_n := \sqrt[n]{4^{n+1} + 5^n}$, (17) $a_n := \dfrac{(2n^2+4n)^2}{4n^4 + 5n + 1}$, (18) $a_n := \left(\dfrac{n-4}{n+4}\right)^n$

Exercise 3. *Construct a real sequence which takes any natural number infinitely many times!*

Exercise 4. *Characterize the real sequences which are convex and concave simultaneously!*