Computer aided mathematics and visualization
Practice (2023)

1. Let us consider the following surface:

$$
\begin{aligned}
x(u,v) &= u - \frac{u^3}{3} + uv^2 \\
y(u,v) &= v - \frac{v^3}{3} + vu^2 \\
z(u,v) &= u^2 - v^2 \\
& u \in [-25, 25]\,, \quad v \in [-25, 25]
\end{aligned}
$$

   Draw the surface! Draw the point corresponding to $u = 10$ and $v = 15$ along with the corresponding isocurves. Draw the normal vector of the surface at P.

2. Let us consider the surface

$$z = \sqrt{1 - x^2 - 0.5y^2}.$$

   Draw the surface! Draw the point on the surface corresponding to $x = 0.5$ and $y = 0.2$.

3. Let us consider the surface

$$\sin(x) + \frac{\cos(y)}{x} - z = 0.$$

   Draw the surface! Draw the intersection of the surface with the xy plane.

4. Let us define the following 3 planes:

$$x + y - z = 0, \qquad x - 2y + 3z = 4, \qquad 2x - 0.5y + 4z = -2.$$

   Draw them with different colors!

5. Let us have curves

$$
\begin{aligned}
p(u) &= (1-u)\ P_1 + u\ P_2 \\
r(u) &= (1-u)\ R_1 + u\ R_2 \\
& u \in [0, 1]
\end{aligned}
$$

   where $P_1 = (0,0,0), P_2 = (0,1,1)$, and $R_1 = (1,0,1)$, $R_2 = (1,1,0)$. Let us define the parametric surface as follows:

$$
\begin{aligned}
s(u,v) &= (1-v)\ p(u) + v\ r(u) \\
& u \in [0,1]\,, \quad v \in [0,1]
\end{aligned}
$$

   Draw the given two curves and the given surface on the same figure.

6. Draw a polynomial curve of degree 4 that goes through the points $(10, 20), (20, 40), (40, 40), (50, 20), (20, 10)$ when the parameter is $0, 1, 2, 3$, and 4 respectively. Draw the tangent vector of the curve when $t = 0.5$

7. Let us define the points $P_1 = (-2,-2), P_2 = (4,0), P_3 = (6,-2), P_4 = (10,2)$. Draw the Hermite arc that goes through these points at the parameter values $-1, 0, 2, 3$ respectively. Draw the tangent vector of the curve when $t = 2$.

8. Let us define the points $P_1 = (-2, -2), P_2 = (6, -2), P_3 = (10, 2)$, and the vector $\mathbf{v} = (6, -4)$. Draw the Hermite arc that goes through these points at the parameter values $0, 1, 1.5$, and whose tangent vector at 0 is vector $\mathbf{v}$.

9. Let us define the points a $P_1 = (-2, -2), P_2 = (6, -2)$, and vectors $\mathbf{v}_1 = (6, -4)$ and $\mathbf{v}_2 = (4, 4)$. Draw the Hermite arc that goes through these points at the parameter values $0, 1$, and whose tangent vector at 0 is $\mathbf{v}_1$ and at 1 is $\mathbf{v}_2$.

10. Draw a Bézier curve with the control points $(10, 20), (20, 40), (40, 40), (50, 20)$, $(20, 10)$. Draw its tangent vectors at its beginning and at its end.

11. Let us consider the curve in Question 10. Draw a Bézier curve of degree 6 which connects to the curve with C^1 continuity.

12. Let us consider the curve in Question 9. Let us join an Hermite arc with C^1 continuity whose starting point is $(6, -2)$, endpoint is $(14, -4)$, and the tangent vector at its endpoint is $(3, 0)$. The parameter at the starting point is 0, and at the endpoint it is 2.

13. Let us consider the curve in Question 7. Let us join an Hermite arc with C^1 continuity whose starting point is $(10, 2)$, endpoint is $(14, -4)$, and the tangent vector at its endpoint is $(3, 0)$. The starting point corresponds to the parameter value -1, its endpoint corresponds to 1.