Preference score aggregation

This document describes an aggregation algorithm for use in preference-based design and decision making. In short, the following two starting principles apply to this algorithm:

1. it should reflect relative scoring as encountered in actual design and decision-making practice.
2. it should adhere to the governing mathematics in a one-dimensional affine space, which is the mathematical model applicable to preference score(s).

The algorithm therefore consists of two operations: (1) normalizing the preference scores of all alternatives per criterion, and (2) finding the representative aggregated preference score P^* for each alternative using the weighted least squares method. These two operations are further elaborated mathematically below.

Normalization

For normalization, the standard score (z-score) method is used. This yields a normalization that preserves information about the population of preference scores and reads as follows:

$$z_{i,j} = \frac{p_{i,j} - \mu_j}{\sigma_j} \text{ for } i = 1, 2, ..., I;\ j = 1, 2, ..., J \tag{1}$$

Here $z_{i,j}$ is the normalized score of alternative i for criterion j; $p_{i,j}$ is the preference score of alternative i for criterion j; μ_j is the mean of all preference scores p for criterion j; σ_j is the standard deviation of all preference scores p for criterion j. By performing this normalization for all criteria J, the preference scores are transformed to a single scale with the same properties ($\mu_J = 0$, $\sigma_J = 1$).

Weighted least squares

Since all $z_{i,j}$ scores are now on a single scale, it is possible to compare all normalized scores per alternative with each other. To find the representative aggregated preference score of an alternative that provides a best fit of all (weighted and relative) scores for each criterion, a minimization of the weighted least squares difference between this aggregated score and each of the (normalized) individual scores on all criteria is applied. This is expressed mathematically as follows:

$$\textit{Minimize } S_i = \sum_{j=1}^{J} w_j * (z_{i,j} - P_i^*)^2 \tag{2}$$

Note that since the search is for a single representative aggregated preference score, the model function $f(x_{i,j}, \beta_i)$ from the classical weighted least square method is replaced by P_i^*. The solution to this minimization can be found by differentiating with respect to P_i^* and equating it to zero. Since $\sum_{j=1}^{J} w_j = 1$, this results in the following analytical expression for the representative aggregated preference score:

$$P_i^* = \sum_{j=1}^{J} w_j * z_{i,j} \tag{3}$$