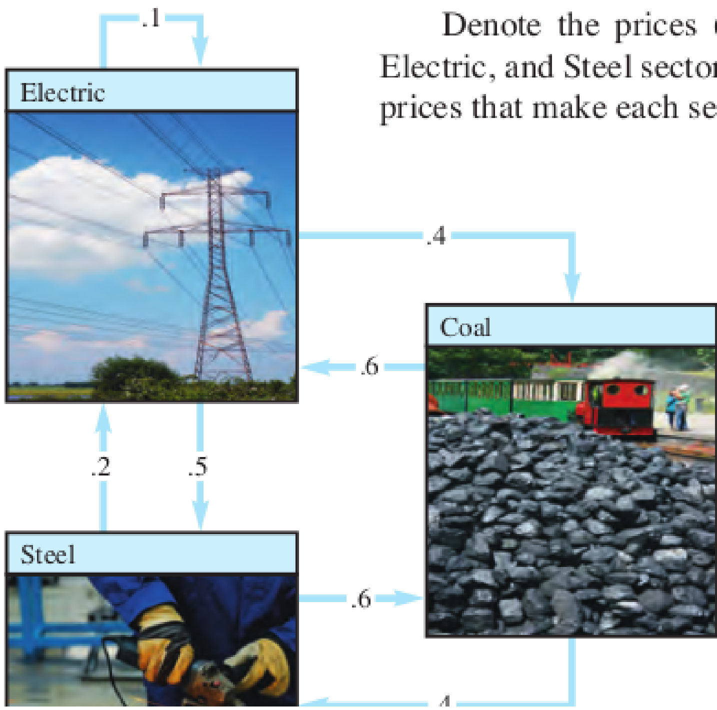
#linear_algebra

A Homogeneous System in Economics

suppose a nation's economy is dividied into many sectors.such as various manufacturing,communication,entertainment,and service industries.
suppose that for each sector we know its total output for one year and we know exactly how this output is divided or "exchanged" among the other sectors of the economy .Let the total dollar value of a sector's output be called the **price** of the output. Leontief proved the following result.
there exist equilibrium prices that can be assigned to the total outputs of the various sectros in such a way that the income of each sector exactly balances it expenses.



Denote the prices (i.e., dollar values) of the total annual outputs of the Coal, Electric, and Steel sectors by p_C, p_E, and p_S, respectively. If possible, find equilibrium prices that make each sector's income match its expenditures.

TABLE 1 A Simple Economy

Distribution of Output from:			
Coal	**Electric**	**Steel**	**Purchased by:**
.0	.4	.6	Coal
.6	.1	.2	Electric
.4	.5	.2	Steel

$$
\begin{aligned}
p_C - .4p_E - .6p_S &= 0 \\
-.6p_C + .9p_E - .2p_S &= 0 \\
-.4p_C - .5p_E + .8p_S &= 0
\end{aligned}
$$

Row reduction is next. For simplicity here, decimals are rounded to two places.

$$
\begin{bmatrix} 1 & -.4 & -.6 & 0 \\ -.6 & .9 & -.2 & 0 \\ -.4 & -.5 & .8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -.4 & -.6 & 0 \\ 0 & .66 & -.56 & 0 \\ 0 & -.66 & .56 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -.4 & -.6 & 0 \\ 0 & .66 & -.56 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\sim \begin{bmatrix} 1 & -.4 & -.6 & 0 \\ 0 & 1 & -.85 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -.94 & 0 \\ 0 & 1 & -.85 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

The general solution is $p_C = .94p_S$, $p_E = .85p_S$, and p_S is free. The equilibrium price vector for the economy has the form

$$\mathbf{p} = \begin{bmatrix} p_C \\ p_E \\ p_S \end{bmatrix} = \begin{bmatrix} .94p_S \\ .85p_S \\ p_S \end{bmatrix} = p_S \begin{bmatrix} .94 \\ .85 \\ 1 \end{bmatrix}$$

Any (nonnegative) choice for p_S results in a choice of equilibrium prices. For instance, if we take p_S to be 100 (or \$100 million), then $p_C = 94$ and $p_E = 85$. The incomes and expenditures of each sector will be equal if the output of Coal is priced at \$94 million, that of Electric at \$85 million, and that of Steel at \$100 million. ■

Network flow

systems of linear equations arise naturally when scientists,engineers,or economists study the flow of some quantity through a network.For instance ,urban planners and traffic engineers monitor the pattern of traffic flow in grid of city streets.

Electrical engineers caculate current flow manufactures to consumers through a network of wholesalers and retailers. For many networks,the systems of equation involve hundred or even thousands of variables and equations.

A network consists of a set of points called *junctions,or nodes* with lines or arcs called *brances* connecting some or all of junctions.the direction of flow in each branch is indicated,and the flow amount(or rate) is either shown or is denoted by a variable.

The basic assumption of network flow is that the total flow into the network equals the total flow out of the network and that the total flow into a junction equals the total flow out of the junction.

for example fig.1 shows 30 units flowing into a junction through one branch.,with x1 and x2 denoting the flows out of the junction through other brances.since the flow is "conserved" at each junction,we must have

$x1 + x2 = 30$.in a similar fashion ,the flow at each junction is described by a linear equation.the problem of network analysis is to determine the flow in each branch when partial information(such as flow into and out of the nework)is known.

EXAMPLE 2 The network in Fig. 2 shows the traffic flow (in vehicles per hour) over several one-way streets in downtown Baltimore during a typical early afternoon. Determine the general flow pattern for the network.

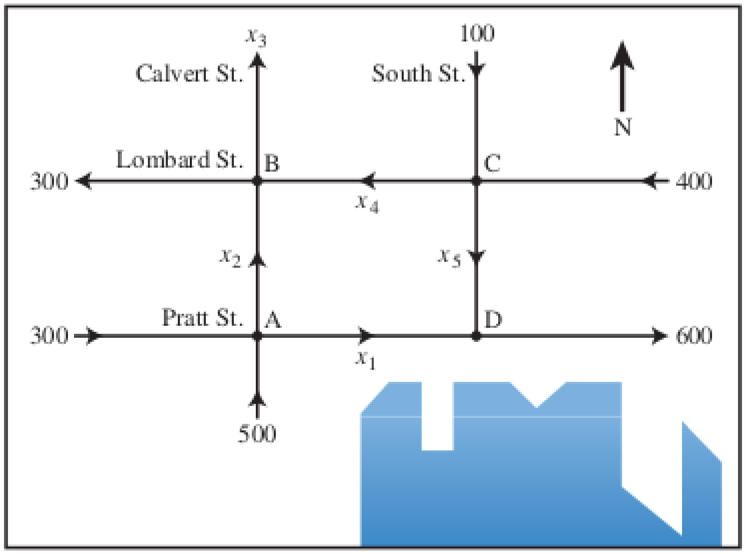


FIGURE 2 Baltimore streets.

SOLUTION Write equations that describe the flow, and then find the general solution of the system. Label the street intersections (junctions) and the unknown flows in the branches, as shown in Fig. 2. At each intersection, set the flow in equal to the flow out.

Intersection	Flow in		Flow out
A	$300 + 500$	$=$	$x_1 + x_2$
B	$x_2 + x_4$	$=$	$300 + x_3$
C	$100 + 400$	$=$	$x_4 + x_5$
D	$x_1 + x_5$	$=$	600

Also, the total flow into the network $(500 + 300 + 100 + 400)$ equals the total flow out of the network $(300 + x_3 + 600)$, which simplifies to $x_3 = 400$. Combine this equation with a rearrangement of the first four equations to obtain the following system of equations:

$$x_1 + x_2 \qquad\qquad = 800$$

$$
\begin{aligned}
x_2 - x_3 + x_4 \qquad &= 300 \\
x_4 + x_5 &= 500 \\
x_1 \qquad\qquad + x_5 &= 600 \\
x_3 \qquad\qquad &= 400
\end{aligned}
$$

Row reduction of the associated augmented matrix leads to

$$
\begin{aligned}
x_1 \qquad\qquad + x_5 &= 600 \\
x_2 \qquad - x_5 &= 200 \\
x_3 \qquad\qquad &= 400 \\
x_4 + x_5 &= 500
\end{aligned}
$$

The general flow pattern for the network is described by

$$
\begin{cases}
x_1 = 600 - x_5 \\
x_2 = 200 + x_5 \\
x_3 = 400 \\
x_4 = 500 - x_5 \\
x_5 \text{ is free}
\end{cases}
$$

A negative flow in a network branch corresponds to flow in the direction opposite to that shown on the model. Since the streets in this problem are one-way, none of the variables here can be negative. This fact leads to certain limitations on the possible values of the variables. For instance, $x_5 \le 500$ because x_4 cannot be negative. Other constraints on the variables are considered in Practice Problem 2. ■

solve all the problems in this section with python