#linear_algebra

imagine placing 2000 random numbers into a 40 * 50 matrix A and then determining both the maximum number of linearly independent columns in A and the maximum number of linearly independent columns in A^t(rows in A).Remarkably the two numbers are the same. their common value is the rank of the matrix.

The row spaces

If A is an $m \times n$ matrix, each row of A has n entries and thus can be identified with a vector in $\mathbb{R}^n$. The set of all linear combinations of the row vectors is called the **row space** of A and is denoted by Row A. Each row has n entries, so Row A is a subspace of $\mathbb{R}^n$. Since the rows of A are identified with the columns of A^T, we could also write Col A^T in place of Row A.

THEOREM 13

If two matrices A and B are row equivalent, then their row spaces are the same. If B is in echelon form, the nonzero rows of B form a basis for the row space of A as well as for that of B.

THEOREM 14

The Rank Theorem

The dimensions of the column space and the row space of an $m \times n$ matrix A are equal. This common dimension, the rank of A, also equals the number of pivot positions in A and satisfies the equation

$$\text{rank } A + \dim \text{Nul } A = n$$

- the Row A and Nul A have only the zero vector in common and are actually**Perendicular** to each other.The same fact will apply to Row A^T=(Col A) and Nul A^T

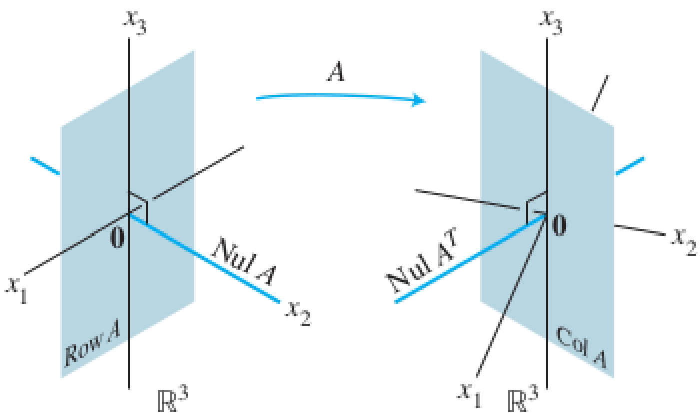

FIGURE 1 Subspaces determined by a matrix A.

applications to systems of Equations

A scientist has found two solutions to a homogeneous system of 40 equations in 42 variables.The two solutions are not multiples and all other solutions ccan be constructed by adding together appropriate multiples of these two solutions.Can the scientist be certain that an associated nonhomogenous system(with the same coefficients)has a solution?

Solution

yes,let A be the 40 * 42 coefficient matrix of the system.The given information implies that the two solutions are linearly independent and span Nul A. so dim Nul A = 2.By the rank theorem,dim col A = 42 - 2=40.Since R^40 is the only subspace of R^40 whose dimension is 40,col A must be all of R^40.This means that every nonhomogeneous equation Ax = b has a solution.

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rank and the invertible matrix Theorem

THEOREM

The Invertible Matrix Theorem (continued)

Let A be an $n \times n$ matrix. Then the following statements are each equivalent to the statement that A is an invertible matrix.

m. The columns of A form a basis of $\mathbb{R}^n$.

n. $\text{Col } A = \mathbb{R}^n$

o. $\dim \text{Col } A = n$

p. $\text{rank } A = n$

q. $\text{Nul } A = \{\mathbf{0}\}$

r. $\dim \text{Nul } A = 0$