#linear_algebra

Parallelogram rule for Addition

if u and V in R^2 are representded as points in the plane,then u+v corresponds to the fourth vertex of the parallelogram whose other vertices are u,0,adn v.

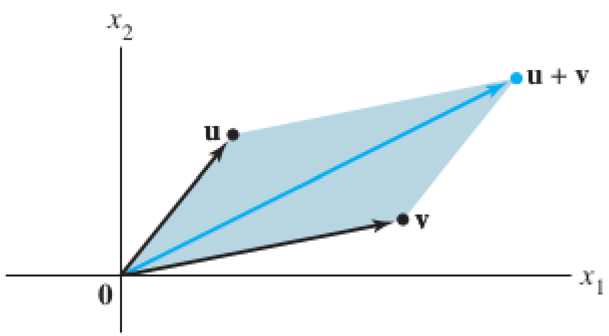
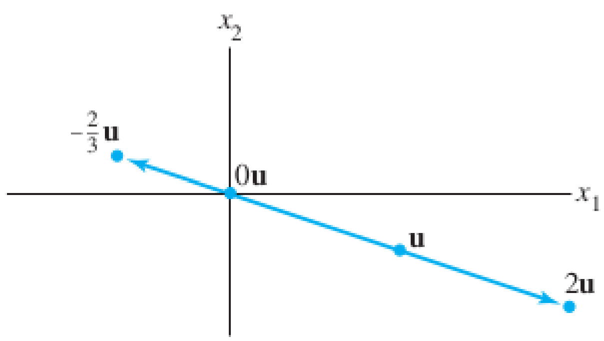
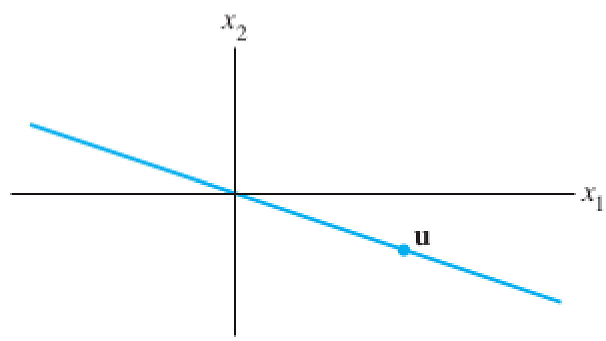


FIGURE 3 The parallelogram rule.

the set of all scalar multiples of one fixed nonzero vector is a line through the origin(0,0).



Typical multiples of **u**



The set of all multiples of **u**

Algebraic Properties of $\mathbb{R}^n$

For all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathbb{R}^n$ and all scalars c and d:

(i) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(ii) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(iii) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

(iv) $\mathbf{u} + (-\mathbf{u}) = -\mathbf{u} + \mathbf{u} = \mathbf{0}$, where $-\mathbf{u}$ denotes $(-1)\mathbf{u}$

(v) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

(vi) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

(vii) $c(d\mathbf{u}) = (cd)(\mathbf{u})$

(viii) $1\mathbf{u} = \mathbf{u}$

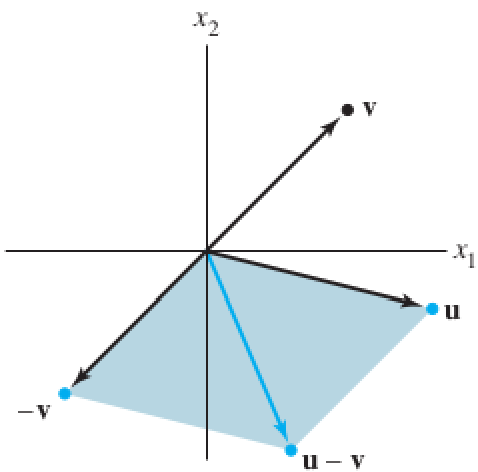


FIGURE 7
Vector subtraction.

Linear combinations

Given vectors v1,v2, ...vp in R^n and given scalaras c1,c2,...cp.the vector y defined by
y = c1v1 +c2v2+...+cpvp
is called a **linear combination** of v1, ...vp with weights c1...cp.

some linear combintaions of vectors v1 and v2 are

$$\sqrt{3}\,\mathbf{v}_1 + \mathbf{v}_2, \quad \tfrac{1}{2}\mathbf{v}_1 \;(= \tfrac{1}{2}\mathbf{v}_1 + 0\mathbf{v}_2), \quad \text{and} \quad \mathbf{0}\;(= 0\mathbf{v}_1 + 0\mathbf{v}_2)$$

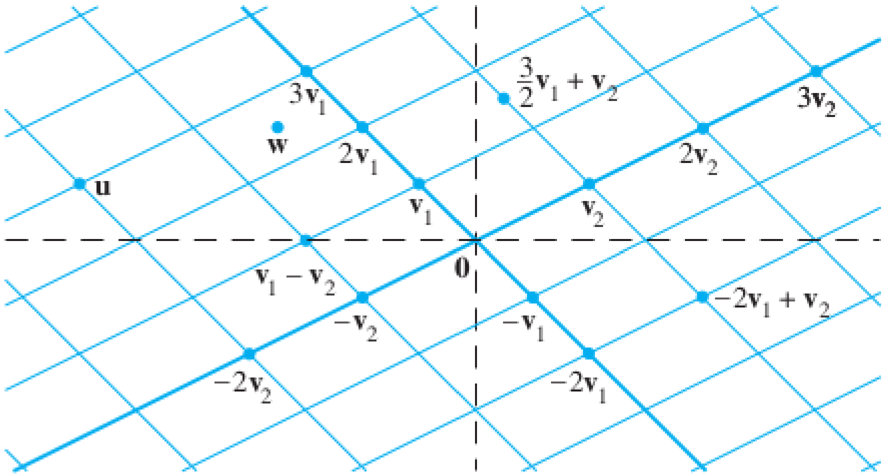


FIGURE 8 Linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$.

A vector equation

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$$

has the same solution set as the linear system whose augmented matrix is

$$\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n & \mathbf{b} \end{bmatrix} \tag{5}$$

In particular, $\mathbf{b}$ can be generated by a linear combination of $\mathbf{a}_1, \ldots, \mathbf{a}_n$ if and only if there exists a solution to the linear system corresponding to the matrix (5).

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