



# PYTHON INTERACTIVE MONTE-CARLO PRICER

Python for Finance  
Group Project

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In this document, we give an overview of 10 different derivative types, and outline the rationale behind our selection of these securities in particular. Simply put, our code enables the use of a tool that has two core purposes: pricing a single derivative, and pricing multiple derivatives or strategies combined at once.

Below, we have outlined each of these derivatives, what their purpose is, and their mathematical denotation.

## European Options

**Definition of a Call:** A call option can refer to an option contract giving the owner the right, but not the obligation, to *buy* a specified amount of an underlying security at a specified price, at a specific date. It must be noted that the seller of an option must fulfill the contract, meaning that they need to deliver the entirety of the underlying asset's value if the option is exercised. A European Call option cannot be sold or traded before maturity date.

**Definition of a Put:** Contrary to a call option, a put option refers to an option contract giving the owner the right, but not the obligation, to *sell* a specified amount of an underlying security at a specified price, at a specific time. In the same way as with a call, the writer of the put option must also fulfill the contract, and it is not sellable before the maturity date.

**Overview of a Call:** The underlying instrument of a call option can range from stocks, bonds, foreign currencies, commodities, or any other traded instruments. To illustrate what the strike price of a call option is: if the underlying's price is above the option's strike price, then the option is "In-the-Money" – if it is below the strike price, then the option is "Out-of-the-Money". The premium of a call is the amount paid to purchase the option. Hence, if you wish to break even, the price of the underlying must exceed the sum of the strike price and the premium paid.

**Overview of a Put:** When buying a put, one believes that the underlying will go below the strike price before its maturity date. The value of a put increases as the price of the underlying stock decreases relative to the strike price. In the opposite way to a call option, a put is "In-the-Money" if the underlying price is beneath the strike price – and vice versa. The premium of a put is the amount paid to purchase the option. Hence, if you wish to break even, the price of the underlying must drop below that of the strike price minus the premium paid.

**Pricing Overview of Call and Put Options:** Components that are used to price an option include: Delta (rate of change of the option compared to its underlying), its Gamma (rate of change of Delta), Theta (time decay of the option), Vega (volatility) and more. To price a call or a put, we can use the Black Scholes & Merton model as it is perhaps the most common options pricing method. It comprises of multiplying the underlying asset's price by the cumulative standard normal probability distribution function, which is outlined further below. Furthermore, the call-put parity states that the premium of a call option implies a certain fair price for the corresponding put option of the same strike price and maturity – and vice versa.

**Mathematical Background:**

$$payoff_{call} = \max(S_T - K, 0) - P \qquad payoff_{put} = \max(K - S_T, 0) - P$$

Where,

$S_T$  = underlying's price at a given time T

K = strike price

P = option premium

**Basic Strategies of Call Options:** As a strategy, being purely long a call insinuates that you believe that the underlying asset should see an upwards price movement before the option's maturity date. An interesting strategy with call options can also be shorting or 'writing' a call. This strategy comprises of selling the option in the hope that the underlying asset falls or stays Out-of-the-Money: this would mean benefitting from a downward price movement, from the theta decay (time decay) of the option, and from the cash flows derived from the premium's sale.

**Basic Strategies of Put Options:** In parallel, put options can help generate profits from the downwards price movements of the underlying asset. It could be more attractive to an investor that to 'write' or sell a call to benefit from a stock's downwards movement, as your losses are capped at 100%. When writing options, the price can potentially rise far above any expected levels, and your losses multiply rapidly. On the flip side, when purchasing a put rather than selling a call you are not receiving cash from the sale of the option's premium.

**Combined Strategies:** Finally, a combination of call and put options could be used to implement various strategies. An example of these is a butterfly spread, which a market neutral strategy that is used to reduce risk but has capped profits. This involves the purchasing of either four calls or four puts with three different strike prices of equal distance from the 'base' or middle strike price. On the other hand, an iron condor option strategy involves two puts and two calls, with one of each being long and short. All options have different strike prices but the same maturity. This strategy is designed to profit from low volatility in the underlying asset.

## Binary Options

**Definition of a Binary Call or Put:** A binary option has very similar structural points as a traditional vanilla option. However, the difference lies within the term 'binary', indicating that the outcome is either "1 or 0". So, taking this into account, buying a binary call means that the investor earns a fixed profit if the underlying rises above the strike price. A binary put means the investor earns a fixed profit if the underlying falls below the strike price. The payoffs are known in advance: either 100% of the investment is lost, or the pre-determined profits are realized.

**Overview of a Binary Call or Put:** By default, binary options are often traded OTC (Over The Counter) and involve a peer to peer agreement between two counterparties. This makes them inherently difficult to price and illiquid.

## Barrier Options

**Definition:** Barrier Options are exotic, path-dependent options, which mean that their values is tied to the one of the underlying assets. In other terms, when crossing a certain price limit fixed in the contract, the option could become active or worthless depending on the type of Barrier Option.

**Pricing & Overview:** The underlying instrument of Barrier Option can be a stock, bond, foreign currency, commodity, or any other traded instrument. There are two main types of Barrier Options: Knock-In and Knock-Out. Knock-in options are the kind of barrier options that are activated when the underlying asset a certain price limit is reached. When it happens, these options become regular vanilla options and can be exercised. However, if

during the period of the contract this price limit is never achieved, then the option expires worthless. There are two sub-types of Knock-In options: up-and-in and down-and-in. Up-and-in are activated when the underlying price rises over the threshold strike price. Down-and-in are activated when the underlying asset price drops below this limit. On the contrary, Knock-Out options cease to exist when the underlying asset reaches a certain limit during the period of the contract. There are two sub-types of Knock-Out options: up-and-out and down-and-out. Up-and-out cease to be valid when the underlying price rises over the threshold strike price. Down-and-out cease to be valid when the underlying asset price drops below this limit.

**Basic Strategies:** All types of Barrier Options can either be a call or a put. Similarly to several exotic options that are adding conditions in the terms of the contract, Barrier Options often offer cheaper premiums than American or European options. Therefore, depending on whether you think a target share price will be reached or not, you can decide to invest in a Knock-In or a Knock-Out option. These options can also be used if you want to hedge a position but only if you think the underlying will reach a certain price (upwards or downwards). Hedging using Barrier Options is an important strategy to consider because of their cheap premiums, making it an interesting alternative to American or European options hedging. For illustrative purposes, if a CEO of a given company XYZ has a contractual agreement to receive stock compensation for share price hitting a given price, an investor could place a speculative bet on a Barrier Option with the same strike price (similar to Elon Musk unlocking compensation with Tesla's share price).

**Pay-off structures of the barrier options:** Maximum and the minimum of the spot price process  $S = S_t, 0 \leq t \leq T$  as  $m_S = \inf_{0 \leq t \leq T} S_t$  and  $M_S = \sup_{0 \leq t \leq T} S_t$ .

**Pay-off at maturity of the Up-and-In option:**

$$\Phi_{UI} S_T, M_S = S_T - K + \mathbf{1}(M_S \geq H)$$

where  $\mathbf{1}(M_S \geq H)$  is an indicator function equal to 1 if  $M_S \geq H$  and zero otherwise.

**Pay-off at maturity of the Up-and-Out option:**

$$\Phi_{UO} S_T, m_S = S_T - K + \mathbf{1}(m_S < H)$$

**Pay-off at maturity of the Down-and-In option:**

$$\Phi_{DI} S_T, m_S = S_T - K + \mathbf{1}(m_S \leq H)$$

**Pay-off at maturity of the Down-and-Out option:**

$$\Phi_{DO} S_T, M_S = S_T - K + \mathbf{1}(M_S > H)$$

## Asian Options

**Definition:** Asian Options, also called average-price options, are Exotic Options and are one of the most popular path dependent options. Unlike more common options such as European and American, Asian options have an idiosyncratic character. In fact, these are not based on

the price of the underlying asset at a given time (at maturity or not) but on the average price of the underlying asset over some prespecified period and prespecified frequency, during the lifetime of the option. Thus, this kind of option allow the buyer (or seller) to buy (or sell) the underlying at the average price and not the spot price.

**Pricing & Overview:** The underlying instrument of Asian Option can be a stock, bond, foreign currency, commodity, or any other traded instrument. There are two main ways of calculating the average-price of the underlying: arithmetic mean and geometric mean. We will focus on Continuous arithmetic average Asian call or put. Generally, as they use the average instead of the spot price, Asian options tend to have a lower volatility than common options such as European and American, they also tend to be less expensive. Since the method to calculate the payoff of these options is quite unique, they are not priced using an algebraic formula. We must use a simulation method to obtain an approximation of their price (eg Monte Carlo simulation). The following formula is one of those used to implement the stock price in this type of simulations:

$$S_{t+1} = S_t \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \cdot \epsilon \sqrt{\Delta t}}$$

Where,

S = price of the stock

$\mu$  = stock's return

$\sigma$  = stock's volatility

$\Delta t$  = one-time step

$\epsilon$  = uncertainty parameter

There are two types of Asian options: call and put. For Asian call, the profit of the buyer is equal to the average of the underlying asset over the period minus the strike. The loss limitation is similar to the one of standard options, it is limited to the premium paid for the contract(s) if the average is below the strike.

$$payoff_{asiancall} = \max\left(\frac{1}{n} \sum_{i=1}^T S_i - K, 0\right)$$

Where,

n = number of observations

$S_i$  = price of the stock

K = strike

T = period of time

For Asian put, the profit of the seller is equal to strike minus the average of the underlying asset over the period. The loss limitation is similar to the one of standard options, it is limited to the premium paid for the contract(s) if the average is over the strike.

$$payoff_{asianpu} = \max\left(K - \frac{1}{n} \sum_{i=1}^T S_i, 0\right)$$

Where,

n = number of observations

$S_i$  = price of the stock

K = strike

T = period of time

**Basic Strategies:** This type of option can be interesting for a business with a very high dependance on a specific exchange rate over time. Asian options can offer a security that reduces the impact of the volatility of this exchange rate. They can also be a hedge against exceptional manipulation of a stock price or even against a highly volatile stock market. The use of the average method is taking into account several values and reduces the effect of a single value on the underlying. All of these strategies are interesting mainly because Asian options are generally less expensive than American or European ones.

## Conclusion

Having outlined which derivatives we use in our code, we can now elaborate the essence of our solution. What we implement is a tool that enables two actions:

- 1) *The simple pricing of a single derivative, through a Monte Carlo simulation*
- 2) *The pricing of any derivative combination at once, in one single block*

The tool is therefore extremely flexible, as it enables us to not only price just one asset, but whichever combination of assets we seem fit, or wish to use.

In the event that an exotic combination of derivatives is deemed the ideal strategy to take advantage of a given market environment, we do not have to worry about the complexity of pricing it. The tool we have implemented does it for us effectively. With this pricing constraint out of the way, we can focus all our attention on which combined strategy we should select (butterfly, iron condor, etc.), and have the freedom to input any combination of derivatives to test our investment thesis.

Finally, the solution also centralizes the process of derivatives investing: an investor can now engage with a single derivative, but also with a combination of exotic derivatives all through one centralized solution.