Sampling Based Inference

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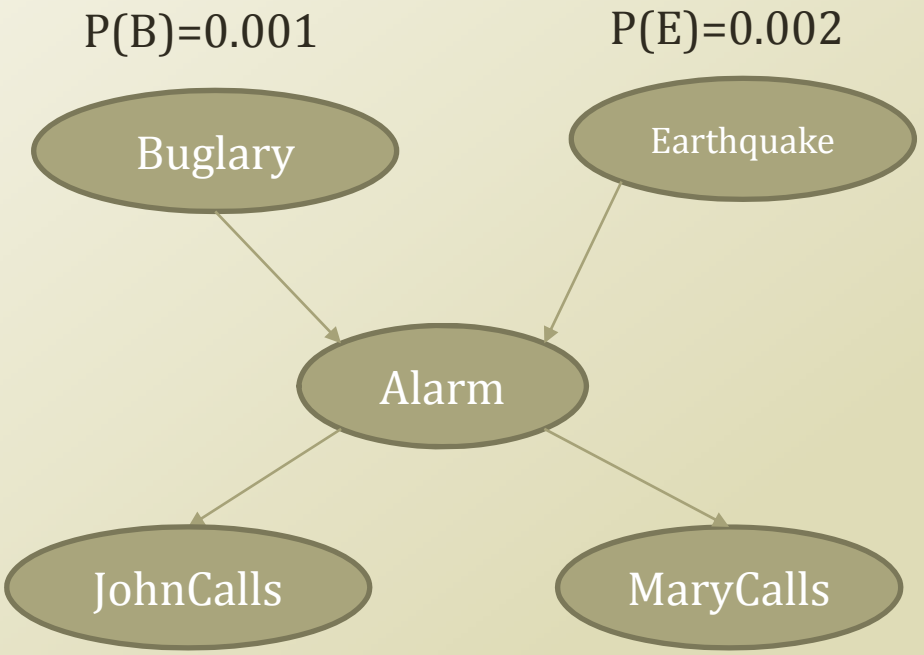
Weekly Objectives

- Learn basic sampling methods
  - Understand the concept of Markov chain Monte Carlo
  - Able to apply MCMC to the parameter inference of Bayesian networks
  - Know the mechanism of rejection sampling
  - Know the mechanism of importance sampling
- Learn sampling based inference
  - Understand the concept of Metropolis-Hastings algorithm
  - Know the mechanism of Gibbs sampling
- Know a case study of sampling based inference
  - Understand the latent Dirichlet allocation model
  - Know the collapsed Gibbs sampling
  - Know how to derive Gibbs sampling formula for LDA

Rejection Sampling

- P(E=T|MC=T,A=F)=?
- RejectionSampling
  - Iterate many times
    - Generate a sample from the Bayesian network
      - Buglary → false
      - Earthquake → true
      - Alarm|B=F,E=T→true
        - If the sample does not follow MC=T, A=F, reject the sampling procedure, and repeat
      - JC|A=T→true
      - MC|A=T→false
  - Return Count(E=T,MC=T,A=F)/# of Samples
- Any problem?



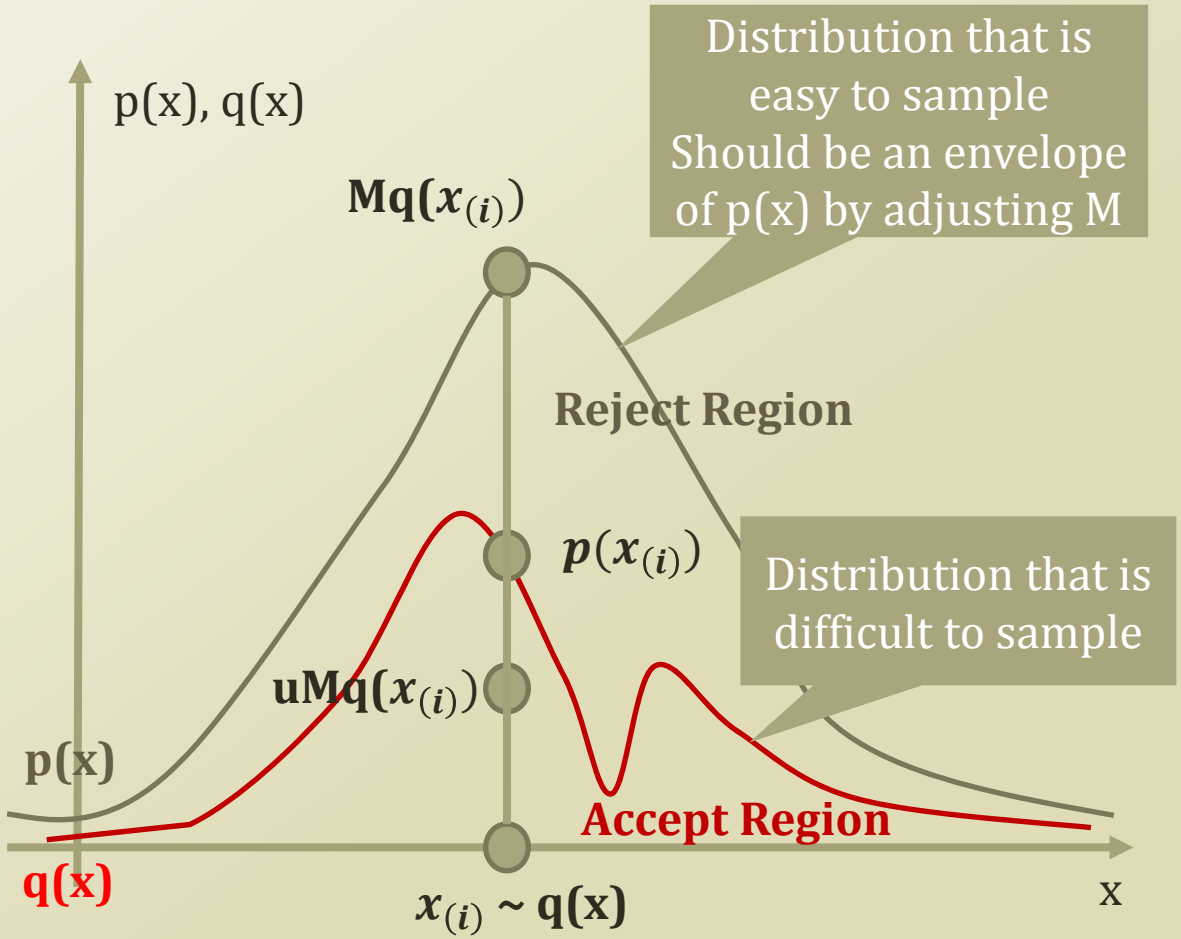
B	E	P(A|B,E)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(J|A)
T	0.90
F	0.05

A	P(M|A)
T	0.70
F	0.01

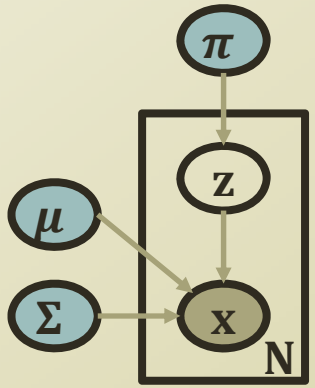
Rejection Sampling from Numerical View

- count = 0
- while count < N
  - Sample $x_{(i)} \sim$ q(x)
  - Sample u ~ Unif(0,1)
  - If $u < \frac{p(x_{(i)})}{Mq(x_{(i)})}$
    - Accept $x_{(i)}$
    - Increase count
  - Else
    - Reject and re-sample



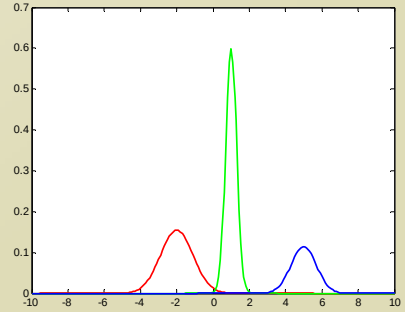
Rejection Sampling in GMM

$$P(x) = \sum_{k=1}^{K} P(z_k)P(x|z)$$
$$= \sum_{k=1}^{K} \pi_k N(x|\mu_k, \Sigma_k)$$

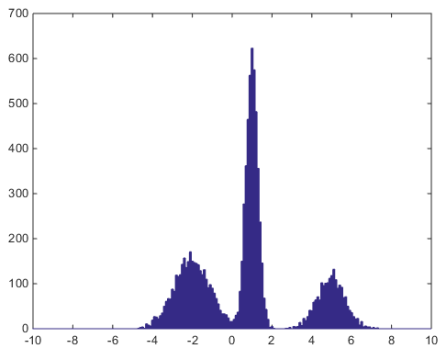


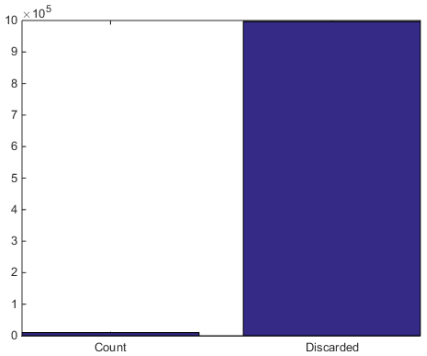
- Rejection sampling of GMM
  - Sample z from {1, 2, 3} with 1/3 change each
  - Sample x from $N(\mu_{q(z)}, \Sigma_{q(z)})$
    - $q(x)$ = The probability drawing x from $N(\mu_{q(z)}, \Sigma_{q(z)})$
  - Sample u from Uniform(0,1)
  - If $M \times u \times q(x) < p(x)$
    - Accept the sample of (z, x)
  - Else
    - Discard the sample

P Mixture
= 0.35*N(-2,0.9),
0.45*N(1,0.3),
0.2*N(5,0.8)



Q Mixture
= 1/3*N(-2,1), 1/3*N(1,1), 1/3*N(5,1)





Q Mixture
= 3 * (1/3 * N(0,1))