Support Vector Machine

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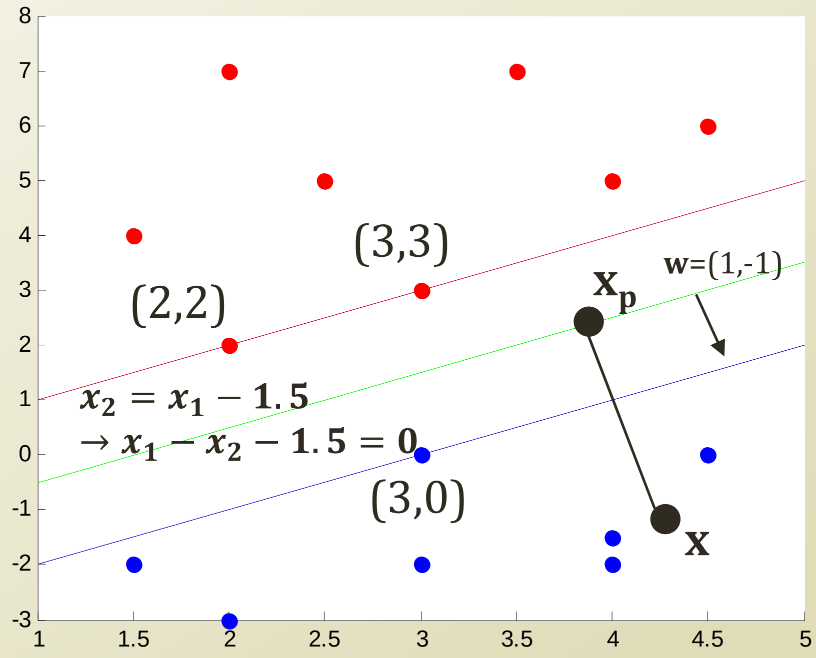
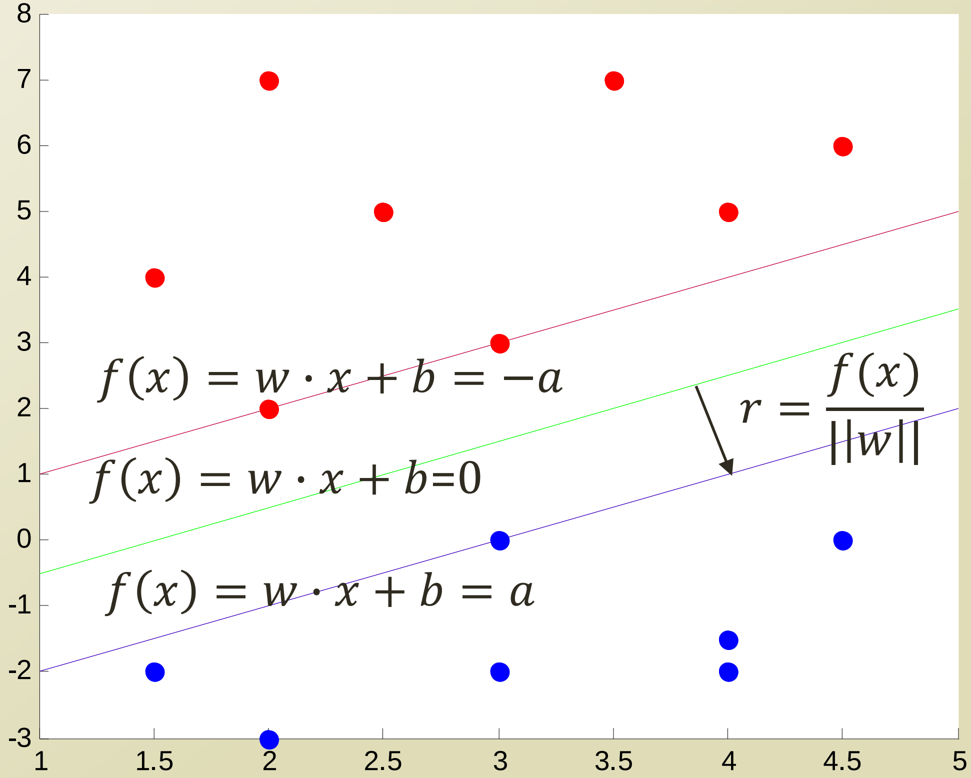
Margin Distance



- Let's say
  - $f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$
  - A point **x** on the boundary has
    - $f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b = 0$
  - A positive point **x** has
    - $f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b = a, a > 0$
- We are going to measure the distance
  - between an arbitrary point **x** and a point $\mathbf{x_p}$ on the boundary and on the perpendicular line from **x** to the boundary
  - $x = x_p + r\frac{w}{||w||}, f(x_p) = 0$
  - $f(x) = w \cdot x + b = w\left(x_p + r\frac{w}{||w||}\right) + b = \overline{wx_p + b} + r\frac{w \cdot w}{||w||} = r||w||$
- The distance is $r = \frac{f(x)}{||w||}$

Maximizing the Margin

- Good decision boundary?
  - Maximum margin!
  - $r = \frac{a}{||w||}$
  - Need to consider the both side
- Optimization problem?
  - $max_{w,b} 2r = \frac{2a}{||w||}$
    $s.t. (wx_j + b)y_j \geq a, \forall j$
- a is an arbitrary number and can be normalized
  - $min_{w,b} ||w||$
    $s.t. (wx_j + b)y_j \geq 1, \forall j$



This becomes a quadratic optimization problem. Why?