Logistic Regression

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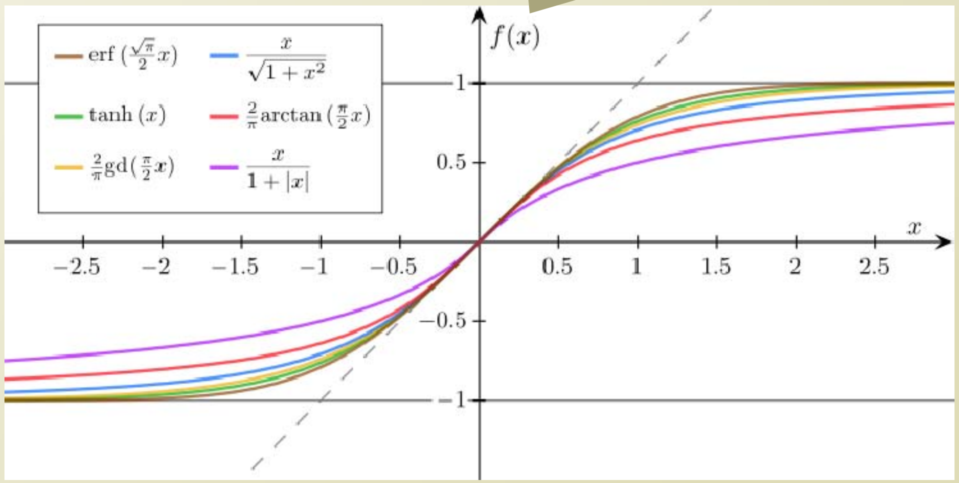
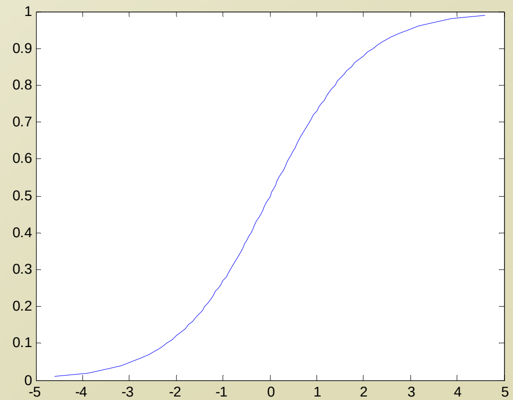
Logistic function

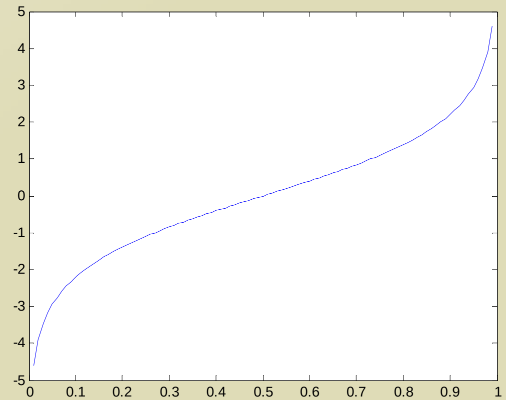
Many types of sigmoid functions



- Sigmoid function is
  - Bounded
  - Differentiable
  - Real function
  - Defined for all real inputs
  - With positive derivative
- Logistic function is
  - $f(x) = \frac{1}{1+e^{-x}}$
  - In relation to the population growth
  - Why is this good?
    - Sigmoid function
    - Particularly, easy to calculate the derivative…



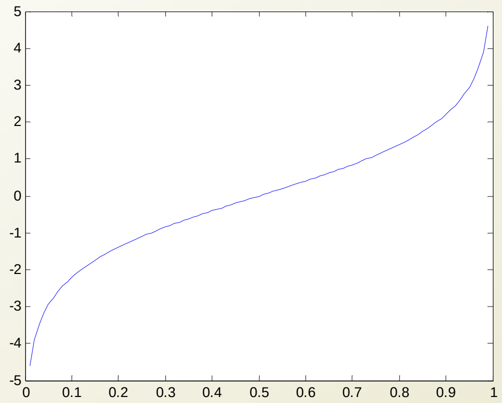
Logistic Function

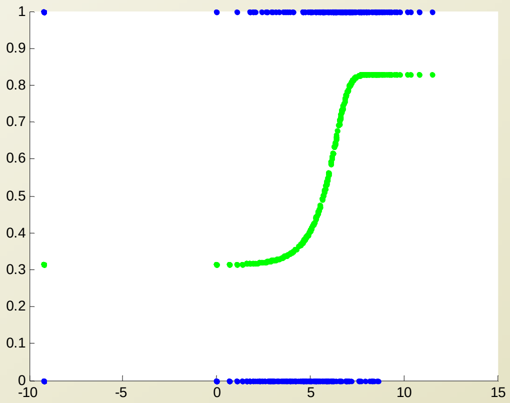


Logit Function

$$f(x) = \log\left(\frac{x}{1-x}\right)$$

Logistic Function Fitting





Linear Regression:

$\hat{f} = X\theta \quad \theta = (X^T X)^{-1} X^T Y$

Very similar to the linear regression.
Turning to the multivariate case

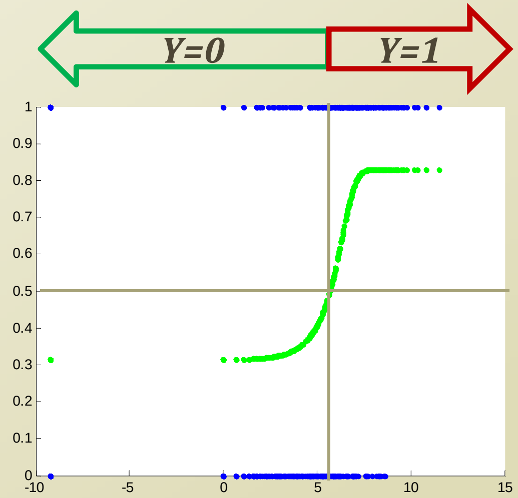
$$f(x) = \log\left(\frac{x}{1-x}\right) \rightarrow x = \log\left(\frac{p}{1-p}\right) \rightarrow ax + b = \log\left(\frac{p}{1-p}\right) \rightarrow X\theta = \log\left(\frac{p}{1-p}\right)$$

Logit→Logistic
Inverse of X and Y
X in Logit is the probability

Linear shift for a better function fitting

- When we are fitting the linear regression to approximate P(Y|X)
  - $X\theta = P(Y|X)$
  - Though, this is not going to keep the probability axiom
- Now we are fitting to the logistic function to approximate P(Y|X)
  - $X\theta = \log\left(\frac{P(Y|X)}{1-P(Y|X)}\right)$
  - From linear to logistic

Logistic Regression



- Logistic regression is a probabilistic classifier to predict the binomial or the multinomial outcome
  - by fitting the conditional probability to the logistic function.
- You can see the problem from the different view.
  - This way is actually closer to the formal definition.
- Given the Bernoulli experiment
  - $P(y|x) = \mu(x)^y(1-\mu(x))^{1-y}$
  - $\mu(x) = \frac{1}{1+e^{-\theta^T x}} = P(y=1|x)$
  - Here, $\mu(x)$ is the logistic function
- From the previous slide,
  - $X\theta = \log\left(\frac{P(Y|X)}{1-P(Y|X)}\right) \rightarrow P(Y|X) = \frac{e^{X\theta}}{1+e^{X\theta}}$

Logistic Function

$$f(x) = \frac{1}{1+e^{-x}}$$

The goal, finally, becomes finding out $\boldsymbol{\theta}$, again