

A Lightweight McEliece Cryptosystem Co-processor Design

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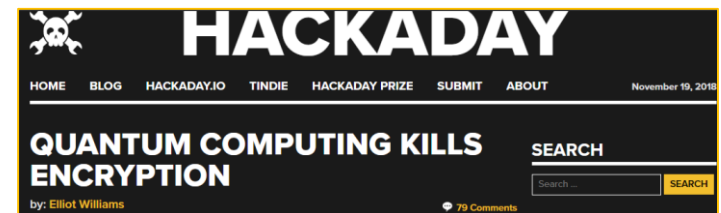
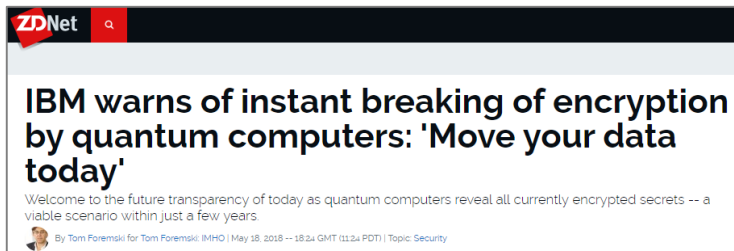
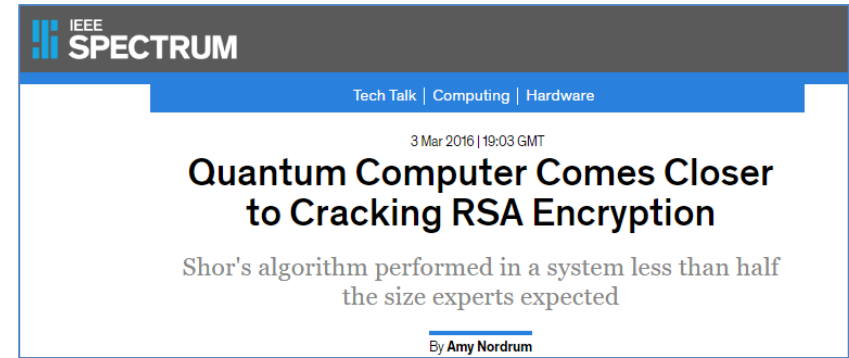
Presentation Flow

- The problem of quantum computers coming real
- How pressing is the problem
- What can we do?
- Public-key systems for post-quantum era
 - Code-based encryption
 - Can we make it lighter & faster?
- Conclusion

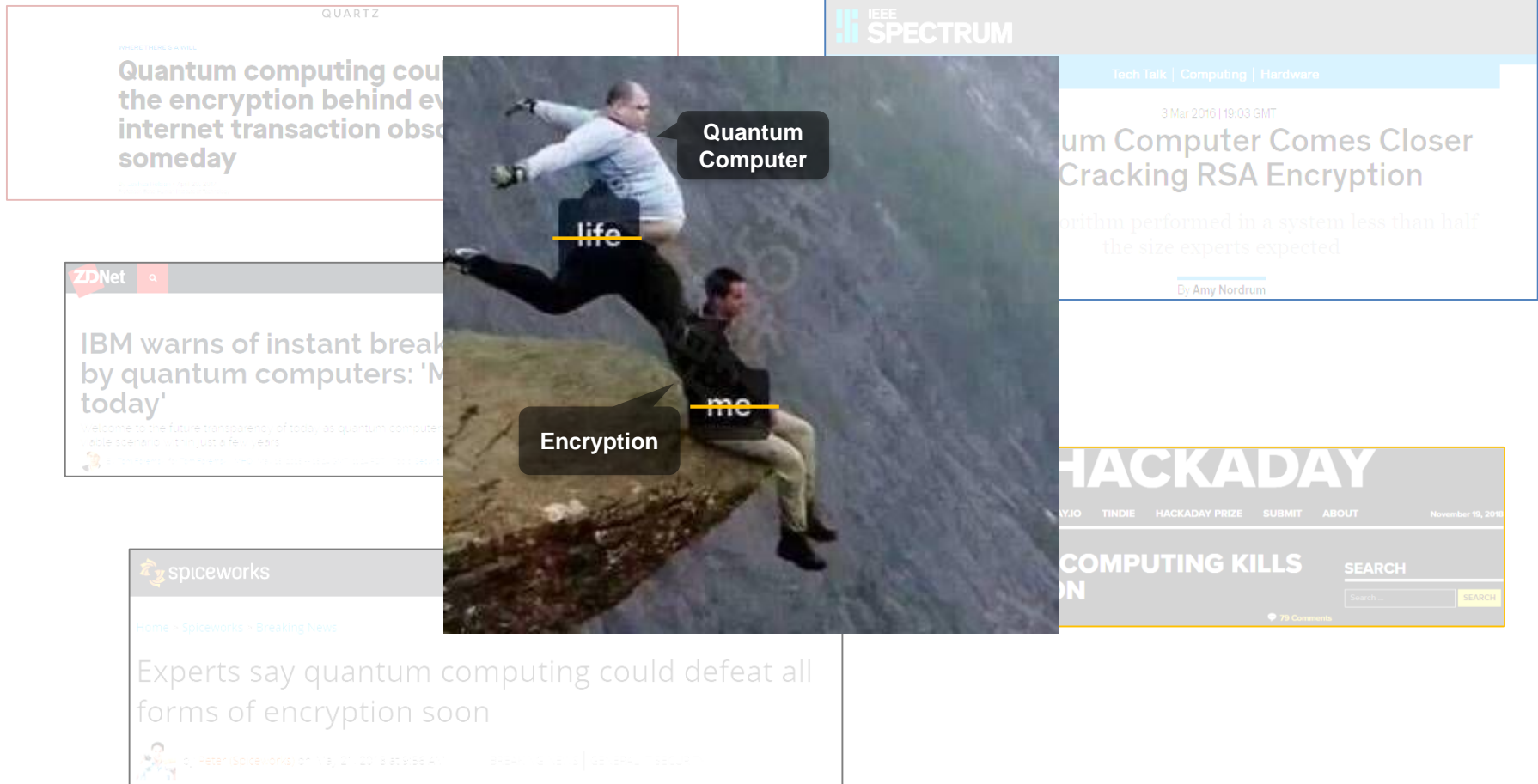
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What Media Have Said About the Coming of Quantum Computers



1. What Some Have Said About Quantum Computers



The Actual Situation Now

- Who is considered as non-post-quantum secure?

Algorithm	Secure in Post-quantum Era?
RSA-1024, -2048, -4096	No
Elliptic Curve Crypto (ECC)-256, -521	No
Diffie-Hellman	No
ECC Diffie-Hellman	No
AES-128, -192	No

[1]

One Question

- Can we increase the key size of some popular encryption schemes, so that they can be post-quantum secure?
 - Maybe yes, maybe no.

Table II. Equivalent Security Levels of AES and RSA under Attacks from Classic and Quantum Computers *

Attack Platform	Symmetric Encryption			Asymmetric (Public-key) Encryption		
	Algorithm	Key Size	Security Level	Algorithm	Key Size	Security Level
Classic Computers	AES-128	128	128	RSA-2048	2,048	112
	AES-256	256	256	RSA-15360	15,360	256

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Quantum Computers	AES-128	128	64	RSA-2048	2,048	25
	AES-256	256	128	RSA-15360	15,360	31

Grover's algorithm

Shor's algorithm

Department of Electrical & Computer Engineering

* TechBeacon, Waiting for quantum computing: Why encryption has nothing to worry about, 2018

Presentation Flow

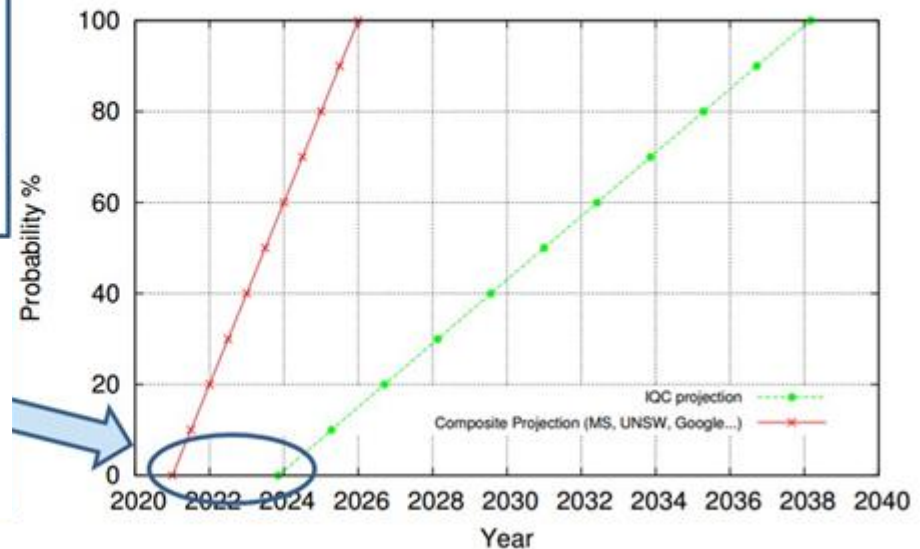
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How Pressing is the Issue?

■ The Timeline

Projected Probability of General Purpose* Quantum Computers Arriving By Year

The green graph is based on data from the IQC (Institute for Quantum Computing) provided earlier in 2015. (19 yr)

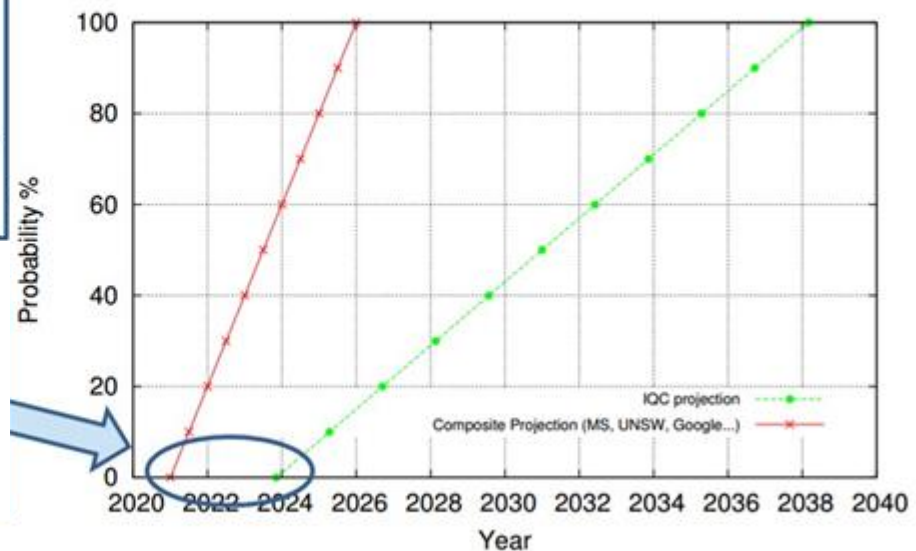


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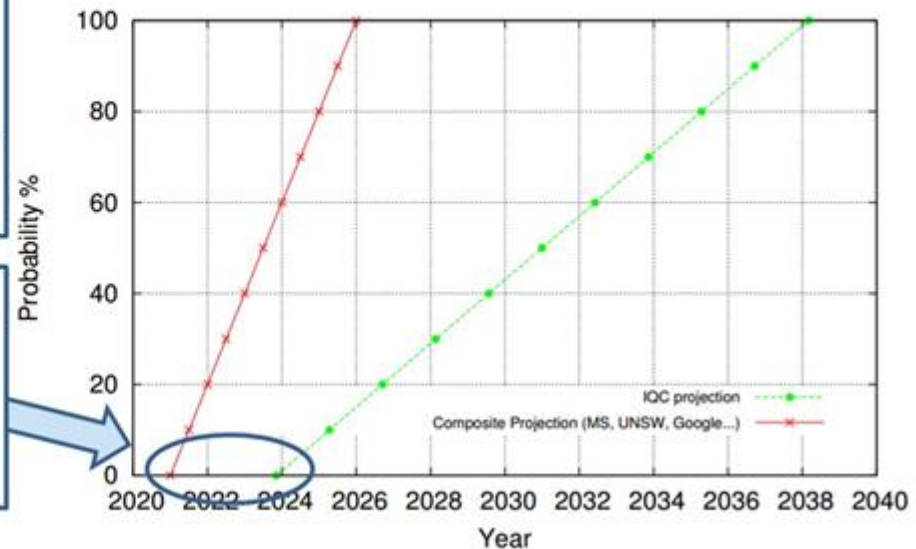
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Critical infrastructure and industries with fiduciary responsibilities MUST be re-tooled when the threat window opens! (1 ~ 5 yr)



How Pressing is the Issue?

- What do companies/institutes say?

- Microsoft Research

- 5 years



- NIST

- 15 years



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Post-Quantum Cryptography Standardization



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- PROJECTS
- POST-QUANTUM CRYPTOGRAPHY

Post-Quantum Cryptography^[1]



Round 1 Submissions

Official comments on the First Round Candidate Algorithms should be submitted using the "Submit Comment" link for the appropriate algorithm. Comments from the [pqc-forum Google group subscribers](#) will also be forwarded to the [pqc-forum Google group list](#). We will periodically post and update the comments received to the appropriate algorithm.

All relevant comments will be posted in their entirety and should not include PII information in the body of the email message.

Please refrain from using OFFICIAL COMMENT to ask administrative questions, which should be sent to pqc-comments@nist.gov

By selecting the "Submitter's Website" links, you will be leaving NIST.gov. We have provided links to submitter web sites because they may have information that would be of interest to you. No inferences should be drawn o.n account of other sites being referenced, or not, from this page. There may be other web sites that are more appropriate for your purpose. NIST does

PROJECT LINKS

- Overview
- FAQs
- News
- Events
- Publications
- Presentations

ADDITIONAL PAGES

- [Post-Quantum Cryptography Standardization](#)
- [Call for Proposals](#)

Post-Quantum Cryptography (PQC) Standardization

■ NIST

- Jan 2017 - present
- Evaluating 69 (5 withdrawn) submissions of PQC, to bring up a standard

(just like AES or RSA):

- 21 lattice-based
- 18 code-based
- Some hash-based
- Some others

[1]

Algorithm	Algorithm Information <i>KAT files are included in zip file unless they were too large</i>	Submitters	Comments
BIG QUAKE	Zip File (4MB) IP Statements Website	Alain Couvreur Magali Bardet Elise Barelli Olivier Blazy Rodolfo Canto-Torres Philippe Gaborit Ayoub Otmani Nicolas Sendrier Jean-Pierre Tillich	Submit Comment View Comments
BIKE	Zip File (10MB) IP Statements Website	Nicolas Aragon Paulo Barreto Slim Bettaleb Loic Bidoux	Submit Comment View Comments
CFPKM	Zip File (<1MB) IP Statements Website	O. Chakraborty J.-C. Faugere L. Perret	Submit Comment View Comments
<u>Classic McEliece</u>	Zip File (<1MB) KAT Files (26MB) IP Statements Website	Daniel J. Bernstein Tung Chou Tanja Lange Ingo von Maurich Rafael Misoczki Ruben Niederhagen Edoardo Persichetti Christiane Peters Peter Schwabe Nicolas Sendrier Jakub Szefer Wen Wang	Submit Comment View Comments
<u>Compact LWE</u>	Zip File (1MB) IP Statements Website	Dongxi Liu Nan Li Jongkil Kim Surya Nepal	Submit Comment View Comments

Submission deadline Nov 30, 2017. List updated Dec 20, 2018.

Why Code-based?

1. It has withstood the test of time
 - Published in 1978, 40 years of examination
2. The security reduction (hardness) is decoding of linear codes without knowing the encoding algorithm
 - NP-hard for quantum computers
3. Although:
 - Its key size is large: 1MB
 - Its decryption is highly parallel: easy for both good & bad guys

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Code-based Encryption

- McEliece cryptosystem
 - Error Correction Code (ECC)
 - A (n, k, t) ECC code C :
 - k : size of your message (information)
 - n : size of the encoded message (codeword)
 - t : # of random errors C can tolerate

Code-based Encryption

- McEliece cryptosystem

- Error Correction Code (ECC)

- A (n, k, t) ECC code C :

- k : size of your message (information)
- n : size of the encoded message (codeword)
- t : # of random errors C can tolerate

- Generating matrix G for C :

$$\begin{matrix} 1 \\ k \end{matrix} \times \begin{matrix} n \\ k \end{matrix} \begin{matrix} G \end{matrix} = \begin{matrix} n \\ C \end{matrix} \begin{matrix} 1 \end{matrix}$$

- When m is encoded by G to C , it is able to be recovered even after being distorted by t errors

- Err-correct $(C + e) = m$

Code-based Encryption

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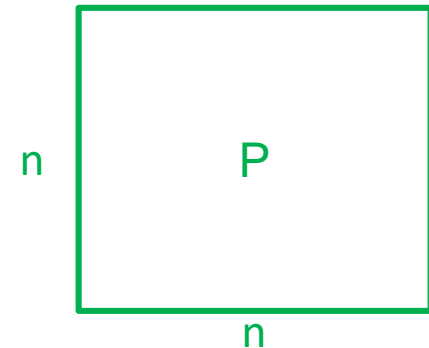
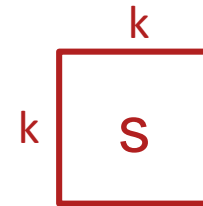
Nothing secret to anyone.

- When m is encoded by G to C , it is able to be recovered even after being distorted by t errors
 - Err-correct $(C + e) = m$

Code-based Encryption

■ Key Generation (Alice)

- Pick two random matrices:
 - $k \times k$ non-singular binary matrix S :
 - $n \times n$ permutation binary matrix P :
- Compute:
 - $k \times n$ matrix $G' = S \times G \times P$



$$\begin{array}{c} \boxed{S} \end{array} \times \begin{array}{c} \boxed{G} \end{array} \times \begin{array}{c} \boxed{P} \end{array} = \begin{array}{c} \boxed{G'} \end{array}$$

For **obfuscation**,
s.t. G 's leaks no
information of G

Code-based Encryption

■ Key Generation

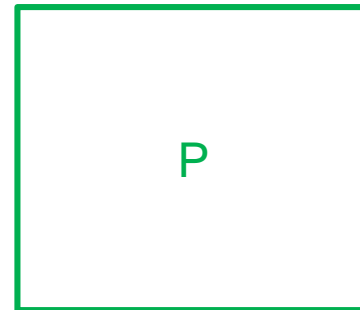
■ Public key (t, G'):

• t,



■ Private key (S, G, P):

• Remember $G' = S \times G \times P$



Code-based Encryption

- McEliece cryptosystem

- Encryption (Bob to Alice):

1. Message (plaintext) encrypted to codeword

$$\boxed{m} \times \boxed{G'} = \boxed{C}$$

2. Codeword obfuscated by error to cipher

$$\boxed{C} + \boxed{e \text{ (t of 1's)}} = \boxed{\text{Cipher}}$$

Code-based Encryption

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Can be **arbitrarily** chosen. Will be eliminated anyways but makes attack **difficult**.

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- Decryption (Alice computes):

- Remember $G' = S \times G \times P$

$$1) \text{ Cipher}' = \text{Cipher} \times P^{-1} = (m \times G' + e) \times P^{-1} = (m \times S \times G) + (e \times P^{-1})$$

$$2) \text{ Err-correct } (m \times S \times G + e') = m \times S$$

$$3) m \times S \times S^{-1} = m$$

Code-based Encryption

■ McEliece cryptosystem

■ Encryption (Bob to Alice):

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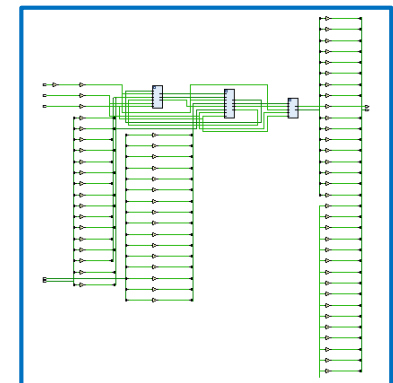
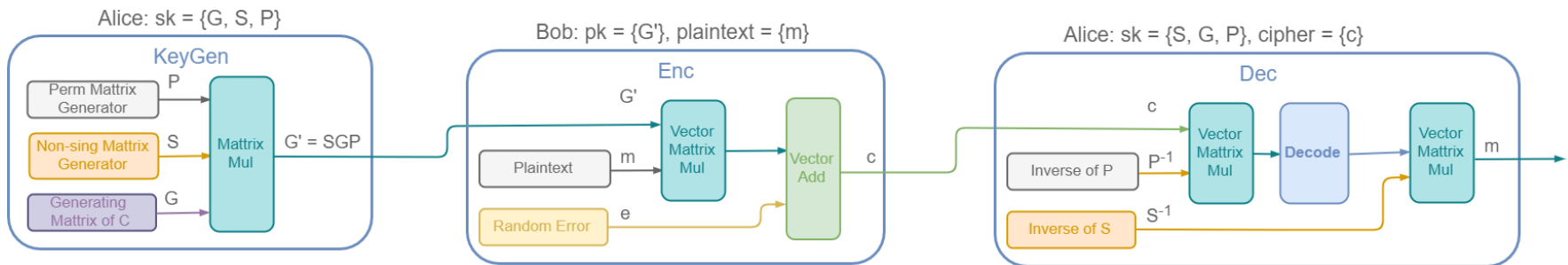
$$2) \text{ Err-correct } (m \times S \times G + e') = m \times S$$

$$3) m \times S \times S^{-1} = m$$

Attacker does not know: S, G, P
Thus he/she cannot compute:
 $S^{-1}, \text{Err-correct}(e), P^{-1}$
Therefore, he/she does not know m

Code-based Encryption

- Architecture
 - Key Generation
 - Encryption
 - Decryption



Code-based Encryption

- Can we make it faster?
 - The original scheme uses binary Goppa codes
 - Encoding & decoding involves
 - Long polynomial division
 - Solving equations over finite fields
 - We propose using Orthogonal Latin Square Codes
 - Encoding & decoding involves
 - Binary operations only
 - Can be fully parallel

- Orthogonal Latin Square Codes (OLSC)

The figure displays two 6x5 grids. The left grid contains 30 chess pieces, and the right grid contains 30 colored squares. The pieces are arranged in a specific pattern, and the colors are arranged in a corresponding pattern.

The chess pieces in the left grid are:

- Row 1: King, Queen, Rook, Bishop, Knight
- Row 2: Rook, Bishop, Knight, King, Queen
- Row 3: Knight, King, Queen, Rook, Bishop
- Row 4: Queen, Rook, Knight, Bishop, King
- Row 5: King, Rook, Bishop, Knight, Queen
- Row 6: Bishop, Knight, King, Queen, Rook

The colored squares in the right grid are:

- Row 1: Black, Red, Blue, Green, Purple
- Row 2: Red, Blue, Green, Purple, Black
- Row 3: Blue, Green, Purple, Black, Red
- Row 4: Green, Purple, Black, Red, Blue
- Row 5: Purple, Black, Red, Blue, Green
- Row 6: Purple, Black, Red, Blue, Green

- $$\bullet \left[\begin{array}{c} M_0 \\ M_1 \\ \dots \\ M_{2m-1} \end{array} \right] I_{2mq}$$

[illegible]

Code-based Encryption

■ OLSC-based McEliece Cryptosystem

- Information: q^2
- Redundancy: $2mq$
- Errors to correct: m

$$\left[\begin{array}{c} M_0 \\ M_1 \\ \vdots \\ M_{2m-1} \end{array} \right] I_{2mq}$$

Algorithm 1: OLSC-based McEliece Cryptosystem

```

1  Let  $G' = SGP$  and  $t$  be the public key, and  $\{G, S, P\}$ 
   the private key, where  $G$  is a  $k \times n$  OLSC
   encoding matrix with random permutation of
   columns, and  $H$  as its corresponding decoding
   matrix. Let each Latin square size  $q \times q$ . Let  $m$ 
   be the plaintext and  $c$  the encrypted cipher.

2

3  Precompute:  $S^{-1}, P^{-1}$  as the inverse to  $S, P$ 
   respectively.

4

5   $c' \leftarrow cP^{-1}$ 

6   $u \leftarrow Hc' \times H$ 
7  for  $i=0$  to  $n$ 
8       $m'_i \leftarrow (u_i > q/2)? \sim c'_i : c'_i$ 
9   $m \leftarrow m'S^{-1}$ 

10

11 return  $m$ 

```

Code-based Encryption

■ OLSC-based McEliece Cryptosystem

Code	Time Complexity
Binary-Goppa	$O(n^2)$
OLSC	$O(1)$

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Recall that ...

- McEliece cryptosystem

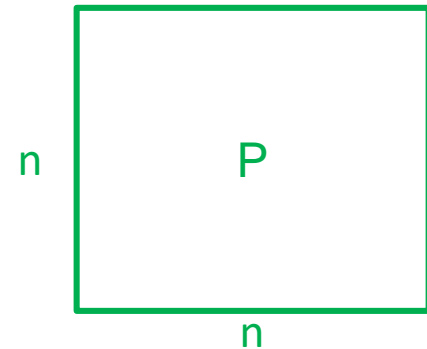
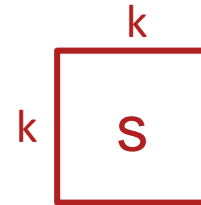
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Code-based Encryption

- G (the private key)
 - From a $t=2$ double error-correcting OLSC code

	Generate				G																															
	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0				
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0				
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0				
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0				
3	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1				
4	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	1	0				
5	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	1				
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0				
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0				
8	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	1				
9	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0				
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	1	0	0				
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0				
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	1	0	0				
13	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0				
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1				
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	1	0				

Code-based Encryption

- S and its inverse (the private key)

	S																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																		
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36

Code-based Encryption

- $G' = S \times G \times P$ the public key

$S \times G \times P = PK$

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	1	1
2	1	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1	1	1
3	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	0	1	1	0	1	0	0	0	0	0	0	1	1
4	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1
5	1	1	0	0	0	0	0	0	1	0	1	1	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1
6	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	1
7	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	1	1	0	0	0	0	0	0
11	0	0	0	0	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0
12	0	0	0	0	0	0	1	1	0	1	1	1	1	1	0	1	1	1	1	0	0	1	0	0	1	1	1	0	0	0	0	0
13	0	0	0	0	0	1	1	1	0	0	1	0	0	1	0	1	1	0	1	0	0	1	0	0	1	1	1	0	0	0	0	0
14	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0
15	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0

Code-based Encryption

■ Key Generation

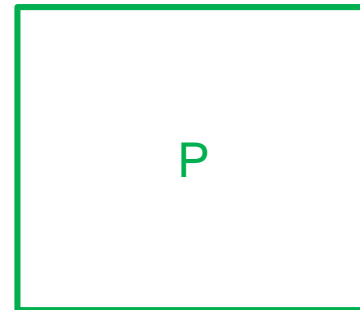
- Public key (t, G'): Alice to Bob

• $t = 2,$



- Private key (S, G, P): Alice to herself

• Remember $G' = S \times G \times P$



Code-based Encryption

■ Encryption (Bob to Alice)

1. Message (plaintext) encrypted to codeword

$$\boxed{m} \times \boxed{G'} = \boxed{C}$$

7777

0
0
0
1
1
1
1
1
1
0
0
1
1
0
0
0
0
1

×

S*G*P=PK																																		
	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0		
0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	1	1	1
2	1	1	1	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1	1	1	1
3	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	0	1	1	0	1	0	0	0	0	0	0	1	1	1
4	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1
5	1	1	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1
6	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1
7	1	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0
12	0	0	0	0	0	0	1	1	0	1	1	1	1	1	0	1	1	1	1	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0
13	0	0	0	0	0	1	1	1	0	0	1	0	0	1	0	1	1	0	1	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0
14	0	0	0	0	0	1	1	1	0	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
15	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0

= 0 1 1 0 1 1 1 1 1 0 1 1 0 0 1 0 0 0 0 1 0 1 1 1 1 0 1 1 0 0 0 0

Code-based Encryption

- Encryption (Bob to Alice)
 1. Message (plaintext) encrypted to codeword
 2. Codeword obfuscated by error to cipher

$$\boxed{C} + \boxed{e \text{ (t of 1's)}} = \boxed{\text{Cipher}}$$

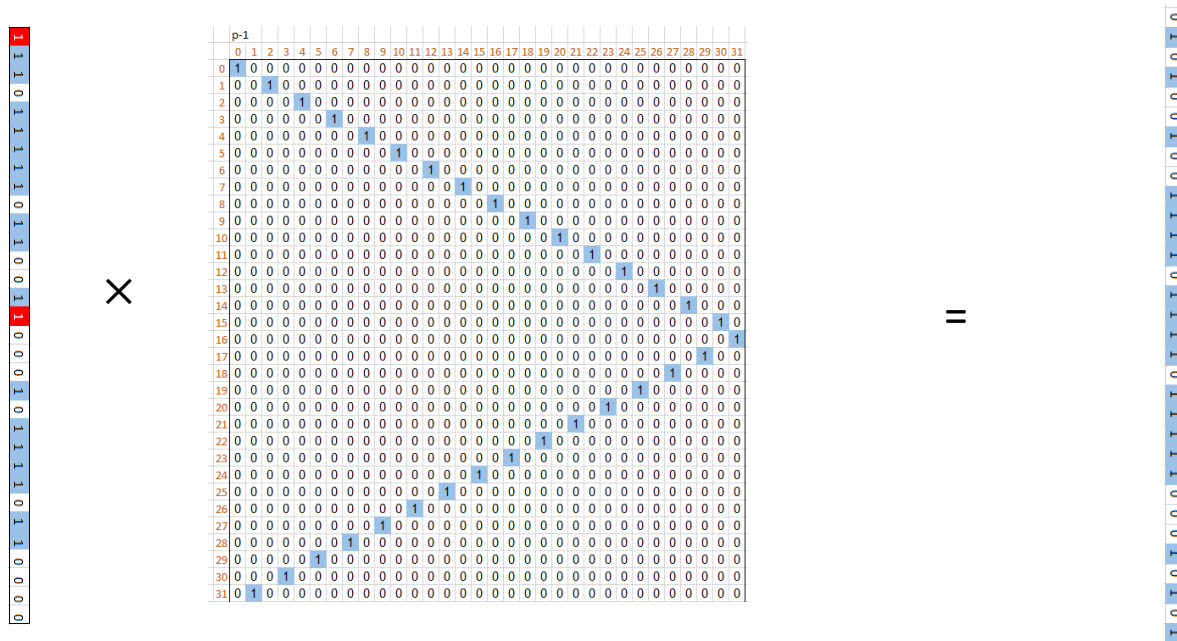
$$\begin{array}{c}
 \boxed{0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0} \\
 + \\
 \boxed{1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1} \\
 || \\
 \boxed{1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0}
 \end{array}$$

Code-based Encryption

■ Decryption (Alice)

(Remember $G' = S \times G \times P$)

$$1) \text{ Cipher} \times P^{-1} = (m \times G' + e) \times P^{-1} = (m \times S \times G) + (e \times P^{-1})$$



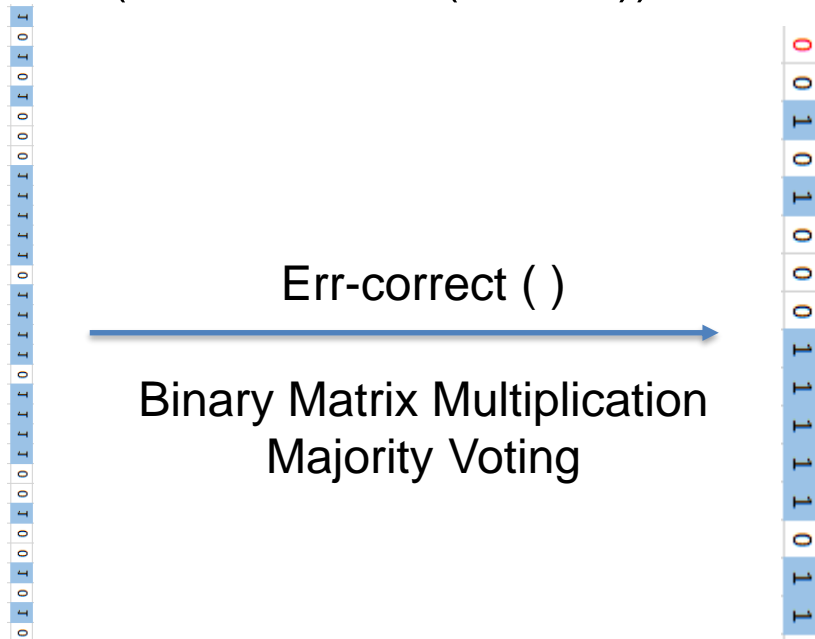
Code-based Encryption

■ Decryption (Alice)

(Remember $G' = S \times G \times P$)

$$1) \text{ Cipher} \times P^{-1} = (m \times G' + e) \times P^{-1} = (m \times S \times G) + (e \times P^{-1})$$

$$2) \text{ Err-correct } (m \times S \times G + (e \times P^{-1})) = m \times S$$



Code-based Encryption

Decryption (Alice)

(Remember $G' = S \times G \times P$)

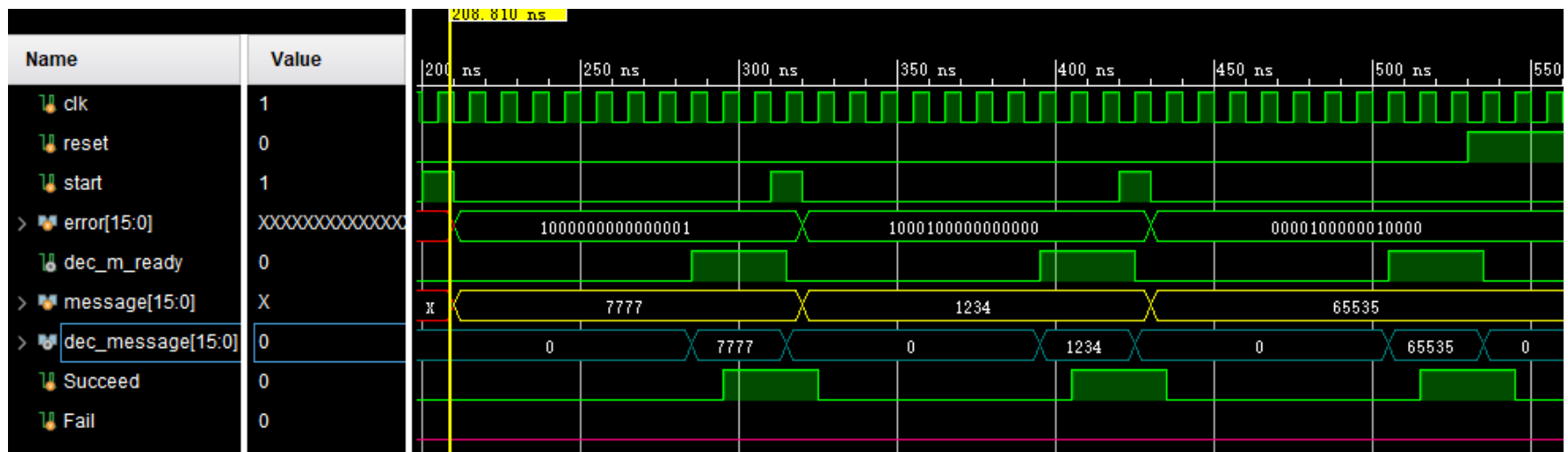
- 1) Cipher $\times P^{-1} = (m \times G' + e) \times P^{-1} = (m \times S \times G) + (e \times P^{-1})$
- 2) Err-correct ($m \times S \times G + (e \times P^{-1})$) = $m \times S$
- 3) $m \times S \times S^{-1} = m$

Attacker does not know: S, G, P
Thus he/she cannot compute:
 $S^{-1}, \text{Err-correct}(), P^{-1}$
Therefore, he/she does not know m

$$\begin{array}{c}
 \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline 1 \\ \hline 0 \\ \hline 1 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 0 \\ \hline 1 \\ \hline 1 \\ \hline \end{array}
 \times
 \begin{array}{c}
 \begin{array}{|c|} \hline S^{-1} \\ \hline \end{array}
 \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline
 & 15 & 14 & 13 & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 3 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 4 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 5 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 6 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 7 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline
 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ \hline
 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ \hline
 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline
 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline
 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 \end{array}
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{|c|} \hline 7777 \\ \hline \end{array}
 \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 0 \\ \hline 0 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline 1 \\ \hline \end{array}
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 \begin{array}{c}
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 \end{array}
 \end{array}$$

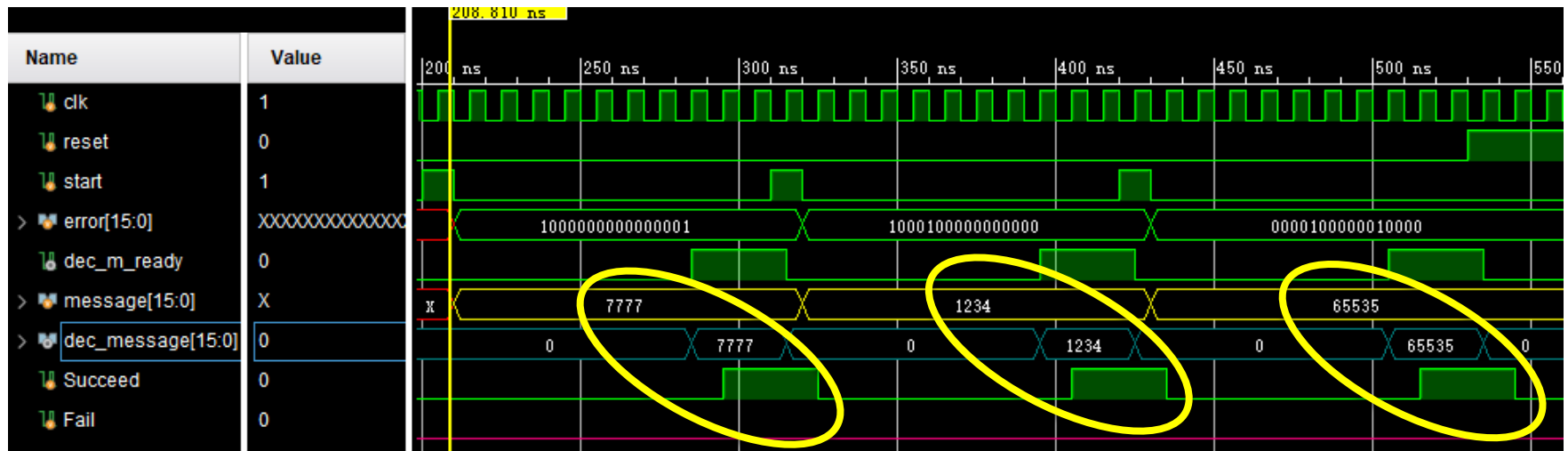
Code-based Encryption

- Simulation successful
 - Able to encrypt & decrypt 16-bit plaintexts



Code-based Encryption

- Simulation successful
 - Able to encrypt & decrypt 16-bit plaintexts



Presentation Flow

- The problem of quantum computers coming real
- How pressing is the problem
- What can we do?
- Public-key systems for post-quantum era
 - Code-based encryption
 - Can we make it lighter & faster?
- **Conclusion**

What needs to be done?

- McEliece cryptosystem
 - Post-quantum secure key size:
 - $k \times n$ matrix G' : $k = 5413$, $n = 6960$, $t = 119$
 - Key size: 1mb for 128-bit security (RSA usually 2048 bits)



- Crypto-analysis
 - Key space (G , S , P)
 - Decoding techniques

Any Questions?

