(Pre-lab and Data Sheets in spate files. You must print those out)

Experiment #5, Series and Parallel Circuits, Kirchhoff's Laws

1 Purpose

(You are expected independently to list the objectives in your lab report. Each objective's success or failure needs to be addressed in your conclusion.)

2 Introduction

As we learned from the Ohm's law experiment, an electrical circuit is any continuous path or array of paths along which current may flow. A circuit usually contains a battery or other sources of EMF (electromotive force) to create the current. Without a source of energy to drive the circuit no current (electric charge) will flow. Between the terminals of our power source can be any combination of elements through which the electrons may pass; anything from a single wire to a complicated collection of wires, diodes, transistors, and other circuit elements as resistors and capacitors.

Whatever the elements that make up the circuit there are some simple rules that must be obeyed. Two of these rules are ***Kirchhoff's laws*** regarding ***current*** (flow of electric charge) and ***voltage*** (electrical potential difference).

Kirchhoff's current law: Since, as far as we know, charge can be neither created nor destroyed, if we pick a single point in a circuit all of the charge that flows into that point must also flow out of it in a steady state, or unchanging situation. Put in terms of currents Kirchhoff's law derived from the conservation of charge states that at a given node (point in the circuit) the sum of the currents flowing into and out of that node will be zero. This is shown schematically in figure 1 where the sum of the three current flowing into and out of the node, signified by the black dot, adds to zero: $I_1 + I_2 + I_3 = 0$. Note in this case two of the currents are negative, that is to say, they flow out of the node, where the node is the low potential point. The negative (-) node dot is the low connection reference point and the three positive (+) dots are the high connection reference points for the voltage measurements in part 3.2.3 of this lab. It should be noted that this node is our arbitrary reference point for making measurements.

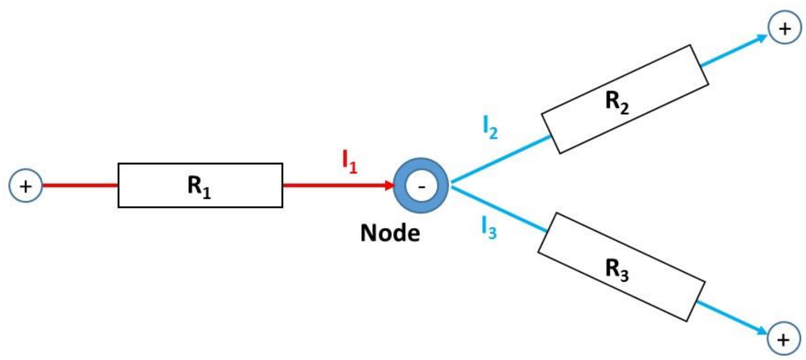


Figure 1: Schematic depicting Kirchhoff's current law

Kirchhoff's voltage law: This law is a consequence of the fact that the electric field exerts a force which is conservative. Thought of in terms of potential energies, the potential energy of a charge at a particular point in space does not depend on the path the charge took to get to that point. Since the potential energy of a charge in a circuit is its charge, Q, times the voltage at the point in the circuit, V, the sum of the voltages measured around any loop in a circuit must add to zero. Otherwise, if the charge went around that loop its energy would be changed when it returned to its starting point. This is shown schematically in Figure 2. Here the sum of the voltages will be zero, $V_{a\text{-}b} + V_{b\text{-}c} + V_{c\text{-}d} + V_{d\text{-}a} = 0$. Assuming that the bottom left hand node "**a**" is at the higher potential V_1, V_2 and V_3 would be positive and $V_{d\text{-}a}$ would be negative as the voltages are measured counterclockwise. In this part of the lab, $V_{emf} = 10.00V$ will be the power supply that is driving the circuit. This yields:

$$V_1 + V_2 + V_3 - 10.00V = 0, \text{ or } V_1 + V_2 + V_3 = 10.00V = V_{emf}$$

$$IR_1 + IR_2 + IR_3 = 10.00V = V_{emf} \text{ (Current I is a constant.)}$$

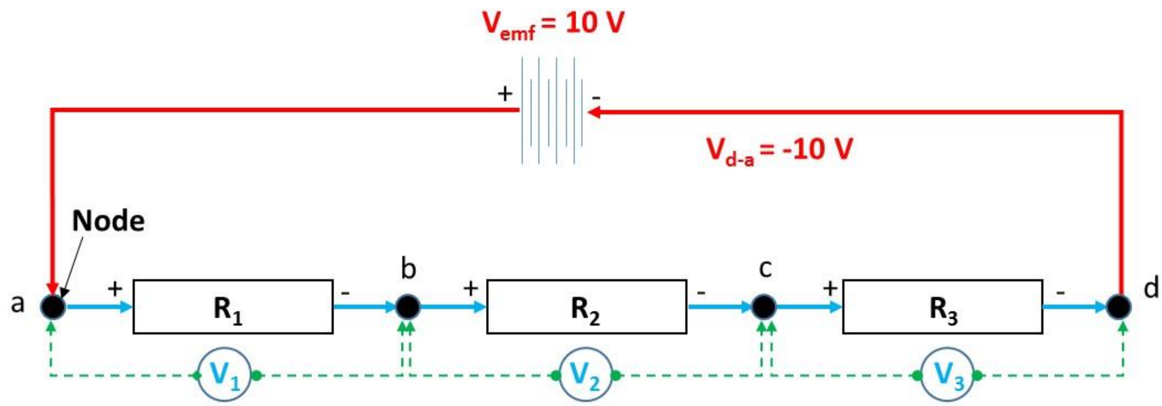


Figure 2: Schematic depicting Kirchhoff's voltage law

Kirchhoff's laws are important because, if we take all the loops and all the nodes in a complicated circuit and apply Kirchhoff's laws to them then we end up with a series of algebraic equations which can be solved to uniquely determine all of the voltages and currents in our problem.

3 Apparatus

1. DC power supply.
2. Agilent 34405A digital volt-ohmmeter (to be used for voltage and resistance measurements).
3. Fluke 75 digital multi-meter (to be used for all current measurements).
4. Carbon resistors and connecting cables.

If you are curious, read the manuals concerning the operation of the power supply and the digital meters or read the short description in the appendix.

4. Procedure

Direct Resistance Measurements with a Digital Ohmmeter.

4.1 Resistance in Series and Parallel. Measurements go in the report data section.

1. Directly measure and record the resistances of the carbon resistors $R_{1,\,dm}$, $R_{2,\,dm}$, $R_{3,\,dm}$ using the digital voltmeter (Agilent 34405A) set to measure resistance in ohms (the button with Ω on it).
2. Connect the resistors in parallel and series and measure and record the equivalent parallel and series resistance of these networks, $R_{p,\,dm}$, and $R_{s,\,dm}$.

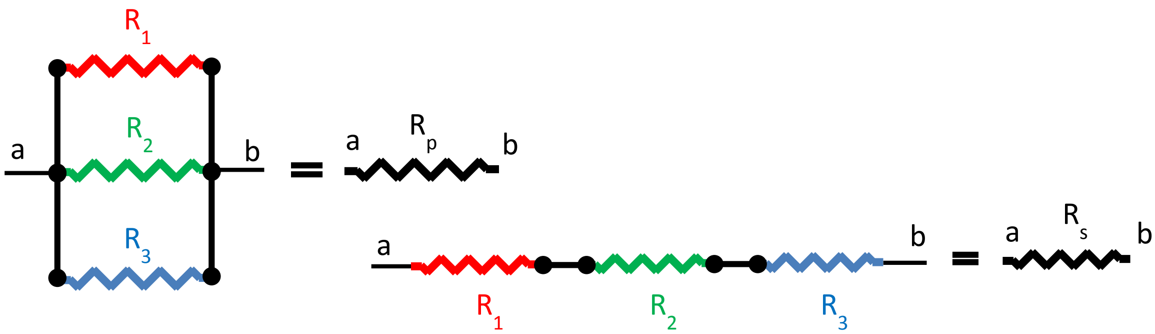
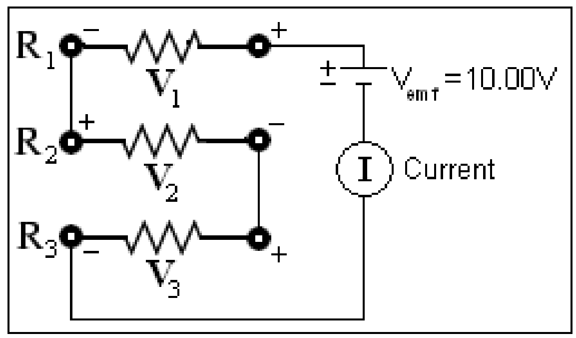


Figure 3: Resistors arranged in parallel and in series

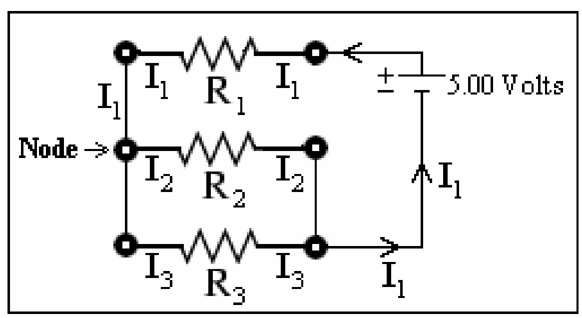
4.2 Kirchhoff's Voltage Law

Assemble the series circuit using three carbon resistors as shown in the figure to the right. Connect the power supply across this network with the milliammeter (Fluke 75) in series. The milliammeter (current meter) measures the current (1 mA = 0.001 A, where A = ampere). The current is a constant in this series circuit. Set the power supply to V_{emf} = 10V. (Record the number you set as it shows on the Agilent, eg V_{emf} = 10.0076V.) Record the current that is being directly measured by the current meter as I_{dm}. Directly measure the voltage drops across each resistor using the voltmeter as $V_{1,dm}$, $V_{2,dm}$, and $V_{3,dm}$.



4.3 Kirchhoff's Current Law

The circuit we will be examining is shown to the right. The node we are going to explore is the point where the three resistors connect. Because we need to insert the current meter in series with each leg of the circuit of interest there will be a series of measurements with the ammeter in different place in the circuit. All measurement will be made with the same voltage, 5.00 volts, applied to the circuit.



First, record the voltage across each of the three resistors with the low side always at the node. As R_2 and R_3 form a parallel connection, v_2 and v_3 should be the same negative voltage. v_1 is a positive voltage. Next, insert the ammeter between the end of the first resistor and the low side which is the node. To do so, cables will have to be removed when the ammeter is connected. Record the current running through the first resistor. Next, insert the ammeter between the second resistor and the node. Record the current running through the second resistor. Finally, insert the ammeter between the third resistor and the node. Record the current running through the third resistor. Both $I_{2,\,dm}$ and $I_{3,\,dm}$ should have a negative reading. $I_{1,\,dm}$ is a positive current

If I_2 and I_3 were measured relative to the direction of the current flow, these two values would have been positive and we would have $I_{1,exp} = I_{2,exp} + I_{3,exp}$. I_2 and I_3 are negative in the experiment as we used the node as common reference point. Same applies to V_2 and V_3.

5 Calculations and Analysis

5.1 Analysis of Resistances in Parallel and Series

Check 1

First we assembled three resistors in a parallel network. What does the theory claim? It claims that the reciprocal of the combined resistance equals the sum of the reciprocals of each resistor. In other words,

$$\frac{1}{R_{p,th}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Giving us our theoretical calculation,

$$R_{p,th} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]^{-1}$$

Now we directly measured this, allowing us to check this theoretical claim with a percent difference.

$$\%\, diff_1 = \left|\frac{R_{p,dm} - R_{p,th}}{R_{p,th}}\right| * 100$$

Check 2

When we have three resistors assembled in a series network, we calculate our theoretically predicted resistance as the sum of each individual resistor.

$$R_{s,th} = R_1 + R_2 + R_3$$

Check this theoretical claim versus what was directly measured.

$$\%\, diff_2 = \left|\frac{R_{s,dm} - R_{s,th}}{R_{s,th}}\right| * 100$$

5.2 Analysis of a series circuit and Kirchhoff's Voltage Law

Check 3

Our theoretical claim is that the current passing through the series network can be calculated using the quotient of the applied emf and series resistance. Calculate this predicted current using the directly measured series resistance.

$$I_{s,th} = \frac{V_{emf}}{R_{s,dm}}$$

Check this claim with the directly measured current.

$$\%\, diff_3 = \left|\frac{I_{s,dm} - I_{s,th}}{I_{s,th}}\right| * 100$$

Check 4

In the series circuit with ~10 volts across it, our theoretical claim is that each voltage drop across the three resistors should sum to the three directly measured voltages.

$$V_{emf,th} = V_{1,dm} + V_{2,dm} + V_{3,dm}$$

Check this claim versus what you directly set, V_{emf}, with

$$\% \, diff_4 = \left|\frac{V_{emf,dm} - V_{emf,th}}{V_{emf,th}}\right| * 100$$

Check 5

Using just the directly measured resistances and current, our next theoretical claim is that those quantities would be enough to find the voltage drop across each resistor.

$$V_{1,th} = R_1 * I_{s,dm} \text{ , } V_{2,th} = R_2 * I_{s,dm} \text{ , and } V_{3,th} = R_3 * I_{s,dm}$$

Check each against what was directly measured,

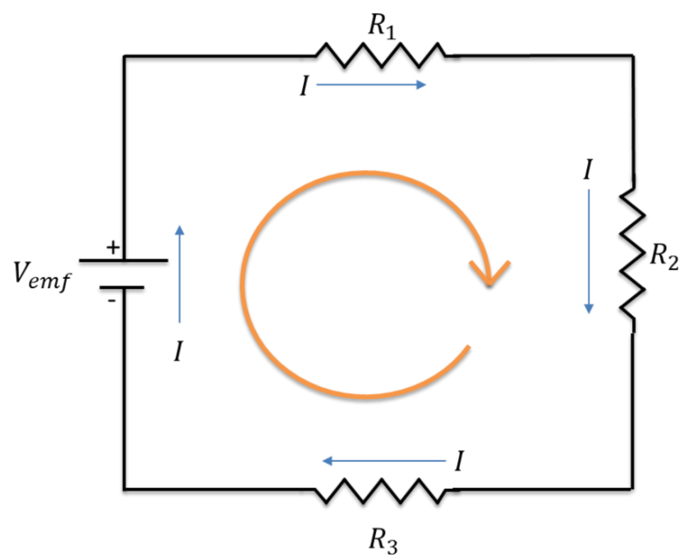
$$\% \, diff_{5a} = \left|\frac{V_{1,th} - V_{1,dm}}{V_{1,th}}\right| * 100$$

$$\% \, diff_{5b} = \left|\frac{V_{2,th} - V_{2,dm}}{V_{2,th}}\right| * 100$$

$$\% \, diff_{5c} = \left|\frac{V_{3,th} - V_{3,dm}}{V_{3,th}}\right| * 100$$

Check 6

Now we are ready to check Kirchhoff's loop law! The claim is that all the voltage drops and rises around any closed loop must sum to zero. Due to this being a series circuit, we have essentially already accomplished this with Check 4; however, the loop law is important enough to merit its own check!



We draw our loop in the same direction as the assumed direction of the current. Starting at the bottom left corner, the loop law claims that

$$V_{emf} - IR_1 - IR_2 - IR_3 = 0$$

Switching to the notations we already used, we check the loop law by seeing how close to zero this calculation comes out to be:

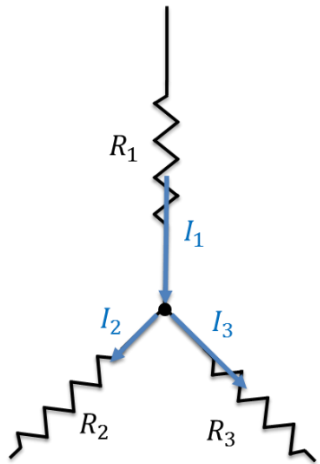
$$V_{emf} - R_1 * I_{s,dm} - R_2 * I_{s,dm} - R_3 * I_{s,dm} = 0?$$

(Just give this result to two number after the decimal place in both your lab report and quick sheet.)

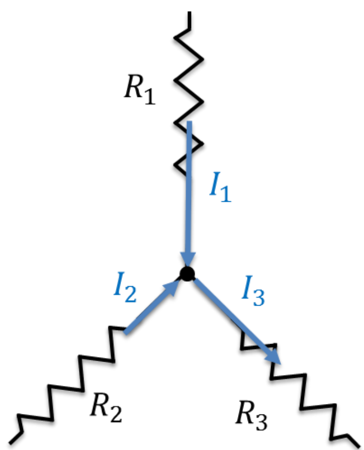
5.3 Analysis of Kirchhoff Current Law.

Check 7

The last two checks deal with Kirchhoff's node law. In essence, any junction that has current flowing in, must have the same current flowing out. First, we zoom in on our node.



When directly applying the node law, as in a text book problem, we start with $I_1 + I_2 + I_3 = 0$. Then we would assume the numerical value of I_1 to be positive as we believe it to be entering the node, and then we would assume the numerical values of both I_2 and I_3 to be negative as we believe they are leaving the node. If we were incorrect about any of these positive/negative assumptions, the algebra would ultimately correct us. However, as we are trying to confirm these assumptions experimentally, we accomplish this same confirmation by always leaving the low end of our volt and current meters on the node. For example, by leaving the low end at the node, you should have measured v_1 as positive and v_2 and v_3 as both negative—confirming our assumption about what currents are going in and out of the node. Suppose that you measured v_1 and v_2 as both positive and only v_3 as negative. This would let us know that our original assumption was wrong and the current flow is instead,



So to start our checks, we want to theoretically predict our three currents based on our three voltages.

$$I_{1,th} = \frac{v_1}{R_1},\ I_{2,th} = \frac{v_2}{R_2},\ \text{and } I_{3,th} = \frac{v_3}{3}$$

Test each against what was directly measured,

$$\%\,diff = \left|\frac{I_{1,th} - I_{1,dm}}{I_{1,th}}\right| * 100,\ \%\,diff = \left|\frac{I_{2,th} - I_{2,dm}}{I_{2,th}}\right| * 100, \text{ and}$$

$$\%\,diff = \left|\frac{I_{3,th} - I_{3,dm}}{I_{3,th}}\right| * 100$$

Check 8

Confirm Kirchhoff's node law!

$$I_{1,dm} + I_{2,dm} + I_{3,dm} = 0?$$

(Just give this result to two number after the decimal place in both your lab report and quick sheet.)

6 Questions

1. Let $R_{big} = 54321\ \Omega$ and $R_{small} = 0.54321\ \Omega$.

   a) Find the series and parallel combinations of these two resistors.

   b) When there is a large difference in two resistor's sizes, what useful approximations can be used when considering their series and parallel combinations? *(This is a handy thing to know in circuit design!)*

2. In the circuit used to verify Kirchhoff's current law, R_2 and R_3 are connected in parallel.

   a. Calculate the equivalent parallel resistance R_p using the measured values of R_2 and R_3.

   b. Your value of R_p, calculated above in part 2.a, is in series with R_1. Calculate R_s using the measured value of R_1 and your calculated value of R_p in part 2.a.

   c. Using the value of R_s from part 2.b above, and 5.00 volts, calculate the current. Call this current I_{cal}.

   d. Use percent difference to show the difference between the calculated current in 2.c and the positive sum of $I_{2,dm}$ and $I_{3,dm}$., i.e. let $I_{2+3} = |I_{2,dm} + I_{3,dm}|$. (Example. If $I_{2,dm}$ = - 0.45 mA and $I_{3,dm}$ = - 0.93 mA, then $I_{2+3} = 1.38\ mA$.) Use the 2.c value as the theoretical current value when doing percent difference.

   e. Why should the positive sum of $I_{2,exp}$ and $I_{3,exp}$ equal the current in 2.c?

7 Discussion

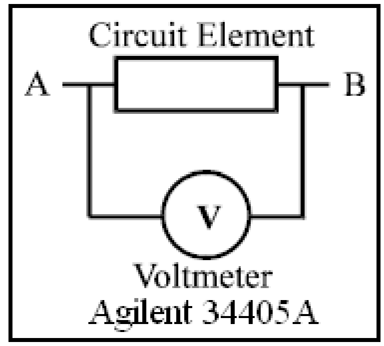
In this open response section of the lab report you have the opportunity to demonstrate that you have gained a comprehensive understanding of all aspects of the experiment. In your own analysis, what were the key elements of the experimental measurement? Are the results intuitive or do they appear in any way to be inconsistent with physical observations in daily life? Are there intrinsic aspects of either the experimental design or the way it was implemented that could introduce systematic errors or fail to account for relevant physical phenomena? A detailed discussion should include analysis of any experimental errors, instrumentation problems or mishaps that occurred, and how these may have impacted the results. Be thoughtful and think critically about these considerations. If an experiment was challenging, a discussion of exactly what made it challenging, and possibly, how it could be conducted differently, should be included. Or, if an experimental measurement went completely smoothly, this should also be discussed. Also this section may include discussion of how the insights from one particular experiment are related or complementary to other experiments conducted in the course. Remember that your discussion should be a thoughtful scientific analysis, not a discussion of how you enjoyed or did not enjoy the lab.

8 Conclusions

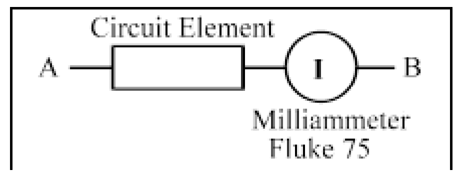
This is where your report should come full circle with your objectives you listed at the beginning. One by one, evaluate the success or failure of each objective.

Instrumentation

1. The **DC Voltmeter** measures the voltage difference between two points to which its terminals are connected. The voltmeter is always connected **in parallel** to the part of the circuit across which the potential difference is to be measured (Figure A1). In this experiment the Agilent 34405A multi-meter (multimenter) is used to measure voltage differences and the resistance of the carbon resistor.



2. The **DC Milliammeter** measures the electric current between any two points to which its terminals are connected. To measure the current in any part of the circuit, the circuit must be broken at that point and the ammeter must be inserted in the gap with loose ends connected to its terminals, i.e., the meter is connected **in series** with that part of the circuit where the current is to be measured (Figure A2). The Fluke 75 battery powered multi-meter is used to measure the current in milliamperes in this experiment.



3. Connecting wire leads. Assumed to be made of a good conducting material, e.g., a metal such as copper, and of a large enough cross-section to have negligible resistance and voltage drop across them.

4. The **DC Power supply** is a device used to supply a constant source of EMF between its output terminals. The electric current flows internally in the power supply from the minus to the plus terminal. In the external circuit it flows from the plus to the minus terminal (Figure A3). The voltmeter on the actual power supply is not accurate and should not be used to determine the output voltage. The Agilent 34405A multi-meter is used to measure voltages on the power supply.