# Capacitors in Series and Parallel

## 1 Introduction

A direct-current (DC) circuit containing a resistor and a capacitor is referred to as an *R-C* circuit. The current and voltage in this circuit is not constant, but varies as a function of time. When a potential difference $V_o$ is applied, the capacitor will charge over time. Switching off the voltage source of $V_o$ will cause the capacitor to discharge. The expressions describing the variations in the current and voltage contain exponential functions. The use of natural logarithms on these functions enables linear analysis of the relationship between the potential difference $V_c$ across the capacitor and the time.

In an *R-C* circuit multiple capacitors may be connected in either series or parallel. A parallel connection results in the summation of the individual capacitance. For a series connection, the reciprocal of the equivalent capacitance equals the sum of the reciprocals of the individual capacitances.

### 1.1 R-C Circuit

Figure 1 shows a resistor and a capacitor connected in a circuit containing a DC voltage source (output voltage $V_o$), a switch and a voltmeter used to measure the voltage across the capacitor ($V_c$).

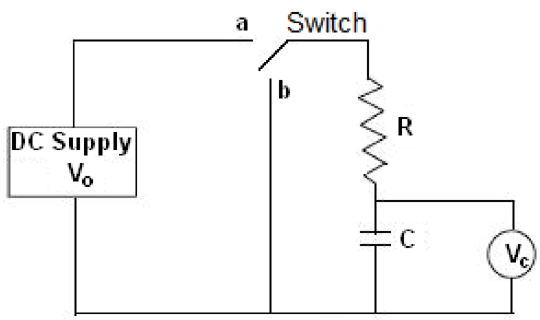



**Figure 1: An R-C Circuit**

In Figure 1, we can examine the voltage which is developed across the capacitor when the switch is closed to terminal "a" and the capacitor charges. When the switch is moved to terminal "b", the discharging of the capacitor can then be examined.

### 1.2 Capacitor Charging:

When the switch is connected to terminal "a", the resistor and the capacitor is connected to the DC voltage supply and a current I is in the circuit which serves to charge the capacitor. If we denote the voltages across the resistor and capacitor by $V_r$ and $V_c$, respectively, then from the loop theorem (Kirchhoff's voltage law) we have:

$$V_o = V_r + V_c \tag{1}$$

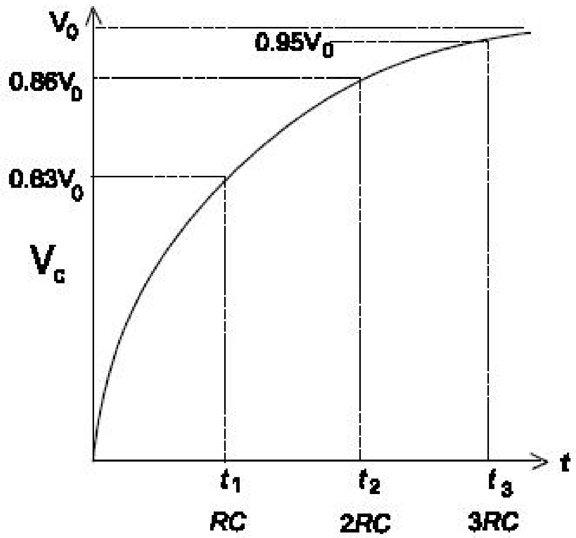
The charge $Q_c$ builds up on the capacitor exponentially:

$$Q_c = CV_o\,(1-e^{-t/RC}) \tag{2}$$

where e (= 2.718..) is the base of natural logarithms. This also leads directly to an expression for the voltage $V_c$ across the capacitor as a function of time, since

$$V_c = Q_c/C = V_o(1-e^{-t/RC}) \tag{3}$$

A plot of either $Q_c$ or $V_c$ against time will have the same general shape, as shown in Figure 2 in the case of $V_c$.




| $n$ | $t = nRC$ | $V_c = V_0(1-e^{-n})$ |
|---|---|---|
| 1 | $RC$ | $0.623V_0$ |
| 2 | $2RC$ | $0.865V_0$ |
| 3 | $3RC$ | $0.950V_0$ |
| 4 | $4RC$ | $0.982V_0$ |

**Figure 2: Voltage vs time for a charging capacitor**

Since the exponent ($-t/RC$) in Equations (2) and (3) must be dimensionless, **the product *RC* must have the dimension of time and is referred to as the *time constant* of the circuit**. If $R$ is measured in ohms and $C$ in Farads, then the product $RC$ is in units of seconds.

If we set the time $t$ equal to integer multiples of $RC$, i.e. if:

$$t = nRC, \text{ where } n = 1, 2, 3, \text{.....etc,}$$

then,

$$V_c = V_o(1-e^{-n}) \tag{4}$$

Values of the voltage $V_c$ for $n$ = 1, 2, 3 and 4 are given in the table alongside Figure 2. We see that after a time interval $t_1 = RC$ after closing the switch, the voltage on the capacitor has risen to 63% of the maximum value of $V_o$; after the interval $t_2 = 2RC$ it is 86% of $V_o$, and so on. By measuring the times taken to reach these values, the product $RC$ can be determined; and if the circuit resistance is known, the capacitance $C$ can be found.

## 1.3 Capacitor Discharging

Suppose that, after the capacitor has been fully charged to a voltage $V_c = V_o$, the switch in the circuit of Figure 1 is connected to terminal "b". With the voltage source now isolated in the circuit, and with the resistor and capacitor alone in the loop, the loop theorem yields the relationship:

$$V_r + V_c = 0 \tag{5}$$

The charge on the capacitor now leaks through the resistor at a rate given by:

$$Q = CV_o\, e^{-t/RC} \tag{6}$$

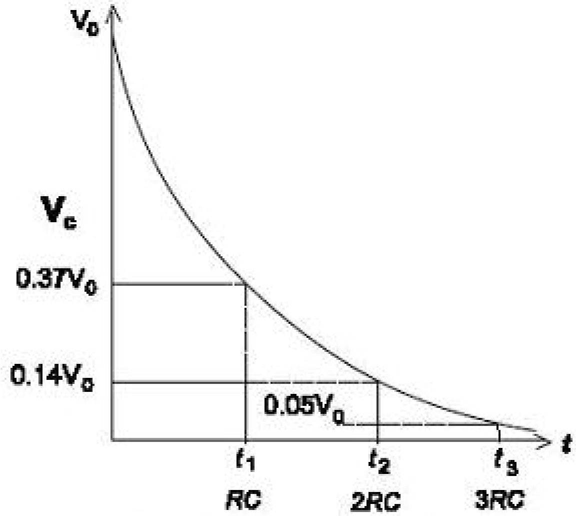
As before, we may write this equation in terms of the voltage $V_c$ across the capacitor,

$$V_c = Q/C = V_o\, e^{-t/RC} \tag{7}$$

This equation shows that the charge and voltage on the capacitor decay exponentially with time, the time-dependence having the general shape shown in Figure 3.

The rate of decay of the voltage (or charge) on the capacitor is again determined by the time constant $RC$ of the circuit, and if as before we choose time intervals which are integer multiples $n$ of $RC$ ($t = nRC$), we may write:

$$V_c = V_o\, e^{-n} \tag{8}$$




| $n$ | $t = nRC$ | $V_c = V_0e^{-n}$ |
|---|---|---|
| 1 | $RC$ | $0.368V_0$ |
| 2 | $2RC$ | $0.135V_0$ |
| 3 | $3RC$ | $0.050V_0$ |
| 4 | $4RC$ | $0.018V_0$ |

**Figure 3: Voltage vs time in a discharging capacitor**

Values of $V_c$ for $n$ = 1, 2, 3 and 4 are given in the table alongside Figure 3. We see that after a time interval $t_1 = RC$ after closing the switch to terminal "b", the voltage on the capacitor has dropped to 36.8% of its initial value $V_o$, after an interval $t_2 = 2RC$ it is just 13.5% of $V_o$, and so on. Figure 3 shows that the rate of discharge decreases as the voltage decreases, and theoretically $V_c$ never reaches zero. In practice, however, $V_c$ becomes negligibly small after a period equal to a few time constants. Note that the time constant of the circuit ($t_1 = RC$) is the time necessary for the voltage (or charge) to decay to 1/e, or $e^{-1}$, (= 0.368) of its original value $V_0$.

By choosing suitable values for resistance and capacitance, circuits may be designed having a wide range of different time constants. For example, with $R$ = 5 MΩ and $C$ = 10μF, the time constant $t_1 = RC = 5\text{x}10^6$ x $(10\text{x}10^{-6})$ = 50 seconds. If $R$ = 100Ω and $C$ = 0.1μF, then $t_1 = 100(0.1\text{x}10^{-6}) = 10\text{x}10^{-6}$ seconds, or 10μ seconds. Note that 1μF = $1\text{x}10^{-6}$F, where F stands for Farad, the SI unit of capacitance.

## 1.4 Graphical Determination of Capacitance C

The voltage $V_c$ across a capacitor as it discharges is given by $V_c = V_o\,(e^{-t/RC})$.

Our experiment will entail looking at a decaying voltage across a capacitor and then seeing how long it takes for the signal to decay a volt at a time. In other words, time is our dependent variable, so we linearize our equation with that in mind.

$$V_C = V_o e^{-\frac{t}{RC}}$$

Divide both sides by $V_o$.

$$\frac{V_C}{V_o} = e^{-\frac{t}{RC}}$$

For convenience, let's get rid of that minus sign by inverting both sides.

$$\frac{V_o}{V_C} = e^{\frac{t}{RC}}$$

Take the natural logarithm of both side.

$$ln\left(\frac{V_o}{V_C}\right) = ln\left(e^{\frac{t}{RC}}\right)$$

$$ln\left(\frac{V_o}{V_C}\right) = \frac{t}{RC} ln(e)$$

$$ln\left(\frac{V_o}{V_C}\right) = \frac{t}{RC}(1)$$

Now solve for $t$.

$$t = (RC) * \ln\left(\frac{V_o}{V_C}\right)$$

Now we can see the linearized relationship we seek. We need to perform two-dimensional linear statistics on a series of $\left(\ln\left(\frac{V_o}{V_C}\right)\ ,\ t\right)$ pairs.

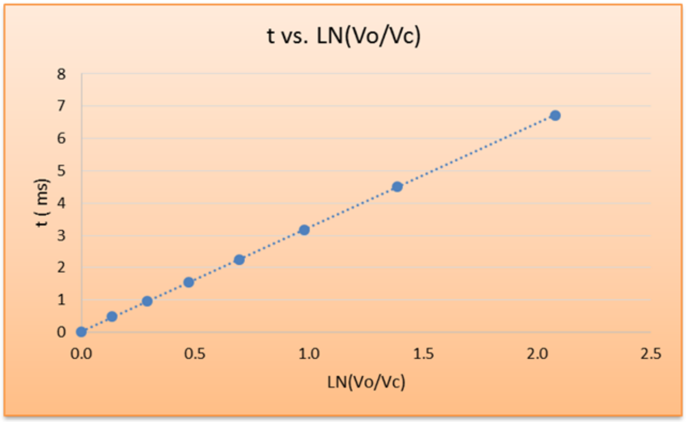



**Figure 4: Linearized relationship for time-dependent voltage across capacitor**

Therefore $slope = RC$. Using a directly measured value for R, we can experimental determine the capacitance,

$$C = \frac{slope}{R}$$

## 1.5 Determination of Capacitance C by Measuring Half-Life.

A related quantity is the ***half-life***, $t_{½}$, which is the time required for the voltage (charge) to decrease to just one-half the original value.

So starting with the decay equation,

$$V_C = V_o e^{-\frac{t}{RC}}$$

We seek the time when half of what we started with decays away.

$$\frac{1}{2}V_o = V_o e^{-\frac{t_{1/2}}{RC}}$$

Divide both sides by $V_o$.

$$\frac{1}{2} = e^{-\frac{t_{1/2}}{RC}}$$

Invert both sides.

$$2 = e^{\frac{t_{1/2}}{RC}}$$

Take the natural logarithm of both sides.

$$\ln 2 = \ln e^{\frac{t_{1/2}}{RC}}$$

$$\ln 2 = \frac{t_{1/2}}{RC}$$

Finally solving for the half-life,

$$t_{1/2} = RC \ln 2$$

$$t_{1/2} \approx RC(0.693)$$

As indicated in Figure 5, when the potential difference across the capacitor, $V_c$, is half of its initial maximum voltage $V_o$, the ***half-life time***, $t_{½}$, can be directly determined by the corresponding time under the exponential function at that point.

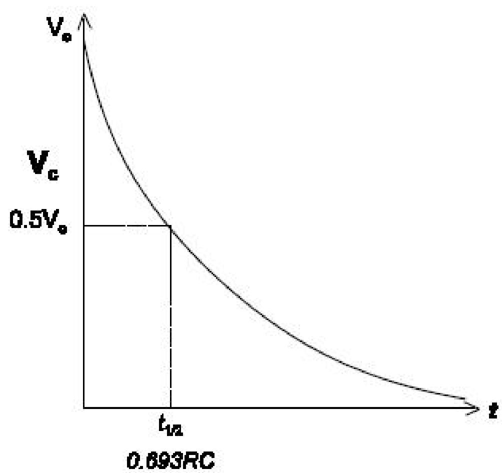



**Figure 5: Capacitor half-life**

The concept of half-life is widely known in describing radioactivity and other exponential decay processes.

## 1.6 RC Time Constant:

To measure short time constants, say < 5 ms (1 ms = 1 millisecond = $1 \times 10^{-3}$ s), it is necessary to use a voltmeter capable of responding to very rapid changes in voltage level, a timer capable of measuring down to 1μs, and a fast-acting switch which can be synchronized with the timer. A cathode ray oscilloscope, CRO, can serve both as a voltmeter and a timer for such measurements.

A function generator capable of generating a square-wave voltage output can also be used to combine the functions of both the fast-acting switch and the power supply. The output voltage of frequency *f = 1/T* from such a square wave generator is depicted in Figure 6(a). T is the time for one period of the square wave. If such a voltage is applied to a capacitor-resistor pair then the capacitor will alternately charge and discharge through the resistor, its voltage varying with time according to Figure 6(b). It is assumed in this figure that the period of the generated waveform *T* is approximately equal to 16 *RC*, so that the capacitor becomes nearly fully charged or discharged during successive half-periods. In this experiment, *T* is approximately 20 *RC* in order to make measurements of capacitors in both series and parallel connections.

If the oscilloscope is connected across the capacitor, the capacitor voltage of Figure 6(b) can be displayed on the oscilloscope. The sweep frequency of the oscilloscope time base may be adjusted so that just one full period, *T*, of the function generator output is displayed. If the time-axis is calibrated, the time constant of the circuit can be measured by measuring the time over which the voltage across the capacitor drops to 1/e value of the maximum value of the voltage across the capacitor.

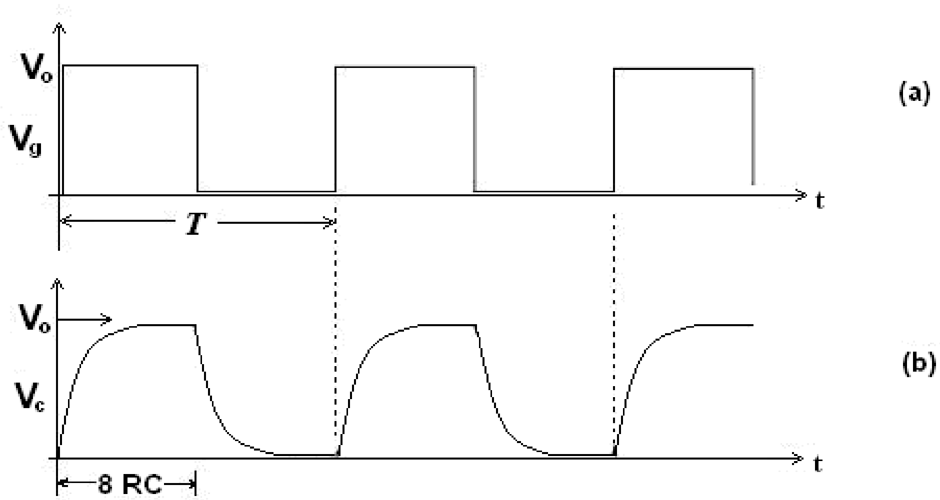



**Figure 6: (a) A square wave voltage applied to an RC circuit, and (b), the resulting capacitor voltage as a function of time, where T ~ 16RC.**

# 2 Experimental Apparatus and Procedure

## 2.1 Experimental Apparatus

1. Circuit board
2. Resistor and two 0.10μF capacitors
3. Agilent multimeter
4. Tektronics oscilloscope
5. Function generator (50 Ohm impedance)

## 2.2 Procedure

### 2.2.1

1. Directly measure and record the resistance of the resistor, $R_1$, which will be used in this experiment. Directly measure and record the value of each capacitor $C_{1,dm}$ and $C_{2,dm}$. Capacitor $C_1$ is already connected to the resistor on the circuit board. Measurements should be in μF ($10^{-6}$ Farads). Use the 1 μF range on the multimeter when measuring capacitance in this experiment.

2. Connect the two capacitors in parallel and directly measure this combined capacitance, $C_{p,dm}$. Connect the two capacitors in series and directly measure this combined capacitance, $C_{s,dm}$.

3. Using the resistor $R_1$ and the first 0.10μF capacitor $C_1$, construct the series $R$-$C$ circuit as shown in Figure 6. **Be careful not to mix up ground terminals.** The low side **( - )** of the function generator and one side of the oscilloscope terminals are grounded - thus the circuit must be connected exactly as shown in Figure 6, i.e., one side of the capacitor must be common with the ground **( - )** terminals of the scope and function generator.

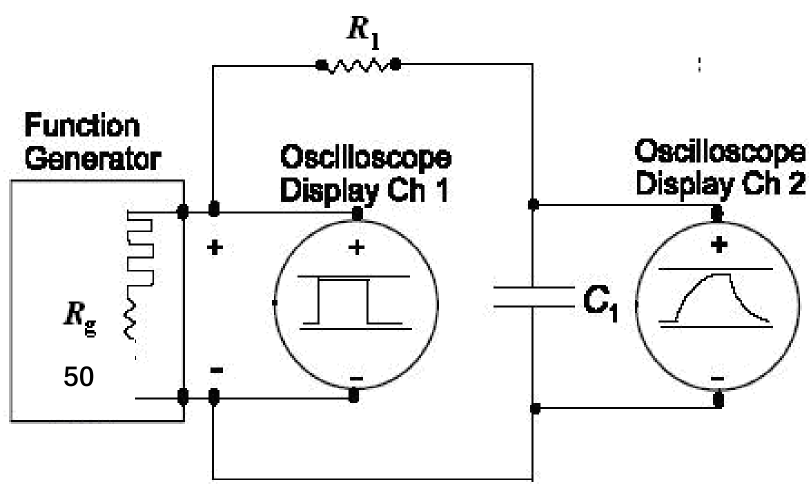
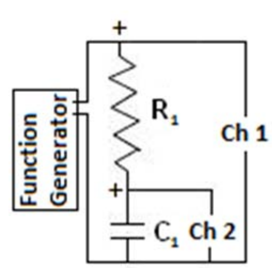



**Figure 7: Measurement of voltage and time in an RC circuit**




4. Set both Channel 1 (Ch.1) and Channel 2 (Ch.2) to **DC, 1 volt/div,** and the sweep rate to **5 msec/div.**
5. Set the function generator to a square wave function.

6. Connect the function generator to the circuit board with the high potential at the resistor $R_1$. Ch.1 of the oscilloscope is connected across the circuit (across both $R_1$ and $C_1$ or output of the function generator) and Ch. 2 of the oscilloscope is just connected across the capacitor $C_1$. Set the vertical mode to "**both**", "**chop**". Set the trigger

controls to **internal**, **Ch.1** and **Auto**.

7. Set the output of the function generator at sufficiently low signal frequency, approximately **15 Hz**, so that you observe the complete discharge of the capacitor before the next cycle of the square wave begins. The frequency range button goes to 50Hz (Hertz).

8. Adjust the amplitude of the function generator so that the square wave occupies the total height of the display on the oscilloscope. At 1 volt/div, this translates to voltage $V_o$ = 8 volts. Use the position knob to display the total vertical component of the square wave on the display.

9. The display on the oscilloscope can be stabilized by adjusting the trigger level.

### 2.2.2 Measurement of the Capacitance of a Single Capacitor

1. Using the circuit constructed in **3.1.1**, confirm that the square wave has an amplitude of $V_o$ = 8 volts, and that the top and bottom of the display corresponds to the top and bottom horizontal line of the display.

2. Display **Ch. 2 only**, and **set the sweep rate to 1 msec/div**. Adjust the slope and trigger to display the decay curve.

3. Adjust the vertical position so that the top of the display is even with the top horizontal line. The bottom of the display **will not** fall on the bottom horizontal line, as the voltage across the capacitor will not decay to zero.

4. Adjust the horizontal position so that the top of the decay curve corresponds to the first vertical line on the display. Once done, the beginning of the decay curve will have a voltage of $V_c = V_o$ = 8 volts and a time of $t$ = 0 msec. Enter this data into the data table. No further adjustments should be made until this portion of the experiment is complete**.** This positioning of the decay curve is done to compensate for the impedance of the oscilloscope which is one of the systematic errors in this experiment.

5. Record the time corresponding to point where the decay curve crosses the horizontal line representing to $V_c$ = 7 volts, and enter your data in the data table. Note that each small mark on the horizontal line corresponds to 0.2 of the horizontal sweep rate of 1 msec/div. Reducing the intensity of the display may improve your ability to estimate your reading.

6. Repeat step 5 and find the times for $V_c$ = 6 V, 5 V, 4 V, 3 V, 2V and 1V.

### 2.2.3 Measurement of the Half-Life of a Single Capacitor

1. From the data above (3.1.2), record the time corresponding to $V_c = ½ V_o$ = 4 volts, where $V_o$ = 8.00 volts. Record this half-life, $t_{\frac{1}{2},C_1,exp}$ .

### 2.2.4 Measurement of the Half-Life of Two Capacitors in Parallel and Series

1. Without disconnecting your circuit, use the small leads to connect the second capacitor, $C_2$, so that they are connected in parallel. No adjustments should be made to the function generator.

2. Adjust the position of the initial point of the decay curve so that it corresponds to 8 volts at $t$ = 0 msec.

3. Record the time corresponding to $V_c = ½ V_o$ = 4.00 volts on your data sheet. This is $t_{\frac{1}{2},p,exp}$ .

4. Repeat steps 2 and 3 with the capacitors connected in series. The circuit will have to be reconstructed to do this part. Check with your TA that the circuit is correct. Measure and record this final half-life, $t_{\frac{1}{2},s,exp}$ .

# Data Sheet

**Directly Measure the Values of Carbon Resistor and Capacitors**

$R_1$ = _______________kΩ

$C_{1,dm}$ = _______________μF    $C_{2,dm}$ = _______________μF

**Directly Measure the Values of the Parallel and Series Capacitors**

$C_{p,dm}$ = _____________ μF  Series  $C_{s,dm}$ = _____________ μF

### 2.2.2 Single Capacitor and Linear Determination of the Capacitance

$V_o$ = 8 volts, 15 Hz square wave.  Find the following times

| $t$ | $V_c$ |
|---|---|
| (ms) | (V) |
| 0 | 8 |
| | 7 |
| | 6 |
| | 5 |
| | 4 |
| | 3 |
| | 2 |
| | 1 |

**Experimentally Measured Value of the Half-Life of a Single Capacitor $C_1$**

$t_{\frac{1}{2},C_1,exp}$ = ____________ ms  (Use data from the above table.)

**Experimentally Measured Values Half-Life of Parallel and Series Equivalent Capacitance**

$t_{\frac{1}{2},p,exp}$ = _____________ ms    $t_{\frac{1}{2},s,exp}$ = _____________ ms

# 3 Calculations, Analysis and Graphs.

## 3.1 Analysis of Capacitances in Parallel and Series

1) We assembled two capacitors in a parallel network and directly measured their combined capacitance. What does the theory claim of such a union? It claims this combined capacitance should be the sum of the two individual capacitors.

$$C_{p,th} = C_{1,dm} + C_{2,dm}$$

Find that and check this claim against what was directly measured.

$$\%\, diff_p = \left|\frac{C_{p,dm} - C_{p,th}}{C_{p,th}}\right| * 100$$

2) When we have two capacitors in series, we calculate our theoretically predicted capacitance as the inverse of the sum of each:

$$\frac{1}{C_{s,th}} = \frac{1}{C_{1,dm}} + \frac{1}{C_{2,dm}}$$

Giving us our theoretical calculation,

$$C_{s,th} = \left[\frac{1}{C_{1,dm}} + \frac{1}{C_{2,dm}}\right]^{-1}$$

Check this against what was directly measured.

$$\%\, diff_s = \left|\frac{C_{s,dm} - C_{s,th}}{C_{s,th}}\right| * 100$$

## 3.2 Graphical Determination of Capacitance

1) There is a 50Ω impendence in the function generator. So let R be the resistance we use for the rest of our calculations where

$$R = R_1 + R_g$$
$$R = R_1 + 0.05\ k\Omega$$

2) Enter in $V_o = 8\ V$, followed by the decaying voltages measured across the capacitor, $V_c$, and the measured times, $t$, each volt decayed.

3) Now we need our two columns we will perform 2D Stats on. The independent variable is $\ln\left(\frac{V_o}{V_C}\right)$ and the independent variable is the times.

4) Graph and perform 2D Stats on your eight $\left(\ln\left(\frac{V_o}{V_C}\right)\ ,\ t\right)$ pairs.

5) Now you are ready to find your experimental value for the capacitor in your RC circuit.

$$C_{1,exp} = \frac{slope}{R}$$

Treat $R$ as a constant, having comparatively negligible imprecision.

$$S_{C_{1,exp}} = C_{1,exp}\sqrt{\left(\frac{S_{slope}}{slope}\right)^2} \Rightarrow \frac{S_{slope}}{R}$$

Test your accuracy against what was directly measured.

$$\%\, diff_{C_1} = \left|\frac{C_{1,exp} - C_{1,dm}}{C_{1,dm}}\right| * 100$$

### 3.3 Half-Lives

1) Using directly measured values, predict the theoretical half-life of the voltage across $C_1$.

$$t_{\frac{1}{2},C_1,th} = R * C_{1,dm} * \ln 2$$

2) Note that this experimentally measured half-life, $t_{\frac{1}{2},C_1,exp}$, was a single measurement of time. Conveniently, we already have a best estimate of this precision, which is $S_t$ from the previous 2D Stats. Now all that is left is testing its accuracy against what was theoretically predicted,

$$\%\, diff_{t_{\frac{1}{2},C_1}} = \left|\frac{t_{\frac{1}{2},C_1,exp} - t_{\frac{1}{2},C_1,th}}{t_{\frac{1}{2},C_1,th}}\right| * 100$$

3) Do the same for the parallel and series half-lives.

## 4 Questions

1. Use Equation (7) to calculate the voltage across the capacitor at time $t = RC$. This yields $e^{-1}$. Use your experimental value of $V_o$. Now compare to your data table. Which of your measured data points comes closest to this value? (Note the voltage and time)

2. Could we have ignored the $R_g = 50\ \Omega$ resistance in the function generator? Use percent difference with $R_1$ as the accepted value, $R$ as the experimental value. Assume that a systematic error from this source can be ignored if it is less than 5%.

3. From $RC = t / \ln (V_o / V_c)$, calculate the time constant $RC$ for the circuit by using:

   $$RC = slope$$

4. Which combination would have the largest equivalent capacitance, two 0.10μF capacitors in parallel or two in series? Show your calculations.

## 5 Discussion

As always, remember to include a discussion section in your lab report. Refer to the Appendix on writing lab reports in the Course Syllabus for guidelines and suggestions for this section.