Purpose

In this experiment we will explore the relationship between an applied force and the elongation of a spring. We will determine the spring constant, *k*, in the linear regime.

The goal of this experiment is to:

- Determine the spring constants and relating displacement

Materials and Equipment

- Spring-mass system
- Assorted standard masses

Theory

A body at rest has no net force acting on it, all the forces applied to it sum to zero.

We use this principal to measure an unknown force by balancing it with a known one. In this case, we will use the force of gravity acting on a known mass to determine the force exerted by a spring extended away from its neutral position.

The relationship between the magnitude of the force, *F*, applied by the spring against whatever is extending by a certain amount, *r*, away from its equilibrium position, r_e, is known as Hooke's law, and is given by the following expression:

$$\vec{F} = -k\Delta\vec{r}$$

The negative sign indicates that the spring force is a restoring force, that is, the spring force opposes the direction of displacement.

The spring constant, *k*, is dependent on the spring itself and indicates how easy difficult it is to stretch or compress the spring.

Consider the spring used in the ballistic pendulum apparatus used earlier this semester. This spring has a relatively large spring constant, indicating a large amount of stiffness. The ballistic pendulum apparatus uses this property to store energy in the compressed spring, and when the trigger is pulled, transfer that energy to the steel projectile.

Another common spring is the Slinky. This toy has a spring constant that is much lower than the spring used in the ballistic pendulum apparatus. It takes comparatively much less force to move a Slinky away from its equilibrium position.

Apparatus

This experiment will measure the spring constant of a spring. We will add mass to the spring causing it to stretch. A r = 0 cm the spring is at equilibrium. We add a known mass, allow the spring to reach a new equilibrium position, and measure the elongation. Since the system is at equilibrium we know the net force acting must be zero:

$$\overrightarrow{F_{net}} = \sum_k \overrightarrow{F_k} = m\vec{a}$$

Since we are looking at this in equilibrium, $\vec{a} = 0$.

$$\sum_i \vec{F_i} = \mathbf{0}$$

Therefore, the force of gravity much be balanced exactly by the force exerted by the spring.

$$F_g + F_k = 0$$

Here, we choose a radial coordinate system along the long axis of spring. The force of gravity pulls in the positive direction, the spring force resists, pulling in the negative direction. We can rewrite our net force equation using this coordinate system:

$$F_g(+\hat{r}) + F_k(-\hat{r}) = 0$$

$$(F_g - F_k)\hat{r} = 0$$

$$\therefore \ |F_g| = |F_k|$$

The force due to gravity is calculated easily, and the spring force is dependent on the displacement from equilibrium:

$$W = k\Delta r$$

$$W = k(r - r_e)$$

Here r_e is the equilibrium position, and r the measured elongation. Note, because the mass does not start at the axis of rotation the initial $r_e \neq 0$.

In order to find a value for the spring constant, *k*, in the first part of this experiment we will measure elongation caused by various masses. We will then use a linearized form of the above relationship and a plot of the data to determine *k*.

$$W = k(r - r_e)$$

$$\frac{1}{k}W = (r - r_e)$$

$$\boxed{\frac{1}{k}W + r_e = r}$$

If the spring constant is truly constant, then graphing (*W, r*) pairs should result in a linear plot with a slope of k^{-1} and intercept value equal to r_e.

However, we do expect some of the data will be non-linear. In general Hooke's Law does not hold for all elongations of a spring. In the unstretched position the coils of the spring are in contact with each other, resisting further contraction. Also, if a spring is stretched too far the material can lose its elasticity. Hooke's Law applies between these two extremes. We are interested in using data in this linear regime to calculate a value for *k* where it is, in fact, constant. You may need to decide to omit non-linear data points on you graph.

Procedure

Record the elongation of the spring for each mass attached to the system and tabulate the data.

Analysis

1. Compute the applied force for each value of mass recorded. ($W = mg$ with $g = 9.8035$ m/s^2) Be sure to convert your masses to kilograms.
2. Convert elongations to meters.
3. Run the 2D Stats macro on the (W, r) pairs. Note the value of R^2.
4. Copy the data to a new set of columns, and begin removing early data points, note how the value of R^2 changes. Use this to determine the data points that lie in the linear regime.
5. Obtain an experimental value for the spring constant, *k1* and its uncertainty.
6. Using this spring constant obtain the mass (μ) of the unknown object.
7. Using this mass, obtain the spring constant (*k*2) for the second spring.

Questions

1. (5 points) For part one, did you remove any points from the final statistical analysis you used to determine $k1$? How did you decide whether or not to remove a point?

2. (5 points) Find $r_e \pm S_{re}$

3. (10 points) After determining values for $k1$ and r_e you are able to solve for the unknown mass (μ) using the equation: $r_\mu = \frac{1}{k1} \cdot W_\mu$ where $W_\mu = \mu g$. Starting with this equation (rearranged to solve for μ) derive a formula for the error (S_μ) by propagating the errors from $k1$ and r_e. For the error in a single length reading (S_{r_μ}) you can use the standard deviation of the dependent variable in your 2D Stats which is labeled S_y.

4. (10 points) Using the value for the unknown mass (μ) and your reading for the elongation of the second spring (r_{k2}) it is possible to calculate the spring constant for this new spring ($k2$) by assuming the equilibrium length (r_e) is the same for both springs. Calculate the error for this new spring constant (S_{k2}) from S_μ , S_{r_e} , and $S_{r_{k2}}$ where, as before, you can use the standard deviation of the dependent variable in your 2D Stats (S_y) as the value for $S_{r_{k2}}$.