Experiment 1: Data Analysis and Graphing using Excel (DV Stats)

Purpose:

Before we start measuring physical quantities we want to spend a little time developing techniques for handling the numbers we will be generating. To that end this is a computer-based exercise in the fitting of non-linear data. The purpose of this Physics 182 experiment is to use Excel in the lab and a calculator for your report to analyze data that have linear and non-linear functional forms. You will plot and analyze graphs (using Excel) to validate theory and also use Excel statistical calculations, including linear regression. Your lab report should not have an Excel graph.

Introduction:

When we make a measurement we want to be able to connect the quantities measured to the mathematical theories that we are trying to test. Usually the quantities we can measure; mass, speed, time, etc., are not the quantities that we are interested in knowing. But, they can be connected to our desired quantities by a theoretical model. Every measurement has associated with it intrinsic errors which distort what we measure from the theoretical ideal. One of the strongest tools we have for understanding how to extract the quantities of interest out of the random noise of these errors is Statistical Analysis. For a simple measurement of a single quantity we arrive at an experimental value by making several measurements and calculating their statistical mean value. At the same time we can also calculate the standard deviation from our data set and use it to derive the standard deviation of the mean which is considered the standard error on the measurement. Sometimes our measurements involve a relationship between something we measure, the dependent quantity, and something we control, the independent quantity or control parameter. In this case simple statistics cannot help us. If there is a linear relationship between dependant and independent variables then there is a more sophisticated statistical protocol, Linear Regression, which can give us an exact determination of the linear equation that describes the connection. For a non-linear relationship the situation is less clear cut.

In general if we are trying to fit a non-linear relationship between dependent and independent variables we use what is called simplectic minimization. Unfortunately, this technique does not always provide a solution. The best way around this problem is to manipulate the data so that the relationship between control parameter and some function of the measured quantity fits a linear equation. For example, let us take the exponential decay of the amount of a drug present in a blood stream as a function of time after its administration. The concentration of drug, C, has a time dependence that takes the functional form,

$$C = Ae^{\frac{-t}{\lambda}}$$

where A is the initial concentration and λ is what is known as the decay time. If we measure C at several times we generate a set of data that should fit this functional form. To check this, using linear regression we would plot the natural logarithm of the concentration, Ln(C), as a function of time. Taking the natural logarithm of both sides of the above equation we get,

$$Ln(C) = Ln(Ae^{\frac{-t}{\lambda}}).$$

Which can be rewritten using the properties of logarithms to take the form?

$$Ln(C) = Ln(A) + Ln(e^{\frac{-t}{\lambda}}).$$

And then,

$$Ln(C) = Ln(A) - \frac{t}{\lambda}Ln(e) = -\frac{t}{\lambda} + Ln(A).$$

The above is now a linear expression of the form Y = mx + b. So if we plot the natural logarithm of our concentration (Y = *Ln(C)*) as a function of time (x = *t*) and perform linear regression on the resulting time series the intercept (b) that comes out of the calculation will be equal to the natural logarithm of the initial concentration. And the slope (m) will be equal to the negative reciprocal of the decay time,

$$b = Ln(A)$$

$$m = \frac{-1}{\lambda}$$

Thus, by performing linear regression on our linearized data we can determine the two quantities that are important in our measurement.

Students should study the appendix on error analysis in the supplementary material entitled **"Error Analysis"** which is on blackboard site. You will need this appendix to answer questions for this experiment. Throughout this course you will be required to prepare various graphs using Excel. You will be provided with information regarding the data required to be plotted for each experiment.

In addition to graphing Excel is our primary source of data entry and statistical analysis. You will be required to have a working copy of Microsoft Excel when you work on labs and you will be required to work in the **DV Stats** workbook for each experiment. This workbook contains pre-compiled code that can perform **1D** and **2D Statistics** which will be required as part of the analysis of most experiments. Depending on the experiment being performed there may be a specifically prepared workbook that has both the macro commands as well as predefined formatting and/or data. It is your responsibility to check Blackboard each week and download any files that are posted in the folder for each experiment. If there are any questions as to what you will need for each experiment you should contact your TA as soon as possible.

Theory:

The first data set (**Part A**) for this Excel experiment uses the Inverse Square Law of Light:

$$\boldsymbol{I} = \frac{P}{r^2}$$ **Equation (1)**

where I is the intensity measured at a distance r from the source (light), and P is the total power emitted by the source (light). The inverse square law states that the measured intensity of the light is proportional to the inverse square of the distance from the source.

The second data set (**Part B**) for this Excel experiment uses Malus's Law which states that the relative intensity of light that passes through two polarizes is proportional to the square of the cosine of the angle between the polarization planes. Malus's Law is given as:

$$I_\theta = I_0 \cos^2\theta$$ **Equation (2)**

where I_0 is the intensity of the light exiting the first polarizer, and I_θ is the intensity of this light exiting the second polarizer which is rotated at an angle θ with respect to the first polarizer.

Both sets of data will result in linear and non-linear graphs in this Excel experiment. A linear graph will have the general form of Y = mx + b, where Y is the dependent variable, m is the slope, x is the independent variable and b the intercept.

Once again, students should study the summary on error analysis to attain information on the process of averaging (statistical mean), determining the standard deviation and calculating the standard deviation of the mean. Please note that in future experiments, theory information is embedded in the write-ups (located on the Web) and they will not have a separate theory section as this experiment presents. If you do a theory section for your report, you access the information through your reading. Your Introductory Physics textbooks are a great source of information as well.

Experimental Apparatus / Procedure + Data:

a) Experimental Apparatus

In most experiments a set of measurements will need to be made prior to analysis: mass, temperature, current, voltage, etc. Microsoft Excel will be your primary means of data entry and analysis. In some cases, measurements will have been made for you with the resulting data then provided to you. Such is the case for this lab where multiple sets of data are provided. In every case you are responsible for understanding how the data was obtained and then what steps are involved to analyze it and then calculate the desired results.

b) Experimental Procedure

This experiment will use two sets of data. One set for the inverse square law and a second set for Malus's Law. You will use Excel to analyze each of these data sets both in class as well as for your lab report.

Inverse Square Law (Part A)

The following distance r and intensity I data (Figure 1) will be used for this part of the computer experiment. Your Excel sheet, when used, should resemble Figure 1 which contains this data. Note that all distance data was collected with a precision of one hundredth of a centimeter, or, two places after the decimal point. For your report, and information you are required to enter into your data section, the error associated with the measurement is $\pm \delta x = 0.05$ cm (centimeters) for the distance r data. Enter this value into your data section. For your lab report, feel free to generate tables using a computer, and Excel if you have it, and pasting them into the report.

	A	B
1	***r***	***I***
2	**(cm)**	**(lux)**
3	7.32	249.76
4	10.20	134.63
5	20.12	34.10
6	28.95	8.64
7	59.45	3.87
8	70.00	2.85
9	80.33	2.19
10	89.98	1.73

Figure 1. Distance *r* and intensity *I* measurements entered in Excel.

Malus's Law (Part B)

The following degree θ and intensity I data (Figure 2) will be used for the second part of the computer experiment. Your Excel sheet, when used, should resemble Figure 2 which contains this data. For this part of the experiment, the random error is the positioning the polarizer. This error, in the values for degree θ, is given as is $\pm \delta\theta = 0.05$ degrees. Enter this value into your data section.

	A	B
1	**θ**	**I_θ**
2	**Degrees**	**(lux)**
3	0.00	24.45
4	10.00	23.71
5	20.00	21.59
6	30.00	18.34
7	40.00	14.35
8	50.00	10.10
9	60.00	6.11
10	70.00	2.86
11	80.00	0.74
12	90.00	0.00

Figure 2. Degrees θ and intensity I_θ measurements entered in Excel.

The data presented above should be entered into your lab report in tables similar to how it is shown. All data has two numbers after the decimal point. Enter errors in

measurements, δx and $\delta\theta$, **in this section of your report.** The digital meter may have a systematic error as nothing is perfectly calibrated. Include this comment in your analysis section.

Calculations and Graph:

Part A:

1. In Excel enter your data for Inverse Square Law data. Your Excel sheet should reflect Figure 1.

2. The first thing we want to do is investigate a possible error in the measurements. This can be done by graphing in Excel a linear representation of the data. To convert Equation (1) to a linear expression, use the following equation:

$$r = \frac{\sqrt{P}}{\sqrt{I}} \qquad \textbf{Equation (3)}$$

In Excel, you now need to add a column for the inverse of the square root of the intensity I, or, $(I)^{-.5}$. Note that $(I)^{-.5}$ is the same as $\frac{1}{\sqrt{I}}$. So too is "$= \frac{1}{\mathrm{SQRT}(I)}$" entered in Excel. The following Figure explains this process.

Clipboard | Font

C3 f_x =B3^-0.5

	A	B	C	D	E
1	r	I	$(I)^{-.5}$		
2	(cm)	(lux)	$(\text{lux})^{-.5}$		
3	7.32	249.76	0.06328		
4	10.20	134.63	0.08618		
5	20.12	34.10	0.17125		
6	28.95	8.64	0.34021		
7	59.45	3.87	0.50833		
8	70.00	2.85	0.59235		
9	80.33	2.19	0.67574		
10	89.98	1.73	0.76029		

Figure 3. Using Excel to calculate the inverse of the square root of intensity *I*.

Now, in Excel, graph r vs $(I)^{-.5}$, where r is the dependent variable Y in the general linear expression Y = mx + b. Here m is the slope, x = $(I)^{-.5}$ is the independent variable and b is the intercept. See the calculator help sheet for additional information on this. **Your Excel graph should be linear, as shown in the following Figure.**

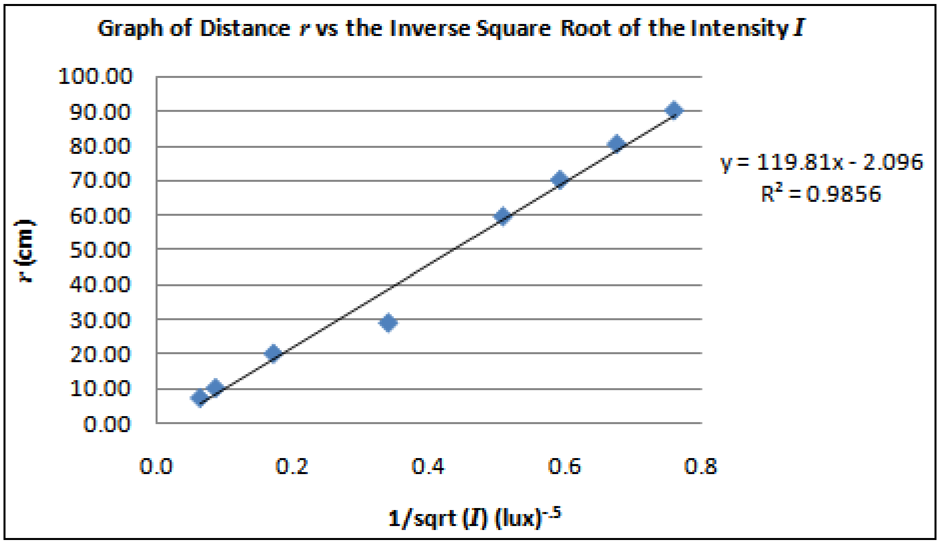

Figure 4. Graph of distance *r* vs the inverse square root of intensity *I*.

In the above Figure, we see that the coefficient of determination is 0.9856. The closer this value is to $R^2 = 1.0000$, the more linear your data. Inspection of this graph shows that one data point appears not to be linear with the rest. This data point should be removed. The following Figure is the same graph with the "non-linear" point removed. **You should do this graph (Figure 5) using Excel for this experiment.**

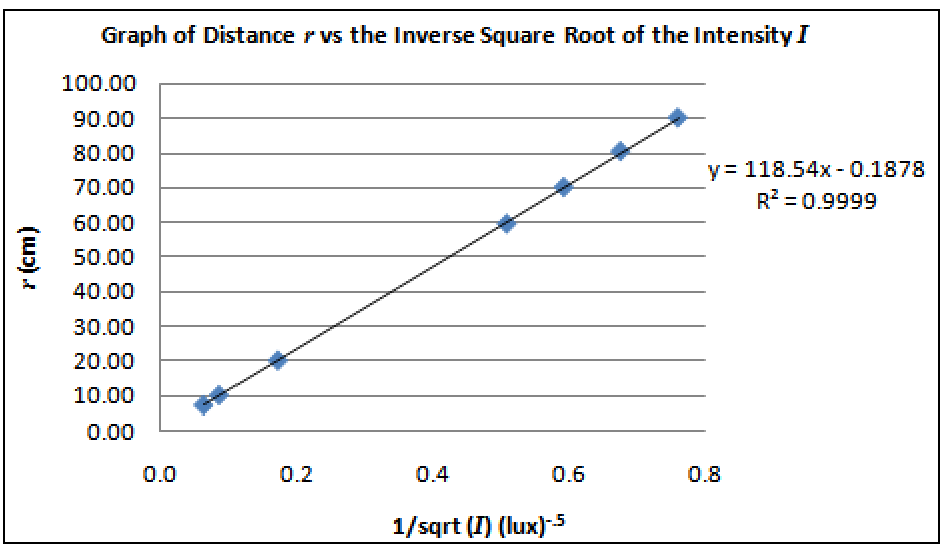

Figure 5. Graph of distance *r* vs the inverse square root of intensity *I* with the

one data point, which appears to fail to be linear, removed.

The coefficient of determination in Figure 5 is now 0.9999, which indicates data that is very linear, and this suggest no large errors in their measurement. Make sure that R^2 is always entered with four numbers after the decimal point.

3. **Use Excel to graph Intensity *I* vs. Distance *r* using data in Figure 1. When you do this graph, be sure it is done with the "non-linear" data point (28.95, 8.64) removed.** When you format the trend line, use the power function and not the linear one. Your graph should reflect the following.

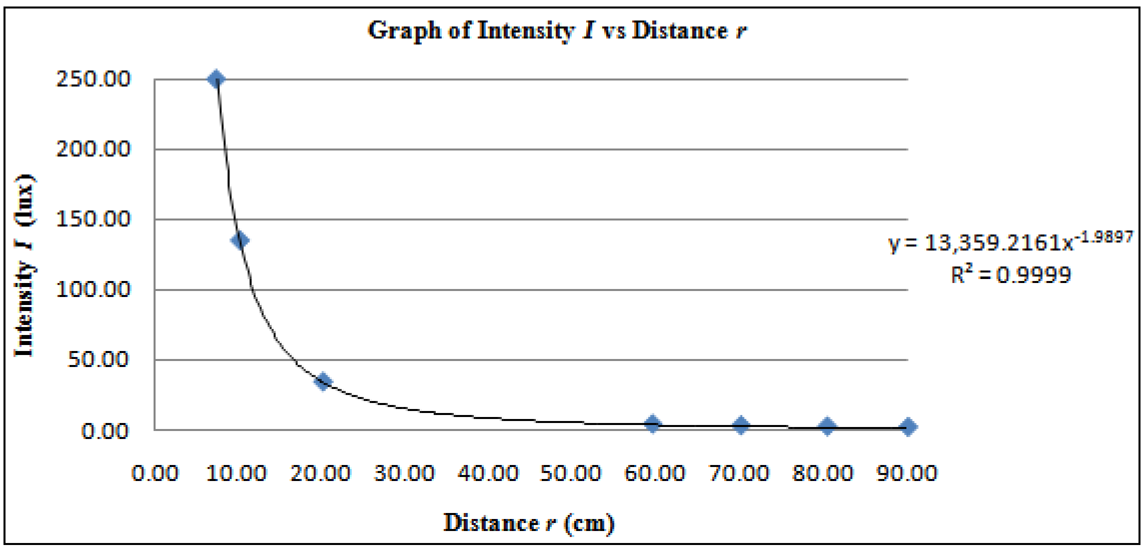

Figure 6. Graph of intensity *I* vs distance *r*.

Using Figure 6, we have Y = 13,359.2161x $^{\mathbf{-1.9897}}$. Applying this to Equation (1), we have:

$$I = \frac{P}{r^{1.9897}} \quad \text{where } P = 13{,}359.2161 \text{ (lux-cm}^2\text{)}$$

or

$$I = \frac{P}{r^{1.9897}} \quad \text{where } P = 1.3359 \times 10^4 \text{ (lux-cm}^2\text{)}$$

The inverse square law states that the measured intensity of the light is proportional to the inverse square of the distance ($1/r^{\mathbf{2}}$) from the source. Our result of $1/r^{\mathbf{1.9897}}$ indicates a strong experimental validation of this inverse square law. Note that R^2, the coefficient of determination, are identical in Figures 5 and 6. Our value of $x^{\mathbf{-1.9897}}$ would be different if we did not remove the "non-linear" data. See Figure below.

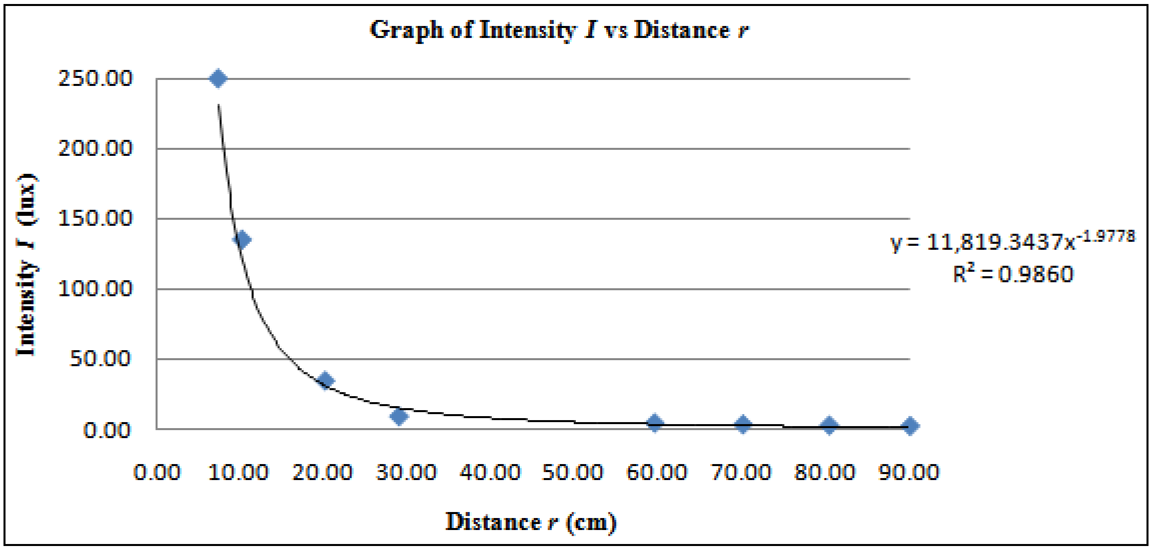

Figure 7. Graph of intensity *I* vs distance *r* with the "Bad" data point included.

Including the "non-linear" data (Figure 7) results in a value with a larger departure from r^2. The power P is less too. Note the similarity of the coefficient of determination in Figures 4 and 7.

When you enter a result, as 13,359.2161, it is best if your final number uses scientific notation to yield 1.3359×10^4 then enter 1.34×10^4. Subsequent calculations should use either 13,359.2161 or 1.3359×10^4 to reduce rounding off errors.

4. **<u>In your lab report</u>**, indicate the percent error between the accepted value (r^2) and the experimental value (**$r^{1.9897}$**), where percent error is given by:

$$\% \text{ error} = \left(\frac{|\text{experimental} - \text{accepted}|}{|\text{accepted}|}\right)*100\%, \text{ so, } \% \text{ error} = \left(\frac{|1.9897 - 2|}{|2|}\right)*100\%$$

From the above expression, use of the absolute value | | results in a difference that is always positive. The percent error is a useful way to measure accuracy.

<u>Part B:</u>

5. In Excel enter your data for Malus Law data. Your Excel sheet should reflect Figure 2.

6. **You will do a graph in Excel of I_θ vs. $\cos^2\theta$**, so you need to, in Excel; create a column of $\cos^2\theta$ data. First, you need to convert degrees to radians, as shown in

Figure 8 below. Once this is done, you will do a column of $\cos^2\theta$ using θ in radians. This is in Figure 9.

C3 | =A3*(PI()/180)

	A	B	C
1	θ	I_θ	θ
2	Degrees	(lux)	Radians
3	0.00	24.45	0.00000
4	10.00	23.71	0.17453
5	20.00	21.59	0.34907
6	30.00	18.34	0.52360
7	40.00	14.35	0.69813
8	50.00	10.10	0.87266
9	60.00	6.11	1.04720
10	70.00	2.86	1.22173
11	80.00	0.74	1.39626
12	90.00	0.00	1.57080

Figure 8. Converting degrees to radians.

D3 | =COS(C3)^2

	A	B	C	D
1	θ	I_θ	θ	$\cos^2\theta$
2	Degrees	(lux)	Radians	
3	0.00	24.45	0.00000	1.0000
4	10.00	23.71	0.17453	0.9698
5	20.00	21.59	0.34907	0.8830
6	30.00	18.34	0.52360	0.7500
7	40.00	14.35	0.69813	0.5868
8	50.00	10.10	0.87266	0.4132
9	60.00	6.11	1.04720	0.2500
10	70.00	2.86	1.22173	0.1170
11	80.00	0.74	1.39626	0.0302
12	90.00	0.00	1.57080	0.0000

Figure 9. Using Excel to calculate $\cos^2\theta$, where θ is in radians.

Please note that in Excel, π is PI(), so when converting degrees to radians, you multiple the degrees by PI()/180. Radians to degrees uses 180/PI(). In Excel, you may too use "=RADIANS(40)" to convert 40 degrees to radians.

7. From Equation (2), and Y = mx + b, we see that x = $\cos^2\theta$ is linear with respect to Y = I_θ. In Equation (2), the intercept b is zero. **In Excel, do a graph of I_θ vs $\cos^2\theta$. Your Excel graph should reflect the one in Figure 10 below.**

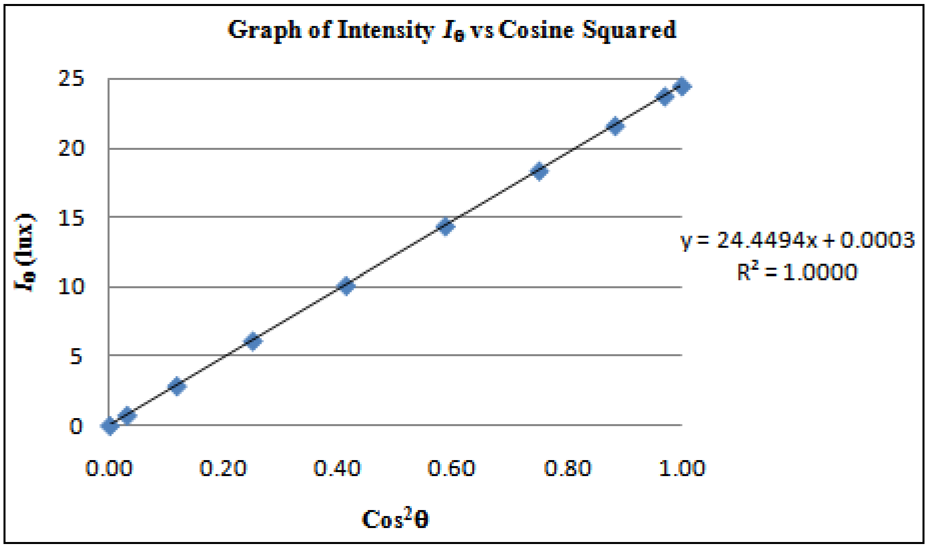

Figure 10. Graph of intensity I_θ vs $\cos^2\theta$.

8. The following is a graph of the data from Figure 2. Note the non-linear relationship.

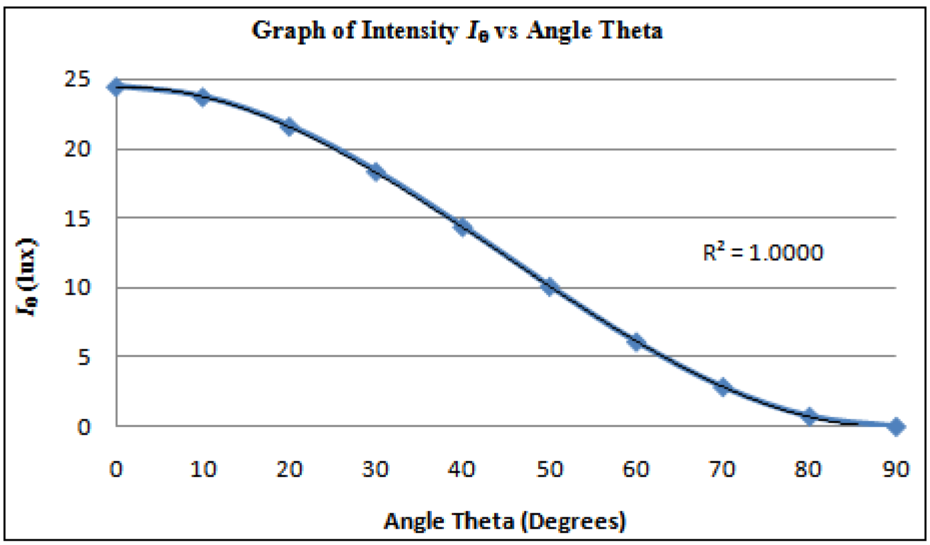

Figure 11. Graph of intensity I_θ vs Angle Theta.

9. **<u>For your lab report</u>, you are required to use 2D stats in Excel to perform a linear regression analysis to find the slope, intercept and coefficient of determination (R^2) for I_θ vs $\cos^2\theta$ data as in Figure 10.** Note in Figure 10 that the intercept approaches the value of zero as reflected in Equation (2).

<u>Caution</u>. When using Excel all trigonometric functions accept inputs only in radians. You can convert from degrees to radians using the built-in function =RADIANS(…) Similarly the results of all trigonometric functions are given in radians which can be converted to degrees using =DEGREES(…) as needed. A good check for degrees vs radians is by ensuring that $\cos 45^o = 0.7071 = \sin 45^o$.

Discussion:

In this open response section of the lab report you have the opportunity to demonstrate that you have gained a comprehensive understanding of all aspects of the experiment. In your own analysis, what were the key elements of the experimental measurement? Are the results intuitive or do they appear in any way to be inconsistent with physical observations in daily life? Are there intrinsic aspects of either the experimental design or the way it was implemented that could introduce systematic errors or fail to account for relevant physical phenomena? A detailed discussion should include analysis of any experimental errors, instrumentation problems or mishaps that occurred, and how these may have impacted the results. Be thoughtful and think critically about these considerations. If an experiment was challenging, a discussion of exactly what made it challenging, and possibly, how it could be conducted differently, should be included. Or, if an experimental measurement went completely smoothly, this should also be discussed. Also this section may include discussion of how the insights from one particular experiment are related or complementary to other experiments conducted in the course. Remember that your discussion should be a thoughtful scientific analysis, not a discussion of how you enjoyed or did not enjoy the lab.

Conclusion:

The report should end with a clear conclusion statement. This is the "bottom-line" experimental result summarizing the main quantitative results of the experiment and the extent to which they are in agreement with theoretical predication and/or an established reference value. When the experiment results in a measurement of a constant (e.g., the acceleration due to gravity at the earth's surface), compare it with its established handbook values for the Boston area. Use percent error to quantify this comparison. To make this comparison meaningful, you should include the impact of the experimental error (random, systematic and any individual investigator mistakes) on your results. This includes errors in plotting and reading linear graphs when determining their slope and intercept.

******* It is advised that you work on reports as soon as possible to allow time to ask your TA for clarification as needed.

Questions (Submit at the end of your report)

1. Both experiments above used a photometer to measure light intensity. What type of error would we have if the photometer was not properly calibrated? Use the Appendix on Error Analysis on Blackboard.

2. Use Equation (1), with P = 13,359.2161(lux-cm^2) and $r^{1.9897}$ to calculate the intensity I at r = 28.95 cm. Use percent error, with the value calculated as the accepted, and indicate the percent error between the calculated intensity and the experimental intensity given in Figure 1 for r = 28.95 cm. **Always show formulas and calculations.**

3. Repeat question number two above with r = 20.12 cm.

4. Give a possible explanation why the percent error is higher in question 2 compared to question 3? Use Figure 4 as a guide in answering this.

5. Use Equation (2), with I_o = 24.45 lux (Figure 10) to calculate I_θ at 30^o. **Ignore the intercept value b in Figure 10 as it is very close to zero.**

6. Does your calculation in question 5 indicate that the data in Figure 2 may yield experimental validation of Equation (2)? Explain your answer.

7. By comparing graphs and trend line data from Figures 7 & 11, what statement would you make regarding the possible presence of any large measurement errors in the data collected for Malus's Law?

8. Using Excel calculate the mean, standard deviation and standard deviation of the mean for the following data. You should use 1D Stats to answer this question (Ctrl+Shift+D): The Appendix on Error Analysis may also be of help.

   Data for statistical calculations: 20.12cm, 20.15cm, 20.18cm, 20.08cm, 20.10cm