Conservation of Momentum in One Dimension

Purpose

The purpose of this experiment is to verify the conservation of momentum in an inelastic collision. We will make independent measurements of the momentum both before, and after an inelastic impact between two masses. The initial momentum will be calculated by measuring distance travelled in a parabolic trajectory under the influence of gravity. The final momentum will be calculated by measuring the maximum height reached by a massive pendulum and the law of conservation of energy.

Materials and Equipment

- Blackwood ballistic pendulum
- Plumb bob
- 30 cm ruler
- 2-meter stick
- Combination square
- Mass balance

Theory

The momentum of an object is defined as the product of its mass times its velocity, $\boldsymbol{P} = m\boldsymbol{v}$. Mass is a scalar quantity (magnitude, but no direction) and velocity is a vector quantity (both magnitude *and* direction). The product of mass and velocity, therefore, will also be a vector quantity. The momentum magnitude, p, and momentum vector, $\boldsymbol{p}$, have the following relationship:

$$p = |\boldsymbol{p}| = m|\boldsymbol{v}| = mv$$

Any system of masses that is isolated from external forces will have a net momentum which is conserved. This conservation of momentum holds for the system as a whole no matter how the masses interact. This conservation of momentum can be written as the equation:

$$\boldsymbol{P}_i = \boldsymbol{P}_f$$

In this experiment we will determine the initial and final momentum of a projectile pendulum system, consisting of a projectile given an initial velocity by a spring-loaded launcher, and a massive pendulum designed to catch and hold the projectile when they collide.

The initial momentum will be determined by measuring the mass of the projectile, and the distance travelled by the projectile under the influence of gravity, which will allow the calculate of the projectile's initial velocity. We will measure the final momentum of the projectile-pendulum system by leveraging the conservation of energy. We will determine the change in potential gravitational energy which will allow the calculation of the velocity of the combined projectile-pendulum system.

We will then compare the initial and final momentum of the system, which should be equal in an isolated system.

Apparatus

The apparatus for this experiment, shown in Fig. 1, consists of a steel ball and a pendulum.

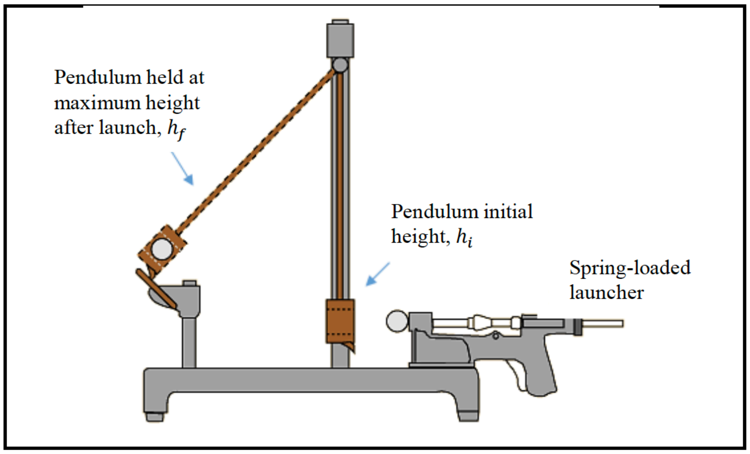

Figure 1: Blackwood ballistic pendulum apparatus with spring launcher

In the first part of the experiment, the pendulum is held out of the way so that it does not interfere with the projectile. The projectile is launched at an initial velocity which will be determined from a measurement of its initial height above the floor, y, and the horizontal range, x, to the point of impact. The momentum of the projectile before the collision is this velocity, v_i, times its mass, m. The velocity of the stationary pendulum before the collision with the moving projectile is zero as it is not moving, thus its momentum is also zero. Therefore, the initial momentum, p_i, of the projectile-pendulum system before the collision is just the momentum of the projectile.

$$p_i = mv_i$$

In the second part of the experiment, the projectile is launched horizontally and embedded in the pendulum. The pendulum then rises from an initial height, h_i to final height, h_f. Using the change in height, Δh, and the law of conservation of energy, the velocity, v_f, of the projectile-pendulum immediately after impact is determined. Knowing this velocity and the mass of the projectile-pendulum system, the final momentum, p_f, can be determined.

$$p_f = (M+m)v_f$$

Part 1: Projectile Motion

Here we will determine the mass of the projectile and its initial launch velocity. After the ball is launched by the spring gun with velocity v_i, it follows the trajectory approximated by the curved dotted line, as shown in figure 2.

At the instant the ball is launched horizontally from the spring gun, it begins to fall. While it is falling, its horizontal velocity v_i is unaffected by the downward acceleration of gravity. Note that the vertical downward velocity is zero at the instant the ball leaves the gun. When air resistance is negligible, the vertical and horizontal motions of a projectile are separable. The net result is the accelerated vertical motion (falling) combined with the horizontal motion at a constant horizontal velocity. This gives the trajectory depicted above in Fig 2. Distance of fall y and range X measurements will be used to determine the initial velocity of the ball, v_i.

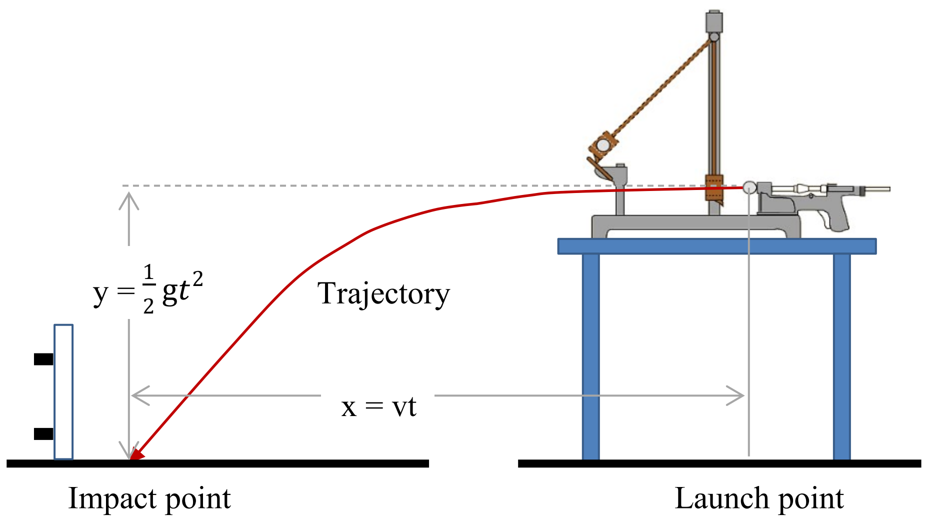

Figure 2: Pendulum swung up out of the way.

A ball released from rest will fall vertically with the acceleration of gravity g. The ball will fall through a distance y in a time t related by:

$$y = \frac{1}{2}gt^2 \ ; t = \sqrt{\frac{2y}{g}}$$

In this time t, a ball launched with a horizontal velocity v will cover a horizontal distance X to the point of impact on the floor. This distance is given by:

$$X = v_i t = v\sqrt{\frac{2y}{g}}$$

The velocity of the ball is found to be:

$$v_i = X\sqrt{\frac{g}{2y}}$$

This yields an initial momentum given as:

$$P_i = mv_i = mX\sqrt{\frac{g}{2y}}$$

Part 2: Conservation of Momentum

Now we will determine the velocity immediately after the instant of impact. We will place the pendulum in the path of the projectile and allow them to collide. At the moment of collision, the momentum of the projectile becomes the momentum of the projectile-pendulum system. The system has some momentum and kinetic energy:

$$p = (M + m)v_f, \qquad KE = \tfrac{1}{2}(M + m)v_f^2$$

After the collision, the pendulum will swing about its center of support from its initial height to some maximum height. At the instant the pendulum reaches this maximum height, the kinetic energy will be completely transferred to gravitational potential energy. By conservation of energy we can determine the final velocity:

$$\tfrac{1}{2}(M + m)v_f^2 = (M + m)g\Delta h \implies v_f = \sqrt{2g\Delta h}$$

where g is the acceleration of gravity. Substituting, one obtains:

$$p_f = (M + m)\sqrt{2g\Delta h}$$

The ballistic pendulum apparatus is designed with a pawl on the bottom of the pendulum and a mounted notched rack which will secure the system at the highest point of the pendulum arc.

Procedure

NOTE: Your TA will instruct you in the proper method to safely use the spring launcher and the correct techniques for each measurement. Most of the data collection will require a partner to assist.

Part 1: Initial Momentum

1. Ensure that the base of the ballistic pendulum is secure and level before beginning the experiment. If you determine that it is not, alert your TA to make the necessary adjustments. It is necessary that the apparatus remain stationary for the entire experiment, so be careful not to bump into, lean on, or otherwise move the table during the experiment!
2. Move the pendulum up onto the rack out of the path of the projectile.

   Caution: Make sure no one is in the path of the steel ball in order to prevent injury to other students or instructors.
3. Prepare the launcher. Hold the trigger handle with one hand, and with the other, push the projectile toward the handle to compress the spring until the shaft collar engages the trigger mechanism. This compresses the spring a fixed amount.
4. Determine the impact zone by making a few test launches. There will be some variation in the location of impact. Once you have determined the general area, place two pieces of paper on the floor in that area, and tape them in place. The impact of the projectile will create a visible mark on the paper, this is how we will record the horizontal travel distance. Launch the projectile once more to check the placement of the paper, and adjust if necessary.

5. Launch the projectile ten times. After *each launch* retrieve the projectile, and highlight the impact mark on the paper by placing a small square around it. When taking measurements, we will measure to the center of the impact mark. Do ***not*** remove the paper from the floor.
6. When you have completed the ten trials confirm the spring is in the released position, and place the projectile on the end of the launch shaft.
7. Use the plumb bob to determine the point on the floor directly beneath the launch point of the projectile (place the projectile on the launch shaft while it is extended, not compressed).
8. Using the 2-meter stick measure from the point on the floor marked by the plumb bob to a spot on the paper approximately 10cm before the impact zone. Draw a horizontal line on the paper and record the distance between the launch point and the horizontal line. You may now remove the paper from the floor.
9. Measure the distance from the horizontal line on the paper to each impact mark, add this to the distance to the horizontal line recorded above to determine the total distance travelled by the projectile, x.
10. Place the projectile on the end of the spring-launcher shaft, then measure the fall height, y. This is the distance between the floor and the bottom of the projectile. Record at least five measurements.

Part 2: Final Momentum

1. Release the pendulum from the rack and allow it to hang freely. Identify the index pointer which marks the center of mass of the pendulum. Carefully measure the initial height above the base of the apparatus, h_i, using the combination square. Make and record at least five measurements.
2. Move the pendulum up onto the rack, then prepare the launcher.
3. Release the pendulum from the rack and allow it to hang freely. When the pendulum is at rest launch the projectile. The projectile will embed itself in the pendulum and the system will swing up and stop at the maximum height. Use the combination square to measure the final height, h_f, to the same index pointer used for h_i.
4. To release the projectile gently push it out of the pendulum.
5. Repeat these steps until you have ten measurements of the final height.
6. Record the mass of the pendulum, M, which is listed on the base of the apparatus. Since this was not measured directly, assume $S_A = 0.1\ g$.
7. Measure and record the mass of your projectile, m, using five different mass balances.

Calculations and Analysis of Data

1. Find the average value for projectile mass, launch height, horizontal travel distance, initial height, and final height, $(\bar{m}, \bar{y}, \bar{x}, \bar{h}_i, \bar{h}_f)$
2. Calculate initial velocity and momentum, v_i, p_i.
3. Calculate change in pendulum height, Δh.

4. Calculate final velocity and momentum, v_f, p_f.

5. Determine the percent fractional error: $PFE = 100 * \frac{|p_i - p_f|}{|p_i|}$

   Note: Conservation of momentum means initial and final momentum should be equal, and therefore their difference should be zero. You should calculate the uncertainty in your initial and final momentum: S_{pi}, S_{pf}, and then calculate their difference, $p_i - p_f$, and the uncertainty associated with that difference: S_{pi-pf}.

   Use these values to perform a precision vs accuracy test. Does the difference between initial and final momentum equal zero, within the bounds of your experimental precision? If not, identify the factors that shifted your values out of this range.

Questions

1. (6 points) Calculate the kinetic energy of the system prior to collision. $KE = \frac{1}{2}mv^2 = p^2/2m$
2. (6 points) Calculate the kinetic energy of the system immediately after collision.
3. (6 points) Calculate the fractional energy loss: $|(KE_f - KE_i)/KE_i|$
4. (6 points) Compute the ratio of the mass of the pendulum to the mass of the projectile pendulum system and compare with the fractional energy loss.
5. (6 points) Is there a violation of the law of conservation of energy in this inelastic collision (is there energy missing)? Explain.