Resonance in LRC Circuits

1 Introduction

In this experiment we build on previous study of circuits, incorporating the concept of self-inductance in an AC circuit. If a changing current passes through a coil of wire with inductance *L*, the EMF produced in the coil will also change in proportion to the rate of change of the current:

$$\mathcal{E} = -L\frac{\Delta I}{\Delta t}.$$

When appearing as a circuit element the coil of wire is called an inductor, with inductance L quantified in units of Henry, H. We already know from the previous experiment that voltage across a capacitor is time-dependent. AC circuits that combine both inductance and capacitance display particularly interesting and important properties which we will study in this experiment. A circuit containing at least one resistor (R), capacitor (C) and inductor (L) is referred to as an *LRC* (or often *RLC*) circuit.

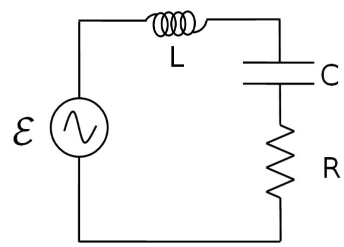

Figure 1: An LRC Series Circuit

Analysis of current flow in AC circuits containing these components requires special consideration. Since the voltage drops across both inductors and capacitors depend on the frequency of the AC current, we write the AC analogs of Ohm's Law: $V_R = IR$, $V_L = I X_L$, $V_{C\,max} = I_{max} X_C$, where X_L and X_C are the frequency-dependent AC analogs of resistance called the inductive reactance and the capacitive reactance, respectively. The capacitive reactance and the inductive reactance are measured in ohms and given by:

$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

where ω is the angular frequency of the AC current: $I = I_{max} \cos(\omega t)$. The effective resistance of the whole circuit is called the impedance, given by Z, such that V = IZ.

In an LRC circuit we also need to pay attention to the phase relationship between the voltage across each element. The voltage across the resistor will be in phase with the current, the voltage across the inductor leads the current by 90º and the voltage across the capacitor lags by 90º. Adding the three voltages, results in a total voltage that varies sinusoidally, but has a phase shift φ with respect to the current and is given by $V = V_{max} \cos(\omega t + \varphi)$. The phase relationship between V_L, V_R, V_C and net voltage, V is illustrated conveniently by a phasor diagram:

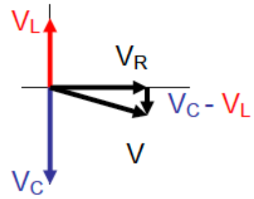

Figure 2: A phasor diagram for a series LRC circuit

From the phasor diagram we can see clearly that the net voltage is not simply the algebraic sum of the voltage across each element, but is instead given by:

$$V = \sqrt{{V_R}^2 + (V_L - V_C)^2}$$

Also, from above, V= IZ, and V_L and V_C are frequency-dependent so there will be a frequency at which impedance Z is minimized and current flow in the circuit is maximized. This is called the resonance frequency, f_0 where $f_0 = \omega_0 / 2\pi$.

The impedance can be written as follows:

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

where we can see that Z is minimized when $X_L = X_C$. Therefore, at resonance

$$\omega_0 L = \frac{1}{\omega_0 C}$$

So,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

or

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The goal of this experiment is to measure the resonance frequency in a LRC circuit. By using a function generator, we can vary the frequency of the input signal and identify the frequency at which the voltage in the circuit is maximized. We will repeat the measurement three times using different arrangements of two capacitors.

2 Experimental Apparatus and Procedure

2.1 Experimental Apparatus

1. Circuit board
2. Resistor and two 0.10μF capacitors
3. 63 mH inductor
4. Agilent multimeter
5. Tektronics oscilloscope
6. Function generator (50 Ohm impedance)

2.2 Procedure

2.2.1 Circuit setup and direct measurement of capacitance values

1. Directly measure and record the value of each capacitor $C_{1,dm}$ and $C_{2,dm}$. Capacitor C_1 is already connected to the resistor on the circuit board. Measurements should be in μF (10^{-6} Farads). Use the 1 μF range on the multimeter when measuring capacitance in this experiment.

2. Connect the two capacitors in parallel and directly measure this combined capacitance, $C_{p,dm}$. Connect the two capacitors in series and directly measure this combined capacitance, $C_{s,dm}$.

3. Using the inductor *L,* resistor *R* and the first (only) 0.10μF capacitor C_1, construct the series *LRC* circuit as shown in Figure 1. Connect the oscilloscope Channel 1 to the output of the function generator to measure V; connect the oscilloscope Channel 2 to measure the voltage drop across the resistor V_R. Use the external trigger from the function generator to set **Ext Trigger**.

5. Set the function generator to a sinusoidal wave function. **Wait for your TA to check all connections in your circuit before connecting the function generator to the circuit.** Your TA will help optimize the function generator output and oscilloscope settings.

2.2.2 Measurement of f_0 for LRC circuit with a single capacitor

1. Once your circuit is set up you can begin the main part of the measurement by recording the voltage amplitude on the oscilloscope as a function of the frequency of the AC signal. Start with a frequency of approximately 500 Hz on the function generator and increase in increments of 500 Hz up to about 4kHz. For each frequency, examine the sinusoidal signal on the oscilloscope monitor. You will need to adjust the number volts/DIV as you change frequencies so maximize the sinusoidal amplitude on the display. For each frequency record the resulting voltage amplitude (peak to peak) on the oscilloscope. Plot the obtained V_R versus *f.*

2. Now, go back and look at the graph and table of voltage amplitudes. You should find that as you increased frequency towards the resonance frequency the voltage went up. Then at some point you will have gone past the resonance frequency and the measured voltage amplitudes started going down. Now go back and fine tune within this range to try to find the frequency that maximizes the voltage on the oscilloscope. Record this value. This is your experimentally measured resonance frequency f_0.

2.2.3 Measurement of f_0 for LRC circuit with capacitors in series

1. Now go back and modify your circuit to incorporate the second capacitor in series with the first capacitor. All other connections should remain the same.

2. Once the second capacitor is in the circuit you can proceed to repeat the same procedure as above to determine the resonance frequency. Again start around 500 Hz and sweep the function

generator upwards to map out the approximate resonance curve. Then go back to fine tune the frequency around the peak to identify the experimentally measured resonance as best you can.

2.2.4 Measurement of f_0 for LRC circuit with capacitors in series

1. For the final part of the measurement go back and modify your circuit to incorporate the second capacitor in parallel with the first capacitor. All other connections should remain the same.
2. Once the second capacitor is in the circuit you can proceed to repeat the same procedure as above to determine the resonance frequency. Again start around 500 Hz and sweep the function generator upwards to map out the approximate resonance curve. Then go back to fine tune the frequency around the peak to identify the experimentally measured resonance as best you can.

Data Sheet

Directly Measure the two individual capacitors

$C_{1,\mathrm{dm}}$ = _______________μF $C_{2,\mathrm{dm}}$ = _______________μF

Directly Measure the Values of the Parallel and Series Capacitors

$C_{p,dm}$ = _____________ μF Series $C_{s,dm}$ = _____________ μF

2.2.2 Measurement of resonance for LRC circuit with a single capacitor

f	V_R
(Hz)	(V)

Experimentally Measured Resonance Frequency for LRC circuit with single capacitor:

f_0 = ____________ Hz

2.2.3 Measurement of resonance with capacitors in series

f	V_R
(Hz)	(V)

Experimentally Measured Resonance Frequency for LRC circuit with capacitor in series:

f_0 = ___________ Hz

2.2.4 Measurement of resonance with capacitors in parallel

f	V_R
(Hz)	(V)

Experimentally Measured Resonance Frequency for LRC circuit with capacitors in parallel:

f_0 = ___________ Hz

3. Calculations, Analysis and Graphs.

In this experiment we measured the resonance frequency for three different LRC circuits. Each one had the same inductance L, and resistance R, but we changed the capacitance, and therefore the resonance frequency, by using three different configurations of our two capacitors (individual, series and parallel). Now, using all three data points we will analyze resonance frequency as a function of capacitance for our three data points. We will perform 2D stats and compare the inductance, *L* calculated from the slope again the value quoted by the manufacturer of 63 +/- 3 mH.

1) First compile your three data points for resonance frequency in a new data table in a DV stats worksheet with one column for f_0 and one column for the corresponding capacitance.

Recall that the formula for resonance frequency is:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

So in order to linearize our data we will have to introduce a third column, for $C^{-1/2}$. Compute this value for all three capacitance. This will be our independent variable for performing 2D stats

2) Now graph and perform 2D Stats on your three $(C^{-1/2},\ f_0)$ pairs. <u>This graph must be included in your lab report.</u>

3) Now you are ready to find your experimental value for the inductor in your LRC circuit. Comparing the resonance frequency formula to slope intercept form, and solving for *L*, we get:

$$L_{exp} = \frac{1}{(2\pi * slope)^2}$$

Use error propagation to determine $S_{L,exp}$
Test your accuracy against the quoted inductance of 63 mH.

4 Questions

1. In our calculations we did not end up directly using information about the resistance in the circuit. How does the resistance in the circuit impact the height and width of the resonance curve? (If the resistance were to increase would the height change? Would the width? If so, how?)

2. Calculate the capacitive reactance X_C at your measured resonance frequency for each of the three capacitor configurations used in this experiment.

3. Calculate the inductive reactance X_L at your measured resonance frequency for each of the three capacitor configurations used in this experiment. Which circuit had the largest voltage drop across the inductor at resonance?

5. Discussion

As always, remember to include a discussion section in your lab report. Refer to the Appendix on writing lab reports in the Course Syllabus for guidelines and suggestions for this section.