Purpose

This experiment will observe and analyze an elastic collision of two pucks on an air table to verify the law of conservation of momentum. This experiment differs in two ways from the one dimensional case: The collision here is elastic, and the system interacts in two-dimensions. Thus, two-dimensional vector arithmetic is a skill necessary to complete this analysis.

The goal of this experiment is to:
• Compute the total initial and total final momentum of a two-dimensional system
• Verify the law of conservation of momentum in elastic collisions
• Add vectors graphically using a hand-drawn graph

Materials and Equipment

- Plastic pucks (2)
- Air table
- Recording and analysis software
- Mass balance
- Meter stick

Theory

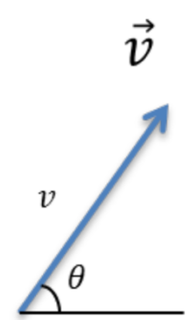
Many physical quantities (mass, time, temperature, etc.) can be described using a number independent of spatial orientation. These quantities are *scalars*. Other quantities (velocity, acceleration, momentum, etc.) are described using both a number and some directionality. These quantities are *vectors*. A vector is usually represented in text as either a bold character ($\boldsymbol{v}$) or with an arrow ($\bar{v}$). In diagrams vectors are represented as arrows (the forces drawn in a free body diagram are vectors).

Characterizing a two-dimensional vector requires two numbers: the *magnitude* and the *direction*

Here, vector $\bar{v}$ has magnitude $|\bar{v}| = v$. It is also necessary to indicate the direction of the vector. Usually direction is measured counterclockwise from the horizontal position. The vector $\bar{v}$ has direction $\theta = 30°$.

Polar vs Cartesian Coordinates

There are a variety of ways to indicate direction. Above, using the angle θ, the vector is described in polar coordinates: a magnitude (which gives the length of the vector), and an angle (which gives direction). Using polar coordinates, the above vector could be written as



$$\boldsymbol{v} = (v, \theta)$$

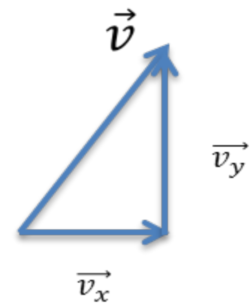
or as

$$\boldsymbol{v} = v\hat{r} + \theta\hat{\theta}$$

In the second case $\hat{r}$ and $\hat{\theta}$ symbolize *unit vectors*, vectors of length one pointing in the *radial* and one in the *angular* direction. The coefficients v and θ give the magnitude in each direction.

Another way to describe a vector is using the perpendicular components of that vector (you should recall breaking vectors into components in the Inclined Plane experiment).

In Cartesian coordinates the same vector $\boldsymbol{v}$ can be written as



$$\boldsymbol{v} = (v_x, v_y)$$

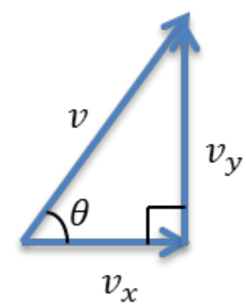
or as

$$\boldsymbol{v} = v_x\hat{x} + v_y\hat{y}$$

Breaking a vector into two perpendicular components is often the most convenient way to perform calculations.

It is also necessary to convert between these two equivalent notations.

Using trigonometry, we can see



$$cos\theta = \frac{v_x}{v}$$
$$sin\theta = \frac{v_y}{v}$$

Therefore, to convert from polar representation to Cartesian representation:

Go $(v, \theta)_P \longrightarrow (v_x, v_y)_C$ with:

$$v_x = v * cos\theta$$

$$v_y = v * sin\theta$$

Using the Pythagorean theorem, we can convert from Cartesian to polar:

$$v^2 = v_x^2 + v_y^2$$
$$tan\,\theta = \frac{v_y}{v_x}$$

Therefore:

Go $(v_x, v_y)_C \longrightarrow (v, \theta)_P$ with:

$$v = \sqrt{v_x^2 + v_y^2}$$

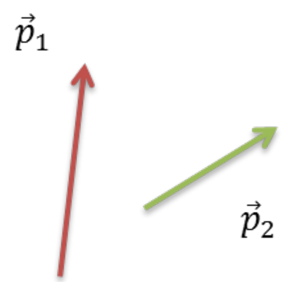
$$\theta = arctan\left(\frac{v_y}{v_x}\right)$$

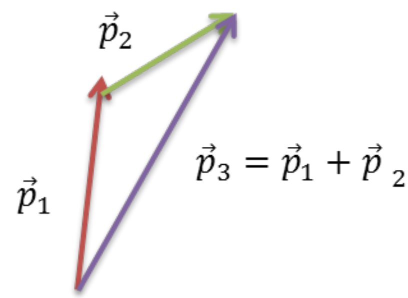
Adding Vectors

Adding vectors can be done algebraically or graphically. The sum of two vectors is the sum of their components.

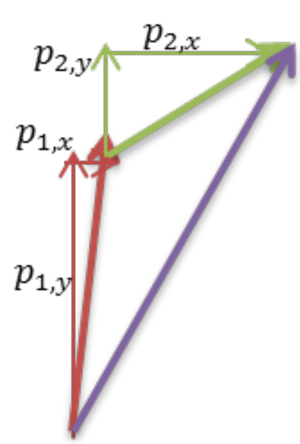
$$\boldsymbol{p_1} = 2\hat{x} + 3\hat{y}$$
$$\boldsymbol{p_2} = 4\hat{x} + 7\hat{y}$$
$$\boldsymbol{p_1} + \boldsymbol{p_2} = 6\hat{x} + 10\hat{y}$$

To add vectors graphically first draw one vector starting at the origin, then draw the other vector starting at the tip of the first vector. The sum of these two vectors is the arrow drawn from the origin to the tip of the second vector. This is often called "tail-to-tip" addition.

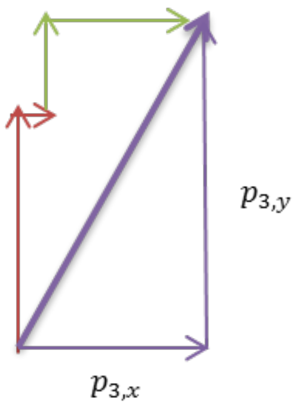


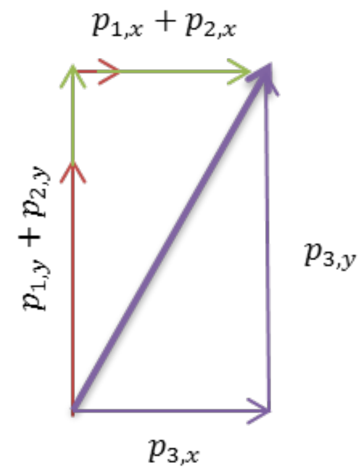


It does not matter which vector you draw first, as the resulting vector will be the same either way. This process works even if both vectors are broken into their Cartesian components and then summed:



The sum of the horizontal components and the sum of the vertical components results in the same total vector.





If

$$\vec{p}_3 = \vec{p}_1 + \vec{p}_2$$

Then

$$p_{3,x} = p_{1,x} + p_{2,x}$$

$$p_{3,y} = p_{1,y} + p_{2,y}$$

$$\vec{p}_3 = (p_{3,x}\,,\ p_{3,y})_C$$

Adding Cartesian vectors just requires adding their corresponding components.

$$\vec{A} + \vec{B} = (A_x, A_y)_C + (B_x, B_y)_C$$

$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)_C$$

Example:

If $\vec{A} = (3.63\ N, 1.69\ N)_C$ and $\vec{B} = (4\ N, -3\ N)_C$, and we want to find $\vec{C} = \vec{A} + \vec{B}$.

Then

$$\vec{C} = (A_x + B_x, A_y + B_y)_C = (3.63\ N + 4\ N, 1.69\ N - 3\ N)_C$$

$$\vec{C} = (7.63, -1.31\ N)_C$$

We can now convert this sum to polar form if that is preferred.

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(7.63)^2 + (-1.31)^2}\ N = 7.74\ N$$

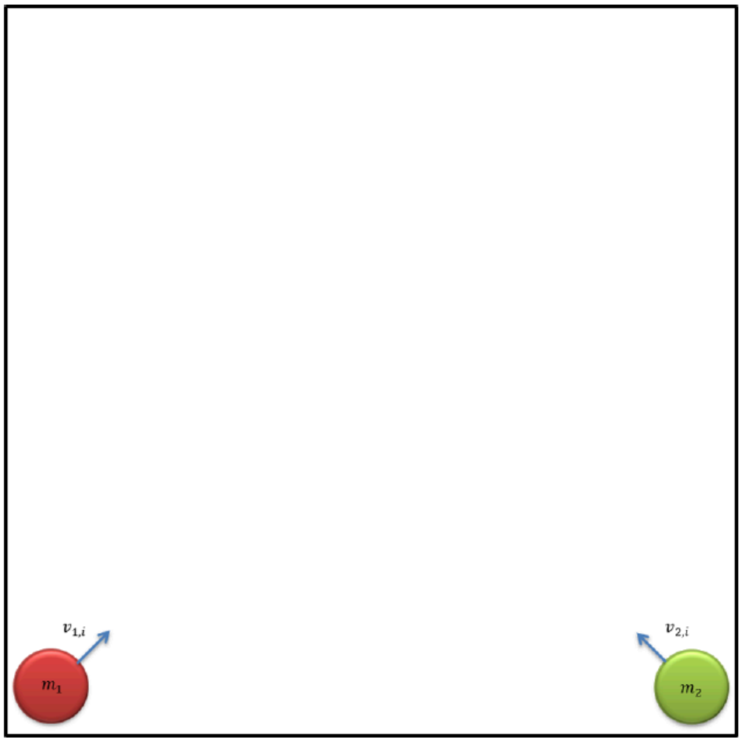
$$\theta = arctan\left(\frac{C_y}{C_x}\right) = arctan\left(\frac{-1.31}{7.63}\right) = -9.7^0 => 350.3^0$$

Method

In an elastic collision momentum is conserved. In this experiment we will measure the momentum vector of two pucks prior to a collision, then measure the momentum vectors after the collision. The sum of the initial momentum should equal the sum of the final momentum.

The momentum referred to so far is *linear* momentum, and not *angular* momentum (the momentum associated with rotation). This distinction may play an important role when analyzing your results.

The experiment will take place on an air table in order to reduce friction, and will be recorded as a video which can allows measurement with a software tool provided. Your TA will demonstrate how to use the software correctly. Once recorded, you will transfer your data to Excel for analysis.



The particular system of interest for us is two pucks of differing masses, m_1 and m_2, moving toward each other with differing constant velocities, $v_{1,i}$ and $v_{2,i}$, on an air track.

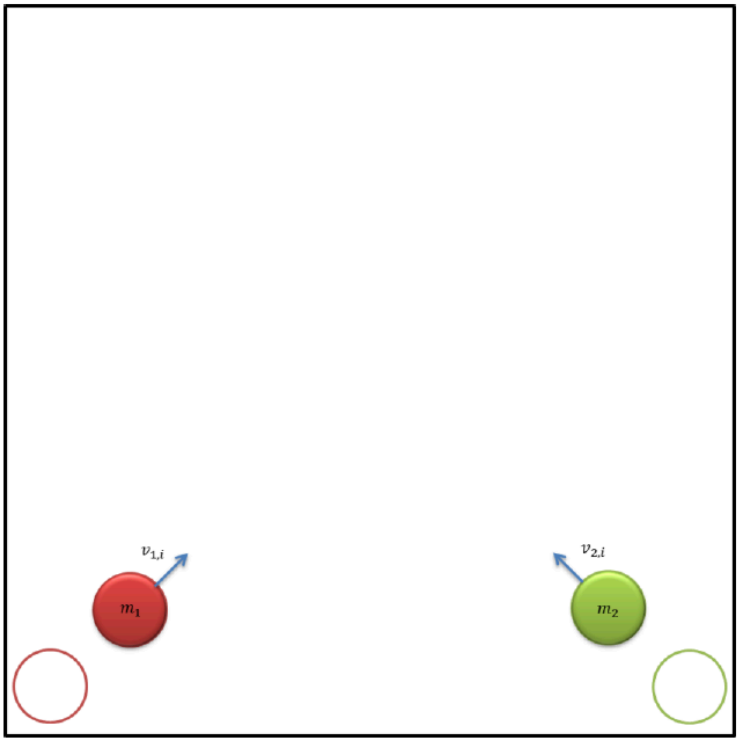
Each puck will have momenta

$$\vec{p}_{1,i} = m_1\vec{v}_{1,i}$$

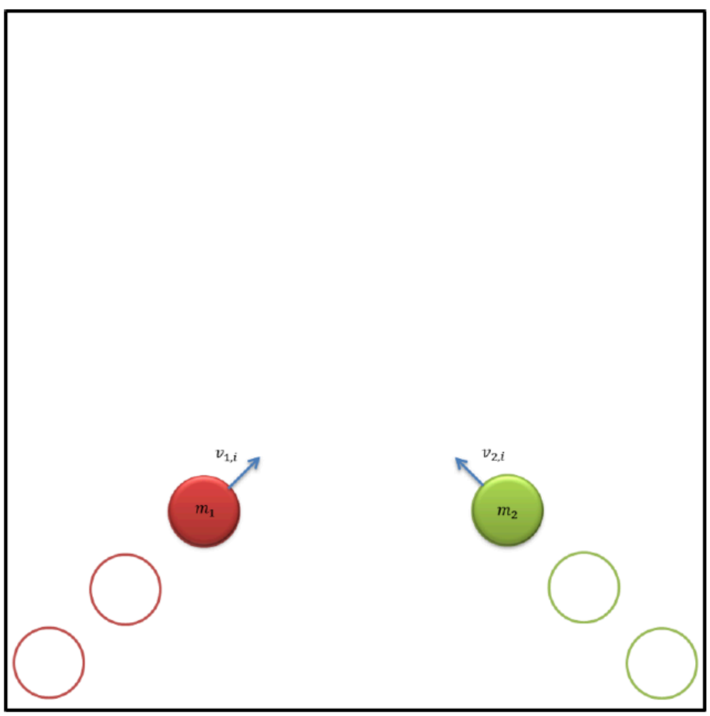
$$\vec{p}_{2,i} = m_2\vec{v}_{2,i}$$

The total initial momentum will be the sum of the two individual momenta.

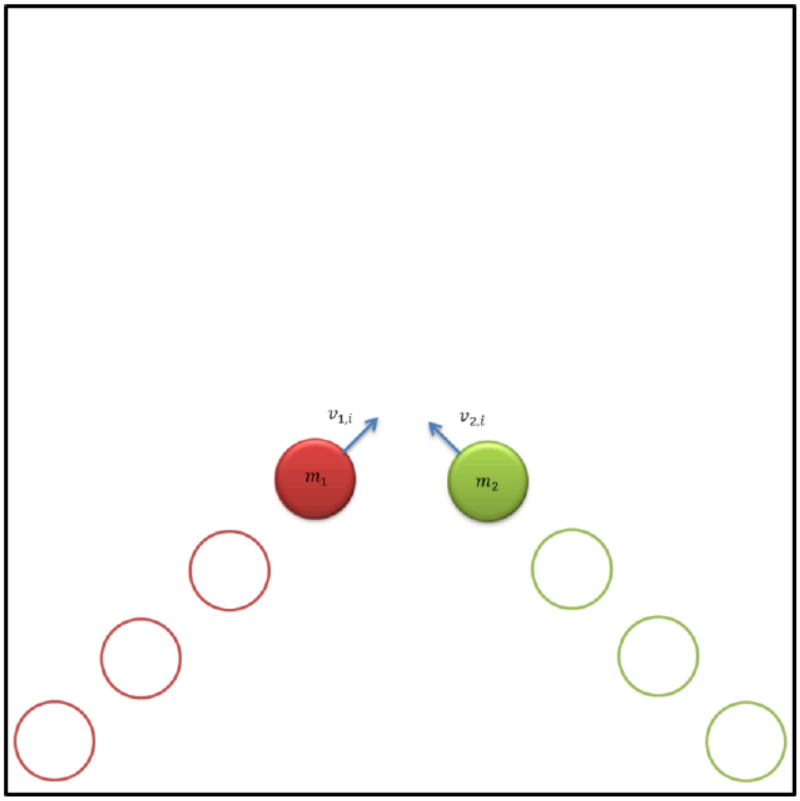
$$\vec{p}_i = \vec{p}_{1,i} + \vec{p}_{2,i}$$



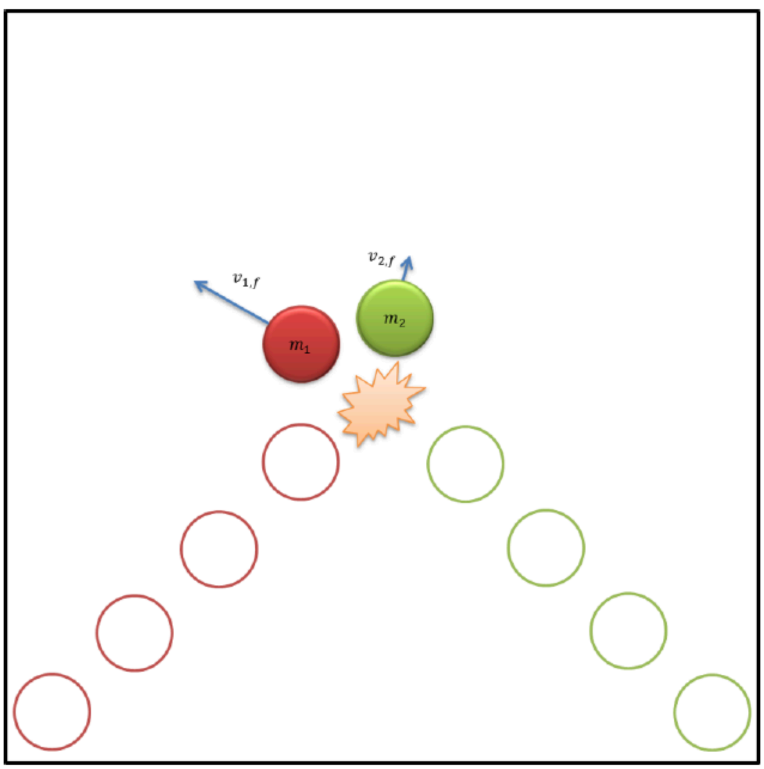
Looking at the system a particular Δt later, and leaving the ghost image of where the pucks were before.



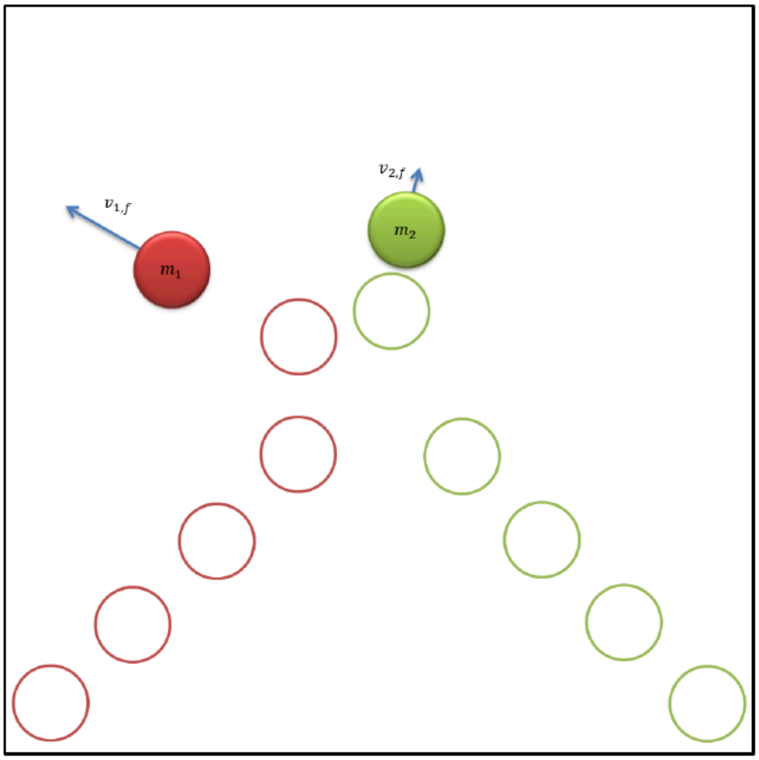
Another Δt later with more ghost images.



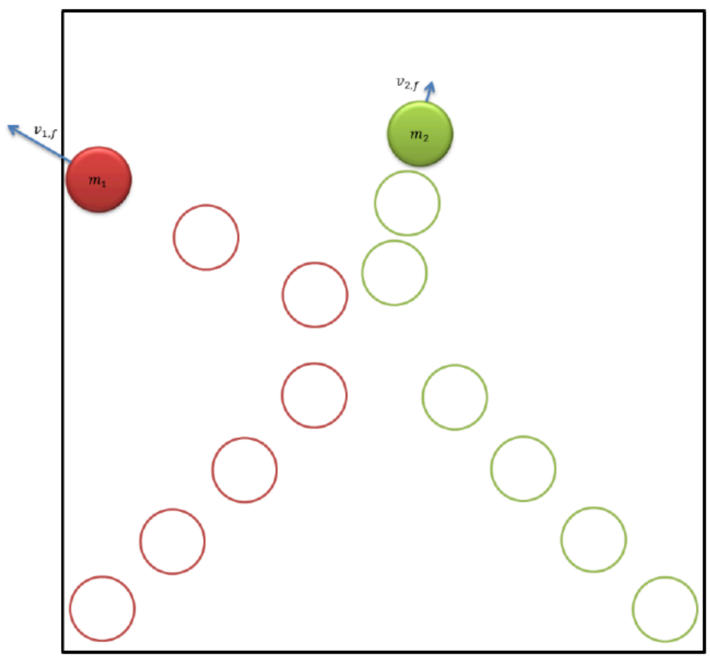
After another time Δt.



Sometime in the next Δt interval, the pucks collide elastically. Now they continue on with their different final velocities, $v_{1,f}$ and $v_{2,f}$.



Continuing on another Δt later.



So after the collision, each puck will have their respective final momenta

$$\vec{p}_{1,f} = m_1\vec{v}_{1,f}$$

$$\vec{p}_{2,f} = m_2\vec{v}_{2,f}$$

The total initial momentum will be the sum of the two individual momenta.

$$\vec{p}_f = \vec{p}_{1,f} + \vec{p}_{2,f}$$

Procedure

1. Measure and record the mass of each puck.
2. Measure the length of the marked square on the air table.
3. Turn on the air pump.

4. Place the pucks in the launch position.
5. Begin recording.
6. Launch the pucks so that they collide within the marked square on the air table.
7. Review the video to ensure the collision was captured.
8. Fix the origin position on the screen and enter the measured side length to establish a scale.
9. Measure the position of the first puck before and after collision. You should have at least five positions prior to, and five positions after collision.
10. Transfer this data to Excel
11. Measure the position of the second puck before and after collision in the same way.
12. Transfer this data to Excel

Analysis

Using the Excel data template provided, determine the following for *each* puck, *before* and *after* collision:

1. The horizontal velocity.
2. The vertical velocity.
3. The total velocity and angle of the velocity vector.
4. The speed (the magnitude of the velocity vector).
5. The horizontal and vertical momentum.
6. The total momentum vector and angle of the momentum vector.

Then, determine the magnitude of the total initial momentum in the horizontal direction, the magnitude of the total initial momentum in the vertical direction, and then the magnitude of the total initial momentum.

Determine the same for the final momentum, and compare this to the total initial momentum by calculating a percent difference.

Calculate the percent difference between final and initial momentum. Use initial momentum as the accepted value.

Calculate the difference in initial and final momentum, Δp. This quantity should be close to zero. Finally, calculate the percent fractional error: $PFE_{\text{P}} = \left|\frac{\Delta p}{\ }\right| * 100_{\text{PR}}$

Error Analysis

In this experiment there are a large number of sources for systematic error. Rather than spend time attempting to quantify each and then propagate their effect on the final result, we will instead use the Questions section to initiate a guided discussion about the sources of error. Therefore, you may omit the error propagation calculations from your lab report for this experiment.

<u>**Hand-drawn Graph**</u>

For this hand-drawn graph you **do not** need to use 80% of the graph paper. Instead your plot should have the same scale for the x-axis and the y-axis. Your origin should be placed near the center of the plot (you will be plotting both positive and negative values). Your horizontal range will be determined by the maximum of your two initial horizontal momentum, and the maximum of your two final horizontal momentum.

Add the initial momentum vector for puck 1 to the initial momentum vector for puck 2, and draw the resulting total initial momentum vector (remember, the second vector should start from the tip of the first vector, **not** the origin). Then, add the final momentum vector for puck 1 to the final momentum vector for puck 2, and draw the resulting total final momentum vector.

(You may find it easier to use the components of these vectors to find the sum, rather than plotting the vector directly with the correct angle.)

Are these vectors the same? Should they be? Determine the difference between these two vectors by measuring the difference between their tips, and convert using the scale of the graph. How does this compare to the difference calculated using Excel?

Questions

1. (20 points) Since we are unable to obtain a best estimate for experimental precision there is some difficulty in interpreting the experimental result. Δp is not quite zero, but why? We expect some of the fluctuation around zero due to random error, but there are certainly also systematic errors present.
   a) Our system consists of two pucks colliding. In general, when is *linear* momentum not conserved in a system?
   b) Discuss any systematic error that might cause linear momentum to not be conserved in this experimental setup. (*Hint: What environmental factors introduce non-conservative forces? Is the track truly frictionless? Was all of the energy transferred as motion? Is* ***linear*** *momentum the only momentum we should be measuring?)*
   c) Based on the discussion of systematic error in Part (b) predict whether you expect the initial momentum to be less than or greater than the final momentum in this experiment.
   d) Is your prediction in Part (c) true for your experimental results?
   e) Would your experimental result have been better if you had only used two positions before and after the collision to measure velocity? Which systematic errors would this reduce? Which might it increase?
2. (10 points) The collision in this experiment is an elastic collision. In a true elastic collision both momentum *and* energy are conserved.
   a) Calculate the initial and final kinetic energies of the system.

$$KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 \qquad KE_f = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

   b) Calculate

$$\Delta KE = KE_i - KE_f$$

   c) Find the percent fractional error

$$PFE = \left|\frac{\Delta KE}{KE_i}\right| * 100$$