Introduction

In this experiment we will once again study the motion of a falling object (a glider) in order to calculate acceleration due to gravity and compare the experimental result to the accepted value of 980.35 cm/s^2. This glider, however, will not be in freefall, but instead falling along a smooth inclined air track with minimal, but non-negligible friction. We will also measure the coefficient of friction, μ, between the glider and track.

The goal of this experiment is to:
- Compute the acceleration due to gravity an inclined plane
- Compute the coefficient of friction between the glider and track

Materials and Equipment

- Metal glider
- Air track
- Air pump
- Digital angle gauge
- Electronic photogates (5)
- Digital oscilloscope
- Mass balance
- 30cm ruler

Theory

An object moving down an incline under the force of gravity will fall more slowly than an object in free fall from the same height, since the force of gravity must move the object both down and across.

Consider a glider on track inclined at an angle, θ, see (fig. 1). The force diagram (assuming negligible air resistance) shows the total forces acting on the glider due to its weight, $\vec{W}$, friction between the track and glider, $\vec{f}$, and the normal force, $\vec{N}$, produce a net force pointing downward along the track resulting in a downward acceleration, $\overrightarrow{a_D}$.

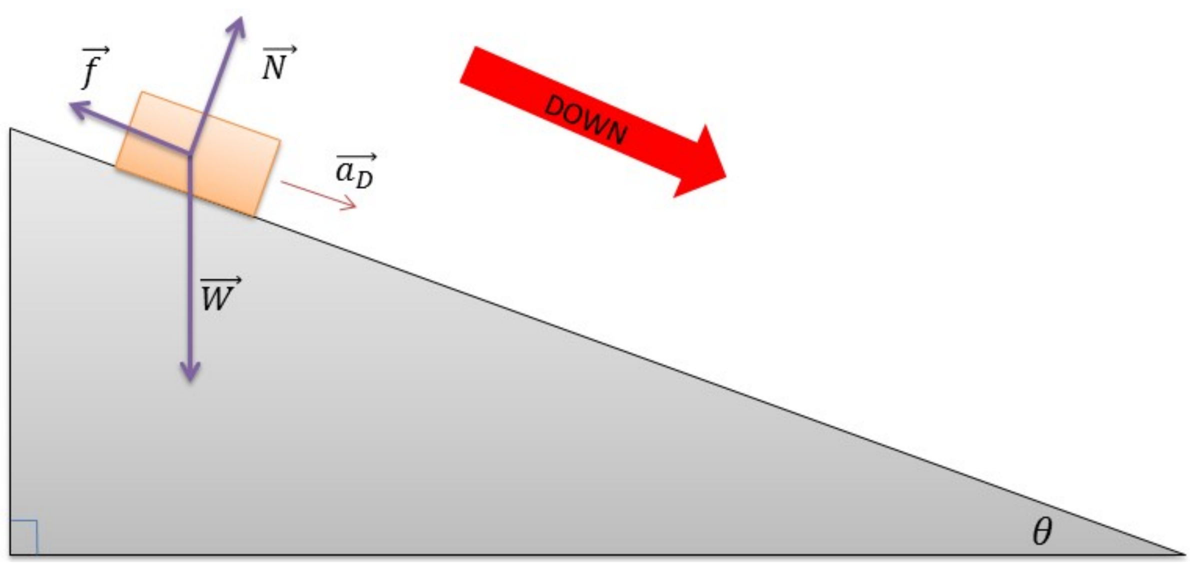


Figure 1

We begin, then, with the following vector equation:

$$\vec{W} + \vec{N} + \vec{f} = m\,\vec{a_D}$$

Since the direction of motion is along the surface of the track it is inconvenient to use the familiar updown, left-right coordinate system. As shown in figure 2, three of the vectors do not lie along the vertical or horizontal axes, which necessitates breaking each vector into two components.

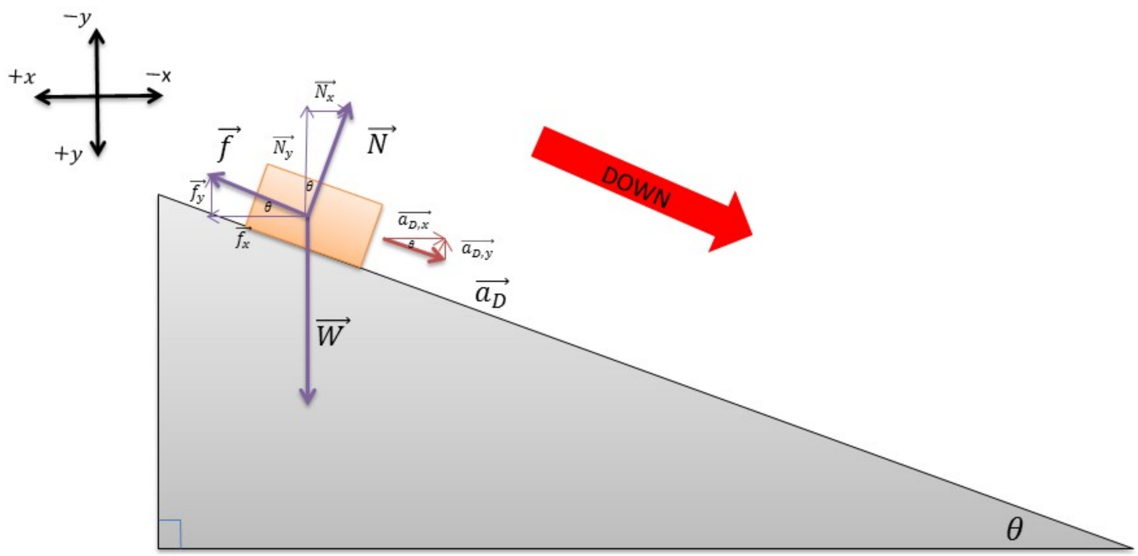


Figure 2

If, instead, we align a coordinate system to the surface of the track, only one of the vectors needs to be broken into components (fig. 3) which allows for easier calculation. (Note, we have chosen the downward direction to correspond to the positive y-direction.)

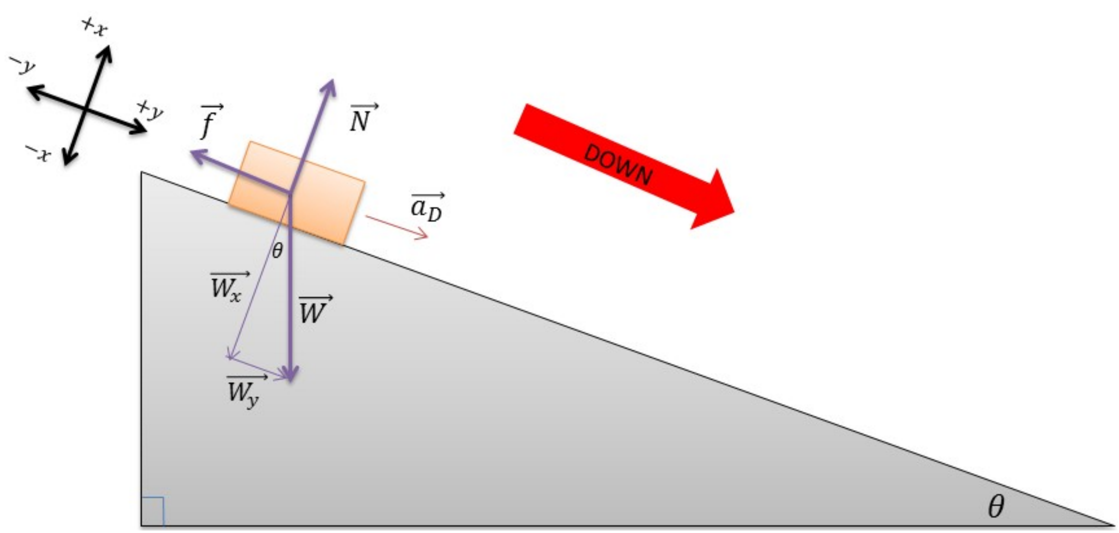


Figure 3

We then separate our vector equation into x-directed and y-directed pieces. Immediately, we notice that the normal force must balance the x-component of weight since there is no motion into or out of the track. We then see that since friction pointing up the track it is negative y-directed, and the y-component of weight is positive y-directed.

$$-W_x + N = 0$$

$$W_y - f = ma_D$$

Turning to trigonometry, we calculate the components of weight, given the angle θ. (fig 4.)

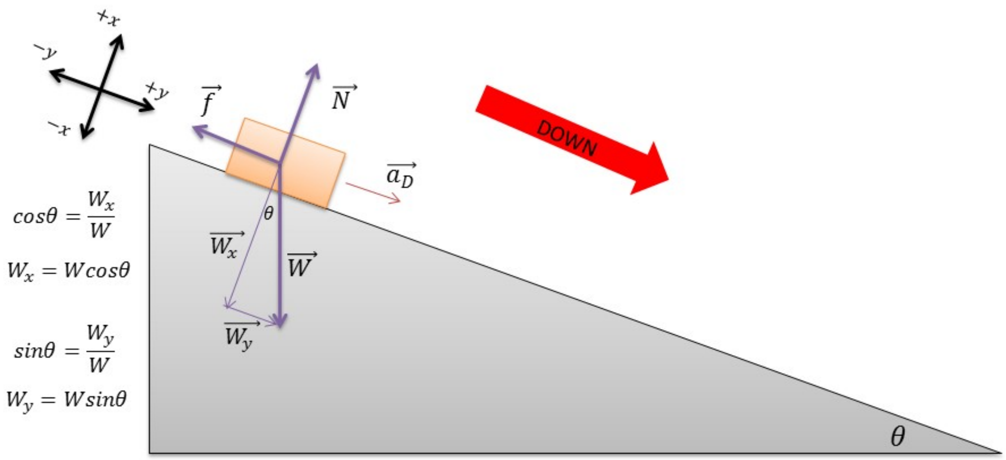


Figure 4

Substituting the results for W_x and W_y into our x- and y-equations we find the following:

$$N = Wcos\theta = mgcos\theta$$

and

$$W\ sin\theta - f = ma_D$$
$$mg\ sin\theta - \mu N = ma_D$$
$$mg\ sin\theta - \mu\, mg\ cos\theta = ma_D$$
$$gsin\theta - \mu g\ cos\theta = a_D$$

Then, solving for g:

$$g = \frac{a_D}{sin\theta - \mu cos\theta}$$

In order to calculate the constant of gravitational acceleration, g, we must measure the angle of incline, θ, and the downward acceleration, a,, (by measuring distances and times similarly to the Kinematics of Freefall experiment), however, we do not have a method to measure the coefficient of friction, μ.

Suppose, then, that the glider changes direction, and is moving up the incline instead. How would this change the free body diagram? Since friction opposes the direction of motion, it would point in the positive y-direction, the component of weight would also point in the negative y-direction. These forces would combine to decelerate the glider as it approaches the top of the track. A bumper placed at the bottom of the track will cause this change of direction.

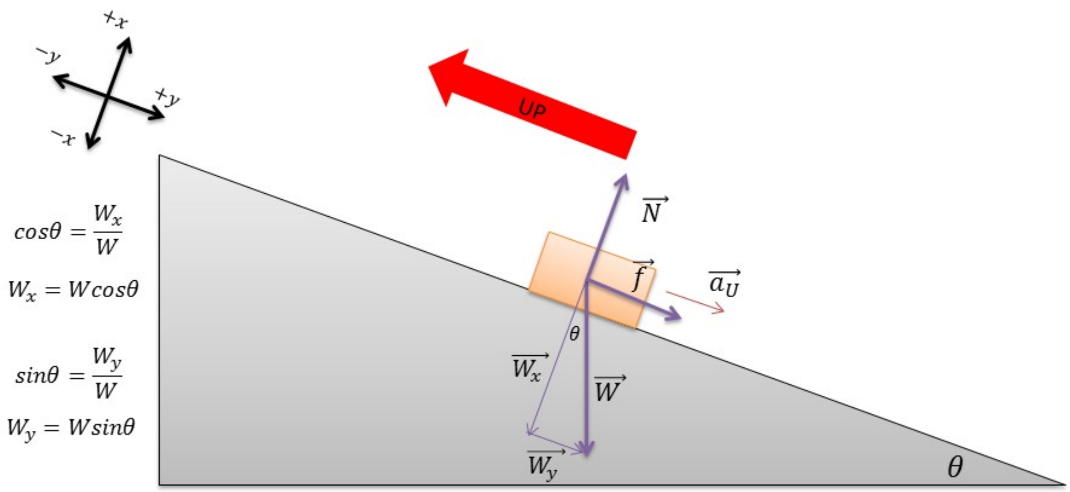


Figure 5: Velocity and acceleration are in different directions. The glider moves up the ramp in the negative y-direction, and the acceleration vector is pointing downward in the positive y-direction. The glider slows as it travels up the ramp.

Using a similar analysis as above (and completed in your PreLab Worksheet for this experiment) we find the following:

$$gsin\theta + \mu gcos\theta = a_U$$

Using the results for both upward and downward accelerations:

$$gsin\theta - \mu gcos\theta = a_D$$

$$gsin\theta + \mu gcos\theta = a_U$$

Adding these equations together allows us to eliminate the friction term and derive an expression for g in terms of the upward, and downward accelerations:

$$2gsin\theta = a_U + a_D$$

$$\boxed{g = \frac{a_U + a_D}{2sin\theta}}$$

Also, subtracting these two equations (as in the PreLab Worksheet) allows us to find an expression for the coefficient of friction:

$$\boxed{\mu = \frac{a_U - a_D}{a_U + a_D} tan\theta}$$

By reversing direction, we are able to isolate both unknown quantities. This technique can be very useful in eliminating systematic error in many contexts (friction, heat loss to environment, etc). By designing an experiment so that a known systematic error contributes in opposite directions we are able to cancel its effect without needing to measure it.

In order to calculate accelerations upward and downward we will use the same procedure as the previous Kinematics of Freefall experiment. Linearizing the kinematics equation, and assuming $y_0 = 0$ results in the following:

$$\frac{y}{t} = \frac{1}{2}a_i t - v_0$$

Which we will solve first for a_D, and then for a_U.

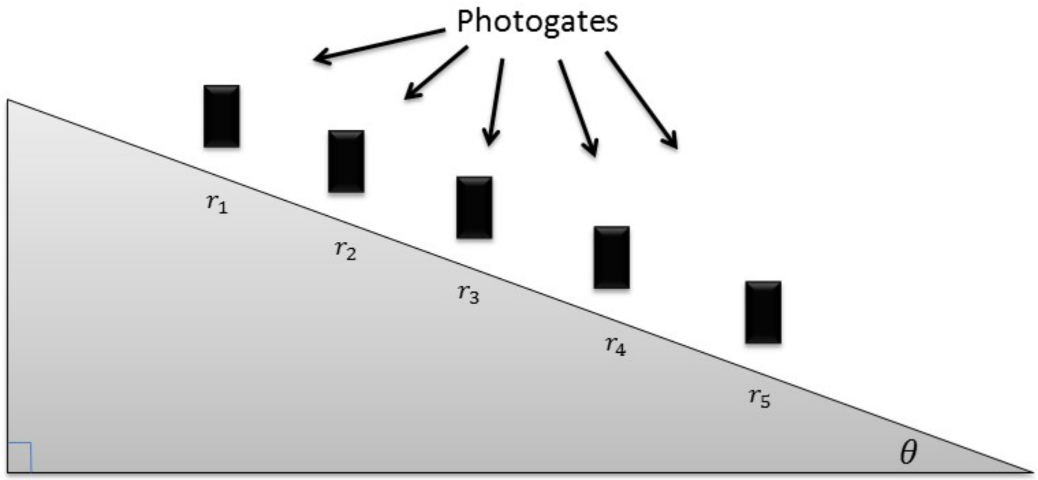
Apparatus

We will use an air track in this experiment to minimize friction experienced by the glider as it travels. The air is supplied by a pump connected by a flexible hose.

Do not place the glider on the track unless the air pump is powered on. Moving the glider along the track without air will cause scratches and damage both the track and the glider.

Five (5) photogates are positioned along the track. The photogate signals when an object (such as the glider) passes through it. These signals will be tracked and recorded using a digital oscilloscope. Your TA will confirm the correct settings for proper operation.

The photogates are powered by a small DC power supply. Do not adjust the knobs on the power supply, as this will damage the photogates. If you think there is a problem with the photogates, please ask your TA to assist.



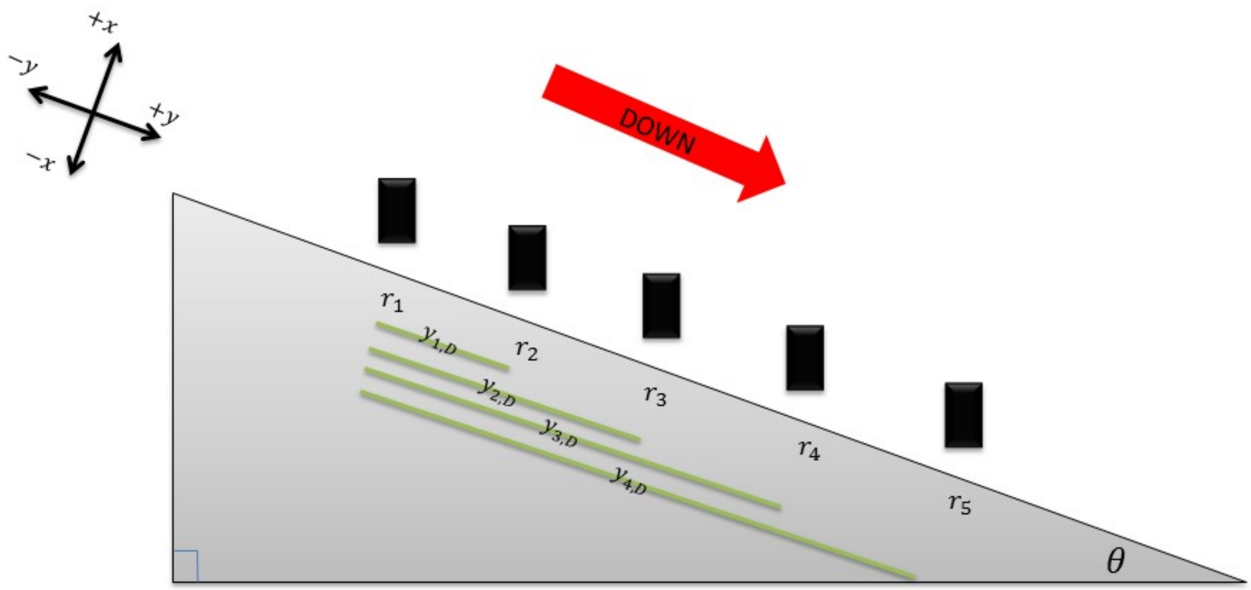
Procedure

- Record the length of the glider and the reading error of the ruler.
- Record the position of the photogates r_1 through r_5.
- Record the angle of the air track, θ.
- Verify that the oscilloscope settings are correct.

- Power on the air pump.
- Place the glider at the top of the track, prepare to press the Run/Stop button on the oscilloscope.
- Release the glider, and press Run/Stop to begin recording.
- Wait for the glider to reach the bottom of the track, bounce, and return.
- When the glider exits the photogate at the top of the track press Run/Stop to end recording.
- Using the Measure function of the oscilloscope record the enter and exit times for each photo gate (G_{1in}, G_{1out}, G_{2in}, G_{2out}, etc). You should have twenty total time entries for this trial.
- Repeat these steps five more times, for a total of six trials.

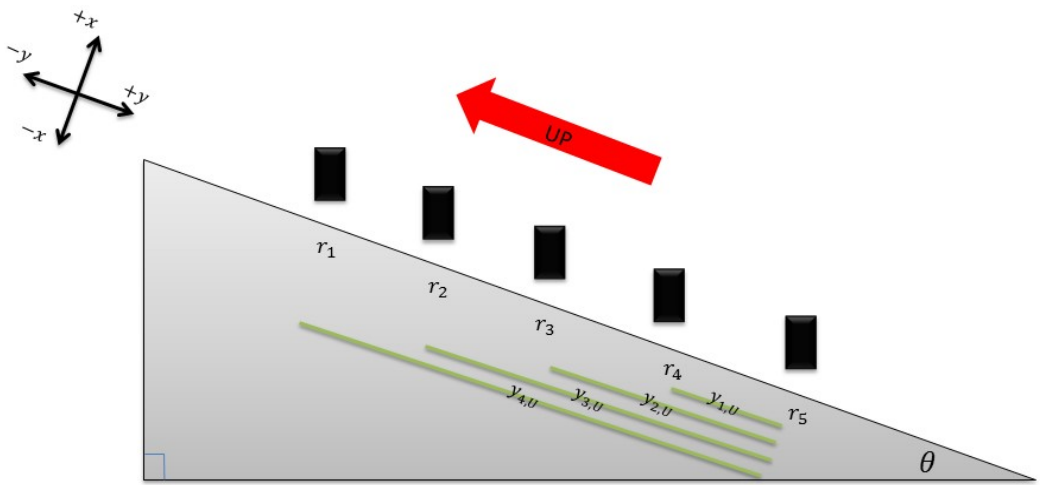
Analysis:

We need to establish the y values used when finding a_D from the photogate positions. As we considered the sixth dot as position zero at time zero in the last experiment, we will treat the first photogate as this point when considering a_D (and the bottom photogate will be this point for a_U). Therefore the (y, t) values will be determined from this point.



$$y_A = |r_2 - r_1|$$
$$y_B = |r_3 - r_1|$$
$$y_C = |r_4 - r_1|$$
$$y_D = |r_5 - r_1|$$

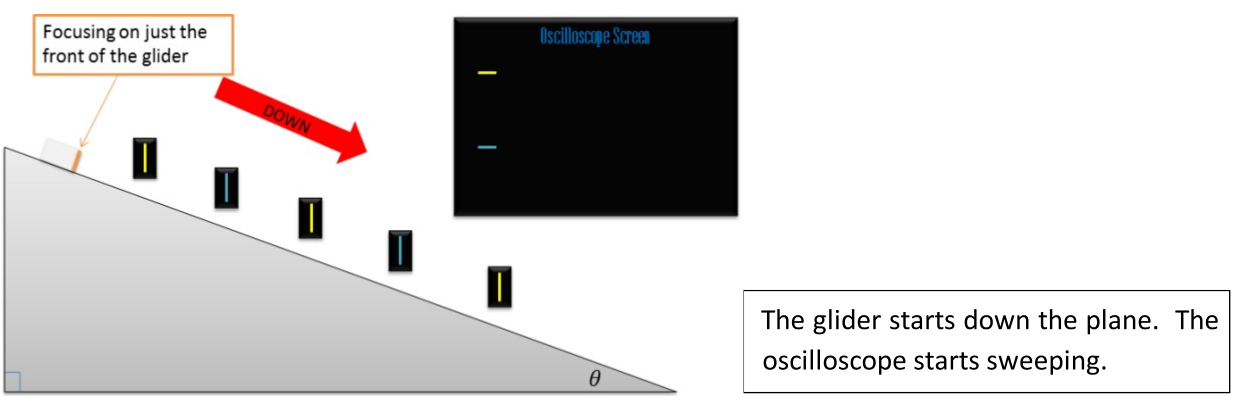
The absolute values are used to make sure these two values are positive, as both are displacements in what we are considering the $+y$ direction. And the way the inclined planes were manufactured starts with the zero position near the bottom and goes upward from there. (E.g. r_1 will be at the 120 cm and r_5 will be at the 30 cm, so $y_{4,D}$ should equal +90 cm.) Let us consider the y values for finding a_U.

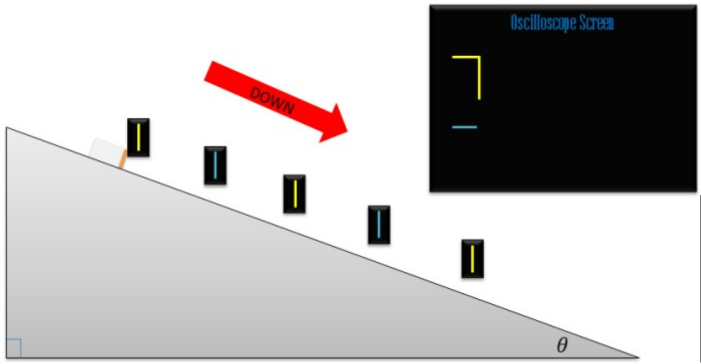


$$y_{A,U} = -|r_4 - r_5|$$
$$y_{B,U} = -|r_3 - r_5|$$
$$y_{C,U} = -|r_2 - r_5|$$
$$y_{D,U} = -|r_1 - r_5|$$

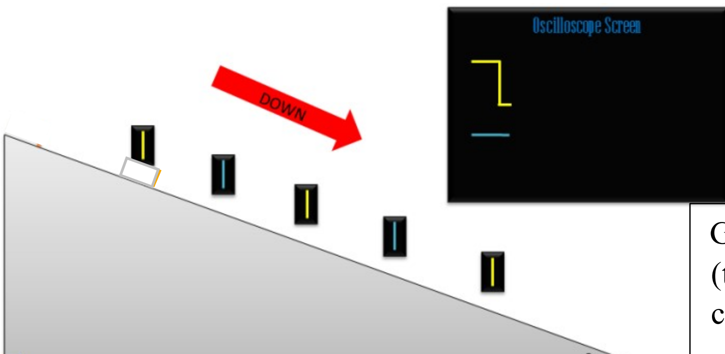
This time we make sure these values are negative, as they represent displacements in what we are considering the $-y$ direction.

To gather the needed times, we will hook the first, third, and fifth photogate to the oscilloscope's channel one; and we will hook the second and fourth photogates to the oscilloscope's channel two. Then we will release the glider at the top and at the same time tell the oscilloscope to watch for photogate dips. Let us focus on just the **front end** of the glider's journey.

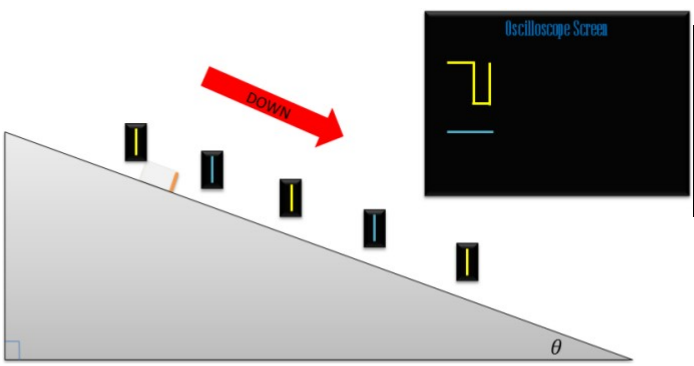




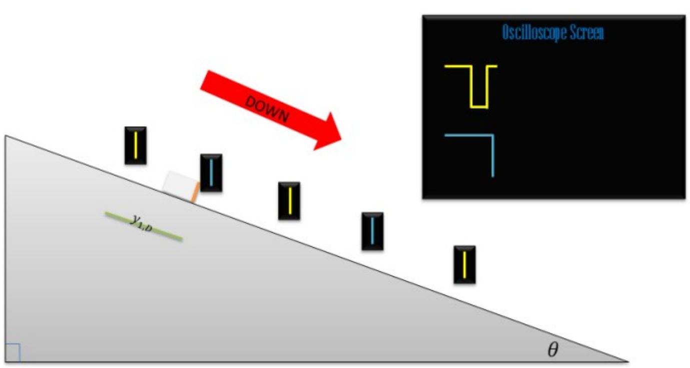
The glider enters the first (top) photogate. The oscilloscope shows the first dip on channel 1, t_1.



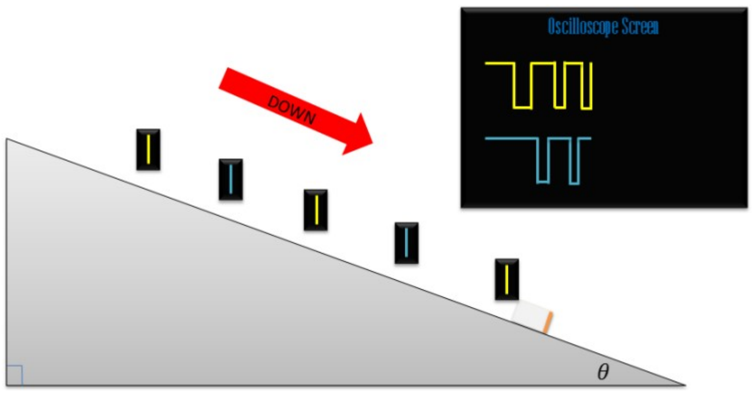
Glider continues through the first (top) photogate. The oscilloscope continues showing a dip.



The glider leaves the top photogate. The oscilloscope shows t_2.

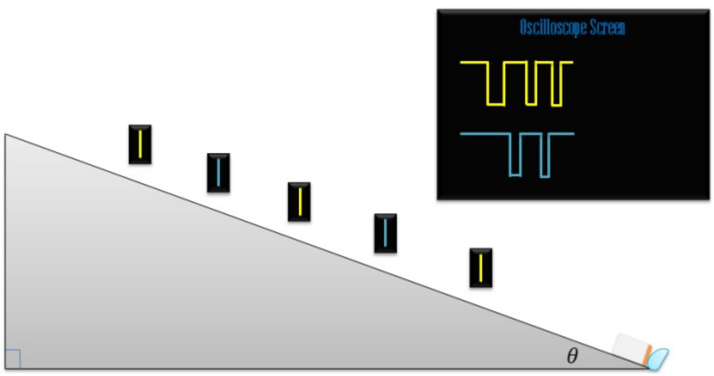


The glider continues on and enters the second photogate. The oscilloscope shows t_3 on channel two.

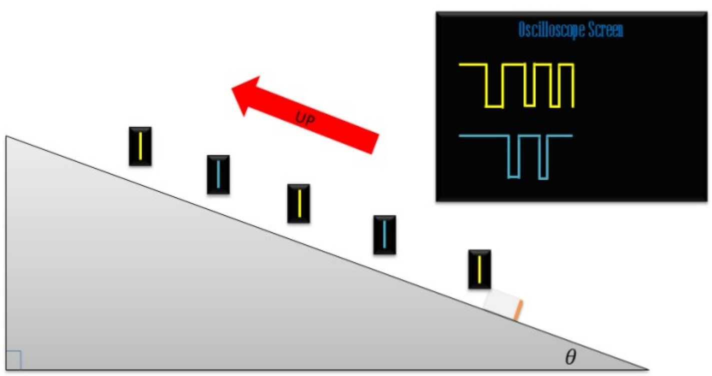


The glider finishes passing through all five photogates. The oscilloscope shows ten times, t_1 to t_{10}.

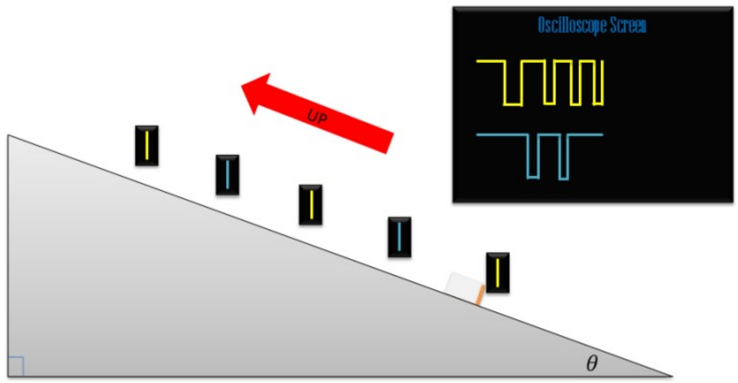
In the experiment, the glider will hit a bumper at the bottom of the inclined plane. This will turn the glider around and send it up the plane. We will obtain all these upward times by continuing our oscilloscope time sweep.



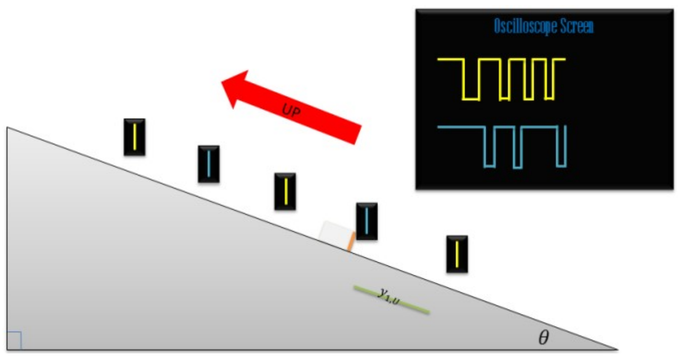
The glider hits a bumper at the bottom of the track and instantaneously comes to rest right before changing direction.



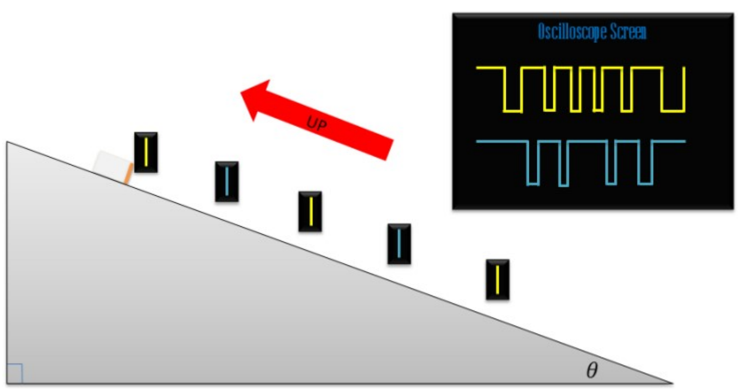
The glider recoils and travels up the plane. The first upward time shown, t_{11}, is when the back of the glider hits the bottom photogate.



Then we see the front of the glider just leaving the bottom photogate for t_{12}.

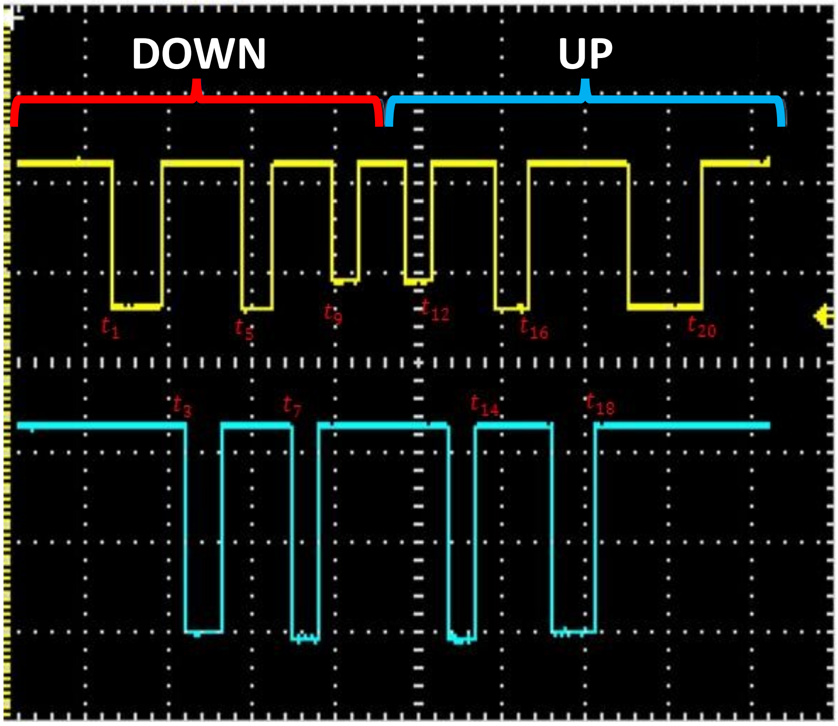


The glider passes through the fourth photogate revealing t_{13} and t_{14} on channel two.



After the front of the glider passes through the top photogate, we have all the upward times, t_{11} to t_{20}.

Summarizing the times used to track the front of the glider passing through the photogates.



- We are interested in the time it takes the middle of the glider to pass from the first photogate to the other gates:

$$t_A = \left(\frac{G_{2in} + G_{2out}}{2}\right) - \left(\frac{G_{1in} + G_{1out}}{2}\right)$$

$$t_B = \left(\frac{G_{3in} + G_{3out}}{2}\right) - \left(\frac{G_{1in} + G_{1out}}{2}\right)$$

$$etc.$$

- Measuring from the first photogate, calculate the distance (y_A, y_B, etc) between the first gate and each subsequent photogate:

$$y_A = r_1 - r_2$$

$$y_B = r_1 - r_3$$

$$etc.$$

- Calculate the velocity of the glider as it enters each photogate:

$$v_A = y_A / t_A$$

- As in our previous Freefall experiment, acceleration can be determined by using the slope of the velocity versus time graph.

$$a_i = 2 * SLOPE(\boldsymbol{v}, \boldsymbol{t})$$

- For each trial use the set of velocities and times and the excel SLOPE() formula to find acceleration down, and then up.
- Average the calculated downward acceleration, and the average upward accelerations using the 1DStats macro, then calculate g and μ.
- Calculate the error associated with g and μ.
- Compare your experimental result with the accepted value of 980.35 cm/s^2.

Remember: Results are not complete without an estimate of the error.

The only new step in calculating error for these equations is the error in the trigonometric formulas. Since our track is elevated to such a small angle ($\theta \ll 1rad$), the small angle approximation (introduced in the Simple Pendulum experiment) applies here:

$$sin\theta \approx \theta \text{ therefore } \boldsymbol{S_{sin\theta}} = \boldsymbol{S_{\theta}}$$

Also, in the small angle approximation:

$$tan\theta \approx sin\theta \approx \theta \text{ therefore } \boldsymbol{S_{tan\theta}} = \boldsymbol{S_{\theta}}$$

Remember also to convert angles to radians when using Excel to calculate trigonometric functions.

Questions

1. (5 points) Is g sensitive to a change in angle of 0.1? Recalculate your experimental value of g by first adding, and then subtracting 0.1 to the angle of the track. How much does your experimental value change?

2. (5 points) Use the accepted value of g and your experimental results for upward and downward acceleration to calculate the inclination of the track. Report your result in degrees with three decimal places. You do not have to calculate the error for this value.

3. (10 points)
   a. What is the speed of the glider as it enters the first gate? (Recall the linearized kinematic equation.)
   b. Calculate the length of the glider used in the experiment. (Hint: We know how fast it was going as it passed through the gate, and can calculate how much time it took to pass through).

4. (10 points)
   a. Suppose the glider was released just before the first photogate, with no time to build up speed. How would that affect your measurements of time, and your final result for g?
   b. Suppose instead you push the glider down the track, increasing its initial velocity before it enters the first gate. How would this affect your measurements of time and final result for g?