Coulomb's Law and Electric Potential

Introduction

A simplified model of an atom is positively charged nucleus is surrounded by electrons. It is said to be electrically neutral when the number of electrons is equal to the number of protons. Both the protons (**positive**) and electrons (**negative**) have equal (and opposite) charge, q. So, a particle or object that has an excess of charge is said to be either positively or negatively charged. For example, protons are subatomic particles with a charge of +1, electrons have a charge of -1, and neutrons are, as you might guess, neutral, with no electric charge.

Another property of charge is that it is always conserved in an isolated system. Charge cannot be *created* or *destroyed*. However, this doesn't mean that charge can't *move*. Opposite charges will *attract* one another, while like charges *repel* each other.

Electricity is flow of electrons – **electric charge**, and the term referring to the rate of flow of electrons (aka electric charge) is called **electric current**. Current is the basis for how electric circuits, and thus how much of society, works. But *why* does charge flow?

Consider the analogy of a stream of water flowing down a hill. The water flows from a higher elevation to a lower elevation. From your studies of kinematics, you know that a difference in height means a difference in gravitational potential energy. The force of gravity then moves the water from the area of higher potential energy to the area of lower potential energy. This is similar to electric current. A difference in **electrical potential energy** (U_E)**,** brought about by a difference in **electrical potential**, V, (analogous to height in our stream example), causes an **electrostatic force**, F_e, to be exerted on the charge. The electric potential difference is called the **voltage,** and is one of the most important properties to consider when building circuits. The equation for electrical potential is

$$U_E = qV \tag{1}$$

and from it we can see that the electric potential can be thought of as the electrical potential energy per unit charge.

Let's take a closer look at the electrostatic force, so named because the particles being considered are, relative to each other, moving very slowly, if at all. How do we calculate it? Fortunately, the electrostatic force between two charged particles q_1 and q_2 can be found using **Coulomb's law**:

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \tag{2}$$

In this equation, r represents the distance between the two particles, and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is a constant called the **permittivity of free space**, or **vacuum permittivity**. The first part of the equation is often called the **Coulomb constant** k_e

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \text{ ,}$$

thus making Coulomb's law

$$F_e = k_e\frac{|q_1||q_2|}{r^2} \tag{3}$$

Note the absolute value signs around q_1 and q_2; the *magnitude* of the electric force is the same for both particles, regardless of whether they are positively or negatively charged.

But how do the particles even sense one another's presence to exert a force on them? It turns out that any charge exerts an **electric field,** *E*, on its surroundings, that causes interaction with other charges found within it. An electric field is an example of a *vector field*, with both magnitude and direction. The magnitude of the electric field of a point charge q is defined in terms of the electrostatic force that field would be exerted on a positive "test charge," q_0, introduced to it, a distance *r* from the point charge

$$E = \frac{F_e}{q_0} = k_e \frac{|q|}{r^2} \tag{4}$$

From this, we can gather that the strength of the electrostatic force depends on (and is proportional to) the strength of the electric field. The direction of the electric field of a point charge (charged particle) depends on the sign of its charge: the field points *away* from the particle if it is positive, and *towards* the particle if it is negative.

Experimental Simulator and Procedure

Experimental Simulator

- PhET Interactive Simulations: Coulomb's Law
- PhET Interactive Simulations: Charges and Fields

Procedure

Part 1: Calculation of the Coulomb constant k_e

Getting started: Go to phet.colorado.edu. On the home scree, go to the tab marked "Simulations" and select "Physics." You should then see a list of simulations. Choose the one titled "Coulomb's Law" and download the simulation. Take some time to familiarize yourself with the simulation's features before beginning the experiment.

1. Fix the values of the charges q_1 and q_2. Record the values of these charges in your data table.
2. Change the distance between the two charges, starting at a distance of 10 cm and decreasing by 1 cm at each step (9 cm, 8 cm, etc.). At each step, record the value of the electrostatic force as shown above each charge in the simulator. Record these values in your data table.

Use this as a model for your data table for Part 1

q_1=__________		**q_2=__________**	
r (cm)	**r^2 (m^2)**	**$1/r^2$ ($1/m^2$)**	**F_E (N)**
10			
9			
8			
7			
6			
5			
4			
3			

Part 2: Calculation of a point charge using the electric field

Getting started: Go to phet.colorado.edu. On the home scree, go to the tab marked "Simulations" and select "Physics." You should then see a list of simulations. Choose the one titled "Charges and Fields" and download the simulation. Take some time to familiarize yourself with the simulation's features before beginning the experiment.

1. Check the "Electric Field" and "Grid" boxes in the simulation control window. Place a single positive charge at a point in the grid (it may be easier to place the charge on the intersection of two major grid lines).
2. Check the "Values" box in the simulation control window. Place four electric field sensors (the yellow dots) around the positive charge at 0, 90, 180, and -90 degree angles. Record the values of the electric field, E, at these points.
3. Reset the simulator with the orange circular arrow button in the bottom right corner of the simulation. Repeat steps 1 and 2, this time using a negative test charge.

Part 3: Verifying the electric field formula using the inverse square law

1. Check the "Grid" and "Values" boxes in the simulation control window. It is advised to not check the "Electric field" box since the field lines may prove to be distracting. Place a positive test charge in the center of the grid. From here on, it may be useful to think of this charge to be at the origin of the grid.
2. Place eight electric field sensors at the x and y coordinates indicated in your Excel sheet. When placed, the sensors should appear to be spiraling out from the test charge.
3. Record the indicated values of the electric field in your data sheet.

Calculations and Analysis

Part 1: Calculation of the Coulomb constant k_e

1. Use Excel to plot the relationship between the electrostatic force F_e and distance r (r is the independent variable).
2. Use 2D stats on your $1/r^2$ and F_e data ($1/r^2$ is the independent variable). Calculate the value for the Coulomb constant k_e with the equation $k_e = \text{slope} \cdot (q_1 q_2)^{-1}$. Include a graph of F_e vs $1/r^2$ in your final report.
3. Calculate the error, S_{ke}, using the equation $S_{ke} = q_1 \cdot q_2 \cdot S_{slope}$ (you can check that this equation is correct by using $S_{q1} = S_{q2} = 0$).
4. Calculate the percent difference for your value of k_e.

Part 2: Calculation of a point charge using the electric field

1. Use the value for the Coulomb constant k_e that you found in part 1 to calculate the value of the positive test charge. Do this **4 times**, once for each value of E that you recorded. Use 1D stats to find the average value of this charge.
2. Repeat the calculations to find the value of the negative test charge. Use 1D stats to find the average value of this charge.

3. Calculate the percent differences of each charge. The accepted values for the positive and negative charges are 1 nC and -1 nC, respectively.

Part 3: Verifying the electric field formula using the inverse square law

1. Use the given x and y coordinates of each sensor to fill in the column *r*, the distance between the sensor and the test charge. As you have probably seen in other courses, r = SQRT($x^2 + y^2$).
2. Create a plot of E vs. r (r is the independent variable). When adding the trendline, select the "power" option from the list of trendline options. Also make sure to display the R-squared value and equation on the graph.
3. Look at the equation of the trendline of your graph. I should be of the form y = Ax^{-exp}, where the value of exp is approximately 2. This makes sense; comparing the equation of your graph to the equation for electric field, you can see that the value of A in your equation is simply the Coulomb constant multiplied by the test charge, $k_e|q|$. Record this value for $k_e|q|$ in your Excel sheet as $k_eq(3)$.
4. Using the k_e value you calculated in part1 and the q value you got in part 2, calculate a second k_eq value, marked $k_eq(1,2)$. Calculate the PFE between these two values.

Questions

1. Look at your graph of F_e vs. r (part 1). What does this say about the relationship between the electrostatic force between two charged particles and the distance between them? Can a similar statement be made regarding the relationship between the electric field of a charged particle and the distance between the particle and the point where the field was measured (look at your E vs. r graph from part 3)? Does this relationship look familiar? (If you did Experiment 1 earlier in the term…it should.) Where else does this relationship appear in physics?

The following questions use the "Charges and Fields" simulation.

2. Create an electric dipole by placing one positive and one negative charge 2 meters away from one another, positive on the left and negative on the right (the axis of the dipole passes through the centers of both charges). Using the accepted values for each charge (1 nC and -1 nC), use vector addition to verify the value of the electric field at the following points. **Show your calculations.**
   - 1 m to the left of the dipole, on the dipole axis
   - 1 m to the right of the dipole, on the dipole axis
   - In between both charges, at the center of the dipole
   - .5 m above the center of the dipole
3. Replace the negative charge on the right with a second positive charge. Repeat the calculations in question 2 to verify the value of the electric field at the same four points. **Show your calculations.**

Source:
Fundamentals of Physics, by David Halliday et al., Wiley, 2013, pp. 609–658.