The Voltage Divider

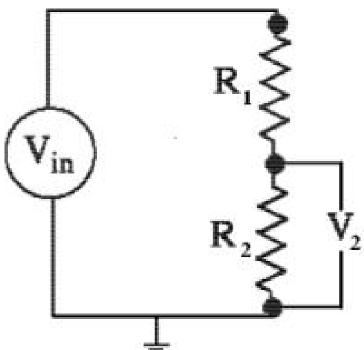
Introduction

In the previous experiment we used Kirchhoff's laws to determine the currents through each circuit element and the voltages at each point in the circuits. This week we will look at a simple circuit, the voltage divider, which is composed of two resistors in series. The voltage divider has an output voltage, V_2, which is a specific fraction of the applied voltage, V_{in}, and is determined by the resistors composing the divider.

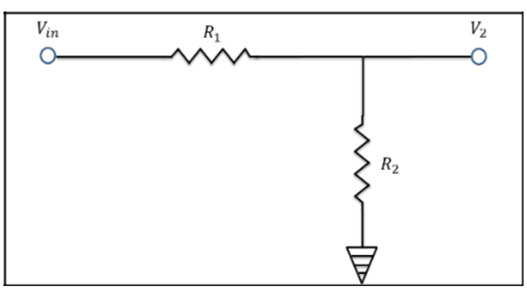
As an added complexity we will be using an oscilloscope to "see" the voltage as a function of time at various places in the circuit. The oscilloscope is one of the most flexible and useful pieces of measurement equipment available to us.

Background

The voltage divider is built by connecting two resistors in series:



An aside on circuit diagram symbols: The dashed, downward arrow at the bottom is for ground, or zero voltage. This is useful shorthand when diagraming circuits, especially as the circuits become more complex and you want to minimize the clutter all the looping to ground would produce. In this manner, the above circuit could be simplified as follows:



If we apply an input voltage V_{in} across the two resistors a current flows, which is equal to the applied voltage divided by the effective resistance of the series combination of the two resistors:

$V_{in} = V_1 + V_2$, or $V_{in} = IR_1 + IR_2$, current flow $I = \frac{V_{in}}{R_1+R_2}$.

As a consequence of the current flowing through the second resistor we can measure a voltage across it, V_2. This voltage is:

$$V_2 = IR_2 = \frac{R_2}{R_1+R_2} V_{in}.$$

V_2 is a fraction of V_{in} determined by the two resistors. Note that we are defining ground, the zero of our electrostatic potential energy, to be at the bottom of the circuit.

We can use this relationship to determine an unknown resistance $R_?$ in series with a known resistance by applying a known voltage and measuring the resulting output voltage across the unknown resistor.

$$V_2 = \frac{R_?}{R_1 + R_?} V_{in}$$

$$V_2(R_1 + R_?) = R_? V_{in}$$

$$V_2 R_1 = R_? V_{in} - R_? V_2$$

$$V_2 R_1 = R_? (V_{in} - V_2)$$

$$R_? = \frac{V_2}{(V_{in} - V_2)} R_1$$

Our experiment will involve connecting a known resistor to an unknown resistor in the voltage divider configuration and measuring V_2 for a known applied V_{in}, and using the resulting data to determine the unknown resistance. In this case, we will not be using the direct formula shown above (although it is good to know), but rather finding it as the slope of a function comparing R_1 and V_{in}/V_2. From the above equation, we can rewrite as

$$\frac{V_{in}}{V_2} = \frac{1}{R_?} R_1 + 1 \, .$$

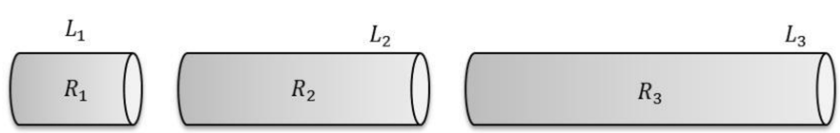
By calculating V_{in}/V_2 for various values of R_1 (V_{in} is constant, and V_2 changes with R_1) we see that the resistance of the second resistor is the inverse of the slope of the equation.

We will also be using this technique to measure the resistivity of a wire. Resistance is related to the resistivity of a material and the geometry of the material. For a sample of a given resistivity, ρ, cross sectional area, A, and length, L, the resistance is:

$$R = \rho \frac{L}{A}.$$

The resistivity of a material is much like a specific heat of a material. Both are fundamentally based on the underlying molecular properties of the material. So it is natural that a material's resistance—its tendency to hinder the flows of changed particles inside it—is dependent its resistivity.

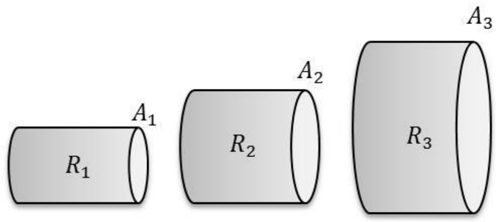
Also, when looking at that relationship, we notice that a wire's resistance is directly dependent on the length of wire being considered. In general, in a metal wire, its current's overall flow from one end to the other end becomes hindered when there are more collisions along the way. A longer wire means there are even more vibrating lattice ions for the flowing electrons to crash into, thus creating more resistance for the current.



$$L_1 < L_2 < L_3$$

$$R_1 < R_2 < R_3$$

Next, we notice the inverse relationship resistance has with the cross-sectional surface area of a wire. In essence, the more room the electrons have to flow, the less likely they are to crash into the vibrating lattice ions, thus creating less resistance for the current.



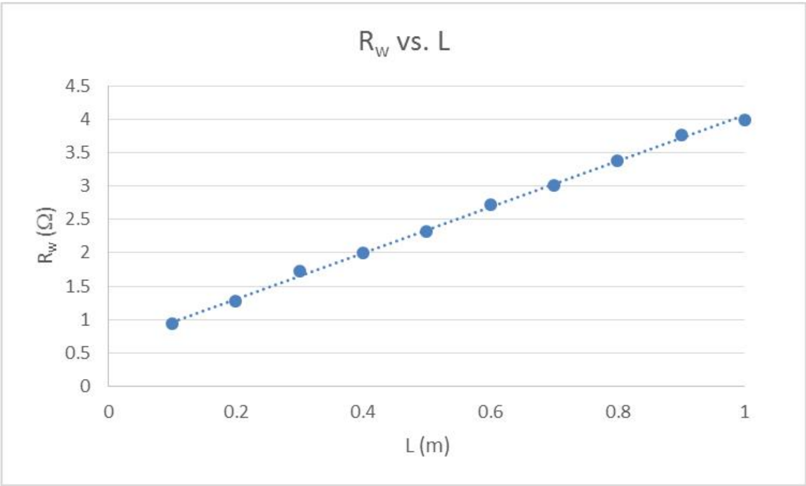
$$A_1 < A_2 < A$$

$$R_1 > R_2 > R_3$$

Our plan is to experimentally measure the resistivity of a nickel-chrome wire, which has an accepted value of $\rho_{acc} = 1.08\ \mu\Omega\ m$. We will be given the wire's diameter (0.63 mm), which will allow us to find the wire's cross sectional area, A. So, we will be considering various lengths, L, of wire and determining that particular length's resistance, R_w. Let us rearrange the equation to emphasize our linearized relationship

$$R_w = (\frac{\rho}{A})L\ .$$

In other words, our goal is to find a series of (L, R_w) pairs, giving us slope = $\rho/_A$.



Thus,

$$\rho = \text{slope} * A\ .$$

Measurements and Calculations

Part 1

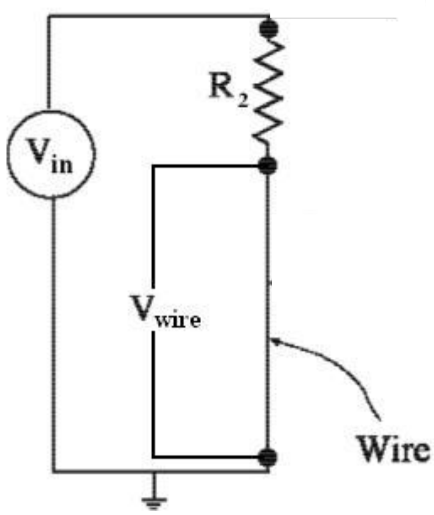
1. Use the Circuit Construction Kit: DC simulation on the PhET Interactive Simulations website to create the voltage divider shown in the diagram above. Instead of a second resistor however, use the pencil available in the simulation. The $R_?$ we wish to measure will be that of the pencil.

2. Set the V_{in} to a value of your choosing by clicking on the battery. This value should not be changed throughout the experiment.

3. You can change the value of R_1 by clicking on the resistor. Choose a value for R_1 and record it in your data sheet. The voltage across the pencil, V_2, should change. Record this value in your data sheet.
4. Repeat the previous step 9 more times, for a total of 10 R_1 values and their corresponding V_2 values.
5. For each value of V_2, calculate V_{in}/V_2. Again, V_{in} should be the same throughout.
6. Use 2D stats on your (R_1, V_{in}/V_2) pairs, and plot their relationship. Find the resistance of the pencil using the equation $R_? = slope^{-1}$. Also calculate the error, $S_{R?}$.
7. Calculate the percent difference between your calculated resistance of the pencil, $R_?$, and its accepted value, $R_{?,acc}$ (the accepted value can be seen by checking the "Values" box in the simulation control window).

Part 2

This second part of the experiment cannot be replicated using the online simulator, so some data will be provided for you. The (theoretical) circuit is shown below. It is essentially the same as the voltage divider in Part 1, however in this case the first resistor has been replaced by a different resistor with a given R value (referred to as R_2; don't get it confused with part 1!), and in place of a second resistor there is now a simple 1 m long nickel-chrome wire. The value of V_{in} will be given to you in your data sheet.



1. You will theoretically be recording the voltage along the wire, V_w, at 10 different points along its length, L (these are given to you). From these V_w values, the resistance of the wire at that point can be calculated using the equation

   $$R_w = \frac{V_w}{V_{in} - V_w} R_2$$

   (this is the same equation seen in the analysis above). Calculate these resistances for all 10 values of L.
2. Graph and perform 2D stats on your (L, R_w) pairs.
3. Using the diameter (D) of the wire provided above, calculate the cross-sectional area of the wire, $A = \pi r^2 = \pi(D/2)^2 = (\pi/4)(D)^2$.

4. Determine your experimental value for the resistivity (ρ) of the nickel-chrome wire, as well as the fractional error S_ρ, and the percent difference.

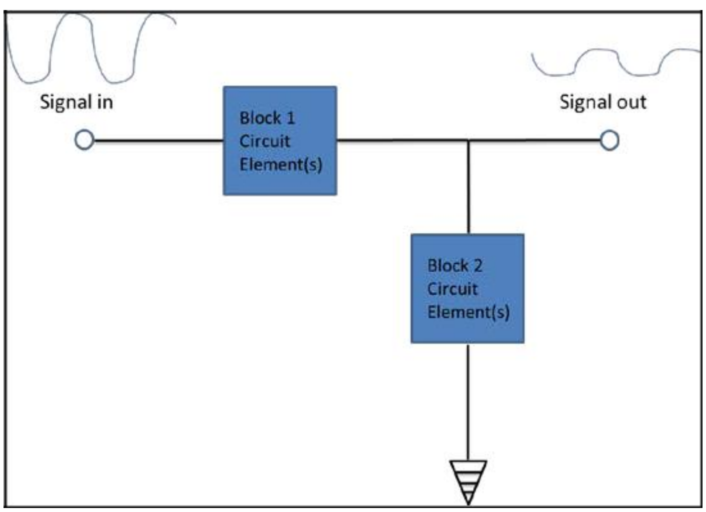
Questions

1. The resistivity of pure copper is 17 nano-Ohm-meters. How much more resistive than copper is the wire used in this experiment? Show numerically.

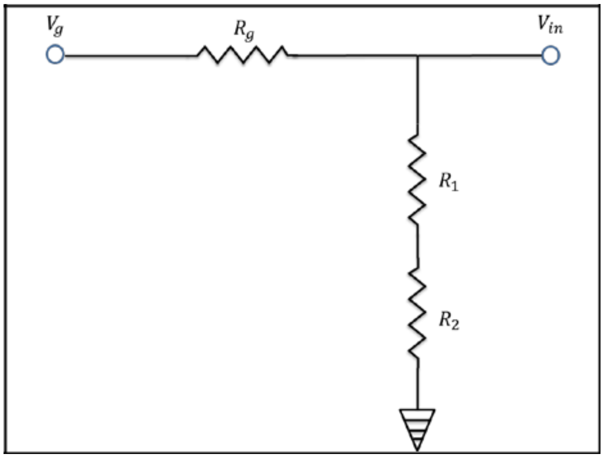
2. (Note: This question has a long preamble, but not to worry… the question itself is straightforward). In general, the devices we use in our circuits have some *internal impedance* (resistance is a specific form of impedance). When we use a current meter or power source in series (as in this experiment) with our circuit elements, we would ideally like these devices to have zero impedance, so they help us obtain measurements without introducing new impedance into the circuit. Similarly, as our voltmeters are in parallel, we would like them to have infinite impedance, for otherwise it will draw some of the current from the rest of the circuit. Of course, there is no such thing as a circuit element as having zero or infinite impedance; this is the circuit equivalent of a frictionless plane or massless rope. So, our devices themselves cause systematic errors, the impact of which depend on the impedance of other elements in the circuit.

   However, we can quantify the power source's internal impedance by treating it as a part of another voltage divider. Indeed, one of the reasons that studying voltage dividers is useful is because they are ubiquitous in circuits, even if it does not appear so at first. This is especially useful when circuits become more complex; if you can reduce part of the circuit to being a voltage divider—even if it has complex combinations of resistors, capacitors, inductors, transistors, and so on—then you can *still* use the same mathematics established in this experiment.

   The general voltage divider is as follows:

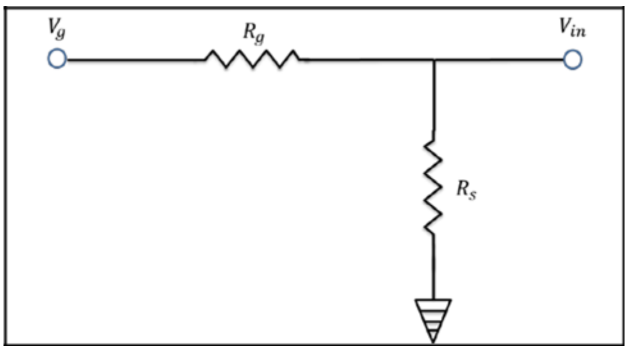


   Now we can use the flexibility of the voltage divider analysis to determine the internal impedance of the power source, R_g. The modified version of Figure 1, accounting for the power source impedance is as follows:



So “Block 1” is just R_g, and “Block 2” is $R_s = R_1 + R_2$.

Thus, we have:



Now we can use the same analysis as before! So

$$R_s = \frac{V_{in}}{V_g - V_{in}} R_g$$

Solving for the power source impedance, we get:

$$R_g = \frac{V_g - V_{in}}{V_{in}} R_s$$

a) In Part 1 did you find that V_{in} is less than V_g? If not, what does this tell you about the impedance of the power source in relation to the precision of your voltage measurements?

b) Depending on the circuit design, the impact of the device impedance in this scenario may be very large, or it may be negligible. Using V_g of 8 volts, what would V_{in} be if $R_s = .01R_g$? Or what if $R_s = 100R_g$? Can you make a general statement?