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9. Revised Simplex method example (Enter your problem)

1. Standard form-1 : Example-1
2. Standard form-1 : Example-2
3. Standard form-1 : Example-3
4. Standard form-2 using Two-Phase method : Example-1
5. Standard form-2 using Two-Phase method : Example-2
6. Standard form-2 using Two-Phase method : Example-3
7. Standard form-2 using Big M method : Example-1
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Other related methods

0. Formulate linear programming model
1. Graphical method
2. Simplex method (BigM method)
3. Two-Phase method
4. Primal to dual conversion
5. Dual simplex method
6. Integer simplex method
7. Branch and Bound method
8. 0-1 Integer programming problem
9. Revised Simplex method

4. Standard form-2 using Two-Phase method : Example-1 (Previous example)

6. Standard form-2 using Two-Phase metho (N

5. Standard form-2 using Two-Phase method : Example-2

Find solution using Revised Simplex (Two-Phase) method
MIN Z = x1 + x2
subject to
x1 + 2x2 >= 7
4x1 + x2 >= 6
and x1,x2 >= 0

Solution:
Problem is

Min $Z = x_1 + x_2$

subject to

$x_1 + 2x_2 \geq 7$

$4x_1 + x_2 \geq 6$

and $x_1, x_2 \geq 0;$

(Enter your problem)

$\therefore$ Max $Z = - x_1 - x_2$

Step-1 :
The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

After introducing surplus,artificial variables

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

After introducing surplus,artificial variables

$$
\begin{array}{rrrrrrrrr}
Z' + & x_1 + & x_2 & & & & & & = 0 \\
- & 5x_1 - & 3x_2 + & S_1 + & S_2 & & & + x_5 & = -13 \\
& x_1 + & 2x_2 - & S_1 & & + A_1 & & & = 7 \\
& 4x_1 + & x_2 & & - S_2 & & + A_2 & & = 6
\end{array}
$$

Now represent the new system of constraint equations in the matrix form

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & -3 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} Z' \\ x_1 \\ x_2 \\ S_1 \\ S_2 \\ A_1 \\ A_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ -13 \\ 7 \\ 6 \end{bmatrix}
$$

or

$$
\begin{bmatrix} 1 & -c \\ 0 & A \end{bmatrix} \begin{bmatrix} Z \\ x \end{bmatrix} = \begin{bmatrix} 0 & b \end{bmatrix}; x \geq 0
$$

where $e = \beta_0, a_4 = \beta_1, a_5 = \beta_2, a_6 = \beta_3$

Step-2 : The basis matrix B_1 of order $(3 + 1) = 4$ can be expressed as

$$
B_1 = \left[\beta_0, \beta_1, \beta_2, \beta_3\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Then, $B_1^{-1} = \begin{bmatrix} 1 & C_B B^{-1} \\ 0 & B^{-1} \end{bmatrix} = 1; B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left[\beta_1, \beta_2, \beta_3\right]; C_B = [0, 0, 0]$

Phase-1

		Basis Inverse B_1^{-1}						Additional	
B	**X_B**	**β_0 Z'**	**β_1 x_5**	**β_2 A_1**	**β_3 A_2**	**y_1 $C_k - Z_k$**	**Min Ratio $\frac{X_B}{y_1}$**	**x_1**	**x_2**
Z'	0	1	0	0	0		---	1	1
x_5	-13	0	1	0	0		---	-5	-3
A_1	7	0	0	1	0		---	1	2
A_2	6	0	0	0	1		---	4	1

Iteration=1 : Repeat steps 3 to 5 to get new solution

$$= \text{Min}\left\{\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 0 & 0\\-5 & -3 & 1 & 1\\1 & 2 & -1 & 0\\4 & 1 & 0 & -1\end{bmatrix}\right\}$$

$$= \text{Min}\left\{\begin{bmatrix}-5 & -3 & 1 & 1\end{bmatrix}\right\}$$

$= -5$ (correspnds to $z_1 - c_1$)

Thus, vector x_1 is selected to enter into the basis, for $k = 1$

Step-4: To select a basic variable to leave the basis, we compute y_k for k=1, as follows

$$y_1 = B_1^{-1} a_1 = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\-5\\1\\4\end{bmatrix} = \begin{bmatrix}1\\-5\\1\\4\end{bmatrix}$$

and $X_B = \begin{bmatrix}0\\-13\\7\\6\end{bmatrix}$

Now, calculate the minimum ratio to select the basic variable to leave the basis

$$\frac{x_{Br}}{y_{rk}} = \text{Min}\left\{\frac{x_{Bi}}{y_{ik}}, y_{ik} > 0\right\}$$

$$= \text{Min}\left\{\frac{0}{1}, \frac{7}{1}, \frac{6}{4}\right\}$$

$$= \text{Min}\left\{0, 7, \frac{3}{2}\right\}$$

$$= \frac{3}{2}\left(\text{correspnds to } \frac{x_{B3}}{y_{31}}\right)$$

Thus, vector A_2 is selected to leave the basis, for $r = 3$

The table with new entries in column y_1 and the minimum ratio

		Basis Inverse B_1^{-1}						Additional	
B	X_B	β_0 Z'	β_1 x_5	β_2 A_1	β_3 A_2	y_1 $C_k - Z_k$	**Min Ratio** $\frac{X_B}{y_1}$	x_1	x_2
Z'	0	1	0	0	0	1	0	1	1
x_5	-13	0	1	0	0	-5	---	-5	-3
A_1	7	0	0	1	0	1	7	1	2

(**Enter your problem**)

For this we apply the following row operations in the same way as in the simplex method

	X_B	β_1	β_2	β_3	y_1
R_1	0	0	0	0	1
R_2	-13	1	0	0	-5
R_3	7	0	1	0	1
R_4	6	0	0	1	4

+ R_4(new) $=$ R_4(old) $\div 4$

+ R_1(new) $=$ R_1(old) - R_4(new)

+ R_2(new) $=$ R_2(old) + $5R_4$(new)

+ R_3(new) $=$ R_3(old) - R_4(new)

The improved solution is

		Basis Inverse B_1^{-1}						Additional	
B	**X_B**	**β_0 Z'**	**β_1 x_5**	**β_2 A_1**	**β_3 x_1**	**y_1 $C_k - Z_k$**	**Min Ratio $\frac{X_B}{y_1}$**	**A_2**	**x_2**
Z'	$-\frac{3}{2}$	1	0	0	$-\frac{1}{4}$		---	0	1
x_5	$-\frac{11}{2}$	0	1	0	$\frac{5}{4}$		---	0	-3
A_1	$\frac{11}{2}$	0	0	1	$-\frac{1}{4}$		---	0	2
x_1	$\frac{3}{2}$	0	0	0	$\frac{1}{4}$		---	1	1

Iteration=2 : Repeat steps 3 to 5 to get new solution
Step-3: To select the vector corresponding to a non-basic variable to enter into the basis, we compute

$$z_k - c_k = \text{Min}\left\{\left(z_j - c_j\right) < 0;\ \right\}$$

$$= \text{Min}\left\{\left(2^{nd} \text{ row of } B_1^{-1}\right)\left(\text{Columns } a_j \text{ not in basis}\right)\right\}$$

$$= \text{Min}\left\{\begin{bmatrix} 0 & 1 & 0 & \frac{5}{4} \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}\right\}$$

(**Enter your problem**)

Thus, vector x_2 is selected to enter into the basis, for $k = 2$

Step-4: To select a basic variable to leave the basis, we compute y_k for k=2, as follows

$$y_2 = B_1^{-1} a_2 = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{4} \\ 0 & 1 & 0 & \frac{5}{4} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ -\frac{7}{4} \\ \frac{7}{4} \\ \frac{1}{4} \end{bmatrix}$$

and $X_B = \begin{bmatrix} -\frac{3}{2} \\ -\frac{11}{2} \\ \frac{11}{2} \\ \frac{3}{2} \end{bmatrix}$

Now, calculate the minimum ratio to select the basic variable to leave the basis

$$\frac{x_{Br}}{y_{rk}} = \text{Min} \left\{ \frac{x_{Bi}}{y_{ik}}, y_{ik} > 0 \right\}$$

$$= \text{Min} \left\{ \frac{-\frac{3}{2}}{\frac{3}{4}}, \frac{\frac{11}{2}}{\frac{7}{4}}, \frac{\frac{3}{2}}{\frac{1}{4}} \right\}$$

$$= \text{Min} \left\{ -2, \frac{22}{7}, 6 \right\}$$

$$= \frac{22}{7} \left(\text{correspnds to } \frac{x_{B2}}{y_{22}} \right)$$

Thus, vector A_1 is selected to leave the basis, for $r = 2$

The table with new entries in column y_2 and the minimum ratio

		Basis Inverse B_1^{-1}					(Enter your problem)	Additional	
B	X_B	β_0 Z'	β_1 x_5	β_2 A_1	β_3 x_1	y_2 $C_k - Z_k$	**Min Ratio** $\frac{X_B}{y_2}$	A_2	x_2

	3				1	3				
A_1	$\frac{11}{2}$	0	0	1	$-\frac{1}{4}$	$\frac{7}{4}$	$\frac{22}{7}$	0	2	
x_1	$\frac{3}{2}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	6	1	1	

The table solution is now updated by replacing variable A_1 with the variable x_2 into the basis.

For this we apply the following row operations in the same way as in the simplex method

	X_B	β_1	β_2	β_3	y_2
R_1	$-\frac{3}{2}$	0	0	$-\frac{1}{4}$	$\frac{3}{4}$
R_2	$-\frac{11}{2}$	1	0	$\frac{5}{4}$	$-\frac{7}{4}$
R_3	$\frac{11}{2}$	0	1	$-\frac{1}{4}$	$\frac{7}{4}$
R_4	$\frac{3}{2}$	0	0	$\frac{1}{4}$	$\frac{1}{4}$

+ R_3(new) = R_3(old) $\times \frac{4}{7}$

+ R_1(new) = R_1(old) - $\frac{3}{4}R_3$(new)

+ R_2(new) = R_2(old) + $\frac{7}{4}R_3$(new)

+ R_4(new) = R_4(old) - $\frac{1}{4}R_3$(new)

The improved solution is

		Basis Inverse B_1^{-1}						Additional		
B	X_B	β_0 Z'	β_1 x_5	β_2 x_2	β_3 x_1	y_2 $C_k - Z_k$	**Min Ratio** $\frac{X_B}{y_2}$	A_2	A_1	
Z'	$-\frac{27}{7}$	1	0	$-\frac{3}{7}$	$-\frac{1}{7}$		---	0	0	
x_5	0	0	1	1	1		---	0	0	
x_2	$\frac{22}{7}$	0	0	$\frac{4}{7}$	$-\frac{1}{7}$		---	0	1	
x_1	$\frac{5}{7}$	0	0	$-\frac{1}{7}$	$\frac{2}{7}$		--- (**Enter your problem**)	1	0	

$x_5 = 0$

Phase-2

B	X_B	β_0 Z'	β_1 x_5	β_2 x_2	β_3 x_1	y_2 $C_k - Z_k$	Min Ratio $\frac{X_B}{y_2}$	S_1
Z'	$-\frac{27}{7}$	1	0	$-\frac{3}{7}$	$-\frac{1}{7}$		---	0
x_5	0	0	1	1	1		---	1
x_2	$\frac{22}{7}$	0	0	$\frac{4}{7}$	$-\frac{1}{7}$		---	-1
x_1	$\frac{5}{7}$	0	0	$-\frac{1}{7}$	$\frac{2}{7}$		---	0

Iteration=1 : Repeat steps 3 to 5 to get new solution
Step-3: To select the vector corresponding to a non-basic variable to enter into the basis, we compute

$$z_k - c_k = \text{Min}\left\{ \left(z_j - c_j\right) < 0; \right\}$$

$$= \text{Min}\left\{ \left(1^{st} \text{ row of } B_1^{-1}\right)\left(\text{Columns } a_j \text{ not in basis}\right) \right\}$$

$$= \text{Min}\left\{ \begin{bmatrix} 1 & 0 & -\frac{3}{7} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$$

$$= \text{Min}\left\{ \begin{bmatrix} \frac{3}{7} & \frac{1}{7} \end{bmatrix} \right\}$$

Since all $Z_j - C_j \ge 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = \frac{5}{7}, x_2 = \frac{22}{7}$

Max $Z = -\frac{27}{7}$

$\therefore$ Min $Z = \frac{27}{7}$

4. Standard form-2 using Two-Phase method : Example-1
(Previous example)

6. Standard form-2 using Two-Phase metho
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