How to Design 10 kHz filter.

(Using Butterworth filter design)

Application notes.

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This application note describes how to build a 5^{th} order low pass, high pass Butterworth filter for 10 kHz signal frequency. The frequency response of the Butterworth Filter approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible in the cut-off frequency range. This filter will pass all frequency signals in the cut-off range but attenuate signals with frequencies higher and lower than the cut-off frequency.

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Introduction

Filters are essential to the operation of most electronic circuits. Filter is a device or process that removes unwanted components or features from a transmitted signal. Most often, this means removing some frequencies and not other to suppress interfering signals and reduce background noise. Filters are classified according to the functions that they are to perform, in terms of ranges of frequencies. In circuit theory, a filter is an electrical network that alters the amplitude and/or phase characteristics of a signal with respect to frequency. Ideally, a filter will not add new frequencies to the input signal, nor will it change the component frequencies of that signal, but it will change the relative amplitudes of the various frequency components and/or their phase relationships. Filters are often used in electronic systems to emphasize signals in certain frequency ranges and reject signals in other frequency ranges.

Background:

The Butterworth filter is a type of signal processing filter designed to have as flat frequency response as possible (no ripples) in the pass-band and zero roll off response in the stop-band. Butterworth filters are one of the most commonly used digital filters in motion analysis and in audio circuits. They are fast and simple to use. Since they are frequency-based, the effect of filtering can be easily understood and predicted. Choosing a cutoff frequency is easier than estimating the error involved in the raw data in the spline methods. However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. The ideal frequency response, referred to as a "brick wall" filter *Figure 1*.

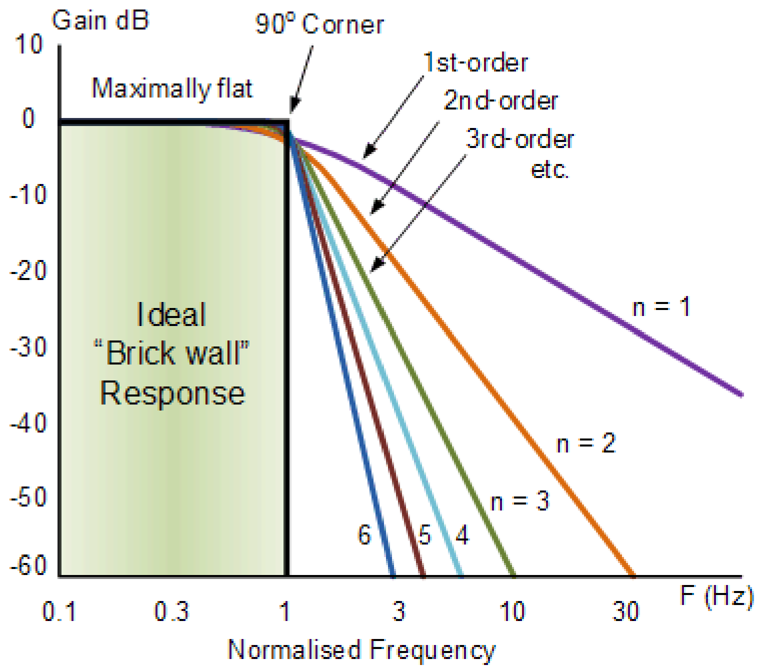
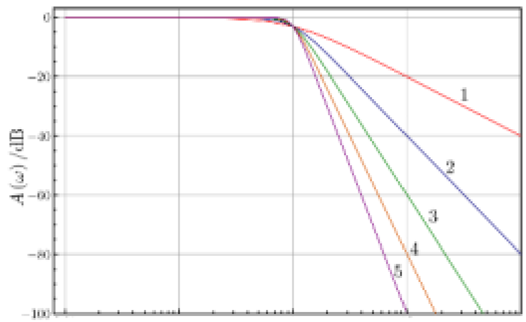


Figure 1: Butterworth Lowpass filter

NOTE: That the higher the Butterworth filter order, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal "brick wall" response. However, in practice this "ideal" frequency response is unattainable as it produces excessive passband ripple.

Procedure

Low pass filter will be used to remove all high order frequencies up to 10.25 kHz from the inputted signal as demonstrated in *Figure 2 (purple line)*.



f_c = 10.25

Figure 2: *Lowpass filter*

Equation 1 is used to calculate capacitor values for the lowpass filter side.

$$Ck = \frac{Dk}{m * 2 * \pi * fc} \qquad (Equation\ 1)$$

Where:

k= order number

f_c = cutoff frequency = 10.25 kHz.

m = magnitude coefficient = R = 10kΩ (standard audio resistance value from 10kΩ to 1MΩ)

D_k = normalized capacitor value. D_k is obtained from a normalized second order low pass polynomial function H(s)=1/Dk (*Table1*).

n	Normalised Denominator Polynomials in Factored Form
1	$(1+s)$
2	$(1+1.414s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.765s+s^2)(1+1.848s+s^2)$
5	$(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$
6	$(1+0.518s+s^2)(1+1.414s+s^2)(1+1.932s+s^2)$
7	$(1+s)(1+0.445s+s^2)(1+1.247s+s^2)(1+1.802s+s^2)$
8	$(1+0.390s+s^2)(1+1.111s+s^2)(1+1.663s+s^2)(1+1.962s+s^2)$
9	$(1+s)(1+0.347s+s^2)(1+s+s^2)(1+1.532s+s^2)(1+1.879s+s^2)$
10	$(1+0.313s+s^2)(1+0.908s+s^2)(1+1.414s+s^2)(1+1.782s+s^2)(1+1.975s+s^2)$

Table 1: *Normalized Denominator Polynomials.*

Using *Table 1* for a fifth order polynomial five poles were obtained from which normalized capacitor (D_k) values were calculated (*Table 2*).

n	D1	D2	D3	D4	D5	D6	D7
2	1.414	0.7071					
3	3.546	1.392	0.2024				
4	1.082	0.9241	2.613	0.3825			
5	1.753	1.354	0.4214	3.235	0.3089		
6	1.035	0.966	1.4144	0.7071	3.863	0.2588	
7	1.531	1.336	0.4885	1.604	0.6235	4.493	0.2225

Table 2: Calculated Normalized capacitor values

Next, high pass filter is designed to attenuate frequencies from 0 to 9.75 kHz. Cut-off frequency is set to 9.75 kHz and standard capacitor value for audio circuit design chosen to be 0.01 micro Farads. To calculate Resistor values for High pass filter *Equation 2* is used.

$$Rk = \frac{m}{Dk} \qquad (Equation\ 2)$$

Where:

m = magnitude coefficient $= \frac{1}{2*\pi*fc*Cs} = 1720.594$

f_c = 9.75 kHz

Cs = 0.01 micro Farads

D_k = 5^{th} order normalized capacitor values (*Table 2*)

Required Components/Parts

4 x LF411 CN: Operational Amplifier.

1 x Audio input jack.

1 x C_o = 0.01 microFarads

1 x R_o = 1 kΩ

$$C1 = \frac{D1}{m * 2 * \pi * fc} = \frac{1.753}{10k * 2 * \pi * 10.25}$$

1 x $C1 = 2.862\,nF \;\approx 3nF$

1 x $C2 = 2.2102\,nF \;\approx 2nF$

1 x $C3 = 687.876\,pF \;\approx 680\,pF$

1 x $C4 = 5.281\,nF \;\approx 5\,nF$

1 x $C5 = 504.236\,pF \;\approx 470\,pF$

$$R1 = \frac{m}{D1} = \frac{1720.594}{1.753}$$

1 x $R1 = 981.514\,\Omega \;\approx 1\,k\Omega$

1 x $R2 = 1.271\,k\Omega \;\approx 1.2\,k\Omega$

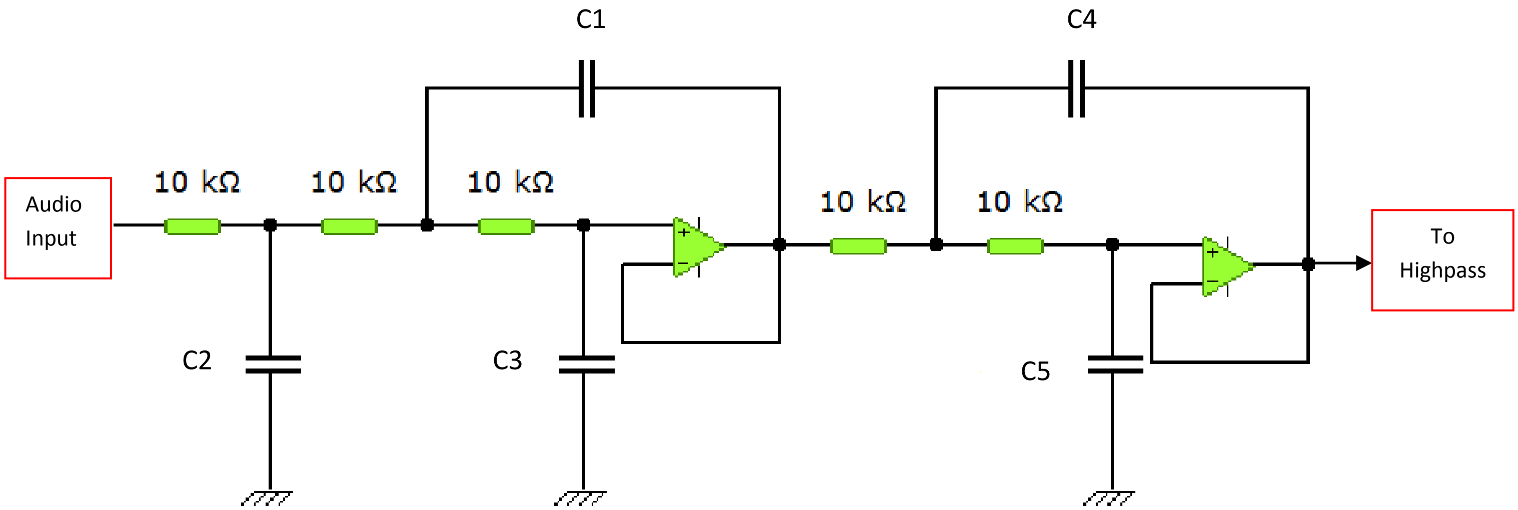
1 x $R3 = 4.083\,k\Omega \;\approx 3.9\,k\Omega$

1 x $R4 = 531.868\,\Omega \;\approx 560\,\Omega$

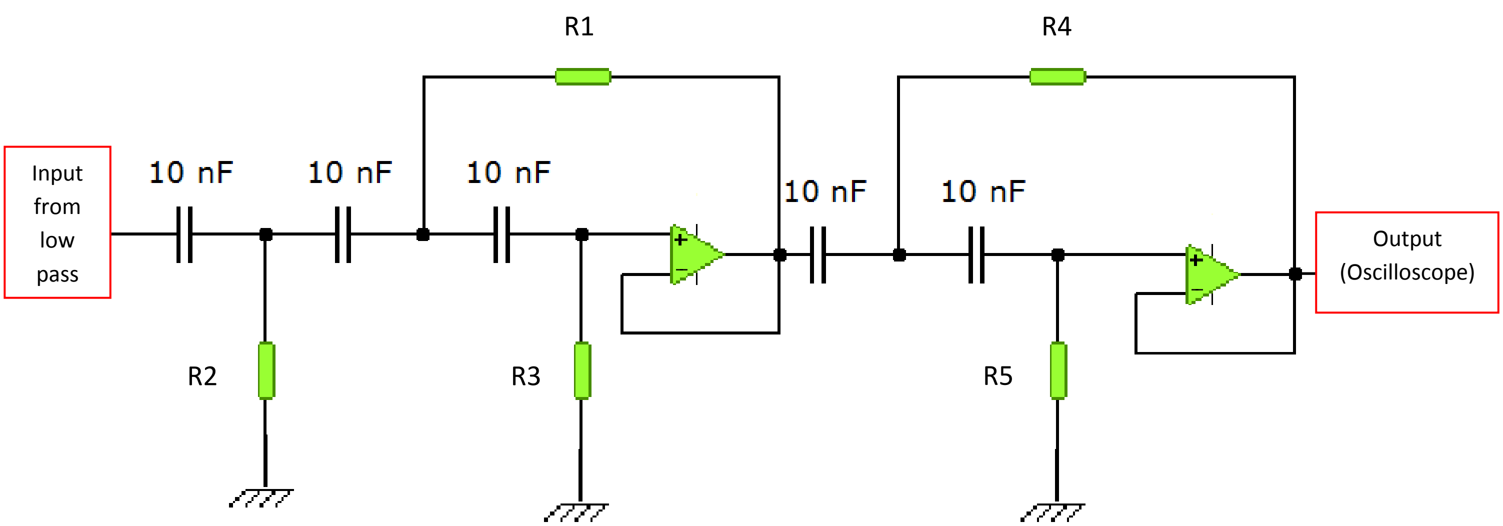
1 x $R5 = 5.570\,k\Omega \;\approx 5.6\,k\Omega$

Circuit Schematics

Lowpass Butterworth Filter.

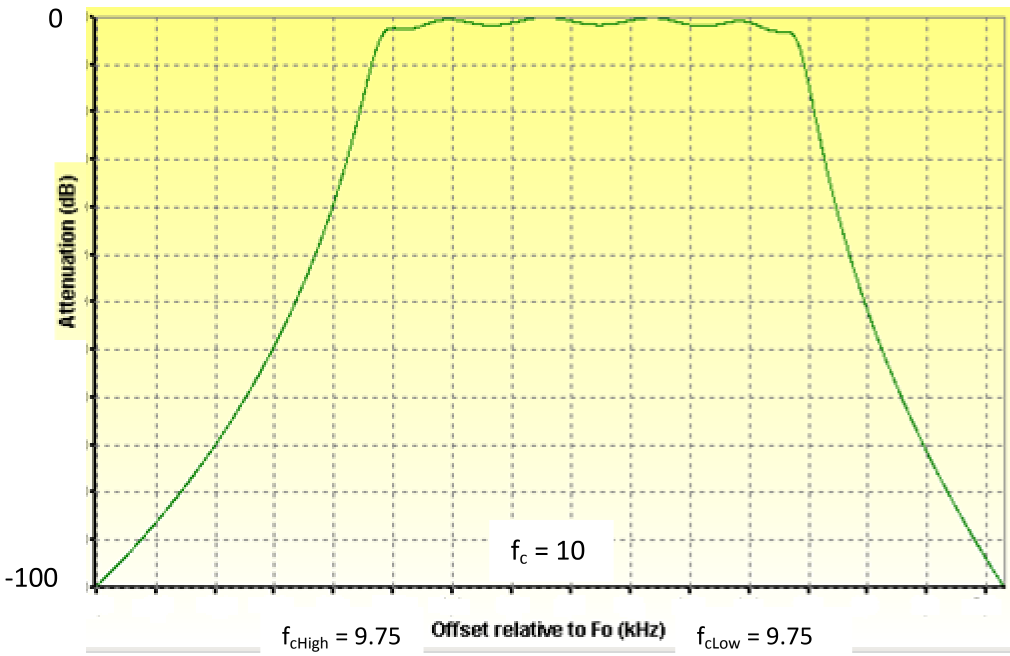


Highpass Butterworth Filter.



Graph

Graph below shows an output of an audio signal with variation of high, low noise frequencies. 10kHz Butterworth filter attenuates frequencies above and below cut-off frequency.



Conclusion

This application note demonstrates how to build 10 kHz filter applying Butterworth filter design. Using the same techniques even better filter can be obtained. By increasing design order, even more noise frequencies can be eliminated from the inputted signal. Also, low noise operational amplifier can be used to remove additional background noises. Butterworth filters are fast and simple to use. They are frequency-based and the effect of filtering can be easily understood and predicted. Choosing a cutoff frequency is easier than estimating the error involved in the raw data in the spline methods.

Reference

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