# **Heap Sort**

# (heap\_sort.cpp/c)

Time Limit: 1 sec , Memory Limit: 262144 KB

There are a number of sorting algorithms which have different characteristics as follows.

Algorithm	Time Complexity (Worst)	Time Complexity (Average)	Stability	Memory Efficiency	Method	Features
Insertion Sort ALDS1_1_A	O(N <sup>2</sup> )	O(N <sup>2</sup> )	0	0	Insertion	Can be fast for an almost sorted array
Bubble Sort ALDS1_2_A	O(N <sup>2</sup> )	O(N <sup>2</sup> )	0	0	Swap	
Selection Sort ALDS1_2_B	O(N <sup>2</sup> )	O(N <sup>2</sup> )	×	0	Swap	
Shell Sort ALDS1_2_D	O(N <sup>2</sup> )	O(NlogN)	×	0	Insertion	
Merge Sort ALDS1_5_A	O(NlogN)	O(NlogN)	0	×	Divide and Conqure	Fast and stable. It needs an external memory other than the input array.
Counting Sort ALDS1_6_A	O(N + K)	O(N + K)	0	×	Bucket	Fast and stable. There is a limit to the value of array elements.
Quick Sort ALDS1_6_B	O(N <sup>2</sup> )	O(NlogN)	×	Δ	Divide and Conqure	Fast and almost-in-place if it takes a measure for the corner cases. Naive implementation can be slow for the corner cases.
Heap Sort ALDS1_9_D (This problem)	O(NlogN)	O(NlogN)	×	0	Heap structure	Fast and in-place. It is unstable and performs random access for non-continuous elements frequently.

The Heap Sort algorithms is one of fast algorithms which is based on the heap data structure. The algorithms can be implemented as follows.

```
1 maxHeapify(A, i)
    1 = left(i)
3
       r = right(i)
       // select the node which has the maximum value
4
5
       if 1 \leq \text{heapSize} and A[1] > A[i]
6
           largest = 1
7
       else
8
          largest = i
9
       if r \leq \text{heapSize} and A[r] > A[largest]
           largest = r
10
11
       if largest \neq i
12
13
           swap A[i] and A[largest]
           maxHeapify(A, largest)
14
15
16 heapSort(A):
17
       // buildMaxHeap
18
       for i = N/2 downto 1:
19
           maxHeapify(A, i)
20
       // sort
21
       heapSize ← N
22
       while heapSize \geq 2:
23
           swap(A[1], A[heapSize])
24
          heapSize--
25
           maxHeapify(A, 1)
```

On the other hand, the heap sort algorithm exchanges non-continuous elements through random accesses frequently.

Your task is to find a permutation of a given sequence A with N elements, such that it is a max-heap and when converting it to a sorted sequence, the total number of swaps in maxHeapify of line 25 in the pseudo code is maximal possible.

# Input (heap\_sort.in)

In the first line, an integer N is given. In the next line,  $A_i$  ( $i=0,1,\ldots,N-1$ ) are given separated by a single space character.

# Output (heap\_sort.out)

Print N integers of the permutation of A separated by a single space character in a line.

This problem has multiple solutions and the judge will be performed by a special validator.

#### Constraints

- $1 \le N \le 200000$
- $0 \le \text{each element of } A \le 1000000000$
- The elements of A are all different

# Sample Input and Output

# Sample Input 1

8 1 2 3 5 9 12 15 23

# Sample Output 1

23 9 15 2 5 3 12 1