# POJ 2663 Tri Tiling

# **Description**

In how many ways can you tile a 3xn rectangle with 2x1 dominoes? Here is a sample tiling of a 3x12 rectangle.



### **Input**

Input consists of several test cases followed by a line containing -1. Each test case is a line containing an integer  $0 \le n \le 30$ .

## **Output**

For each test case, output one integer number giving the number of possible tilings.

# **Sample Input**

2 8 12 -1

# **Sample Output**

3 153 2131

### **Source**

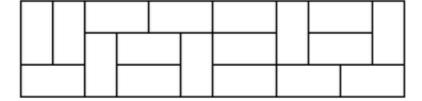
Waterloo local 2005.09.24

动态规划的一道题

根据Stanford CS97SI课件

# POJ 2663: Tri Tiling

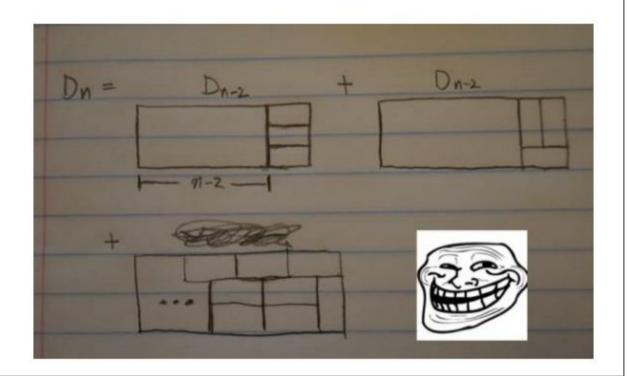
- $\square$  Given n, find the number of ways to fill a  $3 \times n$  board with dominoes
- $\ \square$  Here is one possible solution for n=12



# POJ 2663: Tri Tiling

- Define subproblems
  - lacksquare Define  $D_n$  as the number of ways to tile a 3 imes n board
- □ Find recurrence
  - □ Uuuhhhhh...

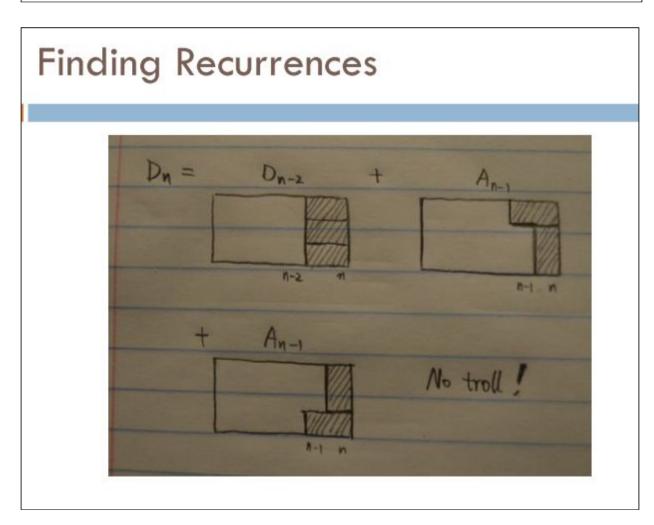
# **Troll Tiling**



# Defining Subproblems

- Obviously, the previous definition didn't work very well
  - $\ \square \ D_n$ 's don't relate in simple terms
- What if we introduce more subproblems?

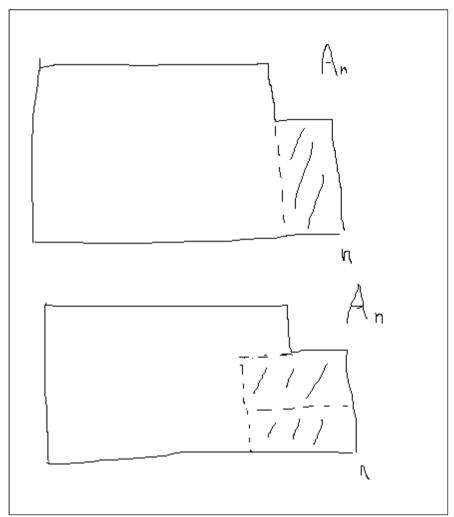
# Defining Subproblems



# Finding Recurrences

- □ Consider different ways to fill the *n*th column
  - And see what the remaining shape is
- Exercise:
  - $\square$  Finding recurrences for  $A_n$ ,  $B_n$ ,  $C_n$
  - lacksquare Understanding why  $E_n$  is always zero
- □ Extension: solving the problem for  $n \times m$  grids, where n is small ( $n \le 10$ )
  - How many subproblems should we consider?

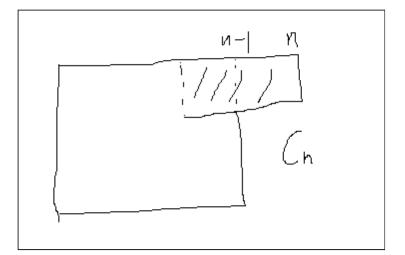
然后我们再分析 A(n)的情况:



可以得到:有两种排列方式

A(n)=D(n-1)+C(n-1)

再分析C(n):



可得:

### C(n)=A(n-1)

所以,我们有3个公式:

D(n)=D(n-2)+2\*A(n-1)

A(n)=D(n-1)+C(n-1)

C(n)=A(n-1)

### 化简第一个公式得:

D(n)=D(n-2)+2\*A(n-1)

= D(n-2)+2\*(D(n-2)+C(n-2))

= D(n-2)+2\*(D(n-2)+A(n-3))

= 3\*D(n-2)+2\*A(n-3)

= 3\*D(n-2)+2\*(D(n-4)+C(n-4))

= 3\*D(n-2)+2\*D(n-4)+2\*C(n-4)

= 3\*D(n-2)+2\*D(n-4)+2\*A(n-5)

= 3\*D(n-2)+2\*D(n-4)+2\*(D(n-6)+C(n-6))

= 3\*D(n-2)+2\*D(n-4)+2\*D(n-6)+2\*C(n-6)

= . . .

### 其中base case有:

D(0)=1, D(2)=3

```
#define RUN
#ifdef RUN
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <assert.h>
#include <string>
#include <iostream>
#include <sstream>
#include <map>
#include <set>
#include <vector>
#include <list>
#include <cctype>
#include <algorithm>
#include <utility>
#include <math.h>
using namespace std;
int n;
int buf[31];
int main(){
        memset(buf, sizeof(buf), 0);
        buf[0] = 1;
        buf[2] = 3;
        while(scanf("%d",&n)==1 && n!=-1){
                if(n%2 != 0){
                        printf("0\n");
                        continue;
                }
                for(int i=4; i<=n; i+=2){
                        int tmp = 3*buf[i-2];
                        for(int j=4; j<=i; j+=2){
                               tmp += 2*buf[i-j];
                        buf[i] = tmp;
               }
                printf("%d\n", buf[n]);
        }
}
#endif
```