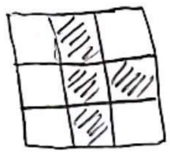
Re-exam 2 : FFR135

[redacted]

1) Convolutional net

Choose the following filter :



Legend:

□ corresponds to $w = 0$

■ corresponds to $w = 1$

- Pattern $x^{(1)}$ can be represented as:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

The local fields of the feature map of pattern $x^{(1)}$ can be determined as follows (assume the threshold θ is zero) :

Consider the top left 3x3 block of the inputs

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{3x3 block of inputs}} \underset{\text{multiply}}{\times} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{kernel}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{sum everything}} 2$$

[redacted]

Sweep the kernel over the 2-D input matrix with stride [1,1], padding 0
It follows that the local fields of the feature map of pattern $x^{(1)}$ are:

$$\begin{bmatrix} 2 & 2 \\ 1 & 2 \\ 1 & 1 \\ 1 & 2 \\ 2 & 2 \\ 1 & 2 \end{bmatrix}$$

- Now, pattern $x^{(2)}$ can be represented as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Similarly, the local fields of the feature map of pattern $x^{(2)}$ can be determined as follows:

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{\text{3x3 block of inputs}} \underset{\text{multiply}}{\times} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{kernel}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{} \underset{\text{sum everything}}{2}$$

[redacted]

Sweep the kernel over the 2-D input matrix with stride [1, 1], and padding [0, 0, 0, 0]. It follows that the local fields of the feature map of pattern $x^{(2)}$ are:

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

The ReLU - activation function does not exert any effect, since all local fields are possible. Thus, the feature maps are therefore equal to the above local fields.

- Apply max-pooling operation on the resulting feature map of pattern $x^{(1)}$.

$$\begin{bmatrix} 2 & 2 \\ 1 & 2 \\ 1 & 1 \\ 1 & 2 \\ 2 & 2 \\ 1 & 2 \end{bmatrix}$$

For 2×2 max pooling, the maximum element within the top 2×2 block is 2

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Sweep over the resulting feature map with stride 2 and padding 0

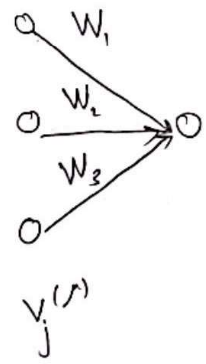
The output of the max-pooling layer is:

$$\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

- Similarly for pattern $x^{(2)}$, the output of the max-pooling layer is

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

- To determine the weights W_k and threshold Θ, the layout of the output of the network can be represented as follows:



$$O^{(\mu)} = \theta_H\left(\sum_{j=1}^{3} W_j V_j^{(\mu)} - \Theta\right)$$

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where

$$V^{(1)} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \quad \text{for pattern } x^{(1)}$$

$$V^{(2)} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \quad \text{for pattern } x^{(2)}$$

To classify the 2 patterns we need the output of the network to be

$$O^{(1)} = 0 \quad \text{for} \quad x^{(1)}$$

$$O^{(2)} = 1 \quad \text{for} \quad x^{(2)}$$

This can be done by setting the output of the network to be

$$O^{(\mu)} = \Theta_H \left(- V_1^{(\mu)} - V_2^{(\mu)} - V_3^{(\mu)} + 5 \right)$$

where $W = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$ and $\Theta = -5$

In this case we get

For $\mu = 1$: $O^{(1)} = \Theta_H(-2-2-2+5) = \Theta_H(-1) = 0$

For $\mu = 2$: $O^{(2)} = \Theta_H(-2-1-1+5) = \Theta_H(+1) = 1$ Verified