In[]:=
```
Clear["Global`*"]
ω = {4, -1};
fx = D[{-x^3, -x^2}, x];
fxx = D[fx, x];
fxxx = D[fxx, x];
fxy = D[fx, y];
fxyy = D[fxy, y];
fy = D[{-x^3, -x^2}, y];
fyy = D[fy, y];
gx = D[ {2 y^3, 2 x^2}, x];
gxx = D[gx, x];
gxxy = D[gxx, x];
gxy = D[gx, y];
gy = D[{2 y^3, 2 x^2}, y];
gyy = D[gy, y];
gyyy = D[gyy, y];
Solve[
 16 a == fxxx[[1]] + fxyy[[1]] + gxxy[[1]] + gyyy[[1]] + 1 / ω[[1]] * (fxy[[1]] * (fxx[[1]] + fyy[[1]]) -
       gxy[[1]] * (gxx[[1]] + gyy[[1]]) - fxx[[1]] * gxx[[1]] + fyy[[1]] * gyy[[1]]), a]
Solve[16 a == fxxx[[2]] + fxyy[[2]] + gxxy[[2]] + gyyy[[2]] +
   1 / ω[[2]] * (fxy[[2]] * (fxx[[2]] + fyy[[2]]) -
       gxy[[2]] * (gxx[[2]] + gyy[[2]]) - fxx[[2]] * gxx[[2]] + fyy[[2]] * gyy[[2]]), a]
```

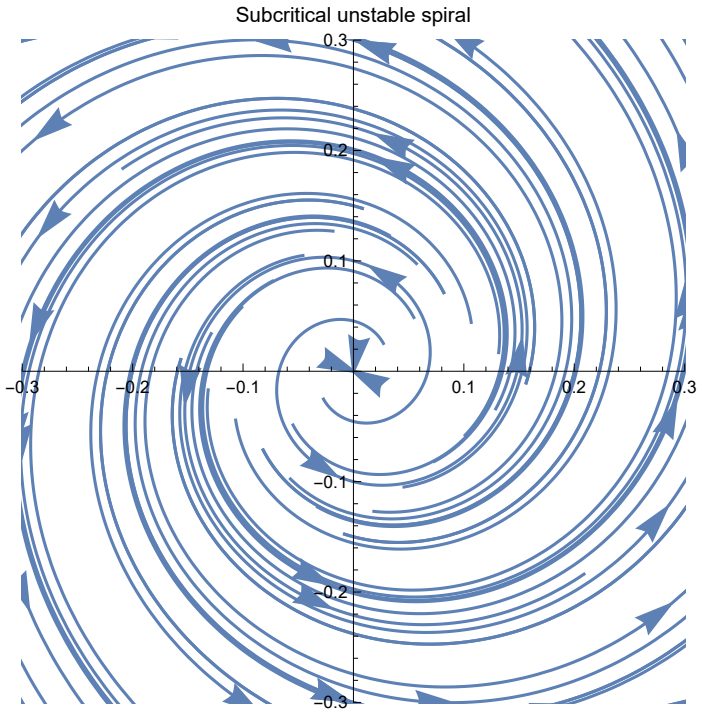
Out[]= $\left\{\left\{a \to \frac{3}{8}\right\}\right\}$

Out[]= $\left\{\left\{a \to -\frac{1}{2}\right\}\right\}$

In[281]:=
```
Clear["Global`*"]
minx = -0.3;  maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] == μ * x[t] - 4 y[t] - x[t]^3, y'[t] == 4 x[t] + μ * y[t] + 2 y[t]^3,
     x[0] == x0, y[0] == y0} /. μ → 1, {x, y}, {t, -1, 1}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
   Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
   Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]]], {t, -1, 1},
    PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
  Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
 PlotLabel → "Subcritical unstable spiral"]
```

NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.

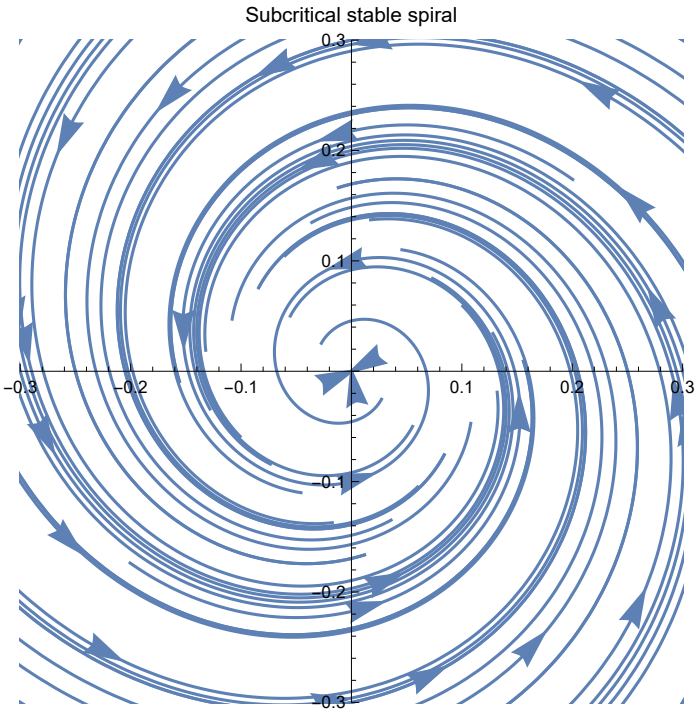
Out[285]=



In[ ]:=
```
Clear["Global`*"]
minx = -0.3;  maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] == μ * x[t] - 4 y[t] - x[t]^3, y'[t] == 4 x[t] + μ * y[t] + 2 y[t]^3,
     x[0] == x0, y[0] == y0} /. μ → -1, {x, y}, {t, -1, 1}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
   Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
   Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1, 1},
    PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
  Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
 PlotLabel → "Subcritical stable spiral"]
```

NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.

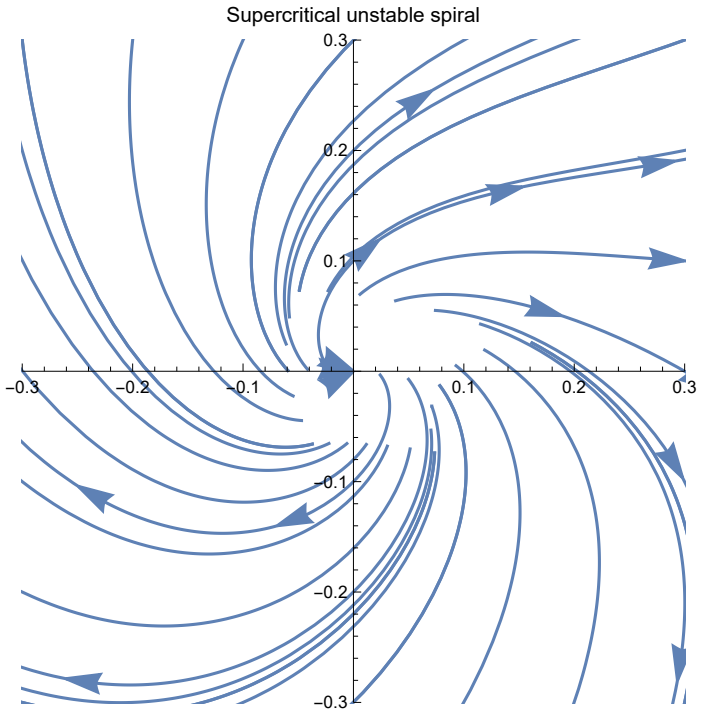
Out[ ]=



In[ ]:=
```
Clear["Global`*"]
minx = -0.3;  maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 x[t]^2,
     x[0] == x0, y[0] == y0} /. μ → 1, {x, y}, {t, -1.5, 1.5}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
   Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
   Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1.5, 1.5},
    PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
  Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
 PlotLabel → "Supercritical unstable spiral"]
```

NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.

Out[ ]=



In[276]:=
```
Clear["Global`*"]
minx = -0.3;  maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 x[t]^2,
     x[0] == x0, y[0] == y0} /. μ → -1, {x, y}, {t, -1.5, 1.5}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
   Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
   Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
      s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1.5, 1.5},
    PlotRange → {{minx, maxx}, {miny, maxy}}], {i, Length[initialCondition]}] /.
  Line[x_] :> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
 PlotLabel → "Supercritical stable spiral"]
```

NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.

NDSolve: At t == −1.10177, step size is effectively zero; singularity or stiff system suspected.

InterpolatingFunction: Input value {−1.49994} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {−1.49994} lies outside the range of data in the interpolating function. Extrapolation will be used.

NDSolve: At t == −1.17309, step size is effectively zero; singularity or stiff system suspected.

InterpolatingFunction: Input value {−1.49994} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

NDSolve: At t == −1.26898, step size is effectively zero; singularity or stiff system suspected.

General: Further output of NDSolve::ndsz will be suppressed during this calculation.

Out[280]=