

```

In[ ]:= Clear["Global`*"]
ω = {4, -1};
fx = D[-x^3, -x^2], x];
fxx = D[fx, x];
fxxx = D[fxx, x];
fxy = D[fx, y];
fxyy = D[fxy, y];
fy = D[-x^3, -x^2}, y];
fyy = D[fy, y];
gx = D[{2 y^3, 2 x^2}, x];
gxx = D[gx, x];
gxy = D[gx, y];
gy = D[{2 y^3, 2 x^2}, y];
gyy = D[gy, y];
gyyy = D[gyy, y];
Solve[
  16 a == fxxx[[1]] + fxyy[[1]] + gxyy[[1]] + gyyy[[1]] + 1 / ω[[1]] * (fxy[[1]] * (fxx[[1]] + fyy[[1]]) -
    gxy[[1]] * (gxx[[1]] + gyy[[1]]) - fxx[[1]] * gxx[[1]] + fyy[[1]] * gyy[[1]]), a]
Solve[16 a == fxxx[[2]] + fxyy[[2]] + gxyy[[2]] + gyyy[[2]] +
  1 / ω[[2]] * (fxy[[2]] * (fxx[[2]] + fyy[[2]]) -
    gxy[[2]] * (gxx[[2]] + gyy[[2]]) - fxx[[2]] * gxx[[2]] + fyy[[2]] * gyy[[2]]), a]

Out[ ]:= {{a -> -3/8}}


Out[ ]:= {{a -> -1/2}}

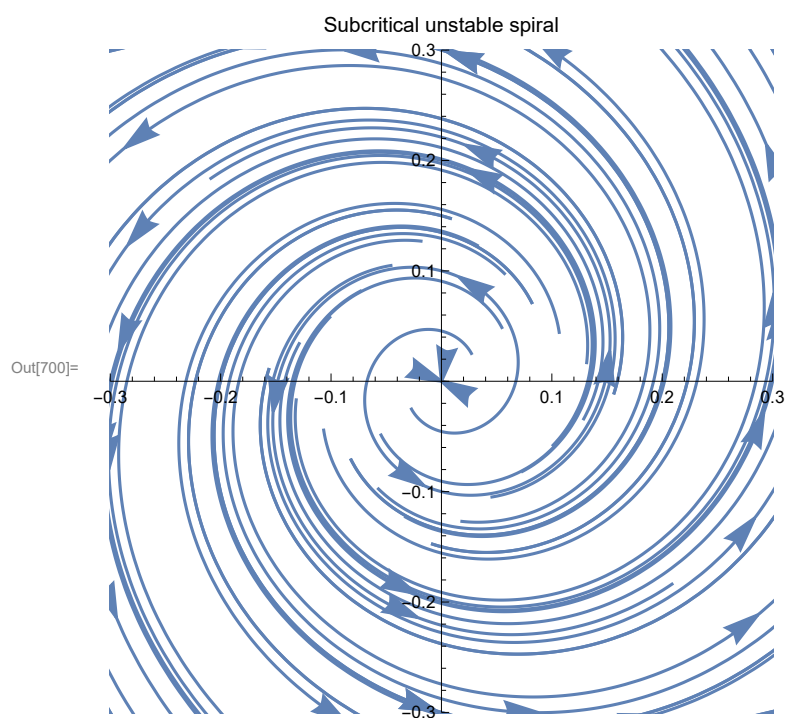
```

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In[696]:= Clear["Global`*"]
minx = -0.3; maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] ==  $\mu$  * x[t] - 4 y[t] - x[t]^3, y'[t] == 4 x[t] +  $\mu$  * y[t] + 2 y[t]^3,
    x[0] == x0, y[0] == y0} /.  $\mu \rightarrow 1$ , {x, y}, {t, -1, 1}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
  Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
  Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
  s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1, 1},
  PlotRange -> {{minx, maxx}, {miny, maxy}}, {i, Length[initialCondition]}] /.
  Line[x_] -> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
  PlotLabel -> "Subcritical unstable spiral"]

```

 **NDSolve:** Initial condition x0 is not a number or a rectangular array of numbers.

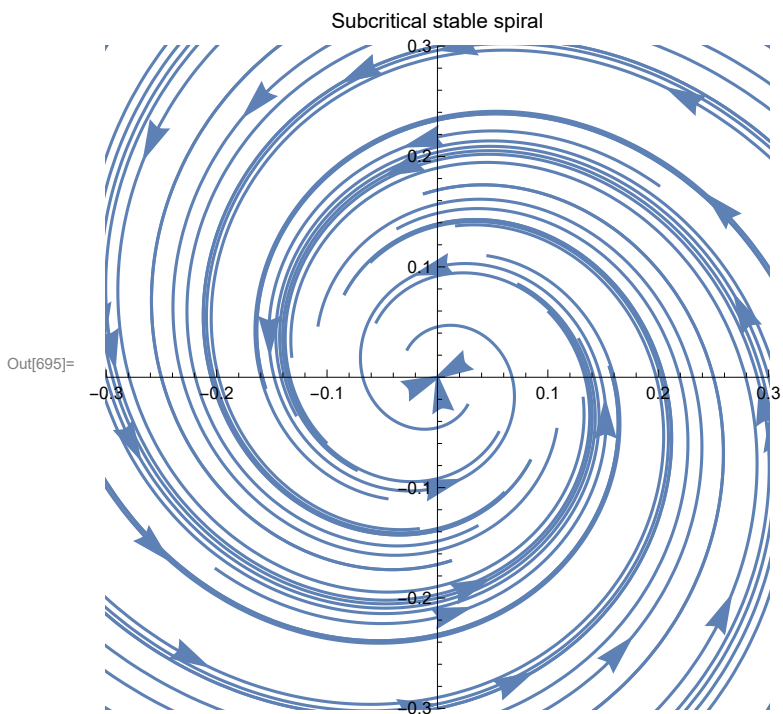


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In[691]:= Clear["Global`*"]
minx = -0.3; maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] ==  $\mu$  * x[t] - 4 y[t] - x[t]^3, y'[t] == 4 x[t] +  $\mu$  * y[t] + 2 y[t]^3,
    x[0] == x0, y[0] == y0} /.  $\mu \rightarrow -1$ , {x, y}, {t, -1, 1}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
  Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
  Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
  s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1, 1},
  PlotRange -> {{minx, maxx}, {miny, maxy}}, {i, Length[initialCondition]}] /.
  Line[x_] -> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]},
  PlotLabel -> "Subcritical stable spiral"]

```

... **NDSolve:** Initial condition x0 is not a number or a rectangular array of numbers.

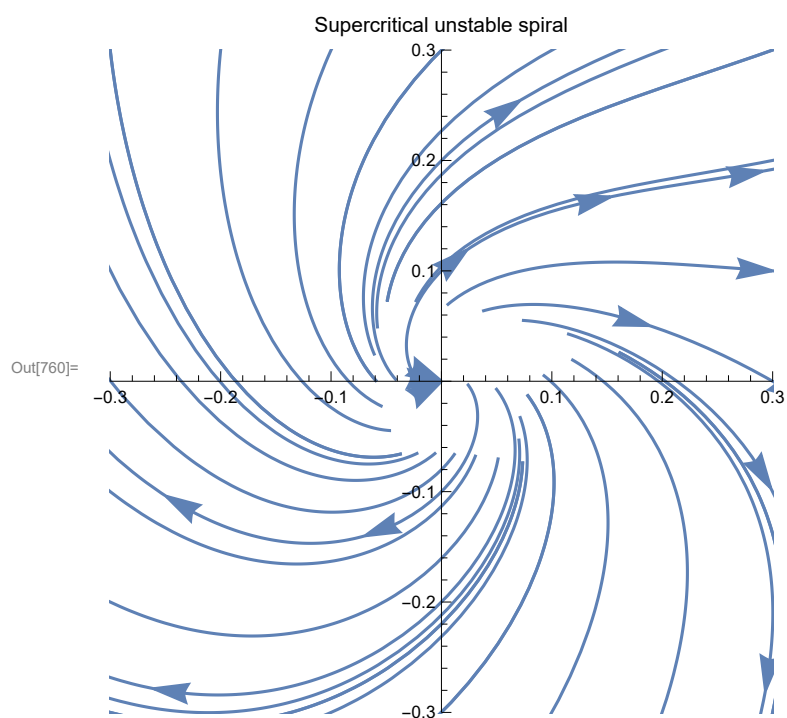


```

In[756]:= Clear["Global`*"]
minx = -0.3; maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] ==  $\mu$  * x[t] + y[t] - x[t]^2, y'[t] == -x[t] +  $\mu$  * y[t] + 2 x[t]^2,
    x[0] == x0, y[0] == y0} /.  $\mu \rightarrow 1$ , {x, y}, {t, -1.5, 1.5}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
  Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
  Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
  s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1.5, 1.5},
  PlotRange -> {{minx, maxx}, {miny, maxy}}, {i, Length[initialCondition]}] /.
  Line[x_] -> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]}],
  PlotLabel -> "Supercritical unstable spiral"]

```

... NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.



```

In[751]:= Clear["Global`*"]
minx = -0.3; maxx = 0.3; miny = -0.3; maxy = 0.3;
s[x0_, y0_] =
  NDSolve[{x'[t] ==  $\mu$  * x[t] + y[t] - x[t]^2, y'[t] == -x[t] +  $\mu$  * y[t] + 2 x[t]^2,
    x[0] == x0, y[0] == y0} /.  $\mu \rightarrow -1$ , {x, y}, {t, -1.5, 1.5}];
initialCondition = Join[Table[{0, y}, {y, miny, maxy, 0.1}],
  Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
  Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /.
  s[initialCondition[[i, 1]], initialCondition[[i, 2]]], {t, -1.5, 1.5},
  PlotRange -> {{minx, maxx}, {miny, maxy}}, {i, Length[initialCondition]}] /.
  Line[x_] -> {Arrowheads[{0., 0.05, 0.05, 0.05, 0.}], Arrow[x]}],
  PlotLabel -> "Supercritical stable spiral"]

```

... NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.

... NDSolve: At t == -1.10177, step size is effectively zero; singularity or stiff system suspected.

... InterpolatingFunction: Input value {-1.49994} lies outside the range of data in the interpolating function. Extrapolation will be used.

... InterpolatingFunction: Input value {-1.49994} lies outside the range of data in the interpolating function. Extrapolation will be used.

... NDSolve: At t == -1.17309, step size is effectively zero; singularity or stiff system suspected.

... InterpolatingFunction: Input value {-1.49994} lies outside the range of data in the interpolating function. Extrapolation will be used.

... General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

... NDSolve: At t == -1.26898, step size is effectively zero; singularity or stiff system suspected.

... General: Further output of NDSolve::ndsiz will be suppressed during this calculation.

