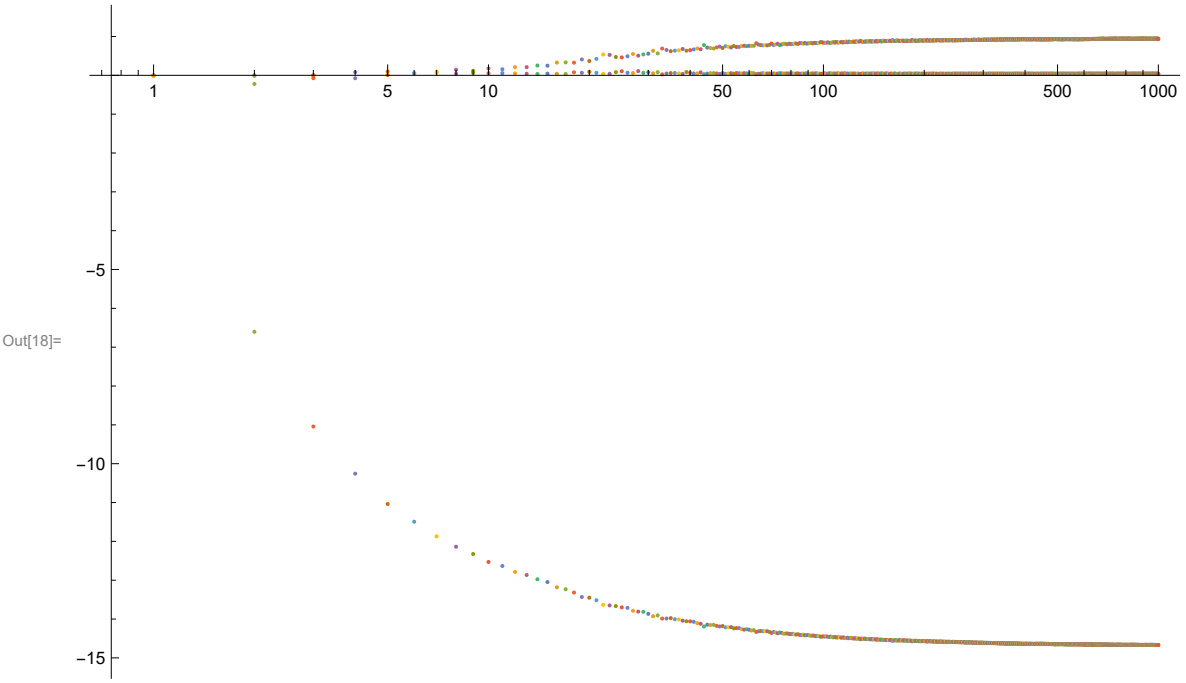
In[1]:=
```
Clear["Global`*"]
(* Initialization *)
σ = 10; b = 8 / 3; r = 28;
f[x_, y_, z_] := σ * (y - x);
g[x_, y_, z_] := r * x - y - x * z;
h[x_, y_, z_] := x * y - b * z;
J[x_, y_, z_] := {{D[f[x, y, z], x], D[f[x, y, z], y], D[f[x, y, z], z]},
  {D[g[x, y, z], x], D[g[x, y, z], y], D[g[x, y, z], z]},
  {D[h[x, y, z], x], D[h[x, y, z], y], D[h[x, y, z], z]}}
Tmax = 1000; Tmin = 1;
s = NDSolve[{x'[t] == σ * (y[t] - x[t]), y'[t] == r * x[t] - y[t] - x [t] * z[t],
    z'[t] == x[t] * y[t] - b * z[t], x[0] == y[0] == z[0] == 0.01},
   {x, y, z}, {t, 0, Tmax}, Method → "ExplicitRungeKutta"];
dt = 0.001;
Q = IdentityMatrix[3];
λ = ConstantArray[0, 3];
lambdaList = {{0, 0, 0}};
plotIndex = 0;
(* Main loop *)
For[i = Tmin, i < Tmax, i = i + dt,
 trajI = {Evaluate[x[i] /. s], Evaluate[y[i] /. s], Evaluate[z[i] /. s]};
    Jxt = J[x, y, z] /. {x → trajI[[1]][[1]], y → trajI[[2]][[1]], z → trajI[[3]][[1]]};
    M = IdentityMatrix[3] + Jxt * dt;
    QR = QRDecomposition[M.Q];
    Qnew = Transpose[QR[[1]]];
    λ[[1]] = λ[[1]] + 1 / Tmax * Log[Abs[QR[[2]][[1]][[1]]]];
    λ[[2]] = λ[[2]] + 1 / Tmax * Log[Abs[QR[[2]][[2]][[2]]]];
    λ[[3]] = λ[[3]] + 1 / Tmax * Log[Abs[QR[[2]][[3]][[3]]]];

    If[Mod[plotIndex, 1000] == 0, lambdaList = AppendTo[lambdaList,
    {{i, Tmax / i * λ[[1]]}, {i, Tmax / i * λ[[2]]}, {i, Tmax / i * λ[[3]]}}];, f];
    plotIndex = plotIndex + 1;

    Q = Qnew;
    ]
λ
Total[λ] (* Total should be -13.67 *)
```

Out[15]= {0.940715, 0.0375423, -14.6679}

Out[16]= -13.6896

In[17]:=
```
Transpose[lambdaList];
ListLogLinearPlot[lambdaList]
```

Out[18]=




```
(* Looks like the Lyapunov exponents have converged. *)
```

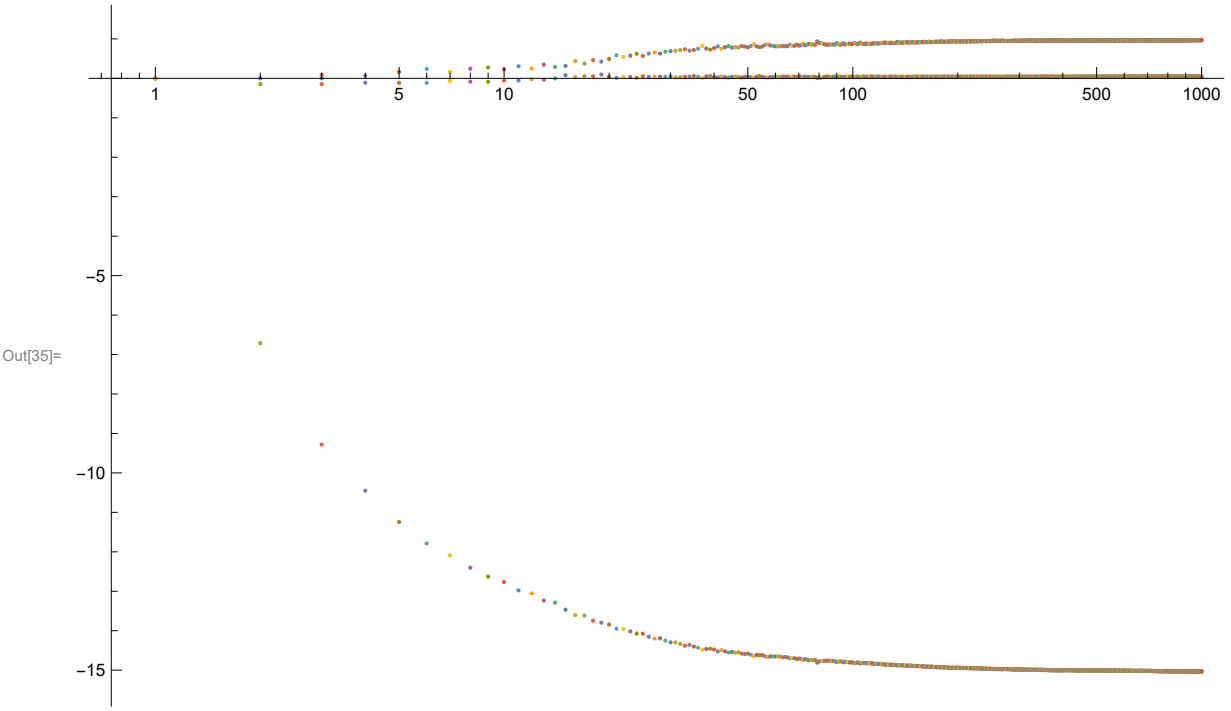
In[19]:=
```
(* Initialization *)
σ = 10; b = 3; r = 28;
f[x_, y_, z_] := σ * (y - x);
g[x_, y_, z_] := r * x - y - x * z;
h[x_, y_, z_] := x * y - b * z;
J[x_, y_, z_] := {{D[f[x, y, z], x], D[f[x, y, z], y], D[f[x, y, z], z]},
   {D[g[x, y, z], x], D[g[x, y, z], y], D[g[x, y, z], z]},
   {D[h[x, y, z], x], D[h[x, y, z], y], D[h[x, y, z], z]}};
Tmax = 1000; Tmin = 1;
s = NDSolve[{x'[t] == σ * (y[t] - x[t]), y'[t] == r * x[t] - y[t] - x[t] * z[t],
    z'[t] == x[t] * y[t] - b * z[t], x[0] == y[0] == z[0] == 0.01},
   {x, y, z}, {t, Tmin, Tmax}, MaxSteps → 1000000];
dt = 0.001;
Q = IdentityMatrix[3];
λ = ConstantArray[0, 3];
lambdaList2 = {{0, 0, 0}};
plotIndex = 0;
(* Main loop *)
For[i = Tmin, i < Tmax, i = i + dt,
 trajI = {Evaluate[x[i] /. s], Evaluate[y[i] /. s], Evaluate[z[i] /. s]};
    Jxt = J[x, y, z] /. {x → trajI[[1]][[1]], y → trajI[[2]][[1]], z → trajI[[3]][[1]]};
    M = IdentityMatrix[3] + Jxt * dt;
    QR = QRDecomposition[M.Q];
    Qnew = Transpose[QR[[1]]];
    λ[[1]] = λ[[1]] + 1 / Tmax * Log[Abs[QR[[2]][[1]][[1]]]];
    λ[[2]] = λ[[2]] + 1 / Tmax * Log[Abs[QR[[2]][[2]][[2]]]];
    λ[[3]] = λ[[3]] + 1 / Tmax * Log[Abs[QR[[2]][[3]][[3]]]];

    If[Mod[plotIndex, 1000] == 0, lambdaList2 = AppendTo[lambdaList2,
    {{i, Tmax / i * λ[[1]]}, {i, Tmax / i * λ[[2]]}, {i, Tmax / i * λ[[3]]}}];, f];
    plotIndex = plotIndex + 1;

    Q = Qnew;
    ]
λ
Total[λ] (* Total should be -14 *)
```

Out[32]= {0.969316, 0.0394433, −15.0319}

Out[33]= −14.0231

In[34]:=
```
Transpose[lambdaList2];
ListLogLinearPlot[lambdaList2]
```

Out[35]=




```
(* Looks like the Lyapunov exponents have converged. *)
```

In[36]:=
```
(* Initialization *)
σ = 16; b = 5; r = 350;
f[x_, y_, z_] := σ * (y - x);
g[x_, y_, z_] := r * x - y - x * z;
h[x_, y_, z_] := x * y - b * z;
J[x_, y_, z_] := {{D[f[x, y, z], x], D[f[x, y, z], y], D[f[x, y, z], z]},
   {D[g[x, y, z], x], D[g[x, y, z], y], D[g[x, y, z], z]},
   {D[h[x, y, z], x], D[h[x, y, z], y], D[h[x, y, z], z]}};
Tmax = 2000; Tmin = 1;
s = NDSolve[{x'[t] == σ * (y[t] - x[t]), y'[t] == r * x[t] - y[t] - x[t] * z[t],
    z'[t] == x[t] * y[t] - b * z[t], x[0] == y[0] == z[0] == 0.01},
   {x, y, z}, {t, Tmin, Tmax}, MaxSteps → 10000000];
dt = 0.0001;
Q = IdentityMatrix[3];
λ = ConstantArray[0, 3];
lambdaList3 = {{0, 0, 0}};
plotIndex = 0;
(* Main loop *)
For[i = Tmin, i < Tmax, i = i + dt,
 trajI = {Evaluate[x[i] /. s], Evaluate[y[i] /. s], Evaluate[z[i] /. s]};
    Jxt = J[x, y, z] /. {x → trajI[[1]][[1]], y → trajI[[2]][[1]], z → trajI[[3]][[1]]};
    M = IdentityMatrix[3] + Jxt * dt;
    QR = QRDecomposition[M.Q];
    Qnew = Transpose[QR[[1]]];
    λ[[1]] = λ[[1]] + 1 / Tmax * Log[Abs[QR[[2]][[1]][[1]]]];
    λ[[2]] = λ[[2]] + 1 / Tmax * Log[Abs[QR[[2]][[2]][[2]]]];
    λ[[3]] = λ[[3]] + 1 / Tmax * Log[Abs[QR[[2]][[3]][[3]]]];

    If[Mod[plotIndex, 10000] == 0, lambdaList3 = AppendTo[lambdaList3,
    {{i, Tmax / i * λ[[1]]}, {i, Tmax / i * λ[[2]]}, {i, Tmax / i * λ[[3]]}}];, f];
    plotIndex = plotIndex + 1;

    If[Mod[plotIndex, 1500000] == 0, Print[i];, f];

    Q = Qnew;
    ]
λ
Total[λ] (* Total should be -22 *)
```

```
151.
301.
451.
601.
751.
901.
1051.
1201.
1351.
1501.
1651.
1801.
1951.
```

Out[49]= {0.11855, -2.99782, -18.9849}

Out[50]= -21.8642

In[51]:=
```
Transpose[lambdaList3];
ListLogLinearPlot[lambdaList3]
```

Out[52]=