Assignment 2

Jonas Lauri

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Problem a)

We are given $v_L(t) = v_0(t/t_1)$ and $v_R(t) = v_0(t/t_2)$ where v_0, t_1, t_2 are constants. Additionally, we have the following formulae from the lecture on kinematics

$$x_1 = x_0 + \int_{t_0}^{t_1} V_x(t)dt = x_0 + \int_{t_0}^{t_1} \frac{v_R(t) + v_L(t)}{2} \cos\phi(t)\, dt$$
$$y_1 = y_0 + \int_{t_0}^{t_1} V_y(t)dt = y_0 + \int_{t_0}^{t_1} \frac{v_R(t) + v_L(t)}{2} \sin\phi(t)\, dt$$
$$\phi_1 = \phi_0 + \int_{t_0}^{t_1} \dot{\phi}\, dt = \phi_0 + \int_{t_0}^{t_1} \frac{v_R(t) - v_L(t)}{2R} dt.$$

Insert the expressions for v_L and v_R into the equations above to get

$$x_1 = x_0 + \int_{t_0}^{t_1} \frac{v_0 \frac{t(t_1+t_2)}{t_1 t_2}}{2} \cos\phi(t)\, dt = x_0 + \frac{v_0(t_1+t_2)}{2t_1t_2} \int_{t_0}^{t_1} t\cos\phi(t)\, dt.$$

For ϕ, we have

$$\phi_1 = \phi_0 + \int_{t_0}^{t_1} \frac{v_R(t) - v_L(t)}{2R} dt = \phi_0 + \int_{t_0}^{t_1} \frac{v_0(t/t_2 - t/t_1)}{2R} dt = \phi_0 + \frac{v_0(t_1 - t_2)}{2t_1t_2R} \int_{t_0}^{t_1} t\, dt.$$

With $B = \frac{v_0(t_1-t_2)}{2t_1t_2R}$ and $\int t\, dt = t^2/2 + C_1$ we have the last equation

$$\phi(t) = \phi_0 + B\left(\frac{t^2}{2} + C_1\right).$$

Use our initial condition $\phi(0) = \phi_0$ to get $C_1 = 0$ and insert to get

$$\phi(t) = \phi_0 + B\frac{t^2}{2} = \frac{5t^2}{48}. \tag{1}$$

Write $A = \frac{v_0(t_1+t_2)}{2t_1t_2}$ and insert $\phi(t)$ into the expression for x_1

$$x(t) = x_0 + A\int_0^t t\cos\left(\phi_0 + B\frac{t^2}{2}\right) dt.$$

Integrate

$$x(t) = x_0 + \frac{A}{B}\left(\sin\left(\phi_0 + B\frac{t^2}{2}\right) - \sin(\phi_0) + C_2\right).$$

Use the initial condition $x(0) = x_0$ get

$$\begin{aligned}
x(0) &= x_0 + \frac{A}{B}\left(\sin\left(\phi_0 + B\frac{0^2}{2}\right) - \sin(\phi_0) + C_2\right) \\
x(0) &= x_0 + \frac{A}{B}\left(\sin(\phi_0) - \sin(\phi_0) + C_2\right) \\
x(0) &= x_0 + \frac{A}{B}C_2 \longrightarrow C_2 = 0.
\end{aligned}$$

The resulting expression $x(t)$ with inserted constants is

$$x(t) = x_0 + \frac{A}{B}\left(\sin\left(\phi_0 + B\frac{t^2}{2}\right) - \sin(\phi_0)\right) = \frac{9}{25}\sin\left(\frac{5t^2}{48}\right). \tag{2}$$

Finally, $y(t)$

$$y_1 = y_0 + \frac{v_0(t_1+t_2)}{2t_1t_2}\int_{t_0}^{t_1} t\sin\phi(t)\,dt.$$

Write $A = \frac{v_0(t_1+t_2)}{2t_1t_2}$ and insert $\phi(t)$

$$y(t) = y_0 + A\int_0^t t\sin\left(\phi_0 + B\frac{t^2}{2}\right)\,dt.$$

Integrate

$$y(t) = y_0 + \frac{A}{B}\left(\cos(\phi_0) - \cos\left(\phi_0 + B\frac{t^2}{2}\right) + C_3\right).$$

Again, use the initial condition $y(0) = y_0$ get

$$\begin{aligned}
y(0) &= y_0 + \frac{A}{B}\left(\cos(\phi_0) - \cos\left(\phi_0 + B\frac{0^2}{2}\right) + C_3\right) \\
y(0) &= y_0 + \frac{A}{B}\left(\cos(\phi_0) - \cos(\phi_0) + C_3\right) \\
y(0) &= y_0 + \frac{A}{B}C_3 \longrightarrow C_3 = 0.
\end{aligned}$$

The resulting expression for $y(t)$ is

$$y(t) = y_0 + \frac{A}{B}\left(\cos(\phi_0) - \cos\left(\phi_0 + B\frac{t^2}{2}\right)\right) = -\frac{9}{25}\cos\left(\frac{5t^2}{48}\right) + \frac{9}{25}. \tag{3}$$

An example trajectory can be seen in figure 1.

The MATLAB-code

The MATLAB-code can be found at: https://github.com/BotLauri/TME290.

Problem b)

See Figure 2 for a visualisation of the path and **source_code_lauri.zip** for the source code.

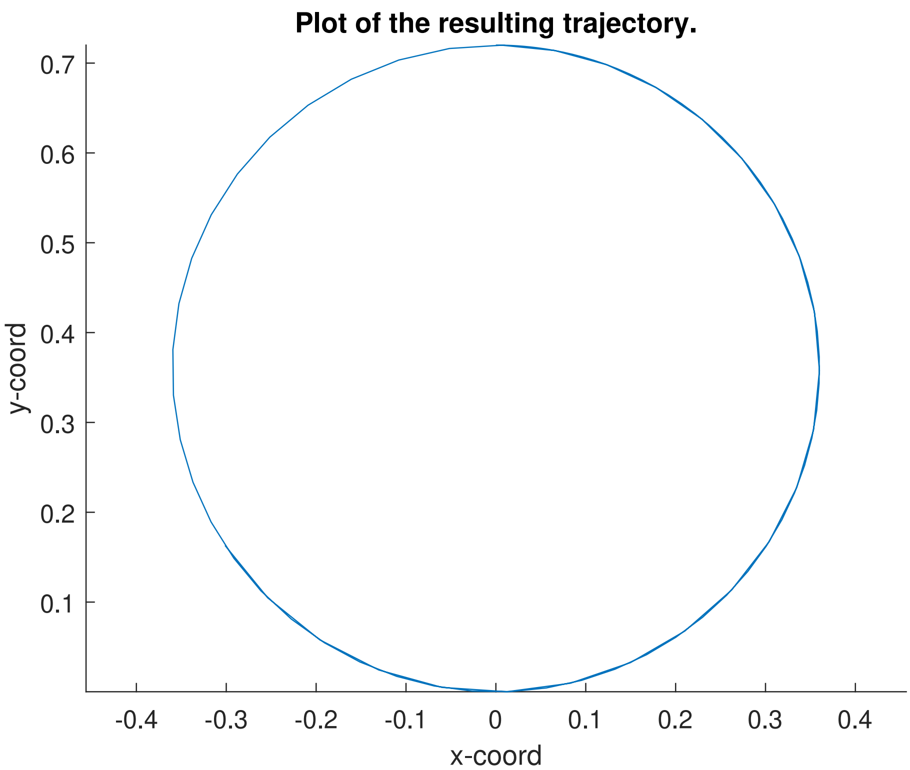

Figure 1: A plot of an example trajectory for $R = 0.12, v_0 = 0.5, t_1 = 10$ and $t_2 = 5$.

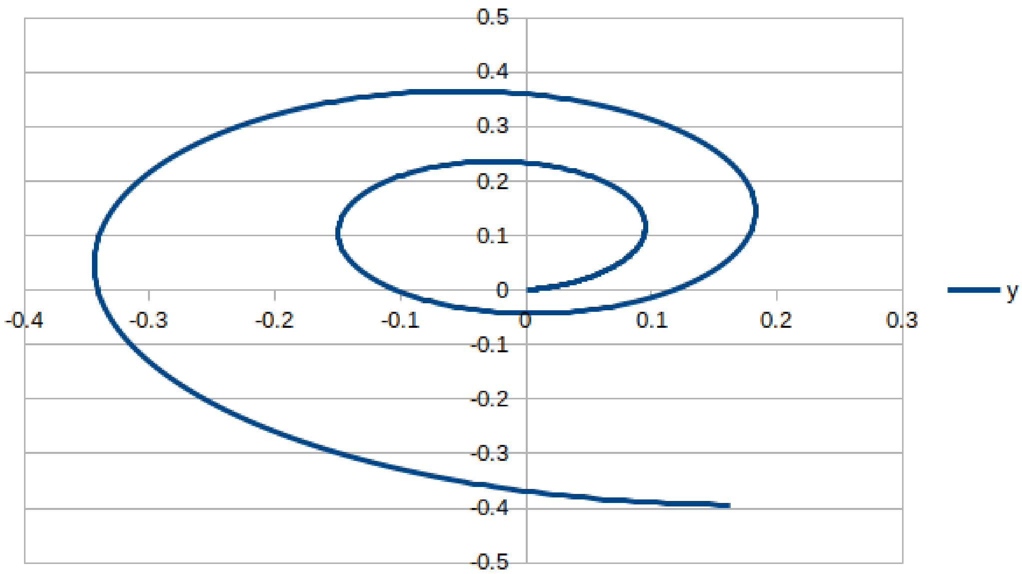

Figure 2: A plot of the trajectory from the .rec file.