

• Notes on Revised Submission for Homework:

After the solution is released,

Step 1: Self-grade (Evaluation for initial submission)

Step 2: Do a revision

Step 3: Self-grade for the second time

(Full marks if finished, and this is your final grade for the homework)

Note: both two self-grade should be reported in the first page of your homework.

— More examples on Counting Methods:

52 cards \Rightarrow 4 suits $\left\{ \begin{array}{l} \text{Spades} \\ \text{Clubs} \\ \text{Hearts} \\ \text{Diamonds} \end{array} \right.$

each suit \Rightarrow 13 ranks (Ace, 2, ..., 10, J, Q, K)

Poker hands consist of 5 cards out of 52

Thus, $\binom{52}{5}$ ways to choose (without ordering)

Now, we consider the following events:

- $A_0 = \{ \text{Poker hands has exactly one pair} \}$
 one pairs = two cards with the same rank but different suits.

Step 1: Choose the pairs

$\binom{13}{1}$ ways to choose the rank

$\binom{4}{2}$ ways to choose the suits

Step 2: For the remaining three single cards

$\binom{12}{3}$ ways to choose the ranks

4^3 ways to choose the suits

$$\text{so } \#A_0 = \binom{13}{1} \binom{4}{2} \binom{12}{3} \cdot 4^3$$

- $A_1 = \{ \text{Poker hands has exactly two pairs with diff rank} \}$

Step 1: For the two pairs

$\binom{13}{2}$ ways to choose the ranks

$\binom{4}{2} \binom{4}{2}$ ways to choose the suits

Step 2: For the remaining one single cards

$\binom{11}{1}$ ways to choose the ranks

4 ways to choose the suits

$$\#A = \binom{13}{2} \binom{4}{2}^2 \binom{11}{1} \cdot 4$$

- $A_2 = \{ \text{Five cards with ranks in a sequence but not all in the same suit} \}$

Step 1: Choose the ranks

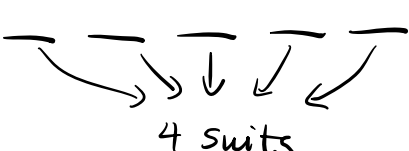
10 sequences in total {

| | | | | |
|----|---|---|---|---|
| A | 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 | 6 |
| 3 | 4 | 5 | 6 | 7 |
| : | : | : | : | : |
| 10 | J | Q | K | A |

\Rightarrow 10 ways to choose the rank

Step 2: Choose the suits.

{ \star In total: Each card has 4 suits,

5 cards in total:  $\Rightarrow 4^5$ ways

{ \star All in the same suits: 4 suits in total \Rightarrow 4 ways

\Rightarrow Not all in the same suits: $4^5 - 4$ ways (Addition rule)

$$\#A_2 = 10(4^5 - 4)$$

- $A_3 = \{ \text{Five cards of the same suits, but not in a sequential ranks} \}$

Step 1: Choose the suits

4 suits in total \Rightarrow 4 ways to choose

Step 2: Choose the ranks

$\left\{ \begin{array}{l} \star \text{ In total: } \binom{13}{5} \text{ ways to choose a rank} \\ \star \text{ In a sequential rank: 10 ways to choose} \end{array} \right.$

\Rightarrow Not in a sequential rank: $\binom{13}{5} - 10$ ways (Addition rule)

$$\#A_3 = 4 \left(\binom{13}{5} - 10 \right)$$

— Multinomial Coefficients

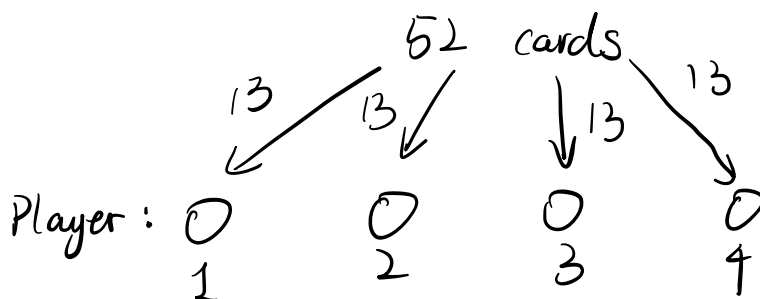
n items in total, assigning labels $1, 2, \dots, r$ to n items s.t.

$$\begin{cases} k_1 \text{ items receive label 1} \\ k_2 \text{ items receive label 2} \\ \vdots \\ k_r \text{ items receive label } r \end{cases}
 \begin{aligned} & \sum_{i=1}^r k_i \\ &= k_1 + k_2 + \dots + k_r \\ &= n \end{aligned}$$

Then, the number of ways to label is

$$\binom{n}{k_1 \ k_2 \ \dots \ k_r} = \frac{n!}{(k_1!)(k_2!) \dots (k_r!)}$$

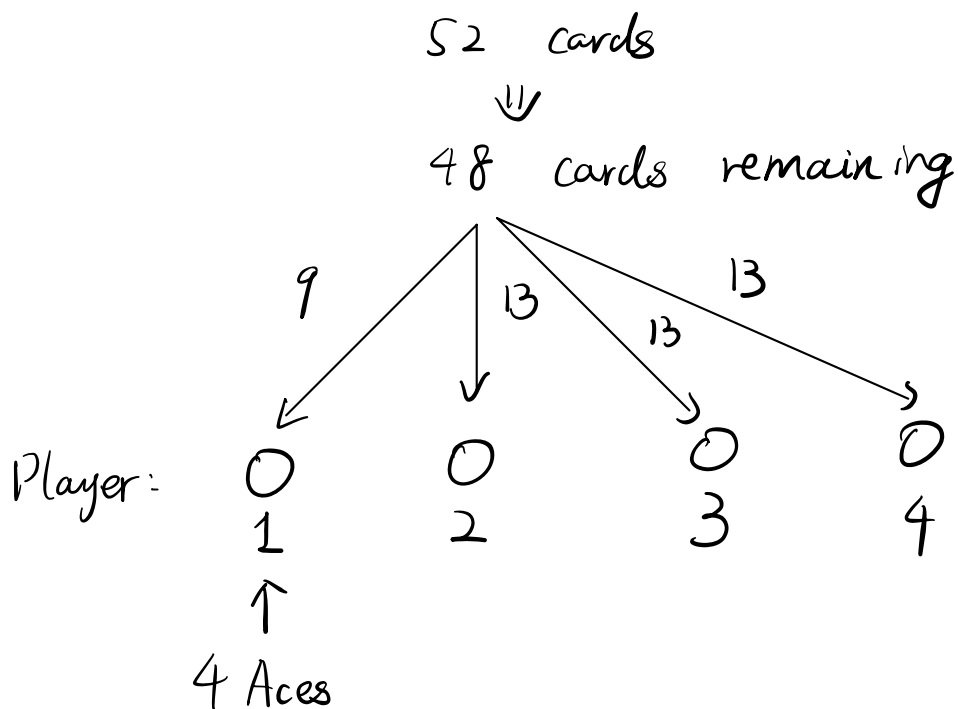
e.g. 52 cards in total, 4 players with each one holding 13 cards.



of ways to distribute cards is

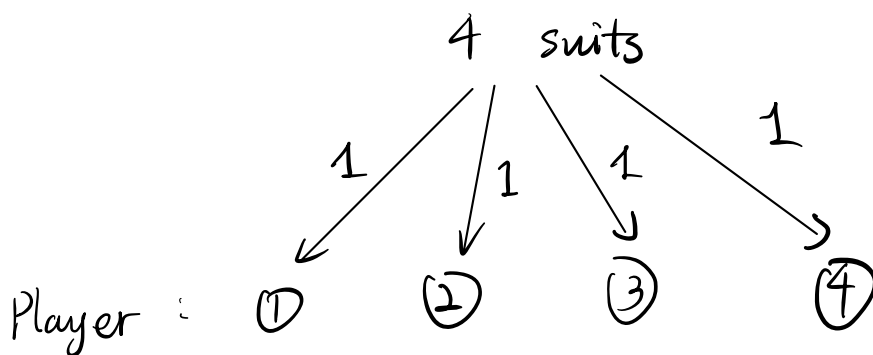
$$\binom{52}{13 \ 13 \ 13 \ 13} = \frac{52!}{(13!)^4}$$

- Now, $B_1 = \{\text{Player 1 receive all 4 Aces}\}$



of ways to distribute cards is $\binom{48}{9 \ 13 \ 13 \ 13}$

- Now, $B_2 = \{\text{Each player receives 13 cards of the same suit}\}$



of ways to distribute is $\binom{4}{1 \ 1 \ 1 \ 1} = 4!$

-Q&A on Homework 1 (if time permits)

Next time: Conditional Probability & HW 2.