

# Practice Final

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## Part 1

Refer to the following exercises in the textbook: 3.37, 3.67, 5.10, 5.22, 6.6, 6.10, 6.32, 8.6, 8.13

## Part 2

Check out these additional exercises:

1. Suppose that  $X_1$  and  $X_2$  are independent random variables, and  $X_i$  has the exponential distribution with parameter  $\beta_1$  and  $\beta_2$ , respectively. Then, for each constant  $k > 0$ ,
  - (a) Identify the distribution of  $\min\{X_1, kX_2\}$ .
  - (b) Show that the probability

$$P(X_1 > kX_2) = \frac{\beta_2}{k\beta_1 + \beta_2}$$

2. Suppose that a sequence of independent tosses are made with a coin for which the probability of obtaining a head on each given toss is  $1/30$ .
  - (a) What is the expected number of tosses that will be required in order to obtain five heads?
  - (b) What is the variance of the number of tosses that will be required in order to obtain five heads?
3. Suppose that a book with  $n$  pages contains on the average  $\lambda$  misprints per page. What is the probability that at least one page which contains exactly  $k$  misprints?
4. Let  $X$  have a density function

$$f(x) = \begin{cases} cx^2e^{-4x} & x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

- (a) Find the value of  $c$  that makes  $f$  a valid density?
  - (b) Calculate the mean and variance of  $\frac{1}{X}$ .
5. Suppose that  $Y$  is a random variable with mean 10 and variance 2. Let  $X$  be a random variable with moment generating function  $M_X(t) = \frac{1}{27}(2e^t + 1)^3 M_Y(t)$ , where  $M_Y(t)$  is the moment generating function  $Y$ , then calculate the mean and variance of  $X$ . (Hint: If two random variables have the same moment generating function, then they have the same distribution. Now, you can consider a random variable  $Z \sim \text{Bin}(3, 2/3)$ , which is independent of  $Y$ )
6. Let the random variables  $X, Y$  have joint density

$$f(x, y) = \begin{cases} 3(2-x)y & \text{if } 0 < y < 1 \text{ and } y < x < 2-y \\ 0 & \text{otherwise} \end{cases}$$

- (a) Verify that it is a valid joint density function.
  - (b) Calculate

$$P(X + Y \leq 1/2)$$

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**Note:** This study guide is used for Botao Jin's sections only. Comments, bug reports: b\_jin@ucsb.edu