Practice Final

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1. Suppose that X_1 and X_2 are independent random variables, and X_1 and X_2 have the exponential distribution with parameters β_1 and β_2, respectively. Then, for each constant $k > 0$,
   (a) Identify the distribution of $\min\{X_1, kX_2\}$.
   (b) Show that the probability
   $$P(X_1 > kX_2) = \frac{\beta_2}{k\beta_1 + \beta_2}$$
2. Suppose that a sequence of independent tosses are made with a coin for which the probability of obtaining a head on each given toss is 1/30.
   (a) What is the expected number of tosses that will be required in order to obtain five heads?
   (b) What is the variance of the number of tosses that will be required in order to obtain five heads?
3. Suppose that a book with n pages contains on the average λ misprints per page. What is the probability that at least one page which contains exactly k misprints?
4. Let X be a density function
   $$f(x) = \begin{cases} cx^2 e^{-4x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$
   (a) Find the value of c that makes f a valid density function.
   (b) Calculate the mean and variance of $\frac{1}{X}$.
5. Suppose that Y is a random variable with mean 10 and standard deviation 2. Let X be a Bernoulli random variable with $p = \frac{1}{2}$, independent of Y. Now, consider a random variable $Z = X + Y$.
   (a) Let $M(t)$ be the moment generating function (MGF) of Y, calculate the moment generating function of Z, denoted as $M_Z(t)$.
   (b) Using the result from part a, derive the mean and variance of Z.
6. Let the random variables X, Y have joint density
   $$f(x,y) = \begin{cases} 3(2-x)y & \text{if } 0 < y < 1 \text{ and } y < x < 2 - y \\ 0 & \text{otherwise} \end{cases}$$
   (a) Verify that it is a valid joint density function.
   (b) Derive the marginal density for X.
   (c) Calculate
   $$P(X + Y \leq 1)$$
7. Suppose that 20 percent of the students who took a certain test were from school A and that the arithmetic average of their scores on the test was 80. Suppose also that 30 percent of the students were from school B and that the arithmetic average of their scores was 76. Suppose, finally, that the other 50 percent of the students were from school C and that the arithmetic average of their scores was 84. If a student is selected at random from the entire group that took the test, what is the expected value of her score? Hint: Use conditional expectation.

Note: This study guide is used for Botao Jin's sections only. Comments, bug reports: b_jin@ucsb.edu