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OH: Monday 4-5 p.m.

Section Material: Course Material in GitHub web.

Reading: App B, App C, Chapter 1.2

Week 1: Set notation and operations

Experiment — A random experiment is a process in which all possible outcomes are known in advanced, but we can't predict which outcomes will occur.

Sample space (Ω) — the set of all possible outcomes in the experiment

e.g. Experiment: toss a coin

$$\Omega = \{H, T\}$$

e.g. Experiment: roll a die

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

1. Notation: Ω - set ; A, B — subset of Ω

ω - elements of Ω.

(1) $\omega \in A$ (ω is a member of A)

$\omega \notin A$ (ω is NOT a member of A)

$\in$ — "belong to"

(2) $A \subseteq B$: A is a subset of B

(Every elem of A is an elem of B)

$A \nsubseteq B$: At least one elem in A is NOT in B.

(3) ϕ : Empty set (A set w. no elems)

e.g. $\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$, $B = \{1, 2, 3, 4\}$, $C = \{1\}$

Easy to see:

$2 \in A$.

$5 \notin A$

$C \subseteq B$

$C \nsubseteq A$

$A \nsubseteq B$

2. Set Operator

(1) Union: $A \cup B = \{\omega \in \Omega : \omega \in A \text{ or } \omega \in B\}$

(2) Intersection: $A \cap B = \{\omega \in \Omega : \omega \in A \text{ and } \omega \in B\}$

(3) Complement: $A^c = \{\omega \in \Omega : \omega \notin A\}$

(4) Difference: $A \setminus B = \{\omega \in \Omega : \omega \in A \text{ and } \omega \notin B\}$

e.g. Back to the previous Example:

$A \cup B = \{1, 2, 3, 4, 6\}$

$A \cap B = \{2, 4\}$

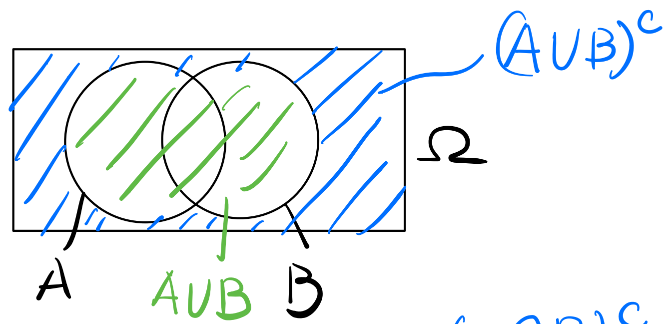
$A \cap C = \phi$ —— A and C are disjoint

$A \setminus B = \{6\}$

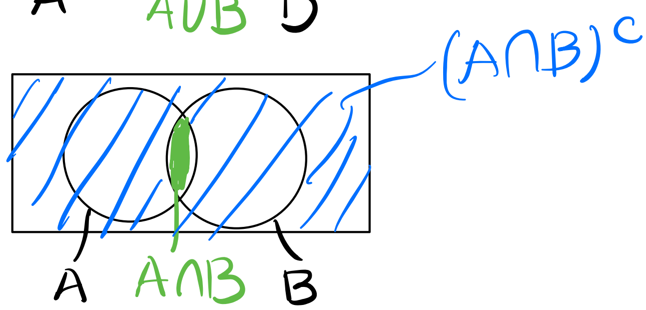
$B^c = \{5, 6\}$

3. De Morgan's laws: A, B – subsets of Ω

(1) $(A \cup B)^c = A^c \cap B^c$

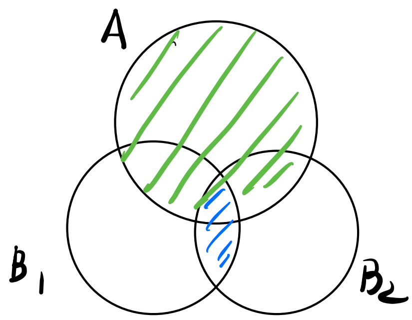


(2) $(A \cap B)^c = A^c \cup B^c$

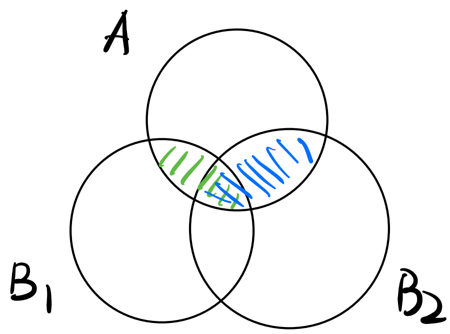


4. Distributive Property: A, B_1, B_2 — subsets of Ω

(1) $A \cup (B_1 \cap B_2) = (A \cup B_1) \cap (A \cup B_2)$



(2) $A \cap (B_1 \cup B_2) = (A \cap B_1) \cup (A \cap B_2)$



Rmk: Sets A_1 and A_2: If $A_1 \subseteq A_2$, then

$$A_1 \cap A_2 = A_1, \quad A_1 \cup A_2 = A_2$$

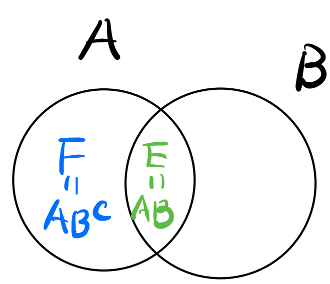
e.g. A, B — subsets of Ω, $E = A \cap B$, $F = A \cap B^c$

(a) Show that E and F are disjoint ($E \cap F = \phi$).

Pf: $E \cap F = (A \cap B) \cap (A \cap B^c)$

$$= A \cap \underbrace{B \cap B^c}_{\phi} = A \cap \phi = \phi$$

(b) Show that $A = E \cup F$.

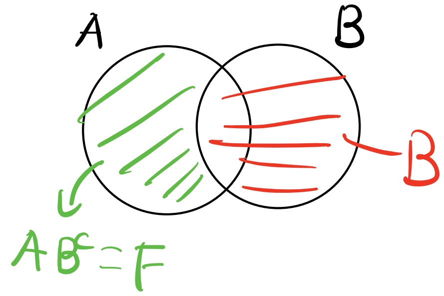


Pf: $A = A \cap \Omega$

$= A \cap (B \cup B^c)$

$= (A \cap B) \cup (A \cap B^c)$ (Distributive law)

$= E \cup F$



(c) Show that $B \cup A = B \cup F$.

Pf: $B \cup F = B \cup (A \cap B^c)$

$= (B \cup A) \cap \underbrace{(B \cup B^c)}$ (Distributive Law)

$= (B \cup A) \cap \Omega = B \cup A$ $\square$