Special Distribution for Discrete R.V.

Brian is shooting a basketball, and all shoots are independent and the probability that he successfully makes the shot is p $(0<p<1)$.

(1) Brian shoots the ball for n times,

X = the number of shots he make. For $k = 0, 1, \cdots, n$

$$\mathbb{P}(X=k) = \mathbb{P}(\text{Brian makes } k \text{ out of } n \text{ shots})$$

$$= \binom{n}{k} p^k (1-p)^{n-k}$$

$X \sim \text{Binomial}(n, p) \qquad S_X = \{0, 1, \cdots, n\}$

(3) Y = the number of shots taken to get the first shot made.

$$\mathbb{P}(Y=k) = \underbrace{(1-p)^{k-1}}_{\text{failure in the first } (k-1) \text{ shots}} p$$

success in the k-th shot

failure in the first $(k-1)$ shots

$Y \sim \text{Geometric}(p) \qquad k = 1, 2, 3, \cdots$

(3) Z = the number of shots needed to get the k-th shot.

$Z \sim NegBin(k, p)$ k represents the # of successes he needed.

For $n \geq k$,

$\mathbb{P}(Z=n) = \binom{n-1}{k-1} p^k (1-p)^{n-k} \quad n = k, k+1, k+2, \cdots$

(4) Suppose Brian continues shooting the ball for two hours, R = the number of shots he makes in 2 hrs.

λ = the mean number of shots he make.

$R \sim Poi(\lambda)$ and $\mathbb{P}(R=k) = e^{-\lambda} \frac{\lambda^k}{k!} \quad k = 0, 1, 2, \cdots$