120A Practice Final

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University of California, Santa Barbara — February 27, 2025

1. Suppose that X_1 and X_2 are independent random variables, and X_1 and X_2 have the exponential distribution with parameters β_1 and β_2, respectively. Then,
   (a) Identify the joint density function $f(x_1, x_2)$ of X_1 and X_2.
   (b) Use part a to show that the probability
   $$P(X_1 > X_2) = \frac{\beta_2}{\beta_1 + \beta_2}$$
2. Suppose that a sequence of independent tosses are made with a coin for which the probability of obtaining a head on each given toss is 1/30.
   (a) What is the expected number of tosses that will be required in order to obtain five heads?
   (b) What is the variance of the number of tosses that will be required in order to obtain five heads?
3. Suppose a book contains n pages, and on average there are λ misprints per page. The number of misprints on each page is independent of the others. What is the probability that at least one page contains no misprints?
4. Let X be a discrete uniform random variable with support $S_X = \{-2, -1, 0, 1, 4, 6\}$, i.e. $P_X(k) = \frac{1}{6}$ for all $k \in S_X$. Find the Moment generating function of $(X + 1)^2$.
5. Suppose that Y is a random variable with mean 10 and standard deviation 2. Let X be a Bernoulli random variable with $p = \frac{1}{2}$, independent of Y. Now, consider a random variable $Z = X + Y$.
   (a) Let $M(t)$ be the moment generating function (MGF) of Y, calculate the moment generating function of Z, denoted as $M_Z(t)$.
   (b) Using the result from part a, derive the mean and variance of Z.
6. Let the random variables X, Y have joint density
   $$f(x, y) = \begin{cases} 3(2 - x)y & \text{if } 0 < y < 1 \text{ and } y < x < 2 - y \\ 0 & \text{otherwise} \end{cases}$$
   (a) Verify that it is a valid joint density function.
   (b) Derive the marginal density for X.
   (c) Calculate
   $$P(X + Y \leq 1)$$
7. Suppose that 20 percent of the students who took a certain test were from school A and that the arithmetic average of their scores on the test was 80. Suppose also that 30 percent of the students were from school B and that the arithmetic average of their scores was 76. Suppose, finally, that the other 50 percent of the students were from school C and that the arithmetic average of their scores was 84. If a student is selected at random from the entire group that took the test, what is the expected value of her score? Hint: Use conditional expectation.

Note: This study guide is used for Botao Jin's sections only. Comments, bug reports: b_jin@ucsb.edu