- Notes on Revised Submission for Homework:

After the solution is released,

Step 1: Self-grade (Evaluation for initial submission)

Step 2: Do a revision

Step 3: Self-grade for the second time

(Full marks if finished, and this is your final grade for the homework)

Note: both two self-grade should be reported in the <u>first page</u> of your homework.

- More examples on <u>Counting Methods</u>:

52 cards $\Rightarrow$ 4 suits $\begin{cases} \text{Spades} \\ \text{Clubs} \\ \text{Hearts} \\ \text{Diamonds} \end{cases}$

each suits $\Rightarrow$ 13 ranks (Ace, 2, ---, 10, J, Q, K)

Poker hands consist of 5 cards out of 52

Thus, $\binom{52}{5}$ ways to choose (without ordering)

Now, we consider the following events:

• $A_0 = \{$ Poker hands has exactly one pair $\}$

One pairs = two cards with the same rank but different suits.

Step 1 : Choose the pairs

$\binom{13}{1}$ ways to choose the ranks

$\binom{4}{2}$ ways to chose the suits

Step 2 : For the remaining three single cards

$\binom{12}{3}$ ways to choose the ranks

4^3 ways to choose the suits

so $\#A_0 = \binom{13}{1}\binom{4}{2}\binom{12}{3}\cdot 4^3$

• $A_1 = \{$ Poker hands has exactly two pairs with diff rank $\}$

Step 1 : For the two pairs

$\binom{13}{2}$ ways to choose the ranks

$\binom{4}{2}\binom{4}{2}$ ways to choose the suits

Step 2 : For the remaining one single cards

$\binom{11}{1}$ ways to choose the ranks

4 ways to choose the suits

$\# A = \binom{13}{2}\binom{4}{2}^2\binom{11}{4}\cdot 4$

- $A_2 = \{$ Five cards with ranks in a sequence but not all in the same suit $\}$

Step 1: Choose the ranks

10 sequences in total $\begin{cases} A\ 2\ 3\ 4\ 5 \\ 2\ 3\ 4\ 5\ 6 \\ 3\ 4\ 5\ 6\ 7 \\ \vdots \\ 10\ J\ Q\ K\ A \end{cases}$

$\Rightarrow$ 10 ways to choose the rank

Step 2: Choose the suits.

☆ In total: Each card has 4 suits,

5 cards in total: _ _ _ _ _ $\Rightarrow 4^5$ ways

4 suits

☆ All in the same suits: 4 suits in total $\Rightarrow$ 4 ways

$\Rightarrow$ Not all in the same suits: $4^5 - 4$ ways (Addition rule)

$\# A_2 = 10(4^5 - 4)$

- $A_3 = \{$ Five cards of the same suits, but not in a sequential ranks $\}$

Step 1: Choose the suits

4 suits in total $\Rightarrow$ 4 ways to choose

Step 2: Choose the ranks

$\begin{cases} \star \text{ In total: } \binom{13}{5} \text{ ways to choose a rank} \\ \star \text{ In a sequential rank: } 10 \text{ ways to choose} \end{cases}$

$\Rightarrow$ Not in a sequential rank: $\binom{13}{5} - 10$ ways (Addition rule)

$$\# A_3 = 4\left(\binom{13}{5} - 10\right)$$

— Multinomial Coefficients

n items in total, assigning labels $1, 2, \cdots, r$ to n items s.t.

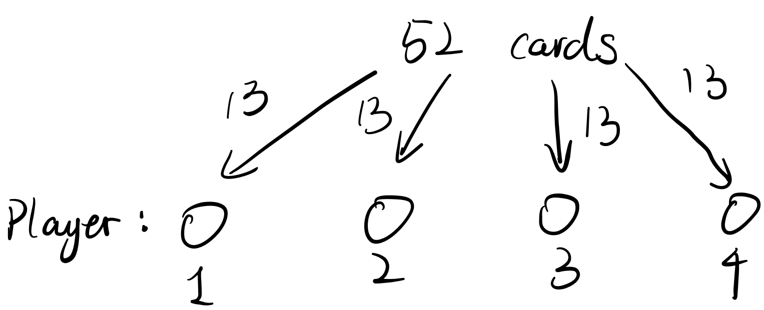
$$\begin{cases} k_1 \text{ items receive label } 1 \\ k_2 \text{ items receive label } 2 \\ \vdots \\ k_r \text{ items receive laber } r \end{cases}$$

$$\sum_{i=1}^{r} k_i = k_1 + k_2 + \cdots + k_r = n$$

Then, the number of ways to label is

$$\binom{n}{k_1 \ k_2 \ \cdots \ k_r} = \frac{n!}{(k_1!)(k_2!)\cdots(k_r!)}$$

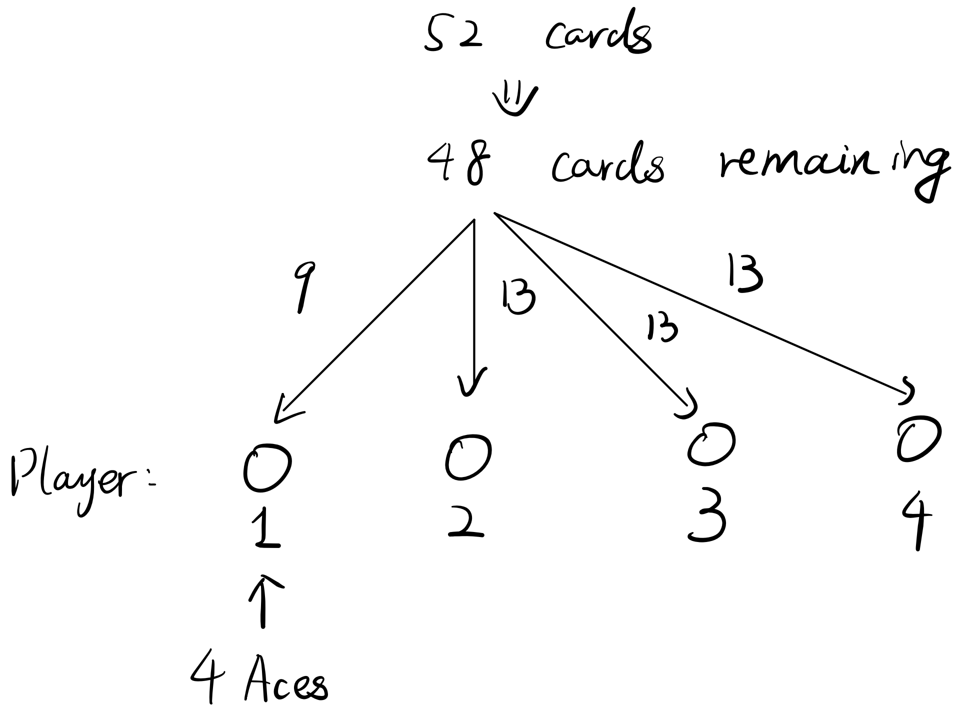
e.g. 52 cards in total, 4 players with each one holding 13 cards.



# of ways to distribute cards is

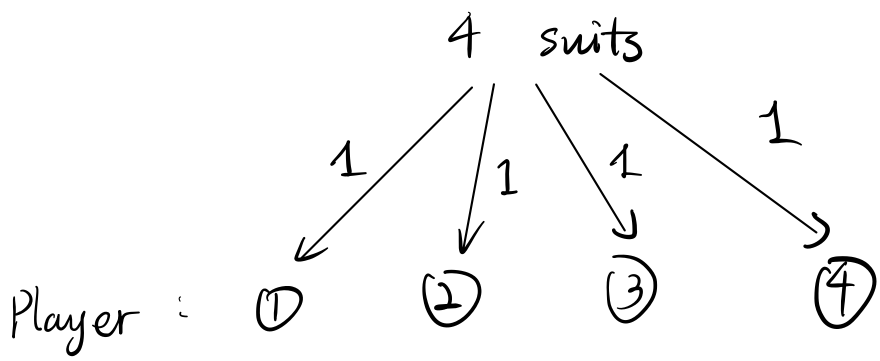
$$\binom{52}{13 \ 13 \ 13 \ 13} = \frac{52!}{(13!)^4}$$

- Now, $B_1 = \{\text{Player 1 receive all 4 Aces}\}$



\# of ways to distribute cards is $\binom{48}{9 \quad 13 \quad 13 \quad 13}$

- Now, $B_2 = \{\text{Each player receives 13 cards of the same suit}\}$



\# of ways to distribute is $\binom{4}{1 \quad 1 \quad 1 \quad 1} = 4!$

- Q&A on Homework 1 (if time permits)

Next time: Conditional Probability & HW 2.