

Week 9 Study Guide

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Joint Distributions

1. Discrete case: See **Exercise 6.2, 6.19** in the textbook.
2. Continuous case: See **Exercise 6.5, 6.35** in the textbook and the **Extra Practice Problem(s) below**:

- (1) Let the random variables X, Y have joint density

$$f(x, y) = \begin{cases} 3(2-x)y & \text{if } 0 < y < 1 \text{ and } y < x < 2-y \\ 0 & \text{otherwise} \end{cases}$$

- a. Verify that it is a valid joint density function.
- b. Derive the marginal density for X .
- c. Calculate

$$P(X + Y \leq 1)$$

3. Independence: See **Exercise 6.27, 6.32, 7.3, 7.5, 8.9** in the textbook and the **Extra Practice Problem(s) below**:

- (1) Suppose that Y is a random variable with mean 10 and standard deviation 2. Let X be a Bernoulli random variable with $p = \frac{1}{2}$, independent of Y . Now, consider a random variable $Z = X + Y$.
 - a. Let $M(t)$ be the moment generating function (MGF) of Y , calculate the moment generating function of Z , denoted as $M_Z(t)$.
 - b. Using the result from part a, derive the mean and variance of Z .

4. Expectation: See **Exercise 8.4, 8.7, 8.11** in the textbook.

5. Special Distributions: See **Exercise 6.6, 6.11, 6.12** in the textbook and the **Extra Practice Problem(s) below**:

- (1) Suppose that X_1 and X_2 are independent random variables, and X_1 and X_2 have the exponential distribution with parameters β_1 and β_2 , respectively. Then,
 - a. Identify the joint density function $f(x_1, x_2)$ of X_1 and X_2 .
 - b. Use part a to show that the probability

$$P(X_1 > X_2) = \frac{\beta_2}{\beta_1 + \beta_2}$$

Note: This study guide is used for Botao Jin's sections only. Comments, bug reports: b_jin@ucsb.edu