

# PSTAT 171 Week 1&2 Extra Practice

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## Problem 1

David can receive one of the following two payment streams:

- (i) 100 at time 0, 200 at time  $n$ , and 300 at time  $2n$ .
- (ii) 600 at time 10

At an effective annual interest rate of  $i$ , the present values of the two streams are equal. Given  $v^n = 0.75941$ , determine  $i$ .

Solution:  $i = 3.51\%$

## Problem 2

Ed buys a TV from Al for 480 by paying 50 in cash, 10 every three months for one year (four payments of 100), and a final payment in 15 months (three months after the final quarterly payment). Find the amount of the final payment if Al earns a 3-month compound interest rate of 3%. What is the final payment if Al earns a one-month rate of 1%?

Solution:  $X = 67.57$  and  $X = 67.98$ .

## Problem 3

Eric deposits  $X$  into a savings account at time 0, which pays interest at a nominal rate of  $i$ , compounded semiannually. Mike deposits  $2X$  into a different savings account at time 0, which pays simple interest at an annual rate of  $i$ . Eric and Mike earn the same amount of interest during the last 6 months of the 8<sup>th</sup> year. Calculate  $i$ .

Solution:  $i = 9.45988\%$

## Problem 4

Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns an effective annual discount rate of  $d$ . The amount of interest earned in Bruce's account during the 11th year is equal to  $X$ . The amount of interest earned in Robbie's account during the 17th year is also equal to  $X$ . Calculate  $X$ .

Solution:  $X = 38.35$

## Problem 5

Tawny makes a deposit into a bank account which credits interest at a nominal interest rate of 10% per annum, convertible semi-annually. At the same time, Fabio deposits 1000 into a different bank account, which is credited with simple interest. At the end of 5 years, the forces of interest on the two accounts are equal, and Fabio's account has accumulated to  $Z$ . Determine  $Z$ .

Solution:  $Z = 1953$

## Problem 6

Dottie receives payments of  $X$  at the end of each year for  $n$  years. The present value of her annuity is 493. Same receives payments of  $3X$  at the end of each year for  $2n$  years. The present value of his annuity is 2748. Both present values are calculated at the same effective annual interest rate. Determine  $v^n$ .

## Problem 7

Sally lends 10000 to Tim. Tim agrees to pay back the loan over 5 years with monthly payments at the end of each month. Sally can reinvest the the monthly payments from Tim in a savings account paying interest at 6% compounded monthly. The yield rate earned on Sally's investment over the five-year period turned out to be 7.45%, compounded semi-annually. What nominal rate of interest, compounded monthly, did Sally charge Tim on the loan?

Solution:  $i^{(12)} = 8.801\%$



## Problem 8

Susan invests  $Z$  at the end of each year for seven years at an effective annual interest rate of 5%. The interest credited at the end of each year is invested at an effective annual rate of 6%. The accumulated value at the end of seven years is  $X$ . Lori invested at the end of each year for 14 years at an effective annual interest rate of 2.5%. The interest credited at the end of each year is invested at an effective annual rate of 3%. The accumulated value at the end of 14 years is  $Y$ . Calculate  $Y/X$ .