PSTAT 171 Week 3 Extra Practice Soln

Section 1 P1 : loan 250,000

(i) 30 annual payments, amortization w. $i = 12\%$,

$$250000 = X\, a_{\overline{30}|(.12)} = X \cdot \frac{1 - 1.12^{-30}}{.12}$$

annual payments $X = 31035.9144$

(ii) Sinking fund :

① Interest $I = 250000\,(.1) = 25000$

② Sinking fund: $(X - I)\, s_{\overline{30}|j} = 250000$

i.e. $$\frac{(1+j)^{30} - 1}{j} = \frac{250000}{X - I}$$

solve for j //

Section 3 P2

$$250000 = X\, a_{\overline{360}|(.09)/12}$$

$$= X \cdot \frac{1 - \left(1 + \frac{.09}{12}\right)^{-360}}{(.09)/12}$$

$$X = 2011.56$$

(a) At time 0, $OB_0 = 250000$

At yr 1, $OB_{12} = X\, a_{\overline{348}|(.09)/12} = 248292.0073$

OB_{12}: 12 months; $a_{\overline{348}|}$: 29 × 12 months remaining

so Amount Paid = $2011.556542 \times 12 = 24138.6785$

PR (principal Repaid) $= 250000 - 248292.0073 = 1707.9927$

so Interest Paid = Amount Paid − PR

$= 24138.6785 - 1707.9927 = 22430.6858$

(b) At yr 29, $OB_{348} = X\, a_{\overline{12}|(.09)/12} = 23001.9734$

so PR $= 23001.9734$

Interest Paid = Amount Paid − PR

$= 24138.6785 - 23001.9734$

$= 1136.7051$ //

Section 1 P3 20-yr loan of 1000, $i = 10\%$

The first 10 yrs: payment = 1.5 Interest

$\Rightarrow PR_t = 0.5\,\text{Interest} = (.5)(10\%)\,OB_{t-1} = (.05)\,OB_{t-1}$

$OB_t = OB_{t-1} - PR_t = (.95)\,OB_{t-1}$

i.e. the OB decreases by 5% each year, so

$OB_{10} = OB_0 \cdot (.95)^{10} = 598.7369$

Last ten years: level payment X

$OB_{10} = X\, a_{\overline{10}|10\%} = X \cdot \dfrac{1 - 1.1^{-10}}{(.1)} = 97.44.$ //

Section 2 P1

① Borrows X, repay $X(1.07)^{10}$ at the end of yr 10.

② pays X to purchase a bond

par value — \$1000

coupons — \$1000 × 10%/2 = 50 semi-annually

yield — $i^{(2)} = 8\%$, $i^{(2)}/2 = 4\%$

so $X = 50\, a_{\overline{20}|4\%} + 1000\left(1+\frac{i^{(2)}}{2}\right)^{-20}$

$= 1135.9$

$\Rightarrow$ Repayment at yr 10: $X(1.07)^{10} = 2234.5$

③ Reinvested coupons:

$50 \cdot s_{\overline{20}|(.06/2)} = 1343.5$

Net Gain at yr 10: $1000 + 1343.5 - 2234.5 = 109$

(1000: redemption value; 1343.5: earned by coupons; 2234.5: amount repaid)

Section 2 P2

$P = Fr\, a_{\overline{n}|j} + F v^n$

Premium $P - F = Fr\, a_{\overline{n}|j} + F(v^n - 1)$

$= Fr\, a_{\overline{n}|j} - Fj\,\frac{1-v^n}{j}$

$= F(r-j)\, a_{\overline{n}|j}$

where $j = \frac{j^{(2)}}{2} = 3.5\%$ (semi-annually), $P-F = 36$

$$36 = 100(r-3.5\%)\cdot\frac{1-v^n}{3.5\%} \quad ①$$

Premium in the 5th coupon: $F(r-j)\,v^{n-5+1}$

$$1 = 100(r-3.5\%)\,v^{n-4} \quad ②$$

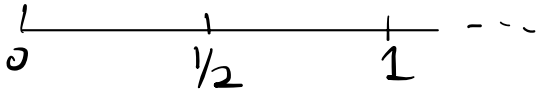
$$\frac{①}{②} \Rightarrow 36 = \frac{1-v^n}{v^{n-4}(.035)}$$

$$36(1.035)^4(.035) = \frac{1-v^n}{v^n} = \frac{1}{v^n}-1$$

solve for $v^n = .4088$

$$n = -\frac{\ln(.4088)}{\ln(1.035)} \approx 26$$

Section 2 P3



90 $\rightsquigarrow 90(1+\frac{i^{(2)}}{2})$

$= 90(1.033) = 92.97$

Coupon: $Fr/2 = \frac{100\times5}{2} = 2.5$

$OB_{1/2} = 92.97 - 2.5 = 90.47$