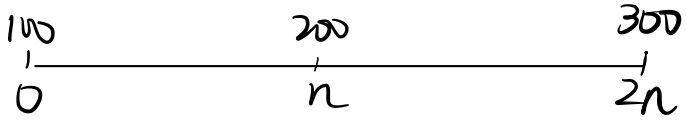
PSTAT 171 Extra Practice Soln

P1: (i)



$PV: \quad 100 + 200\, v^n + 300\, v^{2n}$

(ii)

$PV = 600\, v^{10}$

Two streams have the same PV

$$\Rightarrow \quad 600\, v^{10} = 100 + 200\, v^n + 300\, v^{2n}$$

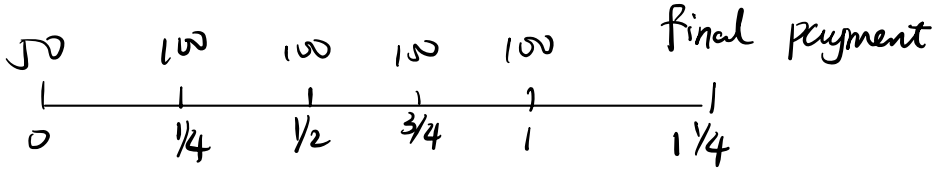
$$= 100 + 200\,(.7594) + 300\,(.7594)^2$$

$$= 424.893$$

So $v^{10} = .708$

$$i = \frac{1}{v} - 1 = 3.51\%$$

P2: Cost for TV: 480



$i^{(4)}/4 = 3\%$

$$PV = 480 = 50 + 100\, a_{\overline{4}|.03} + X(1+.03)^{-5}$$

$$X = (480 - 50 - 100\, a_{\overline{4}|.03})(1.03)^5 = 67.57$$

Moreover, if the interest rate changes to one-month rate $\bar{i}^{(12)}/12 = 1\%$, then three-month effective interest rate is

$$(1.01)^3 - 1 = .0303 = 3.03\%$$

$$X = (480 - 50 - 100\, a_{\overline{4}|3.03\%})(1.0303)^5 = 67.98.$$

P3: Interest for Eric:

$$X\left(1+\frac{i}{2}\right)^{16} - X\left(1+\frac{i}{2}\right)^{15} = X\frac{i}{2}\left(1+\frac{i}{2}\right)^{15}$$

Interest for Mike:

$$2X(1+8\bar{i}) - 2X(1+7.5\bar{i}) = 2X\cdot\frac{\bar{i}}{2} = X\bar{i}$$

They have the same interest

$$\Rightarrow X\frac{i}{2}\left(1+\frac{i}{2}\right)^{15} = Xi$$

so $1+\frac{i}{2} = 2^{1/15}$

$$i = 9.45\%$$

P4: Set $Y=50$, effective rate $i = \frac{d}{1-d}$

- Bruce: deposit $2Y$, interest earned in the 11^{th} yr

$$2Y(1+i)^{11} - 2Y(1+i)^{10} = 2Yi(1+i)^{10} = X$$

- Rubbic: deposit Y, interest earn in the 17^{th} yr

$$Y(1+i)^{17} - Y(1+i)^{16} = Yi(1+i)^{16} = X$$

Thus, $2Yi(1+i)^{10} = Yi(1+i)^{16}$

$$2 = (1+i)^6 \qquad i = 2^{1/6} - 1$$

$$X = 50\left(2^{1/6} - 1\right) 2^{16/6} = 38.87$$

P5: Tawny: $i^{(2)} = 10\%$, convertible semi-annually

Fabio: deposit 1000 w. simple interest j

$t = 5$, $\delta_t^T = \delta_t^F$

where $\delta_t^T = \frac{\frac{d}{dt}\left((1+i^{(2)}/2)^{2t}\right)}{(1+i^{(2)}/2)^{2t}} = 2\log 1.05$

$\delta_t^F = \frac{\frac{d}{dt}(1+tj)}{1+tj} = \frac{j}{1+tj}$ $\left(\delta_t = \frac{a'(t)}{a(t)}\right)$

Thus, $2\log 1.05 = \frac{j}{1+5j}$, $j = \frac{2\log 1.05}{1-10\log 1.05}$, $Z = 1000(1+5j)$

$= 1953$

P6. $PV_{Pottie} = 493 = X a_{\overline{n}|i} = X \cdot \frac{1-v^n}{i}$

$PV_{sam} = 2748 = 3X a_{\overline{2n}|i} = 3X \cdot \frac{1-v^{2n}}{i}$

$\Rightarrow \frac{493}{2748} = \frac{1-v^n}{3(1-v^{2n})} = \frac{1}{3(1+v^n)}$

$v^n = .8580$

P7: Investment at the beginning: \$10000

yield rate: $i^{(2)} = 7.45\%$

After 5 yrs: $AV = 10000\left(1+\frac{.0745}{2}\right)^{5\times 2} = 14415.66$

Note that AV directly comes from the re-investment of repayment from Tim:

i.e. $AV = X s_{\overline{12\times 5}|6\%/12}$

$= X \frac{(1.005)^{60}-1}{.005}$

Repayment from Tim $= X = 206.617$

Interest rate on Tim:

$10000 = X a_{\overline{60}|j} = X \frac{1-(1+j)^{-60}}{j}$

Use TI-30 Calculator $\Rightarrow i^{(12)} = 12j = 8.801\%$

P8: Susan: At the end of

yr 1: Z

yr 2: $Z + i_s Z + Z$ (interest; extra deposit)

yr 3: $2Z + 2i_s Z + Z$

$\vdots$

yr 7: $6Z + 6i_s Z + Z$

$$X = 7Z + (i_s Z)(Ia)_{\overline{6}|i_s}$$

Similarly: $Y = 7Z + (i_L Z)(Ia)_{\overline{6}|i_L}$

$i_s = 5\%$

$i_s' = 6\%$

$i_L = 2.5\%$

$i_L' = 3\%$