171 Wk5 Worksheet Soln

1. 2 bonds, $F = 100$

$$\begin{cases} P_1 + P_2 = 240 \\ P_1 - P_2 = 24 \end{cases} \Rightarrow \begin{cases} P_1 = 132 \\ P_2 = 108 \end{cases}$$

(1) $P_1 = F(2r)\,a_{\overline{n}|j} + Fv^n = 200\,r\,a_{\overline{n}|.015} + 100\,v^n_{.015}$

(2) $P_2 = Fr\,a_{\overline{n}|j} + Fv^n = 100\,r\,a_{\overline{n}|.015} + 100\,v^n_{.015}$

(1) − (2) ⇒

$$24 = 100\,r\,a_{\overline{n}|.015} = \frac{100r}{.015}(1 - v^n)$$

so $$v^n = 1 - \frac{24(.015)}{100r} = 1 - \frac{.0036}{r}$$

Thus, (2) ⇒

$$108 = 100r \cdot \frac{.0036/r}{.015} + 100\left(1 - \frac{.0036}{r}\right)$$

$$= 24 + 100 - \frac{.36}{r}$$

$$-16 = -\frac{.36}{r} \Rightarrow r = .0225$$

$$\left.\begin{aligned} r_2 &= .0225 \\ r_1 &= .045 \end{aligned}\right\} \text{coupon rate for 6 months.}$$

2.

	1/1	7/1	12/31
Balance	10	12	X
Deposit		X	

Time-weight return:

$$i_T = \frac{12}{10} \cdot \frac{X}{12+X} - 1 = 0$$

$$\frac{X}{12+X} = \frac{10}{12}$$

$$X = 10 + \frac{5}{6}X$$

$$\frac{1}{6}X = 10 \Rightarrow X = 60$$

Dollar-weight return:

$$A_0 = 10$$

$$C_1 = 60 \quad (\text{in } 1/2 \text{ yrs})$$

$$A_1 = 60$$

$$i_D = \frac{A_1 - C_1 - A_0}{A_0 + C_1(1-1/2)} = \frac{60-60-10}{10+60(1/2)} = -\frac{10}{40}$$

$$= -.25$$

3.

	2005/1/1	2005/4/1	2006/1/1	2007/1/1
Account:	100000	103000	A	103992
Deposit:		-8000		

2005: $x = \dfrac{A-(100000-8000)}{100000-8000(1-\frac{9}{12})}$ (dollar-weight MTD)

2006: $x = \dfrac{103992}{A} - 1$ (time-weight MTD)

Thus, $x = \dfrac{A-92000}{94000} = \dfrac{103992}{A} - 1$

$$\begin{cases} A = 92000 + 94000x \\ (1+x)A = 103992 \end{cases}$$

$(1+x)(92000+94000x) = 103992$

$\Rightarrow x = 6.25\%$

4.

	1/1	3/15	6/1	6/30	10/1	12/31
Account:	50	40	80	157.5	175	X
Deposit:		20	80		75	

1/1 – 6/30:

$$\frac{40}{50}\cdot\frac{80}{40+20}\cdot\frac{157.5}{80+80}-1=.05$$

6/30 – 12/31:

$$\frac{175}{157.5}\cdot\frac{X}{175+75}=1.05$$

$\Rightarrow X = 236.25$

5. (a) 10 yr bonds, $F=100$, coupon: semi-annually

Bond 1: $85.12 = \sum_{t=1}^{20} \frac{100(.02)}{(1+S_0(t/2))^{t/2}} + \frac{100}{(1+S_0(10))^{10}}$ (1)

Bond 2: $133.34 = \sum_{t=1}^{20} \frac{100(.05)}{(1+S_0(t/2))^{t/2}} + \frac{100}{(1+S_0(10))^{10}}$ (2)

(1) $\times 5$ − (2) $\times 2$:

$85.12 \times 5 - 133.34 \times 2$

$= \frac{500-200}{(1+S_0(10))^{10}}$

$\Rightarrow S_0(10) = .0646$

(b) See section notes.

6 (a) (d) : See section notes.

(b) $\text{RHS} = (1+S_0(k))^k$

$= (1+i_0(k-1,k))(1+S_0(k-1))^{k-1}$

$= (1+i_0(k-1,k))(1+i_0(k-2,k-1))(1+S_0(k-2))^{k-2}$

$= \cdots$ (You can verify by induction)

$= \prod_{j=1}^{k} (1+i_0(j-1,j))$

(c) Use formula:

$$i_0(k-1, k) = \frac{(1+s_0(k))^k}{(1+s_0(k-1))^{k-1}} - 1$$

7. See the next page.

8. See section notes.

Duration of a Portfolio

For portfolio A and B, with values P_A and P_B, and Duration D_A and D_B.

Then, $D_A = -(1+i)\dfrac{dP_A/di}{P_A}$, $D_B = -(1+i)\dfrac{dP_B/di}{P_B}$

$$-(1+i)\frac{dP_A}{di} = D_A \cdot P_A, \quad -(1+i)\frac{dP_B}{di} = D_B \cdot P_B$$

For this portfolio, aggregated Present Value of the portfolio $P = PV_A + PV_B$ and the Duration is

$$D_P = -(1+i)\frac{dP/di}{P}$$

$$= \frac{-(1+i)\,dP_A/di - (1+i)\,dP_B/di}{P}$$

$$= \frac{D_A P_A + D_B P_B}{P} = w_A P_A + w_B P_B$$

weight $w_A = \dfrac{P_A}{P_A + P_B}$ $\quad w_B = \dfrac{P_B}{P_A + P_B}$

Another fact: For one specific bond, if $F = C$ (face = redemption), then $\dfrac{F}{P}$ is a fixed value.

You are supposed to do Problem 7 now:

Step 1: Suppose the face amount $F = 100$, then

Bond 1: $P_1 = 88.35$, $D_1 = 12.7$

Bond 2: $P_2 = 130.49$, $D_2 = 14.6$

Now, we buy the portfolio:

Bond 1 has face F_1 and price P_1'

Bond 2 has face F_2 and price P_2'

Since the ratio is fixed,

$$\frac{P_1}{F} = \frac{P_1'}{F_1} \quad ; \quad \frac{P_2}{F} = \frac{P_2'}{F_2}$$

Step 2: Duration of portfolio $D_P = 13.5$

$$\begin{cases} w_1 D_1 + w_2 D_2 = 13.5 \\ w_1 + w_2 = 1 \end{cases} \Rightarrow \begin{cases} w_1 = 11/19 \\ w_2 = 8/19 \end{cases}$$

In addition, $w_1 = \dfrac{P_1'}{P_1' + P_2'}$ $\quad w_2 = \dfrac{P_2'}{P_1' + P_2'}$

$P_1'/P_2' = w_1/w_2 = 11/8$

Step 3: $F_1 = \dfrac{P_1}{F P_1'}$ $\quad F_2 = \dfrac{P_2}{F P_2'}$

so $\dfrac{F_1}{F_2} = \dfrac{P_1/P_1'}{P_2/P_2'} = \dfrac{11}{8} \cdot \dfrac{130.49}{88.35} = 2.0308$

Note that $\begin{cases} F_1 + F_2 = 100 \\ F_1 = 2.0308 F_2 \end{cases}$

so $F_2 = 32.9943$

$F_1 = 67.0057$

The total value for the portfolio:

$$P_1' + P_2' = \frac{F_1 P_1 + F_2 P_2}{F} = 102.65$$

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