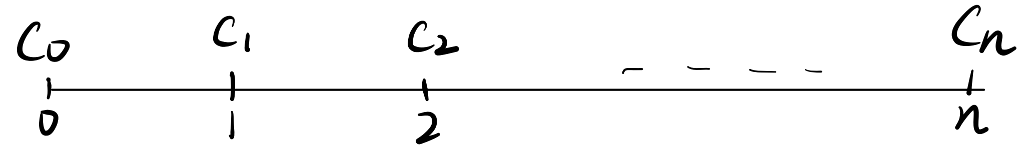
Ch5: Investment Yield

1. Internal Rate of Return



C_k: cash flows at time k $\begin{cases} C_k > 0: \text{ flows in} \\ C_k < 0: \text{ flows out} \end{cases}$

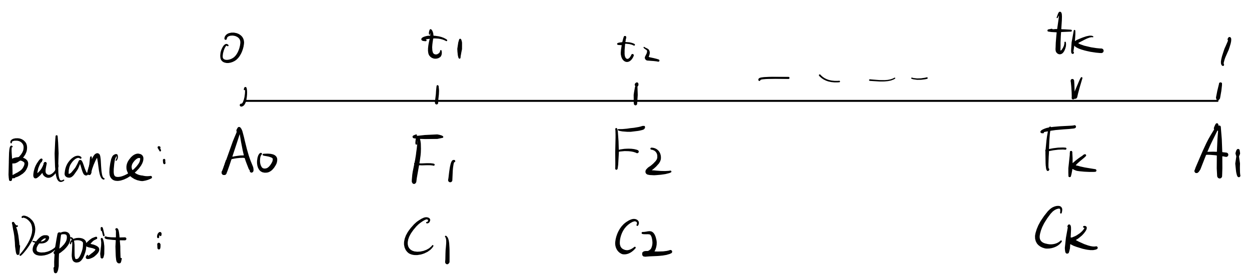
Equivalence Principle: Choose i s.t.

$$C_0 + \frac{C_1}{1+i} + \frac{C_2}{(1+i)^2} + \cdots + \frac{C_n}{(1+i)^n} = 0$$

solution i = IRR

Two ways to measure IRR:

(1) The time-weighted rate of return (Compound Interest)



① i_1: IRR from 0 to t_1

$$F_1 = (1+i_1)A_1 \Rightarrow \frac{F_1}{A_1} = 1+i_1$$

② i_2: IRR from t_1 to t_2

$$(F_1+C_1)(1+i_2) = F_2 \Rightarrow \frac{F_2}{F_1+C_1} = 1+i_2$$

③ $\tilde{i}_{k+1}$: IRR from t_k to 1.

$$(F_k + C_k)(1+\tilde{i}_{k+1}) = A_1 \Rightarrow \frac{A_1}{F_k + C_k} = 1 + \tilde{i}_{k+1}$$

To summerize :

time weighted rate of return $0 \to 1$:

$$(1+\tilde{i}_1)(1+\tilde{i}_2)\cdots(1+\tilde{i}_{k+1}) - 1$$

$$= \frac{F_1}{A_0} \cdot \frac{F_2}{F_1 + C_1} \cdot \cdots \cdot \frac{A_1}{F_k + C_k} - 1$$

(2) The dollar-weighted rate of return (Simple Interest)

	0	t_1	t_2	$\cdots$	t_k	1
Deposit :	A_0	C_1	C_2	$\cdots$	C_k	
yr 1 :	$A_0(1+\tilde{i})$	$C_1(1+(1-t_k)\tilde{i})$	$C_2(1+(1-t_2)\tilde{i})$	$\cdots$	$C_k(1+(1-t_k)\tilde{i})$	

By Equivalence Principle:

$$A_1 = A_0(1+\tilde{i}) + \sum_{j=1}^{k} C_j (1 + (1-t_j)\tilde{i})$$

$$= \left(A_0 + \sum_{j=1}^{k} C_j\right) + \left(A_0 + \sum_{j=1}^{k} C_j (1-t_j)\right)\tilde{i}$$

$$\Rightarrow \quad \tilde{i} = \frac{A_1 - (A_0 + \sum C_j)}{A_0 + \sum C_j (1-t_j)}$$

Dollar-weighted rate of return

P3:

	1/1	2/1	9/1	12/31
Balance:	100,000	98000	100000	105000
Deposit:		-10000	10000	

a. Time-weighted rate of return:

$$\frac{98000}{100000} \times \frac{100000}{98000-10000} \times \frac{105000}{100000+10000} - 1$$

$= 6.3\%$

b. Dollar-weighted rate of return:

$A_0 = 100000$ $\quad C_1 = -10000$ $\quad t_1 = 1/12$

$C_2 = 10000$ $\quad t_2 = 8/12$

$A_1 = 105000$

$$i = \frac{A_1 - (A_0 + C_1 + C_2)}{A_0 + \frac{11}{12}C_1 + \frac{4}{12}C_2} = 5.31\%$$

2. Net Present Value

Given rate of return/effective rate i,

$$NPV = C_0 + \frac{C_1}{1+i} + \frac{C_2}{(1+i)^2} + \cdots + \frac{C_n}{(1+i)^n}$$

C_k : cash flows in/out at time k

$+ \quad -$

P11: time 0: invest 50000

At end of each yr from yr 3-10: returns X

$i = 10\%$, $NPV = 2500$, find X.

Soln: $NPV = 2500$

$C_0 = -50000$ $\quad C_3 = \cdots = C_{10} = X$, $C_1 = C_2 = 0$

$2500 = -50000 + X a_{\overline{8}|10\%} \cdot v^2$

$$X = \frac{52500}{v^2 a_{\overline{8}|.01}} = 11907.38$$

Ch 6: Term Structure

1. Spot Rate: Yield rate on an zero-coupon bond.

Given a t-year maturity zero-coupon bond:

$S_0(t)$ = spot rate

K = maturity value / redemption value

$\Rightarrow P = K/(1+S_0(t))^t$

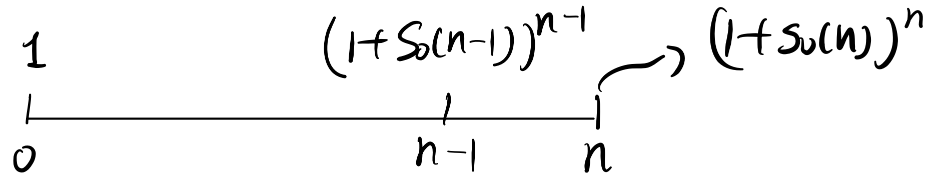
In general:

time	t_1	t_2	---	t_n
CF	C_1	C_2	--	C_n

$$PV_{CF} = \frac{C_1}{(1+S_0(t_1))^{t_1}} + \frac{C_2}{(1+S_0(t_2))^{t_2}} + \cdots + \frac{C_n}{(1+S_0(t_n))^{t_n}}$$

2. Forward rate of interest

$\tilde{i}_0(n-1, n)$: interest rate from time $n-1$ to n.



From $n-1$ to n:

$$(1+S_0(n-1))^{n-1}(1+\tilde{i}_0(n-1,n)) = (1+S_0(n))^n$$

$$\Rightarrow \quad \tilde{i}_0(n-1,n) = \frac{(1+S_0(n))^n}{(1+S_0(n-1))^{n-1}} - 1$$