Week 5 Worksheet

Botao Jin

University of California, Santa Barbara — September 3, 2024

1. Two bonds, each of face amount 100, are offered for sale at a combined price of 240. Both bonds have the same term to maturity but coupon rate for one is twice that of the other. The difference in price of the two bonds is 24. Prices are based on a nominal annual yield rate of 3%. Find the coupon rate of two bonds.

2. You are given the following information about an investment account:

Date	**Value Immediately Before Deposit**	**Deposit**
Jan 1st	10	
July 1st	12	X
Dec 31st	X	

   Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y. Calculate Y.

3. On Jan 1st, 2005, an investment account is worth $100,000$. On April 1st, 2005, the value has increased to $103,000$ and $8,000$ is withdrawn. On Jan 1st, 2007, the account is worth $103,992$. Assuming a dollar-weighted method for 2005 and a time-weighted method for 2006, the effective annual interest rate was equal to x for both 2005 and 2006. Calculate x.

4. An investor deposits 50 in an investment account on Jan 1st. The following summarizes the activity in the account during the year:

Date	**Value Immediately Before Deposit**	**Deposit**
March 15	40	20
June 1	80	80
Oct 1	175	75

   On June 30, the value of the account is 157.50. On Dec 31, the value of the account is X. Using the time-weighted method, the equivalent effective annual yield during the first 6 months is equal to the (time-weighted) effective annual yield during the entire 1-year period. Calculate X.

5. a. You are given the following information about two $10-$year bonds. Both bonds have face amount 100 and coupon payable semi-annually, with next coupon due in 6 months.
   - Bond 1: Coupon rate 4% per year, price \$85.12.
   - Bond 2: Coupon rate 10% per year, price \$133.34.

   Find the yield rate for a $10-$year zero coupon bond.

   b. You are given the following term structure (effective annual interest rates) for zero coupon bond maturities up to n years:

$$s_0(1) = s_0(2) = \cdots = s_0(n-1) < s_0(n)$$

   (flat term except the $n-$year maturity). An $n-$year bond have coupon rate $r > 0$ and annual coupons. Show that the yield to maturities for the bond j must satisfy $s_0(n-1) < j < s_0(n)$.

6. a. Find an expression for $i_0(k-1,k)$ in terms of $\{s_0(t)\}_{t\geq 0}$.

b. Show that for $k = 1, 2, \ldots, n$, we have

$$(1 + i_0(0,1))(1 + i_0(1,2)) \ldots (1 + i_0(k-1,k)) = (1 + s_0(k))^k.$$

c. Show that

$$\frac{d}{ds_0(k)} i_0(k-1,k) > 0$$

and

$$\frac{d}{ds_0(k-1)} i_0(k-1,k) < 0.$$

d. Show that if $s_0(k) > s_0(k-1)$, then $i_0(k-1,k) > s_0(k)$.

7. Under the current market conditions Bond 1 has a price (per 100 of face amount) of 88.35 and a Macaulay duration of 12.7, and Bond 2 has a price (per 100 of face amount) of 130.49 and Macaulay duration of 14.6. A portfolio is created with a combination of face amount F_1 of Bond 1 and face amount F_2 of Bond 2. The combined face amount of the portfolio is $F_1 + F_2 = 100$, and the Macaulay duration of the portfolio is 13.5. Find the portfolio value.

8. Liability payments of 100 each are due to be paid in $2, 4$, and 6 years from now. Asset cashflow consists of A_1 in 1 year and A_5 in 5 years. The yield for all payments is 10%. An attempt is made to have asset cash flow immunize the liability cashflow by matching present value and duration.

   a. Find A_1 and A_5.

   b. Determine whether or not the conditions for Redington immunization are satisfied.

Note: This study guide is used for Botao Jin's sections only. Comments, bug reports: b_jin@ucsb.edu