PSTAT 171 Week 3 Section 2

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Written Answer Questions

Problem 1

A $1,000 par value bond has 7.5% semiannual coupons and matures on July 1, 2017 at $1,050. Find the actual selling price of this bond on Nov 15, 2013 and the price that would bequote in a financial newspaper on the same date, based on a nominal annual yield rate of 5.80% compounded semiannually. Use the actual number of days to compute the accrued interest.

$F = \$1000 \quad r = 7.5\% \text{ (semi-annual)}, \quad j = 5.8\%, \text{ Maturity: } 7/1/2017$

$7/1/2013: \quad P = 1000(.0375) a_{\overline{8}|j/2} + 1000 \cdot \left(1 + \tfrac{j}{2}\right)^{-8}$

$\qquad = 1099.70$

$7/1/2013 \xrightarrow{\text{accumulate}} 11/15/2013$

$\left.\begin{array}{l} 7/1/2013 - 11/15/2013 : 137 \text{ days} \\ 7/1/2013 - 1/1/2014 : 184 \text{ days} \end{array}\right\}$ Actuarial price on 11/15/2013:

$\qquad\qquad (1099.7)\left(1 + \tfrac{.058}{2}\right)^{\frac{137}{184}}$

$\qquad\qquad = 1123.36$

Notes (Continued)

Accrued Interest on 11/15/2013: $\left(\dfrac{137}{184}\right) \cdot (37.5) = 27.92$

$\downarrow$

coupon

Price on financial newspaper: $1123.36 - 27.92 = 1095.44$

Written Answer Questions

Problem 2

A company buys a 100 par value bond with 5% annual coupons. The company pays a price that will give a yield rate of 4% effective if the bond matures at par at the end of 7 years. The company receives all coupons when due. However, at the end of 7 years, the company receives a maturity value of only 90, due to the bankruptcy of the issuer of the bond. The company's effective annual yield rate over the 7-year period is i. Determine i.

Notes

$$P = 100(.05) a_{\overline{7}|.05} + 100 \, v^7 = 106 \qquad v = \frac{1}{1.05}$$

At maturity : Redemption value reduces to 90.

$$P = 106 = 100(.05) a_{\overline{7}|i} + 90(1+i)^{-7}$$

$$\Rightarrow \bar{i} = 2.78\%$$

Notes (Continued)

Written Answer Questions

Problem 3

There are 10 bonds, each one with par value $100, 4% semiannual coupons and redemption value of $120. The bonds are purchased for $106 each. One bond will mature in 11 years, the second in 12 years, ans so on, with the 10th bond mature in 20 years. How many of bonds will earn a numinal yield rate of at least 4.5% per annum compounded semiannually?

$$P = 106 < 120 = C$$

discount : $n \downarrow$ implies yield rate $i \uparrow$

$$P = Fr\, a_{\overline{n}|i} + C\, v^n$$

$$= Fr\, a_{\overline{n}|i} + C v^n - C + C$$

$$= Fr\, a_{\overline{n}|i} + Ci\, \frac{v^n - 1}{i}$$

$$= C + (Fr - Ci)\, a_{\overline{n}|i}$$

$$106 = 120 + (2 - (120)(.0225)) \, a_{\overline{n}.0225}$$

$$\Rightarrow \quad a_{\overline{n}.0225} = \frac{14}{.7} = 20$$

solve for n.

Multiple Choice

Problem 2

Among a company's assets and accounting records, an actuary finds a 15-year bond that was purchased at a premium. For the records, the actuary has determined the following:

(i) The bond pays semiannual interest.

(ii) The amount for amortization of the premium in the 2nd coupon payment was 977.19.

(iii) The amount for amortization of the premium in the 4th coupon payment was 1046.79.

What is the value of the premium?

premium in the k-th coupon PMTs:

$$P(r-j) \, v^{n-k+1} \qquad n = \# \text{ of coupons in total} = 2\times 15 = 30$$

(ii): $P(r-j) \, v^{n-1} = 977.19$

(iii): $P(r-j) \, v^{n-3} = 1046.79$

$$\Rightarrow \quad v^2 = \frac{977.19}{1046.79} \qquad i = 3.5\%$$

Premium:

$$P - F = P(r - j) \, a_{\overline{n}|j}$$

$$= \frac{977.19}{v^{n-1}} \cdot \frac{1 - v^n}{i}$$

$$= 48739.29$$

Extra Practice

Problem 1

An investor borrows an amount at an annual effective interest rate of 7% and will repay all interest and principal in a lump sum at the end of 10 years. She uses the amount borrowed to purchase a 1000 par value 10-year bond with 10% semiannual coupons bought to yield 8% convertible semiannually. All coupon payments are reinvested at a nominal rate of 6% convertible semiannually. Calculate the net gain to the investor at the end of 10 years after the loan is repaid.

Extra Practice

Problem 2

A bond of face amount 100 is purchased at a premium of 36 to yield 7%. The amount for amortization of premium in the 5th coupon is 1.00. What is the term of the bond?

Note: Coupon are paid semi-annually.

Problem 3

The amortization schedule for a 100, 5% bond with yielding a nominal annual rate of $i^{(2)} = 6.6\%$ gives a value of 90 for the bond at the beginning of a certain 6-month period just after a coupon has been paid. What is the book value at the start of the next 6-month period?