171 Wk5 Worksheet Soln

1. 2 bonds, $F = 100$

$$\begin{cases} P_1 + P_2 = 240 \\ P_1 - P_2 = 24 \end{cases} \Rightarrow \begin{cases} P_1 = 132 \\ P_2 = 108 \end{cases}$$

(1) $P_1 = F(2r)\, a_{\overline{n}|j} + F v^n = 200\, r\, a_{\overline{n}|.015} + 100\, v^n_{.015}$

(2) $P_2 = F r\, a_{\overline{n}|j} + F v^n = 100\, r\, a_{\overline{n}|.015} + 100\, v^n_{.015}$

(1) $-$ (2) $\Rightarrow$

$$24 = 100\, r\, a_{\overline{n}|.015} = \frac{100r}{.015}(1 - v^n)$$

so $$v^n = 1 - \frac{24(.015)}{100r} = 1 - \frac{.0036}{r}$$

Thus, (2) $\Rightarrow$

$$108 = 100r \cdot \frac{.0036/r}{.015} + 100\left(1 - \frac{.0036}{r}\right)$$

$$= 24 + 100 - \frac{.36}{r}$$

$$-16 = -\frac{.36}{r} \Rightarrow r = .0225$$

$\left.\begin{array}{l} r_2 = .0225 \\ r_1 = .045 \end{array}\right\}$ coupon rate for 6 months.

2.

	1/1	7/1	12/31
Balance	10	12	X
Deposit		X	

Time-weight return:

$$i_T = \frac{12}{10} \cdot \frac{X}{12+X} - 1 = 0$$

$$\frac{X}{12+X} = \frac{10}{12}$$

$$X = 10 + \frac{5}{6}X$$

$$\frac{1}{6}X = 10 \quad \Rightarrow X = 60$$

Dollar-weight return:

$A_0 = 10$

$C_1 = 60$ (in 1/2 yrs)

$A_1 = 60$

$$i_D = \frac{A_1 - C_1 - A_0}{A_0 + C_1(1-1/2)} = \frac{60-60-10}{10+60(1/2)} = -\frac{10}{40}$$

$$= -.25$$

3.

	2005/1/1	2005/4/1	2006/1/1	2007/1/1
Account:	100000	103000	A	103992
Deposit:		−8000		

2005: $x = \dfrac{A - (100000 - 8000)}{100000 - 8000\left(1 - \frac{9}{12}\right)}$ (dollar-weight MTD)

2006: $x = \dfrac{103992}{A} - 1$ (time-weight MTD)

Thus, $x = \dfrac{A - 92000}{94000} = \dfrac{103992}{A} - 1$

$$\begin{cases} A = 92000 + 94000x \\ (1+x)A = 103992 \end{cases}$$

$(1+x)(92000 + 94000x) = 103992$

$\Rightarrow x = 6.25\%$

4.

	1/1	3/15	6/1	6/30	10/1	12/31
Account:	50	40	80	157.5	175	X
Deposit:		20	80		75	

1/1 – 6/30:

$$\frac{40}{50} \cdot \frac{80}{40+20} \cdot \frac{157.5}{80+80} - 1 = .05$$

6/30 – 12/31:

$$\frac{175}{157.5} \cdot \frac{X}{175+75} = 1.05$$

$$\Rightarrow X = 236.25$$

5. (a) 10 yr bonds, $F=100$, coupon: semi-annually

$$\text{Bond 1: } 85.12 = \sum_{t=1}^{20} \frac{100(.02)}{(1+s_0(t/2))^{t/2}} + \frac{100}{(1+s_0(10))^{10}} \qquad (1)$$

$$\text{Bond 2: } 133.34 = \sum_{t=1}^{20} \frac{100(.05)}{(1+s_0(t/2))^{t/2}} + \frac{100}{(1+s_0(10))^{10}} \qquad (2)$$

$(1)\times 5 - (2)\times 2$:

$$85.12\times 5 - 133.34\times 2$$

$$= \frac{500-200}{(1+s_0(10))^{10}}$$

$$\Rightarrow s_0(10) = .0646$$

(b) See section notes.

6 (a)(d): See section notes.

(b) $\text{RHS} = (1+s_0(k))^k$

$$= (1+i_0(k-1,k))(1+s_0(k-1))^{k-1}$$

$$= (1+i_0(k-1,k))(1+i_0(k-2,k-1))(1+s_0(k-2))^{k-2}$$

$= \cdots$ (You can verify by induction)

$$= \prod_{j=1}^{k}(1+i_0(j-1,j))$$

(c) Use formula:

$$i_0(k-1,k) = \frac{(1+s_0(k))^k}{(1+s_0(k-1))^{k-1}} - 1$$

7.
$$\begin{cases} F_1 + F_2 = 100 \\ \dfrac{F_1 D_1 + F_2 D_2}{F_1 + F_2} = 13.5 \quad (*) \end{cases}$$

$D_1 = 12.7 \qquad D_2 = 14.6$

$\Rightarrow \; 12.7F_1 + 14.6F_2 = 1350$

so $F_1 = 57.89 \qquad F_2 = 42.11$

value: $P_1 = 88.35, \; P_2 = 130.49$ (per 100 of face)

$$V = \frac{F_1 P_1 + F_2 P_2}{100} = 106.095 \quad (**)$$

$(*)$: For portfolio A and B, with values

PV_A and PV_B, and Duration D_A and D_B.

Then,
$$\begin{cases} PV_A \cdot D_A = \sum_{k=1}^{n} k v^k CF_k^{(A)} \\ PV_B \cdot D_B = \sum_{k=1}^{n} k v^k CF_k^{(B)} \end{cases}$$

where $CF_k^{(A)}$: cashflows for asset A at time k,

$CF_k^{(B)}$: cashflows for asset B at time k.

Then Duration for portfolio is

$$D_P = \frac{\sum_{k=1}^{n} k v^k CF_k^{(A)} + \sum_{k=1}^{n} k v^k CF_k^{(B)}}{\sum_{k=1}^{n} v^k CF_k^{(A)} + \sum_{k=1}^{n} v^k CF_k^{(B)}}$$

$$= \frac{PV_A \cdot D_A + PV_B \cdot D_B}{PV_A + PV_B}$$

(**): Fact : If $F=C$ (face = redemption), then

$\frac{F}{PV}$ is fixed.

For Bond 1 : $\frac{100}{P_1} = \frac{F_1}{V_1} \Rightarrow V_1 = \frac{F_1 P_1}{100}$

For Bond 2 : $\frac{100}{P_2} = \frac{F_2}{V_2} \Rightarrow V_2 = \frac{F_2 P_2}{100}$

Value for portfolio:

$$V_1 + V_2 = \frac{F_1 P_1 + F_2 P_2}{100}$$

8. See Section Notes.