



# **Bulanık Mantık**

## **(MÜH 425 – Bilgisayar Müh. Böl.)**

Prof.Dr. Yaşar BECERİKLİ

Hafta-4  
Bulanık Kümeler- Bulanık İşlemler

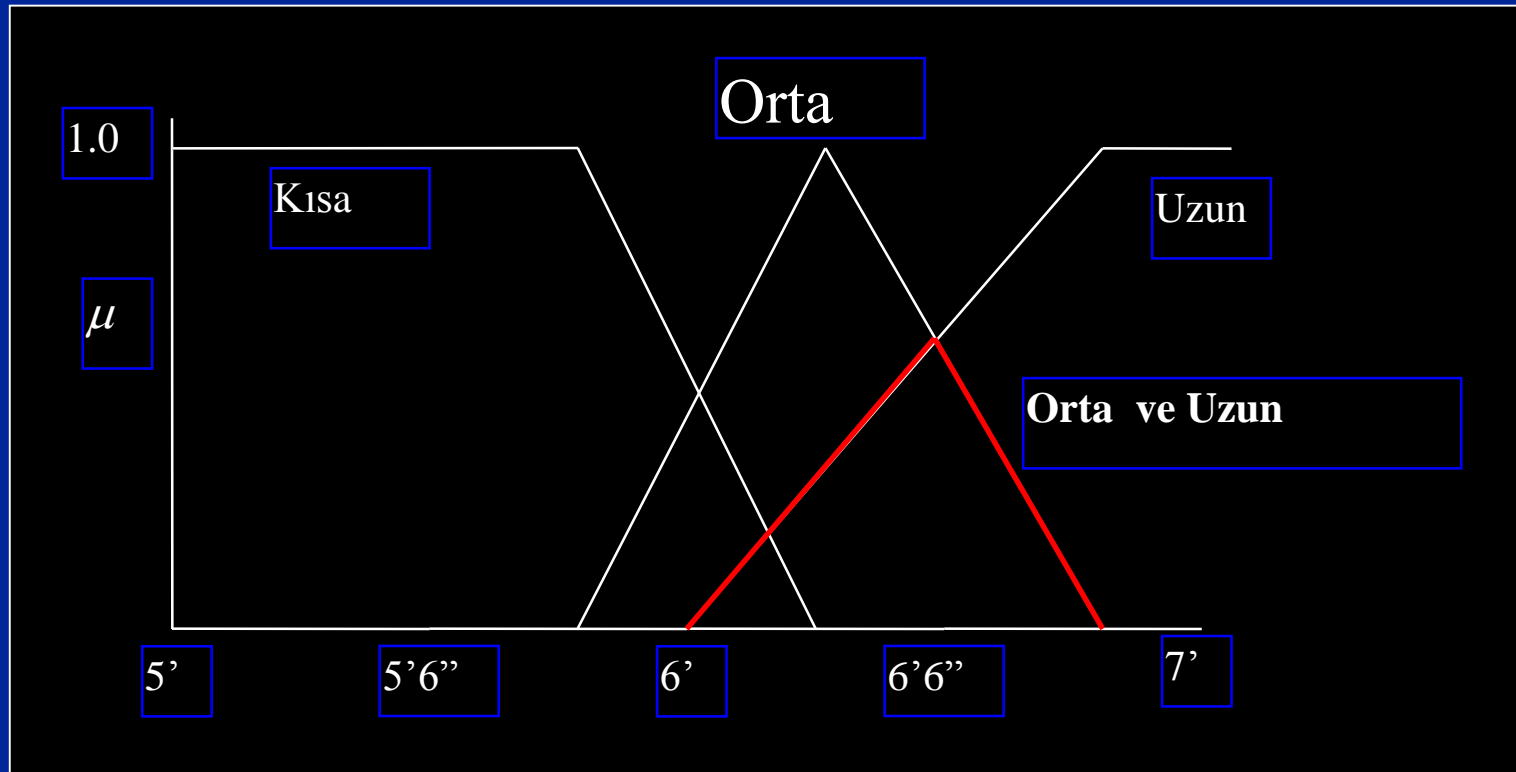
# İÇERİK

- Teorinin mucidi: Lutfi Asker Zadeh
- Bulanık Mantığa Giriş
- Bulanık Kümeler
- Temel İşlemler
- Kural Tabanı
- Bulandırma, Durulama
- Üyelik Fonksiyonları
- Çıkartım Sistemleri
- FAM tablosu,
- Uygulamalar

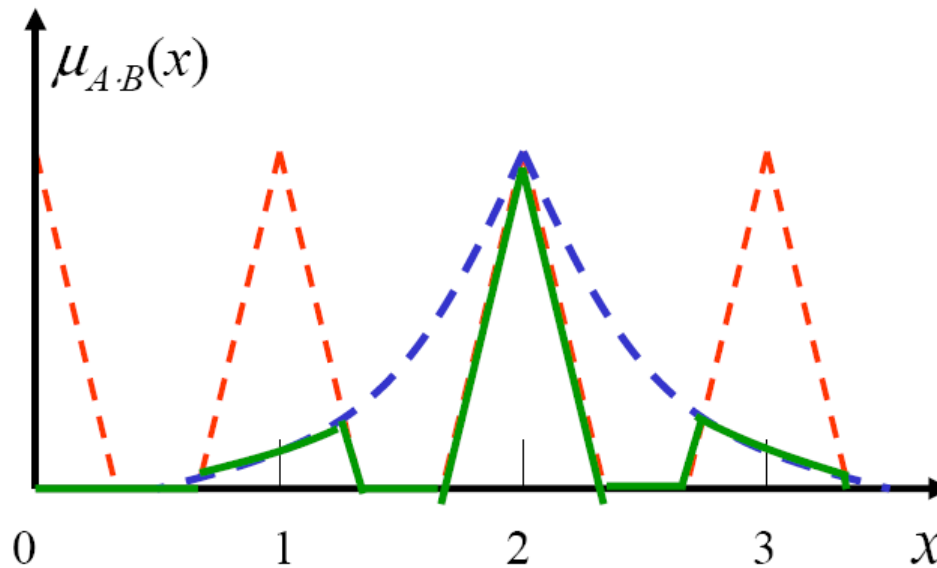
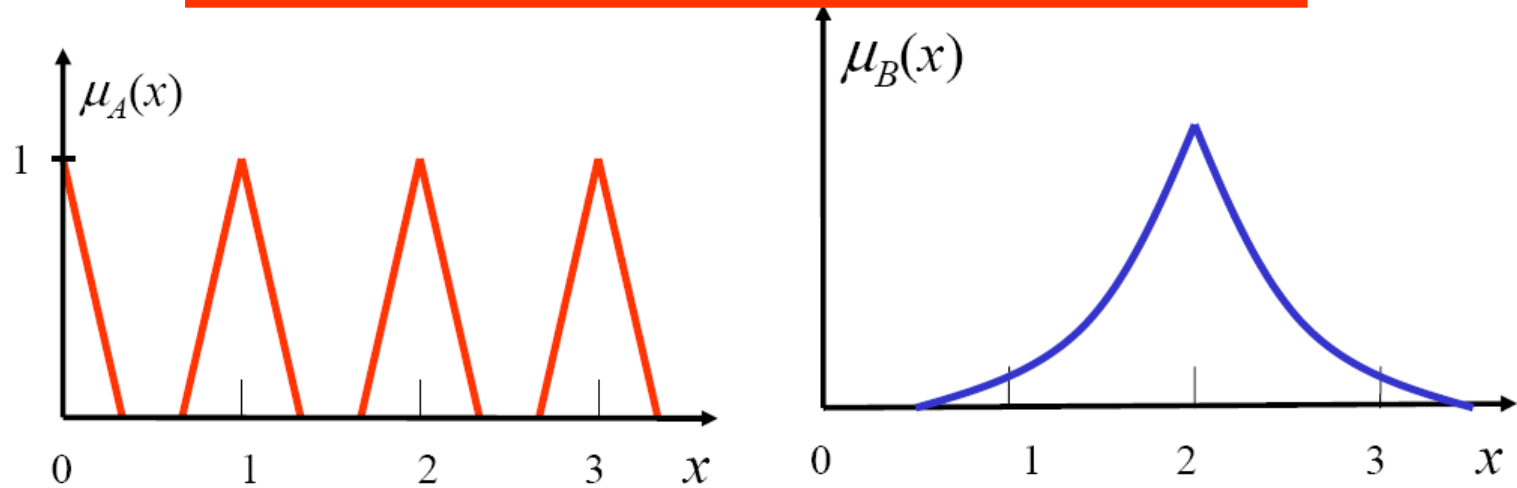
# Kesişim (t-normu tipi)

$$\mu_{A.B}(x) = \mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

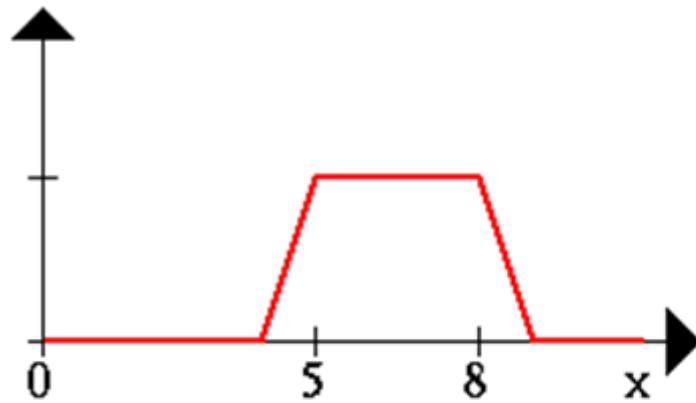
- Örnek: Orta ve Uzun



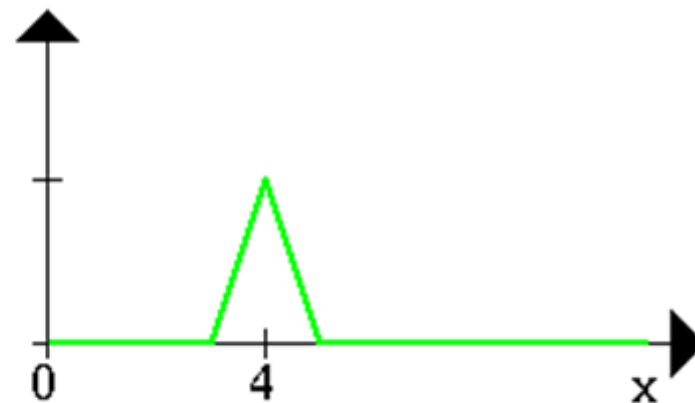
$$\mu_{A.B}(x) = \min(\mu_A(x), \mu_B(x))$$



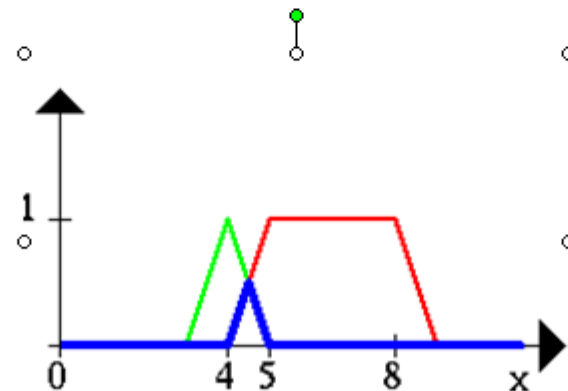
$$\mu_{A.B}(x) = \min(\mu_A(x), \mu_B(x))$$



A

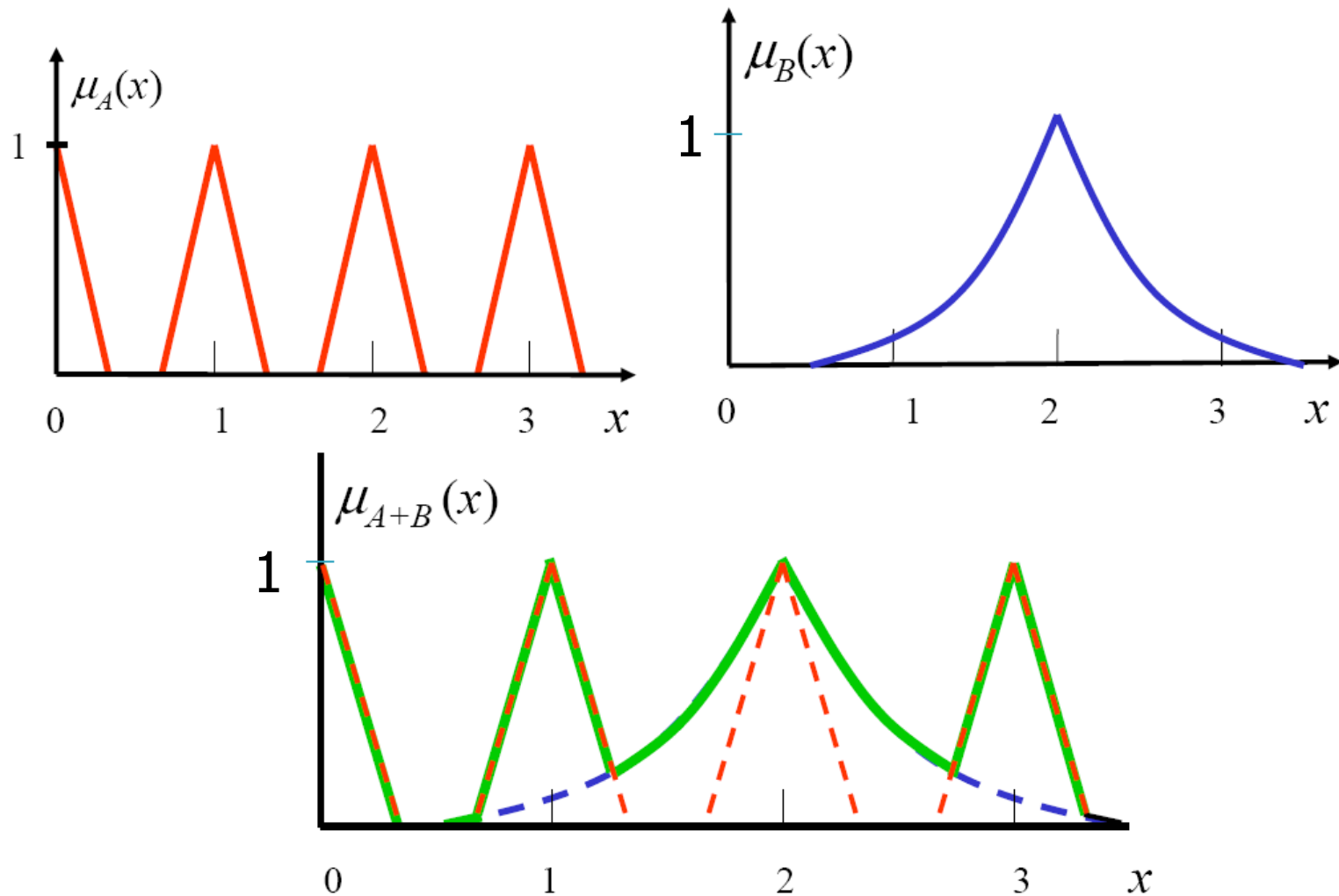


B

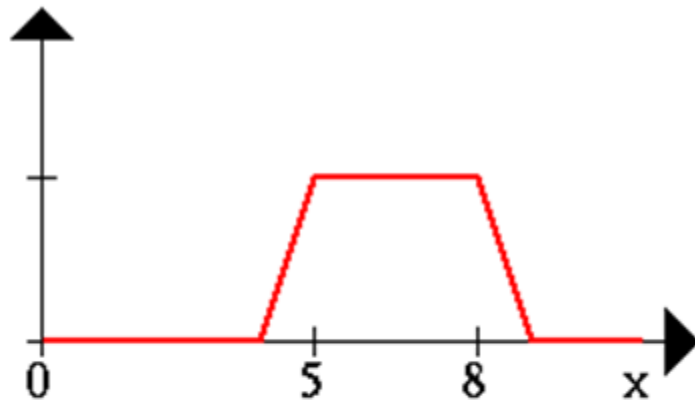


## Birleşim (s-normu tipi)

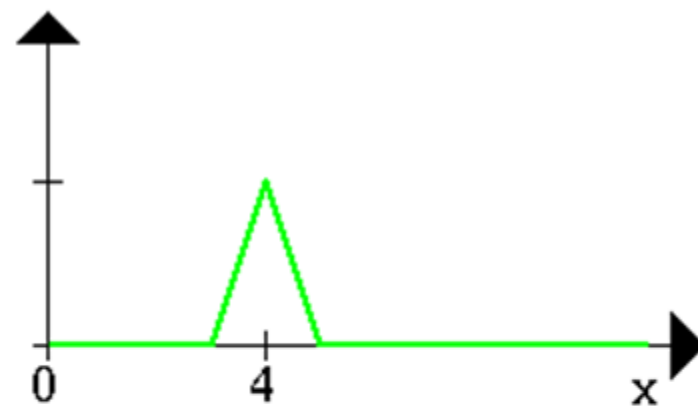
$$\mu_{A+B}(x) = \mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$



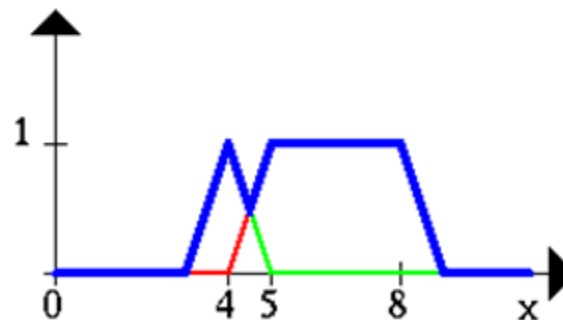
$$\mu_{A+B}(x) = \max(\mu_A(x), \mu_B(x))$$



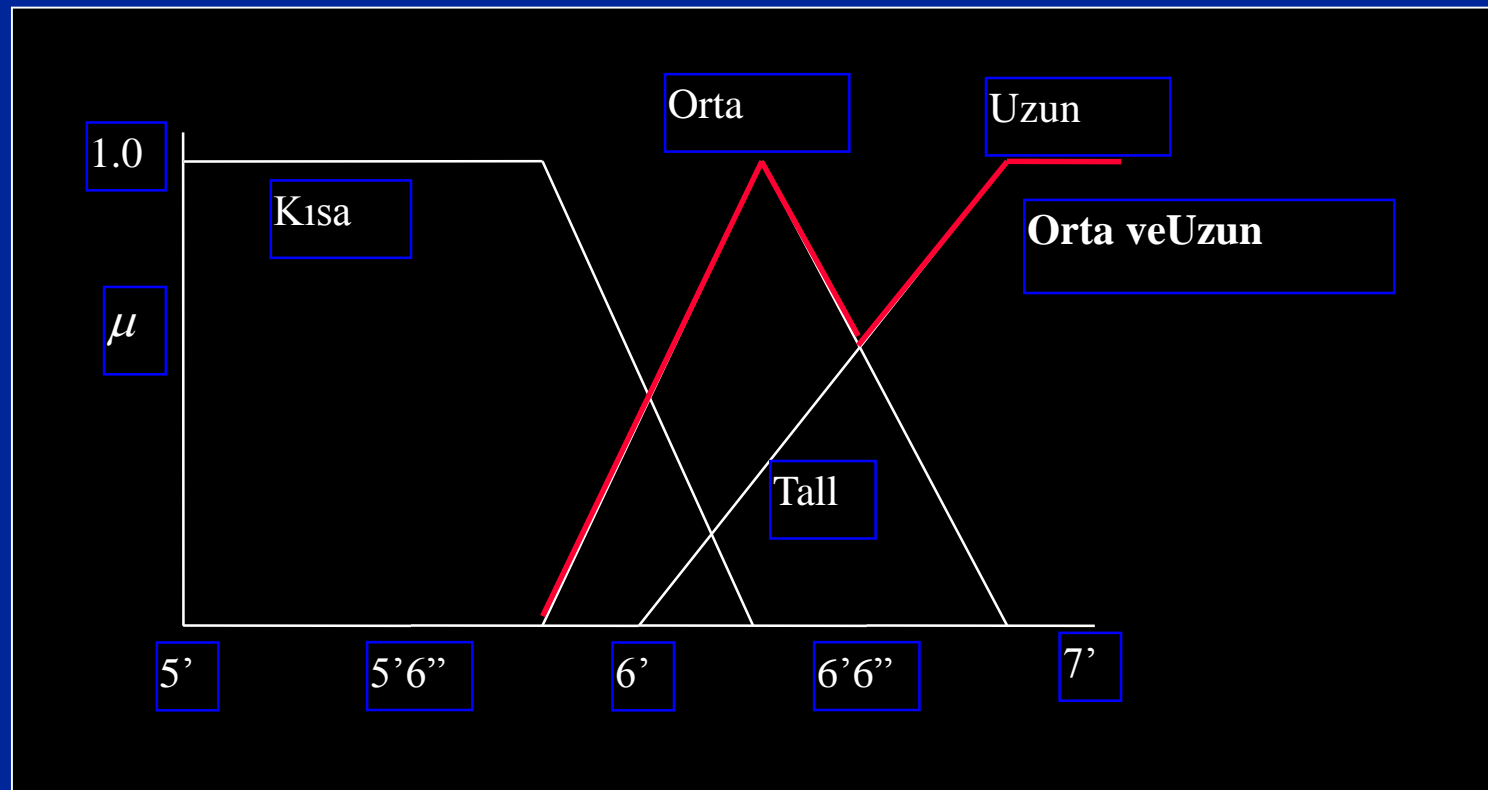
A



B

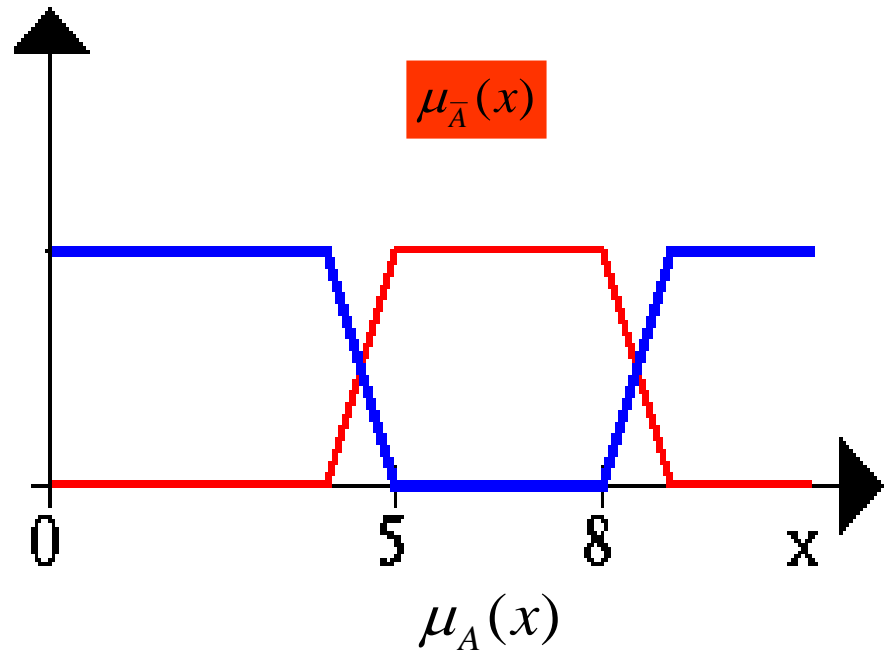
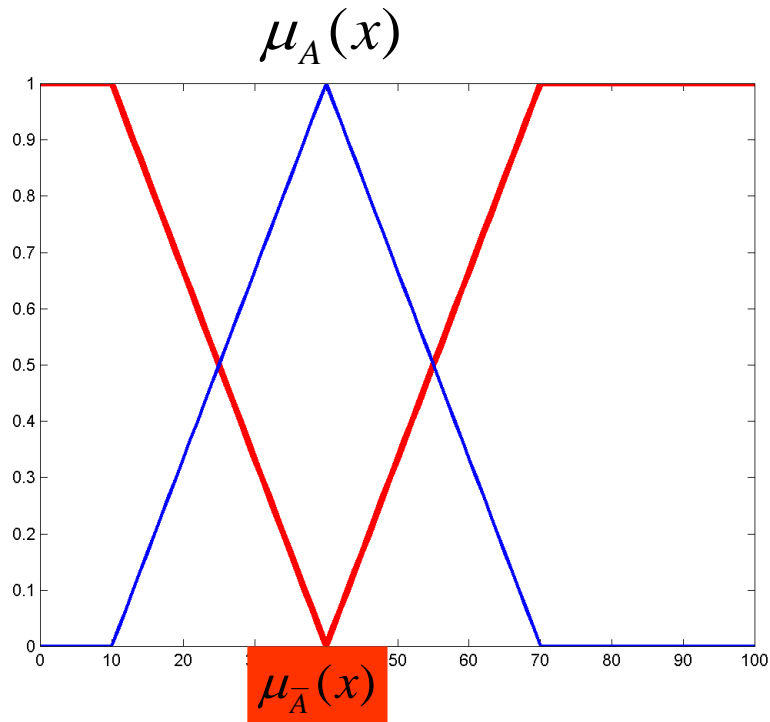


$$\mu_{A+B}(x) = \max(\mu_A(x), \mu_B(x))$$



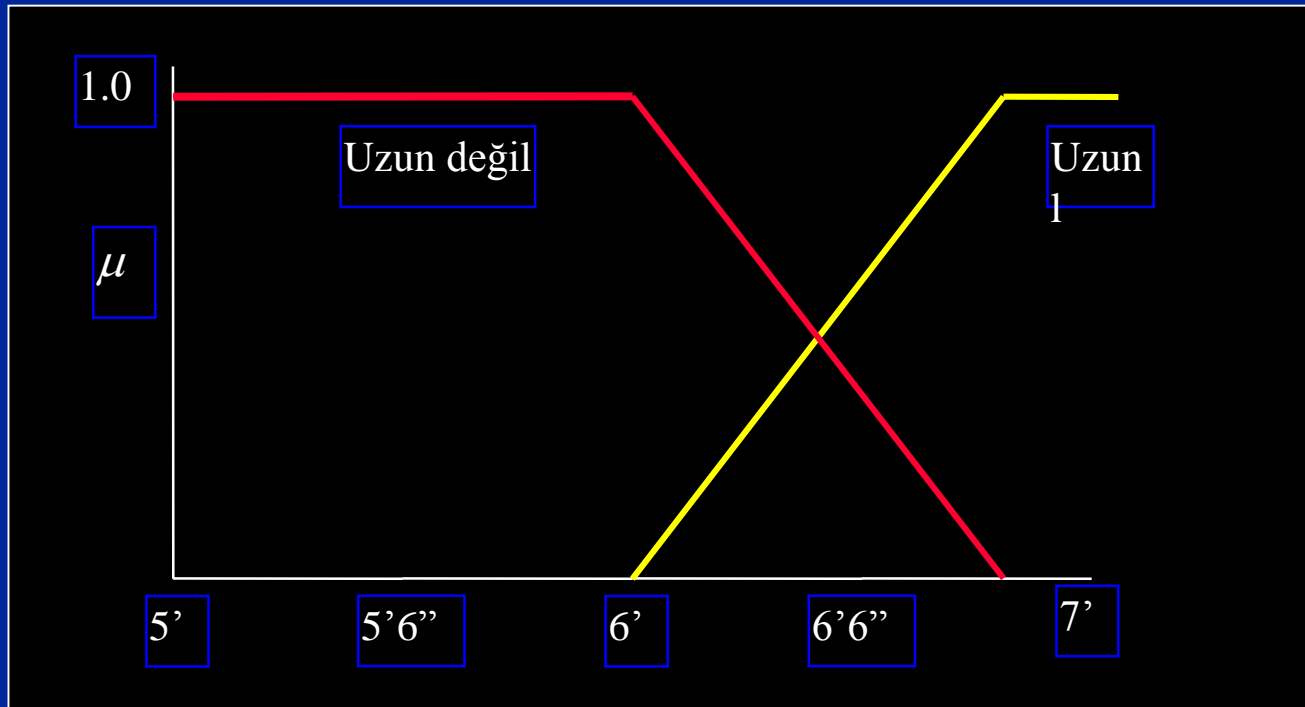


$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$



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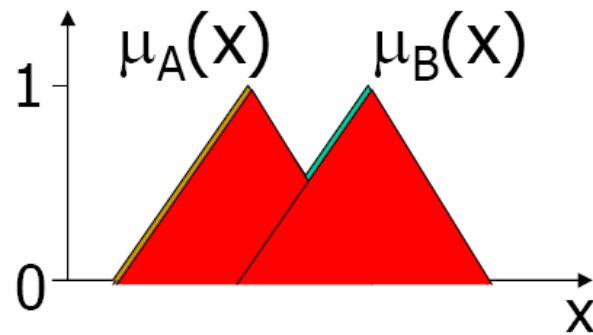
*Tümleme (Negation:)*  $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$



# Bulanık Mantık İşlemleri

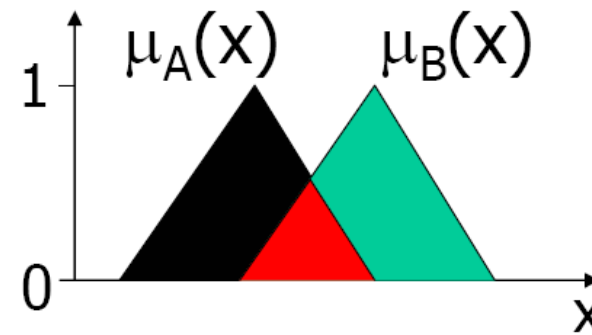
Union

$$\mu_{A \vee B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

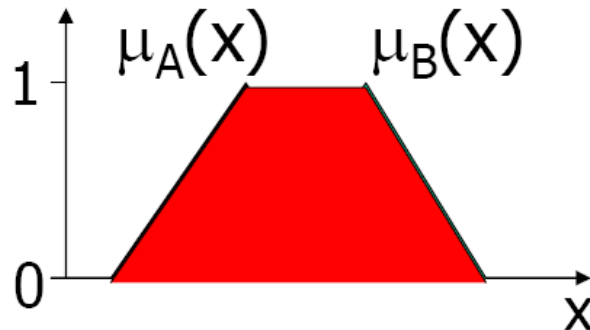


Intersection

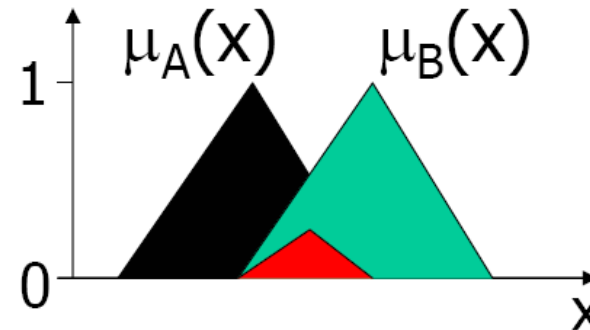
$$\mu_{A \wedge B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$



$$\mu_{A \vee B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$$



$$\mu_{A \wedge B}(x) = \mu_A(x) \cdot \mu_B(x)$$



# Bulanık Mantıkta AND (t-normu) işlemleri

$$\mu_A(X) \cdot \mu_B(X)$$

$$\frac{\mu_A(X) \cdot \mu_B(X)}{\mu_A(X) + \mu_B(X) - \mu_A(X) \cdot \mu_B(X)}$$

$$\max\{0, \mu_A(X) + \mu_B(X) - 1\}$$

$$\frac{\mu_A(X) \cdot \mu_B(X)}{2 - [\mu_A(X) + \mu_B(X) - \mu_A(X) \cdot \mu_B(X)]}$$

# Bulanık Mantıkta OR (s-normu) işlemleri

$$\mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

$$\frac{\mu_A(x) + \mu_B(x) - 2\mu_A(x) \cdot \mu_B(x)}{1 - \mu_A(x) \cdot \mu_B(x)}$$

$$\frac{\mu_A(x) + \mu_B(x)}{1 + \mu_A(x) \cdot \mu_B(x)}$$

$$\min\{1, \mu_A(x) + \mu_B(x)\}$$

if  $\langle \text{bulanık önerme} \rangle$ , then  $\langle \text{bulanık önerme} \rangle$

" $x, A$  ise (ve)  $y, B$  ise" ;  $A \cap B$

$$\mu_{A \cap B} = t(\mu_A(x), \mu_B(y)) \begin{cases} \rightarrow \min(\mu_A, \mu_B) \\ \rightarrow \mu_A \cdot \mu_B \\ \rightarrow \text{diğer} \end{cases}$$

" $x, A$  ise (veya)  $y, B$  ise" ;  $A \cup B$

$$\mu_{A \cup B} = s(\mu_A(x), \mu_B(y)) \begin{cases} \rightarrow \max(\mu_A, \mu_B) \\ \rightarrow \mu_A + \mu_B - \mu_A \cdot \mu_B \\ \rightarrow \text{diğer} \end{cases}$$

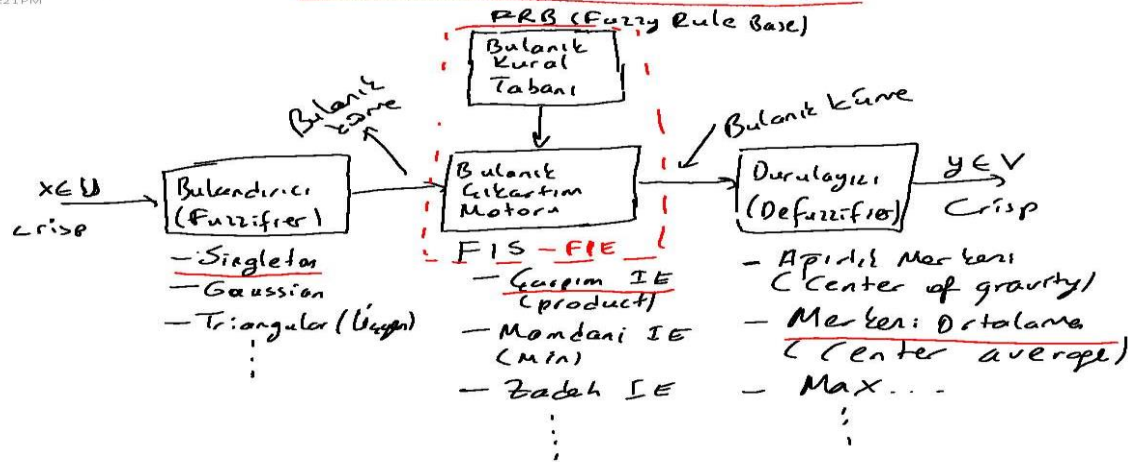
" $x, A$  değil ise",  $\Rightarrow$  DEĞİL

$$\mu_{\bar{A}}(x) = d(\mu_A(x)) = 1 - \mu_A(x)$$

Örnek: " $(x_1, A$  ise ve  $x_2, B$  değilse) veya  $x_3, C$  ise" önermesini yazalım.

$$s[t(\mu_A(x_1), d(\mu_B(x_2))), \mu_C(x_3)]$$

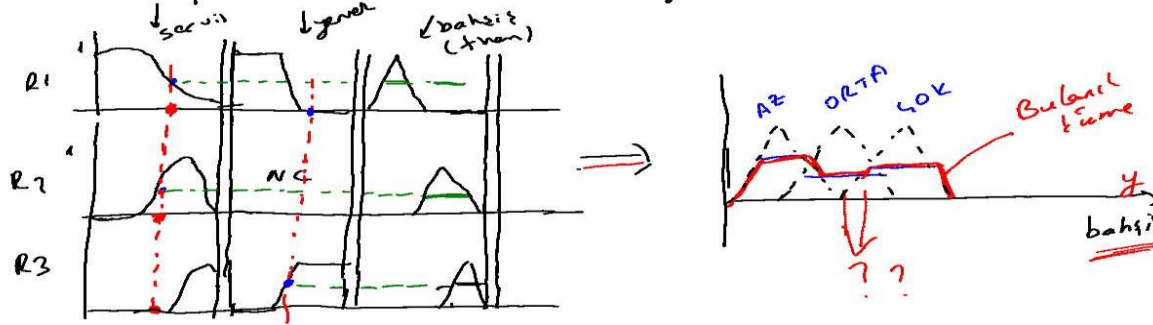
## Genel Bulanik Sistem Yapısı



Standard Bulanik Sistem (SBS): Singleton Bulanıklaştırıcı, Göçüm İE ve Merkez Ortalama Durulaştırıcı kullanan Bulanik Sistemlerdir.

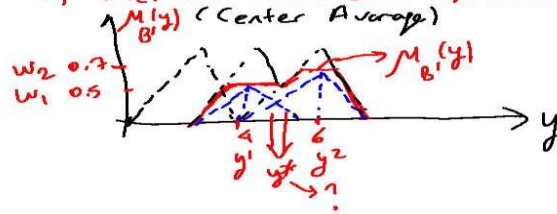
## Bulanik Çıkartım Motoru (FIS-FIE) (S-ağı Çıkartma)

- R1 : Eğer servis KÖTÜ veya yavaş BAYAT ise bahşiş AZ.  
R2 : if servis ORTA then bahşiş ORTA  
R3 : Eğer servis İYİ veya yavaş GÜZEL ise bahşiş ÇOK



### DURULAMA

- 1.) Merkezi Ortalama Metodu:



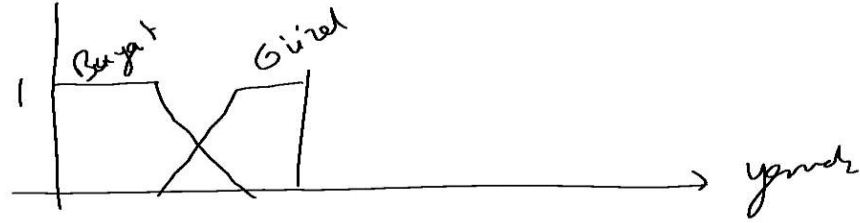
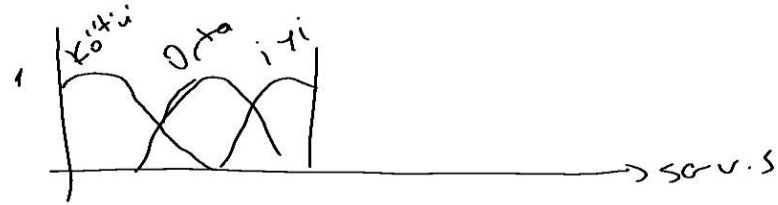
$$y^* = \frac{4 \cdot 0.5 + 6 \cdot 0.7}{0.5 + 0.7} = 5.16 //$$

Genel olarak:

$$y^* = \frac{y^1 \cdot w_1 + y^2 \cdot w_2 + \dots}{w_1 + w_2 + \dots}$$

$$= \frac{\sum_{l=1}^M y^l \cdot w_l}{\sum_{l=1}^M w_l}$$

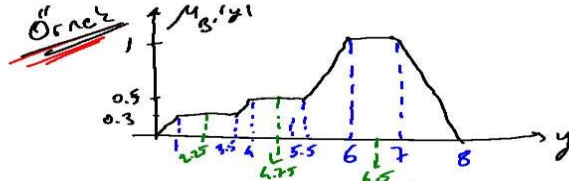




## 2. Ağırlık Merkezi Metodu (Center of Gravity)

$$y^* = \frac{\int_{B'} y \cdot \mu_{B'}(y) dy}{\int_{B'} \mu_{B'}(y) dy}$$

(Genel Formül)



a) Merkezi ortalamalarla

$$y^* = \frac{2.25 \times 0.3 + 4.75 \times 0.5 + 6.5 \times 1}{0.3 + 0.5 + 1} = \underline{\underline{5.22}}$$

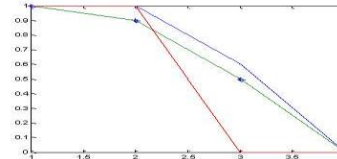
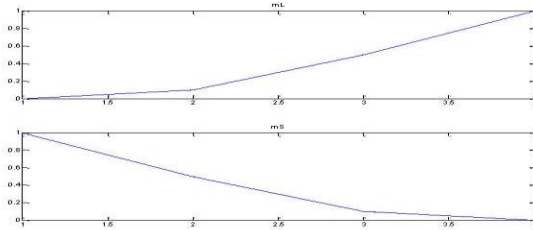
b) Ağırlık Merkezi Yöntemi ile

$$y^* = \frac{\int_0^2 y(0.3y) dy + \int_2^{4.75} y \cdot 0.3 dy + \int_{4.75}^6 y \frac{2}{5} (y - \frac{11}{2}) dy + \int_6^{6.5} y \cdot 0.5 dy + \int_{6.5}^7 y \cdot (y - 5) dy + \int_7^8 y \cdot 1 dy + \int_8^8 y(8 - y) dy}{\int_0^2 0.3y dy + \int_2^{4.75} 0.3 dy + \int_{4.75}^6 \frac{2}{5} (y - \frac{11}{2}) dy + \int_6^{6.5} 0.5 dy + \int_{6.5}^7 (y - 5) dy + \int_7^8 1 dy + \int_8^8 (8 - y) dy} = \underline{\underline{4.9}}$$

27.10.2009

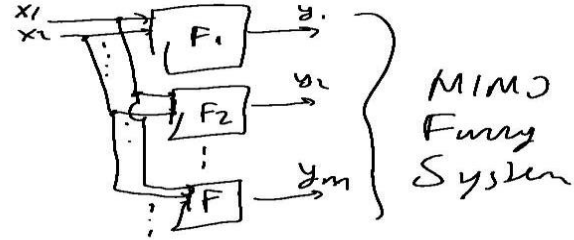
Örnek Matlab.

$u = 1:4;$       $L = \begin{bmatrix} 0 & 0.1 & 0.5 & 1 \end{bmatrix};$   
 $v = 1:4;$       $S = \begin{bmatrix} 1 & 0.5 & 0.1 & 0 \end{bmatrix};$



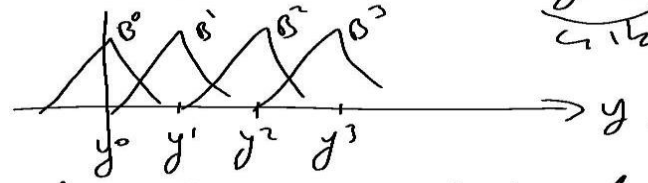
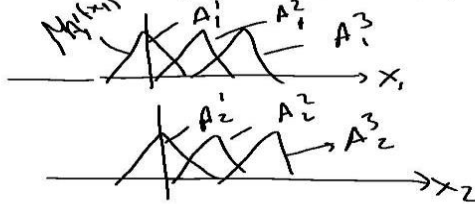
```
clear;
u=1:4; v=1:4;
L=[0 0.1 0.5 1]; S=[1 0.5 0.1 0];
subplot(2,1,1);
plot(u,L);
title('mL');
subplot(2,1,2);
plot(v,S);
title('mS');
pause;
mD=max(1-L,S);
mL=min(1,1-L+S);
mz=max(min(L,S),1-L);
mG=L<S;
close;
plot(u,mD,'*',u,mL,u,mz,u,mG);
pause
```

## Kural Tabanlarının Oluşturulması



Kurallar: (Genel L. kural)

$R^L = ( \text{IF } x_1 \text{ is } A_1^L \text{ and } \dots \text{ and } (x_n \text{ is } A_n^L) \text{ THEN } y = B^L )$



$B^L$ ;  $[0,1]$  aralığında merkezler:  $y^L$  olan  
if yetiri Ponto.

SBS (Standart Bulanık Sistem) : { Singleton fuzzifier  
product IE  
center average defuzzifier.

$x_i$ 'ler girisler olarak sure bir SBS "nonlinear mapping" yapar. Ve buradan;

$$f(x) = \frac{\sum_{l=1}^M y^l \cdot \prod_{i=1}^n \mu_{A_i^l}(x_i)}{\sum_{l=1}^M \prod_{i=1}^n \mu_{A_i^l}(x_i)} = \frac{\sum_{l=1}^M y^l \phi^l(x)}{\sum_{l=1}^M \phi^l(x)}$$

$A_i^l$  : i. degiskenin l. sigmoid fonk.

Baz (bask) fonk. ( $\phi^l$ )

NOT:  $P_i(x)$  : polinom fonk. olan. Herhangi bir  $f(x)$  fonk  $P_i(x)$ 'ler cinsinden ;  $f(x) \approx \sum_{i=1}^N c_i P_i(x)$  yakulabilir. ( $x \in [a, b]$ )

Eger  $P_i(x)$ 'ler ortanormal ise ;  $c_i = \langle f(x), P_i(x) \rangle = \int_a^b f(x) P_i(x) dx$   
(dik+normal) ← iç çarpım

Trigonometrik Fourier Serisi :

$$f(x) = \sum_{n=0}^{\infty} a_n \cos nx + b_n \sin nx$$