



Bulanık Mantık

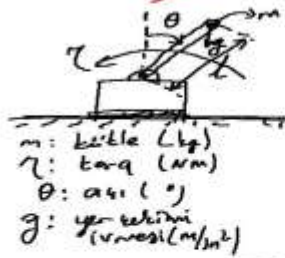
(MÜH 425 – Bilgisayar Müh. Böl.)

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Hafta-8
Bulanık Çıkartım

İÇERİK

- Teorinin mucidi: Lutfi Asker Zadeh
- Bulanık Mantığa Giriş
- Bulanık Kümeler
- Temel İşlemler
- Kural Tabanı
- Bulandırma, Durulama
- Üyelik Fonksiyonları
- Çıkartım Sistemleri
- FAM tablosu,
- Uygulamalar

Örnek : Ters Sarkaç (Inverted pendulum) Problemi

Bir kütük, bir mafsal ile bağlanmıştır. Kütük dik konumda dengede tutulmak isteniyor.
 Bunun için, mafsala bağlı olan motor τ torku üreterek dengeye getirmeye çalışıyor.
 θ açısı algılayıcı vasıtasıyla algılanıp "kontrolcüye" geliyor ve motoru uyandırıyor.
 Bu processi gerçekleştiren "kontrolcüye" bulanık (fuzzy) sistem ile nasıl yansıtılır?

kontrol $u(t) = \tau(t) = -ml^2 \ddot{\theta} + mlg \sin \theta =$, Amaç θ 'yı ve $\dot{\theta}$ 'yi sıfır yapmaktır.

$\theta(t) \ll 1$ çok küçük açı, $\sin \theta \approx \theta$, $\cos \theta \approx 1$
 $\theta = \theta(t)$, $\dot{\theta} = \frac{d\theta}{dt}$, $\ddot{\theta} = \frac{d^2\theta}{dt^2}$

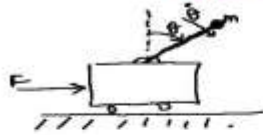
$u(t) = -ml^2 \ddot{\theta} + mlg \cdot \theta$ \Rightarrow $x_1(t) = \theta(t) \Rightarrow \dot{x}_1(t) = x_2(t)$
 $x_2(t) = \dot{\theta}(t) \Rightarrow \dot{x}_2(t) = \ddot{\theta} = \frac{1}{-ml^2} x_1(t) - \frac{1}{ml^2} u(t)$
 Linear dif. denklemler veya anlık hız (°/sn)

$x_1(t)$ ve $x_2(t)$ bulanık değişkenler olarak kullanılıyor. (Δt simülasyon aralığı)
 $\dot{x}_1 = \frac{dx_1}{dt} \approx \frac{x_1(k+1) - x_1(k)}{\Delta t} = x_2(k)$; $\dot{x}_2 = \frac{dx_2}{dt} = \frac{x_2(k+1) - x_2(k)}{\Delta t} = \frac{1}{-ml^2} x_1(k) - \frac{1}{ml^2} u(k)$

$\Delta t = 1$ alınarak : $x_1(k+1) = x_1(k) + x_2(k)$, $x_1(0) = x_{10}$
 $x_2(k+1) = x_2(k) + \left(\frac{1}{-ml^2} x_1(k) - \frac{1}{ml^2} u(k) \right)$, $x_2(0) = x_{20}$ } ilk koşullar

Veriler:

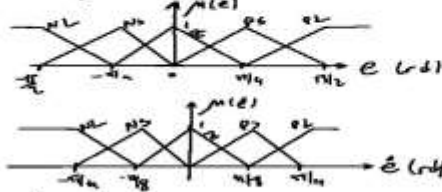
$-2^\circ \leq x_1 \leq 2^\circ$; $-5 \text{ dps} \leq x_2 \leq 5 \text{ dps}$; $-16 \leq u \leq 16$



Answer: surdas dit karamda tutul mada
($\theta = 0, \theta' = 0$ ol mada)

2.1 $\left. \begin{array}{l} -30^\circ \leq \theta \leq 30^\circ \\ -45 \frac{\text{mm}}{\text{min}} \leq \dot{\theta} \leq 45 \frac{\text{mm}}{\text{min}} \\ -30 \text{ N} \leq F \leq 30 \text{ N} \end{array} \right\} \text{Dynamometer}$

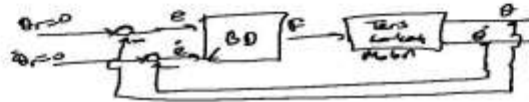
21 Üçelik Font.

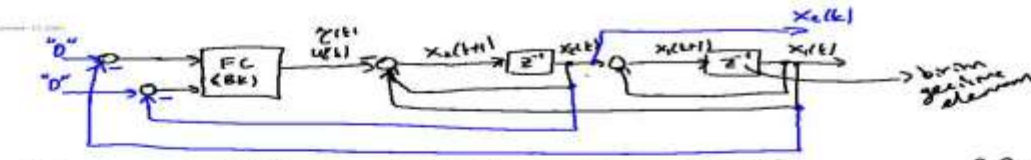


$$\mu(x; a, c, b) = \begin{pmatrix} 0 \\ 0 \\ 0.5 \\ 0 \end{pmatrix}$$

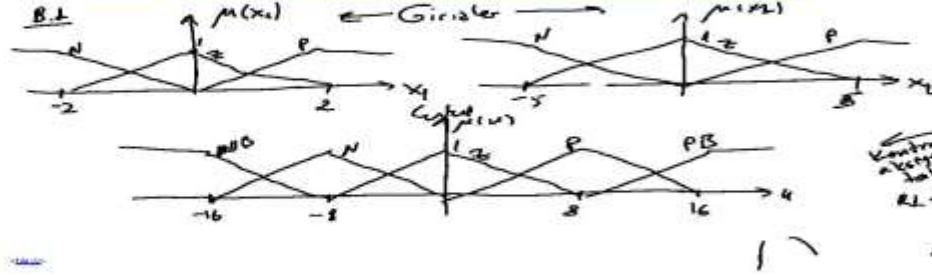
3) FAM tablosu

C	NL	NS	3	PS	PL
NL	PL	PL	PL	PS	3
NS	PL	PL	PS	3	NS
3	PL	PS	3	NS	PL
PS	PS	3	NS	NL	NL
PL	3	NS	NL	NL	NL





8.12.2009



B.2. Kural tablosu
FAM (Fuzzy
Associative
Memory)

$x_1 \backslash x_2$	P	Z	N
P	PB	P	Z
Z	P	Z	N
N	Z	N	NB

← Kontrol
akçyon
tablosu

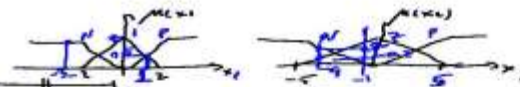
RL: if $x_1=P$ and $x_2=P$ then $u=PB$
...

8.3 Sınırlar (Mamcha'ın ISE)

$x_1(0) = 1$, $x_2(0) = -4$ d.p.c

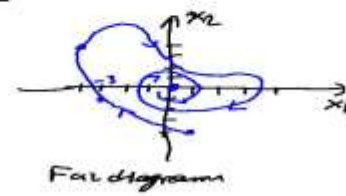
k	(μ)	Fuzzy antecedent (μ_1, μ_2)	μ_k (Fuzzy consequent)
0	$\begin{pmatrix} 1 \\ -4 \end{pmatrix}$	$\frac{0.5}{F} + \frac{0.5}{N}$ (proba) $\frac{0.5}{N} + \frac{0.2}{Z}$ (proba)	$x_1 = F, 0.5$ $x_2 = N, 0.5$ } $u = Z \rightarrow \min(0.5, 0.5) = 0.5Z$ $x_1 = F, 0.5$ $x_2 = Z, 0.2$ } $u = P \rightarrow \min(0.5, 0.4) = 0.2P$ $x_1 = Z, 0.5$ $x_2 = Z, 0.2$ } $u = Z \rightarrow \min(0.5, 0.2) = 0.2Z$ $x_1 = Z, 0.5$ $x_2 = N, 0.8$ } $u = N \rightarrow \min(0.5, 0.8) = 0.5N$
	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	$\frac{1}{N}, \mu(x_1)$ $\frac{0.7}{Z} + \frac{0.2}{N}, \mu(x_2)$	
	$\begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$		

$$\mu(1) = \frac{0.2 \cdot (-16) + 0.8 \cdot (-8)}{0.2 + 0.8} = -3.2 + (-6.4) = -9.6 //$$



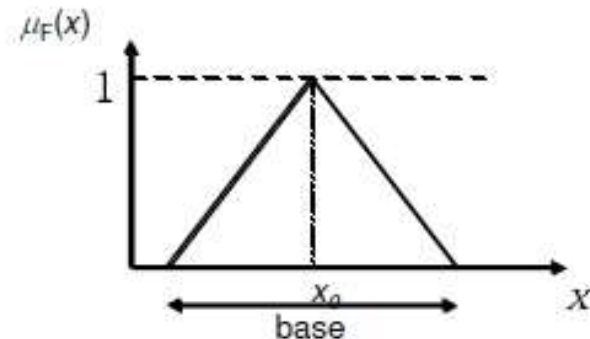
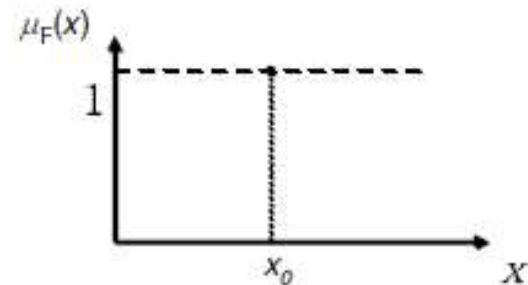
Örnek (CA) :

$$\mu^* = \mu(0) = \frac{0.5 \cdot 0.5 + 0.5 \cdot 0.8}{0.5 + 0.5 + 0.2} = 0.7$$



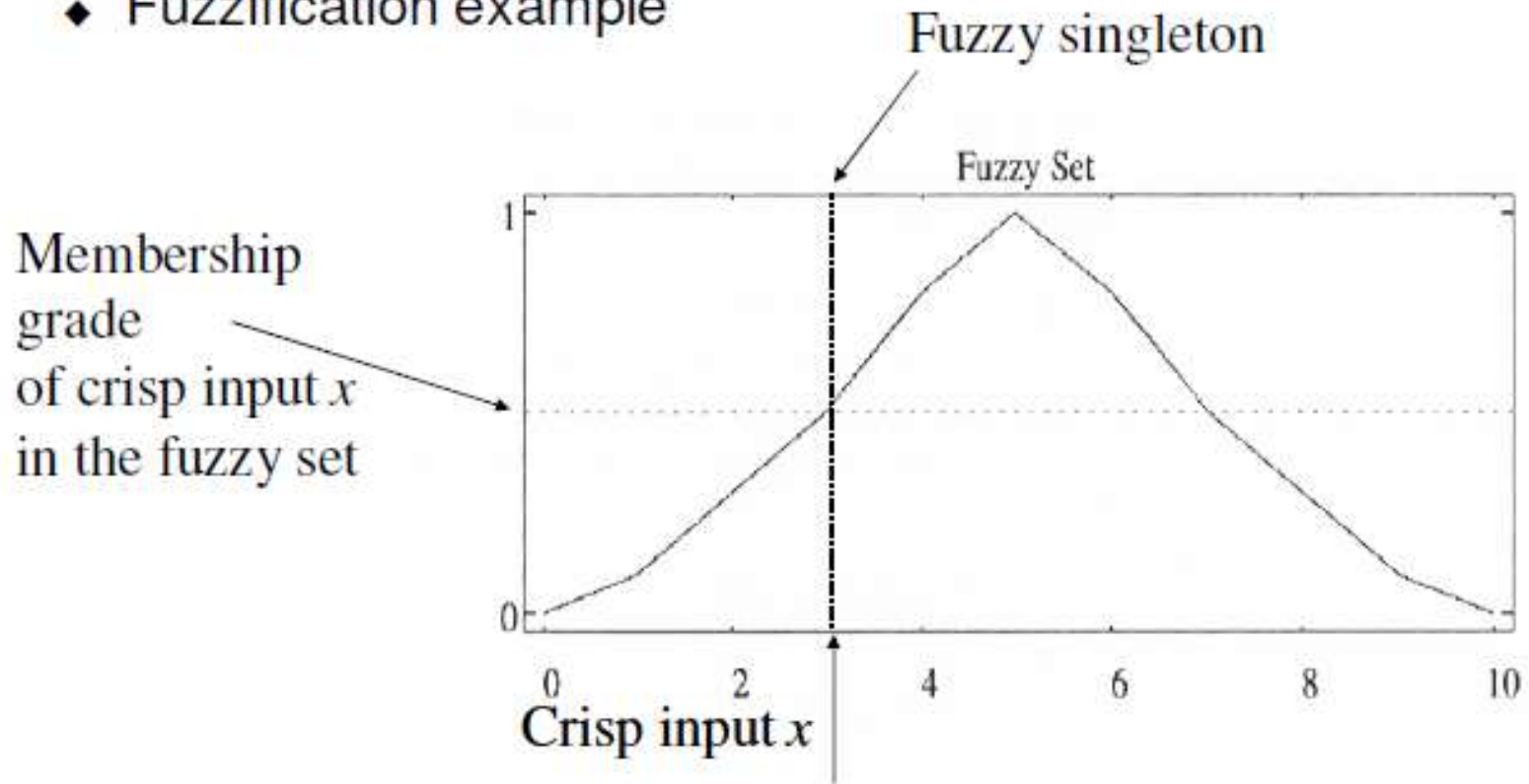
Fuzzification

- ◆ Process of making a crisp quantity fuzzy
- ◆ If it is assumed that input data do not contain noise of vagueness, a fuzzy singleton can be used
- ◆ If the data are vague or perturbed by noise, they should be converted into a fuzzy number

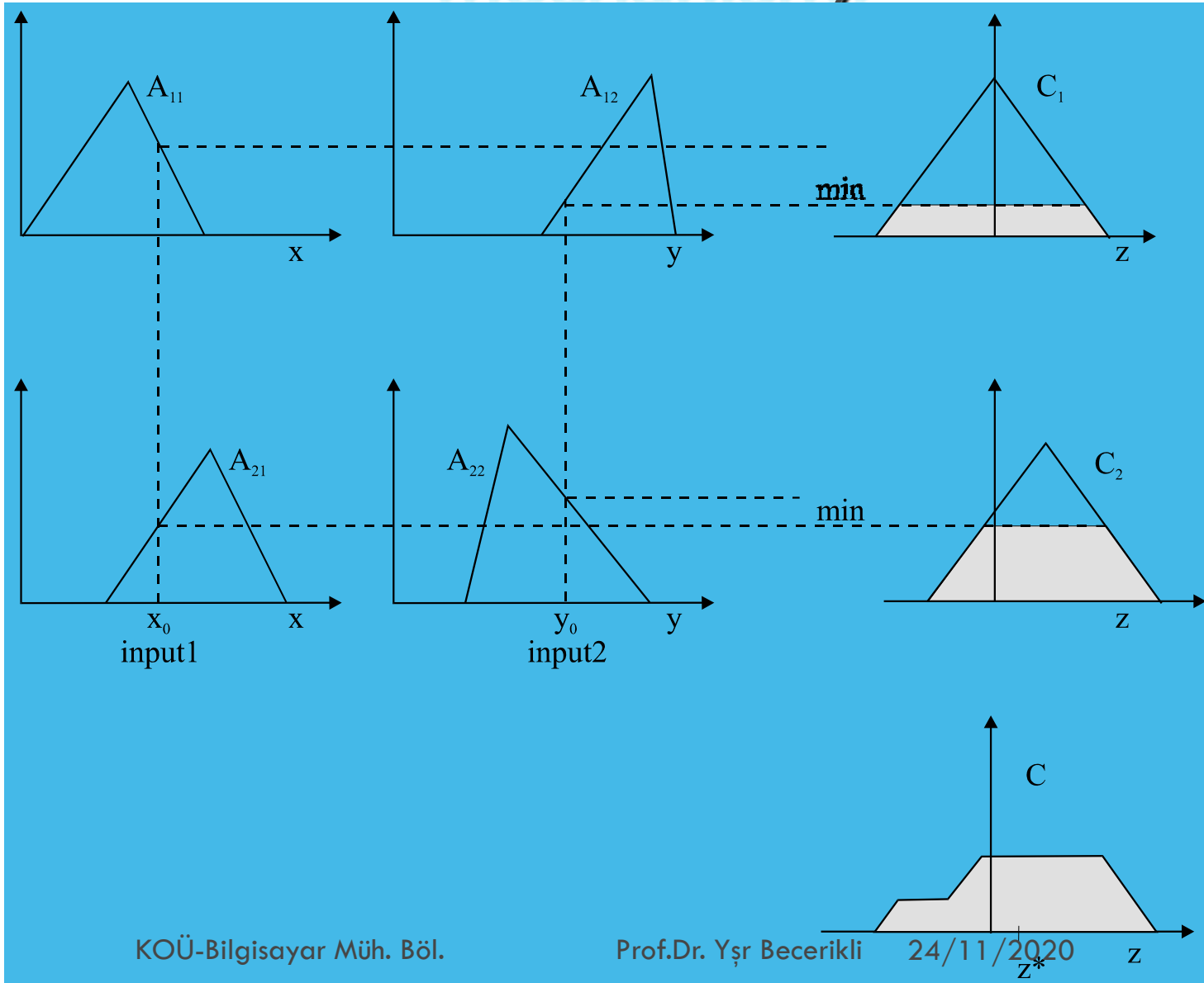


Fuzzification

- ◆ Fuzzification example



Mamdani Çıkartım (min. inference mechanism)



Sugeno Bulanık Çıkartım

- ❑ **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent. A **singleton**, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Devam

Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the **Sugeno-style fuzzy rule** is

IF x is A and y is B THEN z is $f(x,y)$
(eğer) (ise)

Burada, x , y ve z dilsel (linguistic) değişkenler; A ve B tanım aralığında (domain-universe of discourses) bulanık kümelerdir. Ve $f(x, y)$ deterministik/net (crisp) matematiksel fonksiyondur.

Devam

- If $f(.,.)$ is a first order polynomial, then the resulting fuzzy inference is called a (Birinci derece Sugeno) **first order Sugeno fuzzy model**
- If $f(.,.)$ is a constant then it is a (Sıfırıncı derece Sugeno) **zero-order Sugeno fuzzy model** (special case of Mamdani model)
- Case of two rules with a first-order Sugeno fuzzy model
 - Her kural bir kesin (Crisp) çıkışa sahiptir
 - Toplam çıkış ağırlıklı ortalama (**weighted average**)
üzerinden hesaplanır
 - Ayrıca durulamaya gerek yok

Devam

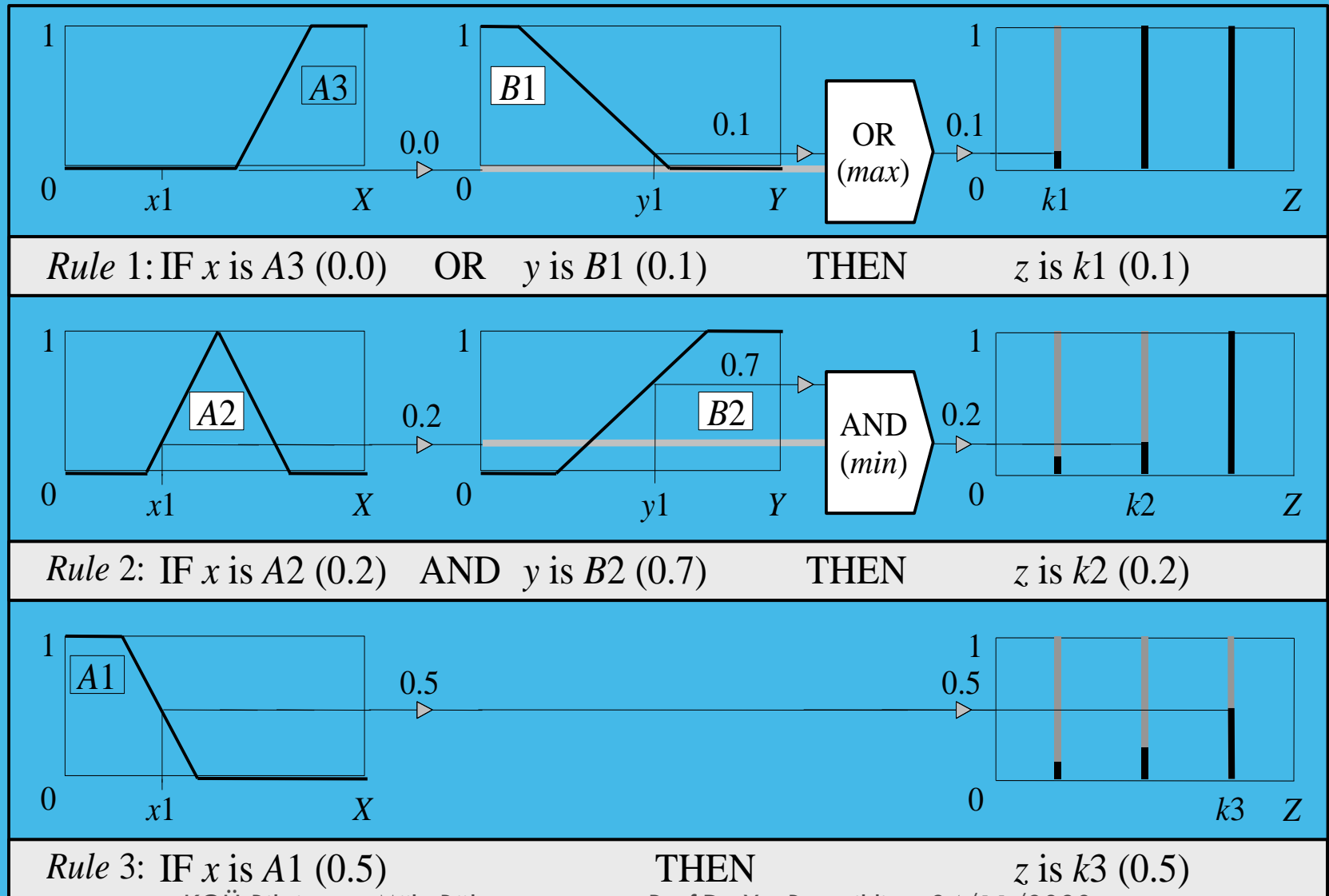
The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

IF x is A AND y is B THEN z is k

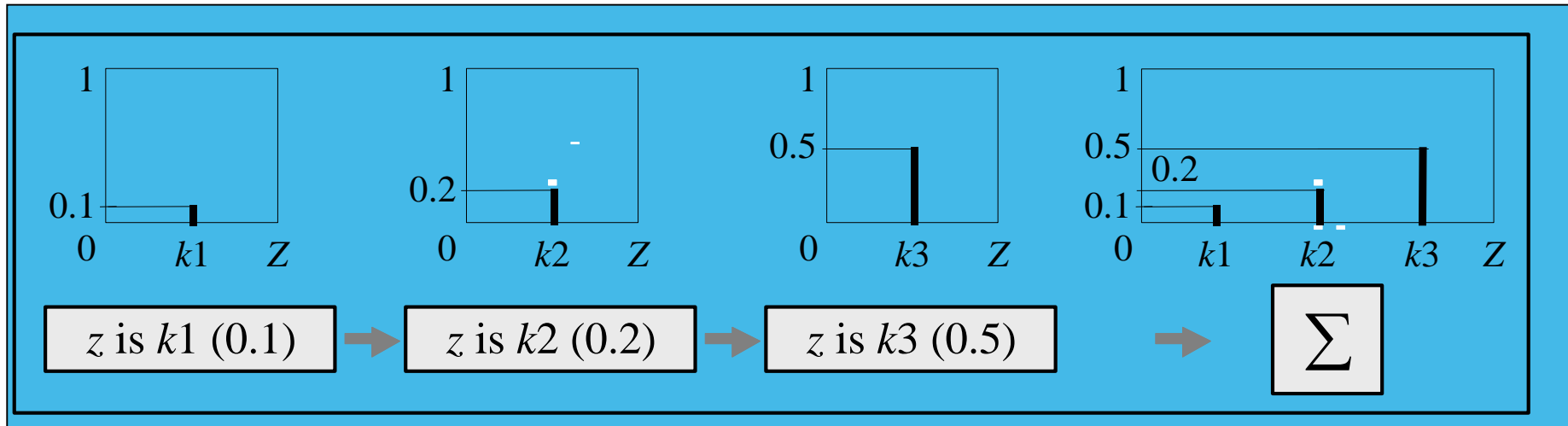
where k is a constant.

In this case, the output of each fuzzy rule is constant. All consequent membership functions are represented by **singleton spikes**.

Sugeno-Türü Kural Gerçekleme (Sıfırıncı Derece)



Sugeno-Türü Kural Çıktılarının Toplanması (aggregation)



Ağırlıklı Ortalama (Weighted average (WA))

$$WA = \frac{\mu(k1) \times k1 + \mu(k2) \times k2 + \mu(k3) \times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65$$

Sugeno-Türü DURULAMA

