Bulanık Mantık

(MÜH 425 – Bilgisayar Müh. Böl.)

Prof.Dr. Yaşar BECERİKLİ

Hafta-3
Bulanık Kümeler- Üyelik Fonksiyonları

İÇERİK

- Teorinin mucidi: Lutfi Asker Zadeh
- Bulanık Mantığa Giriş
- Bulanık Kümeler
- Temel İşlemler
- Kural Tabanı
- Bulandırma, Durulama
- Üyelik Fonksiyonları
- Çıkartım Sistemleri
- FAM tablosu,
- Uygulamalar

Neden Bulanık Mantık

- **Sezgisel and karar/kural tabanlı yapı**
- **Matematiksel modele ihtiyaç yok**
- **Üyeler ve üye olmayan değişkenler arasında düzgün (smooth) bir geçiş sağlar**
- **Göreceli olarak basit, hızlı ve adaptive yapıya sahip**
- **Problem/sistemdeki dalgalanmalardan (fluctuations) az etkilenir (Less sensitive)**
- **dilsel (linguistic) veya tanımsal (descriptive) kurallar ile matematiksel olarak ifade edilmesi güç problemlere genel tasarım amaçları gerçeklenebilir**

Bulanık Mantık Uygulamaları

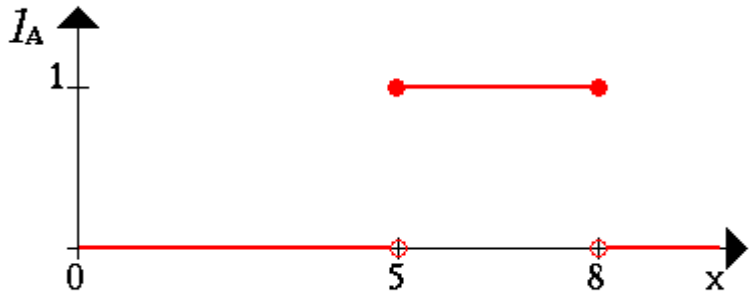
- Örüntü tanıma ve sınıflama
- Bulanık Kümeleme (Fuzzy clustering)
- Görüntü ve ses işleme
- Bulanık sistem ile tahmin
- Bulanık Kontrol (Fuzzy control)
- Bulanık hata takibi (Diagnosis)
- Optimizasyon ve karar verme
- Biyomedikal uygulamalar
- Savunma sanayi
- Hava durumu tahmini
- Otomotiv elektroniği
- Klima sistemler

Crisp Küme

- **Boolean/Crisp set A** is a mapping for the elements of S to the set {0, 1}, i.e., A: S → {0, 1}
- ***Characteristic function:***

$$\mu_A(x) = \begin{cases} 1 & \text{if x is an element of set A} \\ 0 & \text{if x is not an element of set A} \end{cases}$$

Bulanık Küme Tanımı



Classical set A in X is a set of ordered pairs

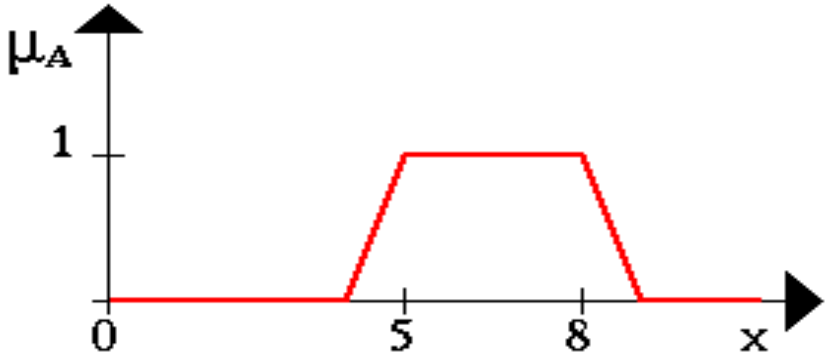
$$A = \{(x, I_A(x)) \mid x \in X\},$$

defined by **indicator function** $I_A(x) \in \{0,1\}$

Fuzzy set A in X is as a set of ordered pairs

$$A = \{(x, \mu_A(x)) \mid x \in X\},$$

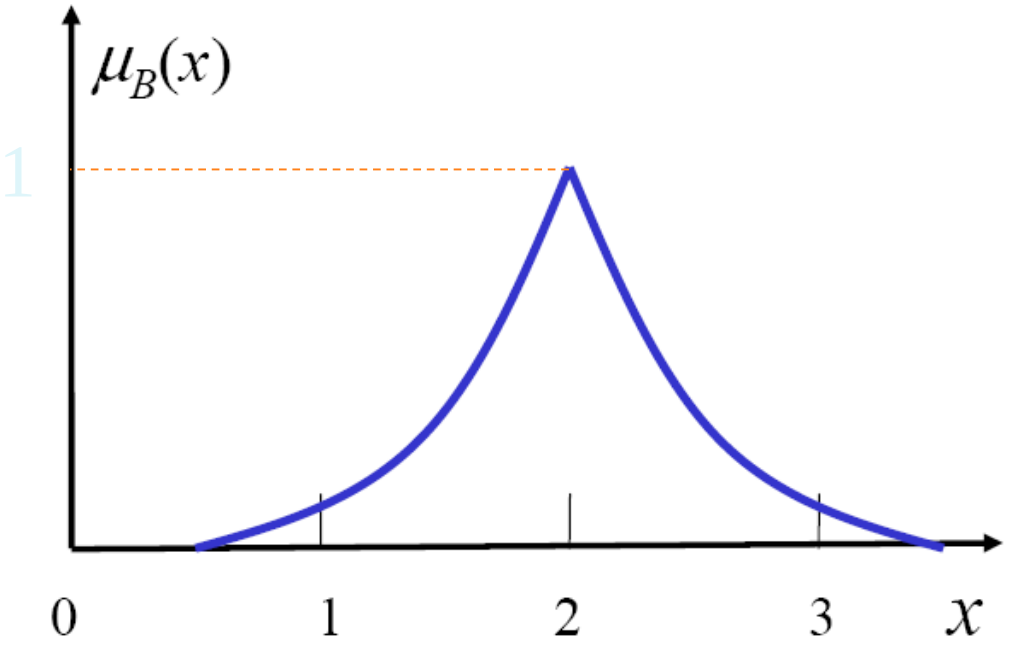
defined by **membership function** $0 \le \mu_A(x) \le 1$

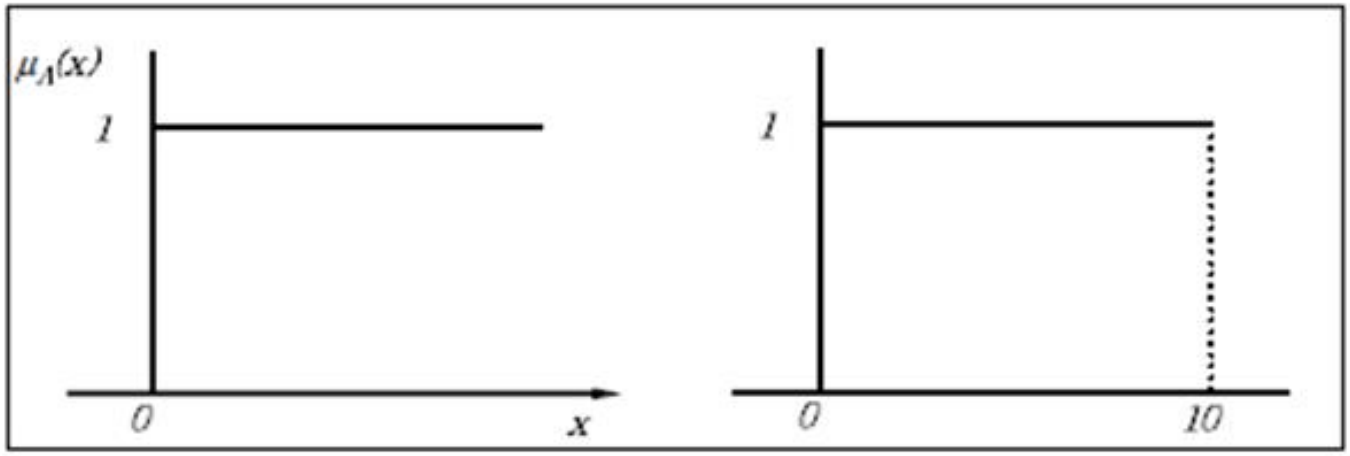


Bulanık üyelik (membership) fonksiyonu

The set, *B*, of numbers *near* to 2 can be represented by a membership function:

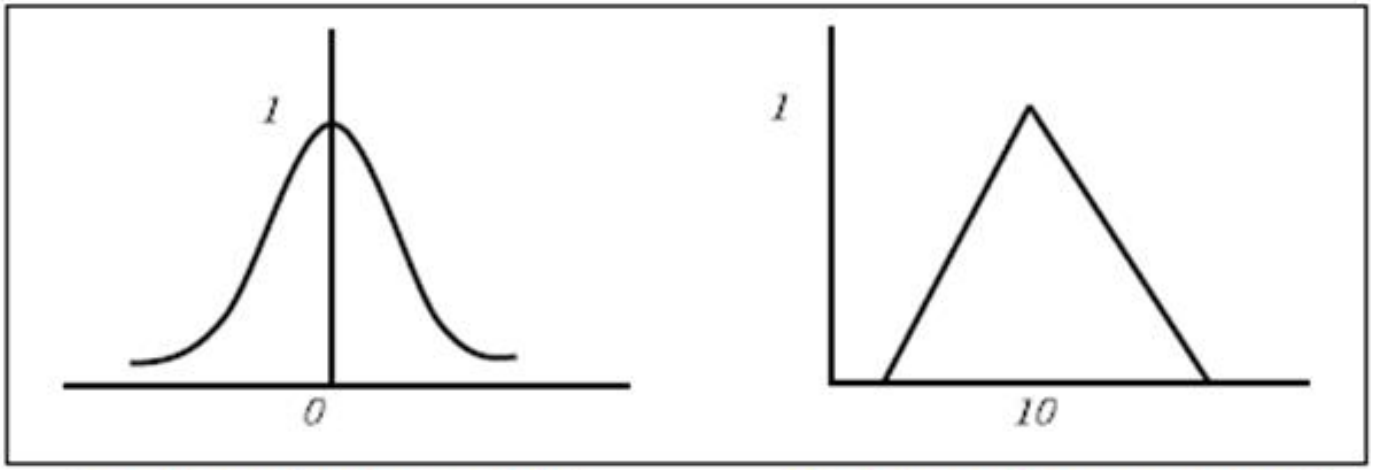
$$\mu_B(x) = e^{-|x-2|}$$





Set "positive number"

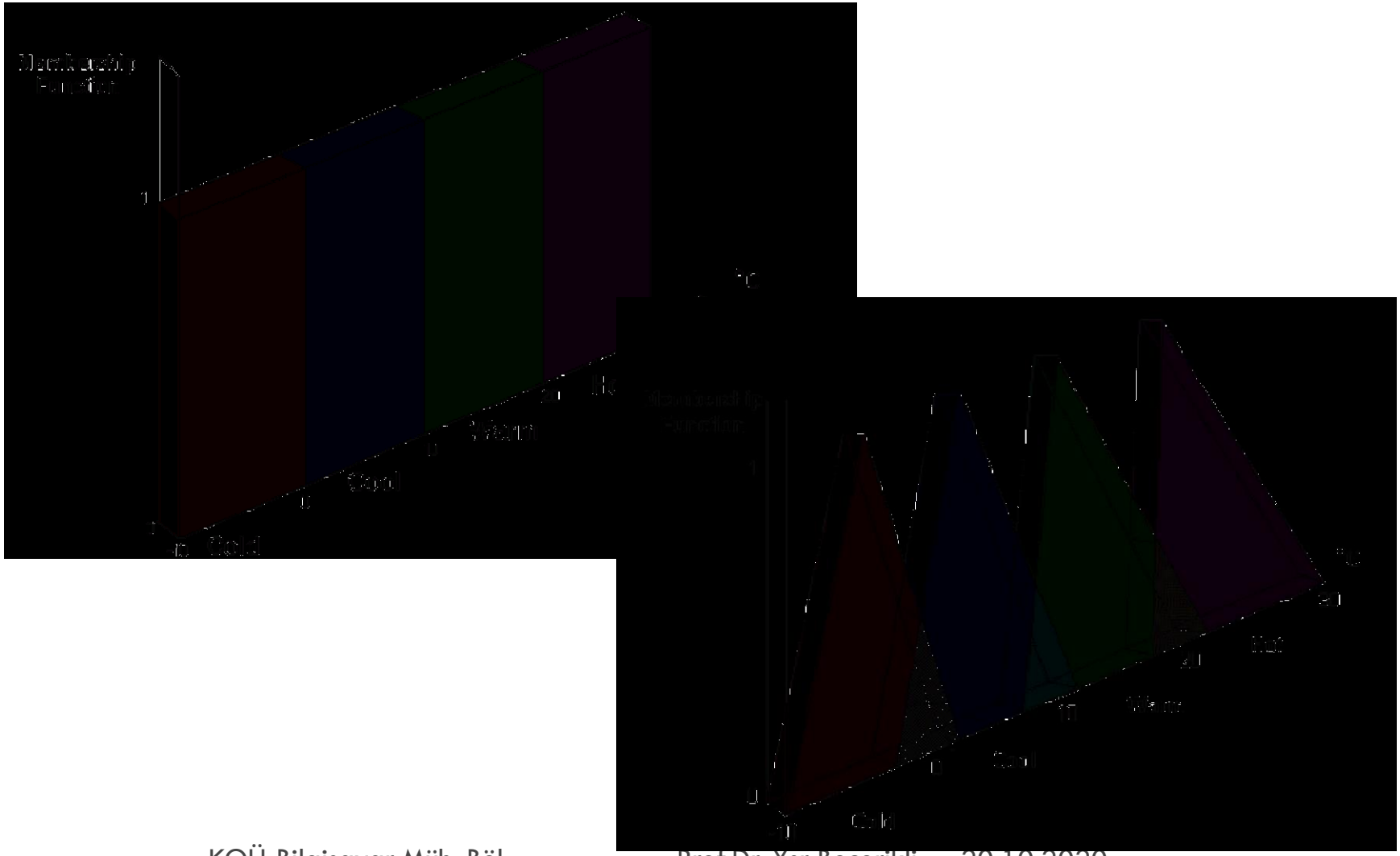
Set "positive number not exceeding 10"



Set "number near 0"

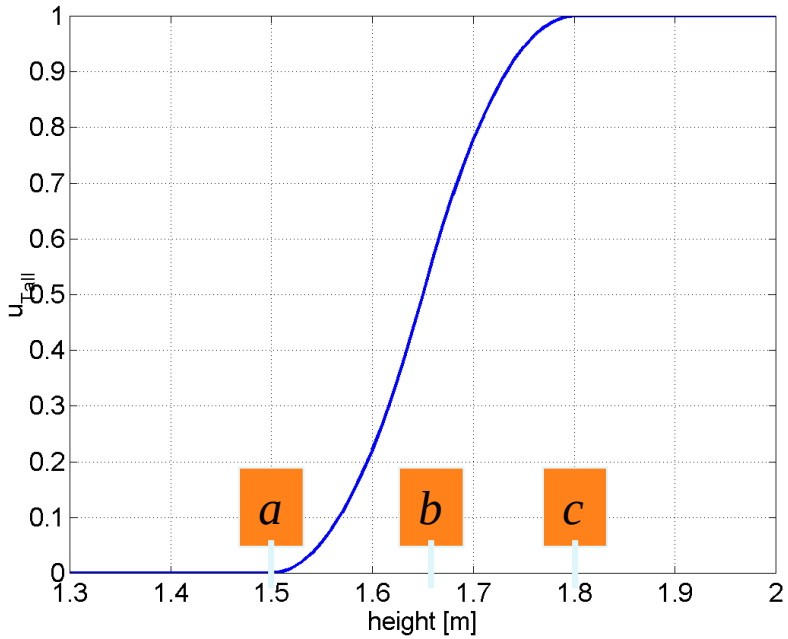
Set "number near 10"

Crisp ve Bulanık üyelik fonksiyonları



Bulanık Üyelik Fonksiyonları: S-funksiyonu

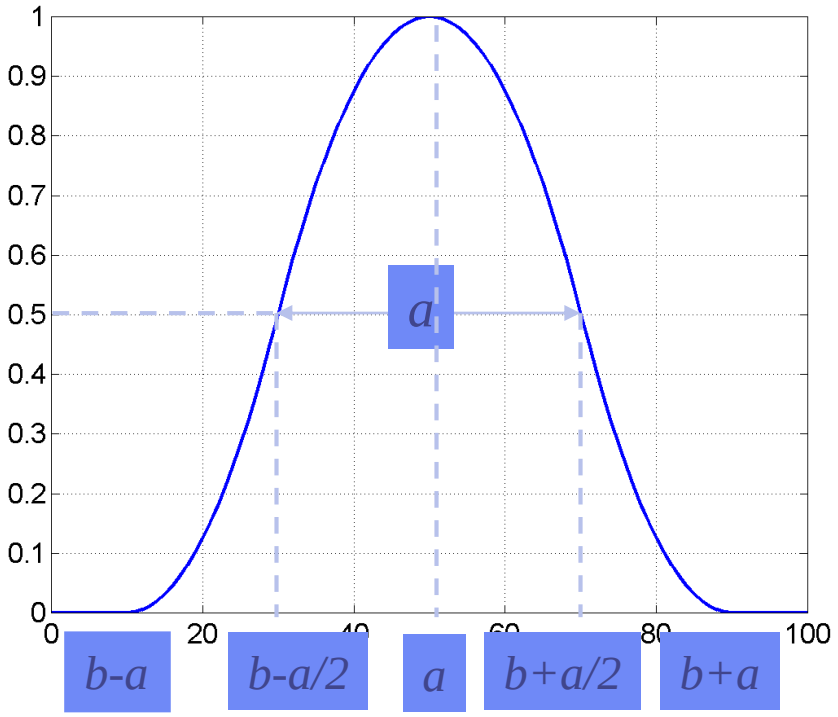
- The S-function can be used to define fuzzy sets
- $S(x, a, b, c) =$
  - 0 for $x \leq a$
  - $2(x\text{-}a/c\text{-}a)^2$ for $a \leq x \leq b$
  - $1 - 2(x\text{-}c/c\text{-}a)^2$ for $b \leq x \leq c$
  - 1 for $x \geq c$



Bulanık Üyelik Fonksiyonları : Π–Funksiyonu

- $\Pi(x, a, b) =$
  - $S(x, b\text{-}a, b\text{-}a/2, b)$ for $x \leq b$
  - $1 - S(x, b, b+a/2, a+b)$ for $x \geq b$

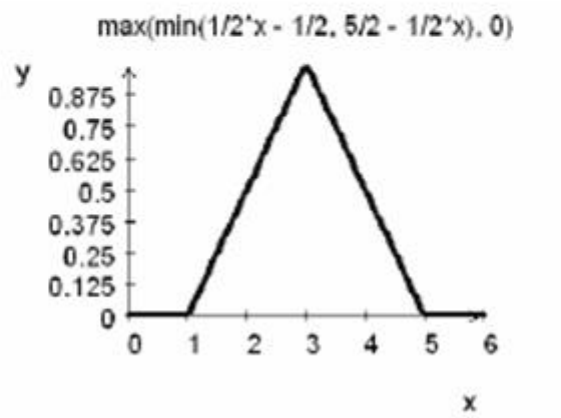
E.g., ***close*** (to a)



Bulanık Üyelik Fonksiyonları
Üçgen ÜF

Triangular MF:

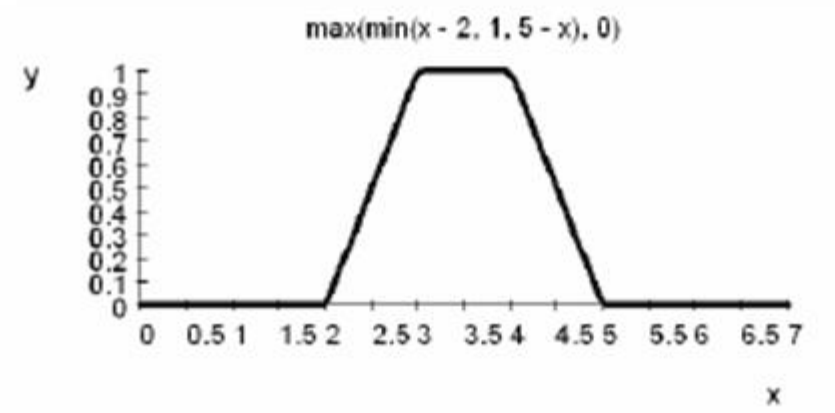
$$\text{triangularmf}(x,\ c,\ h) = \max\left(\min\left(\frac{h-c+x}{h},\ \frac{c+h-x}{h}\right),\ 0\right)$$



Bulanık Üyelik Fonksiyonları
Yamuk ÜF

Trapezoidal MF:

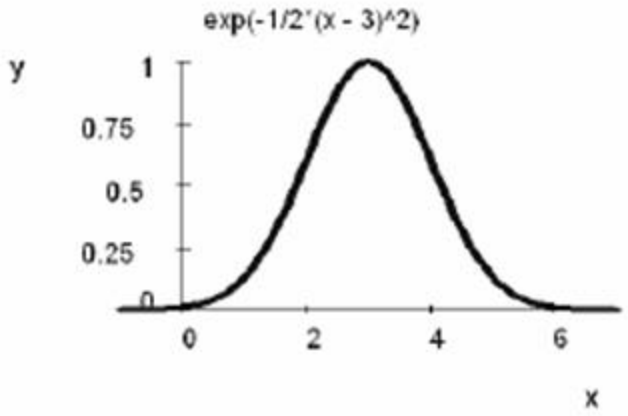
$$\text{trapmf}(x, a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$



Bulanık Üyelik Fonksiyonları
Gauss ÜF

Gaussian MF:

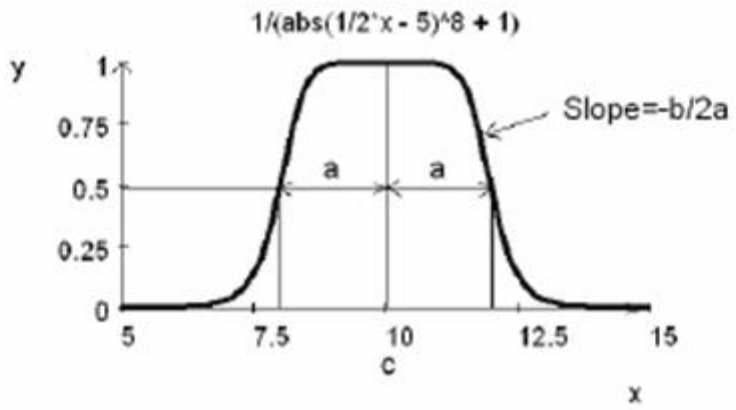
$$\mathrm{gaussmf}(x,\ c,\ s) = e^{-\frac{(x-c)^2}{2\cdot s^2}}$$



Bulanık Üyelik Fonksiyonları
Genelleştirilmiş Bell ÜF

Generalized bell MF:

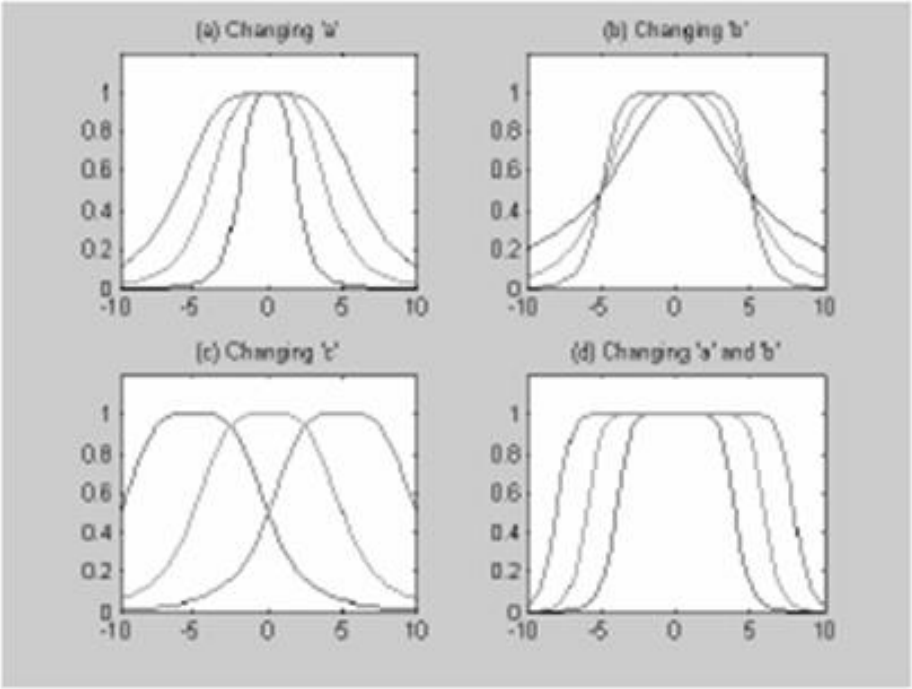
$$\mathrm{gbellmf}(x, a, b, c) = \frac{1}{\left|\frac{x}{a} - \frac{c}{a}\right|^{2 \cdot b} + 1}$$



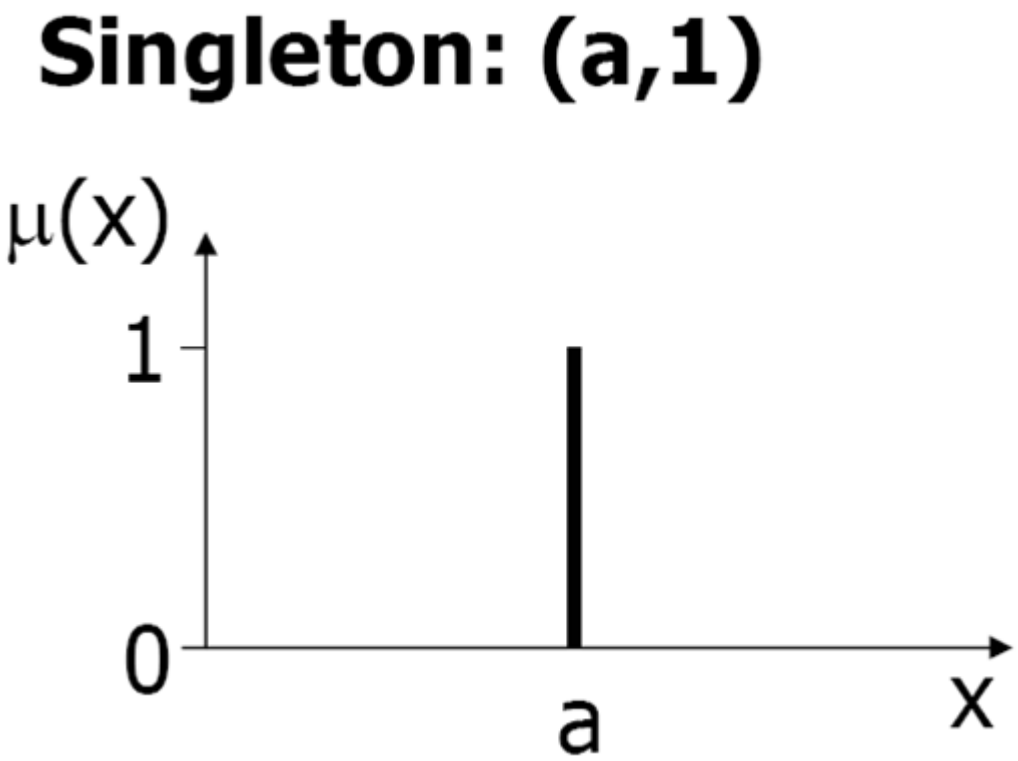
Bulanık Üyelik Fonksiyonları
Genelleştirilmiş Bell ÜF

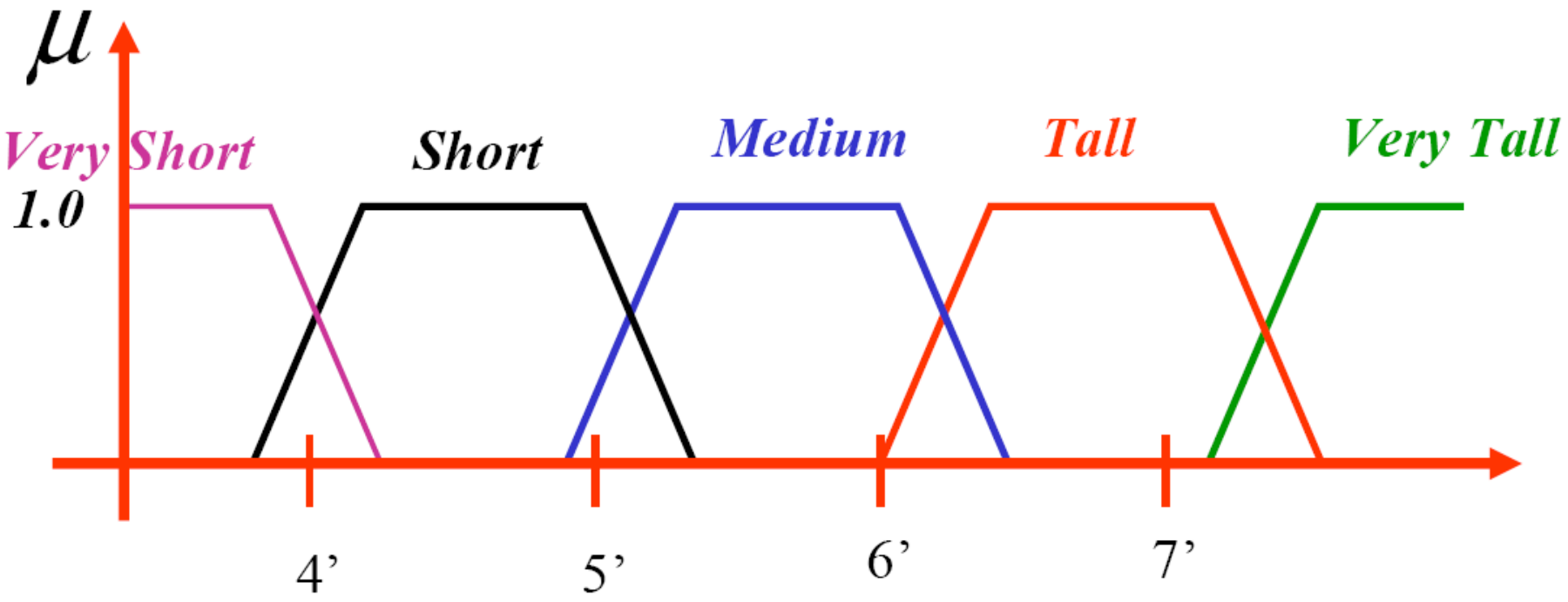
Generalized bell MF:

$$\mathrm{gbellmf}(x, a, b, c) = \frac{1}{\left|\frac{x}{a} - \frac{c}{a}\right|^{2 \cdot b} + 1}$$



Bulanık Üyelik Fonksiyonları
Singleton ÜF





$$\mu = [\mu_{vs}, \mu_s, \mu_m, \mu_t, \mu_{vt}]$$

Bulanık Kümelerde Temel İşlemler

D: "Kocaeli'deki yerli arabalar"
F: " " yerli olmayan arabalar"

Eğer x_1 markalı arabaların %65'i YERLİ ise 0.65 üyelik derecesine sahiptir.

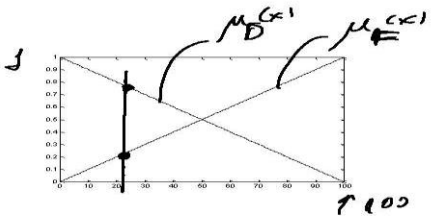
Yerli olmayan arabalar $\mu_F(x_1) = 1 - \mu_D(x_1) = 0.35$ (%35)

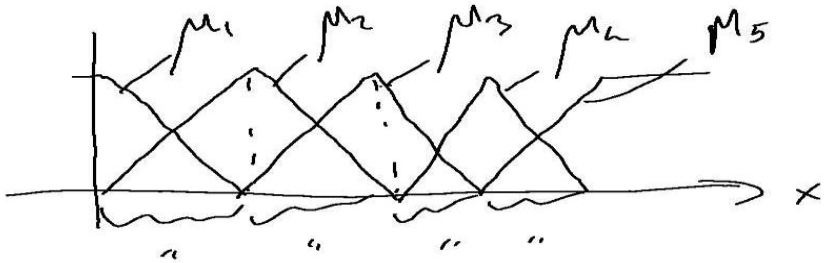
Matlab:

```
x=0:100;
mfD=trimf(x,[0 0 100]);
mfF=1-mfD;
plot(x,mfd,':k',x,mfF,'k');
```

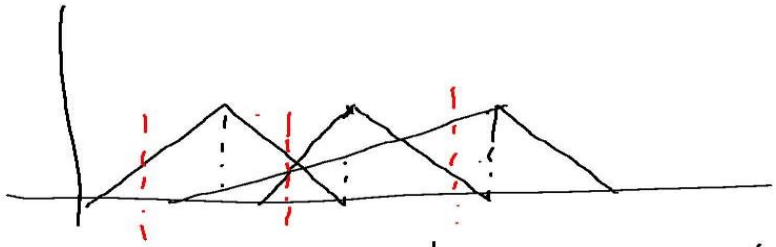
eşik

```
mfF=trimf(x,[0 100 100]);
```





simetrik yerleşim



simetrik olmayan yerleşim

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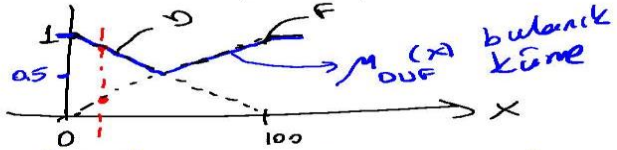
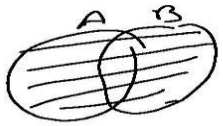
Bulanık Kümelerde Birleşim

$$\mu_{D \cup F}(x) = \max\left(\mu_D(x), \mu_F(x)\right)$$



plot (x, max(mfD, mfF));



$A \cup B = A + B - A \cap B$

```
clear;

x=0:100;
mfd=trimf(x,[0 0 100]);
mfF=1-mfd;
plot(x,mfd,':k',x,mfF,'k');
pause; hold on;
plot(x,max(mfd,mfF));
```

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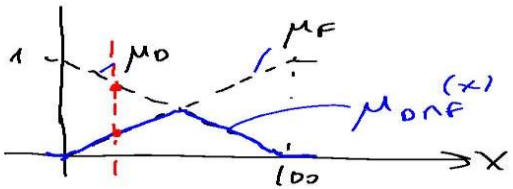
Kesişim (Intersection)

$$\mu_{D \cap F}(x) = \min(\mu_D(x), \mu_F(x))$$




```
clear;

x=0:100;
mfd=trimf(x,[0 0 100]);
mfF=1-mfd;
plot(x,mfd,':k',x,mfF,'k');
pause; hold on;
plot(x,max(mfd,mfF));
pause; hold off;
plot(x,min(mfd,mfF));
```

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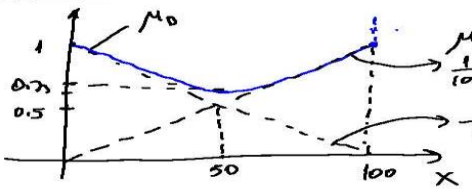
S - Normu

S-Normu bir çeşit birleşimdir.

$$\frac{5\,0^2}{2^2 \cdot 5^2} = 0.25$$

$$\mu_{DUF}(x) = S(\mu_D(x), \mu_F(x)) = \mu_D(x) + \mu_F(x) - \mu_D(x) \cdot \mu_F(x)$$

Not: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$$\mu_{DUF}(x) = -\frac{1}{100}(x-100) + \frac{1}{100}x + \frac{1}{100^2}x \cdot (x-100)$$

$$= 1 + \frac{1}{100^2}x^2 - \frac{x}{100}$$

plot (x, [mfD+mfF-mfD.*mfF]);

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t - Normu

Bir çeşit kesişimdir.

$$\mu_{DnF}(x) = t(\mu_D(x), \mu_F(x)) = \mu_D(x) \cdot \mu_F(x) = -\frac{1}{100^2} x(x-100)$$

$$= -\frac{1}{100^2}x^2 + \frac{x}{100}$$

Not: $P(A \cap B) = P(A) \cdot P(B)$ (bağımsız ise)