Bulanık Mantık

(MÜH 425 – Bilgisayar Müh. Böl.)

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Hafta-7

Bulanık Fonksiyon Modelleme

İÇERİK

- Teorinin mucidi: Lutfi Asker Zadeh
- Bulanık Mantığa Giriş
- Bulanık Kümeler
- Temel İşlemler
- Kural Tabanı
- Bulandırma, Durulama
- Üyelik Fonksiyonları
- Çıkartım Sistemleri
- FAM tablosu,
- Uygulamalar

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Analitik Fonksiyonların Modellenmesinde Kuralların Belirlenmesi:

$g(x)$: Modellenecek Fonk.
$f(x)$: Bulanık sistem (Modelleyici)

1) 1. Türev Metodu: (R^2 de)

$$\| g(x) - f(x) \|_\infty < \left\| \frac{\partial g}{\partial x_1} \right\|_\infty \cdot h_1 + \left\| \frac{\partial g}{\partial x_2} \right\|_\infty \cdot h_2 < \varepsilon$$

2.) 2. Türev Metodu: (R^2 de)

$$\| g(x) - f(x) \|_\infty < \frac{1}{8} \left\| \frac{\partial^2 g}{\partial x_1^2} \right\|_\infty \cdot h_1^2 + \frac{1}{8} \left\| \frac{\partial^2 g}{\partial x_2^2} \right\|_\infty \cdot h_2^2 < \varepsilon$$

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Örnek: $g(x) = \sin x \quad , \varepsilon = 0.2$

1. Türev Metoduna göre:

$h_1 = h_2 = 0.2 \Longrightarrow \frac{6}{0.2} = 30$ aralık
31 kural

$$f(x) = \frac{\sum_{i=1}^{31} \sin(x^i) \cdot \mu_i(x)}{\sum_{i=1}^{31} \mu_i(x)}$$

1. Türev Metoduna Göre:

$$\frac{\partial g}{\partial x} = \cos x \quad , \quad \frac{\partial^2 g}{\partial x^2} = -\sin x \quad ; \quad \left\| \frac{\partial^2 g}{\partial x^2} \right\|_\infty = \sup|-\sin x| = 1$$

$$\frac{1}{8} \cdot \left\| \frac{\partial^2 g}{\partial x^2} \right\|_\infty \cdot h^2 < 0.2 \Longrightarrow \frac{1}{8} \cdot 1 \cdot h^2 < 0.2 \rightarrow h^2 < 1.6$$

$h < 1.23$

$\boxed{h = 1}$ alınabilir

$\frac{6}{1} = 6$ aralık
7 üyelik (kural)

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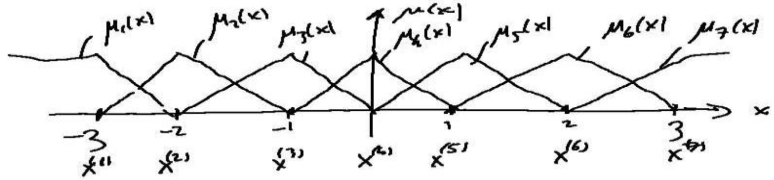

$$f(x) = \frac{\sum_{i=1}^{7} \sin(x^i) \cdot \mu_i(x)}{\sum_{i=1}^{7} \mu_i(x)} =$$

$$= \frac{\sin(-3)\cdot\mu_1(x) + \sin(-2)\cdot\mu_2(x) + \sin(-1)\cdot\mu_3(x) + \sin(0)\cdot\mu_4(x) + \sin(1)\cdot\mu_5(x) + \sin(2)\mu_6(x) + \sin(3)\mu_7(x)}{\mu_1(x) + \mu_2(x) + \mu_3(x) + \mu_4(x) + \mu_5(x) + \mu_6(x) + \mu_7(x)}$$

$$x = -2 \Rightarrow f(-2) = \frac{0 + \sin(-2)\cdot 1 + 0 - \;\; -}{0 + 1 + 0 - \;\; -} = \sin(-2) = g(-2)$$

Tam sağlıyor.

$$x = -1.5 \Rightarrow f(-1.5) = \frac{0 + \sin(-2)\cdot\frac{1}{2} + \sin(-3)\cdot\frac{1}{2} + 0 - - - -}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}\left(\sin(-2) + \sin(-3)\right) \cong g(-1.5)$$

Yaklaşık eşittir

interpolasyon yaklaşımı

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Örnek:

$$x_1 \longrightarrow \boxed{g(x_1,x_2)} \longrightarrow y \quad ; \quad U \in \underbrace{[-1,1]\times[-1,1]}_{\text{domain (universe of discourse)}},\ x \in \mathbb{R}^2$$
$$x_2 \longrightarrow$$

$$g(x_1,x_2) = 0.52 + 0.1\cdot x_1 + 0.28 x_2 - 0.06\, x_1 x_2 \quad (\text{bilineer fonk.})$$

$\varepsilon = 0.1$ için $f(x)$'i tasarlayın.

1. 1. Türev Metodu ile.

$$\frac{\partial g}{\partial x_1} = 0.1 - 0.06 x_2 \quad ; \quad \frac{\partial g}{\partial x_2} = 0.28 - 0.06 x_1$$

$$\left\|\frac{\partial g}{\partial x_1}\right\|_\infty = \sup|0.1 - 0.06 \underset{-1}{x_2}| = 0.16 \qquad \left\|\frac{\partial g}{\partial x_2}\right\|_\infty = \sup|0.28 - 0.06 \underset{-1}{x_1}| = 0.34$$

$$\boxed{0.16\, h_1 + 0.34\cdot h_2 < 0.1}$$

$h_1 = h_2 = h$ düşünülebilir.

$$0.16 h + 0.34 h < 0.1$$
$$0.5 h < 0.1$$
$$h < 0.2$$
$$\boxed{h = 0.2} \text{ seçilebilir} \Longrightarrow$$

$\frac{2}{0.2} = 10$ aralık x_1 için

$\frac{2}{0.2} = 10$ aralık x_2 için

$\rightarrow$ 11'er üyelik $\Rightarrow 11 \times 11 = \boxed{121 \text{ kural}}$

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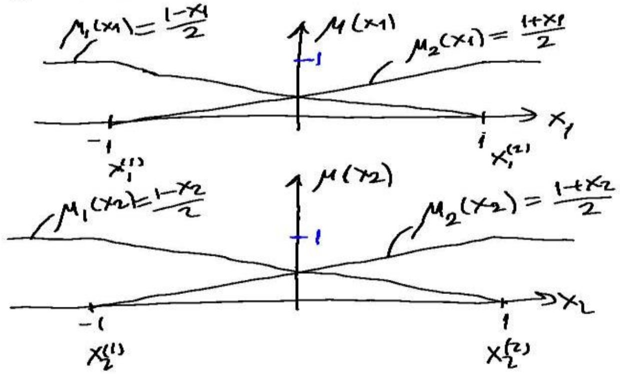
2. II. Türev Metodu ile

$$\left\|\frac{\partial^2 g}{\partial x_1^2}\right\|_\infty = 0 \quad ; \quad \left\|\frac{\partial^2 g}{\partial x_2^2}\right\|_\infty = 0 \Longrightarrow \frac{1}{8}\cdot 0\cdot h_1^2 + \frac{1}{8}\cdot 0\cdot h_2^2 < 0.1$$



$h_1 = h_2 = h$

$0\cdot h^2 < 0.1$

$h = 2$ seçilir (domain $\overbrace{[-1 \quad 1]}^{2 \text{ uzunluk}}$ olduğu için)

$\frac{2}{2} = 1$ aralık 2 değer (x_1 için)

$\frac{2}{2} = 1$ aralık 2 değer (x_2 için)

$\boxed{2\times 2 = 4 \text{ kural}}$

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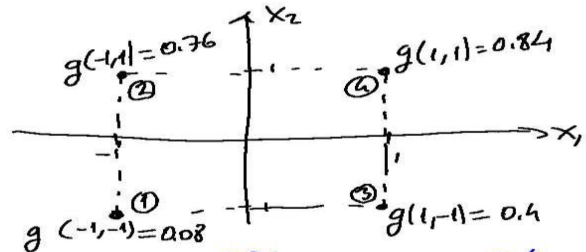

$$f(x)=\frac{\overbrace{g^1(x_1,x_2)}^{0.08}\cdot\mu_1(x_1)\cdot\mu_1(x_2)+\overbrace{g^2(x_1,x_2)}^{0.76}\cdot\mu_1(x_1)\mu_2(x_2)+\overbrace{g^3(x_1,x_2)}^{0.4}\cdot\mu_2(x_1)\cdot\mu_1(x_2)+\overbrace{g^4(x_1,x_2)}^{0.84}\cdot\mu_2(x_1)\cdot\mu_2(x_2)}{\mu_1(x_1)\cdot\mu_1(x_2)+\mu_1(x_1)\cdot\mu_2(x_2)+\mu_2(x_1)\cdot\mu_1(x_2)+\mu_2(x_1)\cdot\mu_2(x_2)}$$

$$f(x_1,x_2)=\frac{0.08\cdot\left(\frac{1-x_1}{2}\right)\left(\frac{1-x_2}{2}\right)+0.76\cdot\left(\frac{1-x_1}{2}\right)\left(\frac{1+x_2}{2}\right)+0.4\cdot\left(\frac{1+x_1}{2}\right)\left(\frac{1-x_2}{2}\right)+0.84\left(\frac{1+x_1}{2}\right)\left(\frac{1+x_2}{2}\right)}{\frac{(1-x_1)(1-x_2)}{4}+\frac{(1-x_1)(1+x_2)}{4}+\frac{(1+x_1)(1-x_1)}{4}+\frac{(1+x_1)(1+x_2)}{4}}$$

$$\frac{(1-x_1)(1-x_2)}{4}+\frac{(1-x_1)(1+x_2)}{4}+\frac{(1+x_1)(1-x_1)}{4}+\frac{(1+x_1)(1+x_2)}{4}\Longrightarrow 1$$

$$\frac{1}{4}\left[(1-\cancel{x_2}-\cancel{x_1}+\cancel{x_1x_2})+(1+\cancel{x_2}-\cancel{x_1}-\cancel{x_1x_2})+(1-\cancel{x_2}+\cancel{x_1}-\cancel{x_1x_2})+(1+\cancel{x_2}+\cancel{x_1}+\cancel{x_1x_2})\right]=\frac{4}{4}=1$$

$$\boxed{f(x_1,x_2)=0.52+0.1x_1+0.28x_2-0.06x_1x_2=g(x_1,x_2)}$$

$$\begin{array}{r}0.08\\0.76\\0.4\\+0.84\\\hline 2.08\end{array}\qquad 2.08/4=0.52$$

$$\begin{array}{r}-0.08\\-0.76\\0.4\\+0.84\\\hline\end{array}\quad \begin{array}{r}1.24\\-0.84\\\hline 0.40\end{array}\qquad 0.4/4=0.1$$

$$\begin{array}{r}-0.08\\+-0.4\\\hline -0.48\end{array}\quad \begin{array}{r}0.76\\+0.84\\\hline 1.60\end{array}\quad \begin{array}{r}1.6\\-0.48\\\hline 1.12\end{array}\qquad 1.12/4=0.28$$

$$\begin{array}{r}+0.08\\+0.84\\\hline 0.92\end{array}\quad \begin{array}{r}-0.76\\+-0.4\\\hline -1.16\end{array}\quad \begin{array}{r}1.16\\-0.92\\\hline -0.24\end{array}\qquad -0.24/4=\boxed{-0.06}$$

SBS bulanık sistem
(simetrik aralıklı üçgen
üyelik fonksiyonları)
Bilineer fonksiyonları
TAM olarak modeller

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R^3'te bilineer fonksiyon

$$\boxed{g(x_1,x_2,x_3) = a + b_1x_1 + b_2x_2 + b_3x_3 + c_1x_1x_2 + c_2x_1x_3 + c_3x_2x_3}$$

$$C(3,2) = \frac{3!}{(3-2)!\,.\,2!} = \frac{6}{2} = 3$$

$$C(4,2) = \frac{4!}{(4-2)!\,2!} = 6$$

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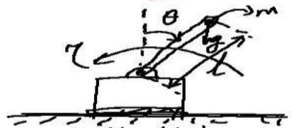
Örnek : Ters Sarkaç (inverted pendulum) Problemi



m: kütle (kg)
τ: tork (Nm)
θ: açı (°)
g: yer çekimi ivmesi (m/sn²)

Bir çubuk, bir mafsal ile bağlanmış. Çubuk dik konumda dengede tutulmak isteniyor. Bunun için, mafsala bağlı olan motor τ torqu üreterek dengeye getirmeye çalışıyor. θ açısı algılayıcı vasıtasıyla algılanıp "kontrolcüye" geliyor ve motoru uyarıyor. Bu processi gerçekleyen "kontrolcüyü" bulanık (fuzzy) sistem ile nasıl gerçekleriz?

kontrol $\boxed{u(t) = \tau(t) = -ml^2\ddot{\theta} + mlg.\sin\theta} \Longrightarrow$ Amaç θ'yı ve $\dot{\theta}$'yı sıfır yapmaktır.

$\theta = \theta(t)$, $\dot{\theta} = \frac{d\theta}{dt}$, $\ddot{\theta} = \frac{d^2\theta}{dt^2}$

$\theta(t) <<<$ çok küçük ise, $\boxed{\sin\theta \cong \theta}$ $\boxed{\cos\theta \cong 1}$

$\boxed{u(t) = -ml^2\ddot{\theta} + mlg.\theta} \Longrightarrow$ $x_1(t) = \theta(t) \Longrightarrow$ $x_2(t) = \dot{\theta}(t) \Longrightarrow$ $\boxed{\begin{array}{l}\dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \ddot{\theta} = \frac{g}{l}x_1(t) - \frac{1}{ml^2}u(t)\end{array}}$

Linear dif. denklem

açı

açısal hız (θ/sn)

$x_1(t)$ ve $x_2(t)$ bulanık değişkenler olarak kullanılacak. (Δt simülasyon aralığı)

$$\dot{x}_1 = \frac{dx_1}{dt} \cong \frac{x_1(k+1) - x_1(k)}{\Delta t} = x_2(k) \quad ; \quad \dot{x}_2 = \frac{dx_2}{dt} = \frac{x_2(k+1) - x_2(k)}{\Delta t} = x_1(k) - u(k)$$

1 . 1 alındı

$\Delta t = 1$ alınarak : fark denklemi
$$\begin{cases} x_1(k+1) = x_1(k) + x_2(k) \quad , & x_1(0) = x_{10} \\ x_2(k+1) = x_1(k) + x_2(k) - u(k) \quad , & x_2(0) = x_{20} \end{cases}$$ ilk koşullar

Verilenler:

$-2° \le \underset{}{\overset{\theta}{x_1}} \le 2°$; $-5\,dps \le \overset{\dot{\theta}}{x_2} \le 5\,dps$; $-16 \le \overset{\tau}{u} \le 16$

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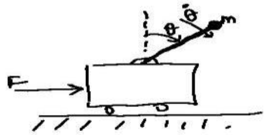
Örnek: **Mobil Ters Sarkaç Problemi**



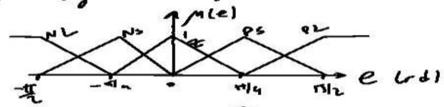
Amaç: sarkaç dik konumda tutulmalı
($\theta = 0$, $\dot{\theta} = 0$ olmalı)

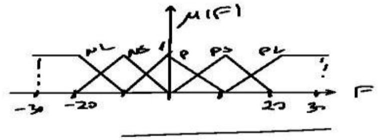
1.)
$$\left.\begin{array}{l} -90^\circ \le \theta \le 90^\circ \\ -45\ {}^\circ/s \le \dot{\theta} \le 45\ {}^\circ/s \\ -30N \le F \le 30N \end{array}\right\} \text{Domain}$$

hata $e = \overset{0}{\theta_r} - \theta = -\theta$

$\dot{e} = -\dot{\theta}$ (açısal hata)

2.) Üyelik Fonk.

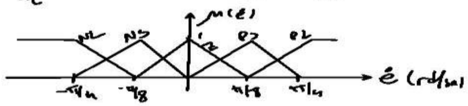




$M(x; a, c, b)$

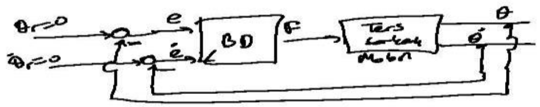
$$a = \begin{pmatrix} -20 \\ -20 \\ -10 \\ 0 \\ 10 \end{pmatrix},\ b = \begin{pmatrix} -10 \\ 0 \\ 10 \\ 20 \\ 20 \end{pmatrix},\ c = \begin{pmatrix} -20 \\ -10 \\ 0 \\ 10 \\ 20 \end{pmatrix}$$

$$M(5; a, c, b) = \begin{pmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$



3.) FAM tablosu

e \ $\dot{e}$	NL	NS	Z	PS	PL
NL	PL	PL	PL	PS	Z
NS	PL	PL	PS	Z	NS
Z	PL	PS	Z	NS	NL
PS	PS	Z	NS	NL	NL
PL	Z	NS	NL	NL	NL



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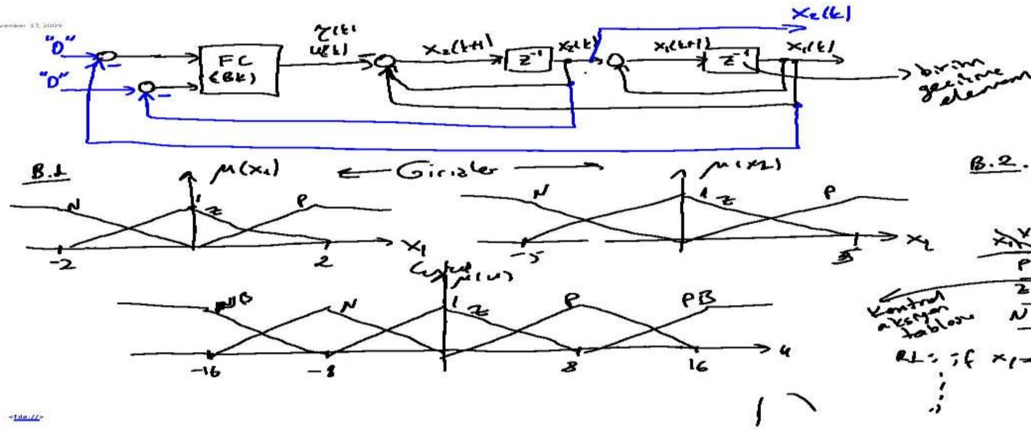

B.2. Kural tablosu
FAM (Fuzzy Associative Memory)

Kontrol aksiyon tablosu

x_1 \ x_2	P	Z	N
P	PB	P	Z
Z	P	Z	N
N	Z	N	NB

R1: if $x_1 = P$ and $x_2 = P$ then $u = PB$
⋮

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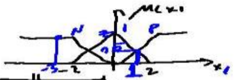
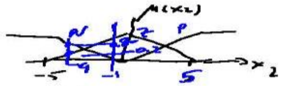
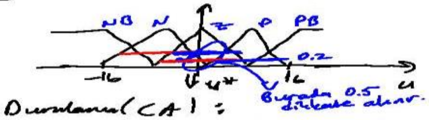
B.3 Simülasyon (Mamdani min IE)

$x_1(0) = 1^\circ$, $x_2(0) = -4$ dps

k	$\binom{x_1}{x_2}$	Fuzzy anticident (x_1, x_2)	u_k (Fuzzy consequent)	
0	$\binom{1}{-4}$	$\frac{0.5}{P} + \frac{0.5}{Z}$, $\mu(x_1)$ $\frac{0.8}{N} + \frac{0.2}{Z}$, $\mu(x_2)$	$x_1 = P$, 0.5; $x_2 = N$, 0.8 $\}$ $u = Z \rightarrow \min(0.5, 0.8) = \underline{0.5Z}$ $x_1 = P$, 0.5; $x_2 = Z$, 0.2 $\}$ $u = P \rightarrow \min(0.5, 0.2) = \underline{0.2P}$ $x_1 = Z$, 0.5; $x_2 = Z$, 0.2 $\}$ $u = Z \rightarrow \min(0.5, 0.2) = \underline{0.2Z}$ $x_1 = Z$, 0.5; $x_2 = N$, 0.8 $\}$ $u = N \rightarrow \min(0.5, 0.8) = \underline{0.5N}$	-2
	$\binom{-3}{-1}$	$\frac{1}{N}$, $\mu(x_1)$ $\frac{0.8}{Z} + \frac{0.2}{N}$, $\mu(x_2)$		-9.6
	$(\ :\ )$			





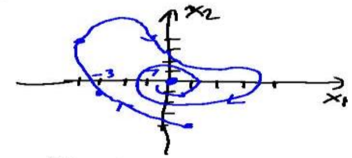


Durulama (CA) :

$$u^* = u(0) = \frac{-8 \cdot 0.5 + 0 \cdot 0.5 + 8 \cdot 0.2 \cdot 8}{0.5 + 0.5 + 0.2} = -2$$

$$u(1) = \frac{0.2 \cdot (-16) + 0.8(-8)}{0.2 + 0.8}$$

$$= -3.2 + (-6.4) = -9.6$$



Faz diagramı

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Giriş-Çıkış Bilgisinden Bulanık Sistem Tasarımı: (Wang-Mendel Yöntemi)

16.12.2009

$g(x)$: bilinmiyor.

Table-Lookup Kullanarak Bulanık Küme Tasarımı:

$(x^{(p)}; y^{(p)})$, $p = 0, 1, 2, \dots n$

$x^p \in U = [\alpha_1, \beta_1] \times [\alpha_2, \beta_2] \times [\ \ ] \dots \times [\alpha_n, \beta_n]$

$y^p \in V = [\alpha_y, \beta_y] \in R$

B.1: $[\alpha_i, \beta_i]$, $i = 1, 2, \dots n$

A_i^j $(j = 1, \dots N_i)$ bulanık küme

B^j $(j = 1, 2, \dots N_y)$ bulanık küme

$[\alpha_y, \beta_y]$'de tanımlıdır.

Örnek: $\underbrace{n=2}_{\text{giriş}}$, $\underbrace{N_1=5}_{\text{birinci giriş üyelik sayısı}}$, $\underbrace{N_2=7}_{\text{ikinci giriş üyelik sayısı}}$, $\underbrace{N_y=5}_{\text{çıkış üyelik sayısı}}$

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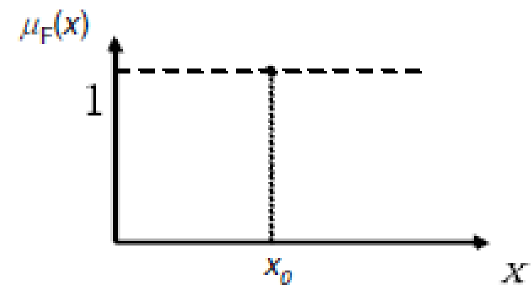
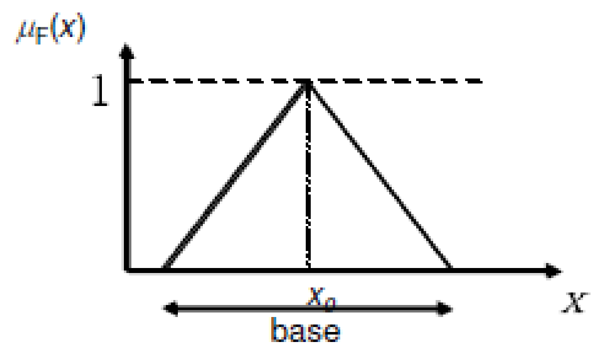
Fuzzification

- Process of making a crisp quantity fuzzy

- If it is assumed that input data do not contain noise of vagueness, a fuzzy singleton can be used

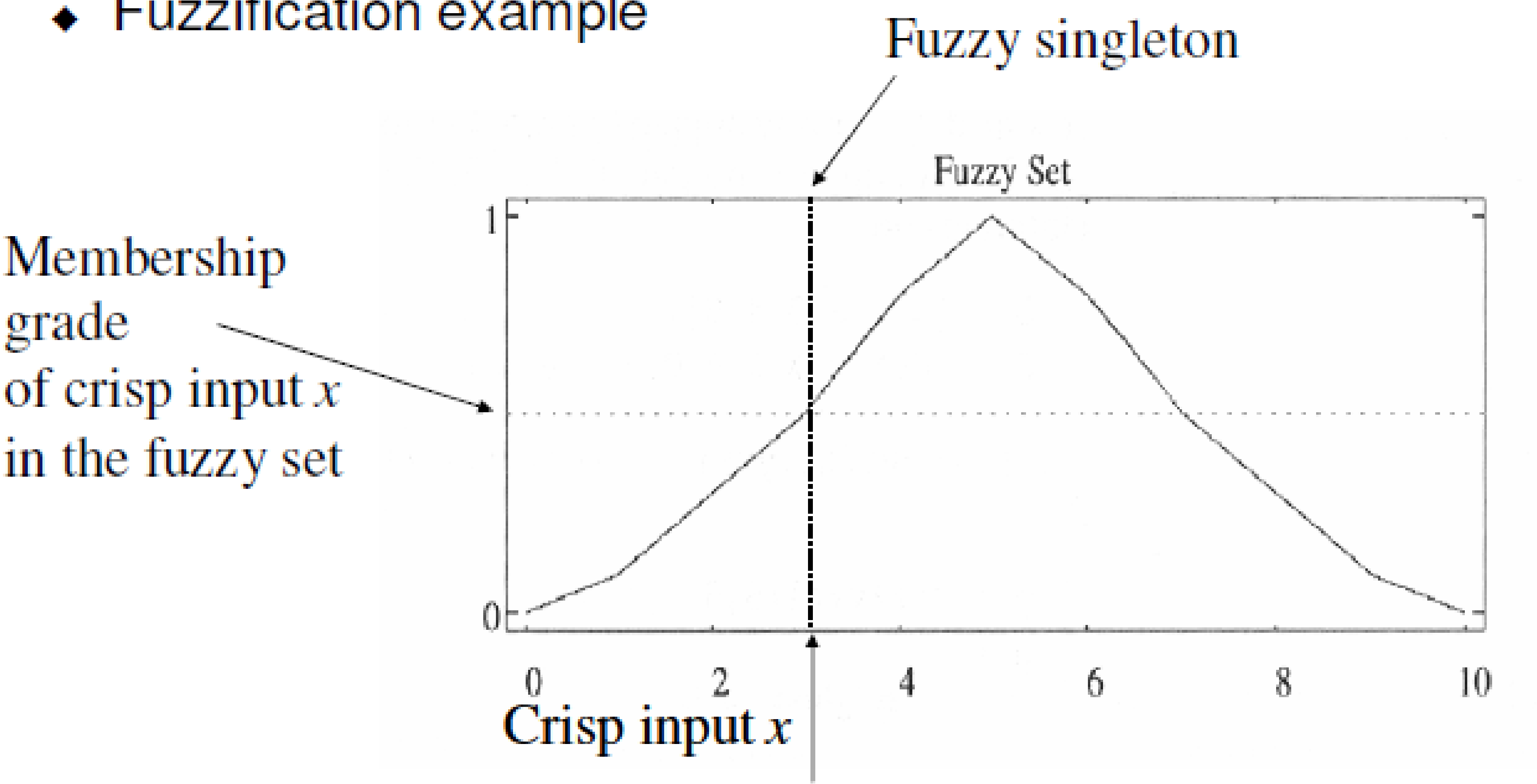


- If the data are vague or perturbed by noise, they should be converted into a fuzzy number

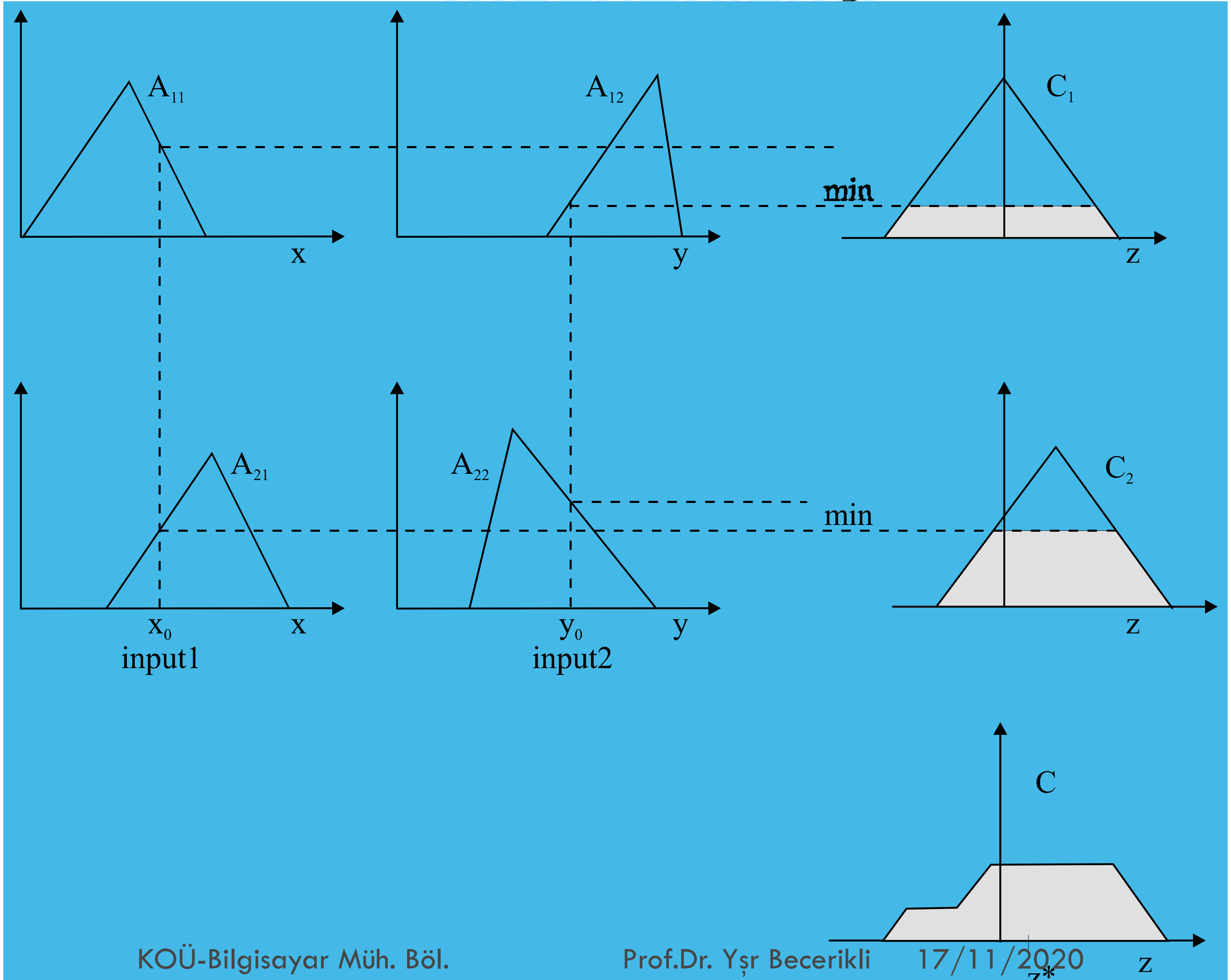


Fuzzification

- Fuzzification example



Mamdani Çıkartım (min. inference mechanism)



Sugeno Bulanık Çıkartım

- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent. A **singleton**, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Devam

Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the **Sugeno-style fuzzy rule** is

IF (eğer) *x* is *A and y is B* *THEN z is f(x,y)* (ise)

Burada, *x*, *y* ve *z* dilsel (linguistic) değişkenler; *A* ve *B* tanım aralığında (domain-universe of discourses) bulanık kümelerdir. Ve *f* (*x*, *y*) deterministik/net (crisp) matematiksel fonksiyondur.