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| *Indices and sets* | |
| *i* | jobs (*i*, *i*′ ∈ *J*) |
| *j* | operations (*j*, *j*′ ∈ *O*) |
| *k* | machines (*k* ∈ *M*) |
| *J* | the set of jobs |
| *M* | the set of machines |
| *O* | the set of operations |
| *Oi* | ordered set of operations of job *i* (*Oi* ⊆ *O*), where Oif(i) is the first and Oiℓ(i) is the last element of *Oi* |
| *Mj* | the set of alternative machines on which operation *j* can be processed, (*Mj* ⊆ *M*) |
| Mj∩Mj′ | the set of machines on which operations *j* and *j*′ can be processed |
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| *Parameters* | |
| *tijk* | the processing time of operation *Oij* on machine *k* |
| *dijk* | Difficulty of operation *Oij* on machine *k* |
| *L* | a large number |
|  | |

|  |  |
| --- | --- |
| *Decision variables* | |
| *Xijk* | 1, if machine *k* is selected for operation *Oij*; 0, otherwise |
| *Sijk* | the starting time of operation *Oij* on machine *k* |
| *Cijk* | the completion time of operation *Oij* on machine *k* |
| Yiji′j′k | 1, if operation *Oij* precedes operation Oi′j′ on machine *k*; 0, otherwise |
| *Ci* | the completion time of job *i* |
| *C*max | maximum completion time over all jobs (makespan) |

The proposed mathematical model is defined as follows:  
Objective function: Minimize   
Constraints:

and

C