Large deviations principle
A probabilistic counterpart to ruin probabilities

Yousra Bouchikhi, Dylan Lebreton, Hamza Belkarkor

2^{nd} year of Modelisation and Artificial Intelligence double degree
at INSA and ENSEEIHT - Toulouse, France

Supervisor: Anthony Reveillac

14 December 2022

ModIA
Promotion 2024

2022 - 2023

Contents

1 Introduction

The Cramér-Lundberg model is a mathematical model used in the field of actuarial science to determine the optimal premium for an insurance policy.

This model can calculate the probability that the insurance company goes bankrupt, also called ruin probability. If the insurance company does not choose a premium higher than a certain threshold, the probability of ruin is equal to one and bankruptcy becomes inevitable. The main object of this project is to determine the minimum premium threshold for which the ruin probability is less than 1.

Under this condition and other assumptions on claim sizes, the large deviation principle allows to find an approximation of the ruin probability and prove it has an exponential behavior for larger values of the initial premium. Insurers use these results to manage their risk of ruin without significantly increasing the value of premiums.

1.1 The Cramér-Lundberg model

The Cramér-Lundberg model suggests the following insurance risk model. Consider a Poisson process $(N_t)_{t\geq 0}$ with rate $\lambda > 0$ defining a number of claims of an insurance firm contract over the time interval $[0, t]$. We denote $(T_n)_{n\in\mathbb{N}}$ the arrival times of the claims and $(W_n)_{n\in\mathbb{N}^*}$ the inter-arrival times as $W_1 = T_1$, $W_n = T_n - T_{n-1}$ for all $n \geq 2$. Let Y_i be the size of the ith claim, u the initial capital and c a constant premium per unit time. The reserves of the insurance firm at time t can be defined with the following risk reserve process:

$$R_t(u) = u + ct - \sum_{i=1}^{N_t} Y_i \tag{1}$$

For the rest of the report, the following hypotheses are considered:

- $(Y_i)_{i\in[\![1,N_t]\!]}$ is a sequence of non-negative, independent and identically distributed (iid) random variables
- $(Y_i)_{i\in[\![1,N_t]\!]}$ is a sequence of Lebesgue integrable random variables, that is $\mathbb{E}(|Y_1|) < +\infty$
- $\forall i \in [\![1, N_t]\!], \forall t \in \mathbb{N} \cup \{+\infty\}, Y_i$ and N_t are independent

1.2 Ruin probability

Considering the Cramér-Lundberg model (1), we define the probability of ruin $\psi(u)$ as the probability that the risk reserve process will drop below zero:

$$\psi(u) = \mathbb{P}(\inf_{t\geq 0} R_t(u) \leq 0) \tag{2}$$

Considering $S_t = u - R_t = \sum_{i=1}^{N_t} Y_i - ct$ the claim surplus process, that is to say the number of claim sizes not compensated by the temporal premiums at time t, the probability of ruin can equivalently be defined as:

$$\psi(u) = \mathbb{P}(\sup_{0\leq t<\infty} S_t > u) \tag{3}$$

2 Determination of the net profit condition

To avoid bankruptcy for the insurance company, the probability of ruin must be less than 1. We see in this section that there is a condition on the premium c for which this is the case. This condition is called the net profit condition.

2.1 Theoretical approach

Proposition 1. *Denoting $\rho = \lambda\,\mathbb{E}(Y_1)$ the average amount of claim per unit time, we have:*

$$\frac{1}{t}\sum_{i=1}^{N_t} Y_i \xrightarrow[t\to+\infty]{a.s.} \underbrace{\lambda\,\mathbb{E}(Y_1)}_{\rho}$$

Proof. Let $N \in \mathbb{N}^*$. According to the strong law of large numbers $\frac{1}{N}\sum_{i=1}^{N} Y_i \xrightarrow[N\to+\infty]{\text{a.s.}} \mathbb{E}(Y_1)$
As shown in [5], we know that $N_t \xrightarrow[t\to+\infty]{\text{a.s.}} +\infty$. Thus, as N_t is independent from Y_i $\forall t$ and $\forall i$, we have $\frac{1}{N_t}\sum_{i=1}^{N_t} Y_i \xrightarrow[t\to+\infty]{\text{a.s.}} \mathbb{E}(Y_1)$. We also know by [5] that $\frac{N_t}{t} \xrightarrow[t\to+\infty]{\text{a.s.}} \lambda$. Then, by product of two continuous functions, we have

$$\frac{N_t}{t}\frac{1}{N_t}\sum_{i=1}^{N_t} Y_i = \frac{1}{t}\sum_{i=1}^{N_t} Y_i \xrightarrow[t\to+\infty]{\text{a.s.}} \underbrace{\lambda\,\mathbb{E}(Y_1)}_{\rho}$$

□

Lemma 1. *The time-averaged claim surplus process $\frac{S_t}{t}$ converges almost surely to $\rho - c$:*

$$\frac{\sum_{i=1}^{N_t} Y_i - ct}{t} = \frac{S_t}{t} \xrightarrow[t\to+\infty]{a.s.} \rho - c$$

Proof. Direct from Proposition 1. □

Lemma 2. *The claim surplus process S_t is almost surely equivalent to $t(\rho-c)$: $S_t \underset{t\to+\infty}{\overset{a.s.}{\sim}} t(\rho-c)$*

Proof. By Lemma 1, we have:

$$\frac{S_t}{t} \xrightarrow[t\to+\infty]{\text{a.s.}} \rho - c \iff \mathbb{P}\left(\lim_{t\to\infty}\frac{S_t}{t} = \rho - c\right) = 1 \iff \mathbb{P}\left(\lim_{t\to\infty}\frac{S_t}{t(\rho-c)} = 1\right) = 1 \iff S_t \underset{t\to+\infty}{\overset{\text{a.s.}}{\sim}} t(\rho-c)$$

□

Theorem 1 (Net profit condition)**.** *The asymptotic behavior of the probability of ruin is dependent on the premium c such that:*

$$\begin{cases} c > \rho \implies \psi(u) < 1 \text{ almost surely for } u \text{ large enough} \\ c < \rho \implies \psi(u) = 1 \text{ almost surely} \end{cases}$$

Proof.

$$\begin{aligned} c > \rho &\implies \lim_{t\to+\infty} t(\rho - c) = -\infty \implies S_t \xrightarrow[t\to+\infty]{\text{a.s.}} -\infty \text{ because } S_t \underset{t\to+\infty}{\overset{\text{a.s.}}{\sim}} t(\rho-c) \text{ by Lemma 2} \\ &\implies \exists u_0 \in \mathbb{R}^+, \forall u > u_0, \sup_{0<t\leq+\infty} S_t < u \text{ almost surely} \\ &\implies \exists u_0 \in \mathbb{R}^+, \forall u > u_0, \quad \underbrace{\mathbb{P}\left(\sup_{0<t\leq+\infty} S_t > u\right)}_{\psi(u) \text{ by (3)}} < 1 \quad \text{almost surely} \end{aligned}$$

$$c < \rho \implies \lim_{t \to +\infty} t(\rho - c) = +\infty \implies S_t \xrightarrow[t \to +\infty]{\text{a.s.}} +\infty \text{ because } S_t \underset{t \to +\infty}{\overset{\text{a.s.}}{\sim}} t(\rho - c) \text{ by Lemma 2}$$

$$\implies \underbrace{\mathbb{P}\left(\sup_{0<t\leq+\infty} S_t > u\right)}_{\psi(u) \text{ by (3)}} = 1 \quad \text{almost surely}$$

□

Theorem 1 shows the existence of a threshold for the premium c. The net profit condition consists then in choosing a value of the premium that's less than this threshold. This results in a ruin probability less than 1 (almost surely) for large enough values of the capital.

2.2 Numerical approach

To illustrate the net profit condition, we see in Figure 1 the evolution of the ruin probability according to the premium c. The estimation of the probability of ruin was done by Monte-Carlo for different values of initial capital u, the parameters used are:

- A Pareto distribution for the claim sizes
- $t \in [0, T_{max} = 2000]$
- 5 iterations for the Monte-Carlo algorithm
- Evaluations of Monte-Carlo iteration for these initial capital values: 10, 20, 30, 40, 50

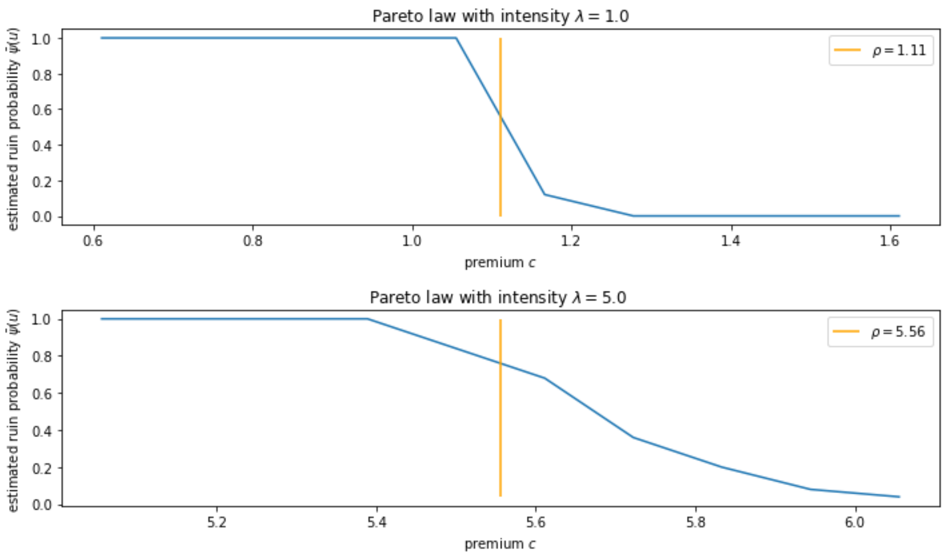

Figure 1: Evolution of estimated ruin probability as a function of the premium c

We see that the estimated probability of ruin decreases from 1.0 to 0.0 for a premium c close to the threshold ρ within 2.10^{-2}. The floating around the threshold may be due to the small number of Monte-Carlo draws or to the approximation of $t \to +\infty$. The numerical results confirm the Theorem 1 for the asymptotic behaviors, the case where $c = \rho$ requires more theoretical investigations and is not treated here.

An insurance company must therefore, to hope to obtain a probability of ruin lower than 1.0, choose a premium c higher than ρ.

3 Expression of ruin probability

The risk model considered in our study assumes that the counting process $(N_t)_t$ is a homogeneous Poisson process, the inter-arrival times $(W_i)_{i\in\mathbb{N}^*}$ are i.i.d and λ- exponentially distributed and $(W_i)_{i\in\mathbb{N}^*}$ are independent of the claims $(Y_i)_i$. Within this risk model, the following result was shown in [2]:

$$\psi(u) \sim \frac{\rho}{1-\rho}\overline{B_s}(u) \tag{4}$$

where $\forall x$, $\overline{B}(x) = 1 - B(x) = \mathbb{P}(Y > x)$ is the tail of the distribution B of claim sizes. Here

$$B_s(x) = \frac{1}{\mathbb{E}(Y)}\int_0^x \overline{B}(y)dy \tag{5}$$

3.1 With Pareto distribution for claims

Definition 1. *If a random variable Y follows a Pareto distribution of parameters (x_m, k), with k a positive real, then the distribution is characterized by:*

$$\mathbb{P}(Y > x) = \begin{cases} \left(\frac{x_\mathrm{m}}{x}\right)^k & x \geq x_\mathrm{m}, \\ 1 & x < x_\mathrm{m}, \end{cases} \tag{6}$$

The expectation of a random variable following a Pareto distribution is: $\mathbb{E}(Y) = \frac{kx_m}{k-1}$

Using (5) and (6), it follows that:

$$\forall x \geq x_m, B_s(x) = \frac{1}{\mathbb{E}(Y)}\int_{x_m}^x \left(\frac{x_\mathrm{m}}{y}\right)^k dy = \frac{x_m^{k-1}(k-1)}{k}\int_{x_m}^x \frac{1}{y^k}dy = \frac{x_m^{k-1}}{k}\left(\frac{1}{x_m^{k-1}} - \frac{1}{x^{k-1}}\right) \tag{7}$$

It was shown in [1] that for efficient programming the following representation of $B_s(x)$ is useful:

$$B_s(x) = \frac{k-1}{x_m k}x\mathbb{1}(x < x_m) + \left(1 - \frac{1}{k}\left(\frac{x_m}{x}\right)^{k-1}\right)\mathbb{1}(x \geq x_m) \tag{8}$$

Then using (4) we show that,

$$\forall u, \psi(u) \sim \frac{\rho}{1-\rho}\left(1 - \left(\frac{k-1}{x_m k}x\mathbb{1}(u < x_m) + \left(1 - \frac{1}{k}\left(\frac{x_m}{u}\right)^{k-1}\right)\mathbb{1}(u \geq x_m)\right)\right) \tag{9}$$

3.2 With log-normal distribution for claims

Definition 2. *The lognormal distribution of parameters μ and σ admits for probability density:*

$$f_Y(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \tag{10}$$

We define the erf function as following: $\forall x$, $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$
By integration of the density function, the distribution function is expressed as a function of the error function erf:

$$\mathbb{P}(Y \leq x) = F_Y(x;\mu,\sigma) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left[\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right] \tag{11}$$

The expectation of a random variable Y following a log-normal distribution is: $\mathbb{E}(Y) = e^{\mu+\sigma^2/2}$
As shown in [3], asymptotically:

$$\bar{B}(x) = \mathbb{P}(Y > x) \sim \frac{\sigma}{\ln x\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\} \tag{12}$$

It was shown in [1] that for efficient programming the following representation of $B_s(x)$ is useful:

$$B_s(u) = \frac{1}{\mathbb{E}(Y)}\left(u - u\Phi(w(u)) + \mathbb{E}(Y)\Phi(w(u) - \sigma)\right) \tag{13}$$

Where $\Phi(\cdot)$ denotes the c.d.f. of a standard normal distribution and $w(x) = \frac{1}{\sigma}(\log(x) - \mu)$.

Then using (4) we show that,

$$\psi(u) \sim \frac{\rho}{1-\rho}\left(1 - e^{-(\mu+\sigma^2/2)}\left(u - u\Phi(w(u)) + e^{(\mu+\sigma^2/2)}\Phi(w(u) - \sigma)\right)\right) \tag{14}$$

3.3 Numerical Simulation

In this section we try to use the result of (9) to see whether the probability of ruin actually decreases exponentially as a function of the initial prime u. In the example below, we used a:

- 1^{st} case: Pareto distribution for the claims $(Y_i)_i$ with parameters $(x_m = 1, k = 10)$
- 2^{nd} case: Log-normal distribution for the claims $(Y_i)_i$ with parameters $(\mu = 0, \sigma = 1)$
- Parameter $\rho = 0.7$
- Initial prime $u \in [0, 50]$

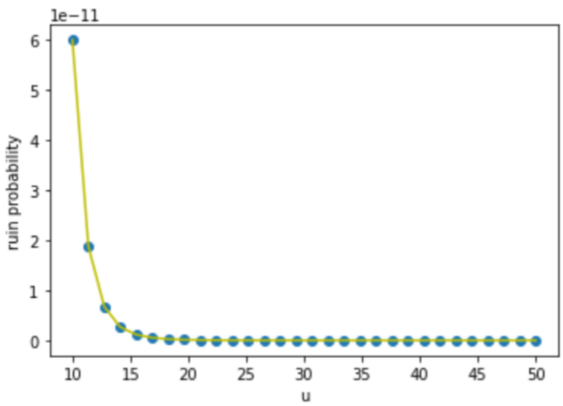

Figure 2: Ruin probability in the 1^{st} case

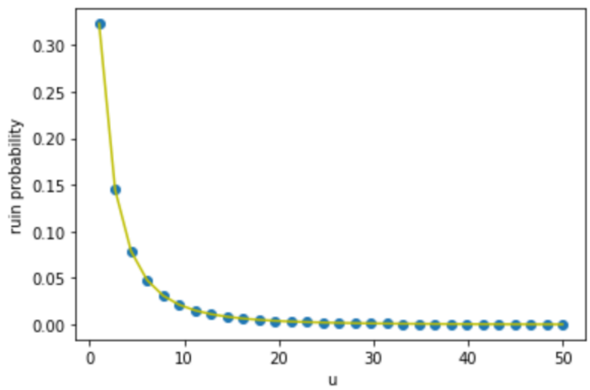

Figure 3: Ruin probability in the 2^{nd} case

We can see in both figures 2 and 3 the exponential decrease of the probability of failure as a function of the initial premium u. With a low initial premium, we can see that the ruin is probable. On the other hand, this probability tends towards 0 for values of u higher than approximately 15.

4 Large deviations principle for the insurance model

The large deviations principle is a mathematical concept used for the study of probabilities of rare events. It states that the probability of a rare event occurring can be approximated by the exponential function of the rate at which the event occurs. This principle is used to calculate the probability of events such as the insolvency of an insurance company, which is the main subject of this paper. In this section we will try to approximate the ruin probability to a decaying exponential function.

4.1 Basic tools and results of Large deviations principle

Definition 3 (Cumulant generating function). *Let X be a real-valued random variable. Suppose that the function $t \mapsto \exp(tX)$ is integrable. The cumulant generating function (cgf) of X is given by :*

$$\Gamma_X(t) = \ln\left(\mathbb{E}\left(\exp(tX)\right)\right)$$

Theorem 2 (Cramér's theorem). *Let $(S_n)_{n\geq 1}$ a sequence of random variables by $S_n = \sum_{i=1}^{n} X_i$, where $X_1, X_2...$ are iid random variables with finite cgf Γ_{X_1}. Then we have :*

$$\lim_{n\to+\infty} \frac{1}{n} ln(\mathbb{P}(S_n \geq nx)) = -\Gamma^*_{X_1}(x)$$

Γ^* *is the Fenchel-Legendre transform defined by :* $\Gamma^*_X(t) = \sup_{x\in\mathbb{R}}(xt - \Gamma_X(x))$

Definition 4 (Stopping time). *Let $X = X_n : n \geq 0$ be a stochastic process. A stopping time with respect to X is a random time such that for each $n \geq 0$, the event $\tau = n$ is completely determined by (at most) the total information known up to time n, $X_1, .., X_n$*

Theorem 3 (Exponential change of measure). *Suppose that $X_1, X_2...X_n..$ are iid random variables with distribution μ . We denote $\mathcal{D}(\Gamma) = \{\theta \in \mathbb{R} : \Gamma(\theta) < \infty\}$ and for any $\theta \in \mathcal{D}(\Gamma)$, we define a probability measure μ_θ defined on $\mathbb{R}$ by $\mu_\theta = \exp(\theta x - \Gamma_X(\theta))\mu$ and let E_θ be the corresponding expectation, then we have :*

$$\mathbb{E}\left(f(X_1, ..., X_\tau)1_{\tau<\infty}\right) = \mathbb{E}\left(f(X_1, ..., X_\tau)\exp(-\theta\sum_{i=1}^{\tau} X_i + \tau\Gamma(\theta))1_{\tau<\infty}\right)$$

With τ a stopping time in $\mathbb{N}$

Proof. One could refer to [4] □

4.2 Approaching the probability of ruin

Considering the risk reserve process defined in (1), we may notice that a ruin can only occur at the arrival of a claim, i.e when the risk process jumps downwards, it's sufficient then to consider the discrete-time process $R_{T_n}(u), n \geq 1$ defined by the jumps of the Poisson process. Let σ_u be the number of claims for which ruin occurs i.e. $\sigma_u = \inf\{n \geq 1 : R_{T_n}(u) < 0\}$.
Let $S_n = u - R_{T_n}(u)$ be the net payout up to the n-th claim, and $Z_i = Y_i - cW_i$ for $i \in \mathbb{N}^*$ then $S_n = Z_1 + ... + Z_n$

Since $W_i \sim \mathcal{E}(\lambda)$ i.e $\mathbb{E}\left(W_1 = \frac{1}{\lambda}\right)$, and as seen before $\eta = \frac{c-\rho}{\rho} > 0$ with $\rho = \lambda\,\mathbb{E}\,(Y_1)$, then

$$\mathbb{E}\,(Z_1) = \mathbb{E}\,(Y_1) - c\,\mathbb{E}\,(W_1) \implies \mathbb{E}\,(Z_1) < -\frac{c}{\lambda} < 0$$

Let Γ_Z , Γ_W and Γ_Y be the c.g.f of Z_i , W_i and Y_i respectively. By independence of Y_i and W_i we have :

$$\Gamma_Z(\theta) = \Gamma_Y(\theta) + \Gamma_W(-p\theta)$$

$$\Gamma_Z(\theta) = \Gamma_Y(\theta) + ln(\frac{\lambda}{\lambda + p\theta}), \theta > -\frac{\lambda}{p} \tag{15}$$

Since σ_u is a stopping time for $(Z_1, ..Zn)$, we consider an exponential change of measure. It follows from Theorem 3 that for all $\theta > -\frac{\lambda}{p}$:

$$\psi(u) = \mathbb{P}(\sigma_u < \infty) = \mathbb{E}\left(1_{\mathbb{R}_+} \circ \sigma_u\right)$$

$$\psi(u) = \mathbb{E}_\theta(1_{\sigma_u<\infty} \exp\left(-\theta S_{\sigma_u} + \sigma_u \Gamma_Z(\theta)\right)) \tag{16}$$

We now assume Y has a light-tailed distribution, i.e. : there exists $\theta_1 > 0$ s.t. for all $\theta < \theta_1, \Gamma_Y(\theta) < \infty$ and $\Gamma_Z(\theta) \to \infty$ as θ approaches θ_1 . In this case, the cgf Γ_Z is finite on $(-\frac{\lambda}{p}, \theta_1)$, and differentiable in 0 with $\Gamma'_Z(0) = \mathbb{E}\,(Z_1) < 0$ under the condition $\eta > 0$. And since $\mathbb{E}\,(Y_1) > 0$ and Y_1 is independent of W_1 , we see that $\mathbb{P}(Z_1 > 0) > 0$, therefore $\Gamma_Z(\theta)$ goes to ∞ when θ goes to θ_1. By convexity of Γ_Z, and since $\Gamma_Z(0) = 0$, we deduce the existence of an unique $\theta_L > 0$ s.t $\Gamma_Z(\theta_L) = 0$, the latter is the solution of the Cramer-Lundberg equation:

$$\Gamma_Y(\theta_L) + ln(\frac{\lambda}{\lambda + p\theta_L}) = 0 \tag{17}$$

By convexity of Γ_Z, we have $\Gamma'_Z(\theta_L) > 0$, it follows ,under the probability measure μ_{θ_L}, and using the expression (16) :

$$\psi(u) = \mathbb{E}_{\theta_L}(\exp\left(-\theta_L S_{\sigma_u}\right)) = \exp\left(-\theta_L u\right)\mathbb{E}_{\theta_L}(\exp\left(-\theta_L(S_{\sigma_u} - u)\right)) \tag{18}$$

For more details, the reader may refer to [4] or [3]
Notice that $S_{\sigma_u} - u$ in nonnegative, this leads to the Lundberg's inequality on the ruin probability :

$$\psi(u) \leq \exp\left(-\theta_L u\right), \forall u > 0. \tag{19}$$

Moreover, by renewal's theory, $S_{\sigma_u} - u$ has a limit (in the sense of weak convergence with respect to μ_{θ_L}) when u goes to infinity, hence $\mathbb{E}_{\theta_L}(\exp\left(-\theta_L(S_{\sigma_u} - u)\right))$ converges to some positive constant C, finally, we obtain the approximation for large values of the initial reserve (the capital) :

$$\psi(u) \approx C \exp\left(-\theta_L u\right), \text{for large values of u} \tag{20}$$

4.3 Interpretation

The result of the previous section may be used to demonstrate how insurers could divide their risk of ruin by n ($n \in \mathbb{N}^*$), without necessarily increasing the value of the premium by too much.

if

$$\psi(u_1) \approx C \exp\left(-\theta_L u_1\right)$$

and

$$\psi(u_2) = \frac{\psi(u_1)}{n} \approx C \exp\left(-\theta_L u_2\right)$$

then

$$\frac{\psi(u_2)}{\psi(u_1)} = \frac{1}{n} \approx C\exp\left(-\theta_L(u_2-u_1)\right)$$
$$\implies u_2 - u_1 \approx -\frac{1}{\theta_L}ln(1/n)$$
$$\implies u_2 \approx u_1 + \frac{1}{\theta_L}ln(n)$$

for example, if the insurance wants to divide its risk by 2, then for a value of $\theta_L = 0.95$ it has to add a value of $\frac{ln(2)}{\theta_L} = 0.729$ to its initial premium.

4.4 Numerical Simulation

In this section we will simulate the ruin probability for different values of u, the initial premium.
In the example below, we used a:

- Binomial distribution for the claims Y_i with parameters (p=0.3, n=100) such that $Y_i = 5$ with probability p and $Y_i = 1$ with probability 1-p
- Poisson process with parameter $\lambda = 1$
- Rate or premium $c = 10$

Here, since $\rho = \lambda\mathbb{E}(Y_1) = \lambda p = 30$, then the net profit condition implies that the rate $c > \rho$ i.e $c > 30$. We chose a rate $c = 10 \leq \rho$ in order to demonstrate ruin.
The simulation of the ruin probabilities is obtained by doing a Monte-Carlo method i.e the ruin probability for a certain value of u is considered as the number of times the risk process is negative for this value of u out of 300 simulations of this risk process.

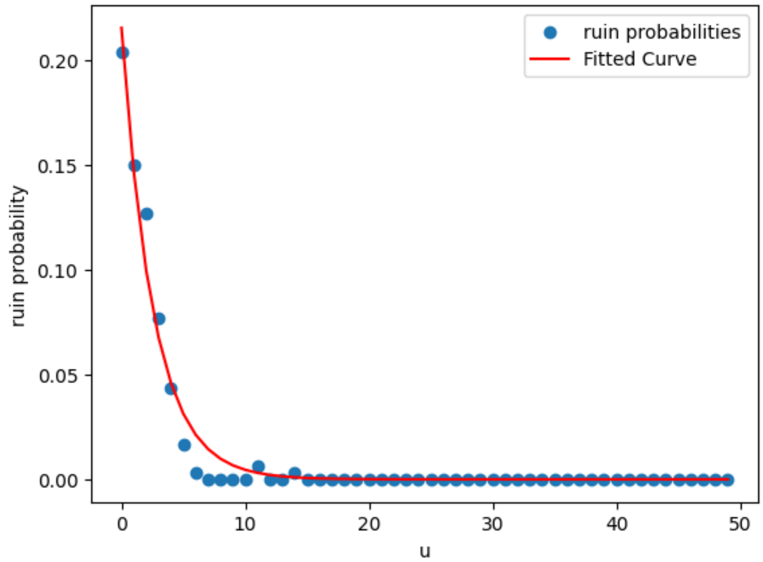

Figure 4: Ruin probability for different values of u.

We observe in figure (4) an exponential behavior of the ruin probability, which decreases as the values of u increase. We interpolated these values of $\psi(u)$ with an exponential function $u \mapsto C\exp\left(-\theta * u\right)$ with optimal parameters $C = 0.239$ and $\theta = 0.428$. This interpolation has a high accuracy as the determination coefficient $R^2 = 0.972$, which supports the results of section 4.2.

5 Conclusion

Ruin probability is the likelihood that a given financial entity, such as an insurance company, will become insolvent or bankrupt.

Under this condition and other assumptions on claim sizes, the large deviation principle allows to find an approximation of the ruin probability and prove it has an exponential behavior for larger values of the initial premium. Insurers use these results to manage their risk of ruin without significantly increasing the value of premiums.

The large deviations principle is a mathematical concept that helps to evaluate the probability of rare events, such as the ruin of an insurance company. Together, these concepts can be used to manage and reduce the risks faced by financial entities, as we showed by adding a logarithmic term to the initial premium, all under the assumption that the net profit condition is met.

References

[1] S. Asmussen and K. Binswanger. “Simulation of Ruin Probabilities for Subexponential Claims”. en. In: *ASTIN Bulletin: The Journal of the IAA* 27.2 (Nov. 1997), pp. 297–318. ISSN: 0515-0361, 1783-1350. DOI: 10.2143/AST.27.2.542054. URL: https://econpapers.repec.org/article/cupastinb/v_3a27_3ay_3a1997_3ai_3a02_3ap_3a297-318_5f01.htm.

[2] Søren Asmussen, Hanspeter Schmidli, and Volker Schmidt. “Tail probabilities for non-standard risk and queueing processes with subexponential jumps”. en. In: *Advances in Applied Probability* 31.2 (June 1999), pp. 422–447. ISSN: 0001-8678, 1475-6064. DOI: 10.1239/aap/1029955142. URL: https://www.cambridge.org/core/product/identifier/S0001867800009186/type/journal_article.

[3] Søren Asmussen. *Ruin probabilities*. eng. OCLC: 827947385. River Edge, NJ: World Scientific, 2000. ISBN: 9789812779311. URL: https://doi.org/10.1142/2779.

[4] Huyen Pham. “Some applications and methods of large deviations in finance and insurance”. Feb. 2007. URL: https://hal.archives-ouvertes.fr/hal-00131578.

[5] Mélisande Albert. *Poisson processes and application to reliability theory and actuarial science*. 2022. URL: https://moodle.insa-toulouse.fr/mod/resource/view.php?id=68240.