CprE/SE/ComS 412, ComS 512 Homework 5

Reminder: present your own work and properly cite any sources used. Solutions should be presented satisfying the *other student viewpoint*. If you need clarification, contact the instructor: `asminer@iastate.edu`.

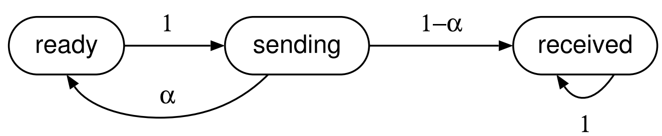
Question 1

The the game of *craps* is played as follows, with two fair, 6–sided dice. If the first throw is 7 or 11, the player wins. If the first throw is 2, 3, or 12, the player loses. Otherwise, the value of the first throw is called the *point* value, and the player repeatedly throws the dice until either the point value is obtained again, in which case the player wins, or a 7 is obtained, in which case the player loses.

1. **(10 points)** Draw a DTMC model for the game of craps.
2. **(10 points)** Based on this DTMC, what is the expected number of throws per game?
3. **(5 points)** Based on this DTMC, what is the probability of winning?

Question 2

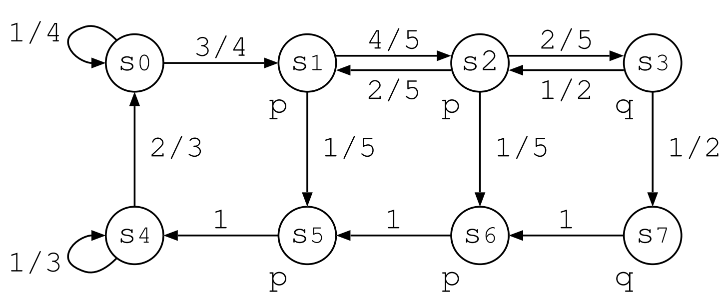
Consider a (tiny) message system that behaves as follows. One unit of time is required to prepare the message for sending. One unit of time is required to send the message. However, with some probability, there is an error and the message must be re-sent (which again requires to prepare the message). The system may be described using the following DTMC, where the initial state is "ready" with probability one:



1. **(5 points)** Does this model satisfy the CTL property $\mathsf{AF}\ \textit{received}$?
2. **(10 points)** What is the average time required for the message to be received?
3. **(10 points, optional for 412)** What is the probability that the message will be received by time t?

Question 3 — 20 points

For the DTMC shown below, which states satisfy $\mathsf{P}_{>1/2}\ p\,\mathsf{U}\,q$?



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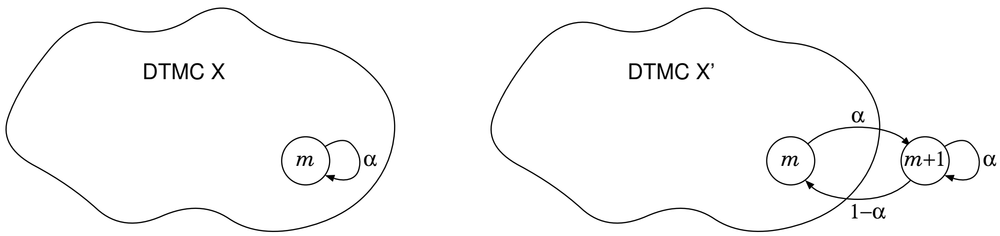
Question 4

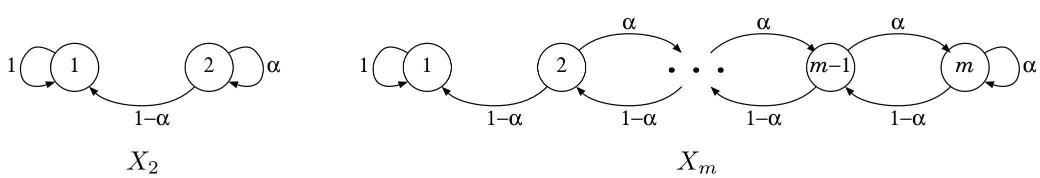
Let X be a DTMC with m states and transition probability matrix $\mathbf{P}$ whose only requirement is that state m has an edge to itself with probability $0 < \alpha < 1$. Construct a new DTMC X' by adding a new state, directing the self–loop from state m to state $m+1$, and putting edges from state $m+1$ to itself with probability α, and back to state m with probability $1-\alpha$, as shown below.



1. **(10 points)** Prove: if X is an absorbing chain, then X' is also an absorbing chain.

2. **(20 points, optional for everyone)** Suppose X is an absorbing chain. Let $\mathbf{p}_0$ be the initial distribution of X, and let $\mathbf{p}'_0 = [\mathbf{p}_0, 0]$ be the initial distribution of X' (i.e., $\mathbf{p}'_0[i] = \mathbf{p}_0[i], \forall i \leq m$, and $\mathbf{p}'[m+1] = 0$). Let $\mathbf{n}$ be the vector of expected number of visits to each state in X, and let $\mathbf{n}'$ be the vector of expected number of visits to each state in X'. Prove that $\mathbf{n}'[i] = \mathbf{n}[i]$ for all $i \leq m$, and that $\mathbf{n}'[m+1] = \frac{\alpha}{1-\alpha}\mathbf{n}[m]$.

3. **(10 points, optional for 412)** Let X_m denote an m–state DTMC as shown below.



Suppose the DTMC starts in state 2 with probability 1. Using the property you are asked to prove in part (2), give an expression for $\mathbf{n}_m$, the vector of expected number of visits to each state in X_m, valid for all $m \geq 2$.

4. **(10 points, optional for everyone)** Let t_m be the mean time to absorption for X_m. Give closed–form expression(s) for t_m, valid for all $0 < \alpha < 1$.

5. **(10 points, optional for everyone)** Let $t_* = \lim_{m\to\infty} t_m$. For which values of α is t_* finite? For those values, what is t_*?