COMS 512 Homework 0

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1. As p is a propositional variable, p can be either True or False.We denote True with tt and False with ff.

   a) $tt \rightarrow p$

      If $p = tt$ then $tt \rightarrow p \equiv tt$,If $p = ff$ then $tt \rightarrow p \equiv ff$.

   b) $p \rightarrow tt$

      If $p = tt$ then $p \rightarrow tt \equiv tt$,If $p = ff$ then $p \rightarrow tt \equiv tt$.

   c) $ff \rightarrow p$

      If $p = tt$ then $ff \rightarrow p \equiv tt$,If $p = ff$ then $ff \rightarrow p \equiv tt$.

   d) $p \rightarrow ff$

      If $p = tt$ then $p \rightarrow ff \equiv ff$,If $p = ff$ then $p \rightarrow ff \equiv tt$.

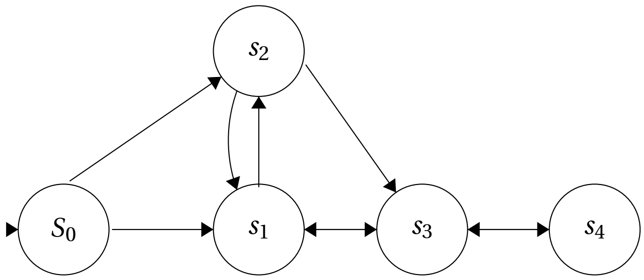
2. We can prove the given expression is a tautology by constructing the truth table of the premises.

A	B	C	A -> B	B -> C	A -> C	(A -> B) ^ (B -> C)	[(A -> B) ^ (B -> C)]->(A -> C
tt	tt	tt	tt	tt	tt	tt	tt
tt	tt	ff	tt	ff	ff	ff	tt
tt	ff	tt	ff	tt	tt	ff	tt
tt	ff	ff	ff	tt	ff	ff	tt
ff	tt	tt	tt	tt	tt	tt	tt
ff	tt	ff	tt	ff	tt	ff	tt
ff	ff	tt	tt	tt	tt	tt	tt
ff	ff	ff	tt	tt	tt	tt	tt

First we create the truth table for $A \rightarrow B$, $B \rightarrow C$ and $A \rightarrow C$.Then we find the truth table for $(A \rightarrow B) \wedge (B \rightarrow C)$.Finally we find the truth table for the given expression.As all the term in the given expression turn out to be True, we can conclude that the expression is a tautology.

3. Let us consider the following Kripke Structure, where $S = \{s_0, s_1, s_2, s_3, s_4\}$,$S = \{s_0\}$ is the initial state, the transition relation,

   $R = \{(s_0, s_1), (s_0, s_2), (s_1, s_2), (s_1, s_3), (s_2, s_1), (s_2, s_3), (s_3, s_1), (s_3, s_4), (s_4, s_3)\}$.

   Lets take $X = \{s_0, s_1\}$.The PreImage function of X will be $PreImage(X, R) = \{s_0, s_2, s_3\}$.Next we need to find the PostImage function of this PreImage set. The PostImage function will be $PostImage(\{s_0, s_2, s_3\}, R) = \{s_1, s_2, s_3, s_4\}$.

   We can see that $\{s_0, s_1\} \not\subseteq \{s_1, s_2, s_3, s_4\}$.i.e $X \not\subseteq PostImage(\{s_0, s_2, s_3\}, R)$.Therefore from this counter-example, we can conclude that $X \not\subseteq PostImage(Preimage(X, R), R)$.

4. By the definition of Kripke structure we know that every state of the Kripke structure must have at least one outgoing edge.

   Now if we consider a set of states $X \subseteq S$ and try to find their PostImage set Y, then every state from X will have at least one state in Y that it can reach in one step.

   i.e. if $s \subseteq X$ then $\exists s' \in Y$ s.t $(s, s') \in R$.

   Hence every state in Y has atleast one predecessor state in X.therefore if we construct the Preimage set of Y it will cover all those states that belong to set X.i.e. $X \subseteq PreImage(Y, R)$.

   Therefore $X \subseteq PreImage(PostImage(X, R), R)$.