CprE/SE/ComS 412, ComS 512 Homework 1

Reminder: present your own work and properly cite any sources used. Solutions should be presented satisfying the *other student viewpoint*. If you need clarification, contact the instructor: `asminer@iastate.edu`.

Question 1 15 points

Write CTL formulas for the following English specifications or questions.

1. "Condition C never holds after condition B holds".
2. "Whenever condition B holds, condition C will eventually hold, and condition D cannot occur between B and C."
3. "Whenever condition B holds, condition C holds within 2 steps".

Question 2 16 points

Given a Kripke structure $M = (\mathcal{S}, \mathcal{S}_0, \mathcal{R}, L)$ where $\mathcal{S} = \{s_0, s_1, s_2, s_3\}$, $\mathcal{R} = \{(s_0, s_1), (s_0, s_2), (s_1, s_1), (s_1, s_2), (s_2, s_3), (s_3, s_2)\}$, and $L(s_0) = \emptyset$, $L(s_1) = \{p\}$, $L(s_2) = \{p, q\}$, $L(s_3) = \{q, r\}$, identify the sets of states that satisfy the following properties:

1. $(\mathsf{AX}\, p) \rightarrow (\mathsf{AX}\, \mathsf{AX}\, p)$
2. $\mathsf{EF}\, \mathsf{EG}\, q$
3. $(\mathsf{AF}\, \mathsf{AG}\, (p \wedge q)) \rightarrow (\mathsf{AG}\, \mathsf{AF}\, (p \wedge q))$
4. $\mathsf{AG}\, \mathsf{AF}\, (\neg p \vee \neg q)$

Question 3 10 points

Define a new CTL temporal operator X^2 that requires a property to hold *two* steps from now. Formally, define AX^2 and EX^2 as follows:

1. $M, s \models \mathsf{AX}^2\phi$ if and only if, for all paths $\pi = (p_0, p_1, p_2, p_3, \ldots)$ with $p_0 = s$, $p_2 \models \phi$.
2. $M, s \models \mathsf{EX}^2\phi$ if and only if, there exists a path $\pi = (p_0, p_1, p_2, p_3, \ldots)$ with $p_0 = s$, such that $p_2 \models \phi$.

Prove or disprove the following conjectures:

$$\mathsf{EX}^2\phi \overset{?}{\equiv} \mathsf{EX}\, \mathsf{EX}\phi$$

$$\mathsf{AX}^2\phi \overset{?}{\equiv} \mathsf{AX}\, \mathsf{AX}\phi$$

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Question 4 (optional for 412) 15 points

A CTL formula ϕ_1 is said to be *at least as strong as* another CTL formula ϕ_2 if, for any Kripke structure $M = (\mathcal{S}, s_0, \mathcal{R}, L)$ and any state $s \in \mathcal{S}$, if $M, s \models \phi_1$ then $M, s \models \phi_2$. Formula ϕ_1 is said to be *stronger than* ϕ_2 if ϕ_1 is at least as strong as ϕ_2, and ϕ_2 is not at least as strong as ϕ_1. Prove or disprove:

$$\mathsf{AF}\,\mathsf{AX}\,\phi \quad \text{is stronger than} \quad \mathsf{AX}\,\mathsf{AF}\,\phi$$

Question 5 10 points

For any set $\mathcal{S}$ and any $\mathcal{Z} \subseteq \mathcal{S}$, define function $u_{\mathcal{Z}} : 2^{\mathcal{S}} \to 2^{\mathcal{S}}$ as

$$u_{\mathcal{Z}}(\mathcal{X}) = \mathcal{X} \cup \mathcal{Z}.$$

1. Prove that $u_{\mathcal{Z}}$ is monotonic.
2. What is the least fixed point of $u_{\mathcal{Z}}$?
3. What is the greatest fixed point of $u_{\mathcal{Z}}$?