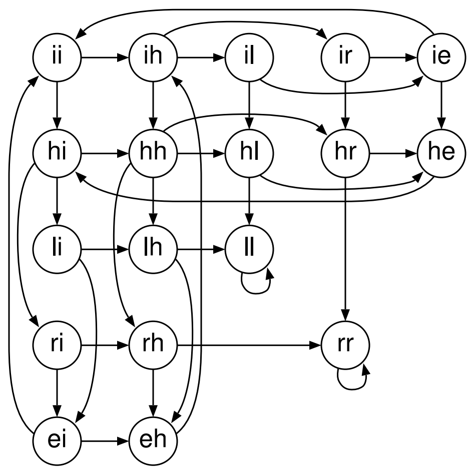
CprE/SE/ComS 412, ComS 512 Homework 2

Reminder: present your own work and properly cite any sources used. Solutions should be presented satisfying the *other student viewpoint*. If you need clarification, contact the instructor: `asminer@iastate.edu`.

Question 1

Consider a Kripke structure model for two dining philosophers, as shown below.



Each state of the Kripke structure is denoted by the states of the two philosophers, where the philosopher states are:

i : Idle

h : Hungry, without any forks

l : Hungry, has left fork but not right

r : Hungry, has right fork but not left

e : Eating (has both forks)

Initially, both philosophers are *Idle*. Determine if the model satisfies the proprties below (remember to show your work).

1. **(5 points)** Whenever Philosopher 1 becomes hungry, he will eventually eat:

$$\mathsf{AG}(\text{Phil.1 hungry} \rightarrow \mathsf{AF}\,(\text{Phil.1 eats})\,)$$

2. **(5 points)** The same property as above, using the fairness constraint $\mathcal{C} = \{ii\}$, i.e., only considering paths where both philosophers are idle, infinitely often:

$$\mathsf{A}_{\mathcal{C}}\mathsf{G}(\text{Phil.1 hungry} \rightarrow \mathsf{A}_{\mathcal{C}}\mathsf{F}\,(\text{Phil.1 eats})\,)$$

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Question 2

Define a new CTL temporal operator EN, that acts like ordinary "until", except it does not require the second property to eventually hold. Formally, for any Kripke structure M and any state s, $M, s \models \mathsf{E}\,\phi_1\,\mathsf{N}\,\phi_2$, if and only if, there exists a path $(p_0, p_1, p_2, \ldots)$ with $p_0 = s$, such that either (a) $p_i \models \phi_1, \forall i$; or (b) $\exists j \geq 0 : p_j \models \phi_2, p_i \models \phi_1, \forall i < j$.

1. **(5 points)** Prove or disprove: $\mathsf{E}\,\mathtt{tt}\,\mathsf{N}\,\phi \;\equiv\; \mathsf{EF}\,\phi$
2. **(5 points)** Prove or disprove: $\mathsf{E}\,\phi\,\mathsf{N}\,\mathtt{ff} \;\equiv\; \mathsf{EG}\,\phi$
3. **(5 points)** Prove: $\mathsf{E}\,\phi_1\,\mathsf{N}\,\phi_2 \;\equiv\; \phi_2 \vee (\phi_1 \wedge \mathsf{EX}\,\mathsf{E}\,\phi_1\,\mathsf{N}\,\phi_2)$
4. **(10 points, Optional for 412)** Prove: $[\![\mathsf{E}\,\phi_1\,\mathsf{N}\,\phi_2]\!]$ is the greatest fixed point of $U_{\phi_1 \phi_2}$, using the definition of $U_{\phi_1 \phi_2}$ as given in Property 4.24.