COMS 512 Homework 3

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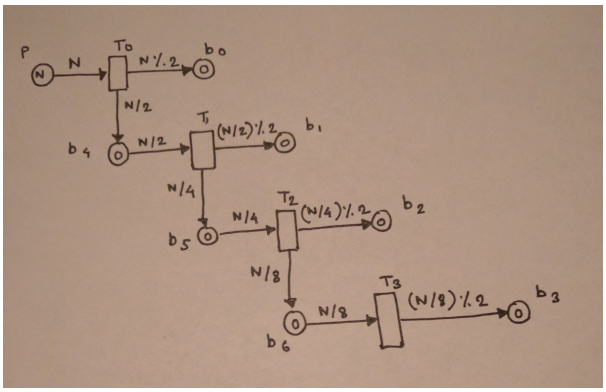
1. a) To draw this Petri net we will use marking dependent arc weights, which is a function of the current marking rather than constant.It is also known as self modifying net.



We have initial number of token N=15. Place p has all 15 tokens. We also have 4 additional places b_0, b_1, b_2, b_3. We need to construct a petri net such that the final marking in the places b_0, b_1, b_2, b_3 is the binary encoding of 15 i.e 1111_b. The construction of the said petri net is given above.

Initially all the places except p has 0 number of tokens.We have constructed it by providing two different marking dependent arc weights in each of it's transition except the last one.First we are putting arc weight as $N\%2$. Where N is the initial token amount in place p.On the other branch of the transition we will put $N/2$.So the place b_1 will have the remainder and we will pass the number of token which is the quotient.

We start from initial marking (15,0,0,0,0,0,0,0).First T_0 will fire. And the following markings are given in the table below,

	p	b6	b5	b4	b3	b2	b1	b0
	15	0	0	0	0	0	0	0
T0	7	0	0	7	0	0	0	1
T1	7	0	3	4	0	0	1	1
T2	7	1	2	4	0	1	1	1
T3	7	0	2	4	1	1	1	1

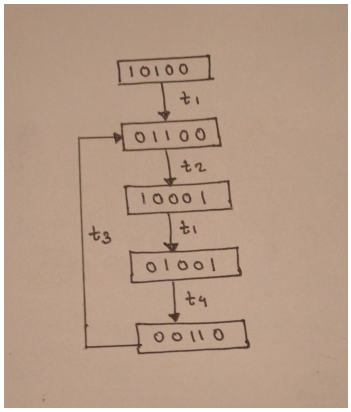
We can observe that in the final marking we have 1,1,1,1 in places b_0, b_1, b_2, b_3 which is the binary encoding of 15 i.e the initial number of tokens.

b) For any arbitrary B>1 we will follow the same procedure that we followed to draw petri net for N=15.In case of $N \leq 15$ we used 4 transitions, because $N = B^4 - 1$. Therefore for any random B the number of transition of the petri net will be the exponent of B.

After creating the transitions the rest of the construction is very straight forward. For each transition we will have two places. From the left place there will be an arc towards transition and from the transition there will be an arc towards the right place. There will be another arc from the transition which will connect a intermediate place. Now we have already described in part (a) how we will construct the marking dependent arc weight. As result of that the places that have no outward arcs will hold the remainder at each step and the other place from the transition will carry the quotient for further operation. This will continue till very last step where the transition will only have arc towards one place which will store the remainder.

Now such places will store remainder in each steps and ultimately will give the binary encoding of the initial number of tokens in the final marking.

2. For the given petri net the initial marking is $m_0 = (1,0,1,0,0)$.The reachability graph is given below:



At the initial marking m_0 transition t_1 is enabled because input places p_1 has token.t_3 is disabled because tough p_3 has token,p_2 does not have any token.

Next marking is $m_1 = (0,1,1,0,0)$.Now the transition t_2 is enabled because p_2, p_3 has tokens.

Next marking is $m_2 = (1,0,0,0,1)$.Now the transition t_1 is enabled because input places p_1 has tokens.

Next marking is $m_3 = (0,1,0,0,1)$.Now the transition t_4 is enabled because input places p_2, p_5 has tokens.

Next marking is $m_4 = (0,0,1,1,0)$.Now the transition t_3 is enabled because input places p_4 has tokens.This transition goes back to the marking m_1.