COMS 512 Homework 1

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1. a) Condition C never holds after condition B holds:

$$AG(B \to AF\neg C)$$

This holds for all states globally hence we started with AG.The CTL syntax inside the parenthesis means for all path negation of C will satisfy in any future state after condition B holds.

b) Whenever condition B holds, condition C will eventually hold and condition D can not occur between B and C:

$$AG(B \to A\neg DUC)$$

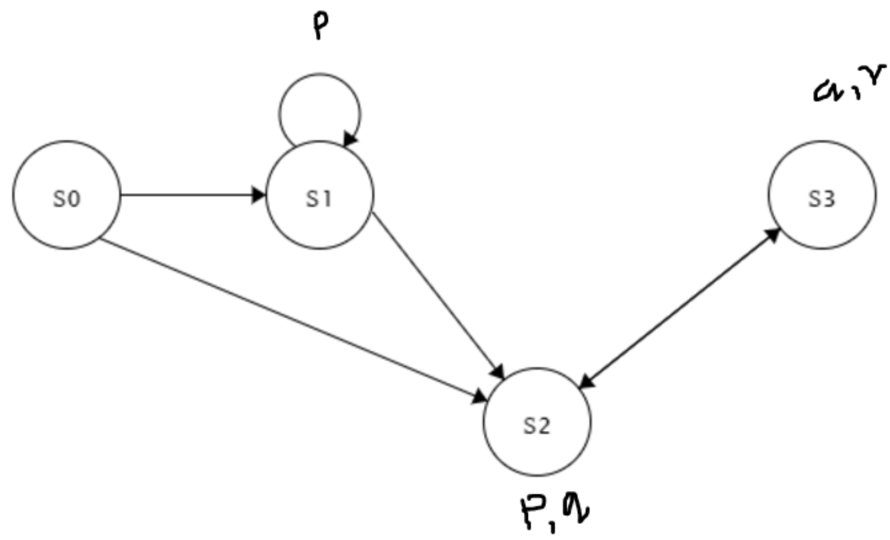
Here we also started with AG because inner CTL is true globally.Inner CTL states that condition B implies for all path condition C will eventually hold and until that negation of D will hold i.e D will not hold until C holds.

c) Whenever condition B holds, conditions C holds within two steps:

$$AG(B \to AXAXC)$$

We started with AG to ensure CTL holds for all states globally.Inner CTL states that occurrence of B means C will hold in exact two steps which is given with two AXs.

2. Below is the state diagram of the given Kripke structure:



a) $(AX_p) \rightarrow (AXAX_p)$:

No.From state S_0 we can go to state S_1 and S_2.Both of them holds p.Hence (AX_p) is true.But if we consider the path $S_0 \rightarrow S_2 \rightarrow S_3$ then S_3 does not hold p.So $(AXAX_p)$ does not hold.Hence S_0 does not satisfy this property.

No.Similarly S_1 also does not satisfy this property because if we consider the path $S_1 \rightarrow S_2 \rightarrow S_3$ then $(AXAX_p)$ is not true.

Yes.S_2 satisfies the property $(AXAX_p)$ because of the path $S_2 \rightarrow S_3 \rightarrow S_2$.But it does not satisfy (AX_p).And $ff \rightarrow tt$ is tt.Hence S_2 satifies the formula.

No.S_3 satisfies the property (AX_p) because of the path $S_2 \rightarrow S_3 \rightarrow S_2$.But it does not satisfy $(AXAX_p)$.As $tt \rightarrow ff$ is ff, therefore S_3 does not satisfy the formula.

b) $EFEG_q$

Yes,because the statement says that there exists a path from S_0 to a state from where it is possible to reach a state that satisfies q and every other state after that on that path will also satisfy q.From the diagram we can find the path $S_0 \rightarrow S_2 \rightarrow S_3 \rightarrow S_2 \rightarrow S_3$... which satisfies the property.

Yes, because there is a path $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_2 \rightarrow S_3$... which satisfies the property.

Yes, because there is a path $S_2 \rightarrow S_3 \rightarrow S_2 \rightarrow S_3$... which satisfies the property.

Yes, because there is a path $S_3 \rightarrow S_2 \rightarrow S_3 \rightarrow S_2$... which satisfies the property.

c) $(AFAG(p \wedge q)) \rightarrow (AGAF(p \wedge q))$

Yes.For state S_0 $(AFAG(p \wedge q))$ is not true because the formula $(AFAG(p \wedge q))$ means the system will eventually reach a state which holds $(p \wedge q)$ and it will continue to hold the property.From S_0 there is no such which holds it.$(AGAF(p \wedge q))$

means $(p \wedge q)$ will hold infinitely often.But if from S_1 system continues to loop at S_1 then the formula will not hold.As $ff \rightarrow ff$ is tt.Hence the overall formula will hold.

Yes.Similarly for s_1 both left and right side of the formula turns out to be false.Hence the overall formula is True.

Yes.For S_2,$(AFAG(p \wedge q))$ is false.But $(AGAF(p \wedge q))$ is true because the property $(p \wedge q)$ holds often on the path $S_2, S_3, S_2, S_3, ...$As $ff \rightarrow tt$ is tt,the overall property holds.

Yes.This is similar as S_2.The left hand side is false,but the right hand side holds for the path $S_2, S_3, S_2, S_3, ...$Hence the formula holds for S_3.

d) $AGAF(\neg p \vee \neg q)$

Yes.This property means it is always possible to reach a state from where condition $(\neg p \vee \neg q)$ always hold or it never enters a state which satisfies either p or q.S_0 satisfies this property because there are two possible paths $S_0 \rightarrow S_1 \rightarrow S_1...$ and $S_0 \rightarrow S_1 \rightarrow S_2 \rightarrow S3....$Both of this holds $(\neg p \vee \neg q)$.Hence S_0 satisfies the property.

Yes, because the path $S_1, S_1, S_1, ...$ will satisfy $\neg q$ and the path $S_1, S_2, S_3, S_2, S_3, ...$ will satisfy $\neg p$.

Yes,because the path $S_2, S_3, S_2, S_3...$ will satisfy $\neg p$.

Yes,because the path $S_3, S_2, S_3, S_2, S_3...$ will satisfy $\neg p$.

3. a) $M, s \models AX^2\phi$

$\Leftrightarrow \forall$ path $\pi = (p_0 = S, p_1, p_2, p_3..), M, p_2 \models \phi$

$\Leftrightarrow \forall$ path $\pi = (p_0 = S, p_1, p_2, ..), M, p_1 \models AX_\phi$

$\Leftrightarrow M, s \models AXAX_\phi$

Therefore $AX^2 \equiv AXAX_\phi$

b) $M, s \models EX^2\phi$

$\Leftrightarrow \exists$ a path $\pi = (p_0 = S, p_1, p_2, p_3..), M, p_2 \models \phi$

$\Leftrightarrow \exists$ a path $\pi = (p_0 = S, p_1, p_2, ..), M, p_1 \models EX_\phi$

$\Leftrightarrow M, s \models EXEX_\phi$

Therefore $EX^2 \equiv EXEX_\phi$

4. To prove or disprove whether one formula is stronger than other let us first try to simplify those formulas.

$M, s \models AFAX\phi$

$\Leftrightarrow \forall \pi = (p_0 = s, p_1, p_2, ...), M, p_i \models AX\phi$ for any i > 0

$\Leftrightarrow \forall \pi = (p_0 = s, p_1, p_2, ...), \forall i, \pi' = (p_0', p_1', p_2', ...), M, p_1' \models \phi$

$\Leftrightarrow \exists i \forall \pi = (p_0 = s, p_1, p_2, ...), M, p_i \models \phi$

$\Leftrightarrow M, s \models AF\phi$

Now from property 4.8 of chapter 4 notes we know that $M, s \models AXAF\phi \equiv M, s \models AF\phi$.

Therefore we can conclude that none of these two formulas are stronger than each other but they are equivalent.i.e $M, s \models AFAX\phi \equiv M, s \models AXAF\phi$

5. a) A function $f : 2^S \rightarrow 2^S$ is monotonic if $X \subseteq Y \rightarrow f(X) \subseteq f(Y), \forall X, Y \subseteq S$.

   Now, $U_Z(X) = X \cup Z$ and $U_Z(Y) = Y \cup Z$.As $X \subseteq Y$, hence $(X \cup Z) \subseteq (Y \cup Z)$.

   Which means, $U_Z(X) \subseteq U_Z(Y)$. Therefore U_Z is monotonic.

   b) According to Tarski-Knaster theorem we know that if a function f is monotonic over 2^S, where |S|=n, then $f^n(\phi)$ is the least fixed point of f.

   Here we have, $U_Z(\phi) = \phi \cup Z = Z$

   $U_Z(Z) = Z \cup Z = Z$

   Hence the least fixed point is Z.

   c) According to Tarski-Knaster theorem we know that if a function f is monotonic over 2^S, where |S|=n, then $f^n(S)$ is the greatest fixed point of f.

   Here we have, $U_Z(S) = S \cup Z = S$

   Hence the greatest fixed point is S.