Advanced Data Analysis HW4

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1.

A national insurance organization wanted to study the consumption patter on cigarettes in all 50 states and the District of Columbia. The variables chosen for the study are

Variable	Definition
Age	Median age of the a person living in a state
HS	Percentage of people over 25 years of age in a state who had completed high school
Income	Per capita personal income for a state in dollars
Black	Percentage of blacks living in a state
Female	Percentage of females living in a state
Price	Weighted average price (in cents) of a pack of cigarettes in a state
Sales	Number of packs of cigarettes sold in a state on a per capita basis

(a)

What would you expect the relationship between Sales and each of the explanatory variables to be? explain.

Answer:

I expect there's a positive correlation between sales of cigarettes and Age, because the people with higher ages tend to smoke more cigarettes, the proportion of people smoking within young people is relatively small.
I expect there's a negative correlation between sales of cigarettes and HS, because people with higher education are more aware of the harm of smoking and are tend to somke less.
I expect there's a positive correlation between sales of cigarettes and Income, because with more money, people can afford more cigarettes.
I don't expect there's a correlation between sales of cigarettes and Black, because there's no supporting evidence that black people are mode likely to smoke more than others.
I expect there's a negative correlation between sales of cigarettes and Female, because there are a lot more men smoking than women, the higher the proportion of the women, the less the proportion of people smoking.
I expect there's a negative correlation between sales of cigarettes and Price, the less the price is, the more people can afford the cigarettes.

(b)

Compute the pairwise correlation coefficient matrix and construct the corresponding scatter plot matrix

Answer:

```
> data = read.table("DATACIGARETTE.txt", header = TRUE)
> mat.data <- data.matrix(data[,2:8])
> cor((mat.data))
```

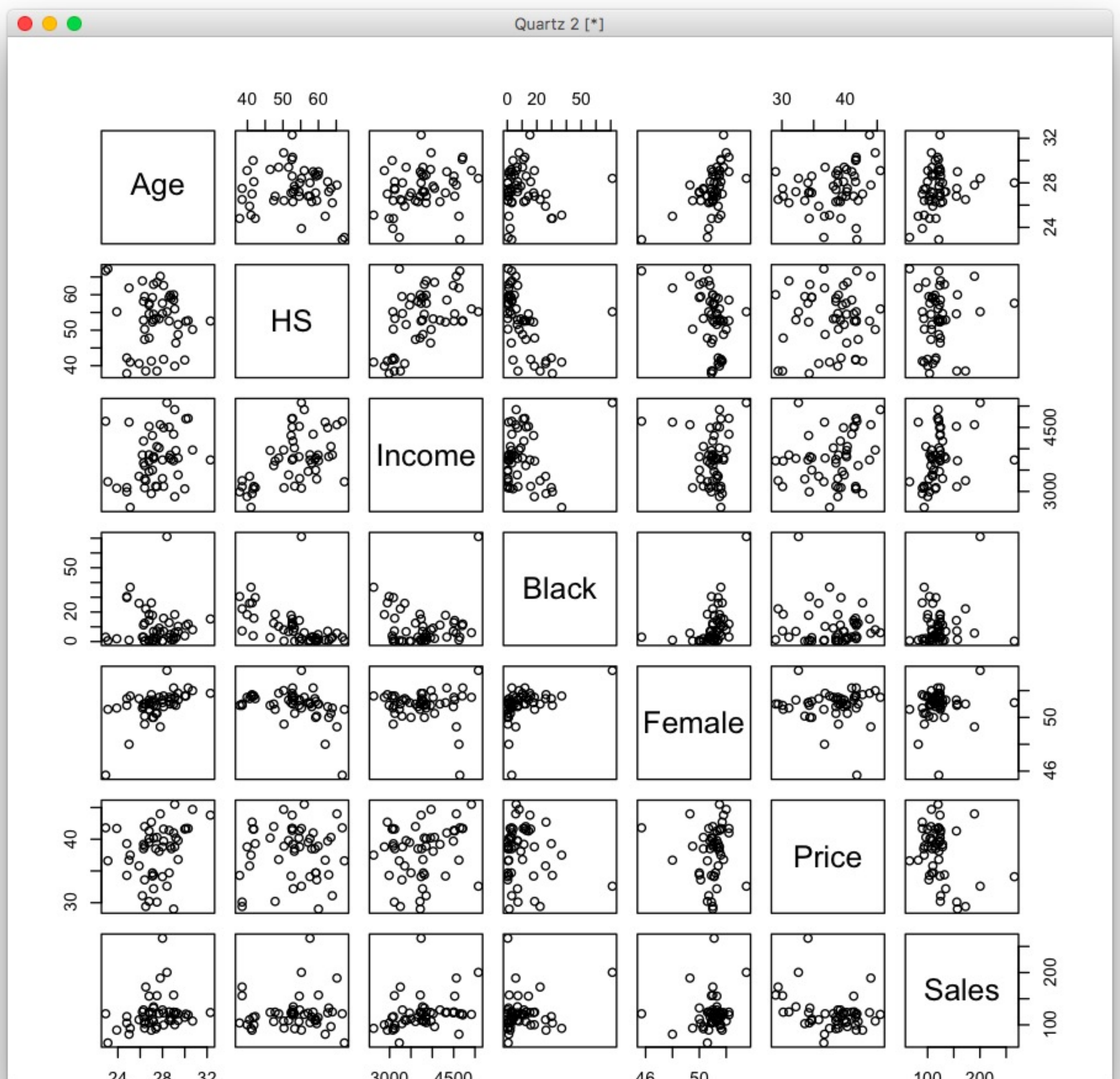
so the pairwise correlation coefficient matrix is:

```
          Age          HS          Income      Black        Female      Price       Sales
Age  1.00000000  -0.09891626  0.25658098  -0.04033021  0.55303189  0.24775673
  0.22655492
```

```
 HS   -0.09891626 1.00000000  0.53400534  -0.50171191 -0.41737794 0.05697473
   0.06669476
 Income  0.25658098  0.53400534  1.00000000  0.01728756  -0.06882666 0.21455717
   0.32606789
 Black -0.04033021 -0.50171191 0.01728756  1.00000000  0.45089974  -0.14777619
   0.18959037
 Female  0.55303189  -0.41737794 -0.06882666 0.45089974  1.00000000  0.02247351
   0.14622124
 Price 0.24775673  0.05697473  0.21455717  -0.14777619 0.02247351  1.00000000
   -0.30062263
 Sales 0.22655492  0.06669476  0.32606789  0.18959037  0.14622124  -0.30062263
   1.00000000
```

```
> pairs(data[,2:8])
```

The corresponding scatter plot is:



(c)

Obtain the six variance inflation factors. What do these results suggest about the effect of multicollinearity?

Answer:

```
> results = lm(Sales ~ Age+HS+Income+Black+Female+Price , data=data)
# install.packages('car')
> library(car)
> vif(results)
```

The VIF for 6 variables are:

```
  Age        HS     Income     Black    Female     Price
2.300617  2.676465  2.325164  2.392152  2.406417  1.142181
```

(d)

Are the there any outlying Sales observations in the regres- sion model relating Sales to the six predictors?

Answer:

Yes, there are outlying Sales observations in the regression model relating Sales to the six predictors.

```
> r = rstudent(results)
> data[abs(r)>3,]
```

```
   State  Age HS    Income   Black  Female   Price  Sales
29  NV   27.8   65.2   4563   5.7  49.3   44.0   189.5
30  NH   28.0   57.6   3737   0.3  51.1   34.1   265.7
```

As we can see from the result, NV and NH are the two states that have outlying Sales observations in the regression model relating Sales to the six predictors.

(e)

Obtain the leverages (the diagonal elements of the hat matrix). Are the there any outlying states in the six predictors?

Answer:

```
> results = lm(Sales ~ Age+HS+Income+Black+Female+Price , data=data)
> lev = hat(model.matrix(results))
> lev
```

```
  0.148834504030601 0.580160351053579 0.0496985364502016 0.13709423363853
   0.0601062187168606 0.130138939805311 0.174671432351856 0.109964259157845
   0.719712840922186 0.2984807799694 0.0992462576906176 0.216903455841289
   0.092060813754798 0.078138923121058 0.0863031077553889 0.0600834204236368
   0.0600531614367214 0.229271714316211 0.147216983914844 0.0592277100717082
   0.082438148474407 0.111768500306865 0.0788600537461464 0.0595424855799236
   0.220268293959589 0.0538126742130819 0.07535749988127 0.0713061951148198
   0.171156408722539 0.0663450574293963 0.112543589203342 0.205163482615724
   0.125210794051503 0.189612219012305 0.0977942068782977 0.0427023659576899
   0.0747312407280888 0.194824611487451 0.110848283936785 0.107728709447137
   0.157387827636549 0.0702636797470909 0.0861059621640753 0.073209300335295
   0.310573704010913 0.0646432821046025 0.121378178796484 0.06441386857486
   0.139413729014416 0.0386706972156815 0.0845573052310269
```

According to the common rule we will flag any observation whose leverage value satisfies

$$h_{ii} > \frac{2p}{n} = \frac{12}{51}$$

```
> data[which(lev>12/51),1]
> lev[lev>12/51]
```

```
AK DC FL UT
 0.580160351053579  0.719712840922186  0.2984807799694  0.310573704010913
```

Thus, according to the rule above, we get 4 states who have high leverage: AK, DC, FL, UT.

(f)

Are there any influential points?

Answer:

Typically, points with Cook's distance greater than 1 are classified as being influential, so we calculate Cook's distance for every state's data:

```
> cook = cooks.distance(results)
> cook
```

Then we get the 51 Cook's distance:

```
 0.000872312260768373  0.11046568368098  0.000231406438616766  0.00533644985289951
     0.00087184813943891  0.00487422897991362  0.000903689085754166  0.0247623942663158
     0.272772360871267  0.00031718427842994  0.00228486844474409  0.149108047215374
     0.00577305260600313  0.000515725336002152  0.000235292422912618  0.0022195814166927
     0.0011742764662855  0.0293108038740074  0.0091907941594545  0.0046197884422297
     0.00853949173434342  5.43666855436729e-05  0.000993543307494363  0.000416583654710988
     0.00357717450931961  0.00181158830548335  0.00222167629931825  0.0106704252779454
     0.226884733564416  0.243073931655456  0.00865807771956428  0.00862556323250348
     0.0128750489379433  0.0514726373485391  0.000694147592466878  0.000198786969647856
     0.000926126973805015  0.000632210801320141  0.0023179250199760 8  3.5406858359337e-07
     0.00559710600247828  0.0021868920733631  0.000167678660476866  0.000276633855805063
     0.0861896935150744  0.00441253690931487  0.0144781318896138  0.00554156138259945
     0.00241683043673791  0.000216940322227759  6.00182901861485e-05
```

No one is greater than 1, so there's no influential points according to the rule of Cook's distance.

(g)

Use log(Sales) instead of Sales and repeat questions d), e) and f).

Answer:

```
> results = lm(log(Sales) ~ Age+HS+Income+Black+Female+Price, data=data)
> r = rstudent(results)
> data[abs(r)>3,]
```

```
   State  Age  HS   Income   Black  Female   Price  Sales
29  NV   27.8   65.2   4563   5.7  49.3   44.0   189.5
30  NH   28.0   57.6   3737   0.3  51.1   34.1   265.7
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As we can see from the result, NV and NH are the two states that have outlying Sales observations in the regression model relating Sales to the six predictors.

```
> lev = hat(model.matrix(results))
> lev
```

```
   0.148834504030601  0.580160351053579  0.0496985364502016  0.13709423363853
     0.0601062187168606  0.130138939805311  0.174671432351856  0.109964259157845
     0.719712840922186  0.2984807799694  0.0992462576906176  0.216903455841289
     0.092060813754798  0.078138923121058  0.0863031077553889  0.0600834204236368
     0.0600531614367214  0.229271714316211  0.147216983914844  0.0592277100717082
     0.082438148474407  0.111768500306865  0.0788600537461464  0.0595424855799236
     0.220268293959589  0.0538126742130819  0.07535749988127  0.0713061951148198
     0.171156408722539  0.0663450574293963  0.112543589203342  0.205163482615724
     0.125210794051503  0.189612219012305  0.0977942068782977  0.0427023659576899
```

```
    0.0747312407280888  0.194824611487451  0.110848283936785  0.107728709447137
    0.157387827636549  0.0702636797470909  0.0861059621640753  0.073209300335295
    0.310573704010913  0.0646432821046025  0.121378178796484  0.06441386857486
    0.139413729014416  0.0386706972156815  0.0845573052310269
```

According to the common rule we will flag any observation whose leverage value satisfies

$$h_{ii} > \frac{2p}{n} = \frac{12}{51}$$

```
> data[which(lev>12/51),1]
> lev[lev>12/51]
```

```
AK DC FL UT
0.580160351053579  0.719712840922186  0.2984807799694  0.310573704010913
```

Thus, according to the rule above, we get 4 states who have high leverage: AK, DC, FL, UT. Typically, points with Cook's distance greater than 1 are classified as being influential, so we calculate Cook's distance for every state's data:

```
> cook = cooks.distance(results)
> cook
```

Then we get the 51 Cook's distance:

```
0.00359045513224516  0.240240855616072  0.00121318532616218  0.00940000348486041
    0.000293993996183841  0.000845445364141786  0.00157627217231795  0.0336654337125128
    0.14134926250704  4.17359595508581e-08  0.00203727701223271  0.270758042867878
    0.00458499785548776  0.000451889520957869  9.31888737750166e-06  0.00184184619189831
    0.000560973161470141  0.0319246707978281  0.0204097590400288  0.00921415105332014
    0.00810736995024783  7.12278615229615e-05  0.00254520116778151  0.0002981214100366
    0.00317957208142376  0.00184483532854467  0.00116873114507587  0.0108404640748746
    0.242062092944229  0.161220045757967  0.0123781360184546  0.0103388197355609
    0.0176912765990876  0.0540788613509783  0.00205060932036119  4.12952664005199e-05
    0.000879330434682632  0.0030802159952135  0.00441873438600284  8.57636871488496e-05
    0.005679380081201  0.00398597075671328  0.000654080555535082  0.00040384707347726
    0.238154414273449  0.00956732233083818  0.0132426216290245  0.00700748304948474
    0.00221365370129141  0.000195642965139178  0.00154531793916012
```

No one is greater than 1, so there's no influential points according to the rule of Cook's distance.

2.

Suppose that North American Oil Company is attempting to develop a regular gasoline that will deliver improved gasoline mileage. As part of its development process, the company would like to study the effect of two qualitative factors on the gasoline mileage obtained by an automobile called the Fire-Hawk. These factors are regular gasoline type (which has levels A, B and C) and gasoline additive type (which has levels M, N, O and P). To carry out the study, the company test drove three Fire-Hawks using each treatment. However upon completion of the experiment the company found that several Fire-Hawks have not been driven under the proper test conditions. Rather than running more tests, the company decided (because of limited time) to analyze the data that had remained after the data for improperly tested Fire-Hawks was dropped from the data set. The remaining data are in Oildata.cvs. Let

$$yij, k = \text{the } k\text{th gasoline mileage obtained when using regular gasoline type } i \text{ and additive type } j$$

A reasonable model to use for this data is

$$y_{ijk} = \mu + \alpha_B D_{i,B} + \alpha_C D_{i,C} + \beta_N D_{j,N} + \beta_O D_{j,O} + \beta_P D_{j,P} + \epsilon_{ij,k}$$

where

$D_{i,B} = 1$ if $i = B$, that is, if we are using gasoline type B and 0 otherwise
$D_{i,C} = 1$ if $i = C$, that is, if we are using gasoline type C and 0 otherwise
$D_{i,N} = 1$ if $i = N$, that is, if we are using additive type N and 0 otherwise
$D_{i,O} = 1$ if $i = O$, that is, if we are using additive type O and 0 otherwise
$D_{i,P} = 1$ if $i = P$, that is, if we are using additive type P and 0 otherwise

(a)

To compare the effects of the regular gasoline type, we need to test $H_0 : \alpha_B = \alpha_C = 0$ versus H_a: at least one of α_B or α_C does not equal zero.

Answer:

In order to test the hypothesis, we have a full model:

$$y_{ijk} = \mu + \alpha_B D_{i,B} + \alpha_C D_{i,C} + \beta_N D_{j,N} + \beta_O D_{j,O} + \beta_P D_{j,P} + \epsilon_{ij,k}$$

and a reduced model:

$$y_{ijk} = \mu + \beta_N D_{j,N} + \beta_O D_{j,O} + \beta_P D_{j,P} + \epsilon_{ij,k}$$

We will reject the Null Hypothesis if the two models are different:

```
> data = read.csv("Oildata.csv")
> colnames(data)
> full = lm(Mileage ~ factor(Rgasoline) + factor(GasolineAd), data = data)
> reduce1 = lm(Mileage ~ factor(GasolineAd), data = data)
> anova(reduce1, full)
```

```
Res.Df  RSS       Df  Sum of Sq  F          Pr(>F)
37      3496.295  NA  NA         NA         NA
35      3466.153  2   30.14234   0.1521834  0.859396
```

Since the p-value of the test is $0.859396 > \alpha = 0.05$, we cannot reject the Null Hypothesis that $\alpha_B = \alpha_C = 0$

(b)

To compare the effects of the gasoline additive type, we need to test $H_0 : \beta_N = \beta_O = \beta_P = 0$ versus H_a: at least one of $\beta_N, \beta_O, \beta_P$ does not equal zero.

Answer:

In order to test the hypothesis, we have a full model:

$$y_{ijk} = \mu + \alpha_B D_{i,B} + \alpha_C D_{i,C} + \beta_N D_{j,N} + \beta_O D_{j,O} + \beta_P D_{j,P} + \epsilon_{ij,k}$$

and a reduced model:

$$y_{ijk} = \mu + \alpha_B D_{i,B} + \alpha_C D_{i,C} + \epsilon_{ij,k}$$

We will reject the Null Hypothesis if the two models are different:

```
 > full = lm(Mileage ~ factor(Rgasoline) + factor(GasolineAd), data = data)
 > reduce2 = lm(Mileage ~ factor(Rgasoline), data = data)
 > anova(reduce2, full)
```

```
Res.Df RSS       Df  Sum of Sq  F          Pr(>F)
38     3592.151  NA  NA         NA         NA
35     3466.153  3   125.9986   0.4240966  0.736912
```

Since the p-value of the test is $0.736912 > \alpha = 0.05$, we cannot reject the Null Hypothesis that $\beta_N = \beta_O = \beta_P = 0$