HW 3

1. (4pt) consider a regression model with p predictors, that is,

$$Y_i = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_p X_{pi} + \epsilon_i, i = 1, 2, \ldots, n$$

   (a) (2pt) Show that

$$F = \frac{n-p-1}{p} \frac{R^2}{1-R^2}$$

   (b) (2pt) If $n = 20, p = 3, R^2 = 0.572$ test $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ against H_a : at least one of them is not zero.

2. (12pt) Comp-U-Systems, a computer manufacturer, sells and services the Comp-Y-Systems Microcomputers. Let

   x_i = the number of microcomputer serviced on the ith service call

   y_i = the number of minutes required to perform service on the ith service call

   the data is in file CompUSys.csv. Suppose the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i, i = 1, 2, \ldots, n$, is used to model the relationship between the number of minutes required to perform a service and the number of microcomputers serviced.

   (a) (2pt) Estimate β_0 and β_1 using the least square method. Interpret the estimate of β_1.

   (b) (2pt) Use a 95% confidence interval to estimate β_1. Interpret your result

   (c) (2pt) Estimate the average time it will take to serve 6 microcomputer using a 95% confidence interval. Interpret your result.

   (d) (2pt) Compute a 95% prediction interval for the amount of time it will take to service 6 microcomputers. Interpret your result.

   (e) (2pt) Use the Bonferroni method and to find a joint confidence intervals for the mean amounts of time it will take to serve 6 and 7 microcomputers.

   (f) (2pt) Test

$$\begin{aligned} H_0 &: E(Y|X = x) = \beta_0 + \beta_1 x \\ H_a &: \text{Not} \quad H_0 \end{aligned}$$

   Using $\alpha = 0.05$.

3. (4pt) International Oil Inc. Is attempting to a develop a reasonably priced minimum unleaded gasoline that will deliver higher gasoline mileage than can be achieved by its current premium unleaded gasolines. As part of its development process, International Oil Inc. wishes to study the effect of one qualitative variable, x_1, premium gasoline unleaded type (A, B, C) and one quantitative variable x_2 amount of gasoline additive VST (0, 1, 2, 3 units) on the gasoline mileage y obtained by an automobile called Encore. For testing purposes a sample of 22 Encores is randomly selected and driven under normal driving conditions. The combination of x_1 and x_2 used in the experiment along with the corresponding values of y are in file mileage.csv. Define $\mu_{[A,x]}, \mu_{[A,x]}$ and $\mu_{[B,x]}$ to be the mean unleaded gasoline mileage by Encore when using AST amount x and premium unleaded gasoline types A, B and C, respectively. Consider the model

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 x_2 + \epsilon_i$$

where $D_{1i} = 1$ gas type is B and 0 otherwise and $D_{2i} = 1$ is gas type is C and 0 otherwise.

   (a) (2pt) Estimate the $beta_i$s and interpret your result (see note for how to fit this model)

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   lm(y ~ factor(x1) + x2)
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   (b) (2pt) Test $H_0 : \beta_1 = \beta_2 = 0$ against H_a : Not H_0 using $\alpha = 0.05$.