Ec142, Spring 2017

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Problem Set 1

Due: February 17th, 2017

Projections

Problem sets are due at 5PM in the GSIs mailbox. You may work in groups, but each student should turn in their own write-up (including a printout of a narrated/commented and executed iPython Notebook). Please also e-mail a copy of any iPython Notebook to the GSI (if applicable).

1. Let X and Y be two mean zero random variables with finite variance (i.e., $\mathbb{E}[X] = \mathbb{E}[Y] = 0$, $\mathbb{E}[X^2] < \infty$ and $\mathbb{E}[Y^2] < \infty$). Let the population correlation coefficient be

$$\rho = \frac{\mathbb{C}(X,Y)}{\sqrt{\mathbb{V}(X)}\sqrt{\mathbb{V}(Y)}}.$$

   Using the Cauchy-Schwarz inequality to prove that $0 \leq \rho^2 \leq 1$.

2. Prove Pythagoras' Theorem.

Production/cost function: theory

Consider a population of firms. A firm with capital k and labor l produces output

$$y(k,l;A) = Ak^{\alpha}l^{\beta}. \tag{1}$$

Here A is firm-specific, capturing heterogeneity in the efficiency with which different firms are able to transform capital and labor into output. Let W denote the wage rate, and R the rental price of a unit of capital, faced by the firm. We assume that a firm seeking to produce output y, while facing input prices r and w, does so in a *cost-minimizing* way; choosing $K(y,r,w;A)$ and $L(y,r,w;A)$ to solve the constrained minimization problem

$$\min_{k,l} rK + wl + \lambda\left[y - Ak^{\alpha}l^{\beta}\right]. \tag{2}$$

1. The *derived demand* for capital and labor equals the amount of each input a firm would choose when seeking to produce y while facing input prices r and w. *Show* these

derived demand schedules, under (2), equal:

$$K(y,r,w;A)=\alpha\left(\frac{y}{A}\right)^{\frac{1}{\eta}}\left(\frac{w}{r}\right)^{\frac{\beta}{\eta}}\left[\alpha^{\alpha}\beta^{\beta}\right]^{-\frac{1}{\eta}} \tag{3}$$

$$L(y,r,w;A)=\beta\left(\frac{y}{A}\right)^{\frac{1}{\eta}}\left(\frac{r}{w}\right)^{\frac{\alpha}{\eta}}\left[\alpha^{\alpha}\beta^{\beta}\right]^{-\frac{1}{\eta}}, \tag{4}$$

where $\eta=\alpha+\beta$. Briefly interpret the two demand schedules [5 to 10 sentences]. Why is η called the returns-to-scale parameter?

2. The cost function equals

$$c(y,r,w;A)=rK(y,r,w;A)+wL(y,r,w;A). \tag{5}$$

Show that under (1) that this function equals

$$c(y,r,w;A)=\eta\left(\frac{y}{A}\right)^{\frac{1}{\eta}}r^{\frac{\alpha}{\eta}}w^{\frac{\beta}{\eta}}\left[\alpha^{\alpha}\beta^{\beta}\right]^{-\frac{1}{\eta}}. \tag{6}$$

Show that (6) is homogenous of degree one in input prices. Provide an economic explanation for this [4 to 6 sentences]? Show that the *cost shares* of capital and labor are, respectively α/η and β/η.

3. Let $i=1,\ldots,N$ index a random sample of firms. For each firm we observe output, Y_i, the input prices W_i and R_i, and total costs C_i. We do not observe the firm-specific productivity parameter, A_i. We assume that firm behavior is governed by (1), (3), (4) and (6). Imposing the restriction that (6) is homogenous of degree one in input prices and taking logs yields

$$\ln C_i-\ln W_i=\kappa_c+\frac{1}{\eta}\ln Y_i+\frac{\alpha}{\eta}\left[\ln R_i-\ln W_i\right]-\frac{1}{\eta}\left(\ln A_i-\mathbb{E}\left[\ln A_i\right]\right)$$

for $\kappa_c=\ln\left[\eta w^{\frac{\beta}{\eta}}\left[\alpha^{\alpha}\beta^{\beta}\right]^{-\frac{1}{\eta}}\right]+\frac{1}{\eta}\mathbb{E}\left[\ln A_i\right]$. Consider the linear regression of log costs onto the (logs of) output and rents minus wages:

$$\mathbb{E}^*\left[\ln C_i-\ln W_i\,\middle|\,\ln Y_i,\ln R_i-\ln W_i\right]=k_0+c_0\ln Y_i+a_0\left[\ln R_i-\ln W_i\right]. \tag{7}$$

Is it likely that $c_0=1/\eta$ and $a_0=\alpha/\eta$? Explain (mathematical calculations are not required; 6 to 12 sentences). So far our analysis has been silent regarding how firms choose their output level. In answering this question it might be helpful to consider two cases. In one case firms *do not* choose their output level (consider an electric utility that must meet demand at regulated prices). In the second case firms *do* choose their output level.

Production/cost function: application

Read the paper "Returns to scale in electricity supply" by Marc Nerlove made available on bCourses (Nerlove, 1963). The dataset printed in the article is available in machine readable form online at `http://people.stern.nyu.edu/wgreene/Text/tables/TableF14-2.txt`. Note that only the first 145 observations in the online archive correspond to the data used by Nerlove. This is the extract you should use in what follows.

1. Compute the least squares fit of the (log of) electricity/fuel prices onto a constant and the logs of wages/fuel prices and rents/fuel prices ratios. Interpret your findings in light of the theoretical analysis completed above (i.e., provide a theoretical interpretation of each coefficient estimate). Construct a table summarizing your results and briefly describe them in a few paragraphs.

2. Compute, and report in a table, the average and standard deviation of the three input cost shares across the 145 utilities in your sample. How do these averages compare with the cost share estimates associated with your analysis in question 1 above?

3. Assume that production continues to be Cobb-Douglas in capital, labor and fuel, but that the input elasticities, α_i, β_i and γ_i may now vary across firms. Let P_i denote the price of fuel faced by firm i. Based on your theoretical analysis above, show that

$$\frac{C_i \left[\frac{\alpha_i}{\eta_i}\right]^{\frac{\alpha_i}{\eta_i}} \left[\frac{\beta_i}{\eta_i}\right]^{\frac{\beta_i}{\eta_i}} \left[\frac{\gamma_i}{\eta_i}\right]^{\frac{\gamma_i}{\eta_i}}}{R_i^{\frac{\alpha_i}{\eta_i}} W_i^{\frac{\beta_i}{\eta_i}} P_i^{\frac{\gamma_i}{\eta_i}}} = \eta \left(\frac{Y_i}{A_i}\right)^{\frac{1}{\eta_i}}.$$

Construct a measure of the expression to the left of the equality for each of the 145 firms in your sample. Call this variable NC_i – for "normalized" costs. Show that

$$\ln NC_i = d + \bar{\eta}^{-1} \ln Y_i + U_i$$

for $d = \mathbb{E}\left[\frac{1}{\eta_i} \ln A_i\right] + \ln \eta$, $\bar{\eta}^{-1} = \mathbb{E}\left[\frac{1}{\eta_i}\right]$ and

$$U_i = \left(\eta_i^{-1} - \bar{\eta}^{-1}\right) \ln Y_i - \left(\frac{1}{\eta_i} \ln A_i - \mathbb{E}\left[\frac{1}{\eta_i} \ln A_i\right]\right).$$

Compute the least squares fit of the (log) of NC_i onto a constant and the (log of) output. Does the coefficient on log output recover $\bar{\eta}^{-1}$ (for a sample size large enough)? Discuss [5 to 10 sentences]. Hint: examine the necessary and sufficient conditions associated with the linear predictor projection associated with this least squares fit.

4. Construct a table summarizing your results and briefly describe them in a few paragraphs. Please compare your results with those found in question 1 above. Construct a scatterplot of Y_i and NC_i. Include your least squares fit in the plot as well as a 90 percent confidence band for the fit.

5. Using your analysis in question 1 above, propose and construct an estimate of A_i for each firm. Plot a histogram of these estimates. Discuss the measured productivity differences across the utilities in your sample. Can you think of any policy implications of your findings?

6. Compute the least squares fit of the logs of output onto those of capital, labor and fuel. Construct a table summarizing your results and briefly describe them in a few paragraphs. Relate your least squares coefficient estimates to the Cobb-Douglas production function parameters as well as your analysis in questions 1 and 2 above.

References

Nerlove, M. (1963). *Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, chapter Returns to scale in electricity supply, (pp. 167 – 198). Stanford University Press: Stanford.