Bruce Campell NCSU ST 534 HW 2

Probems 1.15, 1.20, 1.27, 2.3

Shumway, Robert H.; Stoffer, David S. Time Series Analysis and Its Applications: With R Examples (Springer Texts in Statistics)

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1.15

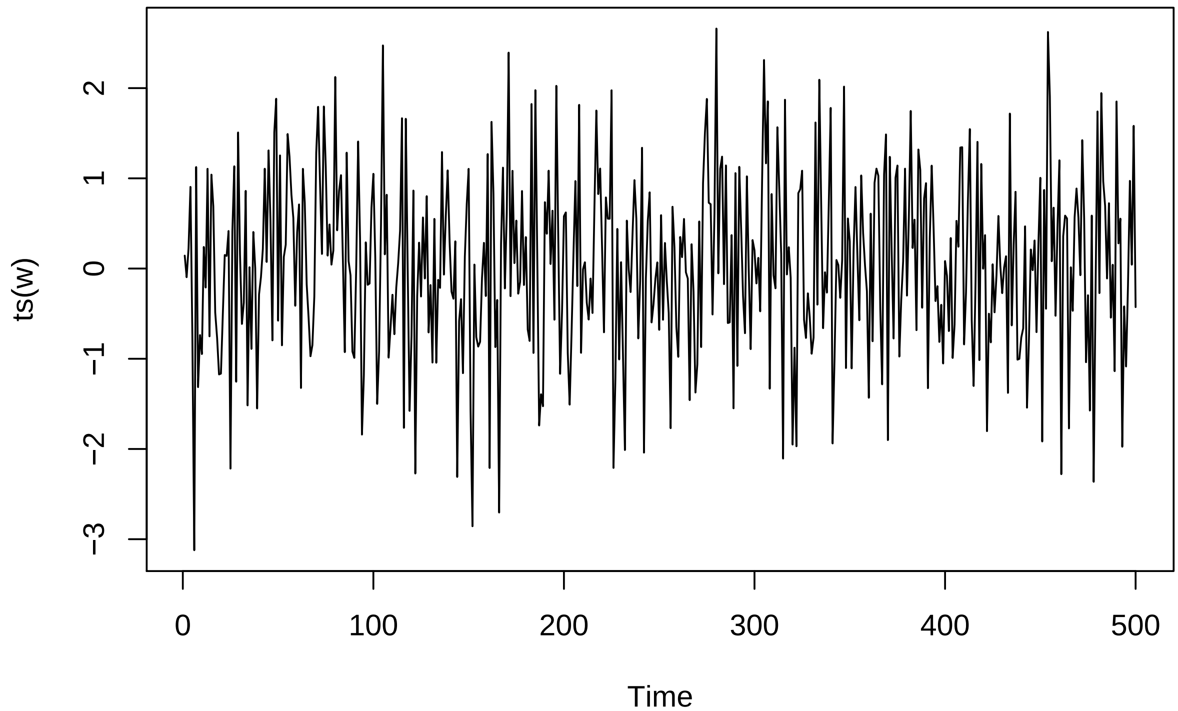
Let w_t, for $t = 0, \pm 1, \pm 2, ...$ be a normal white noise process, and consider the series $x_t = w_t w_{t???1}$. Determine the mean and autocovariance function of x_t, and state whether it is stationary.

1.20

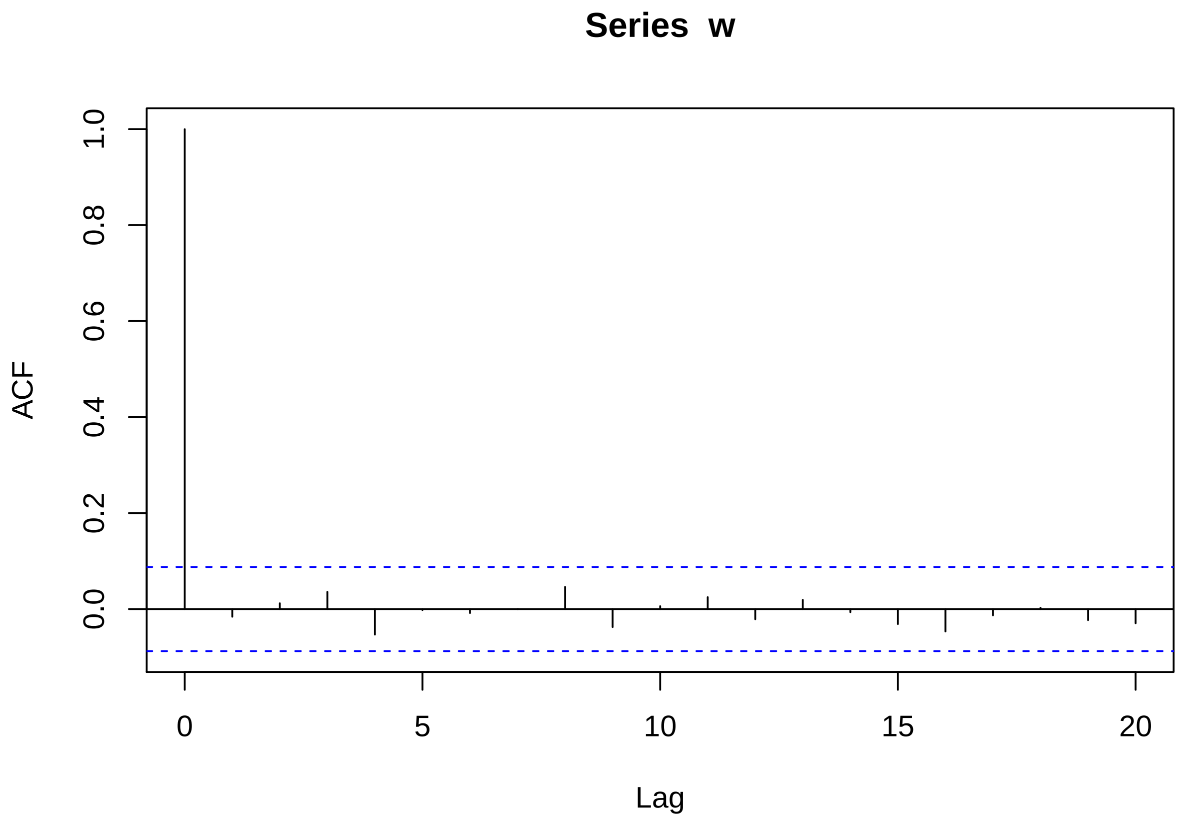
(a) Simulate a series of $n = 500$ Gaussian white noise observations as in Example 1.8 and compute the sample ACF, $\hat{\rho}(h)$, to lag 20. Compare the sample ACF you obtain to the actual ACF, $\rho(h)$.

In the plot the dotted line is at $\pm \frac{z_{(0.05)}}{\sqrt{n}}$, this is the level $\alpha = 0.05$ Wald test for the hypothesis $H_0 : acf(i) = 0$.

```
w = rnorm(500, 0, 1)
plot(ts(w))
```



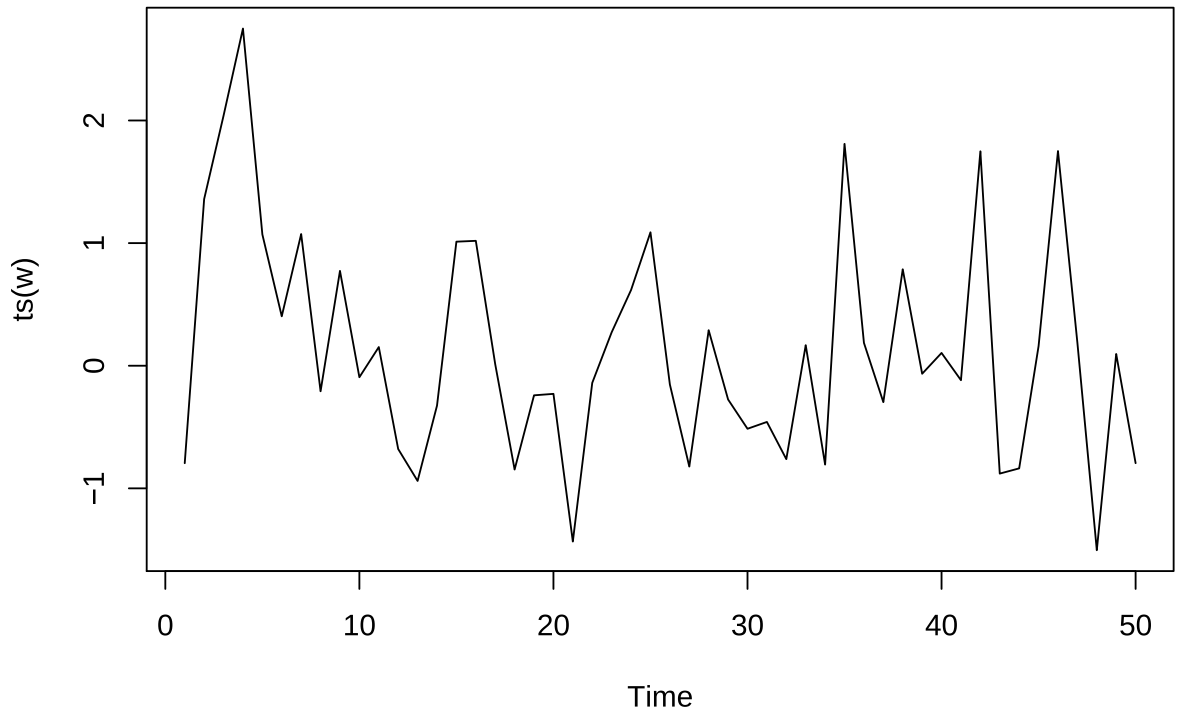

```r
acf(w, lag.max = 20)
```



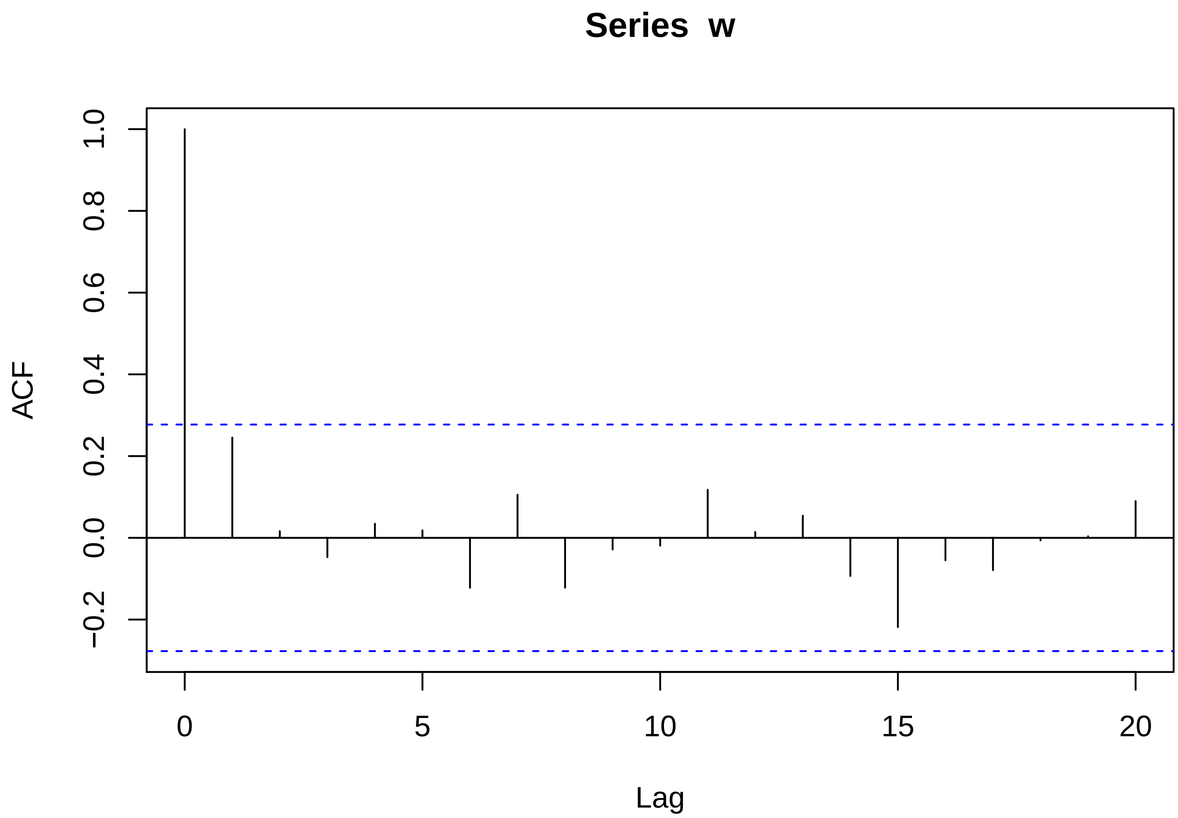
The true acf is zero for all $i \neq 0$ Most of non-zero the entries in the empirical acf from the simulated data are within the bounds of $\pm\frac{z_{(0.05)}}{\sqrt{n}}$.

(b) Repeat part (a) using only n = 50. How does changing n affect the results?

```
w = rnorm(50, 0, 1)
plot(ts(w))
```

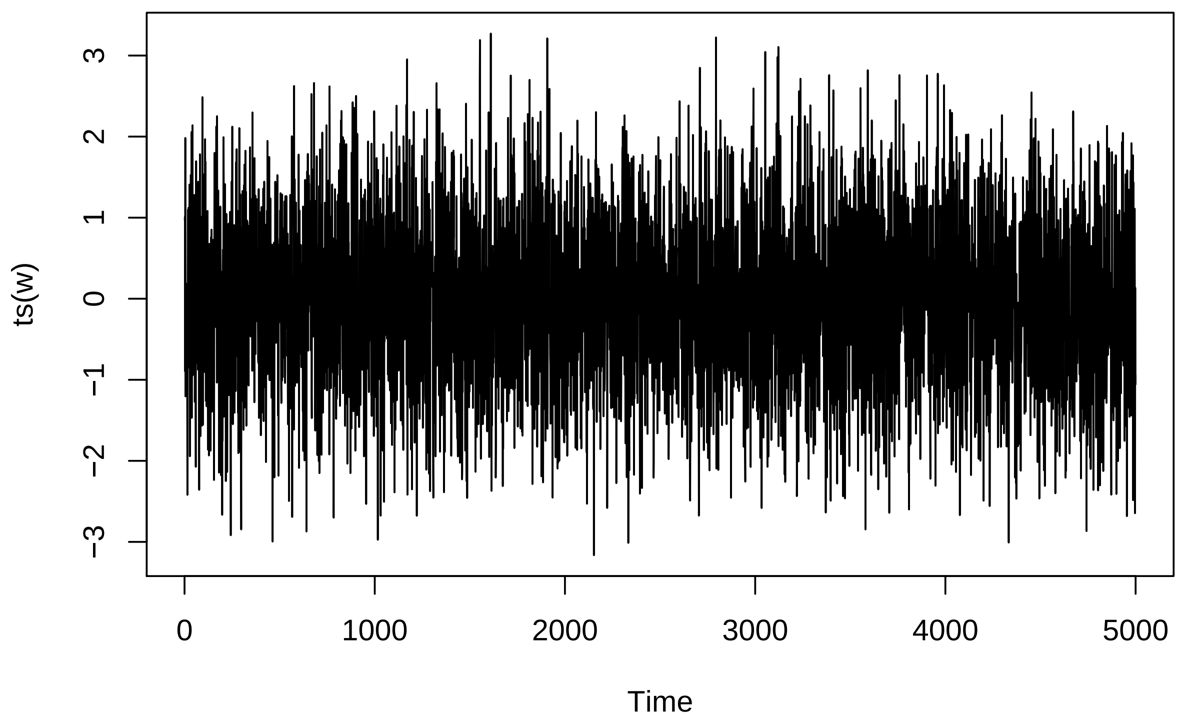



```r
acf(w, lag.max = 20)
```

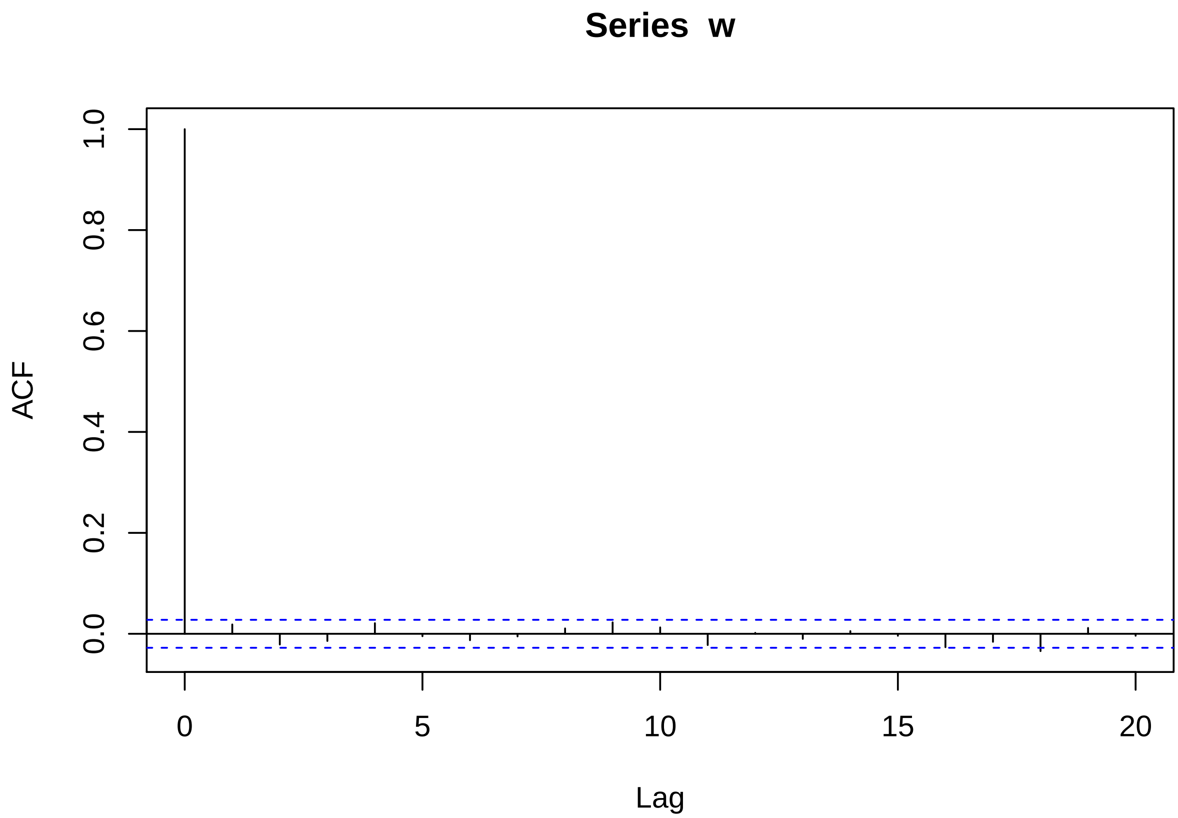


Again, most of non-zero the entries are within the bounds of $\pm\frac{z_{(0.05)}}{\sqrt{n}}$. Note that because n is smaller the scale of $\pm\frac{z_{(0.05)}}{\sqrt{n}}$ is larger and generally the acf values fr $i \neq 0$ are increased. If we do the same experiment for 5000 samples we'll see this effect - in the other direction - more dramatically.

```
w = rnorm(5000, 0, 1)
plot(ts(w))
```




```r
acf(w, lag.max = 20)
```



Since we have more samples, there's more evidence to provide in assesing the correlation between x_t and x_s, and the entries are closer to the true values.

1.27

A concept used in geostatistics, see Journel and Huijbregts (1978) or Cressie (1993), is that of the variogram, defined for a spatial process x_s, $s = (s1, s2)$, for $s1, s2 = 0, \pm 1, \pm 2, ...$, as $V_x(h) = \frac{1}{2}E[(x_s + h???x_s)^2]$, where $h = (h1, h2)$, for $h1, h2 = 0, \pm 1, \pm 2, ...$ Show that, for a stationary process, the variogram and autocovariance functions can be related through $V_x(h) = \gamma(0)???\gamma(h)$, where $\gamma(h)$ is the usual $lagh$ covariance function and $0 = (0, 0)$. Note the easy extension to any spatial dimension.

2.3

Repeat the following exercise six times and then discuss the results. Generate a random walk with drift, (1.4), of length $n = 100$ with $\delta = .01$ and $sigma_w = 1$. Call the data x_t for $t = 1, ..., 100$. Fit the regression $x_t = \beta t + w_t$ using least squares. Plot the data, the mean function (i.e., ȶ$_t = .01t$) and the fitted line, $\hat{x}_t = \hat{\beta}t$, on the same graph. Discuss your results.

```
par(mfcol = c(3, 2))  #set up graphics
for (i in 1:6) {
    x = ts(cumsum(rnorm(10000, 0.01, 1)))  #the data
    reg = lm(x ~ 0 + time(x), na.action = NULL)  #the regression
    plot(x)  #plot data
    lines(0.01 * time(x), col = "red", lty = "dashed")  #plot mean
    abline(reg, col = "blue")
}  #plot regression line
```