# Bruce Campell NCSU ST 534 HW 5

Problems 4.13, 4.16, 4.18*, and 4.27 from CH4

*Shumway, Robert H.; Stoffer, David S. Time Series Analysis and Its Applications: With R Examples (Springer Texts in Statistics)*

*19 November, 2017*

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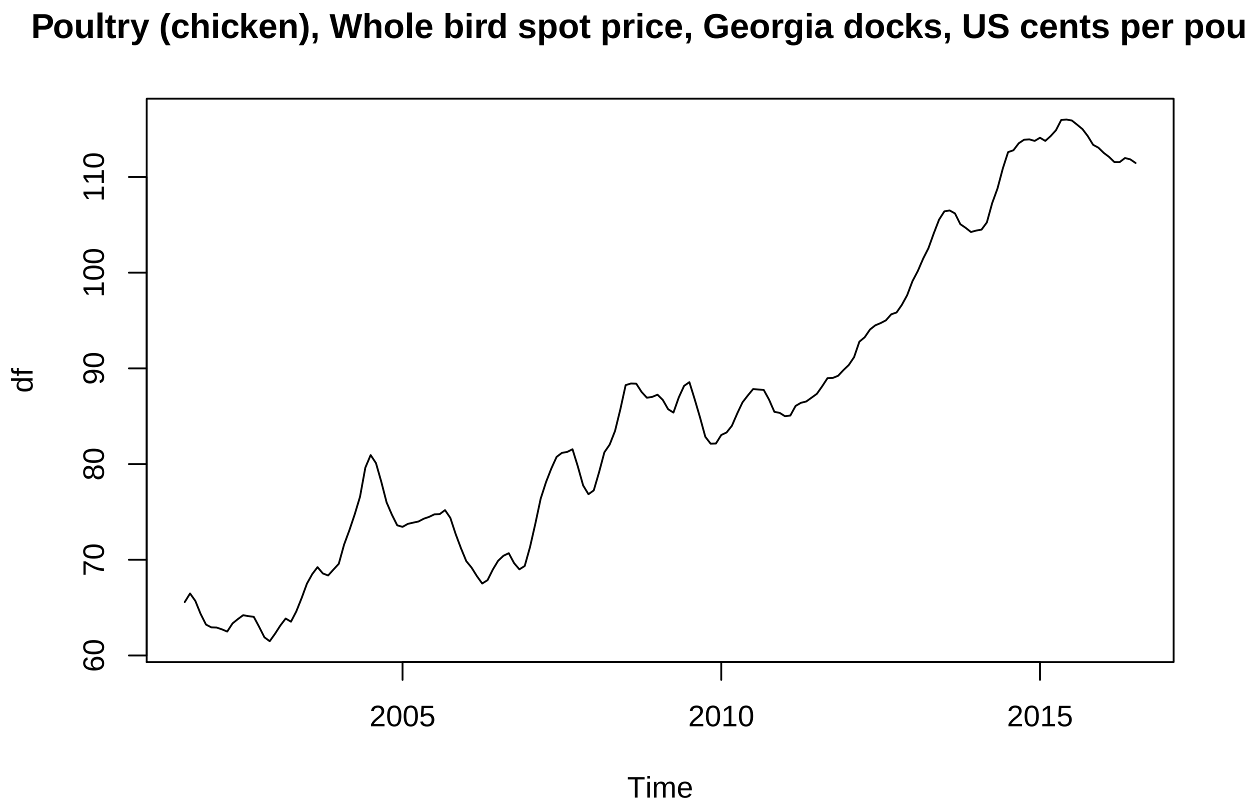
### 4.13 chiken price data.

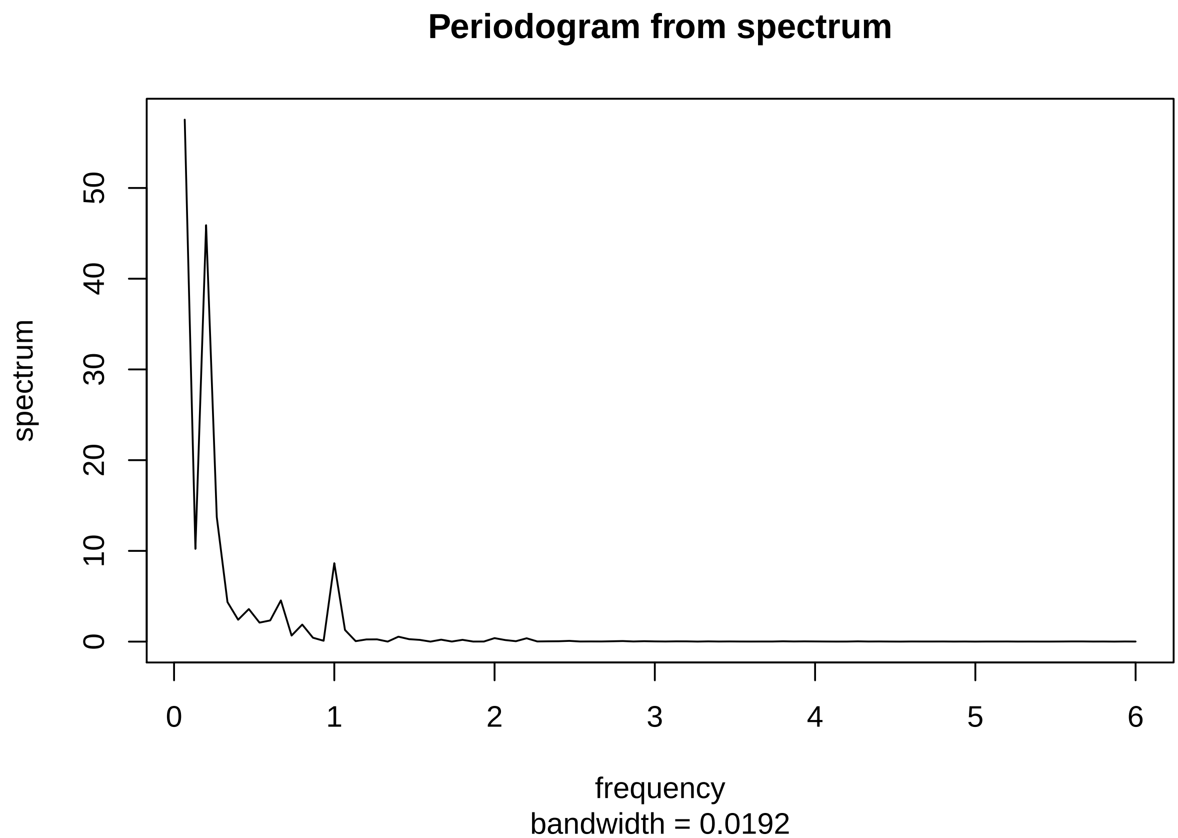
Analyze the chicken price data (chicken) using a nonparametric spectral estimation procedure. Aside from the obvious annual cycle discovered in Example 2.5, what other interesting cycles are revealed?

```
rm(list = ls())
data(chicken, package = "astsa")
df <- chicken
plot(df, main = "Poultry (chicken), Whole bird spot price, Georgia docks, US cents per p
```




Let's calculate the periodogram. There should not be a need to detrend the data since we expect the mean to be represented by the spectrum at $\omega = 0$.

```
df.periodogram.spectrum <- spectrum(df, taper = 0, log = "no", main = "Periodogram from
```




```r
mean(df)
```

```
## [1] 85.67372
```

```r
df.periodogram.spectrum$spec[0]
```

```
## numeric(0)
```

```r
max.spec.loc <- which.max(df.periodogram.spectrum$spec)
max.spec <- df.periodogram.spectrum$spec[max.spec.loc]
max.spec.freq <- df.periodogram.spectrum$freq[max.spec.loc]

pander(data.frame(max.spec.freq = max.spec.freq, max.spec = max.spec), caption = "peak f
```

Table 1: peak from spectrum command We see a long term cycle around 15 months in periodicity There is another peak in the spectrum around around 1

| max.spec.freq | max.spec |
|---|---|
| 0.06667 | 57.53 |

```r
max.spec.loc <- which.max(df.periodogram.spectrum$spec)
max.spec <- df.periodogram.spectrum$spec[15]
max.spec.freq <- df.periodogram.spectrum$freq[15]

pander(data.frame(max.spec.freq = max.spec.freq, max.spec = max.spec), caption = "minor
```

Table 2: minor peak from spectrum command

| max.spec.freq | max.spec |
|---|---|
| 1 | 8.643 |

## 4.16 Cepstral Analysis.

The periodic behavior of a time series induced by echoes can also be observed in the spectrum of the series; this fact can be seen from the results stated in Problem 4.7. Using the notation of that problem, suppose we observe $x_t = s_t + As_{t-D} + n_t$, which implies the spectra satisfy $f_x(\omega) = [1 + A2 + 2Acos(2\pi\ \omega\ D)]f_s(\omega) + f_n(\omega)$. If the noise is negligible ($fn(\omega) \approx 0$) then $logf_x(\omega)$ is approximately the sum of a periodic component, $log[1+A2+2Acos(2\pi\ \omega\ D)]$, and $logf_s(\omega)$. Bogart et al. [27] proposed treating the detrended log spectrum as a pseudo time series and calculating its spectrum, or cepstrum, which should show a peak at a quefrency corresponding to 1/D. The cepstrum can be plotted as a function of quefrency, from which the delay D can be estimated. For the speech series presented in Example 1.3, estimate the pitch period using cepstral analysis as follows. The data are in speech.

### (a)

Calculate and display the log-periodogram of the data. Is the periodogram periodic,as predicted?

### (b)

Perform a cepstral (spectral) analysis on the detrended logged periodogram, and use the results to estimate the delay D. How does your answer compare with the analysis of Example 1.27, which was based on the ACF?

## 4.18 Consider two time series

$x_t = w_t - w_{t-1}$ , $y_t = \frac{1}{2}(w_t + w_{t-1})$, formed from the white noise series $w_t$ with variance $\sigma^2 = 1$.

### (a)

Are $x_t$ and $y_t$ jointly stationary? Recall the cross-covariance function must also be a function only of the lag $h$ and cannot depend on time.

### (b)

Compute the spectra $f_y(\omega)$ and $f_x(\omega)$, and comment on the difference between the two results.

### (c)

Suppose sample spectral estimators $\bar{f}_y(.10)$ are computed for the series using $L = 3$. Find a and b such that

$$P\{a \leq \bar{f}_y(0.10) \leq b\} = 0.90$$

This expression gives two points that will contain 90% of the sample spectral values. Put 5% of the area in each tail.

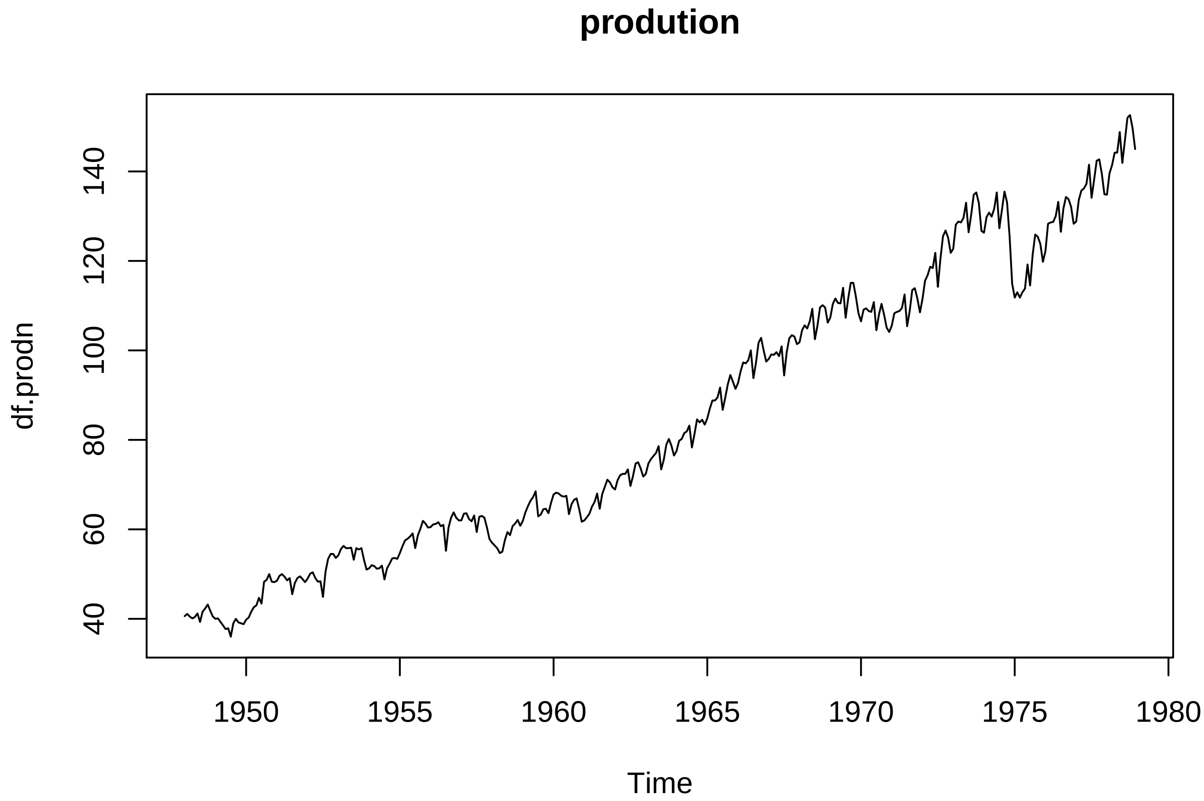
## 4.27 Production vs Unemployment Analysis

Consider the bivariate time series records containing monthly U.S. production (prodn) as measured by the Federal Reserve Board Production Index and the monthly unemployment series (unemp).

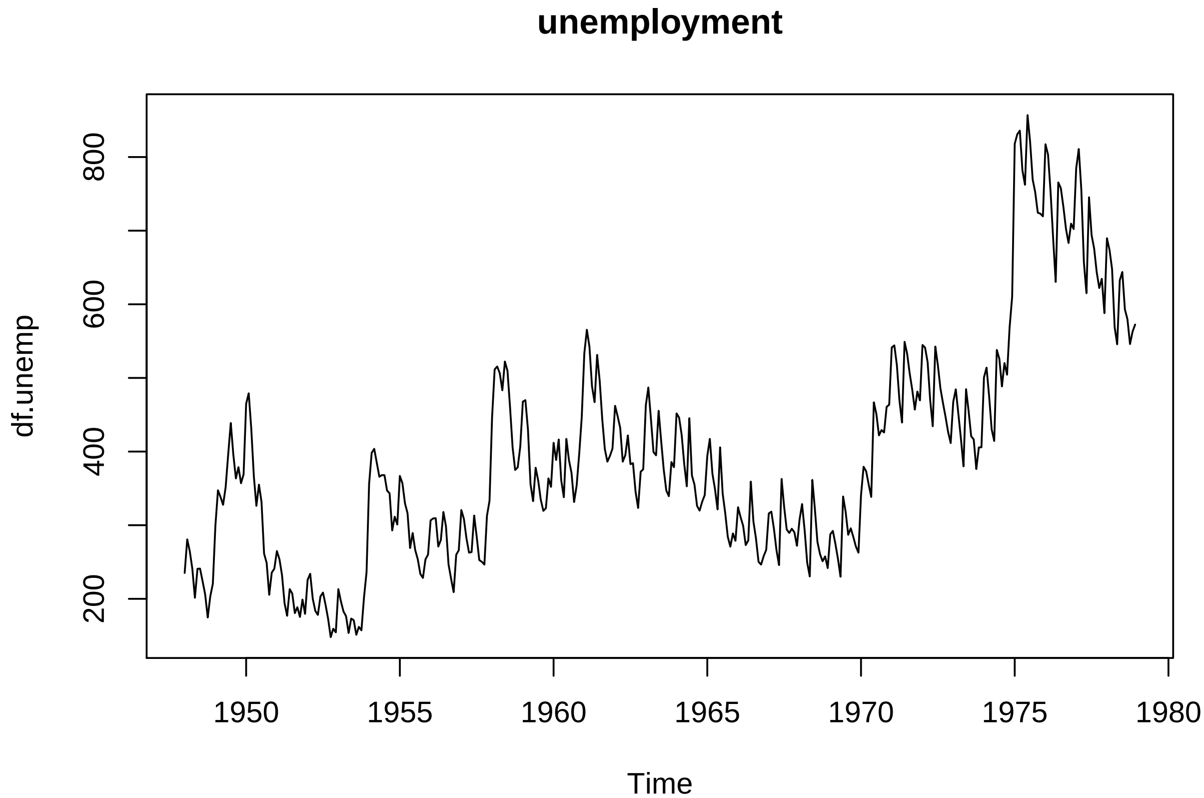
### (a)

Compute the spectrum and the log spectrum for each series, and identify statistically significant peaks. Explain what might be generating the peaks. Compute the coherence, and explain what is meant when a high coherence is observed at a particular frequency.

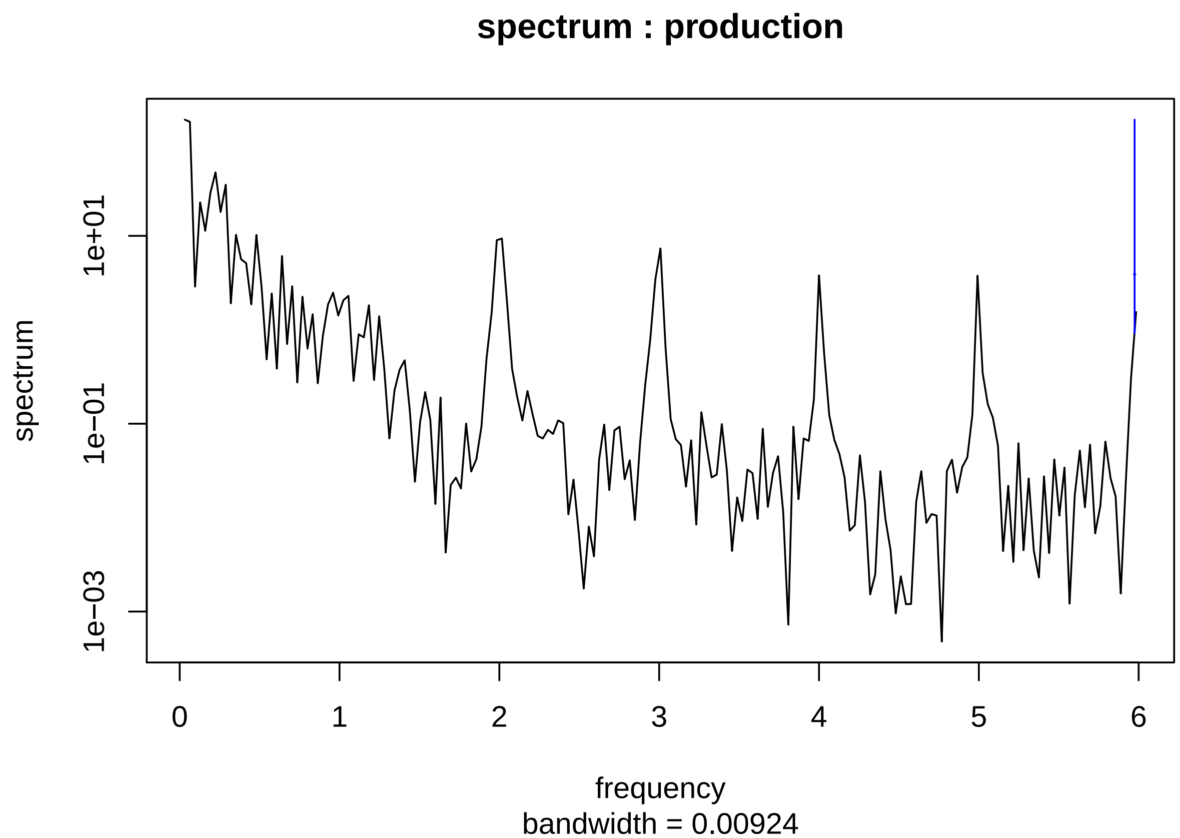
```
rm(list = ls())
library(astsa)
data(prodn, package = "astsa")
df.prodn <- prodn
data(unemp, package = "astsa")
df.unemp <- unemp
plot(df.prodn, main = "prodution")
```



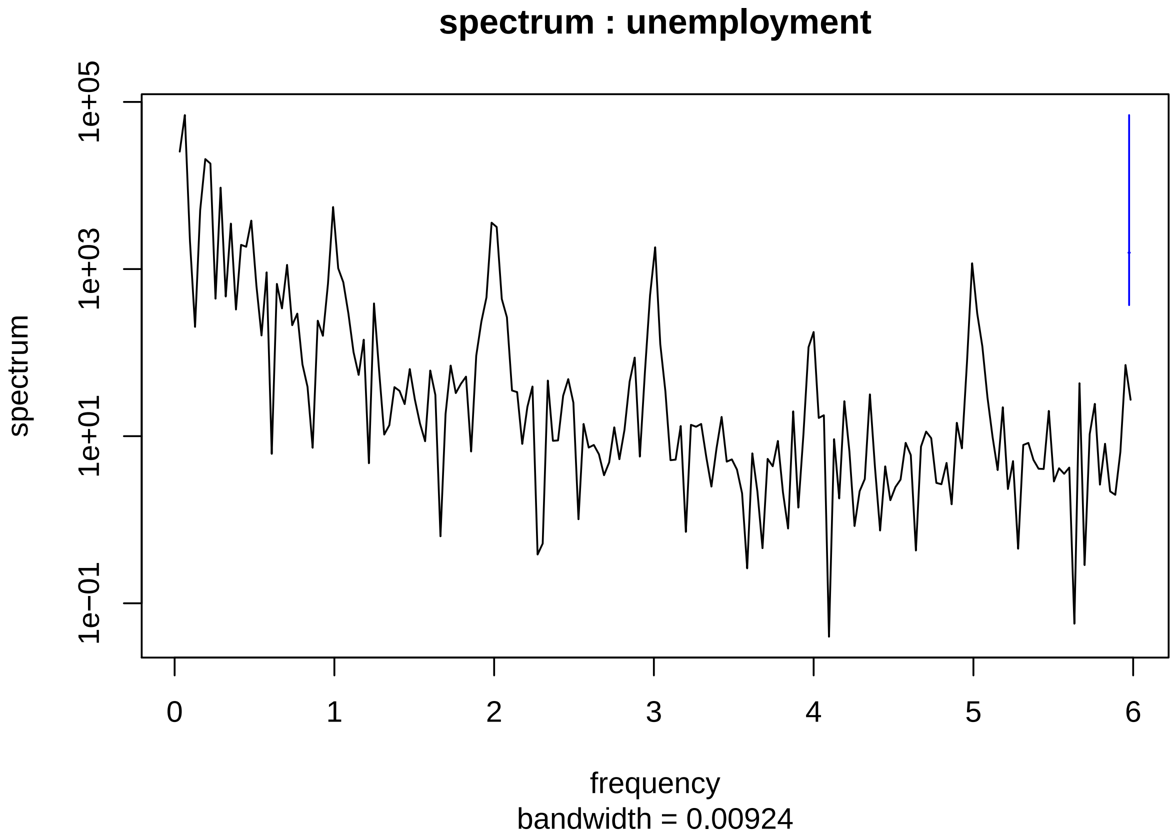

```r
plot(df.unemp, main = "unemployment")
```



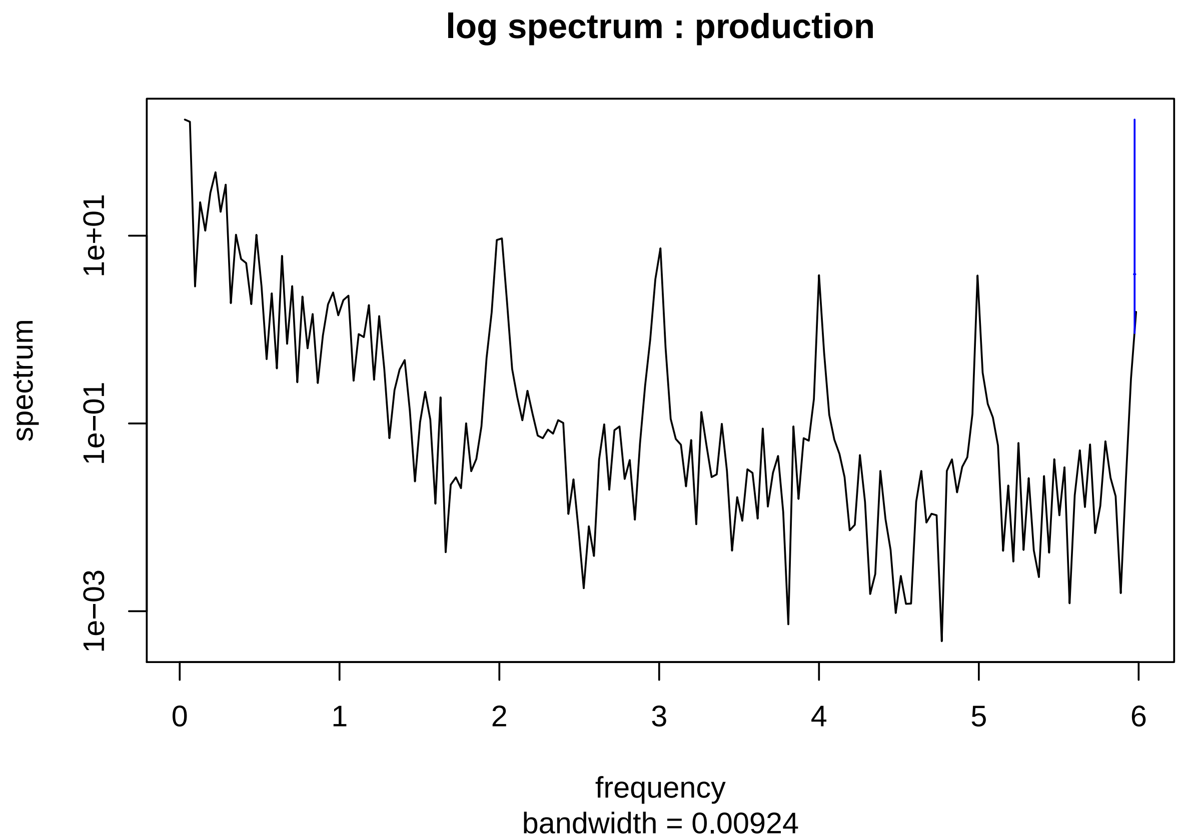

```
spec.prodn <- spectrum(df.prodn, main = "spectrum : production")
```



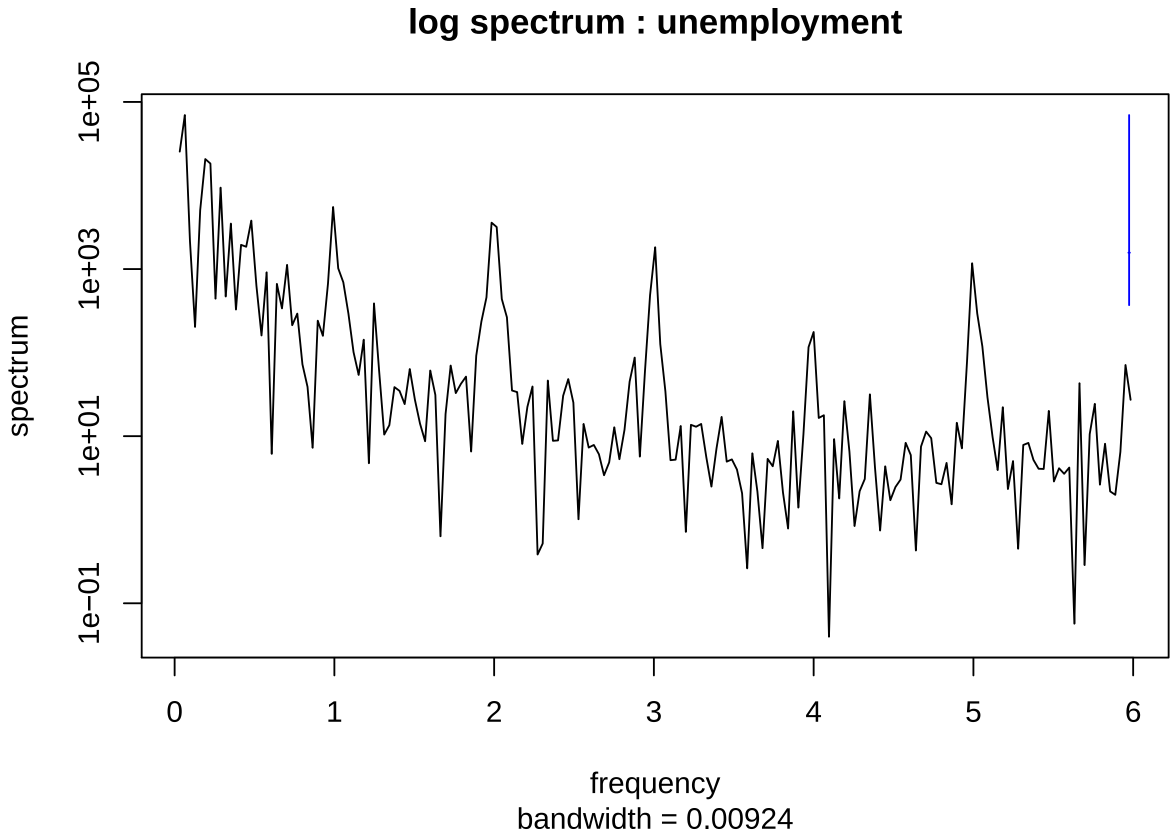

```
spec.unemp <- spectrum(df.unemp, main = "spectrum : unemployment")
```




```
spec.prodn <- spectrum(df.prodn, log = "yes", main = "log spectrum : production")
```




```
spec.unemp <- spectrum(df.unemp, log = "yes", main = "log spectrum : unemployment")
```

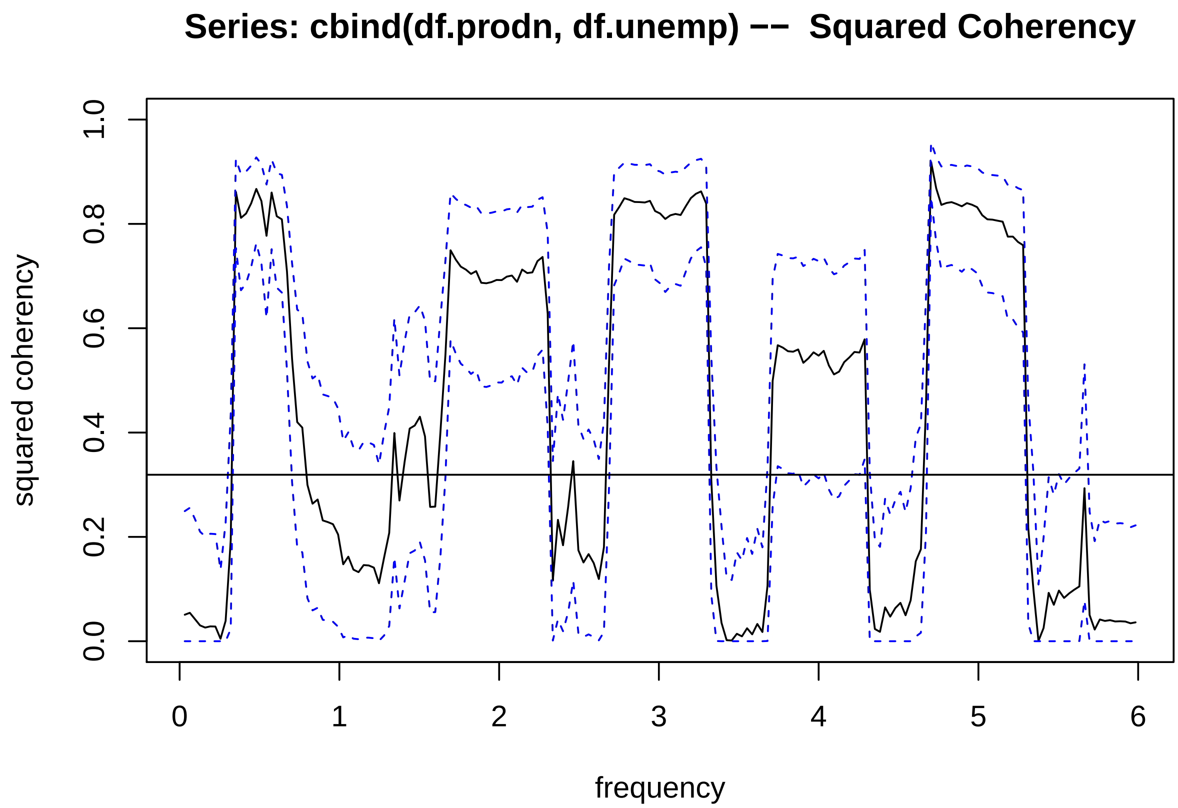



Economic cycles such as recession would be the sources of the strong peaks.

```
sr = spec.pgram(cbind(df.prodn, df.unemp), kernel("daniell", 9), taper = 0,
    plot = FALSE)
sr$df
```

```
## [1] 37.696
```

```
f = qf(0.999, 2, sr$df - 2)
C = f/(18 + f)
plot(sr, plot.type = "coh", ci.lty = 2)
abline(h = C)
```




The areas of large coherence represent frequency ranges where we may predict one series from another via a linear relationship such as a filter. If one series lags another at a particular frequency the phase of the two components are different. The series with the larger phase can be used to perdict the other. Keeping mind that we're talking about projecting the series onto the fourier basis function for a perticualr $\omega$ and using that.

The plot above displays coherence with the line indicating a significance level of $\alpha = 0.001$

### (b)

What would be the effect of applying the filter $u_t = x_t - x_{t-1}$ followed by $v_t = u_t - u_{t-12}$ to the series given above? Plot the predicted frequency responses of the simple difference filter and of the seasonal difference of the first difference.

In applying $u_t$ we'd be attenuating the low frequency components in the time series. Applying $v_t$ would attenuate the components around the sesaonal signal. If there we're a peak in the spectrum at the $\omega$ corresponding to a period 12, the filter $v_t$ would supress that. We can calculate the frequency response of the filter from

Using the notation from the book, and that $u_t$ has coefficients $a_0 = 1,\ a_1 = -1$ and all other $a_j = 0$ in the linear filter, we have

$$|A_{xu}(\omega)| = \sum_{j=-\infty}^{j=+\infty} a_j e^{-2\pi\omega i j} = (1 - e^{-2\pi\omega i})$$

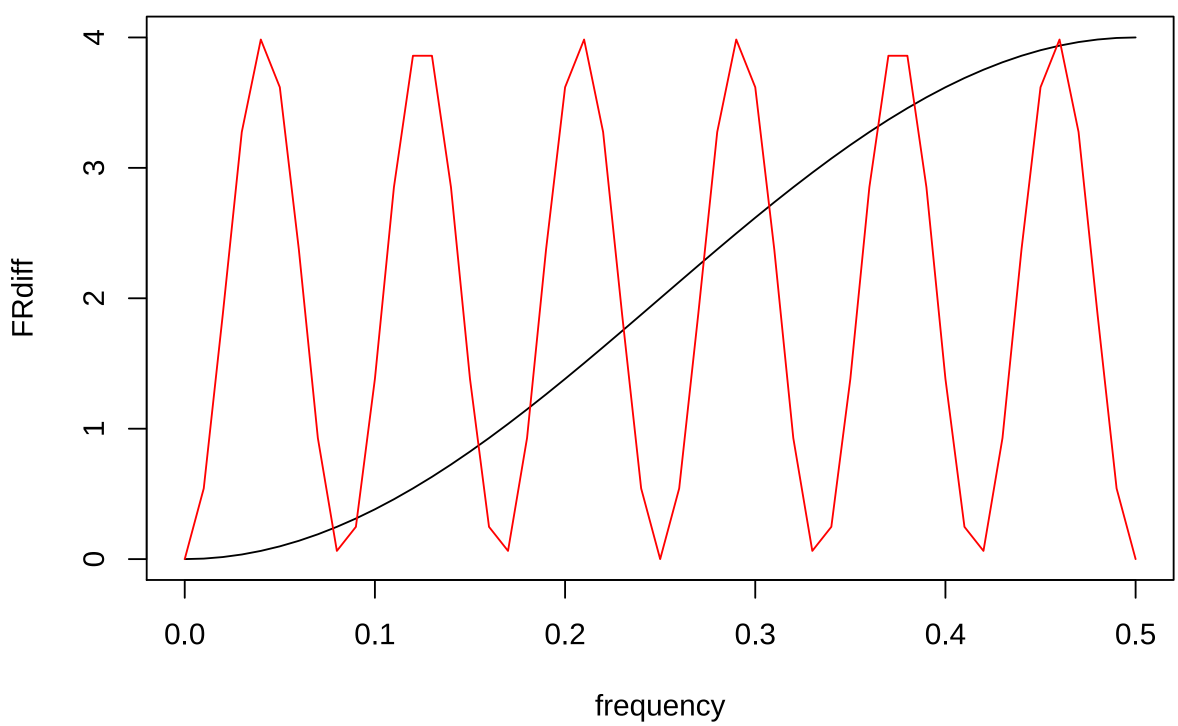
$$|A_{xu}(\omega)|^2 = (1 - e^{-2\pi\omega i}) \times (1 - e^{2\pi\omega i}) = 2(1 - cos(2\pi\omega))$$

Similar arguments for $\nu_t$ give

$$|A_{x\nu}(\omega)|^2 = 2(1 - cos(2\pi 12\omega))$$

The frequency responses are displayed below.

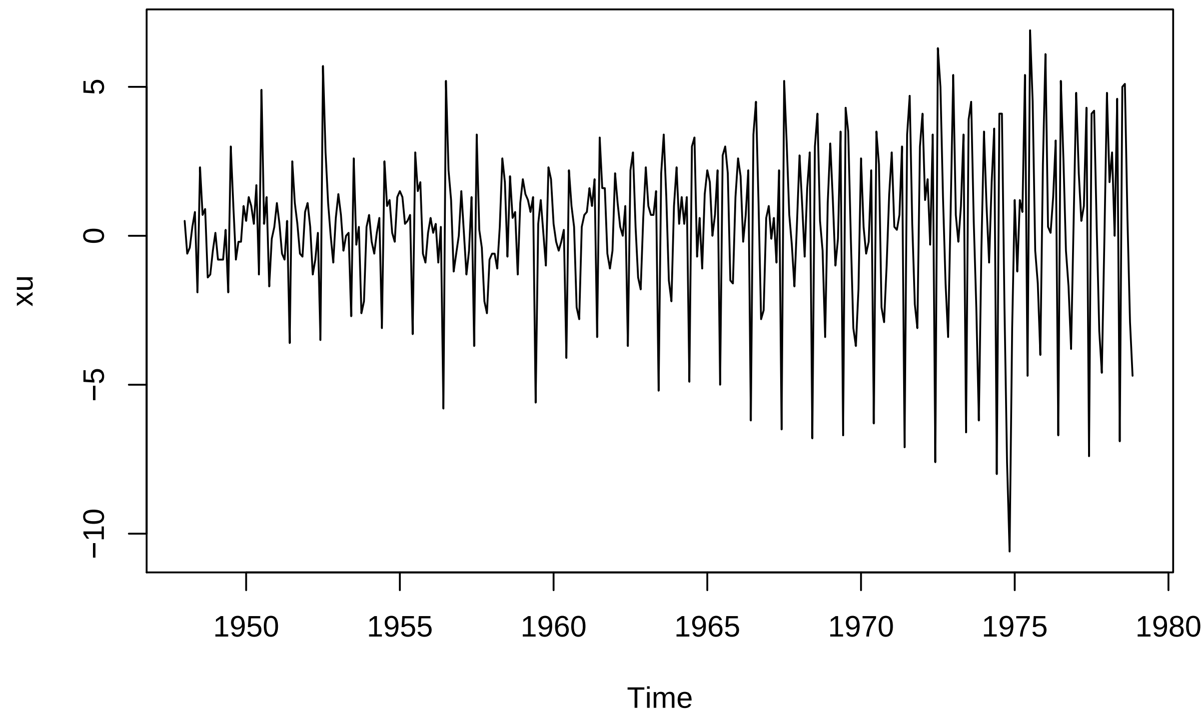
```
w = seq(0, 0.5, by = 0.01)
FRdiff = abs(1 - exp((0+2i) * pi * w))^2
FRdiff12 = abs(1 - exp((0+2i) * pi * 12 * w))^2
plot(w, FRdiff, col = 1, type = "l", xlab = "frequency")
lines(w, FRdiff12, col = 2)
```




**(c)**

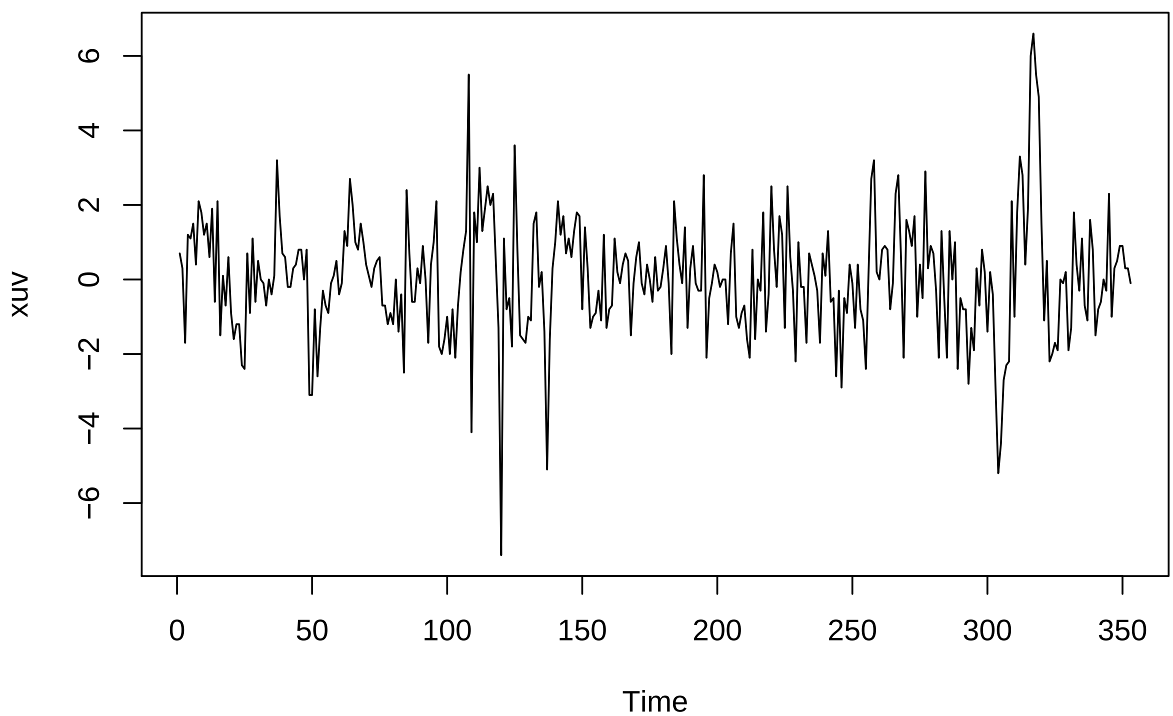
Apply the filters successively to one of the two series and plot the output. Examine the output after taking a first difference and comment on whether stationarity is a reasonable assumption. Why or why not? Plot after taking the seasonal difference of the first difference. What can be noticed about the output that is consistent with what you have predicted from the frequency response? Verify by computing the spectrum of the output after filtering

```
xu <- filter(df.prodn, filter = c(1, -1))
plot(xu)
```



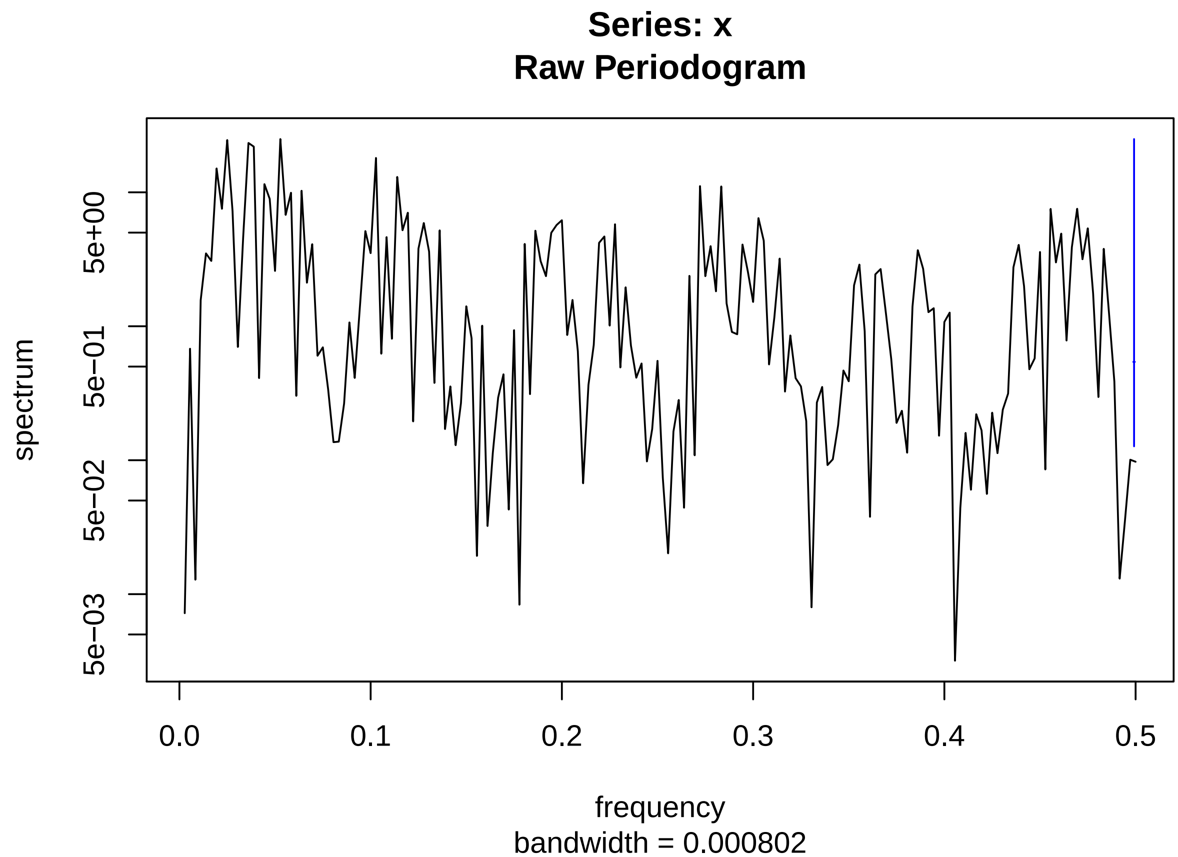

We see the variance is not stable - second order stationarity does not hold for this data. Now we filter the seasonal component.

```
xuv <- filter(xu, filter = c(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1))
xuv <- xuv[-(1:12)]
xuv <- ts(xuv[-(354:360)])
plot(xuv)
```




We seem to have removed the seasonal component of the series. Let's look at the spectrum to verify this.

```
xuv.spec <- spectrum(xuv)
```


Series: x
Raw Periodogram
spectrum
5e+00
5e−01
5e−02
5e−03
0.0
0.1
0.2
0.3
0.4
0.5
frequency
bandwidth = 0.000802