Time Series Analysis And Applications

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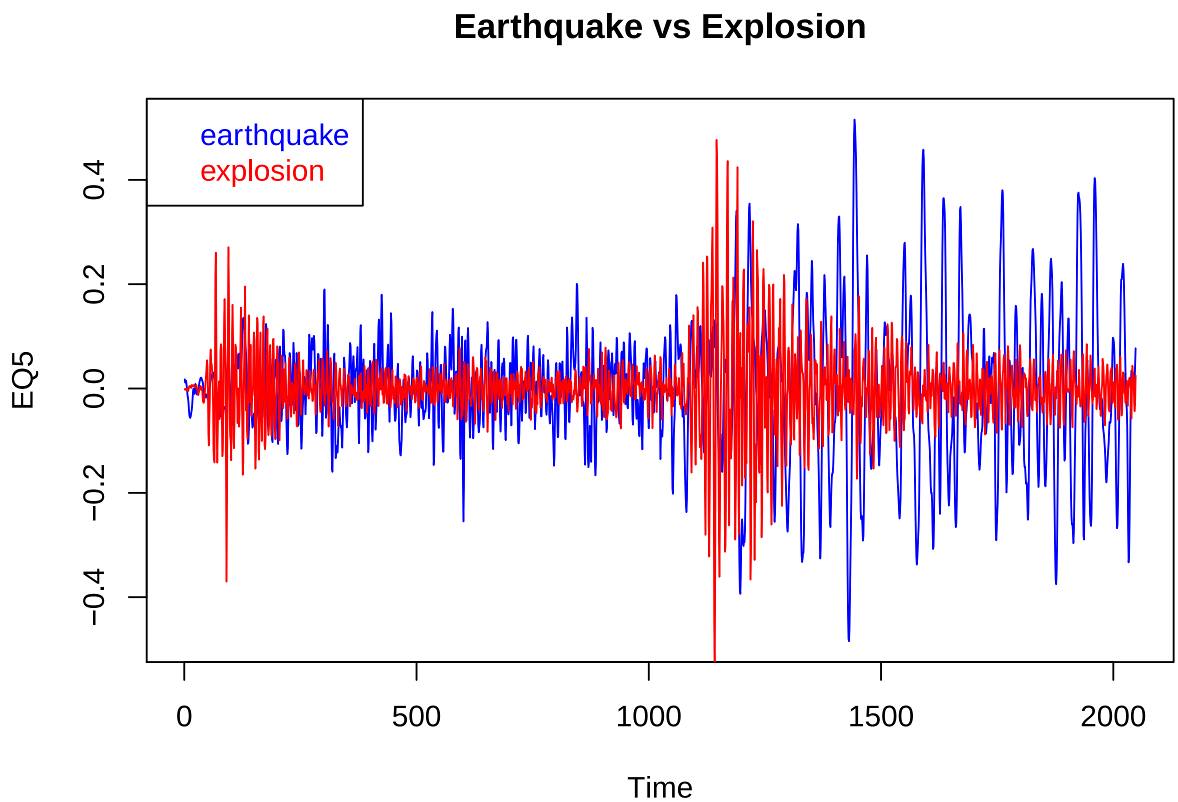
```r
# install.packages('devtools') # only need to do this once
# devtools::install_github('nickpoison/astsa') library(astsa)

# install.packages('latex2exp')

library(latex2exp)
```

1.1 Compare Earthquake and Explsion

To compare the earthquake and explosion signals, plot the data displayed in Figure 1.7 on the same graph using different colors or different line types and comment on the results.

```r
plot(EQ5, col = "blue", main = "Earthquake vs Explosion")
lines(EXP6, col = "red")
legend("topleft", title.col = "black", c("earthquake", "explosion"), text.col = c("blue",
    "red"), text.font = 1, cex = 1)
```



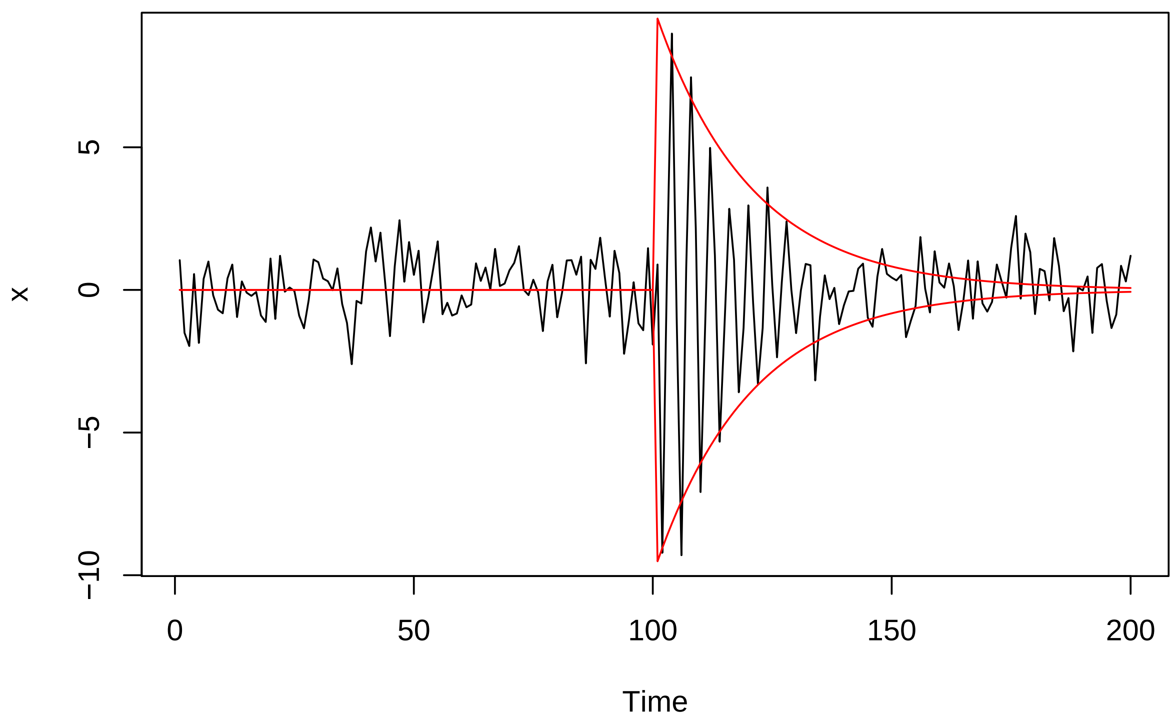
1.2 Plot and Compare signal plus noise model with dampening oscillations.

Consider a signal-plus-noise model of the general form $x_t = s_t + w_t$, where w_t is Gaussian white noise with $\sigma_w^2 = 1$. Simulate and plot $n = 200$ observations from each of the following two models (a) $x_t = s_t + w_t$, for $t = 1, ..., 200$ where

$$s_t = 0, t = 1, ..., 100$$

$$10exp^{\frac{(t100)}{20}} cos(\frac{2\pi t}{4}), t = 101, ..., 200$$

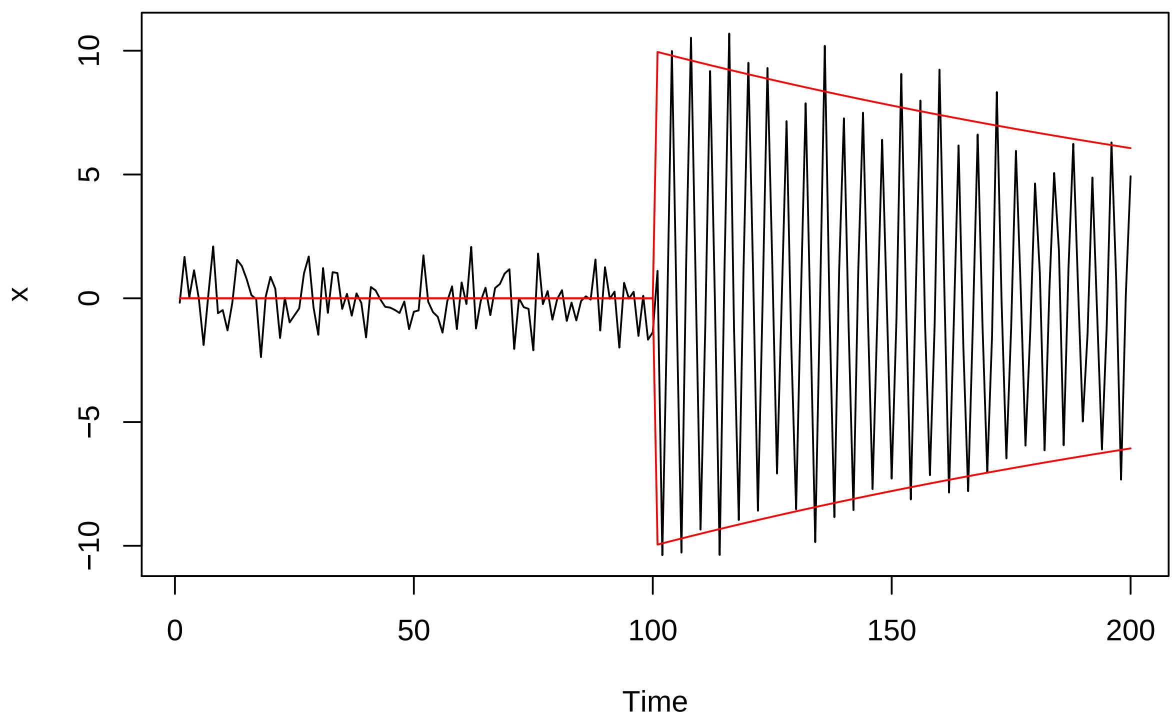
```
s = c(rep(0, 100), 10 * exp(-(1:100)/20) * cos(2 * pi * 1:100/4))
x = ts(s + rnorm(200, 0, 1))
plot(x)

envelopeU = c(rep(0, 100), 10 * exp(-(1:100)/20))
envelopeL = -1 * c(rep(0, 100), 10 * exp(-(1:100)/20))
lines(envelopeU, col = "red")
lines(envelopeL, col = "red")
```



(b) $x_t = s_t + w_t$, for $t = 1, ..., 200$ where

$$s_t = 0 \quad t = 1, ..., 100$$
$$s_t = 10exp^{\frac{(t100)}{200}} cos(\frac{2\pi t}{4}), t = 101, ..., 200$$

```
s = c(rep(0, 100), 10 * exp(-(1:100)/200) * cos(2 * pi * 1:100/4))
x = ts(s + rnorm(200, 0, 1))
plot(x)

envelopeU = c(rep(0, 100), 10 * exp(-(1:100)/200))
envelopeL = -1 * c(rep(0, 100), 10 * exp(-(1:100)/200))
lines(envelopeU, col = "red")
lines(envelopeL, col = "red")
```



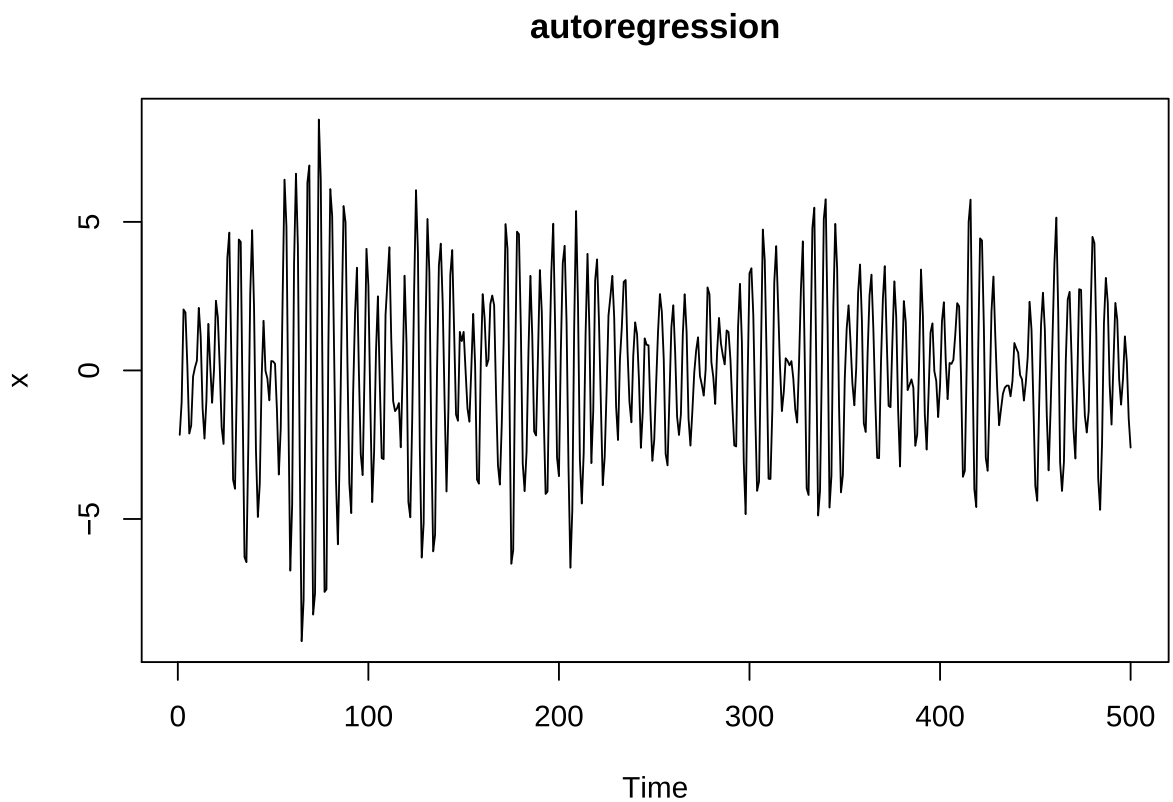
(c) Compare the general appearance of the series (a) and (b) with the earthquake series and the explosion series . In addition, plot (or sketch) and compare the signal modulators.

We see that the time scale of the explosion is shorter than the earthquake. The modulators show the scale of the dampening effect. The second plot is consistent with the slow damping of the oscillations seen in the earthquake.

1.3 Autoregression

Generate n = 100 observations from the autoregression $x_t = -0.9x_{t-2} + w_t$ with $\sigma_w = 1$, using the method described in Example 1.10, page 13.

```
w = rnorm(550, 0, 1)  #50 extra to avoid startup problems
x = filter(w, filter = c(1, -0.9), method = "recursive")[-(1:50)]
plot.ts(x, main = "autoregression")
```

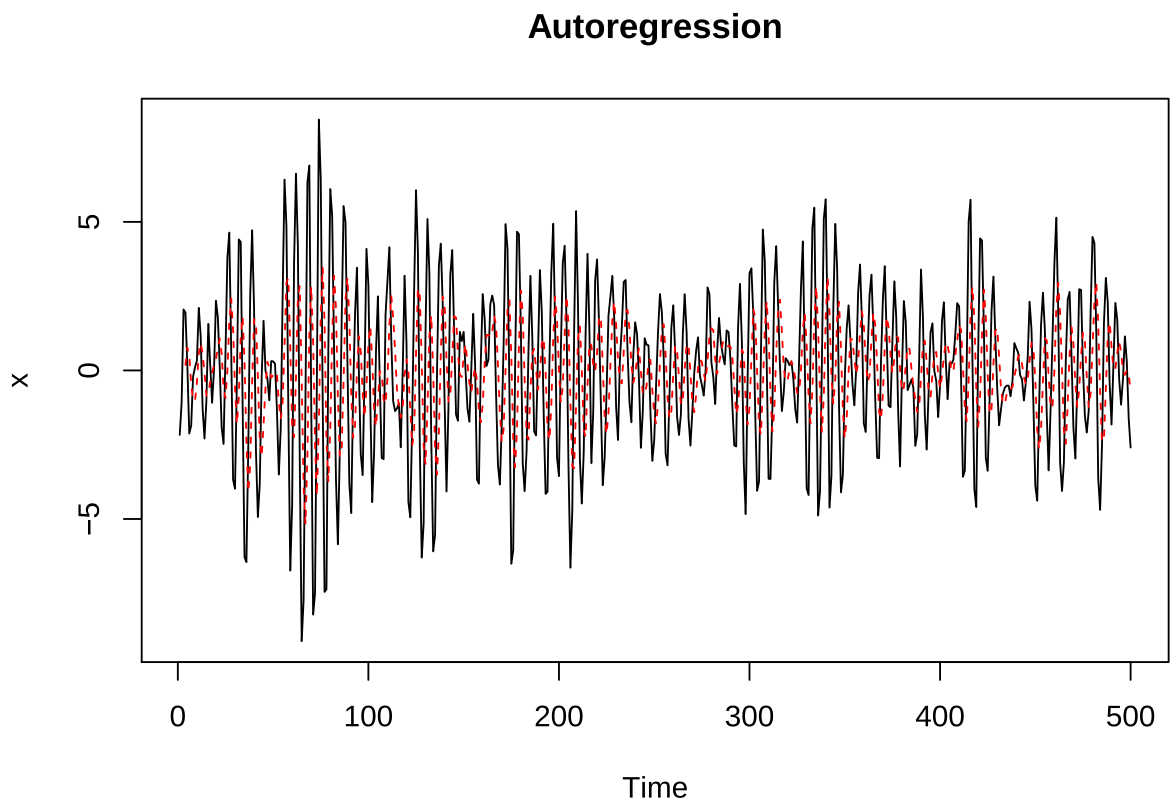


Next, apply the moving average filter

$$v_t = \frac{(x_t + x_{t-1} + x_{t-2} + x_{t-3})}{4}$$

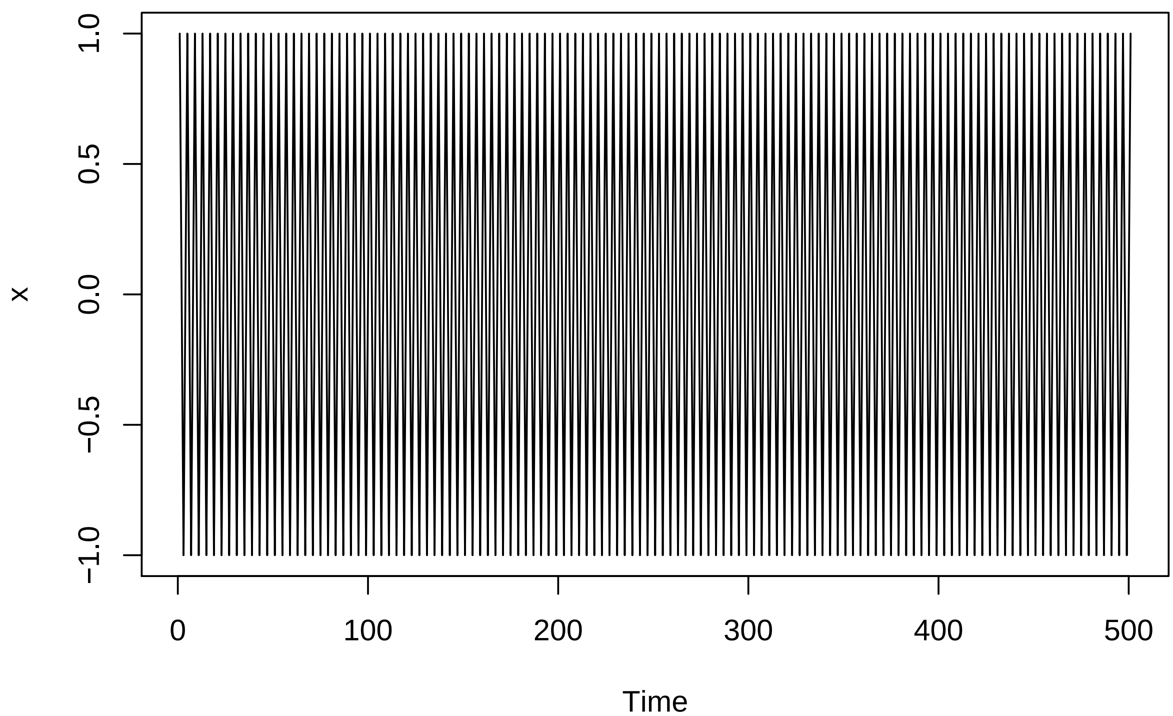
to x_t, the data you generated. Now plot x_t as a line and superimpose v_t as a dashed line. Comment on the behavior of x_t and how applying the moving average filter changes that behavior.

```
v = filter(x, rep(1/4, 4), sides = 1)
plot.ts(x, main = "Autoregression")
lines(v, lty = "dashed", col = "red")
```



(b) Repeat (a) but with $x_t = cos(\frac{2\pi t}{4})$.

```
x = ts(cos(2 * pi * 0:500/4))

plot(x)
```

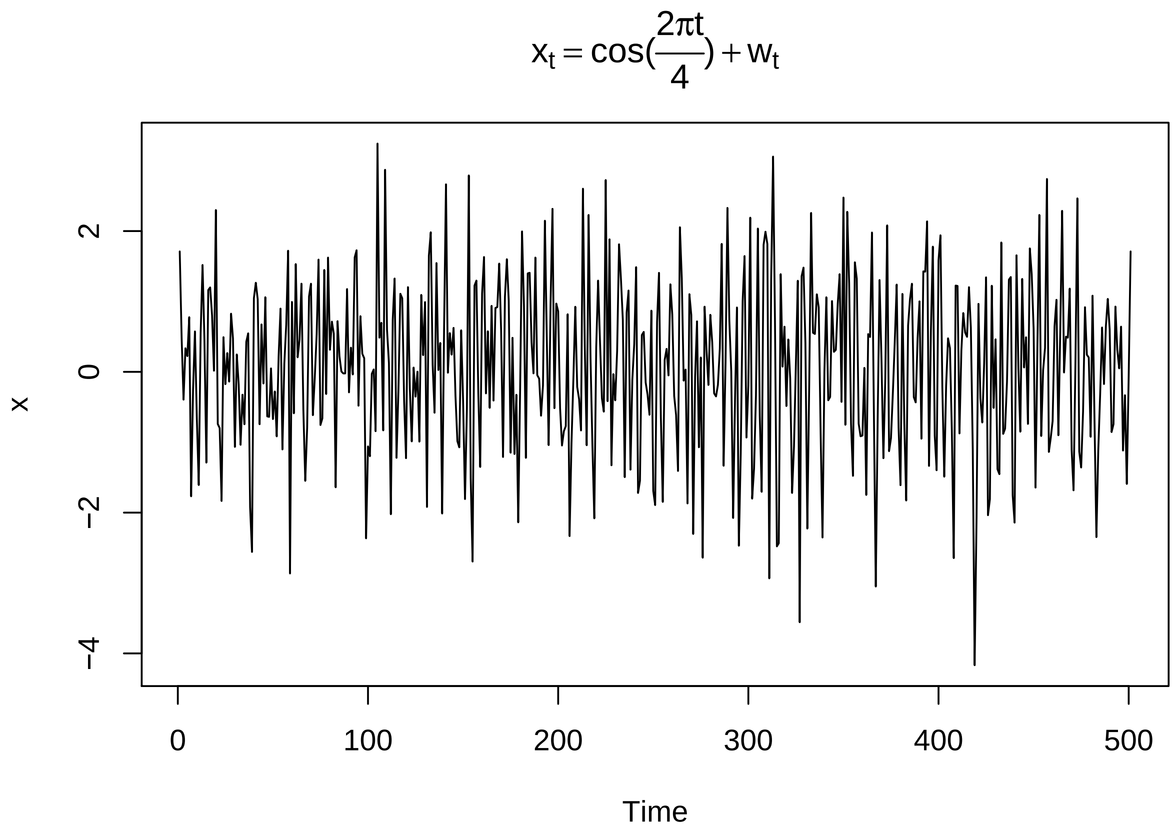


(c) Repeat (b) but with added $N(0, 1)$ noise,

$$x_t = cos(\frac{2\pi t}{4}) + w_t$$

.

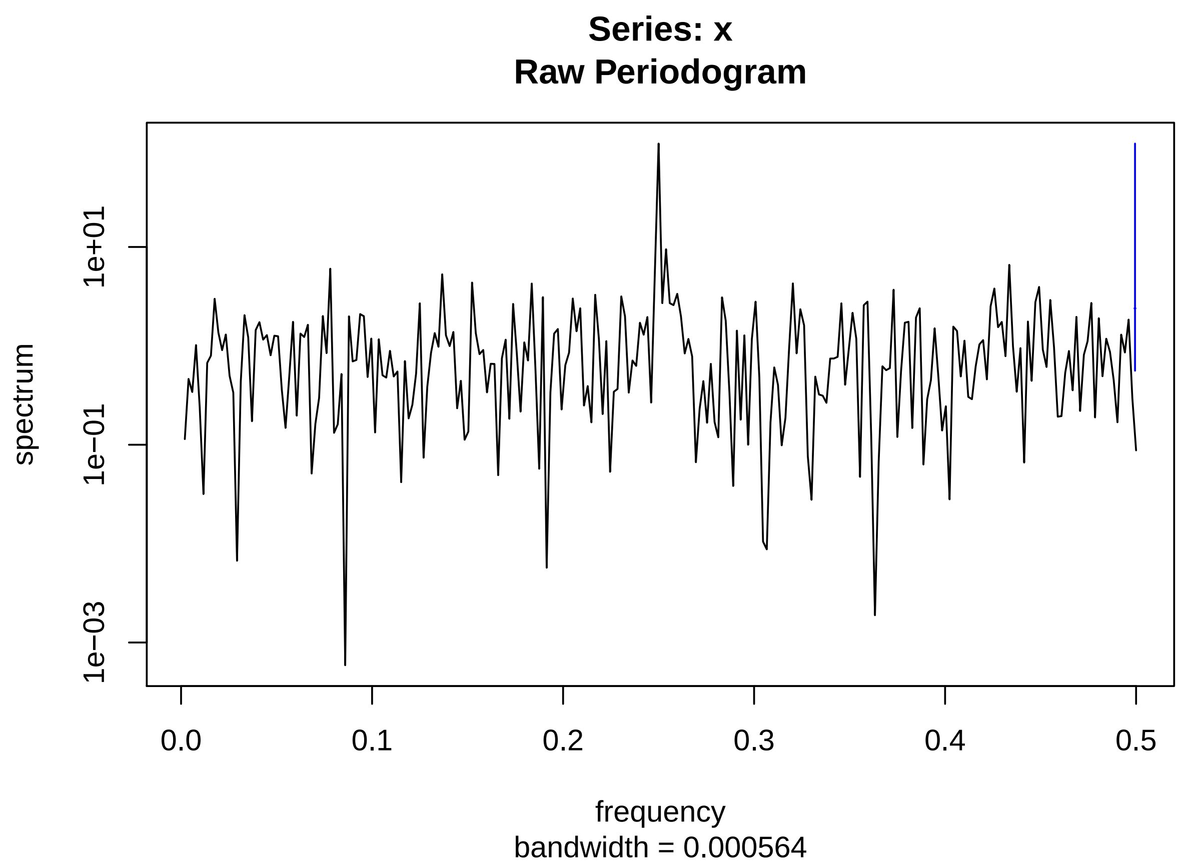
```
library(latex2exp)
expmain <- TeX("$x_t = cos(\\frac{2\\pi t}{4}) + w_t$")

x = ts(cos(2 * pi * 0:500/4) + rnorm(500, 0, 1))

plot(x, main = expmain)
```



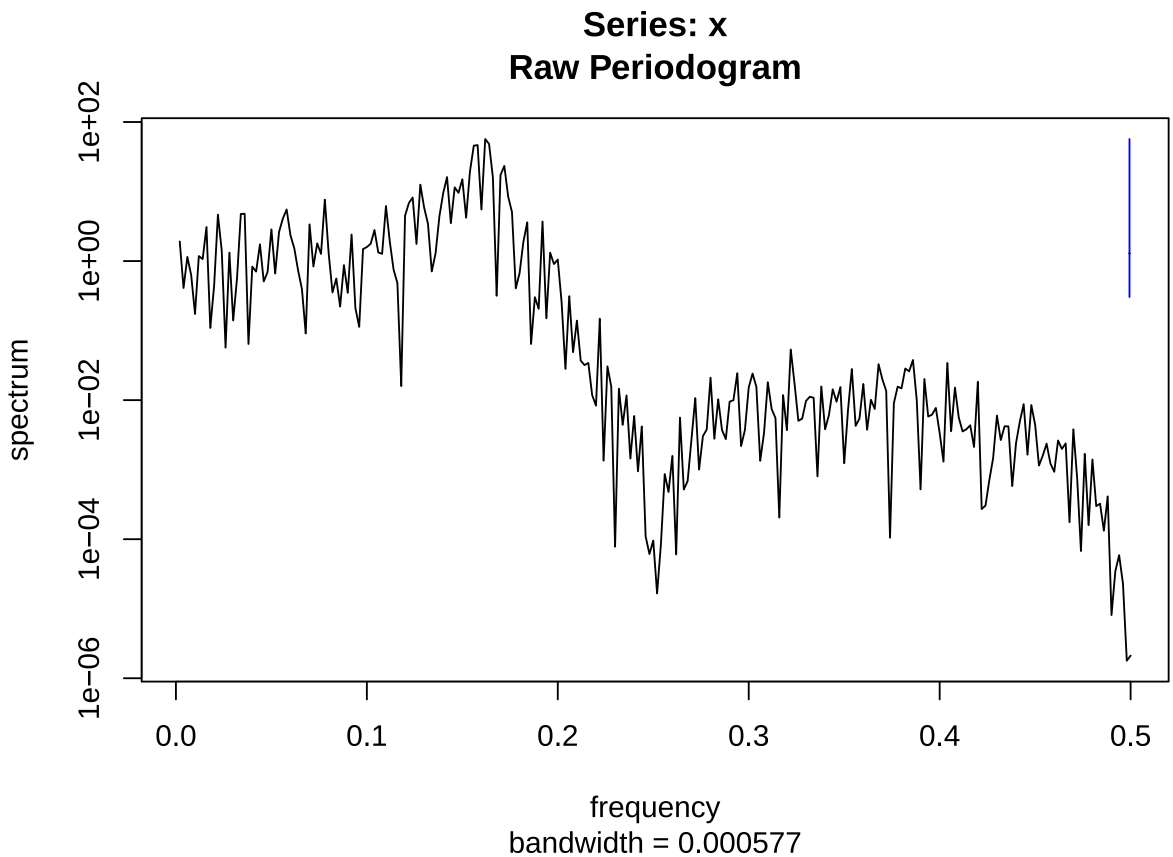
(d) Compare and contrast (a)-(c).

We see that filtering the autoregrssive model smooths the time series and that adding noise to the periodic time series destroys its regularity.

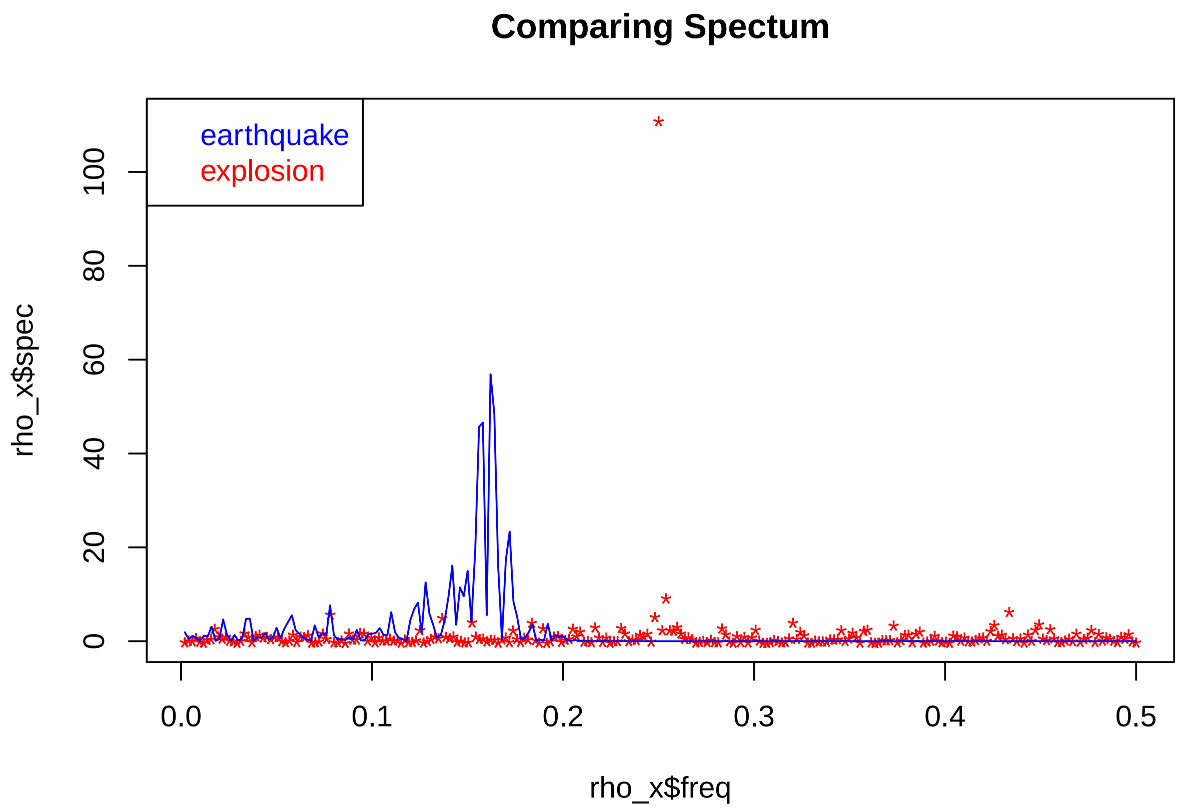
```
rho_x <- spectrum(x)
```




```
rho_v <- spectrum(v[4:500])
```

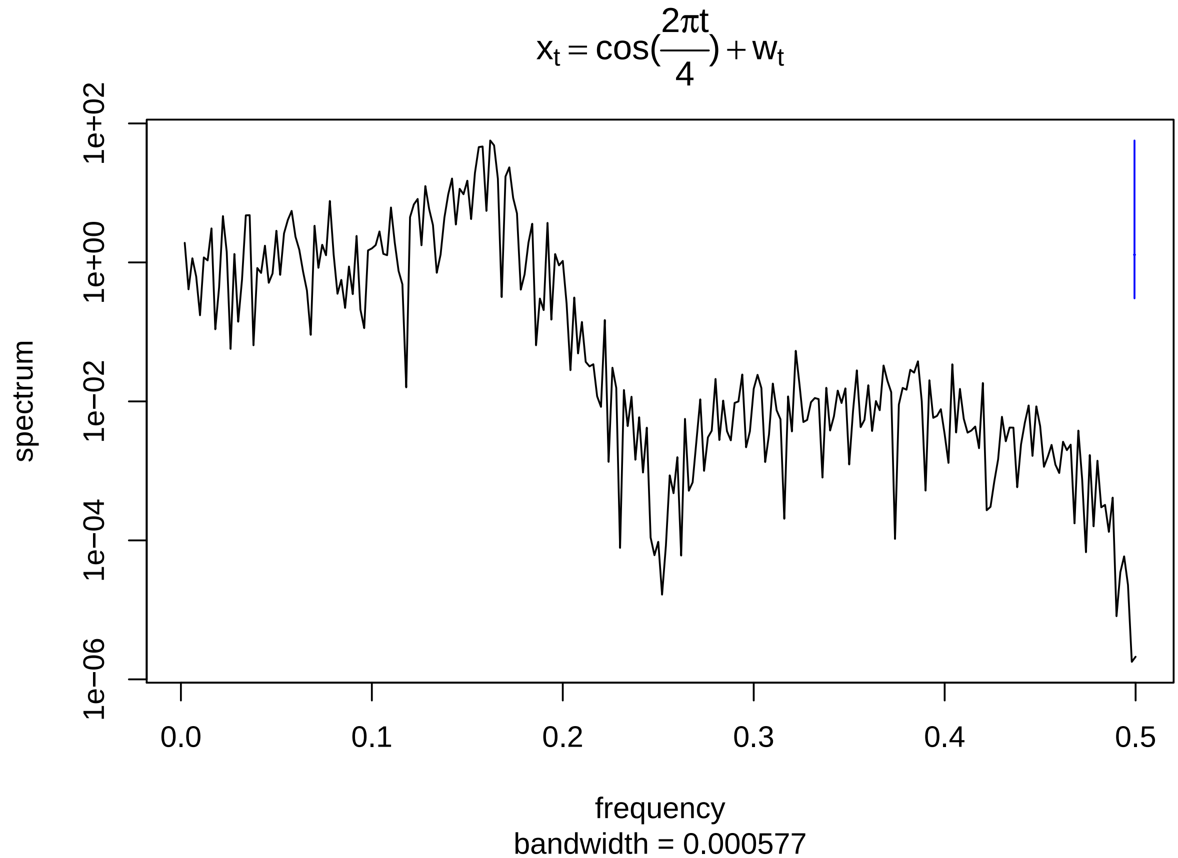



```r
plot(rho_x$freq, rho_x$spec, col = "red", pch = "*", main = "Comparing Spectum")
lines(rho_v$freq, rho_v$spec, col = "blue")
legend("topleft", title.col = "black", c("earthquake", "explosion"), text.col = c("blue",
    "red"), text.font = 1, cex = 1)
```

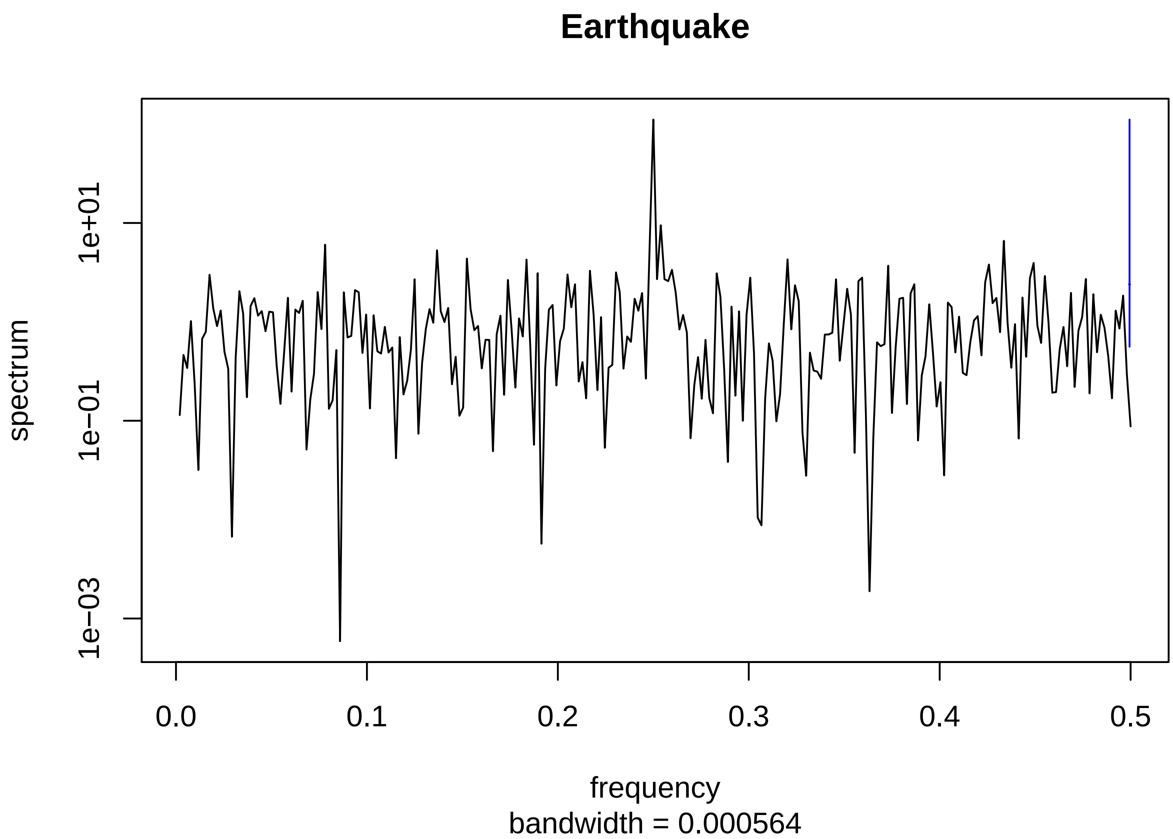


We see the spectral power distribution for the 2 series are different. This plot shows the discriminating capability of spectral mehods. We can have 2 similar looking time series with very different spectrum.

```
plot(rho_v, main = expmain)
```




```
plot(rho_x, main = "Earthquake")
```


Earthquake
spectrum
1e+01
1e−01
1e−03
0.0
0.1
0.2
0.3
0.4
0.5
frequency
bandwidth = 0.000564