Bruce Campell NCSU ST 534 HW 4

Problems 3.32, 3.35(a), and 3.43

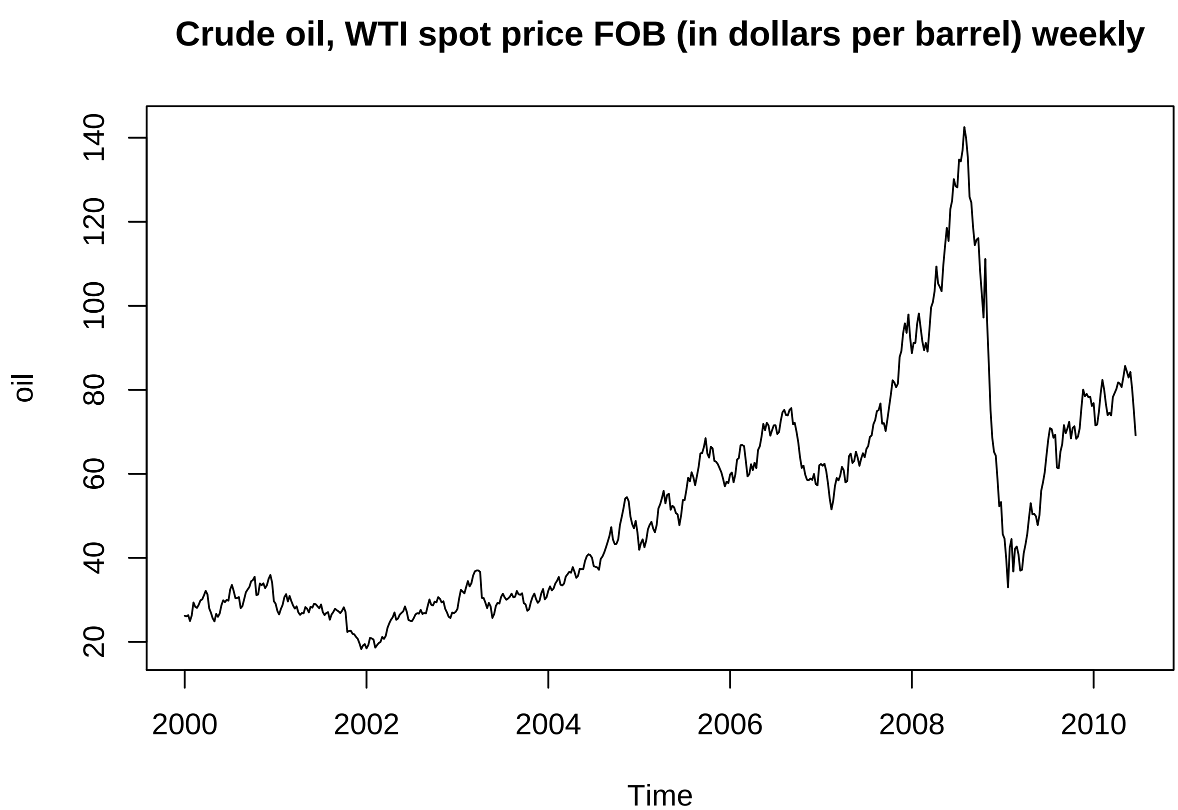
Shumway, Robert H.; Stoffer, David S. Time Series Analysis and Its Applications: With R Examples (Springer Texts in Statistics)

08 November, 2017

3.32 oil time series analysis

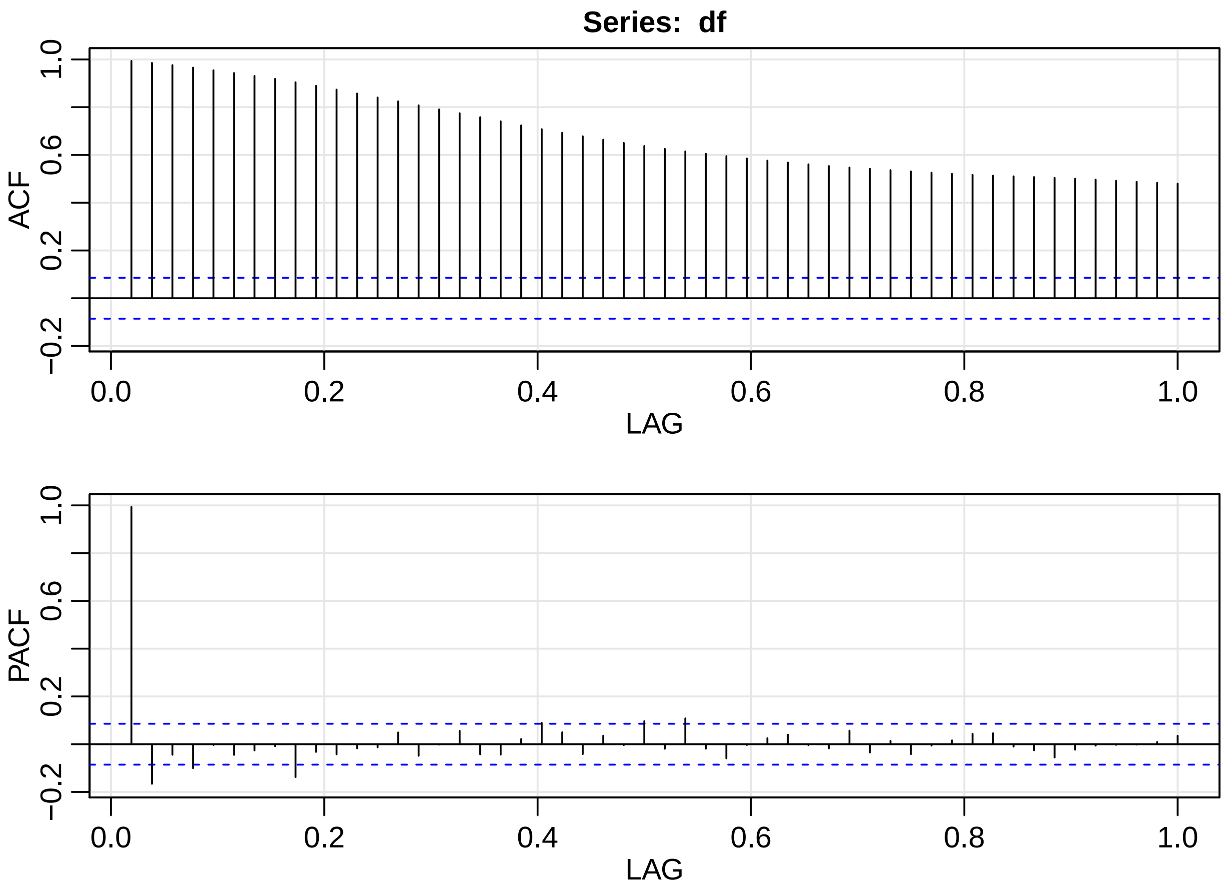
Crude oil prices in dollars per barrel are in oil; see Appendix R for more details. Fit an $ARIMA(p, d, q)$ model to the growth rate performing all necessary diagnostics. Comment.

```
library(astsa)
data(oil, package = "astsa")
df <- oil
plot(oil, main = "Crude oil, WTI spot price FOB (in dollars per barrel) weekly")
```



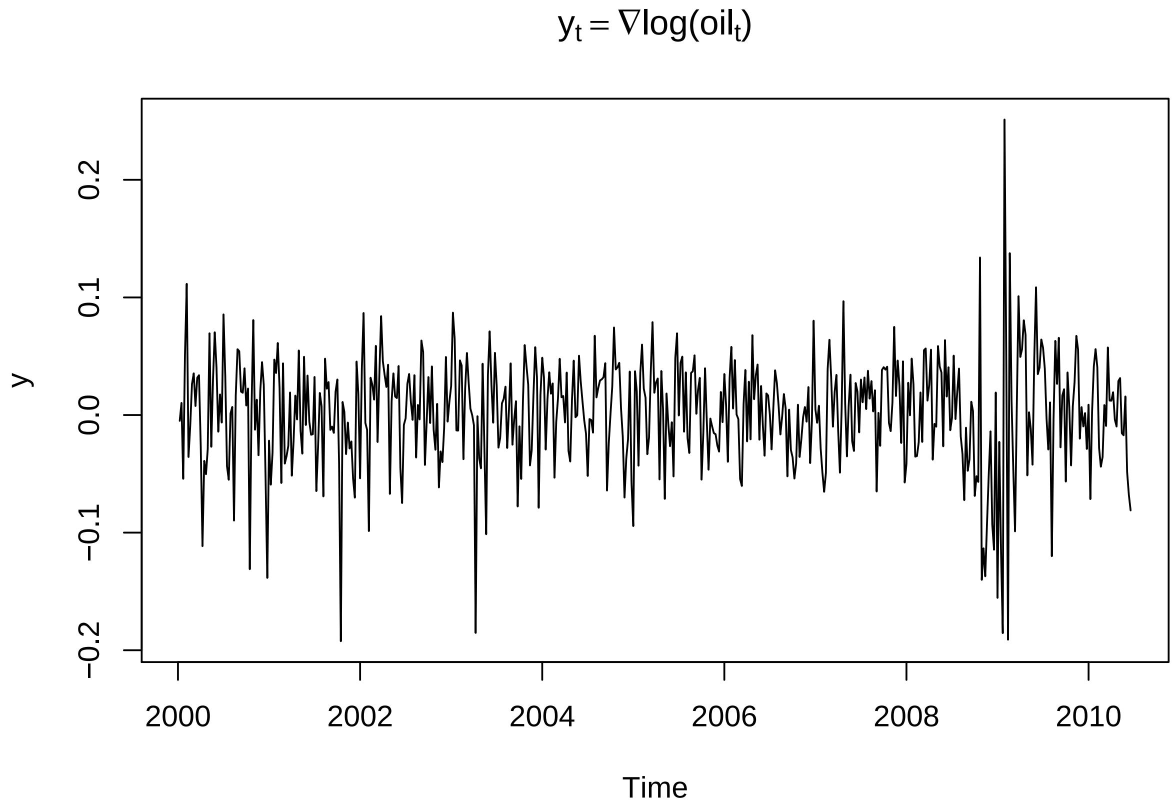
Here is the acf of the oil series.

```r
acf2(df, 52)
```



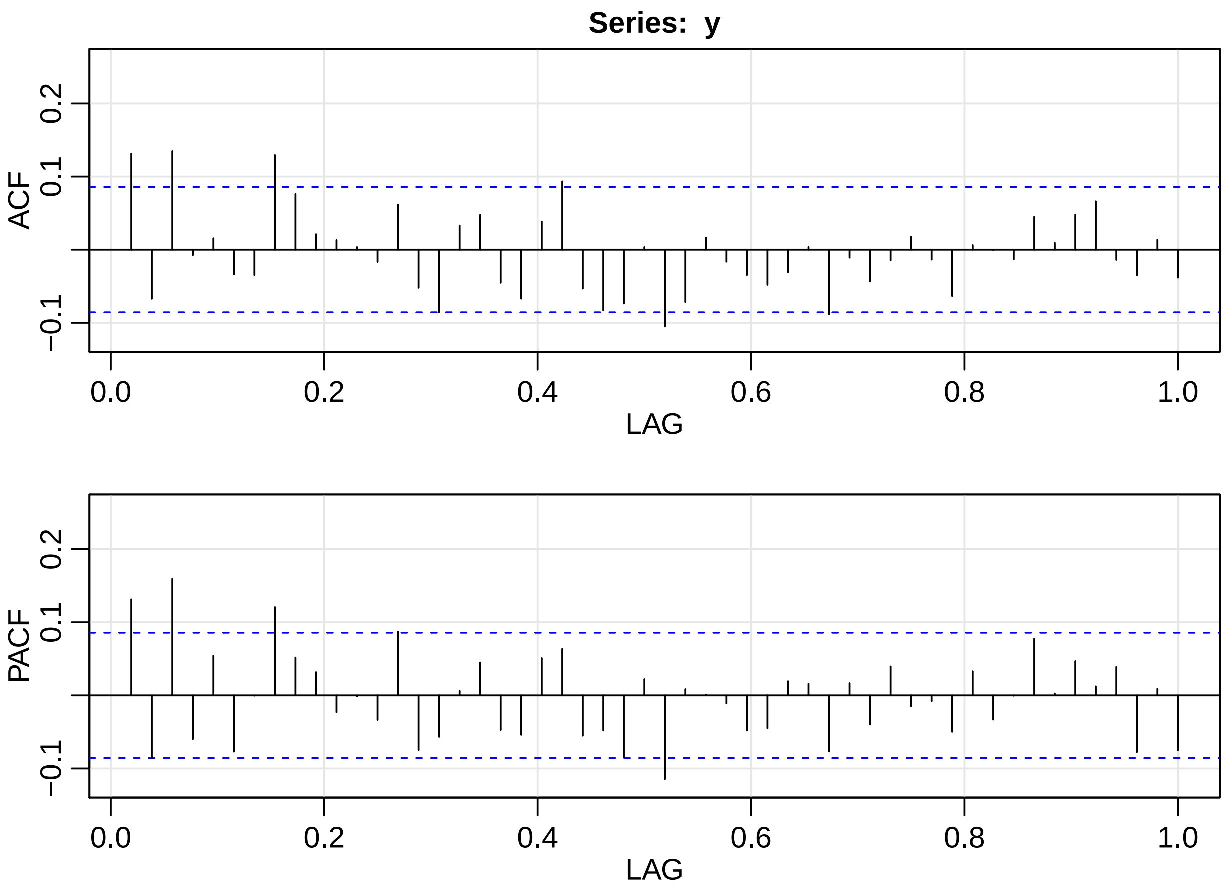
As expected the trend obscures the underlying structure of the fluctuations. We will now calculate $y_t = \nabla log(x_t)$ and display the ACF and PACF.

```r
y <- diff(log(df))
plot(y, main = TeX("$y_t =\\nabla log(oil_t)$"))
```



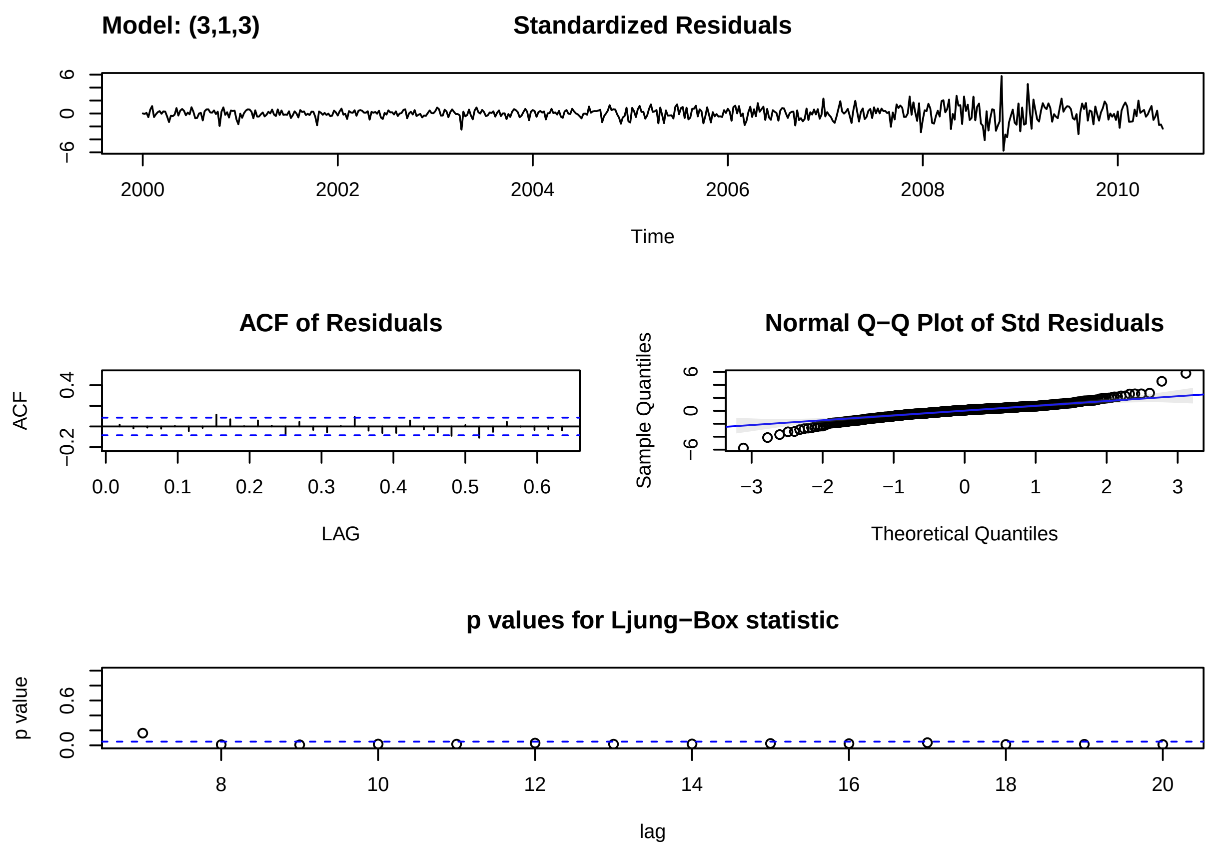
We now see the underlying structure better. There's a period around 2008-2009 that one could argue requires more sophisitcated modelling such as stochastic volatility.

```
acf2(y, 52)
```



Based on the ACF and PACF of the differenced log series - we will try to fit an $ARIMA(3, 1, 3)$ model to $log(x_t)$

```r
invisible(model <- sarima(df, 3, 1, 3))
```

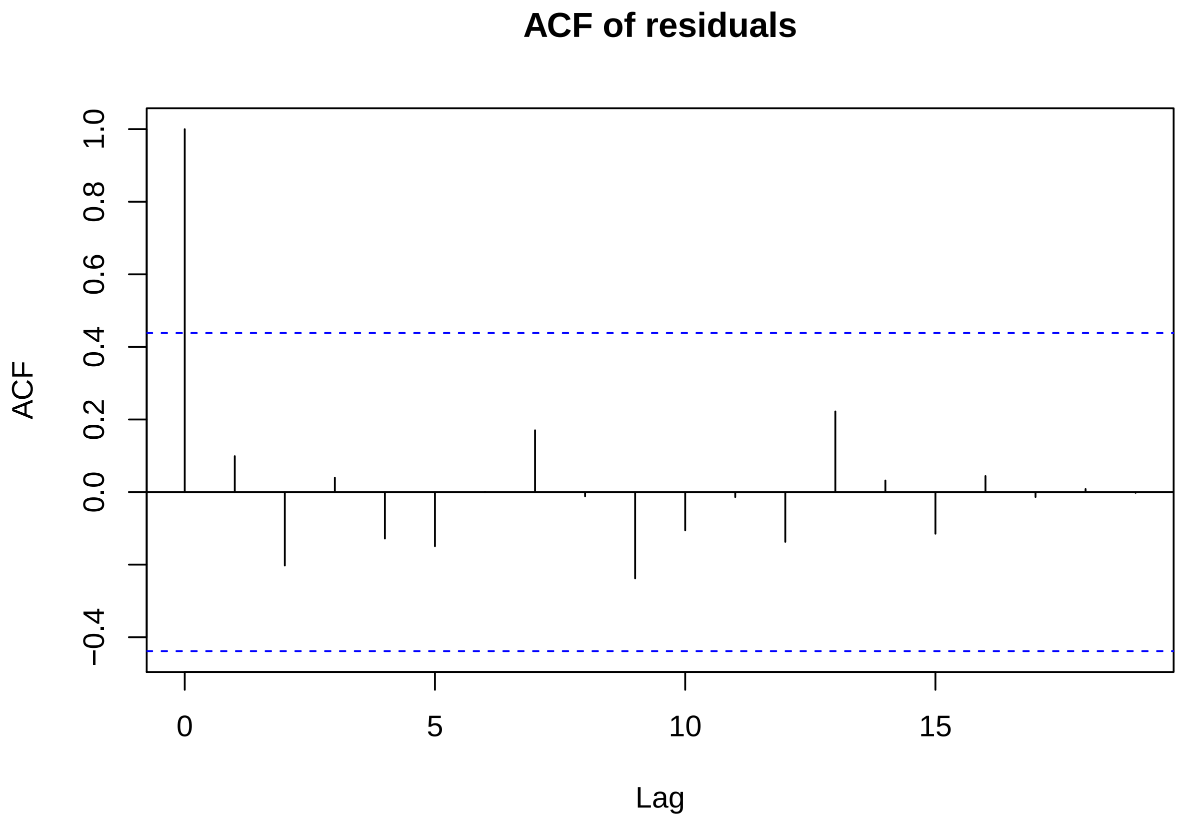



```
model$ttable
```

For lag 1 the Ljung-Box statistic shows significant correlation in the residuals.

For fun let's calculate the Ljung-Box-Pierce Q-statistic to check for systemic autocorrelation in the residuals. We'll extract the residuals and do the calculation by hand - there's and R function for this `Box-Test` that we've experiemnted with. we'll revisit this.

```
n <- length(df)
H <- 20
r <- model$fit$residuals[1:H]
acf.residuals <- acf(r, H, main = "ACF of residuals")
```




```
sum.denominator <- n - seq(H, 1, by = -1)
r.s <- acf.residuals$acf^2/sum.denominator
Q <- n * (n + 2) * sum(r.s)
Q
```

```
## [1] 720.2996
```

We see based on the Q-statistic that we have significant correlation structure remaining in the residuals.

We didn't expect the residuals to be normally distributed. Starting in 2005 there is a change in the volatility.

3.35 Seasonal Model

Consider the ARIMA model $x_t = w_t + \Theta w_{t-2}$.

(a) Identify the model using the notation $ARIMA(p, d, q)\text{Ö}(P, D, Q)s$

(b) Show that the series is invertible for $|\Theta| < 1$, and find the coefficients in the representation $w_t = \sum_{k=0}^{\infty} \pi_k x_{t-k}$.

(c) Develop equations for the m-step ahead forecast, $\tilde{x}_{n+m}$, and its variance based on the infinite past, $xn, xn???1, ...$

3.43 Use Theorem B.2 and B.3 to verify (3.116).