Bruce Campell NCSU ST 534 Exam 1

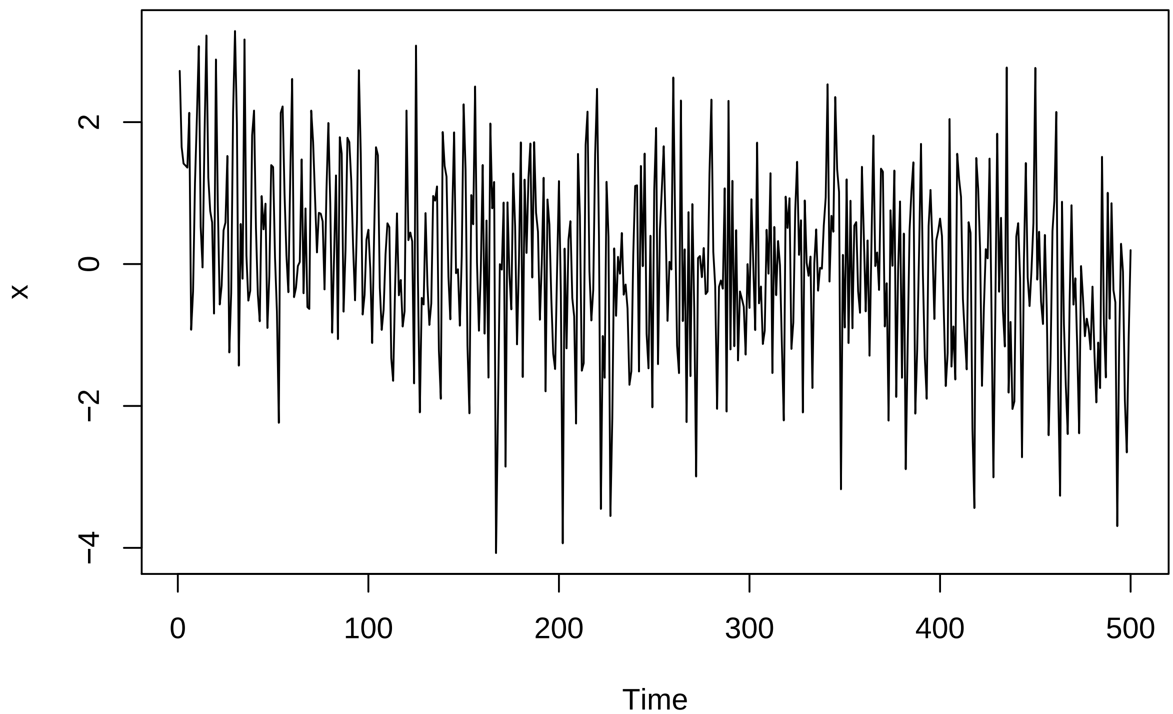
03 October, 2017

1

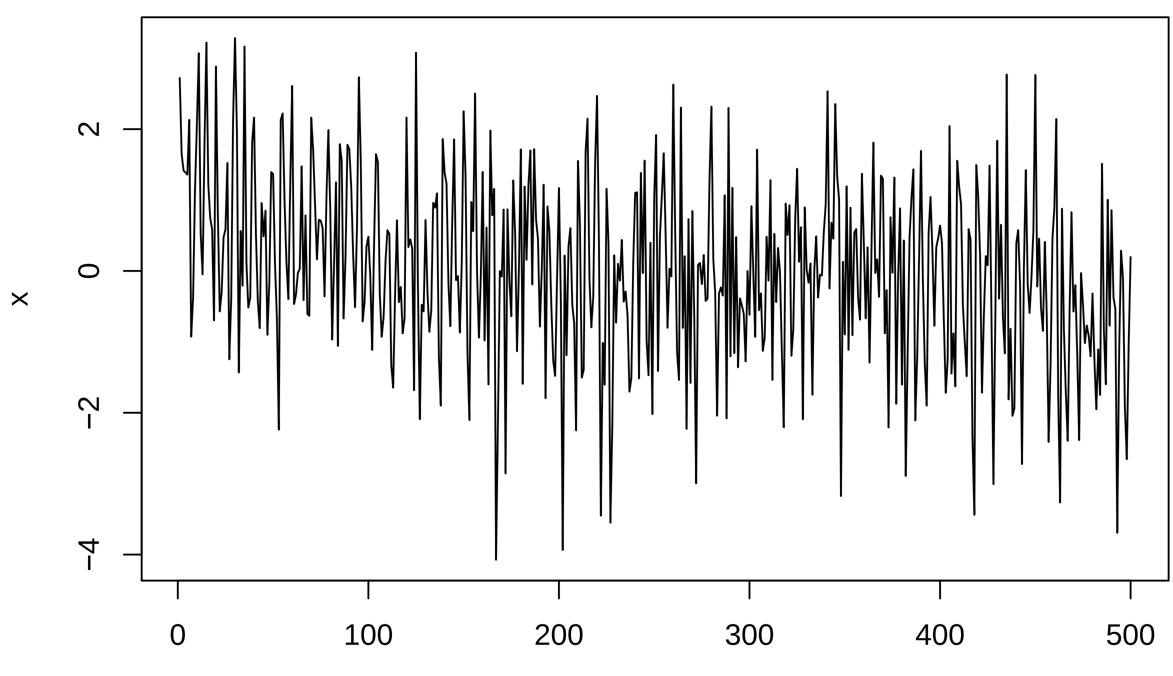
Generate a time series x of length 500 using the following R commands:

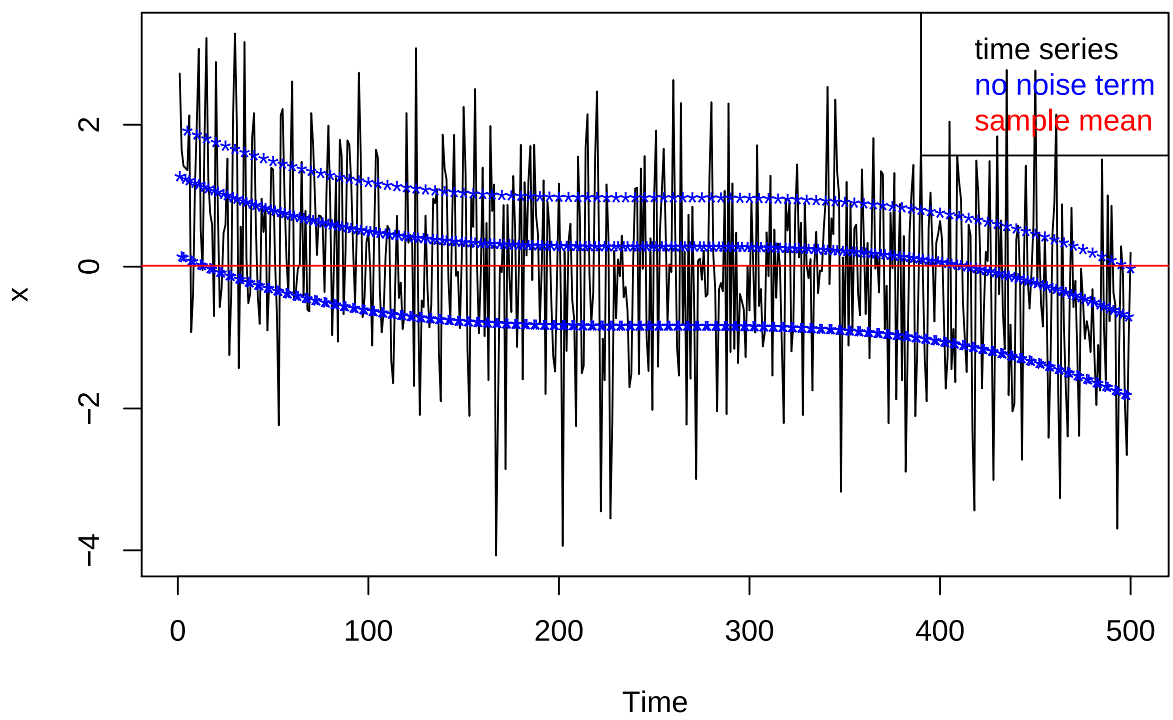
```
t <- seq(1:500)
x <- cos(2 * pi * 0.2 * t) + (1 - t/250)^3 + rnorm(500)
x <- ts(x)
x.no.noise <- cos(2 * pi * 0.2 * t) + (1 - t/250)^3
```

(a) Plot x. Does the plot appear to be stationary?



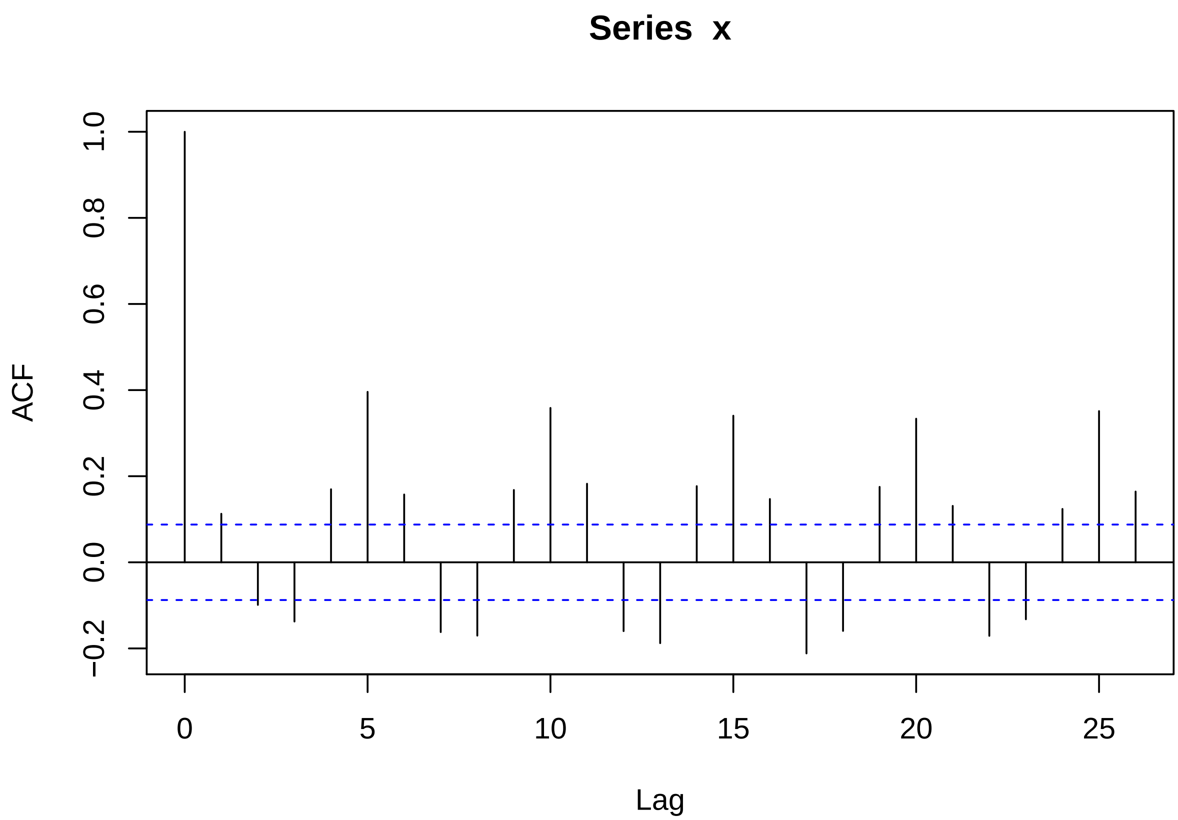
time series along with sample mean and underlying noisless series





This time series does not appear stationary, The mean function apperears to have drift and the variance does not appear constant.

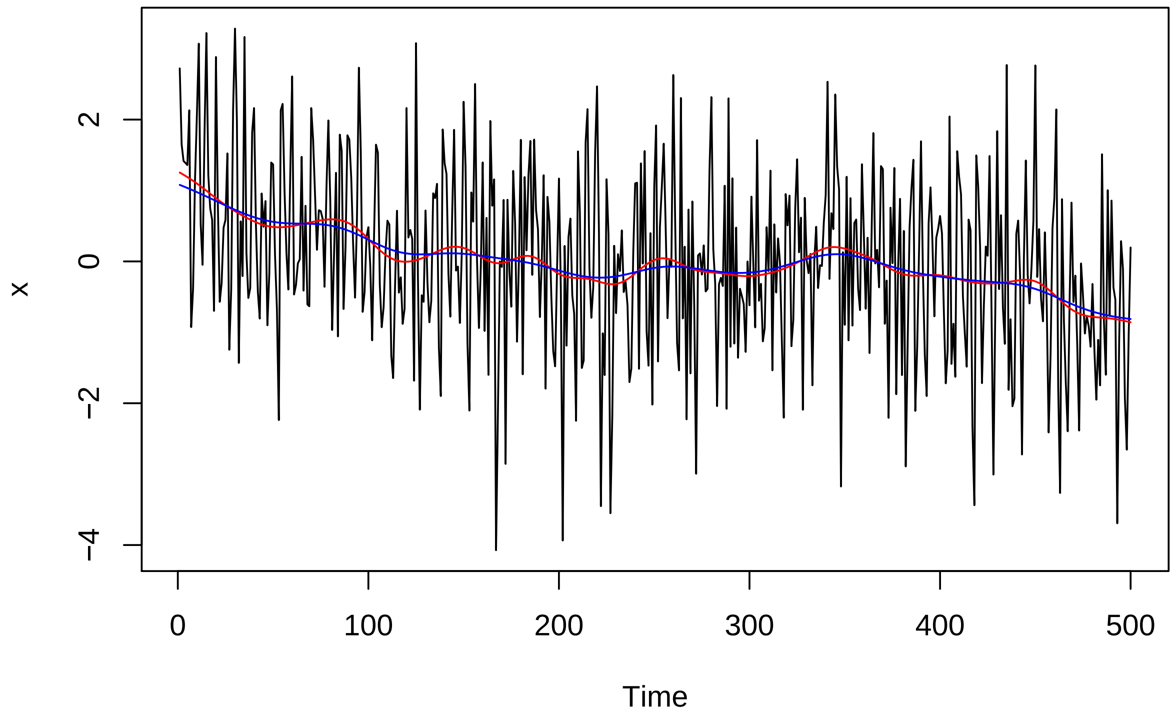
(b) Plot the ACF of x. What feature(s) do(es) the acf-plot reveal ?



We see negative autocorrelation at lags 2,3 and lags $l_i : l_i \ni 2mod5 = 0, l_i \ni 3mod5 = 0FIX$. We see positive autocorrelation at lags 4,5,6 with the strongest peak at lag 5.

(c) Plot x and overlay two kernel smoothed curves using the Gaussian ("normal")

kernel with two different choices of the bandwidth b = 30; 50.



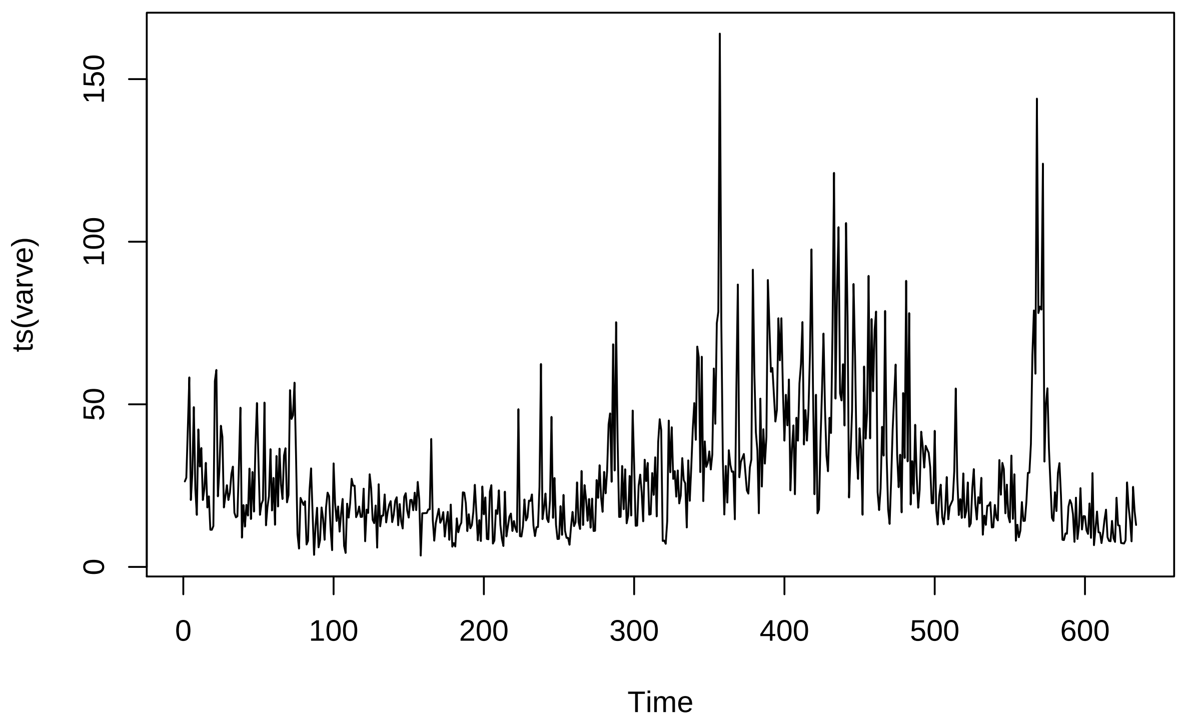
(d) Which of the two smoothed curves (or the bandwidths) gives a better description of the "trend function" $g(t) = (1 - \frac{t}{250})$? Justify your answer by relating the bandwidth choice with the period of the cyclical component.

The kernel with the larger bandwidth will capture the trend better.

2 Consider the time series varve given in the package ASTSA.

(a) Show that the varve series is heteroscedastic by computing the sample variances over the first and the second half of the data.

```
rm(list = ls())
library(astsa)
data(varve, package = "astsa")
plot(ts(varve))
```




```
n.tics <- length(varve)

left.half <- window(varve, start = 1, end = floor(length(varve)/2))
right.half <- window(varve, start = floor(length(varve)/2) + 1, end = length(varve))

mean.left <- mean(left.half)
mean.right <- mean(right.half)
pander(data.frame(mean.left = mean.left, mean.right = mean.right), caption = "Sample mea
```

Table 1: Sample means for right and left half of varve ts

mean.left	mean.right
20.86	34.89

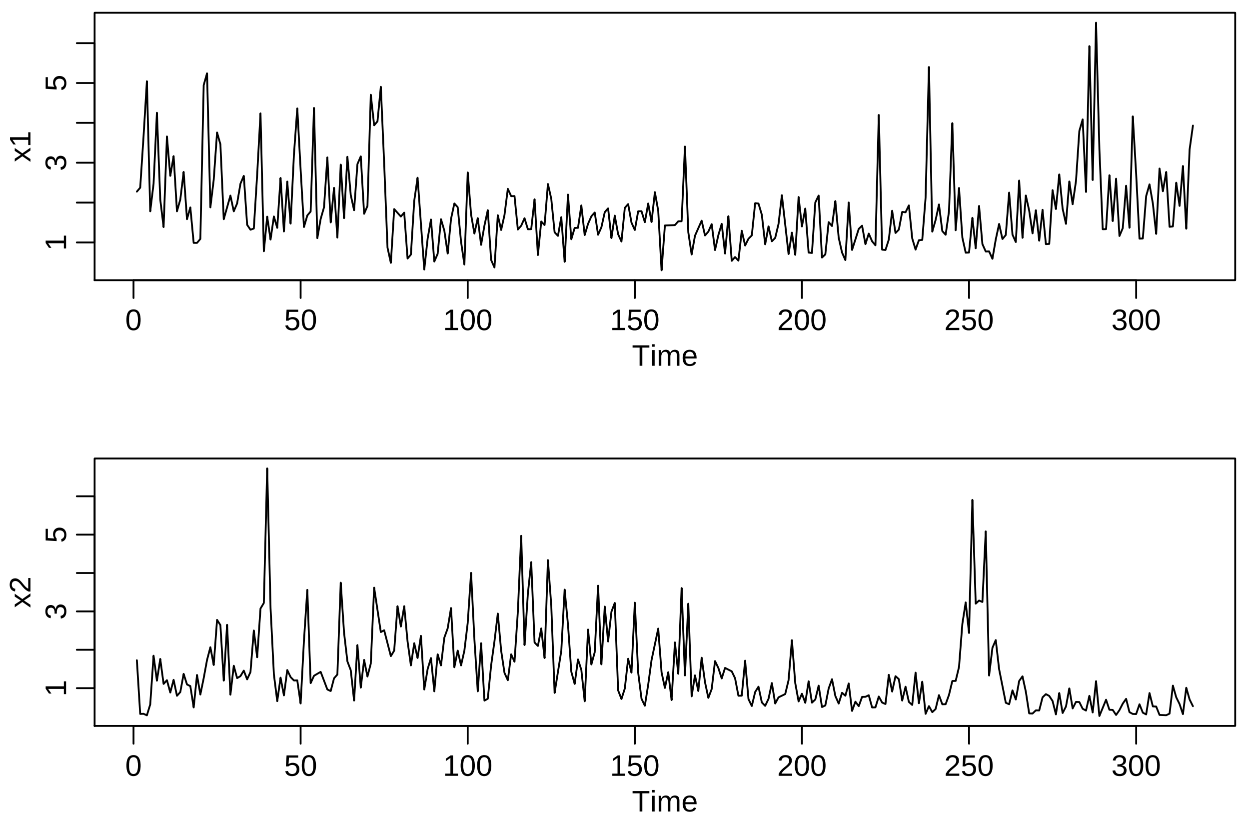
```
var.left <- var(left.half)
var.right <- var(right.half)

pander(data.frame(var.left = var.left, var.right = var.right), caption = "Sample varianc
```

Table 2: Sample variances for right and left half of vearve ts

var.left	var.right
133.5	594.5

(b) Let x1 denote the first half of the varve series scaled by the sample standard deviation of the first half, and similarly, let x2 denote the second half of the varve series scaled by the sample standard deviation of the second half. Plot the two subseries x1 and x2 in two panels using the plotting function mfrow=c(2,1).

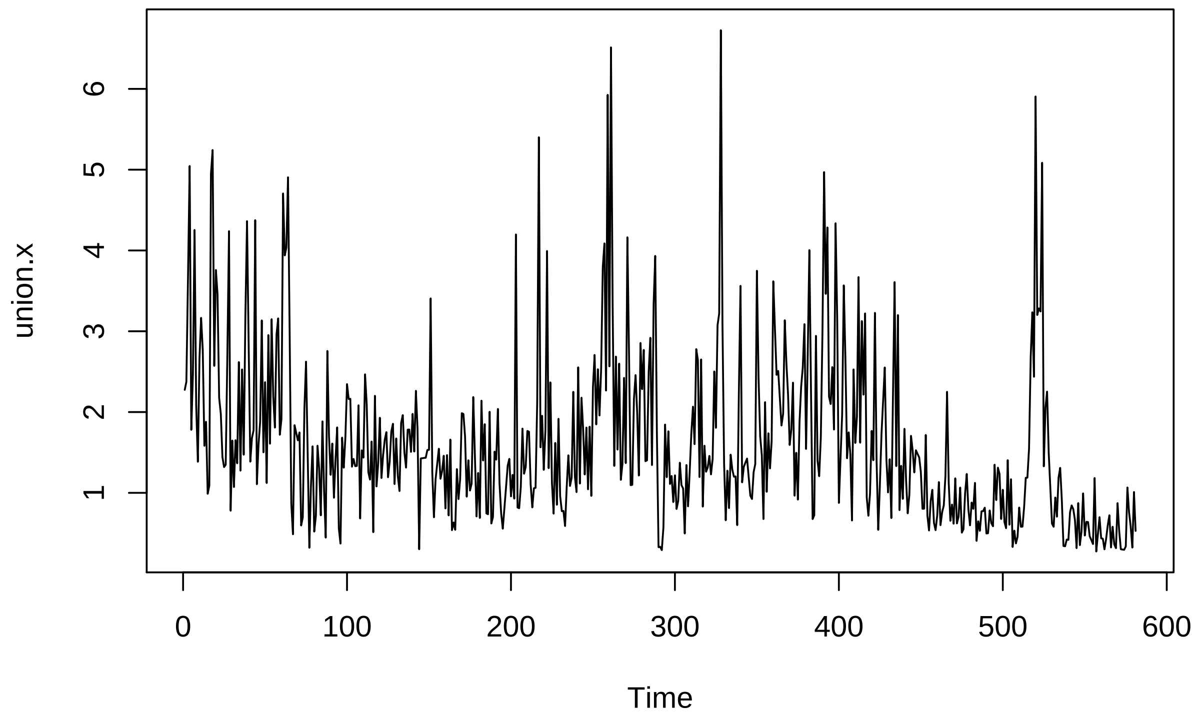
```r
x1 <- left.half
x2 <- right.half
x1.scaled <- scale(left.half, center = FALSE, scale = sqrt(var.left))
x2.scaled <- scale(right.half, center = FALSE, scale = sqrt(var.right))

par(mfrow = c(2, 1), mar = c(3, 2, 1, 0) + 0.5, mgp = c(1.6, 0.6, 0))
plot(ts(x1.scaled), ylab = "x1")
plot(ts(x2.scaled), ylab = "x2")
```



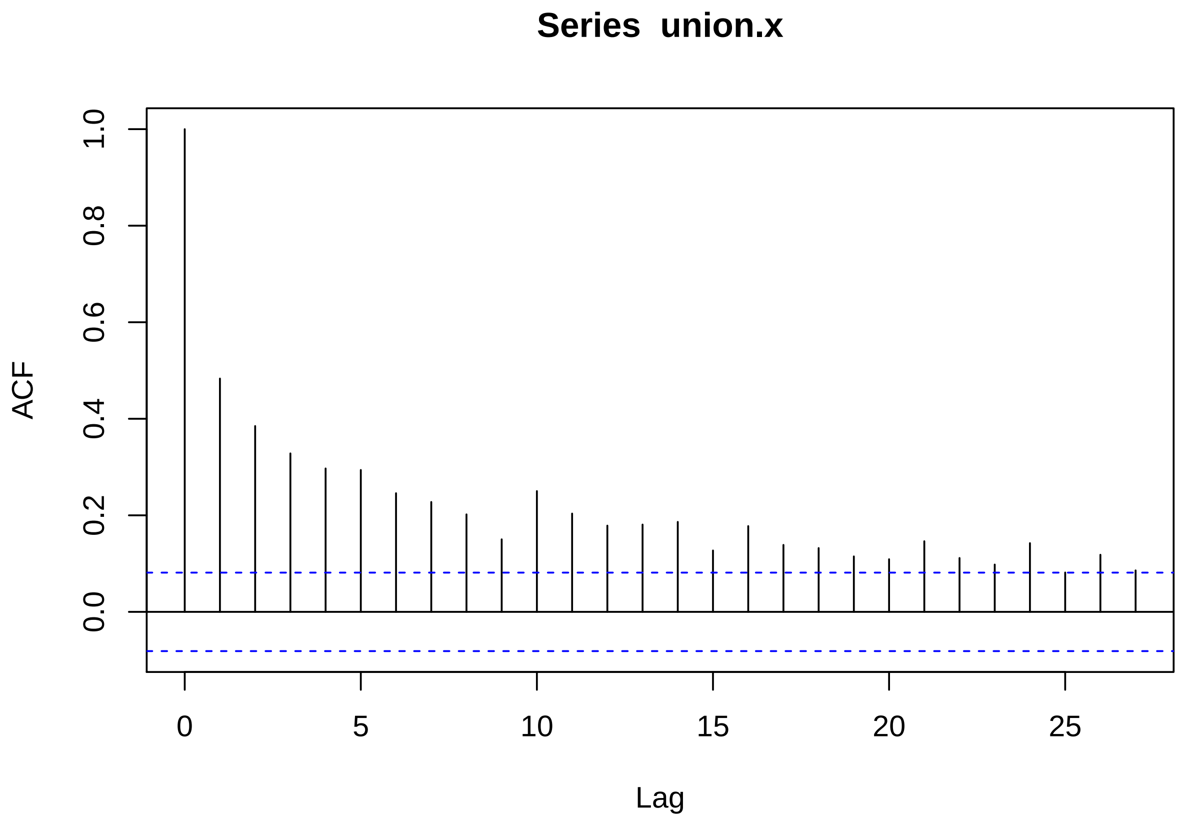
(c) Now combine the two scaled series $x1$ and $x2$, and call it xt. Plot the ACF of the xt

series and comment on its (non-)stationarity properties!

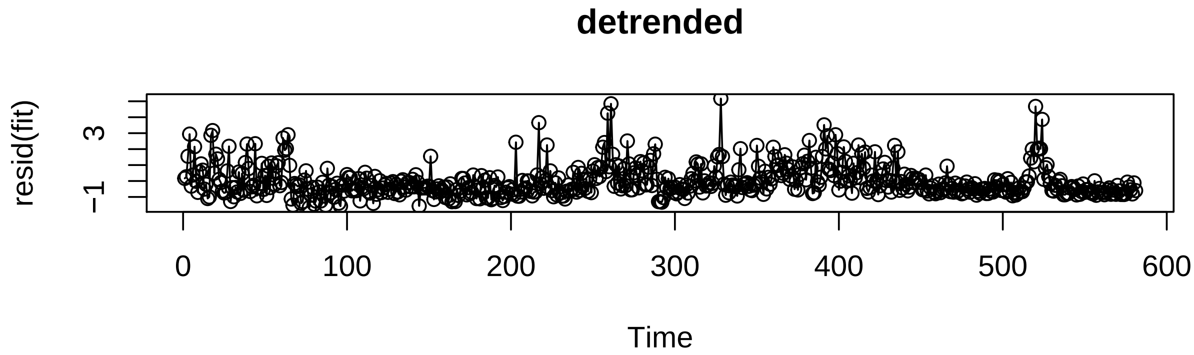
```
union.x <- ts(union(x1.scaled, x2.scaled))
plot(union.x)
```

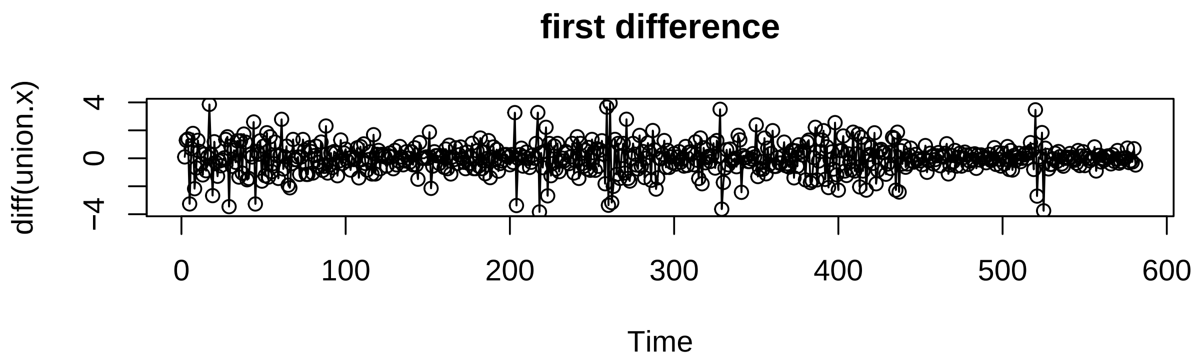



```
acf(union.x)
```

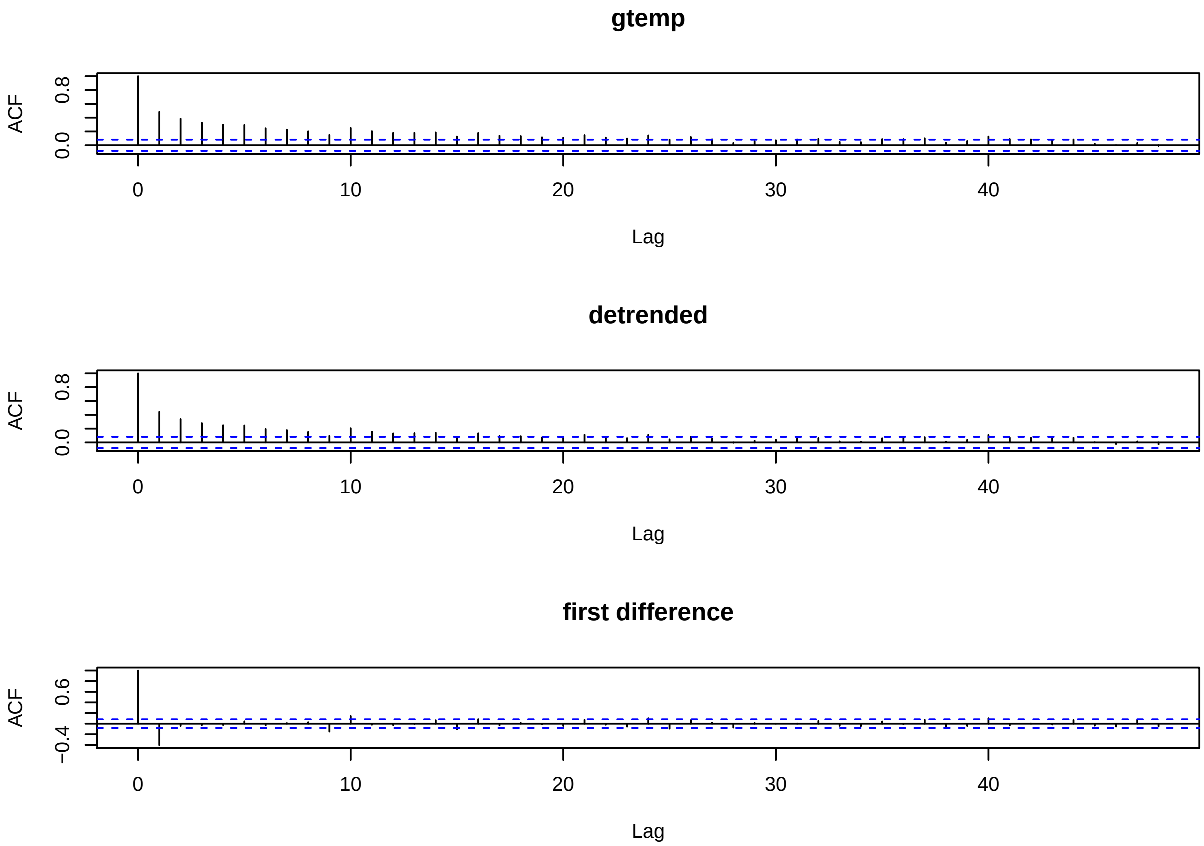



```r
fit = lm(union.x ~ time(union.x), na.action = NULL)
par(mfrow = c(2, 1))
plot(resid(fit), type = "o", main = "detrended")
plot(diff(union.x), type = "o", main = "first difference")
```






```
par(mfrow = c(3, 1))  #plot ACFs
acf(union.x, 48, main = "gtemp")
acf(resid(fit), 48, main = "detrended")
acf(diff(union.x), 48, main = "first difference")
```



Shumway, Robert H.; Stoffer, David S.. Time Series Analysis and Its Applications: With R Examples (Springer Texts in Statistics) (Page 59). Springer New York. Kindle Edition.

(d) Consider the dfferenced time series $xdiff$ obtained from xt. Show that an $MA(1)$ model is appropriate for xdiff.

(e) The model for $X_t = xdiff$ can be written as

$$X_t = \mu + W_t + \theta_1 W_{t-1}$$

Where $W_t \sim N(-, \sigma_W)$. Find an estimate of μ.

(f) Write down the final model for the varve-series based on this analysis.

3

Suppose that X_t is an $ARMA(p, q)$ process:

$$X_t = .5x_{t-1} + w_t - 0.7w_{t-1} + 0.1W_{t-1}$$

where W_t are IID $N(0, 1)$. (a) Find p and q. (Be sure to check for model redundancy.) (b) Show that X_t is invertible. (c) Find the ACF of x_t explicitly.