Fixed Effects versus Lagged Dependent Variables

→ Fixed Effects and Diff-in-Diff estimators are based on the assumption of <u>time</u> <u>invariant</u> omitted variables.

e.g. Effect of participation in a training program on wages.

a) $Y_{it} = \alpha_i + \beta X_{it} + \delta D_{it} + \varepsilon_{it}$

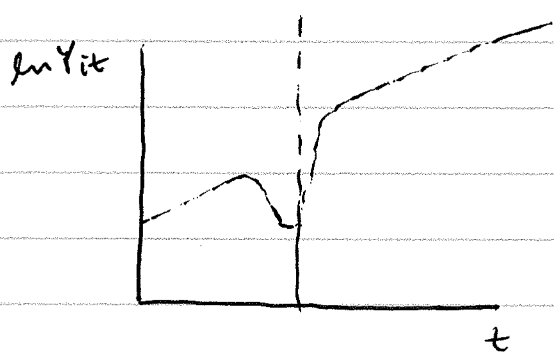
α_i ↓ individual fixed effect

Assumption:

$$E[Y_{0it} \mid \alpha_i, X_{it}, D_{it}] = E[Y_{0it} \mid \alpha_i, X_{it}].$$

i.e., after you've controlled for α_i, X_{it}, there's no need to control for D_{it}. It is randomly assigned conditional on α_i and X_{it}.

b) $Y_{it} = \alpha + \theta Y_{it-h} + \lambda_t + \delta D_t + \beta X_{it} + \varepsilon_{it}$



People enroll in a training program when they suffer some sort of setback: "Ashenfelter Dip."

Assumption:

$$E[Y_{0it} \mid Y_{it-h}, X_{it}, D_{it}] = E[Y_{0it} \mid Y_{it-h}, X_{it}]$$

i.e., after you've controlled for Y_{it-h}, X_{it} there's no need to control for D_{it}.

→ What to do? Fixed Effects and Lagged Dependent Variables estimates have a bracketing property.

If (b) is correct (i.e. pre-program dip) and mistakenly use Fixed Effects then estimates of a positive treatment effect will tend to be too big.

If (a) is correct (i.e., fixed effect) and mistakenly use lagged dependent variables, then estimates of a positive treatment effect will tend to be too small.

Example:

a) suppose (a) is the correct specification, i.e., the correct thing to do is control for individual fixed effects, but instead, we mistakenly control for lagged dependent variables.

The true model is:

$$Y_{it} = \alpha_i + \delta_1 D_{it} + \varepsilon_{it}$$

We also have:

$$Y_{it-1} = \alpha_i + \varepsilon_{it-1}$$

We mistakenly estimate:

$$Y_{it} = \alpha + \theta Y_{it-1} + \delta_2 D_{it} + \varepsilon_{it}$$

The resulting estimate is:

$$\delta_2 = \frac{Cov(Y_{it}, \tilde{D}_{it})}{Var(\tilde{D}_{it})}$$

where $\tilde{D}_{it}$ is the residual from a regression of D_{it} on Y_{it-1}.

regress $D_{it} = \gamma Y_{it-1} + \eta_{it}$

residuals are $\tilde{D}_{it} = D_{it} - \underbrace{\hat{\gamma} Y_{it-1}}_{= \hat{D}_{it}}$.

Now substitute $\alpha_i = Y_{it-1} - \varepsilon_{it-1}$ into the true model to get:

$$Y_{it} = \alpha_i + \delta_1 D_{it} + \varepsilon_{it}$$
$$Y_{it} = Y_{it-1} + \delta_1 D_{it} + \varepsilon_{it} - \varepsilon_{it-1}$$

From here, we get:

$$\delta_2 = \frac{Cov(Y_{it}, \tilde{D}_{it})}{Var(\tilde{D}_{it})} = \delta_1 - \frac{Cov(\varepsilon_{it-1}, \tilde{D}_{it})}{Var(\tilde{D}_{it})}$$

$$= \delta_1 - \frac{Cov(\varepsilon_{it-1}, D_{it} - \gamma Y_{it-1})}{V(\tilde{D}_{it})}$$

$$\delta_2 = \delta_1 + \frac{\gamma \sigma_\varepsilon^2}{V(\tilde{D}_{it})}$$ where σ_ε^2 is the variance of ε_{it-1}.

Since trainees have low Y_{it-1}, then $\gamma < 0$ and resulting estimate, δ_2, is too small.

b) Suppose instead that (b) is the correct specification, i.e., the correct thing to do is control for a lagged dependent variable — treatment is determined by low Y_{it-1}. But we mistakenly control for individual fixed effects by first-differencing.

The true model is:

$$Y_{it} = \alpha + \theta Y_{it-1} + \delta_3 D_{it} + \varepsilon_{it}$$

We mistakenly estimate

$$Y_{it} - Y_{it-1} = \alpha + \delta_4 (D_{it} - D_{it-1}) + (\varepsilon_{it} - \varepsilon_{it-1})$$

Here, the resulting estimate is:

$$\delta_4 = \frac{Cov(Y_{it} - Y_{it-1}, D_{it} - D_{it-1})}{Var(D_{it} - D_{it-1})} = \frac{Cov(Y_{it} - Y_{it-1}, D_{it})}{Var(D_{it})}$$

in the simple case where $D_{it-1} = 0$ for everyone but $D_{it} = \{0, 1\}$, that is, no one is treated in the pre-period, but some individuals are treated in the post period.

If the true model is

$Y_{it} = \alpha + \theta Y_{it-1} + \delta_3 D_{it} + \varepsilon_{it}$, we can subtract Y_{it-1} from both sides to get:

$$Y_{it} - Y_{it-1} = \alpha + \theta Y_{it-1} - Y_{it-1} + \delta_3 D_{it} + \varepsilon_{it}$$

$$Y_{it} - Y_{it-1} = \alpha + (\theta - 1) Y_{it-1} + \delta_3 D_{it} + \varepsilon_{it}.$$

Substituting into the inappropriately differenced model yields:

$$\delta_4 = \frac{Cov(Y_{it} - Y_{it-1}, D_{it})}{V(D_{it})} = \delta_3 + (\theta - 1)\left[\frac{Cov(Y_{it-1}, D_{it})}{Var(D_{it})}\right]$$

In general, $0 < \theta < 1$, otherwise Y_{it} is non-stationary, (i.e., an explosive time series process). Since trainees have low Y_{it-1} (so that $Cov(Y_{it-1}, D_{it}) < 0$), then the resulting estimate, δ_4, is too big.