Demographic Methods

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Week 5: Migration

Overview

- ► Migration intro
  - ► Data sources
  - ► Adding into a cohort component projection framework
- ► Migration models
  - ► Two population Leslie Matrix
  - ► Gravity and log-linear models
  - ► Model age patterns
- ► Interesting data
- ► Next week

Migration

Back to the demographic acounting identity

Recall from week 1:

$$P(t+1) = P(t) + B[t, t+1] - D[t, t+1] + I[t, t+1] - O[t, t+1]$$

or

$$P(t+1) = P(t) + B[t, t+1] - D[t, t+1] + N[t, t+1]$$

where $N[t, t+1]$ is net migration.

- migration is the third component of population change
- generally less contribution to population change, but becoming more important
- we've largely ignored it until now, focusing on natural increase/decrease
- adding migration stuffs up (messes up) our idea of generational renewal, etc

Why is migration important?

From a demographic perspective, migration changes the age and sex structure of a population

- ▶ can offset natural decrease
- ▶ reduce dependency ratios (economic implications)

Old age dependency ratio:

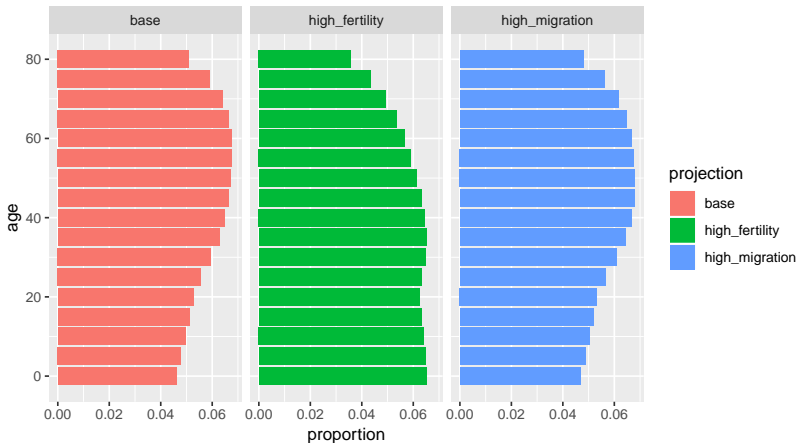$$\frac{\text{Population aged } 65+}{\text{Population aged } 15\text{-}64}$$

Child dependency ratio:

$$\frac{\text{Population aged } 0\text{-}14}{\text{Population aged } 15\text{-}64}$$

Is increasing migration as effective as increasing fertility?

Migration versus fertility effects on population age structure

Canada's population age structure, 2210
Baseline, and fertility and migration increased by 25%

Issues

Definition issues:

- not biological
- geographical element
- depends on intent or subsequent behavior
- migration versus mobility (temporal component)
- contextually dependent (international versus internal)

Measurement issues:

- separate from vital registration
- immigration easier than emigration
- illegality issues

Measurement

$$P(t+1) = P(t) + B[t, t+1] - D[t, t+1] + N[t, t+1]$$

If we knew population size (e.g. from census), births and deaths completely accurately, net migration is just the residual

But in reality, we are usually dealing with

$$P(t+1) = P(t) + B[t, t+1] - D[t, t+1] + N[t, t+1] + \text{error}$$

Measures of migration

Stock: Population that are living in geographic area of interest (state, country) who were not born in that area. Gives idea of overall magnitude but no information about recent changes.

Flows: The number of people moving into (or out of) a geographic area over a time period of interest. Changes in stocks gives some information about flows.

Migration rates: What is the denominator? For emigration it is as before, but for immigration it is not usually the population at risk.

Data sources

Traditional data come from censuses or surveys

- ▶ Inferred from population change
- ▶ Birthplace
- ▶ Place of previous residence
- ▶ Duration of current residence

Example: ACS

15   **a. Did this person live in this house or apartment 1 year ago?**

- [ ] Person is under 1 year old → *SKIP to question 16*
- [ ] Yes, this house → *SKIP to question 16*
- [ ] No, outside the United States and Puerto Rico – *Print name of foreign country, or U.S. Virgin Islands, Guam, etc., below; then SKIP to question 16*

- [ ] No, different house in the United States or Puerto Rico

b. Where did this person live 1 year ago?

Address (Number and street name)

Name of city, town, or post office
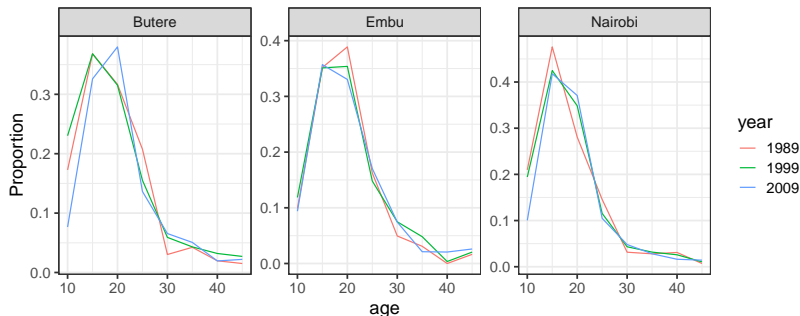
Name of U.S. county or municipio in Puerto Rico

Name of U.S. state or Puerto Rico   **ZIP Code**

Example: Kenya

From the Census we know district of residence one year ago. Can work out net migration by looking at the difference between in and out migrants. Proportion of migrants by age:

Adding migration into a cohort component projection framework

Adding migration into a CCP

Remember from last week, we have the Leslie Matrix population projection set-up:

$$\begin{bmatrix} \text{kids} & \text{kids} & \text{kids} & \ldots & \text{kids} \\ \frac{_5L_{10}}{_5L_0} & 0 & 0 & \ldots & 0 \\ 0 & \frac{_5L_{15}}{_5L_{10}} & 0 & \ldots & 0 \\ \hdotsfor{5} \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}$$

where the kids entries are:

$$_nL_0 \cdot \frac{1}{2}\left(_nF_x + {_nF_{x+n}} \cdot \frac{_nL_{x+n}}{_nL_x}\right) \cdot f_{fab}$$

Adding migration into a CCP

Call this A. Multiply population vector $K(t)$:

$$K(t) = \begin{bmatrix} _5K_0 \\ _5K_5 \\ _5K_{10} \\ \ldots \\ _\infty K_\omega \end{bmatrix}$$

by A to get $K(t+n)$.

$$K(t+n) = AK(t)$$

Adding migration into a CCP

Think about the period t to $t + n$ (e.g. often $n = 5$ so this is a five-year period).

▶ if all migrants arrived at the end of the period, then we would have

$$
K(t+5) = \begin{bmatrix} \text{kids} & \text{kids} & \text{kids} & \dots & \text{kids} \\ \frac{5L_{10}}{5L_0} & 0 & 0 & \dots & 0 \\ 0 & \frac{5L_{15}}{5L_{10}} & 0 & \dots & 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \times K(t) + \begin{bmatrix} {}_5K_0(t){}_5i_0 \\ {}_5K_5(t){}_5i_5 \\ {}_5K_{10}(t){}_5i_{10} \\ \dots\dots\dots \\ {}_\infty K_\omega(t){}_\infty i_\omega \end{bmatrix}
$$

where ${}_n i_x$ is the number of net migrants expressed as a proportion of population size, so that ${}_n K_x \cdot {}_n i_x = {}_n I_x$ is the number of net migrants.

Adding migration CCP

Perhaps a slightly better approximation is assuming that half the migrants arrive at the start of the period and half arrive at the end. Then we have:

$$
K(t+5) = \begin{bmatrix} \text{kids} & \text{kids} & \text{kids} & \ldots & \text{kids} \\ \frac{_5L_{10}}{_5L_0} & 0 & 0 & \ldots & 0 \\ 0 & \frac{_5L_{15}}{_5L_{10}} & 0 & \ldots & 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix} \times \begin{bmatrix} _5K_0(t) + {_5K_0}(t)_5i_0/2 \\ _5K_5(t) + {_5K_5}(t)_5i_5/2 \\ _5K_10(t) + {_5K_1}0(t)_5i_{10}/2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ _\infty K_\omega(t) + {_\infty K_\omega}(t)_5i_0/2 \end{bmatrix}
$$

$$
+ \begin{bmatrix} _5K_0(t)_5i_0/2 \\ _5K_5(t)_5i_5/2 \\ _5K_{10}(t)_5i_{10}/2 \\ \ldots\ldots\ldots\ldots \\ _\infty K_\omega(t)_\infty i_\omega/2 \end{bmatrix}
$$

So half the migrants come in at the start and have time to die/give birth, and half come in at the end of the period.

Adding migrants to CCP

We are assuming that as soon as migrants arrive they experience the same mortality and fertility conditions as the native population.

- In practice, depending on the size of the population, this probably doesn't matter much
- How realistic is this assumption in practice?
    - fertility: usually depends on age at migration, education etc (Adsera and Ferrer (2011) illustrate this for Canada)
    - mortality: depends on migrant group, often observed to be higher in high-skilled migrant groups ('healthy migrant effect')

Example population projection with migration

Canada's population age structure, 2210 with and without migration
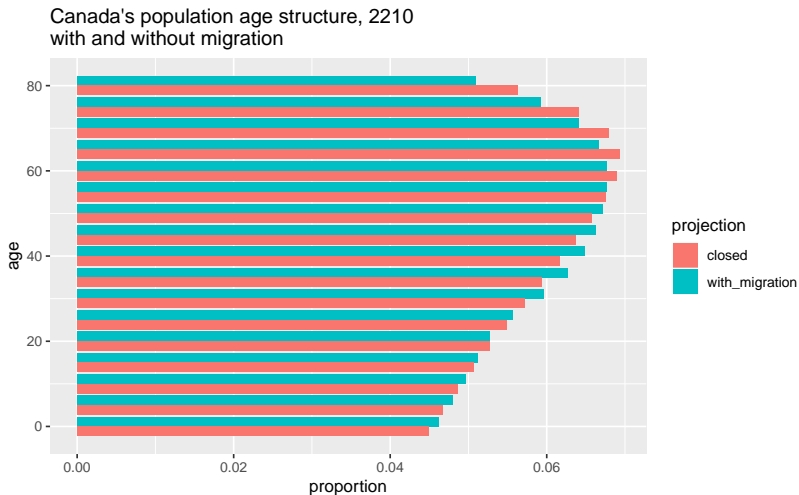
Migration models

Overview

- The focus here is on aggregate models, not individual models
  - But there is an increasing use of agent based models in migration estimation and projection (group at University of Southampton (Bijak et al.))
- Demographers are mostly interested in two dimensions:
  1. net flows into areas (including pairwise relationships between areas)
  2. age patterns of migration

Two population Leslie Matrix

- Interested in capturing movements between two areas (e.g. urban-rural movements)
- Split Leslie Matrix into distinct submatrices based on four populations: urban natives, rural natives, urban migrants and rural migrants.

$$\begin{bmatrix} M^{[\text{city native}]}_{[\text{city native}]} & M^{[\text{city migrant}]}_{[\text{city native}]} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & M^{[\text{city migrant}]}_{[\text{city migrant}]} & M^{[\text{village native}]}_{[\text{city migrant}]} & M^{[\text{village migrant}]}_{[\text{city migrant}]} \\ \mathbf{0} & \mathbf{0} & M^{[\text{village native}]}_{[\text{village native}]} & M^{[\text{village migrant}]}_{[\text{village native}]} \\ M^{[\text{city native}]}_{[\text{village migrant}]} & M^{[\text{city migrant}]}_{[\text{village migrant}]} & \mathbf{0} & M^{[\text{village migrant}]}_{[\text{village migrant}]} \end{bmatrix}$$

- migrant to migrant transitions represent return migration (could simplify by assuming no return migration)
- migrant to native transitions represent births of migrants

Two population Leslie Matrix

$$
\begin{bmatrix}
M_{[\text{city native}]}^{[\text{city native}]} & M_{[\text{city native}]}^{[\text{city migrant}]} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & M_{[\text{city migrant}]}^{[\text{city migrant}]} & M_{[\text{city migrant}]}^{[\text{village native}]} & M_{[\text{city migrant}]}^{[\text{village migrant}]} \\
\mathbf{0} & \mathbf{0} & M_{[\text{village native}]}^{[\text{village native}]} & M_{[\text{village native}]}^{[\text{village migrant}]} \\
M_{[\text{village migrant}]}^{[\text{city native}]} & M_{[\text{village migrant}]}^{[\text{city migrant}]} & \mathbf{0} & M_{[\text{village migrant}]}^{[\text{village migrant}]}
\end{bmatrix}
$$

The elements of each M are: Fertility rates for the four populations; Survival probabilities for the four populations; Probabilities of migration.

Assuming

- survival probabilities are positive
- migration rates are positive
- fertility rates are positive for any two adjacent age groups

the there exists a stable population $\tilde{K}$ such that $M\tilde{K} = \lambda\tilde{K}$.

So can do our eigendecomposition on the whole matrix or each submatrix.

Example
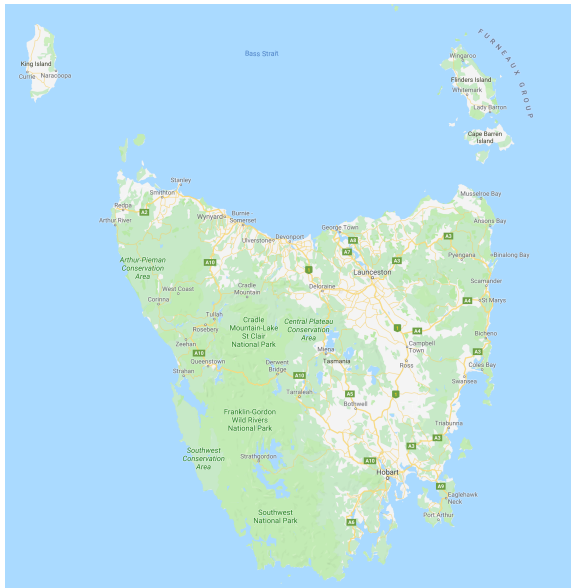
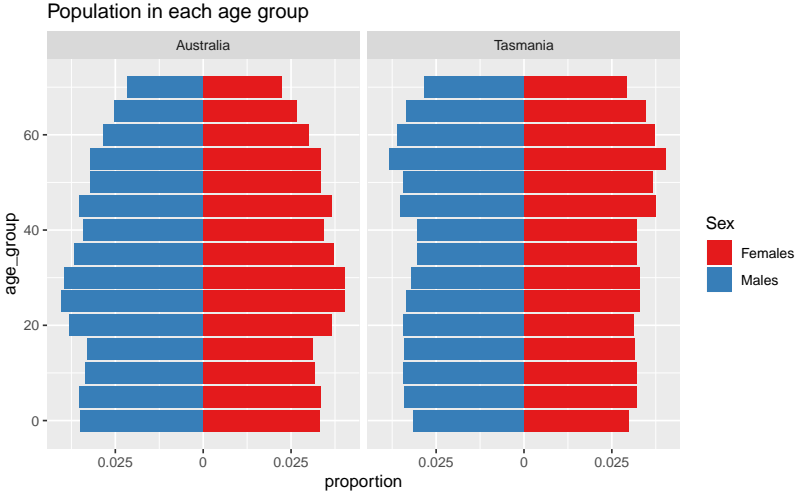# Hobart
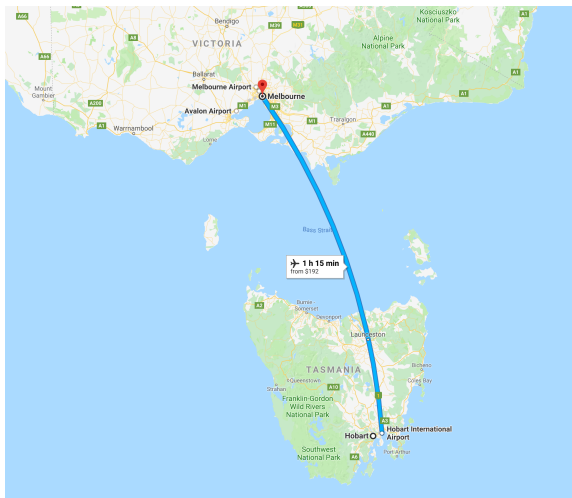
Melbourne

Population age pyramids

Population in each age group

Set up two-population Leslie Matrix

$$
\begin{bmatrix}
M^{[\text{city native}]}_{[\text{city native}]} & M^{[\text{city migrant}]}_{[\text{city native}]} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & M^{[\text{city migrant}]}_{[\text{city migrant}]} & M^{[\text{village native}]}_{[\text{city migrant}]} & M^{[\text{village migrant}]}_{[\text{city migrant}]} \\
\mathbf{0} & \mathbf{0} & M^{[\text{village native}]}_{[\text{village native}]} & M^{[\text{village migrant}]}_{[\text{village native}]} \\
M^{[\text{city native}]}_{[\text{village migrant}]} & M^{[\text{city migrant}]}_{[\text{village migrant}]} & \mathbf{0} & M^{[\text{village migrant}]}_{[\text{village migrant}]}
\end{bmatrix}
$$

- Tasmania is 'village'
- Victoria is 'city'
- Note: assume that migrants are always migrants, but they give birth to natives

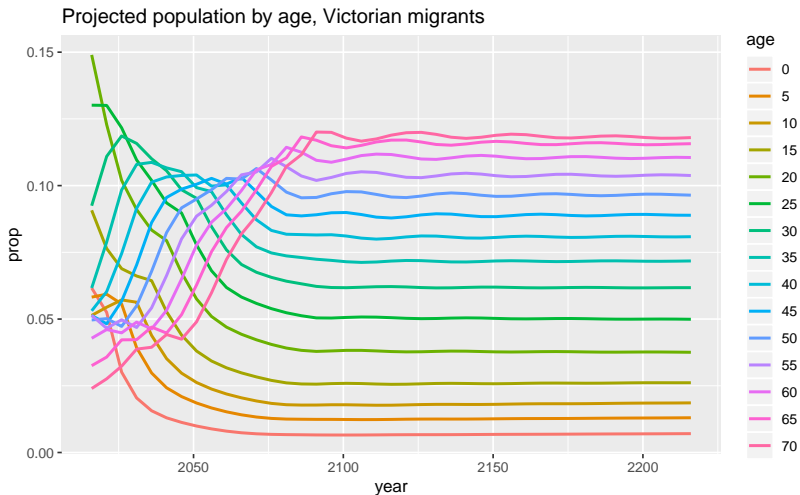# Migration by age

Proportion of population migrating by age
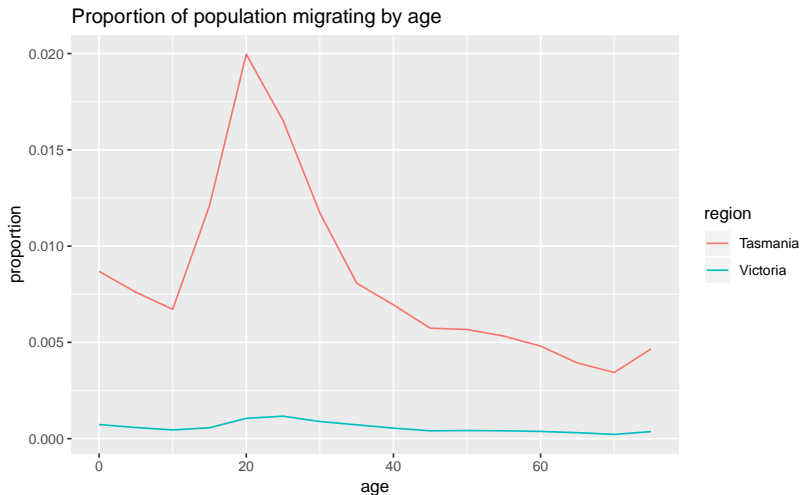
- ▶ majority (95%) of Tasmanians do not return after migrating
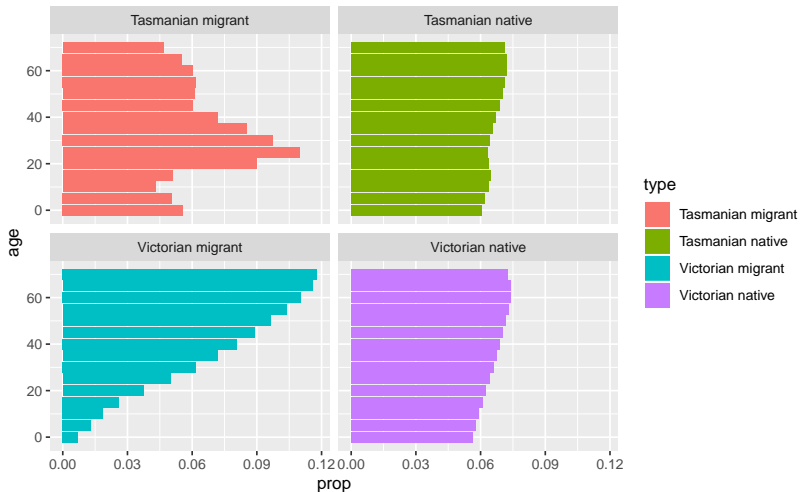- ▶ majority (80%) of Victorians return after migrating

Projections of migrant populations

Projected population by age, Tasmanian migrants

Projections of migrant populations
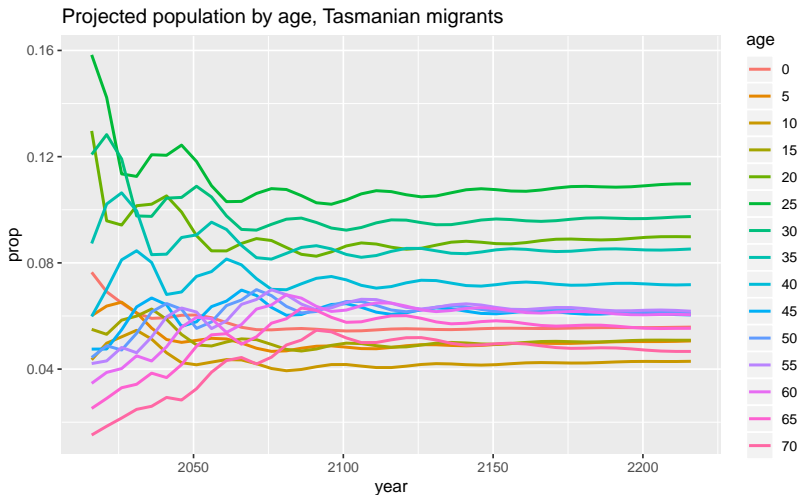
Projected population by age, Victorian migrants

Stable populations

Gravity model

Resembles Newton's law of gravity. The idea that spatial interaction is related to stocks and inversely related to distance.

$$M_{ij} = G \frac{K_i^{\alpha} K_j^{\beta}}{D_{ij}^{\gamma}}$$

- G is some proportionality constant
- K_i is population of area i
- K_j is population of area j
- α is strength of 'push' factors
- β is strength of 'pull' factors
- D_{ij} is distance

Gravity model

On the log scale

$$\log M_{ij} = \log G + \alpha \log K_i + \beta \log K_j - \gamma \log D_{ij}$$

- Easy to fit
- But:
    - predicts symmetric flows between i and j
    - doesn't include push/pull factors other than population and distance

A more general representation

$$\log M_{ij} = \mu + \alpha_i + \beta_j + \gamma_{i,j}$$

for $i \neq j$.

- ▶ Push/pull factors need not just be population size
- ▶ Interaction effect need not be a linear function of distance

Note that this is a saturated model, not really a model, but a way of representing the data.

To make identifiable, set $\alpha_0 = \beta_0 = \gamma_{1j} = \gamma_{i1} = 0$.

Log-linear models

Quasi-independent generally fits reasonably well to large populations:

$$\log M_{ij} = \mu + \alpha_i + \beta_j$$

for $i \neq j$. 'Quasi' because we are not including the diagonal terms in the model.

Could also put some structure on the interaction term, for example

$$\log M_{ij} = \mu + \alpha_i + \beta_j + \gamma d_{ij}$$

$d_{ij} = 1$ when areas are adjacent and zero otherwise.

Example

US inter-regional movements, 2016-2017. Looking at migrants only, so diagonals are 0.

2016	2017 Northeast	Midwest	South	West	Total
Northeast	0	116096	529678	179832	825606
Midwest	92079	0	540847	284665	917591
South	304617	427617	0	471788	1204022
West	128870	246592	496148	0	871610
Total	525566	790305	1566673	936285	3818829

Fit

using `glm` in R:

Inter−regional migration, US 2016−2017
Data from ACS, fitted with log−linear model
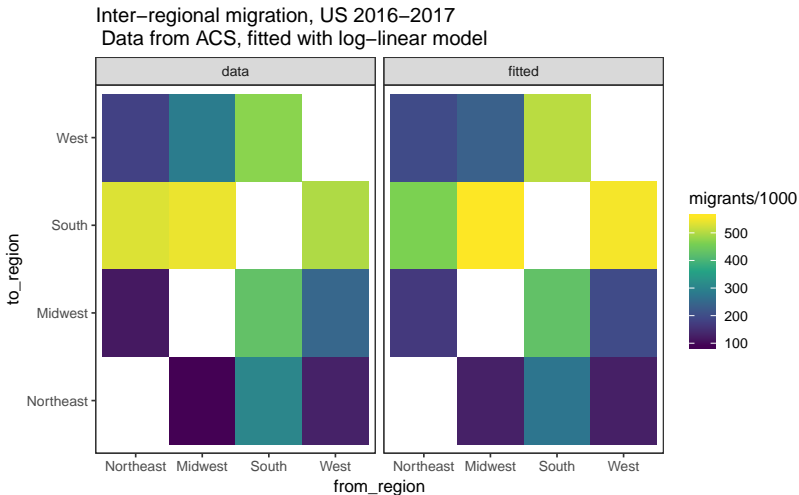
Using log-linear models with missing data

In some cases, we may not have information on all cells.

Example: migration between countries (e.g. Europe)

- probably know marginal totals (i.e. number of migrants in each countries) M_{i+} and M_{+j}
- may only have information on source / destination breakdown for some countries

As long as we know marginals and have some information on the cells, we can use a log-linear model to fill in estimates (Raymer 2007).

Using log-linear models with missing data

Model set-up:

y_{ij} are observed flows with some error.

$$
\begin{aligned}
y_{ij} &\sim N(M_{ij}, \sigma^2) \\
\log M_{ij} &\sim N(\mu + \alpha_i + \beta_j + \gamma d_{ij}, \tau^2) I\left[\sum_j M_{ij} = M_{i+}, \sum_i M_{ij} = M_{+j}\right] \\
\alpha &\sim N(0, \sigma_\alpha^2) \\
\beta &\sim N(0, \sigma_\beta^2) \\
\gamma &\sim N(0, \sigma_\gamma^2)
\end{aligned}
$$

with hyperpriors on variance parameters.

Model age patterns of migration

Migration often occurs in conjunction with some transition in the life course:

- Adult migration peaks at young adult ages (work, education)
- Second peak around retirement
- Child peak associated with parents' migration

Model age patterns of migration

Multi-exponential model of migration (Rogers and Castro, 1981). Migration rate $m(x)$ is modeled as:

$$m(x) = a_1 \exp(-\alpha_1 x) + a_2 \exp\left(-\alpha_2(x - \mu_2) - \exp[\gamma_2(x - \mu_2)]\right)$$
$$+ \ a_3 \exp\left(-\alpha_3(x - \mu_3) - \exp[\gamma_3(x - \mu_3)]\right) + c$$
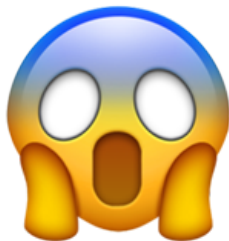
Model age patterns of migration

$$m(x) = a_1 \exp(-\alpha_1 x) + a_2 \exp\left(-\alpha_2(x - \mu_2) - \exp[\gamma_2(x - \mu_2)]\right)$$
$$+ \; a_3 \exp\left(-\alpha_3(x - \mu_3) - \exp[\gamma_3(x - \mu_3)]\right) + c$$

Elderly Ret. Peak and Post-Ret.
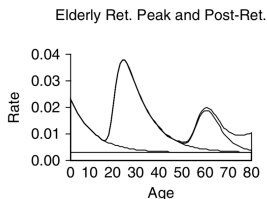
- α's and γ's are rates of descent and ascent
- μ'a are peak ages
- all others are intensity parameters

Example

We can fit this horrible looking model in Stan.

For Florida in 2017: $\mu_1 = 24.5$, $\mu_2 = 60$.

Florida migration age schedule, 2017
Data and Rogers–Castro fit
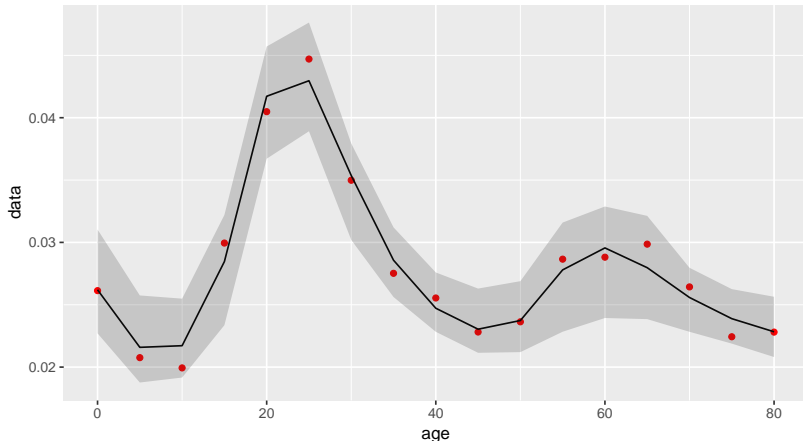
Interesting data sets

Migration data

- Often migration information available from traditional/official sources is sparse or delayed in release
- Lots of examples of researchers using interesting sources of information to try and measure migration and mobility
  - cell phone data (mobility, short-term)
  - school enrollments
  - tax filings
  - social media (Twitter, Facebook/Instagram, LinkedIn)

Social media and migration

x

Example: Mexican migrant age distributions in the USA
Facebook versus ACS
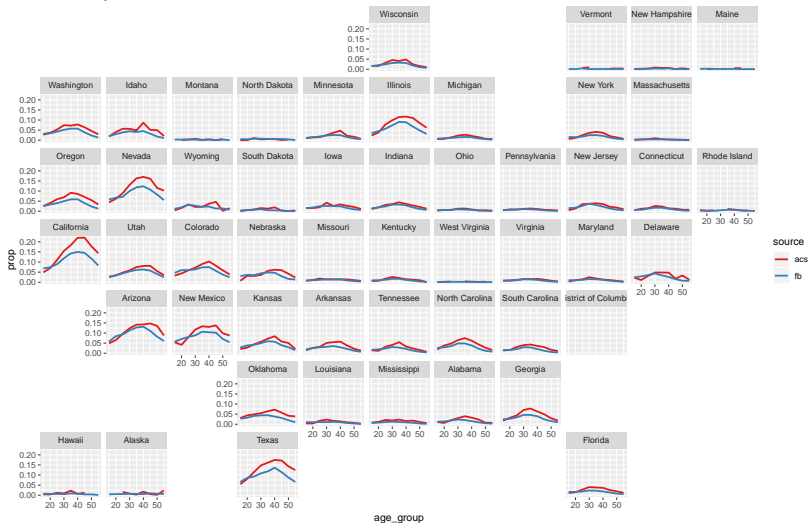
Bias is substantial, but can be modeled

- Adjust Facebook data based on gold standard
- Build time series model incorporating historical trends and new Facebook data
- Projections are informed by both

Next week

Next week

I will probably talk for a bit about interesting methodological issues in demography

Project:

- ▶ Short write up (min 5 pages)
  - ▶ Question of interest
  - ▶ Data
  - ▶ Method
  - ▶ Results
- ▶ Preferably send me Rmd and data so I can reproduce
  - ▶ If data too big etc, give clear instructions about what you did
- ▶ Short presentation ($\sim$ 5 slides, 10 mins)