VARIANCE EFFECTS IN THE BONGAARTS-FEENEY FORMULA*

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Bongaarts and Feeney have recently proposed an adjusted total fertility rate to disentangle tempo effects from changes in the quantum of fertility. We propose an extension to the Bongaarts and Feeney formula that includes variance effects: that is, changes in the variance of the fertility schedule over time. If these variance effects are ignored, the mean age at birth and the adjusted total fertility rate are biased. We provide approximations for these biases, and we extend the TFR adjustment to fertility schedules with changing variance. We apply our method to the Swedish baby boom and bust, and show that variance effects are important for evaluating the relative contributions of tempo and quantum effects to the fertility change from 1985 to 1995.

Bongaarts and Feeney (1998) recently have proposed an *adjusted total fertility rate* (TFR'), which helps to disentangle the tempo and the quantum components of period fertility. Conditional on a set of regularity assumptions, the adjusted total fertility rate equals the TFR that would have been observed if there had been no change in the timing of births. The adjusted total fertility rate, TFR', is calculated separately for each parity as

$$\text{TFR}'_i = \text{TFR}_i \,/\, (1 - r_i), \tag{1}$$

where TFR_i is the observed total fertility rate for births of order i in any given year and r_i is the change in the mean age at childbearing at order i between the beginning and the end of a year. This adjusted TFR is closely related to Ryder's work on the demographic translation (Ryder 1956, 1983; also see Keilman 1994; Zeng Yi and Land 1999) and to more recent studies on the importance of cohort and period influences on fertility (e.g., Calot 1992; Foster 1990; Ní Bhrolcháin 1992; Pullum 1980).

The adjusted TFR, when applied to western and eastern European countries, frequently suggests that the low observed fertility rates are due largely to tempo effects (Bongaarts 1999; Lesthaeghe and Willems 1999; Philipov and Kohler forthcoming). Some recent fertility declines in European countries, however, do not fit the framework analyzed by Bongaarts and Feeney (BF) in an important respect: the increase in the mean age at birth was frequently accompanied by an increase in the variance of the fertility schedule (see Table 1). These increases in variance are relevant from at least two perspectives. First, they may be characteristic for the recent fertility decline in European countries, and thus may indicate aspects of fertility change that are beyond pure tempo effects or mere increases in the mean age at birth (see also Zeng Yi and Land 2001). Second, these variance changes are problematic in the context of the BF adjustment of the total fertility rate: a critical assumption of this adjustment is the absence of age-period interactions or changes in the shape of the fertility schedule. The above-mentioned variance changes, however, constitute a trend toward a different shape of the fertility schedule, and therefore represent a potentially relevant violation of the assumptions that underlie the Bongaarts-Feeney formula.

To investigate these variance effects further, we generalize the approach taken by Bongaarts and Feeney (1998). In particular, we extend the BF formula to allow for a changing variance of the fertility schedule. This extension relaxes the critical assumption, in the BF model, of "no age-period interactions." In our analysis, the tempo effect can vary with age in a systematic manner, and the variance of the fertility schedule can increase or decrease over time. The results indicate that the Bongaarts-Feeney adjustment is sensitive to changes in the variance of the fertility schedule, particularly when the rate of change in the variance increases over time. We show that applying the BF adjustment to periods with an increasing mean age and variance leads to a downward bias in the estimated mean age and the annual change thereof. We provide analytic expressions to approximate these biases, and

TABLE 1. CHANGE IN THE MEAN AGE AT BIRTH AND IN THE VARIANCE OF THE FERTILITY SCHEDULE FOR FIRST AND ALL BIRTHS BETWEEN 1990 AND 1995, SELECTED EUROPEAN COUNTRIES

	Percentage Change Between 1990 and 1995			
	First Births		All Births	
Country	Mean Age	Variance	Mean Age	Variance
Sweden	+3.32	+10.70	+2.28	+0.54
Italy	+4.49	+9.77	+3.08	–0.08
Hungary	+3.10	+18.90	+3.09	+7.78
Czech Republic	+3.80	+27.60	+4.00	+10.40

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we suggest an iterative procedure that includes variance effects in the adjustment of the total fertility rate.

A detailed analysis of the Swedish fertility pattern reveals that increases in the variance largely account for the changes in the shape of the fertility schedule during the baby boom and bust of 1985 to 1995. More important, the incorporation of these variance effects modifies some of the conclusions obtained by adjusting the total fertility rate. The adjusted TFR, according to Bongaarts and Feeney (1998), suggests that a significant part of the fertility increase in the late 1980s was due to a reduced tempo effect for first births. The adjusted TFR with variance effects, on the other hand, suggests that the tempo of fertility for first births remained almost constant throughout the period of the baby boom and bust. After accounting for the changing variance in the fertility schedule, first births still were postponed by about 0.13–0.14 year annually without any major changes in the pace of fertility postponement during 1985 to 1995.

Despite these changes in the interpretation of the Swedish baby boom and bust, our analysis supports the adjustment of the total fertility rate as an important aspect in the evaluation of contemporary low fertility patterns: the adjustment of the TFR can remove distortions in the total fertility rate caused by changes in the tempo of fertility, and thus provides means to assess the quantum of fertility during periods in which births increasingly are postponed to later ages. The incorporation of variance effects, in our opinion, improves the inferences obtained from the adjustment of the TFR, but in many cases the BF-adjusted TFR may provide a close approximation that accounts for the primary effect caused by the postponement of births.

Our analysis of the Bongaarts-Feeney formula in this paper is structured as follows. In the next section we further explain the relevance of variance effects, using data from Sweden. Next we prove the BF adjustment in a more general fashion that allows for age-period interactions. We then use this more general approach to include variance effects: that is, changes in the variance of the fertility schedule in the adjustment of the TFR. In the next section we investigate the relevance of these variance effects during the Swedish baby boom and bust. In the final section we summarize and conclude the paper. An appendix contains proofs and derivations omitted in the text.

CHANGING VARIANCE IN THE AGE AT BIRTH: THE SWEDISH EXAMPLE

Swedish fertility during the 1980s and 1990s has been of great interest to demographers because of its distinctive and unusual pattern. Whereas fertility levels stagnated or declined in many European countries during the 1980s, Sweden experienced a baby boom after 1985 (see Graph 1a, Figure 1).[1] Between 1984 and 1990 the TFR increased from 1.66 to 2.14, exceeding replacement level. Surprisingly, this upsurge in fertility was accompanied by an increase in women's participation in the labor force: between 1985 and 1990, this participation rate rose from an already high level of 78% to 81%, the highest in Europe at the time (OECD 1998). In the 1990s this baby boom was displaced by an equally rapid baby bust, and by 1997 the total fertility rate had declined to a historically low level of 1.53 (Council of Europe 1998).

Graph 1b in Figure 1 shows that the postponement of fertility lost some of its momentum during the late 1980s. The baby boom was due in part to an increase in fertility rates at younger ages, especially below age 25 (Hoem and Hoem 1997a). Between 1975 and 1985, the mean age at first birth rose from 24.5 to 26.1 years, but it increased only marginally, to 26.3 years, during the baby boom period up to 1990. After 1990, the postponement of births regained its momentum as fertility declined, and by 1995 the mean age at first birth had risen to 27.3 years (Council of Europe 1998). The Swedish baby boom and bust therefore are associated with substantial changes in the tempo of fertility. This variation in tempo makes Sweden a particularly appropriate case for the application of the adjusted total fertility rate.

The adjusted TFR in Graph 1a, Figure 1 is relatively constant in the initial phase of the fertility increase, whereas the observed TFR converges toward its adjusted counterpart. For first births the difference between the observed and the adjusted TFR decreases from 0.16 in 1983 to 0.03 in 1988; for all births (not shown in Figure 1) the difference decreases from 0.27 to 0.06 during the same period. The comparison with the adjusted TFR therefore suggests that the increase in fertility during this period is due, to a significant extent, to a decline in the pace of fertility postponement.

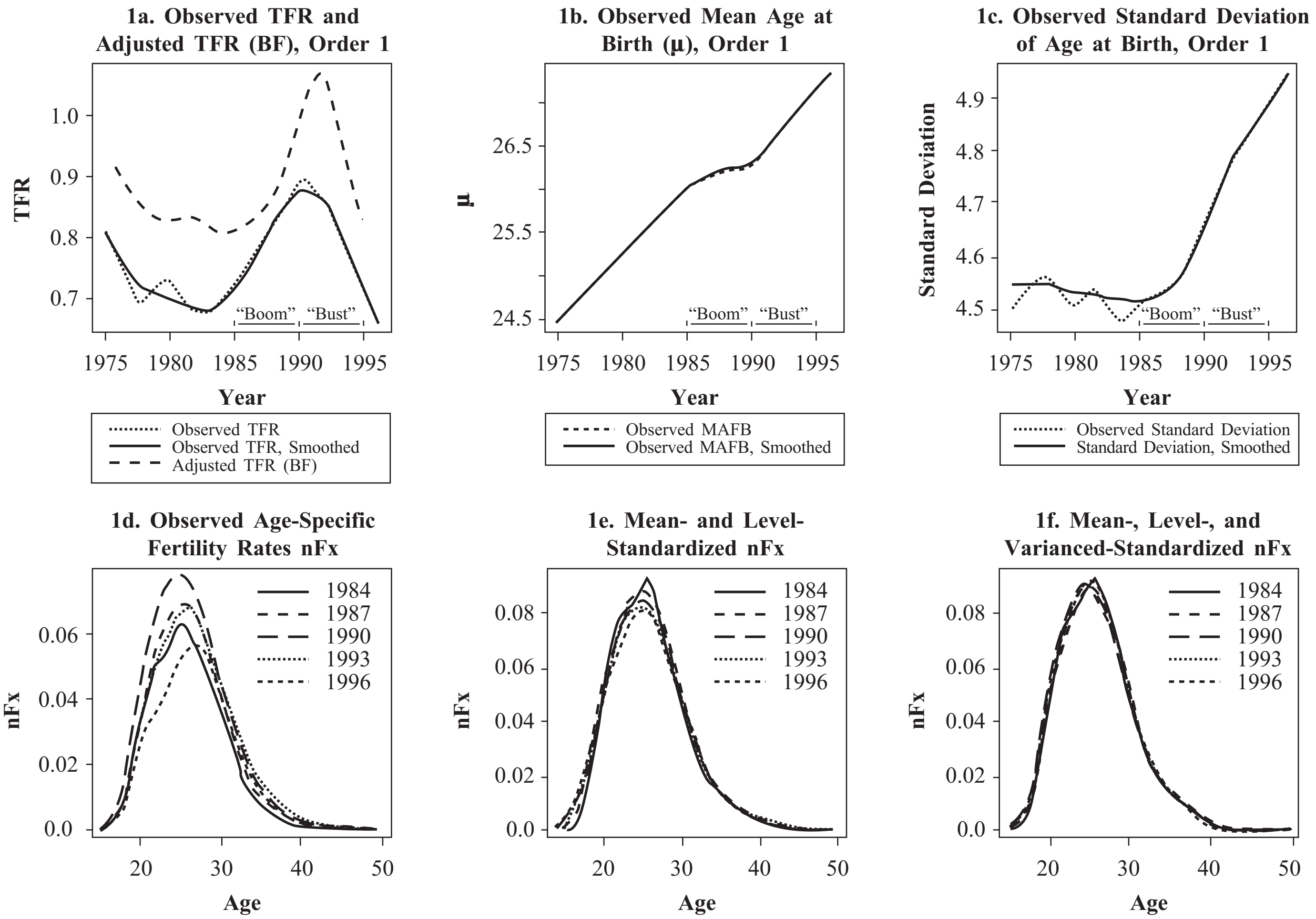
After 1988 the situation changes. The adjusted and the observed TFR diverge considerably as the tempo of fertility begins to increase in the late phase of the baby boom. The adjusted TFR for first births as well as for all births peaks in 1992, two years after the observed TFR has reached its highest level. This means that the initial decline in the TFR after 1990 is apparently due to the recurrence of a strong tempo effect.[2] After 1992, however, the adjusted TFR declines considerably; this suggests the presence of a substantial quantum component in the recent decline in fertility rates.

In the following discussion we analyze whether variance effects are relevant for the above assessment of the Swedish experience in terms of tempo and quantum components. The trend in the standard deviation of the fertility schedule after 1975 is illustrated in Graph 1c, Figure 1. Until about 1985

1. The data for Sweden were obtained from the Eurostat (1998) *New Cronos* database. The age-specific fertility rates are calculated from the number of births by order, mother's age, and the midyear population in the respective years. The series for the TFR, the mean age, and the variance of the fertility schedule are smoothed in all cases (using the robust running-median smoother known as 4(3RSR)2H before the calculation of the adjusted total fertility rates. This smoothing removes minor and apparently random fluctuations from the respective series.

2. The adjusted TFR in 1991–1992 actually exceeds 1. This finding is *not* inconsistent with the fact that within a cohort each woman can have at most one first child. The adjusted TFR is purely a period measure, similar to the usual TFR. Therefore it is not restricted by the possible fertility experience of real cohorts, at least in the short run. Thus it is perfectly consistent with the theory underlying the adjustment of the total fertility rate: that the adjusted TFR for first births exceeds 1 for a limited period. Also see the discussion in Kim and Schoen (2000), Bongaarts and Feeney (2000), and Kohler and Ortega (2001) for the relation between the adjusted TFR and cohort fertility.

FIGURE 1. FIRST BIRTHS IN SWEDEN



Notes: 1a, 1b, and 1c: Observed and BF-adjusted TFR, mean age at first birth, and standard deviation of the fertility schedule. (The full lines represent smoothed series on which all calculations are based.) 1d, 1e, and 1f: Observed age-specific fertility rates from 1984 to 1996, and corresponding standardized fertility schedules. The schedules in 1e imply an equal TFR and mean age; in 1f, in addition, they imply the same variance.

the standard deviation was relatively constant, with some minor fluctuations around the level of 4.55. With the onset of the baby boom in 1985, however, it began to increase significantly and has not ceased to do so since that time.

This increase in the width of the fertility schedule is also visible in a comparison of the observed fertility schedules. Graph 1d, Figure 1 depicts the age-specific fertility rate for first births in Sweden from 1984 to 1996. The sequence of these fertility schedules reflects, on the one hand, the increase in fertility rates, particularly at relatively young ages, from 1984 to 1990. On the other hand, the schedules after 1990 illustrate both the decline and the renewed postponement of fertility during the baby bust.

Graph 1e depicts the fertility schedules after they have been standardized so as to exhibit the same fertility level and the same mean age at birth. Thus, in moving from Graph 1d to Graph 1e, we have standardized for all aspects of the fertility change that are consistent with the assumptions of the BF adjustment of the TFR. Therefore any remaining differences between the fertility schedules shown in Graph 1e constitute deviations from the assumptions underlying the adjusted total fertility rate. The most noticeable difference between the fertility schedules depicted in Graph 1e is the trend toward an increased width, or an increased variance, of the fertility schedule. The schedule for 1984 is concentrated most highly around its peak at age 25; the schedule for 1996 is markedly wider. During the period from 1984 to 1996, fertility became more widely spread and more diversified across ages; this pattern is reflected by the increase in the standard deviation of the fertility schedule after 1985.

Graph 1f shows the fertility schedules after they have been standardized in addition for the changing variance. The schedules in this graph exhibit not only the same fertility level and mean age at birth but also the same standard deviation. Most of the differences between the fertility schedules vanish. Incorporating variance changes therefore is a major step toward including changes in the fertility schedule that extend beyond the two aspects explained by the Bongaarts-Feeney adjustment, namely changes in the fertility level and shifts in the mean age. In combination, however, the three influences—tempo, quantum, and variance changes—provide an accurate characterization of fertility developments, even during periods of substantial transformations in the fertility pattern. In the next sections we focus on quantifying the influence of these variance effects on the adjustment of the TFR.

VARIANCE EFFECTS AND THE TFR ADJUSTMENT

Allowing for Age-Period Interactions in the TFR Adjustment

The analysis of tempo effects in both Ryder's and Bongaarts and Feeney's approach is based on the idea that the postponement of fertility occurs gradually over time or across cohorts. That is, as individuals or cohorts are exposed to different socioeconomic conditions or cultural changes, they progressively delay childbearing to later ages. An important conceptual difference between the approaches of Ryder and of Bongaarts-Feeney is whether "cohort" or "period" is the most important determinant of this postponement. These two approaches are blended by including age-period interactions in the analysis of tempo effects.

In this section we develop a general framework of fertility postponement in which changes in the tempo of fertility can depend on both period and age. Tempo in this context refers to the timing of fertility, and changes in the tempo refer to either a postponement or an advancement of fertility. Tempo effects are the distortions in the observed period TFR and age-specific fertility rates that are due to changes in the timing of fertility. In our discussion, we focus on a delay of childbearing because this is the most common contemporary pattern. The analyses, however, apply analogously to situations where fertility is advanced toward younger ages.

The subsequent framework pertains to births of a given parity and is based on three elements: (1) the cumulated tempo $R(a,t)$, measured in years, which reflects the total amount of postponement that has accumulated for the cohort that is age a at time t; (2) the age- and period-specific tempo change $r(a,t)$, which is derived from $R(a,t)$ and corresponds conceptually to the term r in the Bongaarts-Feeney formula, but also allows for age-period interactions in the pace of fertility postponement; and (3) the adjusted age-specific fertility rate $g'(a,t)$, which represents the fertility rate that would have been observed at age a at time t if there had been no tempo effect. For simplicity we consider the fertility rates and the tempo of fertility in continuous time rather than discrete calendar years.

For the analyses in this section we assume that the cumulated tempo of fertility and the adjusted fertility rates are known, and we derive the observed fertility rates from these elements. It is apparent that this model without further restrictions is not identified empirically: that is, we cannot proceed in the reverse direction from the observed to the adjusted fertility rates. The investigation of variance effects in the next section is one application that is based on a specific, empirically identified functional form for $R(a,t)$ and $r(a,t)$. Nevertheless, the central finding in this section is not restricted to this application. In particular, we establish in Result 3 that the observed fertility rate $g(a,t)$ at age a at time t equals $1 - r(a,t)$ times the adjusted fertility rate $g'(a,t)$. This finding therefore shows that the main insight of the TFR adjustment in Eq. (1) can be extended to age-specific fertility rates: the ratio between the observed fertility rate at age a and at time t, and the hypothetical fertility rate that would have been observed if there had been no tempo effect, equals $1 - r(a,t)$. The age- and period-specific tempo change $r(a,t)$ in this relation can be specified quite generally. For instance, it can depend only on age a or on time t, only on cohort $t - a$, or on quite general interactions between age and period.

The subsequent formal framework is derived by relating the observed age and time of a birth to the age and time when this birth would have occurred if there had been no postponement of fertility. We denote as a and t respectively the observed age and time of a birth, and denote as α and τ the hypothetical age and time at which births would have been observed if fertility had not been postponed.

Our primary indicator of the postponement of fertility is the *cumulated tempo* $R(a,t)$, which measures the total amount of postponement that accumulated for the cohort that is age a at time t. This cumulated tempo establishes the relation between the observed and the hypothetical occurrence of a birth. In particular, we assume that births that occur at age a at time t would have occurred at an age $\alpha = a - R(a,t)$ at time $\tau = t - R(a,t)$ if there had been no postponement of fertility.[3] These two relations reflect the fact that a postponement of fertility delays births to both a higher age and a later time period. The extent of this fertility postponement is measured by the cumulated tempo $R(a,t)$. (Negative values for $R(a,t)$ can represent an advancement of fertility: that is, a pattern in which births tend to occur at earlier rather than at later ages.) Derived from this cumulated tempo is the *age- and period-specific tempo* $r(a,t)$, which measures the incremental postponement of fertility that occurs at age a at time t.

For illustration we consider two specific examples of $R(a,t)$. The first implies the Bongaarts-Feeney approach with a constant tempo r. The second is a simple extension with age-period interactions.

3. The present specification starts with the observed age a and time t of a birth, and expresses the hypothetical α and τ as a function of the observed a and t. In an alternative specification, one can start with the hypothetical age α and time τ, and express the observed age a and time t as a function of α and τ. Although this latter alternative is feasible, the former is more intuitive and somewhat easier to formalize.

In the BF model with constant r, the delay of fertility depends neither on age nor on period. Hence cohorts at all ages, in addition, postpone fertility by an incremental amount of r years per annum (assume $r > 0$). The cumulated tempo is therefore a linear function of time with $R(t) = r \cdot t$. A birth that is observed at age a at time t hence would have occurred at age $\alpha = a - R(t) = a - r \cdot t$ at time $\tau = t - R(t) = t - r \cdot t$ if there had been no postponement of fertility.

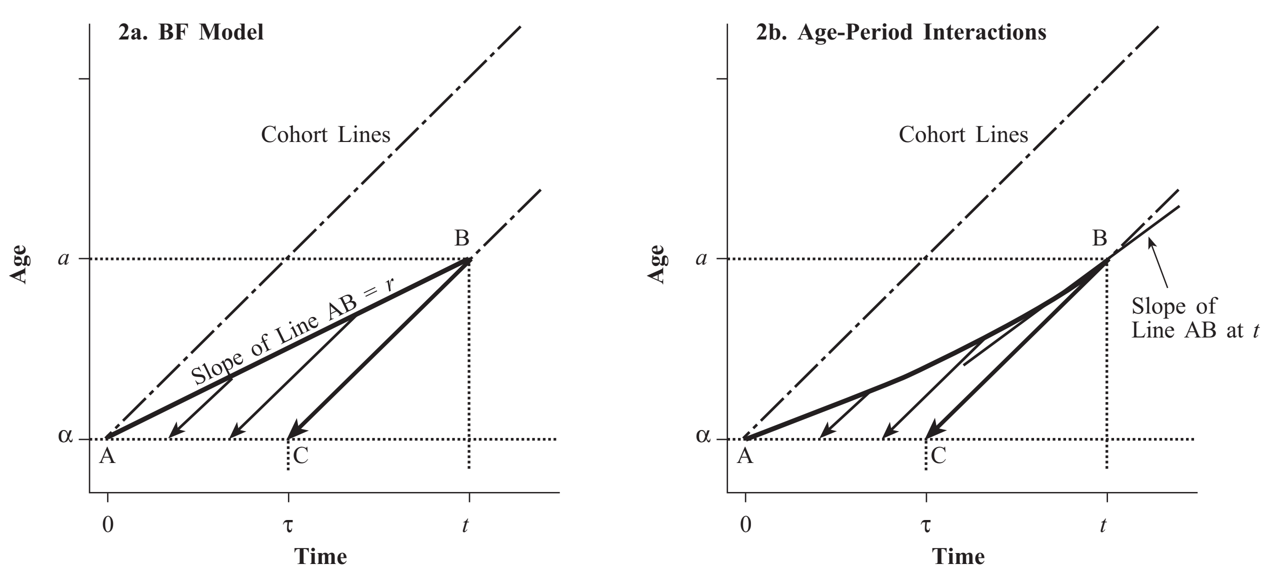
This relation between the observed and the hypothetical occurrence of a birth is indicated in Figure 2a. The figure graphs the observed age (vertical axis) and time (horizontal axis) of births, and it includes diagonal lines that represent the life course of cohorts. Arrow BC in this figure indicates the extent of fertility postponement. It shows how births, which are observed at some age a at time t, would have occurred to the same cohort but at an earlier age α and time τ in the absence of a fertility postponement. Arrow BC therefore reflects the total amount of postponement that has occurred up to time t.

The annual change in the tempo of fertility can be seen in Figure 2a by relating all births that would have occurred at the same age α in the absence of a fertility postponement. These births occur along combinations of a and t that satisfy $a - R(t) = \alpha$, which in this example is given by the straight line AB with slope r. The distance at any time t between line AB and the horizontal line at age α measures the extent to which births that would have occurred at age α have been postponed toward later ages through cumulated tempo effects. At time 0, the cumulated tempo is 0, and no postponement has yet occurred. The postponement of fertility subsequently increases by a constant amount r per unit of time, and line AB increasingly diverges from the horizontal line at age α. The slope of line AB thus measures the change in the tempo of fertility: this slope reflects the incremental postponement of births, which would have occurred at a constant age α, toward higher ages per unit of time.

Figure 2 also provides some insight about the BF adjustment formula itself. Consider births that occur between time 0 and time t along line AB. If there had been no postponement of fertility, these births would have occurred during the period from time 0 to time τ at a constant age α. The postponement of fertility therefore implies that births are spread across a longer period: births that would have occurred between time 0 and time τ actually occur between time 0 and time t. If we assume for the moment a uniform age distribution and a uniform distribution of births, then the ratio of the length of these time periods $\frac{t}{\tau} = \frac{t}{t - r \cdot t} = \frac{1}{1 - r}$ shows that the observed fertility rate along line AB equals $1 - r$ times the fertility rate that would have been observed at an age α if there had been no tempo effects. The adjusted age-specific fertility rates along line AB in this example therefore equal $g'(a,t) = \phi(a - R(t)) = \phi(\alpha)$, where we denote as $\phi(\alpha)$ the fertility rate that would have been observed at age α if fertility had been postponed.

Consider next a simple extension of the above model that is obtained by specifying $R(a,t) = \rho \cdot a \cdot t$, where $\rho > 0$ is a parameter. In this case the cumulated tempo $R(a,t)$ includes an age-period interaction. Despite the presence of this interaction, the interpretation of $R(a,t)$ remains: the cumulated tempo specifies that a birth observed at age a at time t would have occurred at age $\alpha = a - R(a,t) = a - \rho \cdot a \cdot t$ at time $\tau =$

FIGURE 2. POSTPONEMENT OF FERTILITY IN THE BONGAARTS-FEENEY MODEL (2a) AND WITH AGE-PERIOD INTERACTIONS (2b)



$t - R(a,t) = t - \rho \cdot a \cdot t$ if there had been no postponement of fertility.

In Figure 2b the relation between the observed and the hypothetical occurrence of a birth again is indicated by arrow BC. We can also define a line AB, which connects all births that would have occurred at an identical age α in the absence of a fertility postponement. Formally this line is defined by the combinations of a and t that satisfy $a - R(a,t) = \alpha$. In our specific example this condition can be solved as $a = \alpha / (1 - \rho t)$. In contrast to Figure 2a, line AB in Figure 2b is no longer straight but is curved upward. This curvature is a consequence of the age-period interactions in the fertility postponement.

Our earlier discussion of the point that the change in the tempo of fertility is measured by the slope of line AB extends directly to the situation involving age-period interactions. In particular, we define the age- and period-specific tempo $r(a,t)$ as the slope of line AB at age a and time t. This definition is appropriate because line AB traces all births that would have occurred at a constant age α in the absence of a fertility postponement. The slope of line AB thus indicates the incremental postponement of births, which would have occurred at a constant age α in the absence of tempo effects, per unit of time. The curvature of line AB in this example implies that the tempo change $r(a,t)$ increases with age.

It is noteworthy that the term $r(a,t)$ does *not* indicate the increase in the cumulated tempo of cohorts. Increases in cohort tempo are measured by increases of $R(a,t)$ along the diagonal cohort lines in Figure 2. In the presence of age-period interactions, however, the increase of $R(a,t)$ along cohort lines confounds two aspects because of the passage of time and the aging of the respective cohorts. If we measure increases in $R(a,t)$ along line AB, we can avoid this problem. The increase in $R(a,t)$ along this line reflects the increased postponement of births that would have occurred at the same hypothetical age α in the absence of tempo changes. Because these births would have occurred at different times, the tempo change $r(a,t)$ measures the incremental postponement of these births per unit of time without distortions by age effects. Only in situations where $r(a,t)$ does not depend on age (i.e., in the BF approach), it also reflects the incremental increase in the cumulated cohort tempo.

The main finding of this section is that the above definition of the age- and period-specific tempo change $r(a,t)$ leads to a direct extension of the BF approach to age-specific fertility rates: the observed fertility rate $g(a,t)$ equals $(1 - r(a,t))$ times the adjusted age-specific fertility rate $g'(a,t)$.

Below we formalize the notion of age-period interactions in the postponement of fertility. In Assumptions 1–3 we first specify the fertility rates in the absence of any fertility postponement, the period-specific quantum effects (which we ignored in the above discussion), and the relation between the observed and the hypothetical occurrence of births. In Assumption 4 we formalize the idea of adjusted age-specific fertility rates (ASFRs). Subsequently we derive the observed fertility rates as a function of the adjusted ASFRs and the pace of fertility postponement.

Assumption 1: Fertility rates in the absence of any tempo and quantum effects. If tempo and quantum effects are absent at all time periods, the fertility schedule does not change over time and is given by a standardized schedule $\phi(\alpha)$ with $\int \phi(\alpha)d\alpha = 1$.

Assumption 2: Observed fertility rates. The observed fertility schedule can be decomposed as

$$g(a,t) = q(t) \cdot h(a,t), \tag{2}$$

where the term $q(t)$ is a period-specific quantum effect that proportionally increases or decreases fertility rates at all ages. Moreover, we assume that the fertility schedule $h(a,t)$ differs from the standardized fertility schedule $\phi(\alpha)$ only because of the postponement of fertility. The observed total fertility rate is given by $\text{TFR}(t) = \int g(a,t)da$.

The above assumptions specify that in the absence of quantum and tempo effects we would observe a constant fertility schedule $\phi(\alpha)$. Moreover, we specify that quantum changes are pure period effects that proportionally increase or decrease the fertility rates at all ages.

In the next assumption we restate our earlier discussion of the cumulated tempo $R(a,t)$ and how this cumulated tempo relates the observed and the hypothetical occurrence of a birth.

Assumption 3: Cumulated tempo changes. Births that occur at age a at time t would have occurred at an age $\alpha = a - R(a,t)$ at time $\tau = t - R(a,t)$ in the absence of a fertility postponement, where $R(a,t)$ is a smooth function that represents the cumulated tempo changes that have occurred at age a until time t. We require that $a - R(a,t)$ is increasing in a: that is, births that occur at a higher age in the absence of tempo effects also occur at a higher age in the presence of tempo effects.

The fertility schedule $h(a,t)$ in Eq. (2) differs from $\phi(\alpha)$ only because of the effect of tempo changes. The relation between these schedules is therefore determined by the relation between the observed age a and time t of a birth and the corresponding hypothetical age α and time τ in the absence of a fertility postponement. A change of variables from the actual age a and time t of a birth to the hypothetical age α and time τ thus yields the relation between the observed fertility rates $g(a,t)$ and the underlying fertility schedule $\phi(\alpha)$. (The proofs of all subsequent results are given in the appendix.)

Result 1: Observed ASFR. The observed age-specific fertility rates are given by

$$g(a,t) = q(t) \cdot (1 - R_a(a,t) - R_t(a,t)) \cdot \phi(a - R(a,t)), \tag{3}$$

where R_a and R_t denote respectively the partial derivative of $R(a,t)$ with respect to age a and time t.

The observed fertility rate is therefore the product of the period-specific quantum effect $q(t)$, the Jacobian of the transformation from a and t to α and τ, and finally the fertility rate $\phi(a - R(a,t))$. Although Eq. (3) is already a complete formal description of the relation between the observed fertility schedule and the underlying schedule $\phi(\alpha)$, it lacks an interpretation in terms of the tempo of fertility and the adjusted ASFRs. We therefore define in addition:

Assumption 4: Adjusted age-specific fertility rates. Denote as $g'(a,t)$ the adjusted age-specific fertility rates (adjusted ASFRs)

$$g'(a,t) := q(t) \cdot (1 - R_a(a,t)) \cdot \phi(a - R(a,t)), \tag{4}$$

which equal the fertility rates that would be observed in the absence of tempo effects at time t. (The notation ":=" means "defined as.") For the formal derivation it is also useful to define the fertility schedule that would have been observed in the absence of tempo and quantum effects as

$$f(a,t) := (1 - R_a(a,t)) \cdot \phi(a - R(a,t)), \tag{5}$$

with the properties that $g'(a,t) = q(t) \cdot f(a,t)$ and $\int f(a,t)da = 1$.[4]

The adjusted age-specific fertility rate $g'(a,t)$ in Eq. (4) is the product of three terms: the period-specific quantum effect $q(t)$, a correction term $(1 - R_a(a,t))$, and the fertility rate $\phi(\alpha)$ that would have been observed at age $\alpha = a - R(a,t)$ if there had been no fertility postponement and no quantum effects. The correction term is necessary because age-period interactions in the postponement of fertility transform the shape of the fertility schedule. The term $1 - R_a(a,t)$ entails that these transformations in the shape of the schedule itself do not change the level of fertility of the adjusted schedule $g'(a,t)$. All changes in the level of the adjusted TFR are attributed to the period-specific quantum effects $q(t)$, and the adjusted TFR is thus a measure of the quantum of fertility.

Result 2: Adjusted TFR. The adjusted total fertility rate $\mathrm{TFR}'(t)$ measures the quantum of fertility since

$$\mathrm{TFR}'(t) := \int g'(a,t)da = q(t). \tag{6}$$

The difference between the observed and the adjusted total fertility rates is due only to tempo effects.

To establish the relation between the observed fertility rates $g(a,t)$ and the adjusted fertility rates $g'(a,t)$, we need in addition to specify *age- and period-specific tempo* $r(a,t)$, which generalizes the BF approach. As in our discussion in Figure 2, we consider the combinations of a and t that solve the relation

$$a - R(a,t) = \tilde{\alpha} \tag{7}$$

for some given constant $\tilde{\alpha}$. These combinations of a and t trace births that would have occurred at a constant age $\tilde{\alpha}$ in the absence of tempo effects. The age- and period-specific tempo effect $r(a,t)$ is then defined as the derivative da/dt along the combinations of a and t that satisfy Eq. (7): that is, it is the slope of the line that connects all a and all t that satisfy Eq. (7). Applying the implicit function theorem to Eq. (7), we obtain:

Assumption 5: Age- and period-specific tempo. The age- and period-specific tempo $r(a,t)$ is given by

$$r(a,t) := \frac{R_t(a,t)}{1 - R_a(a,t)}. \tag{8}$$

4. Integrating Eq. (5) and changing the variable of integration shows that $\int f(a,t)da = \int (1 - R_a(a,t)) \cdot \phi(a - R(a,t))da = \int \phi(\alpha)d\alpha = 1$, where the last equality follows from Assumption 1.

The formal framework defined by the above assumptions is more general than the framework analyzed by Bongaarts and Feeney (1998) because it allows for age-period interactions in the postponement of fertility. The BF approach emerges as a special case of the above framework when the cumulated tempo $R(a,t)$ depends only on the period t but not on age a. If age-period interactions in the postponement of fertility are present, however, at least three questions are of central importance: (1) What is the relation between the observed fertility schedule $g(a,t)$ and the fertility schedule $g'(a,t)$ that would be observed if tempo effects were absent at time t? (2) How does the cumulated tempo $R(a,t)$ affect the mean age, variance, and possibly other characteristics of the observed and adjusted fertility schedule over time? (3) How sensitive is the Bongaarts-Feeney adjustment to violations of the assumption of no age-period interactions?

We address the second and third questions in the next section, where we make specific assumptions about $R(a,t)$ in order to investigate variance changes. The first question, on the other hand, can be addressed within the very general framework provided in this section, and it provides our first main result:

Result 3: Observed versus adjusted ASFRs. The observed fertility rates $g(a,t)$ and the adjusted fertility rates $g'(a,t)$ are related as

$$g(a,t) = (1 - r(a,t)) \cdot g'(a,t). \tag{9}$$

The observed birth rate in the presence of a tempo effect is therefore only a fraction $(1 - r(a,t))$ of the birth rate that would be observed in the absence of this tempo effect. If no age-period interactions exist—that is, when $r(a,t)$ does not depend on age and can be written as $r(t)$—Result 3 directly yields the Bongaarts-Feeney adjustment formula.

Result 4: The Bongaarts-Feeney adjustment. If the tempo $r(t)$ depends only on period, but not on age, then $\mathrm{TFR}(t) = (1 - r(t)) \cdot \mathrm{TFR}'(t)$.[5]

The Bongaarts-Feeney formula implies that the adjusted $\mathrm{TFR}'(t)$ ultimately increases toward infinity as $r(t) \to 1$, with the observed fertility level $\mathrm{TFR}(t)$ held constant. Moreover, the BF adjustment is not meaningful for $r > 1$ because this would lead to negative values for the adjusted TFR. Fortunately we can show that tempo effects larger than 1 are inconsistent with the TFR adjustment and cannot occur in the present framework.

Result 5: Restrictions on $r(a,t)$. Age-specific tempo effects that are equal to or larger than 1—that is, $r(a,t) \geq 1$—are inconsistent with potential cohort behaviors that underlie the postponement of fertility. In particular, tempo changes of $r(t) \geq 1$ cannot occur under the assumptions of the Bongaarts-Feeney framework.

5. The version of the Bongaarts-Feeney formula given in Result 4 is slightly more general than the version derived in Bongaarts and Feeney (1998). In particular, we do not require that the increases in the mean age must be piecewise linear in time: for example, within a calendar year. We require only that the cumulated tempo $R(t)$ must be a smooth function of time.

Incorporating Variance Changes in the TFR Adjustment

In this section we employ the framework described above and extend the adjustment of the total fertility rate suggested by Bongaarts and Feeney (1998). In particular, we incorporate changes in the variance of the fertility schedule. These variance changes are due to age-period interactions that lead to a dependence of the tempo $r(a,t)$ on age. Here we make assumptions about the nature of these age-period interactions and specify a particular function for the cumulated tempo $R(a,t)$. We then analyze the implications for the observed fertility schedule over time and for the adjustment of the total fertility rate.

Assumption 6: Cumulated tempo change. The cumulated tempo $R(a,t)$ is given by

$$R(a,t) = \gamma t + (a - \bar{a}(t)) \cdot (1 - e^{-\delta t}), \tag{10}$$

where $\bar{a}(t) = \bar{a}_0 + \gamma t$ and $\bar{a}_0$ is the mean age of the fertility schedule $\phi(\alpha)$. (We chose the notation $\bar{a}(t) = \bar{a}_0 + \gamma t$ because we show in Result 6 that this expression describes the mean age of the adjusted fertility schedule $g'(a,t)$ over time.)

The age- and period-specific tempo change $r(a,t)$, which corresponds to the $R(a,t)$ in Eq. (10), is given by[6]

$$r(a,t) = \gamma + \delta(a - \bar{a}(t)). \tag{11}$$

With the above specification of $R(a,t)$ we can obtain the mean age and the variance of the adjusted fertility schedule $g'(a,t)$: that is, the schedule that would be observed at time t in the absence of tempo effects.

Result 6: Mean age and variance in the absence of tempo changes. The mean age and the variance $s^2(t)$ of the adjusted fertility schedule $g'(a,t)$ are given by

(a) $\bar{a}(t) = \int a \cdot g'(a,t)da \,/\, \mathrm{TFR}'(t) = \bar{a}_0 + \gamma t$, and

(b) $s^2(t) = \int (a - \bar{a}(t))^2 g'(a,t)da \,/\, \mathrm{TFR}'(t) = s_0^2 \cdot e^{2\delta t}$,

where $\bar{a}_0$ is the mean age and s_0^2 is the variance of the fertility schedule $\phi(\alpha)$.

The tempo change specified in Assumption 6 therefore leads to a linear increase in the mean age $\bar{a}(t)$, as in the Bongaarts-Feeney framework. More important, if $\delta > 0$, it also leads to an exponential increase in the variance $s^2(t)$ of the adjusted fertility schedule $g'(a,t)$ over time.

The presence of tempo changes at time t distorts the observed fertility rates $g(a,t)$. From Result 3 we know that $g(a,t) = (1 - r(a,t)) \cdot g'(a,t)$. For instance, consider the case with $\gamma > 0$ and $\delta > 0$ in Eq. (11). In this case there is a general postponement of births. The tempo changes $r(a,t)$, however, are less than γ for ages that are below the mean age $\bar{a}(t)$, and $r(a,t)$ exceeds γ for $a > \bar{a}(t)$. Fertility rates at different ages therefore are affected differentially by the postponement of births. In particular, the observed fertility rates $g(a,t)$ for ages below the mean age $\bar{a}(t)$ differ less from the corresponding adjusted fertility rates $g'(a,t)$ than do fertility rates at ages above the mean age.

Despite these age-period interactions in the postponement of fertility, the above framework allows us to establish a relation between the total fertility rate in the presence and the absence of tempo changes.

Result 7: Adjusted TFR. The observed total fertility rate does not depend on the extent of variance changes δ, and equals

$$\mathrm{TFR}(t) = (1 - \gamma) \cdot \mathrm{TFR}'(t) = (1 - \gamma) \cdot q(t). \tag{12}$$

The relation between the observed TFR(t) and the hypothetical TFR'(t) depends only on the annual increase in the mean age at birth γ. It does *not* depend on the annual increase δ in the standard deviation of the adjusted fertility schedule. Hence changes in the fertility level are due to *only* two factors: (1) the period quantum effect $q(t)$, which proportionally increases or decreases fertility rates at all ages, and (2) the annual increase in the mean age $\bar{a}(t)$.

The presence of variance changes, however, affects other aspects of the observed fertility pattern. For $\gamma, \delta > 0$, for instance, the relatively smaller tempo changes below the mean age $\bar{a}(t)$ lead to a *smaller* reduction of the observed fertility rates $g(a,t)$ than do the relatively larger tempo changes above the mean age $\bar{a}(t)$. This differential effect of postponement implies that the observed mean age at birth is biased downward in periods with an increasing mean age and standard deviation. This distortion occurs because the age-specific tempo changes lead to larger reductions in fertility rates at higher ages than at lower ages. The bias in the mean age subsequently leads to an underestimation of tempo effects, which are calculated by taking differences between the mean ages at birth in different years. This effect leads to a downward bias in the adjusted total fertility rate because differences between the observed mean age underestimate the "true" tempo γ, which is necessary for the calculation of TFR'.

Result 8: Distortion in the mean age due to variance effects. Let $\mu(t) := \int a g(a,t)da \,/\, \mathrm{TFR}(t)$ denote the observed mean age at birth at time t. Then

$$\mu(t) = \bar{a}(t) - \frac{\delta}{1-\gamma} s^2(t). \tag{13}$$

The last term in Eq. (13) represents the distortion in the observed mean age that occurs because of age-period interactions when $\delta \neq 0$. Moreover, the dependence of $r(a,t)$ on age distorts the observed variance in the age at birth.

Result 9: Distortion in the variance. Let $\sigma^2(t) := \int (a - \mu(t))^2 g(a,t)da \,/\, \mathrm{TFR}(t)$ denote the observed variance in the age at birth. Then

$$\sigma^2(t) = s^2(t) - \left[\frac{\delta s^2(t)}{1-\gamma}\right]^2 - \frac{\delta}{1-\gamma} \kappa(t), \tag{14}$$

where $\kappa(t) = \int (a - \bar{a}(t))^3 f(a,t)da$ is the centralized third moment of the fertility schedule $f(a,t)$.

6. This specification does not explicitly enforce the restriction that $r(a,t)$ must be less than 1 (see Result 5). To be consistent with possible cohort behaviors, the parameter δ must be chosen to be small enough that $r(a,t)$ in Eq. (11) remains less than 1 for all ages in which the fertility rates are positive.

The first term in Eq. (14) gives the variance of the fertility schedule that would be observed if variance effects are absent at time t. The remaining two terms represent the distortion when $\delta \neq 0$.

Results 8 and 9 are based on the assumption that γ and δ are constant over time. More general nonlinear changes in the mean age and the standard deviation also can be included in the TFR adjustment. In particular, the constants γ and δ in Eq. (11) can be replaced with smooth functions $\gamma(t)$ and $\delta(t)$ of time.

Result 10: Nonlinear tempo and variance changes. Assume that the cumulated tempo $R(a,t)$ defined in Eq. (10) is replaced by $R(a,t) = G(t) + (a - \overline{a}(t)) \cdot (1 - e^{-D(t)})$, where $\gamma(t)$ and $\delta(t)$ are smooth functions of time and $G(t)$ and $D(t)$ are defined as $G(t) := \int_{-\infty}^{t} \gamma(x)dx$ and $D(t) := \int_{-\infty}^{t} \delta(x)dx$. Results 6–9 still hold after γ is replaced with $\gamma(t)$, δ with $\delta(t)$, γt with $G(t)$, and δt with $D(t)$ at all occurrences. Moreover, the change in the observed mean age equals

$$\mu'(t) = \gamma(t) - \frac{s^2(t)}{1-\gamma(t)}\left[\frac{\gamma'(t)\delta(t)}{1-\gamma(t)} + \delta'(t) + 2\delta(t)^2\right]. \quad (15)$$

Biases in the Bongaarts-Feeney Formula Due to Variance Effects

In this section we provide an approximation for the bias in the Bongaarts-Feeney formula that occurs because of variance effects. For this purpose we consider a specific example of Result 10 with $\gamma(t) = \gamma$: that is, a situation in which the mean age of the adjusted fertility schedule increases by a constant amount per annum.

Recall that the tempo change in the BF approach is inferred from differences in the observed mean age at birth. We denote this tempo change as $\hat{r}_{BF}(t) = \mu'(t)$, where $\mu'(t)$ indicates the change in the observed mean age $\mu(t)$. We showed in Result 7 that the appropriate measure for the adjustment of the total fertility rate is γ, which indicates the change in the mean age of the adjusted fertility schedule $g'(a,t)$ per unit of time. The last equation in Result 10, however, shows that the change in the observed mean age at birth $\mu'(t)$ is less than γ whenever $[\delta'(t) + 2\delta(t)^2]$ is positive. (The assumption $\gamma(t) = \gamma$ in our example implies that $\gamma'(t) = 0$ and the corresponding term in Eq. (15) can be dropped.) The term $[\delta'(t) + 2\delta(t)^2]$ is positive, for instance, whenever there is a trend toward an increased variance of the fertility schedule ($\delta(t) > 0$) and the relevance of this variance effect is not diminishing with time ($\delta'(t) \geq 0$). In this case, the tempo change calculated from changes in the observed mean age is less than the appropriate measure γ. The BF formula subsequently leads to an underestimation of the adjusted TFR.

It is possible to approximate the bias that is incurred by applying the BF formula to situations with a changing variance. This approximation allows us, on one hand, to assess the difference between $\hat{r}_{BF}(t)$: that is, the tempo change inferred from increases observed mean age at birth, and the "true" tempo effect γ that would be observed in the absence of variance changes. On the other hand, we can calculate a "bias-corrected" tempo measure as $\tilde{r} = \hat{r}$ – (bias in $\hat{r}$), where the "bias in $\hat{r}$" is obtained from Eq. (16) below.

Result 11: Bias in the BF formula. Consider the nonlinear variance effects introduced in Result 10 with $\gamma(t) = \gamma$, and denote as $\hat{r}_{BF}(t) = \mu'(t)$ the change in the observed mean age at birth. Then

$$\text{bias in } \hat{r}_{BF}(t) := \hat{r}_{BF}(t) - \gamma = \frac{s^2(t)}{1-\gamma}\left[\delta'(t) + 2\delta(t)^2\right]. \quad (16)$$

The bias in the adjusted total fertility rate TFR', calculated as the BF-adjusted TFR minus the adjusted TFR with variance effects, is approximately equal to

$$\text{bias in TFR' at time } t \approx \frac{\text{TFR}(t)}{(1-\gamma)^2} \cdot (\text{bias in } \hat{r}_{BF}(t)). \quad (17)$$

The bias in $\hat{r}_{BF}(t)$ in Eq. (16) also reveals the situations in which the BF formula is most distorted by variance effects. When $\delta'(t) = 0$—that is, when $\delta(t)$ is constant—the bias in $\hat{r}_{BF}(t)$ is proportional to $\delta(t)^2$. Because the values of $\delta(t)$ typically range between 0 and 0.01, and only rarely exceed 0.015 in our experience, this bias is usually quite small.

The above case with $\delta'(t) = 0$ (that is, with a constant rate of increase in the variance of the fertility schedule) is conceptually analogous to the shape changes investigated in Zeng Yi and Land (2001). The formal analyses here thus confirm Zeng Yi and Land's (2001) finding that under certain conditions the BF formula is quite robust with respect to variance changes.

Our formal analyses, however, also show that this conclusion depends strongly on the assumption that variance changes occur at a constant rate. The apparent irrelevance of variance effects vanishes when $\delta(t)$ is not constant over time. For instance, an increasing $\delta(t)$ implies that the downward bias in the observed mean age is increasing over time (see Result 8). This additional effect leads to a considerably larger impact of variance changes on the BF formula. In particular, the bias in $\hat{r}_{BF}(t)$ is approximately proportional to $\delta'(t)$, where values of $\delta'(t)$ typically range between 0 and 0.005.

Consider, for instance, an example with $\gamma = 0.15$, $s^2 = 20$, and $\delta(t) = 0.005$. On the one hand, if $\delta'(t) = 0$, then the bias in $\hat{r}$ is –0.0012 and the tempo change $\hat{r}_{BF}(t)$ underestimates the correct value $\gamma = 0.15$ by less than 1%. On the other hand, if $\delta'(t) = 0.0025$, then the bias in $\hat{r}$ is –0.06 and the estimated tempo change $\hat{r}_{BF}(t)$ is 40% below the correct value $\gamma = 0.15$. Thus when the rate of change in the variance $\delta(t)$ is not constant, variance effects can be considerably important for the application of the adjusted total fertility rate. Variance changes are relevant in the present example for reasons that are not reflected in the simulations by Zeng Yi and Land (2001).

As indicated above, variance changes with $\delta(t) > 0$ lead to a downward bias in the observed mean age (see Result 8). If $\delta(t)$ is constant for all t, the size of this bias—which is reflected in the rightmost term in Eq. (13)—is almost constant over time and changes only because $s^2(t)$ is increasing. When the pace of fertility postponement is calculated by differencing the observed mean age, the bias caused by variance changes is almost differenced out because the change in $s^2(t)$ between two adjacent years is relatively small. Therefore the inference of the change in the tempo of fertility is not strongly affected by the bias occurring in the level of the observed mean age due to variance or similar shape changes.

However, if $\delta(t)$ is increasing over time, as in the above example, then the bias caused by variance changes in the observed mean age (13) grows approximately proportionally with $\delta(t)$. Thus differencing the observed mean age between two adjacent years no longer eliminates this bias, and the tempo of fertility inferred from the observed mean age is underestimated. Variance changes, therefore, affect the TFR adjustment because the inference of the pace of fertility postponement is distorted. In contrast to our formal analyses, the simulations in Zeng Yi and Land (2001) do not address this aspect. Our analyses, however, show that the biases in the BF formula caused by variance changes can be quite relevant when the rate of change in the variance of the fertility schedule is changing over time.

To apply the approximations in Result 11 to observed data, one must estimate the unknown parameters γ, $\delta(t)$, and $\delta'(t)$ in Eq. (16). One possibility is to use the BF estimate, $r_{\mathrm{BF}}(t)$, as an approximation for $\gamma(t)$. The estimation of $\delta(t)$ can then be based on Result 12 below, and the derivative can be calculated as $\delta'(t) = \frac{1}{2}[\delta(t+1) - \delta(t-1)]$.

Estimating and Correcting for Variance Effects in the TFR Adjustment

We can improve the approximations provided in the previous section by a procedure that iteratively corrects the observed mean age and the inferred tempo for the distortions caused by variance effects. For this purpose we initially estimate the variance change δ from the observed variance of the fertility schedule. We then correct the observed mean ages and tempo effects for the distortions caused by variance effects.

Result 12: Estimation of δ. On the basis of the observed variance $\sigma^2(t)$ of the fertility schedule in year t, an estimation of δ is possible as $\hat{\delta}(t) = 0.25 \cdot \log(\sigma^2(t+1) / \sigma^2(t-1))$ for all years t.

Because the variance and mean age series are often subject to random fluctuations, it is advisable to smooth these series before the calculations.[7]

Result 13: Estimation of γ. Given the observed mean age $\mu(t)$, variance $\sigma^2(t)$, and third centralized moment $\kappa(t)$ of the fertility schedule in year t, and given an estimate of $\hat{\delta}(t)$, the estimation of γ is possible in the following steps (the subscript n denotes the number of iterations)

(1) Calculate $\hat{\gamma}_0(t) = \frac{1}{2}\left[\mu(t+1) - \mu(t-1)\right]$ and $\hat{s}_0^2(t) = \sigma^2(t)$ for all years t;

(2) Calculate $\hat{s}_n^2(t) = \sigma^2(t) + \left[\frac{\hat{\delta}(t)}{1-\hat{\gamma}_{n-1}(t)}\hat{s}_{n-1}^2(t)\right]^2 + \frac{\hat{\delta}(t)}{1-\hat{\gamma}_{n-1}(t)}\kappa(t)$ for all years t;

(3) Calculate $\hat{a}_n(t) = \mu(t) + \frac{\hat{\delta}(t)}{1-\hat{\gamma}_{n-1}(t)}\hat{s}_n^2(t)$ for all years t;

(4) Calculate $\hat{\gamma}_n(t) = \frac{1}{2}[\hat{a}_n(t+1) - \hat{a}_n(t-1)]$ for all years t;

(5) Return to step 2, and repeat until the estimates converge;

(6) Calculate the adjusted TFR with variance effects as $\mathrm{TFR}(t) / (1 - \hat{\gamma})$.

The first step initializes the iteration. The next two steps correct the observed variance and mean age for the biases caused by the presence of variance effects (see Results 8 and 9). In step 4, a new estimate of the tempo effect γ is obtained from the corrected mean age instead of the observed mean age. The iteration is repeated until the estimates for $\hat{\gamma}$ converge. (The Swedish data used in this paper and S-PLUS programs for calculating the adjusted TFR with and without variance effects are available at http://user.demogr.mpg.de/kohler.)

The above estimation of δ and γ does not provide estimates for the tempo and variance effect in the first and the last year of a time series. To avoid a shortening of the time series, we suggest using the estimates of the second and second-to-last period in a time series for the first and the last year during the iteration.

AN APPLICATION: THE SWEDISH BABY BOOM AND BUST, 1985 TO 1996, REVISITED

Tempo Versus Quantum: Do Variance Effects Matter?

Figure 3 shows the results obtained from applying the adjusted TFR with variance effects to first births in Sweden after 1975.[8] Graph 3a depicts the estimated change in the standard deviation of the fertility schedule: until 1985 this change is close to 0, and then it increases rapidly to a maximum in 1990. After 1990 it diminishes again, but a trend toward an increasing dispersion in the age at first birth remains, even in 1995.

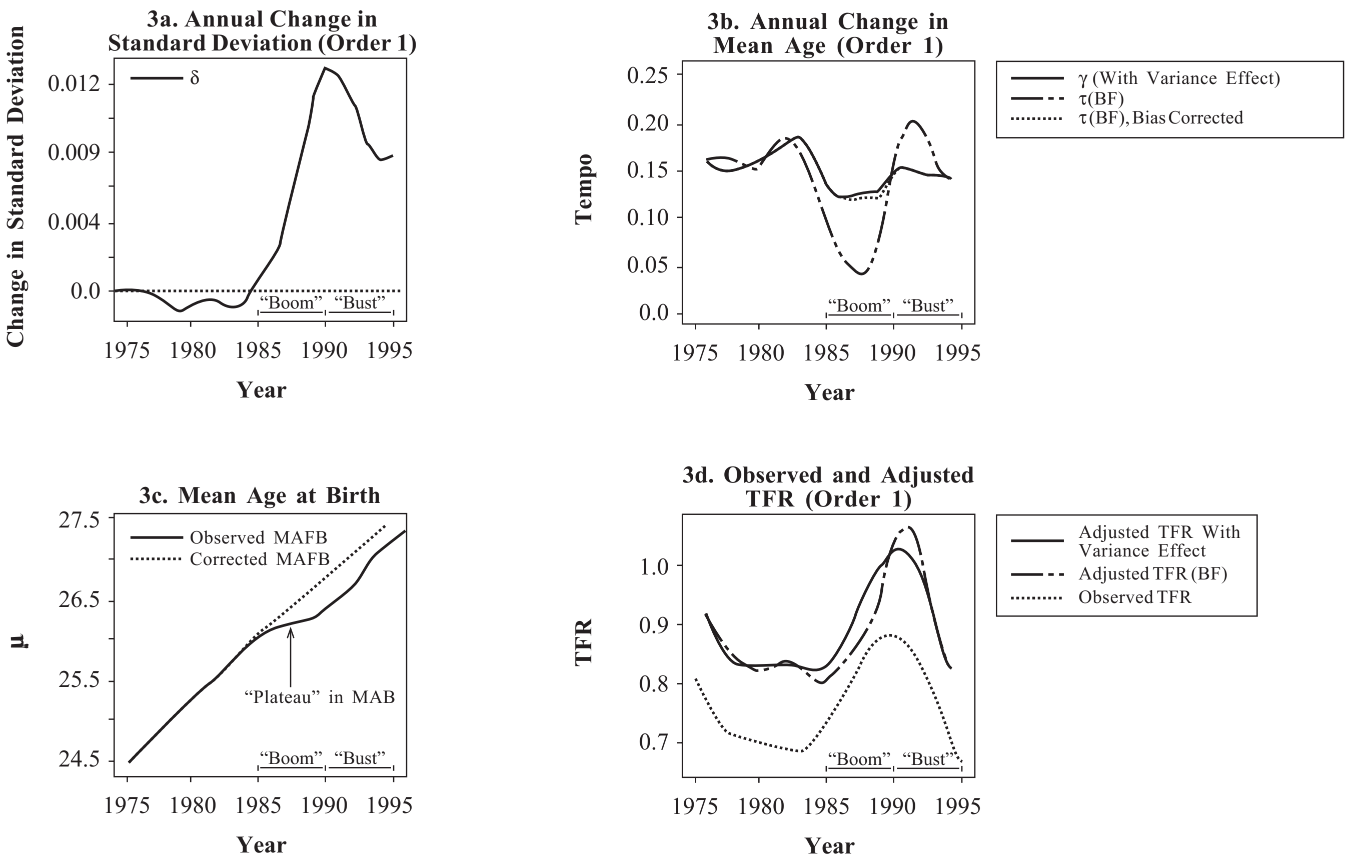
Graph 3b reveals important differences in the tempo of fertility inferred from the TFR adjustment with and without variance effects. The Bongaarts-Feeney approach associates the baby boom from 1985 to 1990 with a substantial deceleration in the delay of first births. This reduction in the pace of postponement corresponds to the "plateau" in the mean age at first birth during this period (see Graph 3c).

After we account for the increases in the variance of the fertility schedule after 1985, this pattern reverses. The estimated tempo change γ in Figure 3 does not decline substan-

7. In our experience, the estimation in Result 12 works well when the observed variance of the fertility schedule is already relatively smooth. In the presence of large fluctuations over time, a regression of the variance on a quadratic function of time is more stable. For this purpose, calculate the ratio $0.5 \cdot \log(\sigma^2(t) / \sigma^2(t_0))$, where $\sigma^2(t_0)$ is the variance at the beginning of the time interval over which the polynomial is fitted. Regress this ratio on $\delta_0 + \delta_1 t + \delta_2 t^2 / 2$. Then calculate $\hat{\delta}(t)$ from the estimated regression coefficients as $\hat{\delta}(t) = \hat{\delta}_1 + \hat{\delta}_2 t$. If necessary, higher-order polynomials can be fitted accordingly.

8. The calculations follow Result 12 for the estimate of $\delta(t)$ and Result 13 for the estimation of $\gamma(t)$ and the adjusted TFR with variance effects.

FIGURE 3. THE INFLUENCE OF VARIANCE EFFECTS ON THE ANNUAL CHANGE IN THE MEAN AGE AND ON THE ADJUSTED TFR IN SWEDEN, FIRST BIRTHS ONLY



tially during the period of the baby boom.[9] Similarly, the mean age at first birth, corrected for variance effects, no longer exhibits a plateau during the period 1985–1990. It continues to increase throughout the baby boom and bust period without major changes in pace. The postponement of births, after variance effects are accounted for, therefore does not noticeably reduce its pace throughout the period 1985–1995.

How is it possible to make this reversal of our earlier conclusion about the tempo changes during the period 1985–1996? The answer pertains to the implications of variance changes. When fertility begins to increase around 1985, the fertility schedule starts to shift at the same time, toward a wider dispersion and increased variance. This onset of increases in the variance of the fertility schedule leads to a downward bias in the observed mean age. We find that the plateau in mean age at first birth, which is depicted in Graph 3c, is due precisely to this bias. Once the bias is removed, the plateau vanishes as well.

Our analyses therefore suggest that the onset of the fertility increase in 1985 was not associated with a reduction in the pace at which first births are postponed. It was, however, associated with the onset of variance changes that led to a larger dispersion of fertility across ages. These variance changes led to a bias in the observed mean age, and the bias manifested itself as a plateau in the development of the mean age after 1985. This plateau, however, is not due to a reduction in the pace of postponement; it is merely a consequence of the distortions caused by variance effects.

Graph 3d in Figure 3 presents the BF-adjusted TFR for first births and the adjusted TFR with variance effects. In contrast to the BF-adjusted TFR, the adjusted TFR with variance effects and the observed total fertility rate rise and fall almost synchronously from 1985 to 1995. The increase and the decrease in fertility for first births during this period are due almost entirely to quantum effects.

9. Graph 3b in Figure 3 depicts the annual increase in the mean age calculated via the iteration in Result 13 (solid line) as well as a “bias-corrected” r calculated from the approximation of the bias in Result 11 (dotted line). A comparison of the solid and the dotted line shows that in many instances the approximation in Result 11 may provide a sufficient correction for variance effects.

Socioeconomic Changes and the Swedish Fertility Pattern

Given the importance of variance effects for understanding the recent fertility change in Sweden, it is necessary to investigate whether the variance changes are systematically related to changes in the socioeconomic context and in family policies in that country. Hoem (1990) and Hoem and Hoem (1997b) argue that the increase in fertility at all birth orders during the 1980s is due to favorable economic conditions and massive investment in social policies directed toward families. Most relevant for the context of this paper are the differential incentives created by the policy changes in the 1980s for the timing of births of different orders. In 1980 and 1986 the Swedish government extended policies favoring a rapid succession of subsequent births after the first or second child. In particular, if the next child was born within 24 months (through 1985) or 30 months (beginning in 1986) of the previous child, the right to receive compensation for income forgone during child rearing was prolonged. Thus parents had a short-term economic incentive to space births closely.[10] Hoem (1990) and Hoem and Hoem (1997b) argue that these policy changes are related causally to the increase in fertility rates relatively shortly after the birth of a previous child.

Because the analyses of policy effects by Hoem (1990) and Hoem and Hoem (1997b) are based on fertility rates by the previous child's age (data that are not commonly available for countries other than Sweden), one should ask whether these policy effects can be identified with order-specific mean ages and with the tempo changes derived from these mean ages. These latter data have the advantage of being available for many more countries, and they are used for adjustment of the total fertility rate.

We argue below that the differential implications of the policy changes for the timing of births are indeed detectable in the order-specific tempo changes, but only after removal of the distortions caused by variance effects.

Consider first Graph 4a in Figure 4, which depicts the annual change *r* for first, second, and third births from 1975 to 1995 obtained from the application of the Bongaarts-Feeney adjustment. In this graph the tempo change for first births is about 0.15 in the late 1970s, with a minor increase around 1980. After 1983, the tempo for first births decreases substantially, leading to the plateau in the observed mean age in Figure 3. In 1988 the pace of postponement for first births attains a minimum, and then increases quite rapidly to a peak in 1992, followed by another decline. The tempo change for second births increases before 1979 and then declines until 1989. The tempo change for third births declines throughout the period 1975–1990.

The most striking aspect of Graph 4a is that the pace of postponement for all births decreases substantially in the years after 1983, without the appearance of any significant differences in the pattern of tempo changes by birth order. One should have expected differences, however, because the policy changes of 1980 and 1986 primarily affected the incentives for the timing of higher-order births.

The estimates for the order-specific tempo change quite substantially after the distortions caused by variance effects are removed. These respective estimates are depicted in Graph 4b in Figure 4. The line for first births confirms our earlier finding: after we adjusted for variance effects, there was no substantial change in the postponement of first births from 1975 to 1995. During the period 1978–1985 the tempo change first increases modestly and then decreases to about 0.12. It is quite striking that during the period of the baby boom and bust (1985–1995), the tempo change does not exhibit any substantial fluctuations. Throughout this period the mean age at first birth, adjusted for variance effects, increases by approximately 0.13–0.14 year per year.

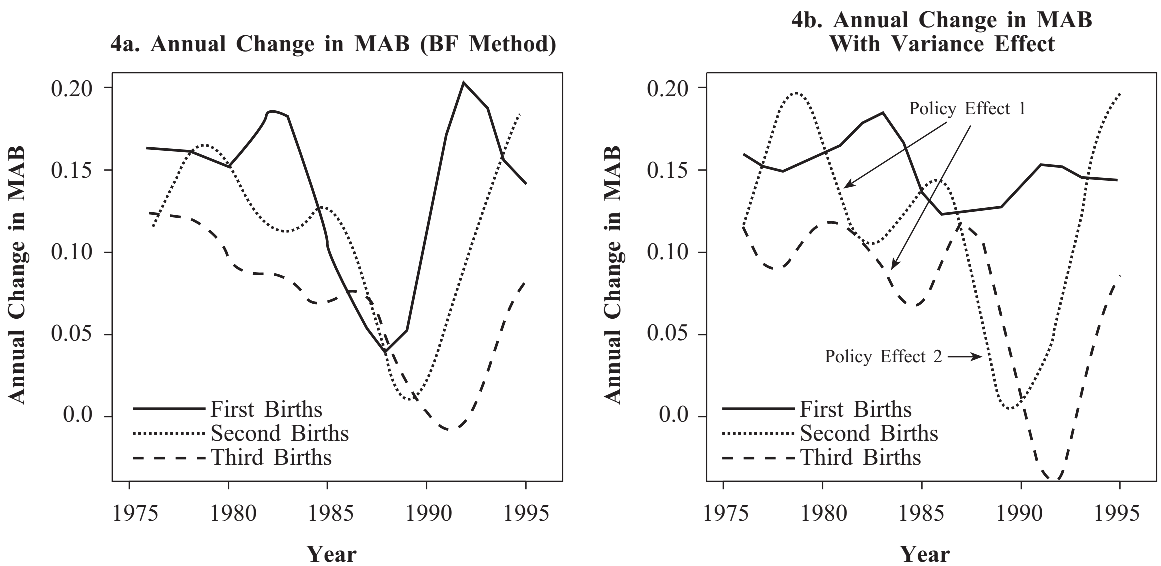
The increase in the mean age for second and third births, however, fluctuates substantially during the 1980s. After 1979, the upward trend that was present in the tempo for second births is reversed. The annual increases in the mean age at second birth decline by about 48% from 1979 to 1982 (indicated by "Policy Effect 1"). The slight increase seen after 1982 is probably an echo effect of the increase in the tempo for first births a few years earlier. After 1986, the pace of postponement again declines substantially, and almost reaches 0 in 1990 (indicated by "Policy Effect 2"). After 1990 the tempo change for second births increases quite rapidly to a level prevailing in the late 1970s. The tempo for third births begins to decline modestly after 1980; this decline is followed by an increase that we believe is also an echo effect. After 1987 the tempo of third births decreases rapidly to a level slightly below 0, and then increases after 1992.

The pattern of the fertility postponement at different birth orders during the 1980s is altered by accounting for the distortions caused by variance effects. More important, the pattern in Graph 4b in Figure 4 (which has been controlled for variance effects) is more consistent with the policy changes during this period. The fluctuations in the tempo change at order 1 are relatively small, in keeping with the fact that the "speed premium" has no direct implication for the timing of the first child. The policy changes, however, provide incentives for shortening the interbirth interval after the first or second child; Graph 4b reflects clear decreases in the pace of postponement for second or third births after 1980 and 1986. Because this pace remains almost constant for first births (after variance effects are taken into account), the postponement pattern after 1980 implies a reduction in the difference between the mean age at first and at second birth, and between the mean age at first and at third birth. Both effects are consistent with the policy changes of the 1980s, and are even an expected consequence.

After variance effects are controlled, the differential effect of the 1985 policy interventions on births of different orders becomes clearly visible. Although the analyses reported do not contain information on interbirth intervals, the

10. The possibility of extending the eligibility interval via a close spacing of births also existed before 1980, but often it was impossible to attain the relatively short interbirth interval of under 12 months (or, with sick leave, 15 months) that was required.

FIGURE 4. ANNUAL CHANGE IN THE MEAN AGE AT FIRST, SECOND, AND THIRD BIRTHS IN SWEDEN, 1975–1995, BASED ON THE BONGAARTS-FEENEY ADJUSTMENT OF THE TFR (4a) AND ON THE TFR ADJUSTMENT WITH VARIANCE EFFECTS (4b)



pattern described above is consistent with a reduction in the interval, especially between the first and the second child. Hoem (1990) and Hoem and Hoem (1996) detected this pattern earlier, using different data, but it was not detectable from the trends in the observed mean ages without adjusting for variance effects.

CONCLUSIONS

The adjusted total fertility rate suggested by Bongaarts and Feeney (1998) provides an additional and very useful measure for analyzing fertility patterns, especially when fertility is subject to strong or fluctuating tempo effects. The adjusted total fertility rate is equivalent to the total fertility rate that would have been observed if there had been no postponement of births during a calendar year. Because the postponement of births has been a characteristic of fertility declines in both western and eastern Europe, the adjusted TFR exceeds—often substantially—the observed total fertility rate in countries with very low fertility levels (e.g., Bongaarts 1999; Lesthaeghe and Willems 1999; Philipov and Kohler forthcoming).

This finding is frequently associated with a cautiously optimistic opinion that women's completed fertility will exceed the level suggested by the observed TFR and that fertility levels will rise once the postponement of births loses its momentum. Moreover, the adjustment for tempo distortions can be extended to occurrence-exposure rates and parity progression ratios (Kohler and Ortega 2001) in order to overcome some of the criticisms raised by Kim and Schoen (2000) and van Imhoff and Keilman (2000) about the adjustment of the TFR. These extensions, however, still support the necessity for adjusting the total fertility rate and related period fertility measures for tempo distortions in order to properly assess the quantum of fertility in many low-fertility settings.

Quite naturally, the adjustment of the total fertility rate must rely on assumptions to permit disentanglement of the quantum and tempo components involved in changes in that rate. Presumably the most severe assumption underlying the TFR adjustment is that there are no age-period interactions or cohort effects. In other words, changes in the tempo of fertility are identical for all ages in any given period, and women of different ages cannot exhibit different paces of fertility postponement.

Variance changes—that is, increases or decreases in the standard deviation of the fertility schedule over time—result from the fact that in some recent European fertility patterns, the tempo of fertility has differed for women of different ages. These changes constitute a deviation from the assumptions underlying the TFR adjustment. In this paper we show that the presence of such variance changes leads to a bias in the mean age and in the adjusted total fertility rate. These distortions in the adjusted TFR and the tempo of fertility caused by variance changes are most relevant when the pace of variance changes is increasing or decreasing over time. This aspect is not reflected in the simulations by Zeng Yi and Land (2001), and our formal analyses do not show that the BF formula is generally robust with respect to variance changes. To make the TFR adjustment applicable to countries with a changing variance in the fertility schedule, we provide an extension to the adjustment of the TFR that includes variance effects.

We apply the adjustment of the total fertility rate with and without variance effects to Sweden for the period 1975–1996. In our analyses we show that the onset of the fertility increase around 1985 was associated with a tendency to spread childbirth more diversely across a wider range of ages: as fertility began to rise, the variance in the fertility schedule began to increase as well. This trend was most pronounced for first births but was also evident, somewhat more moderately, for second and third births. The presence of these variance changes has substantial implications for the interpretation of the Swedish baby boom and bust in terms of tempo and quantum effects. After we account for the distortions caused by variance changes, the plateau in the mean age at first birth in the late 1980s vanishes, and the earlier association of the baby boom with a reduced pace in the postponement of fertility disappears. The tempo change for first births remains almost constant throughout the period 1985–1995, an indication that neither the boom nor the bust in first births in Sweden was due to substantial changes in the postponement of fertility. Therefore, after we account for variance effects, the rise and decline in fertility for first births is due almost entirely to quantum changes.

In this paper we therefore suggest that variance effects are a relevant aspect of fertility change in addition to tempo and quantum effects. Although we fully agree that the adjustment of the total fertility rates provides new insights and is an important step in assessing contemporary low-fertility settings, the inferences can be distorted if the fertility change is associated with age-period interactions that alter the shape of the fertility schedule over time. By correcting for changes in the variance, one can incorporate changes in the fertility schedule in the adjustment of the total fertility rate.

The Swedish fertility pattern is not unique with respect to the presence of variance effects. Increases in the variance of the fertility schedule have also been associated with the fertility decline in Italy and eastern Europe. Although the analysis of these patterns is beyond the scope of the present paper, we suggest that the changes in the variance of the fertility schedule can be a systematic, interesting, and relevant aspect of fertility dynamics—one that deserves further consideration in the analysis of data from other countries as well.

APPENDIX: PROOFS OMITTED IN THE TEXT

Proof of Result 1. Consider births that occur at age a at time t in the presence of tempo changes. In the absence of tempo changes at all periods before time t—that is, if there had been no postponement of fertility before time t—these births would occur at an age $\alpha = a - R(a,t)$ at a time $\tau = t - R(a,t)$. The fertility rates in the absence of tempo and quantum effects are given by $\phi(\alpha)$. We first integrate this fertility schedule over all ages α and times τ, which represent the hypothetical age and time when births occur in the absence of tempo effects. We then change the variable of integration from α and τ to age a and time t, which give the observed age and time in the presence of tempo changes. (R_a and R_t denote the partial derivatives of $R(a,t)$ with respect to age and time respectively.)

$$\iint \phi(\alpha)\, d\alpha d\tau = \iint (1 - R_a(a,t) - R_t(a,t)) \cdot \phi(a - R(a,t))dadt$$
$$= \iint h(a,t)dadt.$$

The first equality follows by changing the variable of integration from α and τ to a and t, and by observing that $1 - R_a(a,t) - R_t(a,t)$ is the Jacobian of the transformation. The second equality follows from Assumption 2, where $h(a,t)$ is defined as the fertility schedule that is observed at time t in the absence of quantum effects. The fertility rates in the absence of quantum effects are therefore given by $h(a,t) = (1 - R_a(a,t) - R_t(a,t))\phi(a - R(a,t))$. Result 1 then follows from Eq. (2).

Proof of Result 2. The result follows directly by integrating $\text{TFR}'(t) := \int g'(a,t)da = q(t) \int (1 - R_a(a,t)) \cdot \phi(a - R(a,t))da = q(t) \int \phi(\alpha)d\alpha = q(t)$. The second equality follows from Eq. (4); the last two equalities are obtained by changing the variable of integration from a to $\alpha = a - R(a,t)$, and by recalling that $\int \phi(\alpha)d\alpha$ equals 1 by assumption.

Proof of Result 3. From Result 1 we obtain

$$g(a,t) = q(t) \cdot (1 - R_a(a,t) - R_t(a,t)) \cdot \phi(a - R(a,t))$$
$$= q(t) \cdot \left(1 - \frac{R_t(a,t)}{1 - R_a(a,t)}\right) \cdot (1 - R_a(a,t)) \cdot \phi(a - R(a,t))$$
$$= (1 - r(a,t)) \cdot g'(a,t).$$

The second equality is obtained by factoring out the term $(1 - R_a(a,t))$. The third line follows from the definition of $g'(a,t)$ in Eq. (4) and from the relation between $R(a,t)$ and $r(a,t)$ in Eq. (8).

Proof of Result 4. When the tempo change is given by $r(t)$, the Bongaarts-Feeney formula follows directly by integrating $\text{TFR}(t) = \int g(a,t)da = \int (1 - r(t))g'(a,t)da = (1 - r(t)) \int g'(a,t)da = (1 - r(t)) \cdot \text{TFR}'(t)$.

Proof of Result 5. Consider births that occur at an age a_1 at time t_1 and births that occur slightly later at time $t_2 = t_1 + \Delta t$ at age $a_2 = a_1 + r(a_1,t_1) \cdot \Delta t$, where Δt denotes an infinitesimally small time period. Because for small Δt the linear approximation $R(a_2,t_2) - R(a_1,t_1) = r(a_1,t_1)\Delta t$ holds, the above births would occur at an identical age $\alpha = a_1 - R(a_1,t_1) = a_2 - R(a_2,t_2)$ if tempo changes are absent. The births, however, would occur at different times, namely at $\tau_1 = t_1 - R(a_1, t_1)$ and $\tau_2 = t_2 - R(a_2, t_2)$. The difference in the hypothetical time of birth $\tau_2 - \tau_1$ is therefore given by

$$\Delta\tau := \tau_2 - \tau_1 = (t_2 - R(a_2,t_2)) - (t_1 - R(a_1, t_1))$$
$$= (1 - r(a,t)) \cdot \Delta t. \tag{18}$$

Tempo changes over time can affect the timing of births, but they cannot affect the temporal order of births that would occur at the same age α in the absence of tempo changes. That is, births at age α that occur earlier on the hypothetical time scale τ must also occur earlier on the observed time scale t.

To see this in the more general framework of this section, note that the difference $\Delta\tau$ in Eq. (18) equals the difference in the birth years of the cohorts reaching age α at time τ_1 and at time τ_2. Result 5 then follows by contradiction: (1) Assume, for example, that the cohort denoted by the subscript 2 is younger than the cohort denoted by the subscript 1: that is, assume that $\Delta\tau > 0$. Then suppose that $r(a,t) > 1$. If Eq. (18) is to hold, this requires that $\Delta t < 0$. This subsequently implies that $a_2 > a_1$ at a time $t_2 < t_1$, which contradicts the assumption that cohort 2 is younger than cohort 1 (i.e., $\Delta\tau > 0$). (2) Suppose now that $r(a,t) = 1$. The relation expressed in Eq. (18) then implies that $\Delta\tau = 0$ for any Δt. This means that births which occur at an identical age and time in the absence of tempo effects occur at different ages and times in the presence of tempo effects. Yet this is inconsistent with the assumption that members of the same cohort experience the same tempo change at each period of time, and that the cumulated tempo $R(a,t)$ is identical for cohort members at each time period t.

The consistency of $r(a,t)$ with underlying cohort behavior therefore requires that $r(a,t) < 1$. The same restriction applies in the Bongaarts-Feeney formula because it is a special case where $r(a,t)$ does not depend on age.

Proof of Result 6. The first part of Result 6 is shown most easily by defining two auxiliary functions based on the cumulated tempo in Eq. (10). The first gives the hypothetical age at birth α in the absence of tempo effects as a function of a for any given t as $\alpha(a,t) = a - R(a,t) = \bar{a}_0 + (a - (\bar{a}_0 + \gamma t))e^{-\delta t}$, where we substituted $\bar{a}_0 + \gamma t$ for $\bar{a}(t)$. The second is the respective inverse function, which yields the observed age at birth a as a function of α for any given t:

$$a(\alpha,t) = \bar{a}_0 + \gamma t + (\alpha - \bar{a}_0)e^{\delta t}. \tag{19}$$

We can now integrate $\int a \cdot g'(a,t)da \,/\, \mathrm{TFR}'(t) = \int a \cdot (1 - R_a(a,t)) \cdot \phi(a - R(a,t))da = \int a(\alpha, t) \cdot \phi(\alpha)d\alpha = \bar{a}_0 + \gamma t$. The first expression is obtained from the definition of the mean age. The first equality then follows from Eqs. (4) and (6); the second equality follows by changing the variable of integration from a to α and using the inverse function defined in Eq. (19). The last equality follows by applying the integration separately to all terms in Eq. (19) and observing that the term $\int (\alpha - \bar{a}_0)\phi(\alpha)d\alpha$ equals 0. The last equality shows that the mean age of the schedule $g'(a,t)$ equals $\bar{a}(t) = \bar{a}_0 + \gamma t$ and therefore establishes the first part of Result 6.

The proof of the second part of Result 6 follows similarly by integrating $s^2(t) = \int (a - \bar{a}(t))^2 g'(a,t)da \,/\, \mathrm{TFR}'(t) = \int (a - \bar{a}(t))^2(1 - R_a(a,t))\phi(a - R(a,t))da = \int (a(\alpha,t) - \bar{a}_0 - \gamma t)^2\phi(\alpha)d\alpha = e^{2\delta t} \int (\alpha - \bar{a}_0)^2\phi(\alpha)d\alpha = e^{2\delta t} s_0^2$. The first equality is obtained from the definition of $s^2(t)$; the second follows from relations expressed in Eqs. (4) and (6). The third equality is obtained by changing the variable of integration from a to α, using the inverse function defined in Eq. (19) and the fact that $\bar{a}(t) = \bar{a}_0 + \gamma t$. The fourth equality follows by replacing $a(\alpha, t)$ with Eq. (19). The final equality follows from the definition of $s^2(0)$.

Proof of Result 7. This result is shown by integrating $\mathrm{TFR}(t) = \int g(a,t)da = \int (1 - r(a,t))g'(a,t)da = q(t) \int (1 - \gamma - \delta(a - \bar{a}(t))f(a,t)da = (1 - \gamma)\mathrm{TFR}'(t) = (1 - \gamma)q(t)$. The second equality follows from Result 3; the third equality follows from the definitions of $g'(a,t)$ and $r(a,t)$ in Eqs. (4) and (11) respectively. The fourth equality follows by observing that the term $\int (a - \bar{a}(t))f(a,t)da = 0$ and by using the definition of $\mathrm{TFR}'(t)$ in Eq. (6); the last equality follows from Eq. (12).

Proof of Result 8. The result is shown by integrating $\mu(t) = \int a \cdot g(a,t)da/\mathrm{TFR}(t) = q(t) \int a \cdot (1 - r(a,t)) \cdot f(a,t)da \,/\, \mathrm{TFR}(t) = \int a \cdot [1 - \gamma - \delta(a - \bar{a}(t))] \cdot f(a,t)da \,/\, (1 - \gamma) = \bar{a}(t) - \dfrac{\delta}{(1-\gamma)} \int a \cdot (a - \bar{a}(t)) \cdot f(a,t)da = \bar{a}(t) - \dfrac{\delta}{(1-\gamma)}[\int a^2 f(a,t)da - \bar{a}(t)^2] = \bar{a}(t) - \dfrac{\delta}{(1-\gamma)} s^2(t)$. The second equality follows from Eq. (4) and Result 3; the third from inserting Eqs. (11) and (12); the fourth equality is obtained by splitting up the term in parentheses and integrating the first part; the fifth and sixth equalities follow by evaluating the respective integrals and observing that the term in squared brackets equals the variance of the fertility schedule $f(a,t)$.

Proof of Result 9. This result is shown by integrating

$$\begin{aligned}
\sigma^2(t) &= \int (a - \mu(t))^2 g(a,t)da \,/\, \mathrm{TFR}(t) \\
&= \int \left[a - \bar{a}(t) + \frac{\delta}{1-\gamma} s^2(t)\right]^2 g(a,t)da / \mathrm{TFR}(t) \\
&= \int \left(a - \bar{a}(t)\right)^2 \frac{g(a,t)}{\mathrm{TFR}(t)} da \\
&\quad + \frac{2\delta s^2(t)}{1-\gamma} \int \left(a - \bar{a}(t)\right) \frac{g(a,t)}{\mathrm{TFR}(t)} da + \left[\frac{\delta s^2(t)}{1-\gamma}\right]^2 .
\end{aligned} \tag{20}$$

The second equality above follows from Result 8; the third equality follows by separating the quadratic term in square brackets.

The integrals in the final line (Eq. (20)) now can be analyzed separately. Consider the first term: $\int (a - \bar{a}(t))^2 g(a,t)da \,/\, \mathrm{TFR}(t) = \int (a - \bar{a}(t))^2(1 - r(a,t))f(a,t)da \,/\, (1 - \gamma) = s^2(t) - \dfrac{\delta}{(1-\gamma)} \int (a - \bar{a}(t))^3 f(a,t)da = s^2(t) - \dfrac{\delta}{(1-\gamma)}\kappa(t)$, where $\kappa(t) := \int (a - \bar{a}(t))^3 f(a,t)da$ is the third centralized moment of the fertility schedule $f(a,t)$. The first equality utilizes Result 3 and Eqs. (4), (5), and (12); the second equality is obtained by inserting Eq. (11) and by integration; the final equality follows from the definition of $\kappa(t)$.

Consider next the integral part of the second term in Eq. (20): $\int (a - \bar{a}(t))g(a,t)da \,/\, \mathrm{TFR}(t) = \int (a - \bar{a}(t))[(1 - \gamma - \delta(a - \bar{a}(t))]f(a,t)da \,/\, (1 - \gamma) = -\dfrac{\delta}{(1-\gamma)} \int (a - \bar{a}(t))^2 f(a,t)da = -\dfrac{\delta}{(1-\gamma)} s^2(t)$. The steps proceed as in the expression above;

in addition, the second equality utilizes the fact that $\int (a - \overline{a}(t))f(a,t)da = 0$.

Combining all terms in Eq. (20) and simplifying then yields Result 9.

Proof of Result 10. This result follows by replacing the cumulated tempo $R(a,t)$ defined in Eq. (10) with $R(a,t) = G(t) + (a - \overline{a}(t)) \cdot (1 - e^{-D(t)})$, and then following the same steps as in the proofs above. After the substitutions in Result 10 are made, Eq. (15) follows by taking the derivative of Eq. (13) with respect to time.

Proof of Result 11. Eq. (16) follows directly from Eq. (15) with $\gamma(t) = \gamma$ and $\gamma'(t) = 0$. The bias in the adjusted total fertility rate TFR' can be approximated by a first-order Taylor expansion of the Bongaarts-Feeney formula (Eq. (1)) evaluated at the "correct" measure γ for the tempo change. The BF-adjusted TFR in year T is then approximately

$$\text{TFR}'(\hat{r}) \approx \frac{\text{TFR}}{1-\gamma} + \frac{\text{TFR}}{(1-\gamma)^2}(\hat{r}-\gamma) = \frac{\text{TFR}}{1-\gamma} + \frac{\text{TFR}}{(1-\gamma)^2}(\text{bias in } r),$$

where $\text{TFR}/(1-\gamma)$ is the adjusted TFR that includes variance effects. Taking the difference $\text{TFR}'(\hat{r}) - \text{TFR}/(1-\gamma)$ yields Eq. (17).

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