Demographic Methods

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Week 3: Fertility

Overview

- Demographic transition
- Measures of fertility
- Measures of reproduction
- Fertility models
- Dataset ideas

The demographic transition

The demographic transition

Over time, human populationsbroadly go through four demographic stages:

1. High mortality / high fertility (Pre-transition)
   - high rate of infectious diseases, poor medical coverage
   - no modern contraception
   - agricultural economies, non-nuclear families
   - population size: low and stable

2. Falling mortality (Early transition)
   - technological progress (vaccinations, modern medicine)
   - improved infrastructure (sanitation)
   - population size: increasing rapidly

3. Falling fertility (Late transition)
   - empowerment of women
   - modern contraception
   - ubranization
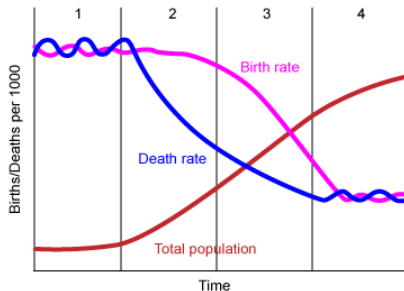   - population size: increasing but deceleration

The demographic transition

4. Low mortality and fertility (Post-transition)
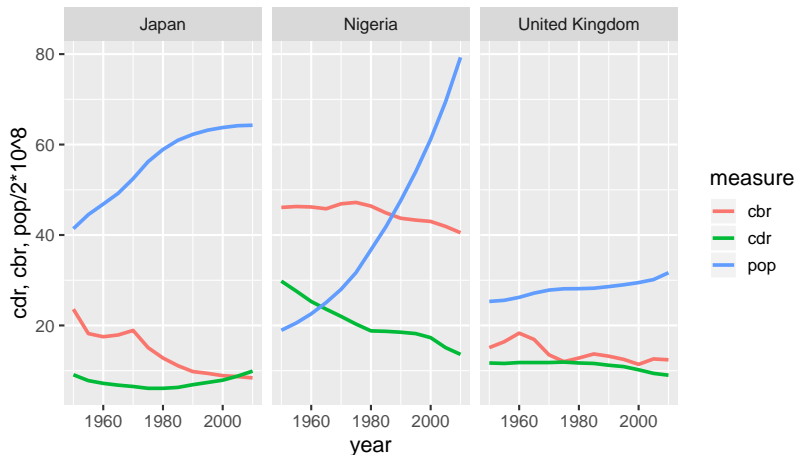   - population size: high and stable

5. (TBD?) fertility $<$ mortality
   - improvements in old-age mortality
   - children are expensive
   - population size: declining

Graph source: IUSSP

Examples

Birth rates, death rates and population, 1950–2015

Measures of fertility

Note: fertility versus fecundity

- Fertility is studying the outcome (circumstances where live births occur)
- Fecundity is the studying the ability to conceive

Crude measures

Crude birth rate:

$$CBR = \frac{\text{Births to women}}{\text{PY lived by population}}$$

But the population at risk: all women aged 15-49.

General fertility rate:

$$GFR = \frac{\text{Births to women}}{\text{PY lived by women of reproductive age}}$$

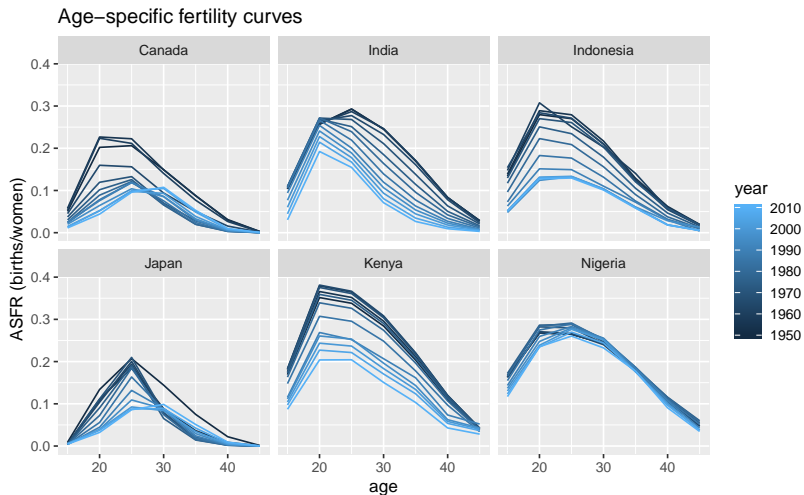Recall discussion about why crude rates are good and bad.

Age-specific fertility rates

$$_nF_x = \frac{\text{Births to women aged } x \text{ to } x + n}{\text{PY lived by women aged } x \text{ to } x + n}$$

Continuous version:

$$f(x) = \frac{B(x)}{P(x)}$$

Age-specific fertility rates

Age−specific fertility curves
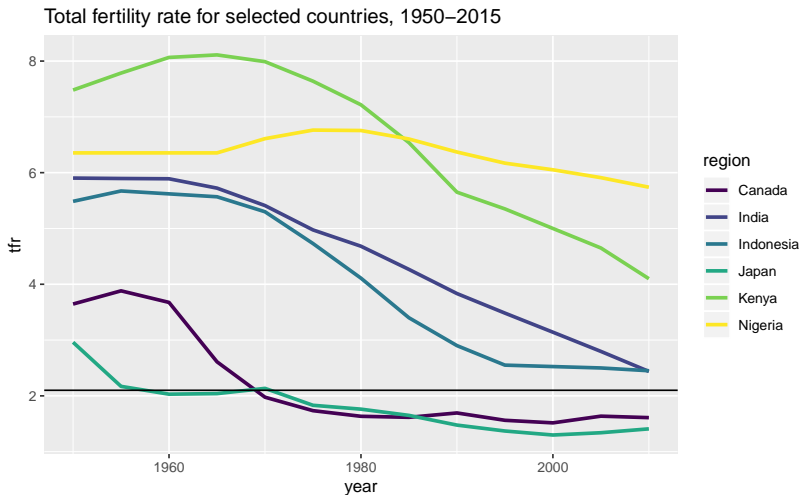
Total fertility rate

TFR is the average number of babies a women would have if she lived through the entire reproductive lifespan.

$$TFR = n \cdot \sum_{15}^{49} {_nF_x}$$

or

$$TFR = \int_{15}^{49} f(a)da$$

Example

Total fertility rate for selected countries, 1950–2015

Replacement level fertility

- The level of fertility required for a generation to replace itself
- TFR $\approx$ 2.1, but varies by region
- If $TFR < 2.1$ does not mean population is declining

Parity

- Sometimes we get data in the form of 'children ever born' to a women (common in surveys)
- **Parity** is the number of live births a woman has had.
- A woman is **nulliparious** if they have parity $= 0$.
- Studying fertility by parity gives us an idea of how women are limiting their fertility
- Cohort measure

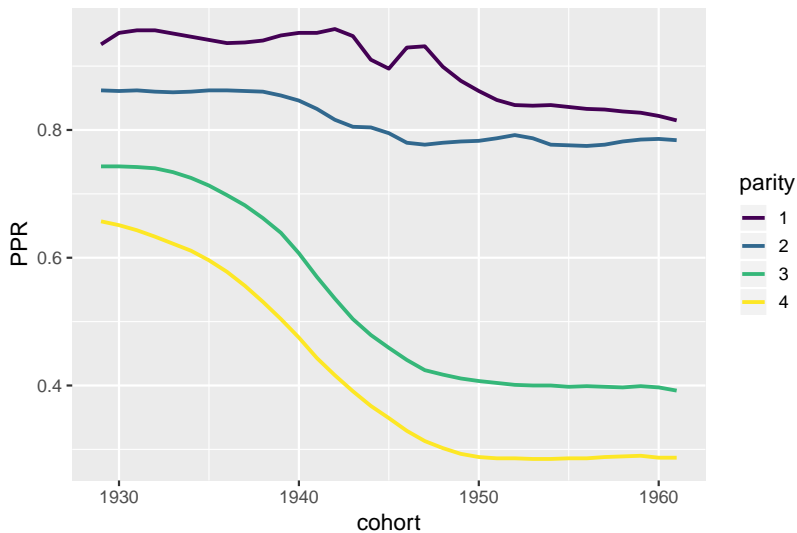Define $w(j)$ to be the number of women at parity j

Parity progression ratios

Parity Progression Ratio at parity j $PPR(j)$ is the fraction of women who, having reached parity j, go on to have at least another child.

$$PPR(j) = \frac{\sum_{j+1}^{\infty} w(i)}{\sum_{j}^{\infty} w(i)}$$

Example

Parity Progression Ratios in Canada

Measures of reproduction

Measures of reproduction

The idea of generational renewel: compare sizes of successive cohorts of women

Only want to consider female babies.

Define fraction female at birth $f_{fab} \approx 0.4886$ i.e. there are slightly fewer girls born c.f. boys.

Gross reproduction ratio

GRR is the average number of **female** babies a women would have if she lived through the entire reproductive lifespan.

$$GRR = n \cdot \sum_{15}^{49} {}_nF_x^{F}$$

Or, if we don't know sex-specific fertility rates

$$GRR = n \cdot \sum_{15}^{49} {}_nF_x \cdot f_{fab}$$

Net reproduction ratio

NRR is the average number of female babies a women would bear if they were subject to the observed age-specific mortality rates.

$$NRR = \sum_{15}^{49} {}_nF_x{}^F \cdot {}_nL_x$$

or

$$NRR = \sum_{15}^{49} {}_nF_x \cdot {}_nL_x \cdot f_{fab}$$

Note the ${}_nL_x$ refers to the survivorship of women.

In continuous form, the product of fertility rates (of female babies) and the survivorship is given its own notation, $\phi(x)$, and called the net maternity function. Then

$$NRR = \int \phi(a)da$$

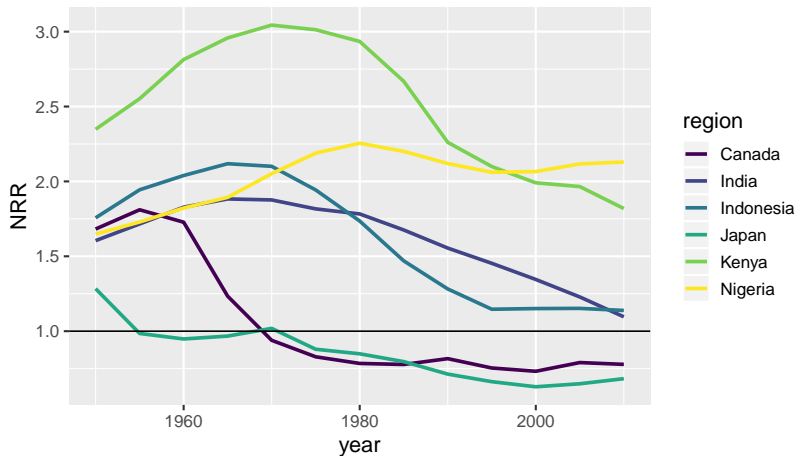# NRR from parity

If we are given data in the form of number of women by parity in a cohort, then we can also calculate the NRR as:

$$NRR = \frac{\sum_0^\infty i \cdot w(i)}{\sum_0^\infty w(i)}$$

Examples

NRRs for selected countries, 1950–2015

How does NRR relate to population growth?

- NRR > 1 implies?
- NRR < 1 ?
- NRR $= 1$?
- How does this relate to replacement level fertility?

More next week (population growth, stable populations)

Mean age at child-bearing

$$\mu = \frac{\sum_x x \cdot {}_nF_x}{\sum_x {}_nF_x}$$

or

$$\mu = \frac{\int_x a f(a) da}{\int_x f(a) da}$$

Fertility models

Fertility distributions are different to mortality

- repeated events
- separated by intervals of non-risk
- individual control

Harder (and less appropriate?) to model at the aggregate level than mortality.

Model fertility schedules

Coale and Trussel (1974). Focus:

- capture the extent to which observed mortality deviates from natural fertility
- use age-specific standards that are empirically derived (similar idea to Brass mortality)
- Parameterize in terms of overall level of fertility and degree of fertility control
- Note: estimating marital fertility. (!)

Model fertility schedules

$$_nF_x = M \cdot n(x) \cdot e^{-m \cdot \nu(x)}$$

- M is the overall level of fertility
- $n(x)$ is the natural fertility age-specific schedule (constant)
- m is the strength of parity-specific limitation
- $\nu(x)$ is the impact of fertility limitation by age (constant)

Natural fertility schedule $n(x)$

Derived from the Hutterites (anabaptists, community of goods, limited use of technology). High fertility rates, closest thing to natural fertility observed (marry young, religious duty)

Interestingly, now evidence Hutterite fertility is declining.

Natural fertility schedule $n(x)$

Age	$n(x)$
15	0.36
20	0.46
25	0.431
30	0.396
35	0.321
40	0.167
45	0.024

What's the implied TFR?

Fertility limitation schedule $\nu(x)$

Derived from empirical data

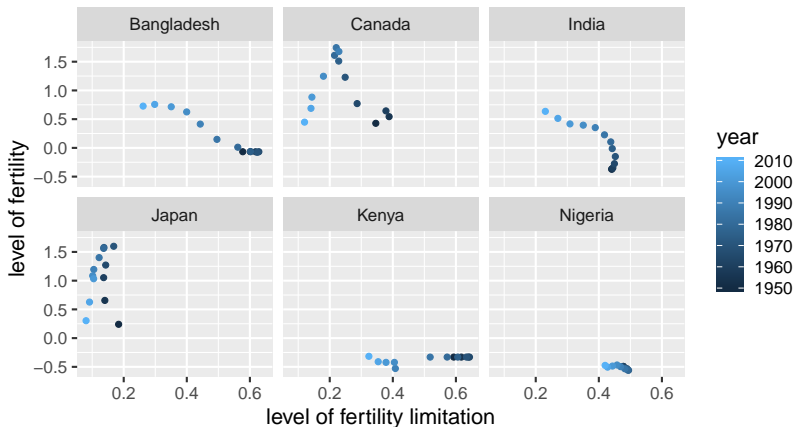| Age	$\nu(x)$
15	0
20	0
25	0.279
30	0.667
35	1.042
40	1.414
45	1.67

Estimating M and m

$$_nF_x = M \cdot n(x) \cdot e^{-m \cdot \nu(x)}$$

implies

$$\log\left(\frac{_nF_x}{n(x)}\right) = \log M - m \cdot \nu(x)$$

Example

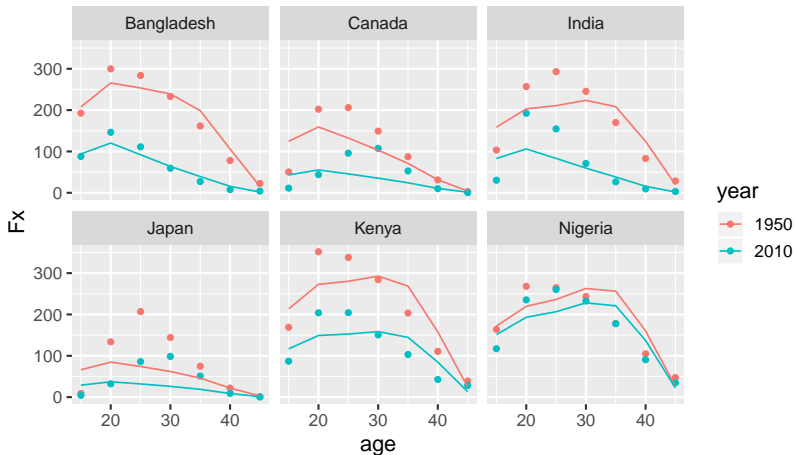Coale and Trussel's M and m for selected countries 1950–2010

Example

BUT the fits are pretty bad

Data and Coale and Trussel fit, 1950 and 2010

..ideas for how to do this differently?

Tempo and quantum

Background

Note: shift in focus of research from high to low fertility.

Most common measure of fertility is the period TFR. Is TFR declining because women are having fewer kids or because they are putting it off until later?

In any given year, fertlity rates can decline because of two reasons:

- Quantum: decline in number of children
- Tempo: shift in age at childbearing (delaying having kids to older ages)

Note: this isn't an issue with cohort TFR, just period TFR

Background

TFR for Canada, 1944–2011
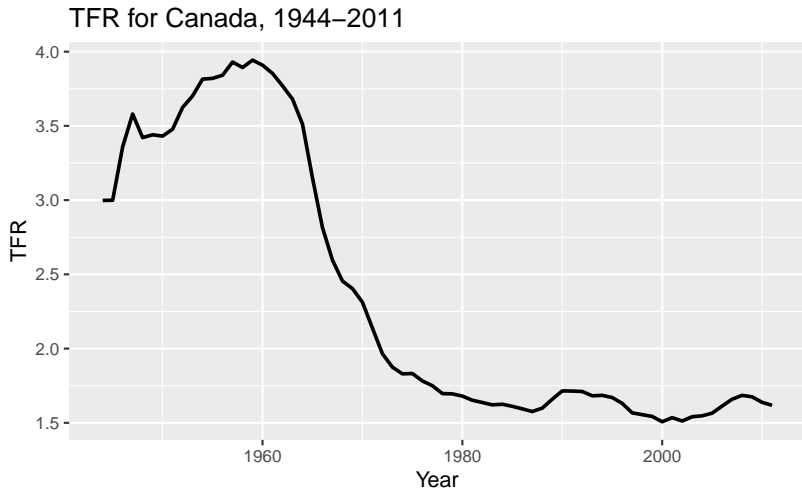
Tempo-adjusted TFR

$$TFR'(t) = \frac{TFR(t)}{1 - \frac{d}{dt}\mu(t)}$$

$\mu(t)$ is the mean age at child-bearing in period t.
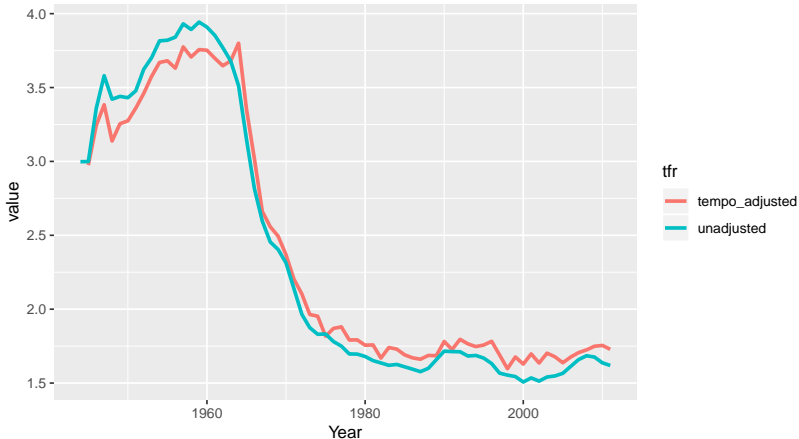
How to calculate this in practice?

Bongaarts and Feeney (1998) calculate the change in the mean age at child-bearing by parity i, r_i:

$$TFR'(t)_i = \frac{TFR_i}{1 - r_i}$$

then $TFR'(t) = \sum_i TFR'_i$

Example

TFR and tempo–adjusted TFR
Canada 1944–2011

Dataset ideas

Dataset ideas

- WPP
- Human Fertility Database
- Current Population Survey (IPUMS)
- DHS (IPUMS)