Demographic Methods

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Week 3: Fertility

Overview

- Measures of fertility
- Measures of reproduction
- Fertility models
- Dataset ideas

Measures of fertility

Note: fertility versus fecundity

- Fertility is studying the outcome (circumstances where live births occur)
- Fecundity is the studying the ability to conceive

Crude measures

Crude birth rate:

$$CBR = \frac{\text{Births to women}}{\text{PY lived by population}}$$

But the population at risk: all women aged 15-49.

General fertility rate:

$$GFR = \frac{\text{Births to women}}{\text{PY lived by women of reproductive age}}$$

Recall discussion about why crude rates are good and bad.

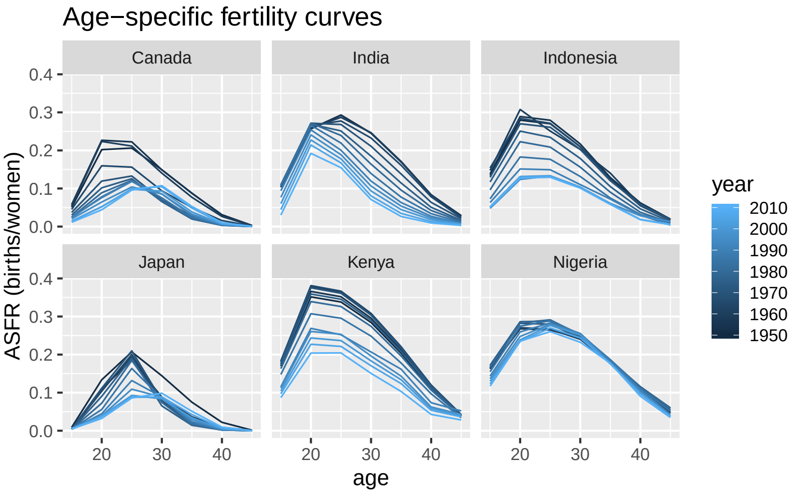
Age-specific fertility rates

$$ {}_nF_x = \frac{\text{Births to women aged } x \text{ to } x+n}{\text{PY lived by women aged } x \text{ to } x+n} $$

Continuous version:

$$ f(x) = \frac{B(x)}{P(x)} $$

Age-specific fertility rates



Total fertility rate

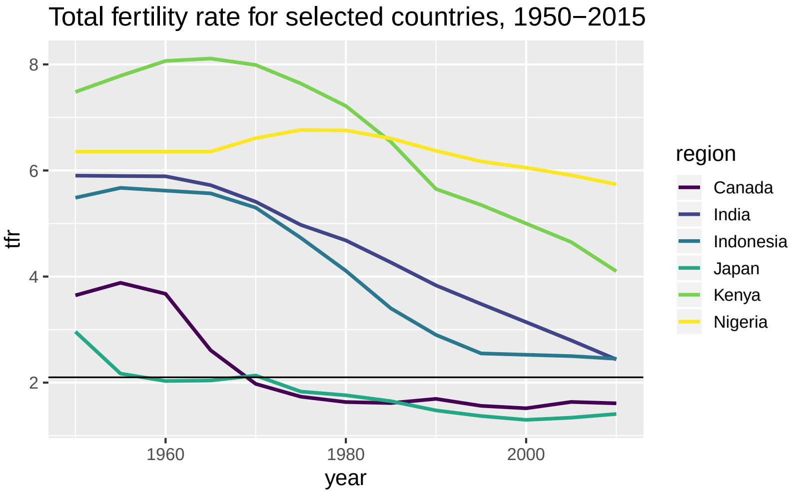
TFR is the average number of babies a women would have if she lived through the entire reproductive lifespan.

$$TFR = n \cdot \sum_{15}^{49} {}_nF_x$$

or

$$TFR = \int_{15}^{49} f(a)da$$

Example



Total fertility rate for selected countries, 1950−2015
8
6
4
2
tfr
1960
1980
2000
year
region
Canada
India
Indonesia
Japan
Kenya
Nigeria

Replacement level fertility

- The level of fertility required for a generation to replace itself
- TFR ≈ 2.1, but varies by region
- If $TFR < 2.1$ does not mean population is declining

Parity

- Sometimes we get data in the form of 'children ever born' to a women (common in surveys)
- **Parity** is the number of live births a woman has had.
- A woman is **nulliparious** if they have parity $= 0$.
- Studying fertility by parity gives us an idea of how women are limiting their fertility
- Cohort measure

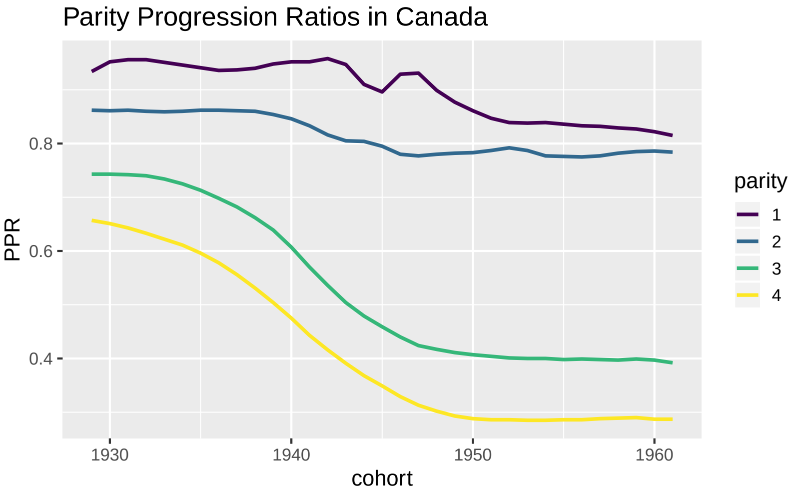
Define $w(j)$ to be the number of women at parity j

Parity progression ratios

Parity Progression Ratio at parity j $PPR(j)$ is the fraction of women who, having reached parity j, go on to have at least another child.

$$PPR(j) = \frac{\sum_{j+1}^{\infty} w(i)}{\sum_{j}^{\infty} w(i)}$$

Example



Parity Progression Ratios in Canada
PPR
0.8
0.6
0.4
1930
1940
1950
1960
cohort
parity
1
2
3
4

Measures of reproduction

Measures of reproduction

The idea of generational renewel: compare sizes of successive cohorts of women

Only want to consider female babies.

Define fraction female at birth $f_{fab} \approx 0.4886$ i.e. there are slightly fewer girls born c.f. boys.

Gross reproduction ratio

GRR is the average number of **female** babies a women would have if she lived through the entire reproductive lifespan.

$$GRR = n \cdot \sum_{15}^{49} {}_nF_x{}^F$$

Or, if we don't know sex-specific fertility rates

$$GRR = n \cdot \sum_{15}^{49} {}_nF_x \cdot f_{fab}$$

Net reproduction ratio

NRR is the average number of female babies a women would bear if they were subject to the observed age-specific mortality rates.

$$NRR = \sum_{15}^{49} {}_nF_x{}^F \cdot {}_nL_x$$

or

$$NRR = \sum_{15}^{49} {}_nF_x \cdot {}_nL_x \cdot f_{fab}$$

Note the ${}_nL_x$ refers to the survivorship of women.

In continuous form, the product of fertility rates (of female babies) and the survivorship is given its own notation, $\phi(x)$, and called the net maternity function. Then

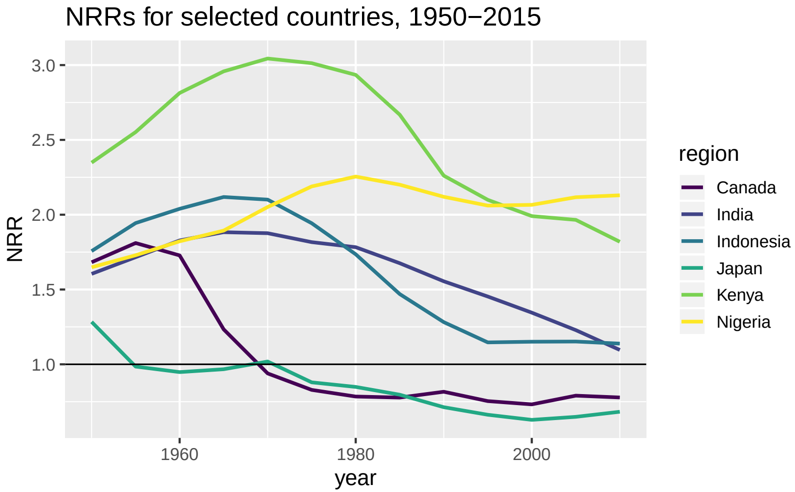
$$NRR = \int \phi(a)da$$

NRR from parity

If we are given data in the form of number of women by parity in a cohort, then we can also calculate the NRR as:

$$NRR = \frac{\sum_0^\infty i \cdot w(i)}{\sum_0^\infty w(i)}$$

Examples


NRRs for selected countries, 1950−2015
NRR
3.0
2.5
2.0
1.5
1.0
1960
1980
2000
year
region
Canada
India
Indonesia
Japan
Kenya
Nigeria

How does NRR relate to population growth?

- NRR > 1 implies?
- NRR < 1 ?
- NRR $= 1$?
- How does this relate to replacement level fertility?

More next week (population growth, stable populations)

Mean age at child-bearing

$$\mu = \frac{\sum_x x \cdot {}_nFx}{\sum_x {}_nF_x}$$

or

$$\mu = \frac{\int_x af(a)da}{\int_x f(a)da}$$

Fertility models

Fertility distributions are different to mortality

- repeated events
- separated by intervals of non-risk
- individual control

Harder (and less appropriate?) to model at the aggregate level than mortality.

Model fertility schedules

Coale and Trussel (1974). Focus:

- capture the extent to which observed mortality deviates from natural fertility
- use age-specific standards that are empirically derived (similar idea to Brass mortality)
- Parameterize in terms of overall level of fertility and degree of fertility control
- Note: estimating marital fertility. (!)

Model fertility schedules

$$ {}_nF_x = M \cdot n(x) \cdot e^{-m \cdot \nu(x)} $$

- M is the overall level of fertility
- $n(x)$ is the natural fertility age-specific schedule (constant)
- m is the strength of parity-specific limitation
- $\nu(x)$ is the impact of fertility limitation by age (constant)

Natural fertility schedule $n(x)$

Derived from the Hutterites (anabaptists, community of goods, limited use of technology). High fertility rates, closest thing to natural fertility observed (marry young, religious duty)

Interestingly, now evidence Hutterite fertility is declining.

Natural fertility schedule $n(x)$

Age	$n(x)$
15	0.36
20	0.46
25	0.431
30	0.396
35	0.321
40	0.167
45	0.024

What's the implied TFR?

Fertility limitation schedule $\nu(x)$

Derived from empirical data

Age	$\nu(x)$
15	0
20	0
25	0.279
30	0.667
35	1.042
40	1.414
45	1.67

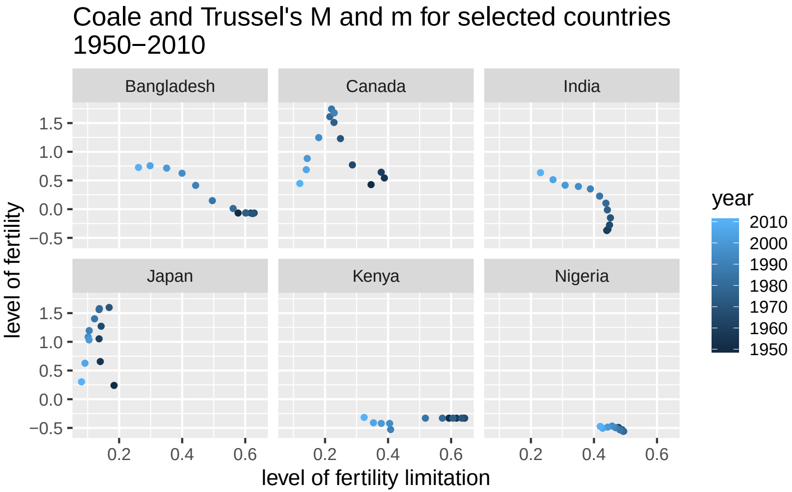
Estimating M and m

$${}_nF_x = M \cdot n(x) \cdot e^{-m \cdot \nu(x)}$$

implies

$$\log\left(\frac{{}_nF_x}{n(x)}\right) = \log M - m \cdot \nu(x)$$

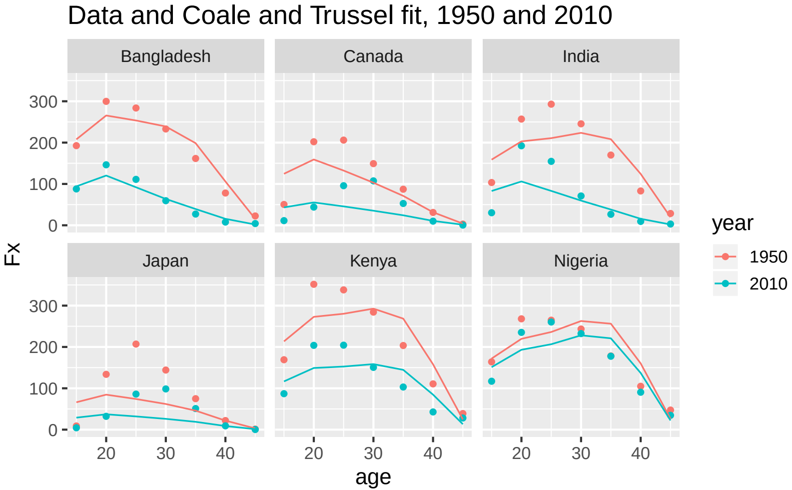
Example



Coale and Trussel's M and m for selected countries
1950−2010
Bangladesh
Canada
India
Japan
Kenya
Nigeria
level of fertility
level of fertility limitation
1.5
1.0
0.5
0.0
−0.5
0.2
0.4
0.6
year
2010
2000
1990
1980
1970
1960
1950

Example

BUT the fits are pretty bad



..ideas for how to do this differently?

Tempo and quantum

Background

Note: shift in focus of research from high to low fertility.

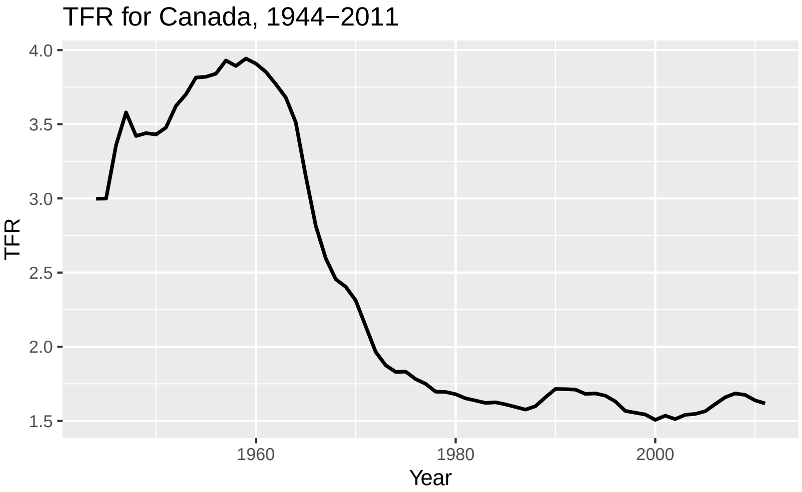
Most common measure of fertility is the period TFR. Is TFR declining because women are having fewer kids or because they are putting it off until later?

In any given year, fertlity rates can decline because of two reasons:

- Quantum: decline in number of children
- Tempo: shift in age at childbearing (delaying having kids to older ages)

Note: this isn't an issue with cohort TFR, just period TFR

Background



TFR for Canada, 1944−2011
4.0
3.5
3.0
2.5
2.0
1.5
TFR
1960
1980
2000
Year

Tempo-adjusted TFR

$$TFR'(t) = \frac{TFR(t)}{1 - \frac{d}{dt}\mu(t)}$$

$\mu(t)$ is the mean age at child-bearing in period t.

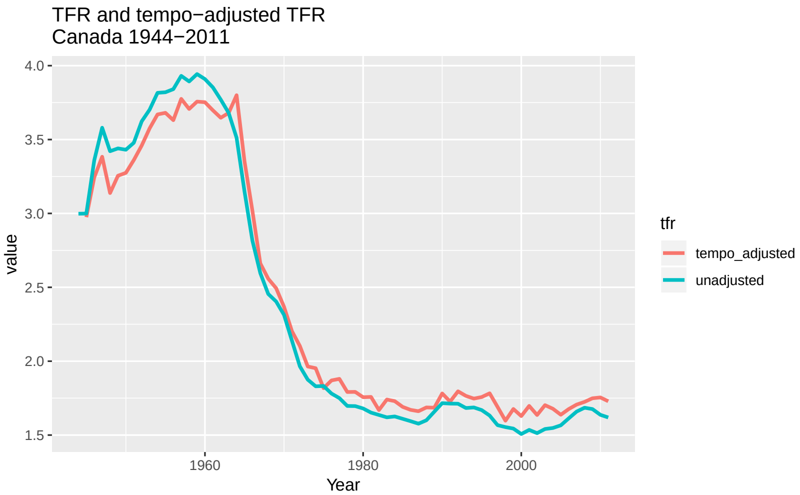
How to calculate this in practice?

Bongaarts and Feeney (1998) calculate the change in the mean age at child-bearing by parity i, r_i:

$$TFR'(t)_i = \frac{TFR_i}{1 - r_i}$$

then $TFR'(t) = \sum_i TFR'_i$

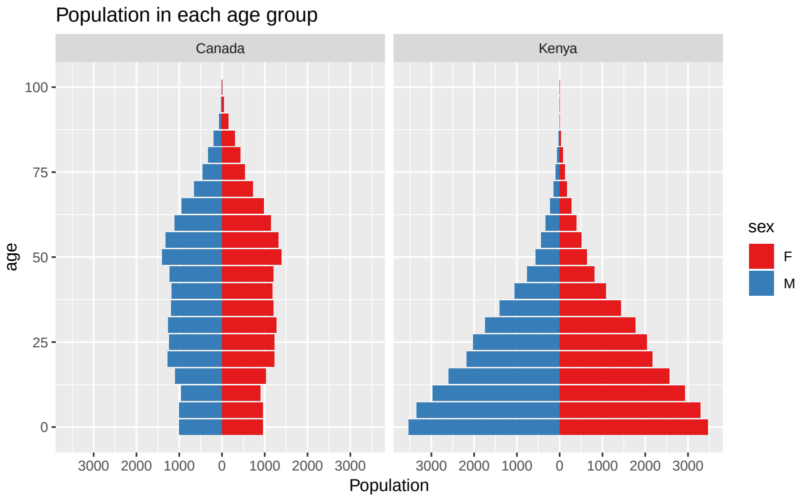
Example



TFR and tempo−adjusted TFR
Canada 1944−2011
4.0
3.5
3.0
2.5
2.0
1.5
value
1960
1980
2000
Year
tfr
tempo_adjusted
unadjusted

Inferring TFR from population pyramids

Population pyramids



Inferring TFR from population pyramids

- Age structure of population tells us something about fertility rates in a population
- TFR is somehow related to the relative numbers of women and children by age group
- Need to take into account mortality (of children and women) as well as fertility

Inferring TFR from population pyramids

Recent work by Matt Hauer and Carl Schmertmann (2018, 2020)

Consider $n = 5$ year age groups. An expression for the expected number of surviving children under age 5 per surviving woman in age group a at the end of a 5 year period, C_a, is

$$C_a = \left[\frac{L_{a-5}}{L_a} F_{a-5} + F_a \right] \frac{L_0}{2}$$

(More on this in the population projections week)

Inferring TFR from population pyramids

We can rearrange this to be

$$\begin{aligned} C_a &= TFR \frac{L_0}{5} \frac{1}{2} \left(\frac{L_{a-5}}{L_a} \phi_{a-5} + \phi_a \right) \\ &= TFR \cdot s \cdot p_a \end{aligned}$$

where ϕ_a is the fraction of lifetime fertility occurring in age group a, s is the expected fraction still alive among children born in the past five years, p_a is the proportion of lifetime fertility experienced over the past five years by females in age group a.

Summing over all ages a we get the expected total number of surviving 0-5 year olds

$$C = \sum_a W_a C_a = W \cdot p \cdot s \cdot TFR$$

Where W_a is the number of women in age group a and W is the total number of women.

Rearrange to get TFR

$$C = W \cdot p \cdot s \cdot TFR$$

Chidren = women x fraction of total births in last 5 years x fraction of births surviving x TFR

Rearranging we get

$$TFR = \frac{1}{s}\frac{1}{p}\frac{C}{W}$$

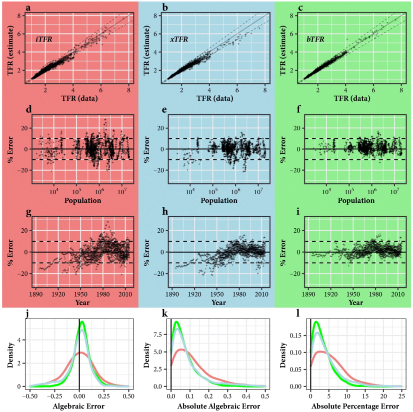
An expression for TFR

$$TFR = \frac{1}{s}\frac{1}{p}\frac{C}{W}$$

- Get C and W from population pyramids
- Hauer and Schmertmann discuss different ways to get values of s and p
- Simplest: $s = 1$ (no child mortality) and $p = 5/35$ which assumes women are uniformly distributed over reproductive ages
- Better: use data to infer likely patterns of p and s (i.e. authors use HFD and HMD) (xTFR, iTFR+. xTFR+)
- Even better: go Bayes, using HMD/HFD as priors (bTFR)


iTFR
xTFR
iTFR+
xTFR+
bTFR

Estimates are pretty good



Reproducible paper and code: https://github.com/mathewhauer/iTFR-replication (At some point I will do an rmd/blog)

Dataset ideas

Dataset ideas

- WPP
- Human Fertility Database
- Current Population Survey (IPUMS)
- DHS (IPUMS)