Econ C142 Problem Set 2

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In [738]:
```
%matplotlib inline
import math
import scipy
import numpy as np
import pandas as pd
import statsmodels.api as sm
import matplotlib.pyplot as plt
```

In [739]:
```
# Import the data
df = pd.read_stata('Brazil_1996PNAD_PS2.dta')
```

Part (a)

In [ ]:
```
# Add a constant
df = sm.add_constant(df)
# Note: we cannot take the log of monthly earnings directly, since it is
 sometimes zero.
# So, we drop these observations
df = df[df['MONTHLY_EARNINGS'] > 0]
# and then take the log
df['LOG_MONTHLY_EARNINGS'] = np.log(df['MONTHLY_EARNINGS'])
# Total number of observations
N = len(df)
# Endogenous variable: log monthly earnings
y = df['LOG_MONTHLY_EARNINGS']
# Exogenous variables...
X = pd.DataFrame()
# ... constant
X['c'] = df['const']
# ... years of schooling
X['YRSSCH'] = df['YRSSCH']
# ... age
X['AgeInDays'] = df['AgeInDays']
# ... age squared
X['AgeInDays_Squared'] = df['AgeInDays']**2
```

```
In [740]: # Compute the least-squares fit
          res1 = sm.OLS(y, X).fit()
          res1.summary()
```

Out[740]:

OLS Regression Results

Dep. Variable:	LOG_MONTHLY_EARNINGS	**R-squared:**	0.420
Model:	OLS	**Adj. R-squared:**	0.420
Method:	Least Squares	**F-statistic:**	1.340e+04
Date:	Fri, 10 Mar 2017	**Prob (F-statistic):**	0.00
Time:	11:12:37	**Log-Likelihood:**	-64167.
No. Observations:	55550	**AIC:**	1.283e+05
Df Residuals:	55546	**BIC:**	1.284e+05
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
c	3.0452	0.043	70.501	0.000	2.960 3.130
YRSSCH	0.1456	0.001	185.872	0.000	0.144 0.147
AgeInDays	0.0893	0.002	37.802	0.000	0.085 0.094
AgeInDays_Squared	-0.0009	3.04e-05	-29.683	0.000	-0.001 -0.001

Omnibus:	1093.915	**Durbin-Watson:**	1.593
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2209.058
Skew:	0.094	**Prob(JB):**	0.00
Kurtosis:	3.959	**Cond. No.**	2.23e+04

We can read off the 95% confidence interval for YRSSCH: $[0.144, 0.147]$.

The expected difference in log earnings for an individual with no schooling and an individual who has completed primary school is $8 \cdot 0.146 = 1.17$, or in linear terms, about a 120% increase in earnings.

Part (b)

In [741]:
```
# Endogenous variable
y = df['LOG_MONTHLY_EARNINGS']
# Exogenous variables...
# ... dummies for years of school
X = pd.get_dummies(df['YRSSCH'], prefix='YRSSCH')
# ... age
X['AgeInDays'] = df['AgeInDays']
# ... age squared
X['AgeInDays_Squared'] = df['AgeInDays']**2
# Compute the least-squares fit
res2 = sm.OLS(y, X).fit()
res2.summary()
```

Out[741]:

OLS Regression Results

Dep. Variable:	LOG_MONTHLY_EARNINGS	**R-squared:**	0.429
Model:	OLS	**Adj. R-squared:**	0.428
Method:	Least Squares	**F-statistic:**	2450.
Date:	Fri, 10 Mar 2017	**Prob (F-statistic):**	0.00
Time:	11:12:37	**Log-Likelihood:**	-63743.
No. Observations:	55550	**AIC:**	1.275e+05
Df Residuals:	55532	**BIC:**	1.277e+05
Df Model:	17		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
YRSSCH_0.0	3.1424	0.044	71.094	0.000	3.056 3.229
YRSSCH_1.0	3.3432	0.047	70.739	0.000	3.251 3.436
YRSSCH_2.0	3.4798	0.045	76.506	0.000	3.391 3.569
YRSSCH_3.0	3.6100	0.045	80.456	0.000	3.522 3.698
YRSSCH_4.0	3.8040	0.044	85.728	0.000	3.717 3.891
YRSSCH_5.0	3.8896	0.044	87.841	0.000	3.803 3.976
YRSSCH_6.0	3.9779	0.045	88.211	0.000	3.889 4.066
YRSSCH_7.0	4.0545	0.045	89.833	0.000	3.966 4.143
YRSSCH_8.0	4.2004	0.044	94.449	0.000	4.113 4.288
YRSSCH_9.0	4.2735	0.049	87.711	0.000	4.178 4.369
YRSSCH_10.0	4.3868	0.048	92.096	0.000	4.293 4.480
YRSSCH_11.0	4.6883	0.046	102.737	0.000	4.599 4.778
YRSSCH_12.0	4.9364	0.058	84.682	0.000	4.822 5.051
YRSSCH_13.0	5.0792	0.058	86.999	0.000	4.965 5.194
YRSSCH_14.0	5.1801	0.058	89.682	0.000	5.067 5.293
YRSSCH_15.0	5.5794	0.047	119.162	0.000	5.488 5.671
AgeInDays	0.0857	0.002	36.306	0.000	0.081 0.090
AgeInDays_Squared	-0.0009	3.03e-05	-29.031	0.000	-0.001 -0.001

Omnibus:	1197.043	**Durbin-Watson:**	1.600
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	2511.034
Skew:	0.098	**Prob(JB):**	0.00

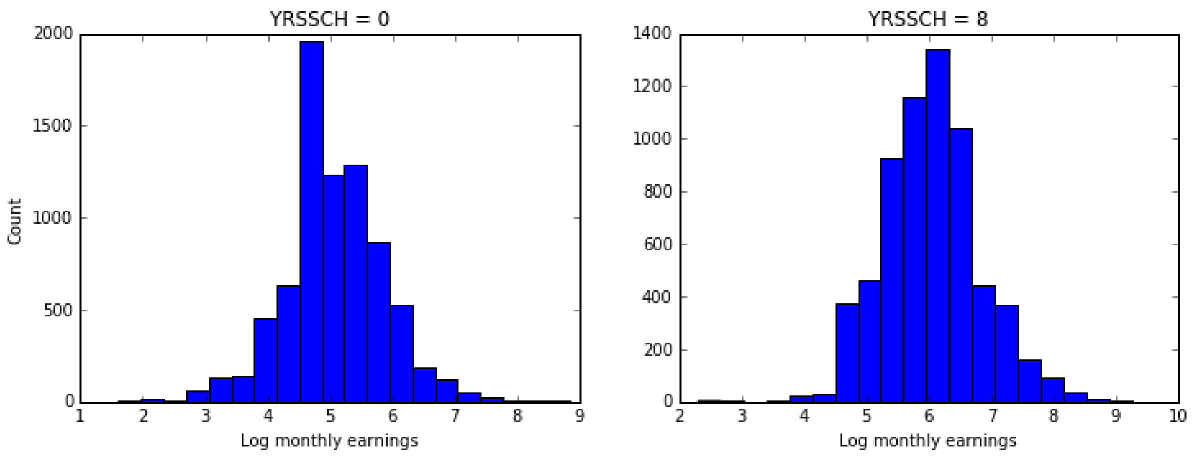
Kurtosis:	4.023	**Cond. No.**	9.14e+04

The expected difference in log earnings for an individual with no schooling and an individual who has completed primary school is $4.20 - 3.14 = 1.06$, or in linear terms, about a 106% increase in earnings. This is comparable to the estimate from the model in part (a).

However, consider the difference between completing 6 years of school and 5 years of school ($4.05 - 3.98 = 0.07$), versus the difference between completing 13 years of school and 12 years of school ($5.07 - 4.93 = 0.14$). This suggests that the effects of an additional year of school are not constant across the population, motivating a quantile regression approach.

Part (c)

In [742]:
```
# Plot histogram
fig = plt.figure(figsize=(12,4))
ax1 = fig.add_subplot(121)
ax1.hist(df[df['YRSSCH'] == 0]['LOG_MONTHLY_EARNINGS'], 20)
ax1.set_xlabel('Log monthly earnings')
ax1.set_ylabel('Count')
ax1.set_title('YRSSCH = 0')
ax2 = fig.add_subplot(122)
ax2.hist(df[df['YRSSCH'] == 8]['LOG_MONTHLY_EARNINGS'], 20)
ax2.set_xlabel('Log monthly earnings')
ax1.set_ylabel('Count')
ax2.set_title('YRSSCH = 8')
plt.show()
```



The distribution of log earnings is centered around 5 for no schooling, and around 6 for 8 years of schooling. Both look approximately normal, but there is an abnormally large bin in the no schooling distribution around 4.8. This is probably due to a clustering effect — perhaps this represents the monthly earnings for a minimum wage job.

Part (d)

In [743]:
```python
def quantile_estimate(data, tau):
    """
    Takes a data series and a quantile and,
    if there are more than 80 observations,
    returns (pi, sigma2, se), where
    - pi is an estimate of the quantile
    - sigma2 is an estimate of the variance,
    - se is an estimate of the standard error.
    """
    # Sort the data
    data = sorted(data)
    n = len(data)
    # If there are fewer than 80 observations, quit
    if n < 80:
        return None, None, None
    # Estimate of the quantile
    pi = np.percentile(data, q=tau*100)
    # z-value for a 95% confidence interval
    z = scipy.stats.norm.ppf(0.975)
    l = z*math.sqrt(n*tau*(1-tau))
    # Index of lower bound
    j = math.floor(n*tau - l)
    # Lower bound of CI
    lbound = data[j-1]
    # Index of upper bound
    k = math.ceil(n*tau + l)
    # Upper bound of CI
    ubound = data[k-1]
    # Variance estimator
    sigma2 = n*(ubound-lbound)**2/(4*z**2)
    # Standard error estimator
    se = math.sqrt(sigma2/n)
    return pi, sigma2, se
```

In [744]:
```python
# We'll store our estimates here
estimates = pd.DataFrame(columns=[
        'L', # age bin
        'K', # school bin
        'p', # proportion of observations in this bin
        'tau', # quantile of interest
        'pi', # quantile estimate
        'sigma2', # variance estimate,
        'se' # standard error estimate
    ])
# Quantiles we're interested in
quantiles = [0.10, 0.25, 0.5, 0.75, 0.90]
# School bins from 0 to 15
schoolbins = range(0, 16)
# Age bins from [20, 25) ... [55, 60)
agebins = range(20, 60, 5)
```

In [745]:
```
for L in agebins:
    for K in schoolbins:
        # Select the subset of data we want
        subset = df[(df.AgeInDays >= L) & (df.AgeInDays < (L + 5)) &
(df.YRSSCH == K)]
        # Series of log earnings from this subset
        earnings_series = subset['LOG_MONTHLY_EARNINGS']
        # Proportion of observations in this bin
        p = len(earnings_series)/N
        # For each quantile of interest
        for tau in quantiles:
            # Compute the estimate and standard deviation
            pi, sigma2, se = quantile_estimate(earnings_series, tau)
            estimates = estimates.append({
                'L': L,
                'K': K,
                'tau': tau,
                'p': p,
                'pi': pi,
                'sigma2': sigma2,
                'se': se
            }, ignore_index=True)
```

In [746]:
```
estimates.head()
```

Out[746]:

	L	K	p	tau	pi	sigma2	se
0	20	0	0.013645	0.10	3.912023	2.456302	0.056925
1	20	0	0.013645	0.25	4.442651	2.456302	0.056925
2	20	0	0.013645	0.50	4.787492	0.234814	0.017601
3	20	0	0.013645	0.75	5.298317	0.000000	0.000000
4	20	0	0.013645	0.90	5.703783	3.006177	0.062976

Part (e)

Let's look at the bins where the standard error estimate is zero.

```
In [747]: zeros = estimates[estimates['se'] == 0]
          print(len(zeros))
          zeros.head()
```

```
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```

Out[747]:

	L	K	p	tau	pi	sigma2	se
3	20	0	0.013645	0.75	5.298317	0	0
16	20	3	0.011377	0.25	4.718499	0	0
25	20	5	0.019586	0.10	4.718499	0	0
30	20	6	0.013195	0.10	4.718499	0	0
35	20	7	0.012979	0.10	4.718499	0	0

There are 11 bins for which the standard error estimate is zero. Let's see what's going on in the first one:

```
In [748]: L = 20
          K = 0
          subset = df[(df.AgeInDays >= L) & (df.AgeInDays < (L + 5)) & (df.YRSSCH
          == K)]
          subset['LOG_MONTHLY_EARNINGS'].value_counts().head()
```

```
Out[748]: 4.718499    84
          4.605170    62
          4.787492    58
          4.382027    57
          5.298317    50
          Name: LOG_MONTHLY_EARNINGS, dtype: int64
```

We have 84 observations with `LOG_MONTHLY_EARNINGS` equal to 4.718499, so `MONTHLY_EARNINGS` is clearly not a continuously-valued random variable; there are certain earnings levels which are shared by many people in the sample. This is causing our standard error estimates to sometimes be zero — if our 95% confidence interval falls within one of these clusters, then the length of the interval and thus the standard error will be 0.

Part (f)

Note: we correct the standard error reported by the WLS program, $se^*(\alpha)$, with the following:

$$se(\alpha) = \frac{se^*(\alpha)}{\sqrt{s^2}}\sqrt{\frac{L}{N}},$$

where s^2 is the standard error of the regression, N is the total number of observations, and L is the total number of bins.

```
In [749]: def quantile_regress(data):
              """
              Takes a data series and returns (alpha, se), where
              - alpha is the coefficient on YRSSCH
              - se is a standard error estimate.
              """
              # Endogenous variable
              y = data['pi']
              # Exogenous variables
              X = pd.DataFrame()
              # Midpoint of the age range
              X['AgeInDays'] = data['L'] + 2.5
              X['AgeInDays_Squared'] = X['AgeInDays']**2
              X['YRSSCH'] = data['K']
              # WLS weights
              weights = data['p']/(data['sigma2'])
              # Fit the model
              res = sm.WLS(y, X, weights).fit()
              # Coefficient on YRSSCH
              alpha = res.params['YRSSCH']
              # Square root of the standard error of the regression
              # (Statsmodels takes care of the degrees of freedom correction
              # so this is an unbiased estimator)
              s = math.sqrt(res.mse_resid)
              # Number of bins
              L = len(data)
              # Corrected standard error for alpha
              se = res.bse['YRSSCH']/s * math.sqrt(L/N)
              return alpha, se
```

```
In [750]: # We'll store our estimates here
          coeff_estimates = pd.DataFrame(columns=[
                  'tau', # quantile
                  'alpha', # coefficient on YRSSCH
                  'se', # standard error of coefficient on YRSSCH
              ])

          for quantile in quantiles:
              # Drop if the standard error is 0
              data = estimates[(estimates.tau == quantile) & (estimates.se > 0)].d
          ropna()
              alpha, se = quantile_regress(data)
              coeff_estimates = coeff_estimates.append({
                      'tau': quantile,
                      'alpha': alpha,
                      'se': se
                  }, ignore_index=True)

          coeff_estimates.head()
```

Out[750]:

	tau	alpha	se
0	0.10	0.129345	0.015231
1	0.25	0.139102	0.007225
2	0.50	0.142337	0.009657
3	0.75	0.145653	0.007234
4	0.90	0.143172	0.013347

The coefficient on schooling increases with the quantile under consideration from the 0.10 to the 0.75 quantile, then decreases again at the 0.90 quantile. So, it seems as though the returns to schooling are greater at higher quantiles in the earnings distribution. If you earn more, schooling is likely to have a larger effect on your earnings; the earnings of those who earn less (and those who earn the very most) are not affected by their schooling levels as much.

The coefficients are all lower than the standard OLS estimate, 0.146. This indicates that the returns to schooling might be very high for some individuals, which is biasing the estimate upwards. The median individual's returns to schooling are lower than the average individual's returns.

Part (g)

Earnings increase with education. On *average*, an extra year of education leads to a 0.145 log point increase in earnings. However, these returns vary dramatically across the year in consideration; for example, the returns from going from 12 to 13 years of school are much greater than the returns from going from 5 to 6 years of school.

The effect of an extra year of education also varies across the earnings distribution. For the lowest earners, the effects of schooling is not as significant (perhaps these are individuals who are unemployed for reasons unrelated to schooling or training); an extra year of education produces about a 0.130 log point increase in earnings. Upper-middle income individuals see the greatest returns to education, about a 0.146 log point increase in earnings for each additional year of education. These might be those with "regular" white-collar jobs for whom a college degree or a postgraduate degree could have a significant impact on their career trajectory. For the *very* highest earners, an extra year of schooling also doesn't have as much of an effect as for those who earn a bit less (think child prodigies and college-dropouts-turned-billionaires). For them, an extra year of education produces about a 0.143 log point increase in earnings.

In general, the fact that the coefficient from the OLS estimate is larger than most of the coefficients from the quantile regression suggests that there are relatively many highly-educated individuals who also earn a lot; the highly-paid and highly-educated professionals are probably biasing the results upwards for the population as a whole, which includes lower-paid workers who experience lower returns to education.

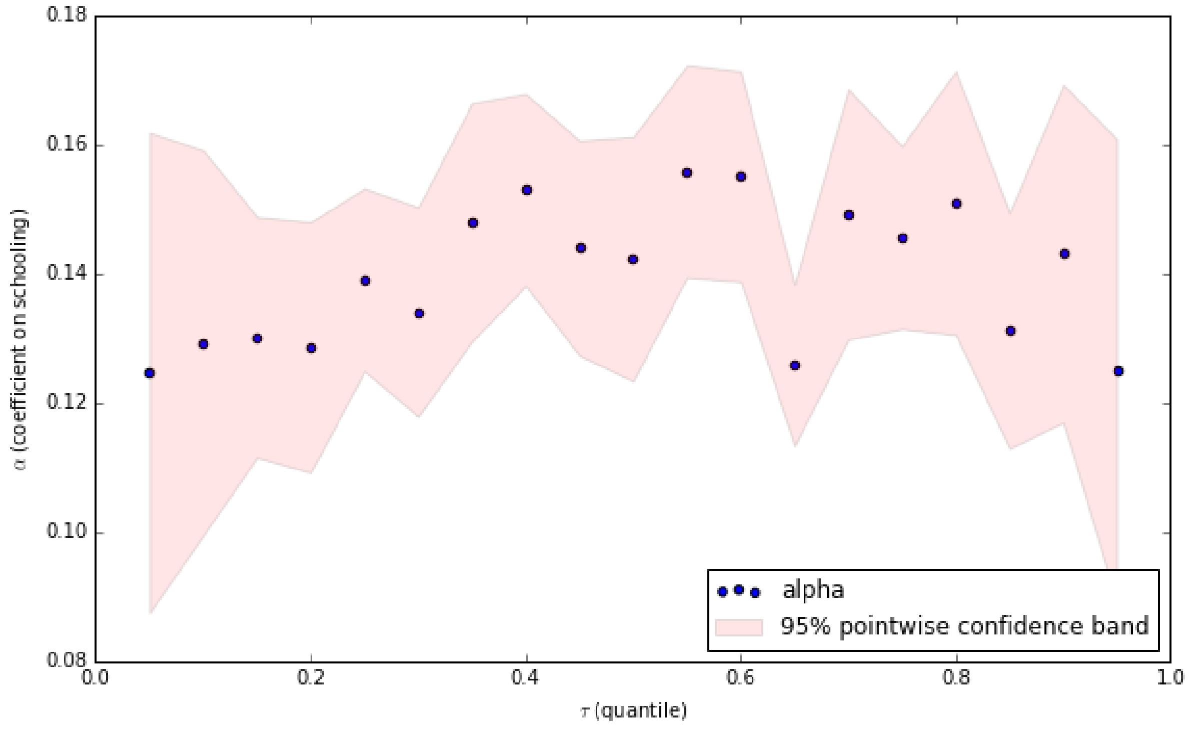
Part (h)

In [751]:
```
# Centiles 0.05 to 0.95
quantiles2 = np.arange(0.05, 1.0, 0.05)

# We'll store our estimates here
estimates2 = pd.DataFrame(columns=[
        'L', # age bin
        'K', # school bin
        'p', # proportion of observations in this bin
        'tau', # quantile of interest
        'pi', # quantile estimate
        'sigma2', # variance estimate
        'se' # standard error estimate
    ])
```

In [752]:
```
for L in agebins:
    for K in schoolbins:
        # Select the subset of data we want
        subset = df[(df.AgeInDays >= L) & (df.AgeInDays < (L + 5)) &
(df.YRSSCH == K)]
        # Series of log earnings from this subset
        earnings_series = subset['LOG_MONTHLY_EARNINGS']
        # Proportion of observations in this bin
        p = len(earnings_series)/N
        # For each quantile of interest
        for tau in quantiles2:
            # Compute the estimate and standard deviation
            pi, sigma2, se = quantile_estimate(earnings_series, tau)
            estimates2 = estimates2.append({
                'L': L,
                'K': K,
                'tau': tau,
                'p': p,
                'pi': pi,
                'sigma2': sigma2,
                'se': se
            }, ignore_index=True)
```

In [753]:
```
# We'll store our coefficient estimates here
coeff_estimates2 = pd.DataFrame(columns=[
        'tau', # quantile
        'alpha', # coefficient on YRSSCH
        'se', # standard error of coefficient on YRSSCH
    ])

for quantile in quantiles2:
    data = estimates2[(estimates2.tau == quantile) & (estimates2.se >
0)].dropna()
    alpha, se = quantile_regress(data)
    coeff_estimates2 = coeff_estimates2.append({
            'tau': quantile,
            'alpha': alpha,
            'se': se
        }, ignore_index=True)
```

In [754]:
```python
# Plot with standard errors
fig = plt.figure(figsize=(10,6))
ax = fig.add_subplot(111)
ax.set_xlim([0,1])
ax.set_xlabel(r'$\tau$ (quantile)')
ax.set_ylabel(r'$\alpha$ (coefficient on schooling)')
# Plot scatter of normalized costs against log output
ax.scatter(coeff_estimates2['tau'], coeff_estimates2['alpha'])
# z-value for a 95% confidence interval
z = scipy.stats.norm.ppf(0.975)
# CI lower and upper bounds
lower = coeff_estimates2['alpha'] - z*coeff_estimates2['se']
upper = coeff_estimates2['alpha'] + z*coeff_estimates2['se']
# Plot confidence band
ax.fill_between(
    coeff_estimates2['tau'],
    lower,
    upper,
    facecolor='red',
    alpha=0.1,
    label='95% pointwise confidence band'
)
plt.legend(loc='lower right')
plt.show()
```



This plot looks roughly U-shaped; it's certainly not monotonically increasing. It appears as though the returns to schooling increase from the lowest quantiles to about the median of the earnings distribution, after which the returns stay approximately constant and even decrease towards the upper quantiles. The data is relatively noisy, though — given the size of the standard errors, it could be that the returns are flat across the population.