

Unified Growth Theory and Comparative Development

Ömer Özak

Department of Economics
Southern Methodist University

Economic Growth and Comparative Development



Fundamental Research Questions

- What is the origin of the vast inequality in income per capita across countries and regions?
- What are the forces that triggered the transition from stagnation to growth?
- What accounts for the divergence in per-capita income across countries in the past two centuries?
- What are the factors that inhibited the convergence of poor economies toward richer ones in the past decades?
- What is the role of deep-rooted factors in explaining the observed patterns of comparative development?

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Limitations of Non-Unified Growth Theory

- Inconsistent with the growth process over most of human history:
- Not designed to shed light on the:
 - Historical origins of vast and persistent inequality across countries
 - Forces that triggered the transition of DCs from stagnation to growth
 - Hurdles faced by LDCs in their take-off from stagnation to growth
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Unified Growth Theory

- Captures the:
 - Process of development in its entirety
 - Forces that permitted DCs to transition from the Malthusian Epoch to sustained growth
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- A unified theory of economic growth that accounts for the:
 - Epoch of Malthusian stagnation
 - Take-off from the Malthusian Regime
 - Emergence of human capital as a significant factor in the growth process
 - Demographic transition
 - Shift to sustained economic growth
 - Emergence of inequality in income per capita across countries

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Origins of the Phase Transition

- Design of a dynamical system that permits a phase transition:
 - Escape from a stable Malthusian equilibrium:
- Hypothetical mechanisms:
 - Shock in an economy with multiple stable equilibria
 - Inconsistent with a gradual increase in TFP growth
 - A gradual escape from an absorbing (stable) equilibrium
 - Contradiction to the essence of a stable equilibrium
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- Evolution of a latent state variable – the demand for human capital
 - Ultimately changes the dynamical system qualitatively:
 - The Malthusian equilibrium vanishes endogenously
 - The economy gravitates towards the emerging Modern Growth Regime

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Characteristics of the Main Transitions

- Transition from Malthusian to Post-Malthusian Regime:
 - Faster rate of technological progress
 - Faster rate of population growth
 - Insignificant investment in human capital
 - Onset of growth in income per capita
- Transition from the Post-Malthusian to Modern Growth Regime:
 - Faster rate of technological progress
 - Faster rate of human capital accumulation
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Suggestive Evidence

- The underlying forces that govern these transitions:
 - The effect of changes in the technological environment on:
 - population size and quality
 - The effect of changes in the size and the quality of the population on:
 - the rate of technological progress

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The Basic Structure of the Model

- Overlapping-generations economy
- $t = 0, 1, 2, 3 \dots$
- One homogeneous good
- 2 factors of production:
 - Labor (measured in efficiency units)
 - Land

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Factor Supply

- Land is fixed over time
 - Surface of planet earth
- Efficiency units of labor evolves endogenously
 - Determined by households' decisions about the number and level of human capital of their children

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The Malthusian Structure

- A subsistence consumption constraint
- Positive effect of income on population
 - $y \uparrow \implies L \uparrow$
- Fixed factor of production – Land
 - $L \uparrow \implies AP_L \downarrow \implies y \downarrow$
- Output per capita fluctuates (with a negligible trend) around a constant level in the long-run
 - Reflecting diminishing returns to labor & positive effect of income on population

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Production

- The output produced in period t

$$Y_t = H_t^\alpha (A_t X)^{1-\alpha}$$

- $H_t \equiv$ efficiency units of labor
- $A_t \equiv$ technological level
- $X \equiv$ land

- Output per worker produced at time t

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The Malthusian Structure – Effects of Technological Progress

- Very short-run (for a given population):
 - $A_t \uparrow \implies y_t \uparrow$ (above \bar{y})
- Short-run (initial adjustment of population):
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- Long-run (population reaches a new steady-state):
 - $L_t \uparrow \implies y \downarrow$ (back to \bar{y})

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Sources of Technological Progress

- Earlier stages of development

- Population size positively affects technological progress:

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- Channels:

- Supply of innovations
 - Demand for innovations
 - Diffusion of knowledge
 - Division of labor
 - Extent of trade

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- Later Stages of Development

- Human capital positively affects technological progress

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Technological Progress

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(e_t, L_t)$$

- $g_{t+1} \equiv$ rate of tech progress
- $e_t \equiv$ education
- $L_t \equiv$ population size

Technological Progress

$$g_{t+1} = g(e_t, L_t)$$

- $g_e(e_t, L_t) > 0$ and $g_{ee}(e_t, L_t) < 0$
 - Education has a positive and diminishing effect on technological progress
- $g_L(e_t, L_t) > 0$ and $g_{LL}(e_t, L_t) < 0$
 - The scale of the economy has a positive and diminishing effect on technological progress
- $g(0, L) > 0$ for $L > 0$
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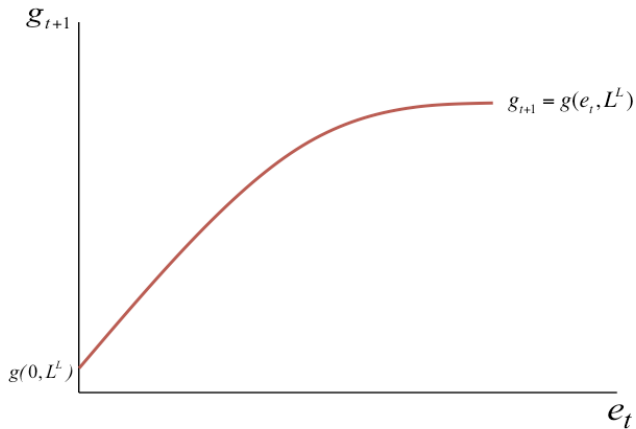
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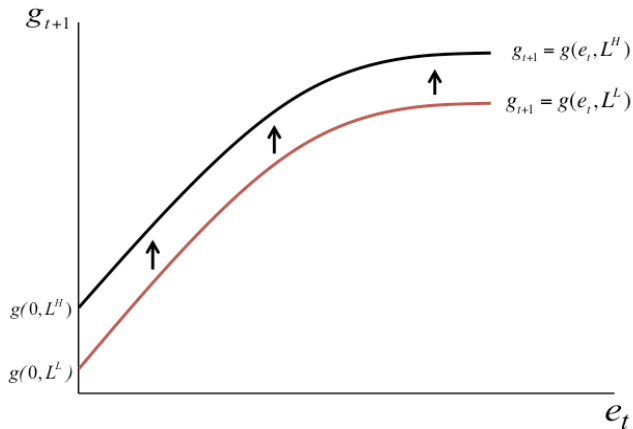
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Technological Progress



The Effect of Population Size on Technological Progress



Origins of Human Capital Formation

- The increase in the rate of technological progress increases the demand for human capital
 - Human capital permits individuals to better cope with the changes in the technological environment
 - The introduction of new technologies is skill-biased in the short-run, although the nature of the technology can be skill-biased or skill-saving in the long run

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Human Capital Formation

Human capital of an individual who joins the labor force in period $t + 1$

$$h_{t+1} = h(e_{t+1}, g_{t+1})$$

- $e_{t+1} \equiv$ the individual education level (determined by parental investment, subject to their subsistence constraint, in period t)
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- $h_g(e, g) < 0$ and $h_{gg}(e, g) > 0$
 - Obsolescence of HC in a changing technological environment
- $h_{eg}(e, g) > 0$
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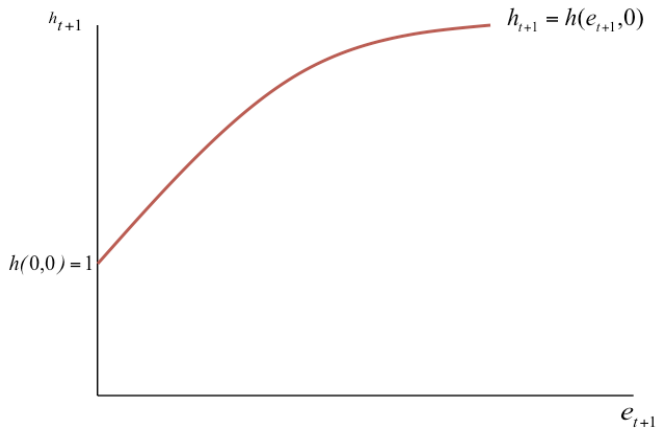
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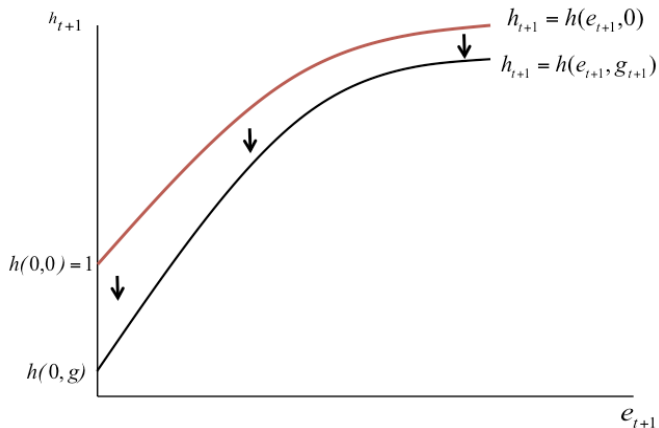
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Human Capital Formation



Triggers of the Demographic Transition

- The rise in the demand for human capital induces parents to substitute quality for quantity of children
- The rise in income along with the rise in the potential return to human capital generates:
 - An income effect – more income to spend on children
 - Substitution effects
 - The opportunity cost of raising children increases
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Triggers of the Demographic Transition

- Early part of the second phase of industrialization:
 - The income effect dominates (moderate demand for human capital):
 - Population growth & human capital formation increase:
- Later part of the second phase of industrialization:
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Individuals

- Live for 2 periods
- Childhood (1st Period):
 - Consume a fraction of parental time endowment
 - The required time increases with child quality
 - $\tau \equiv$ time required to raise a child, regardless of quality
 - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
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 - Allocate the time endowment between childrearing and work
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Preferences

- The utility function of individual t (adult at time t)

$$u^t = (1 - \gamma) \ln(c_t) + \gamma \ln(n_t h_{t+1})$$

- $c_t \equiv$ consumption of individual t
- $n_t \equiv$ number of children of individual t
- $h_{t+1} \equiv$ level of human capital of each child

Budget and Subsistence Consumption Constraints

$$z_t n_t (\tau + e_{t+1}) + c_t \leq z_t$$

- $z_t \equiv$ potential income of individual t
- $\tau \equiv$ time required to raise a child, regardless of quality
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$$z_t \equiv y_t = h_t^\alpha x_t^{1-\alpha} = z(e_t, g_t, x_t)$$

- Subsistence consumption constraint:

$$c_t \geq \tilde{c}$$

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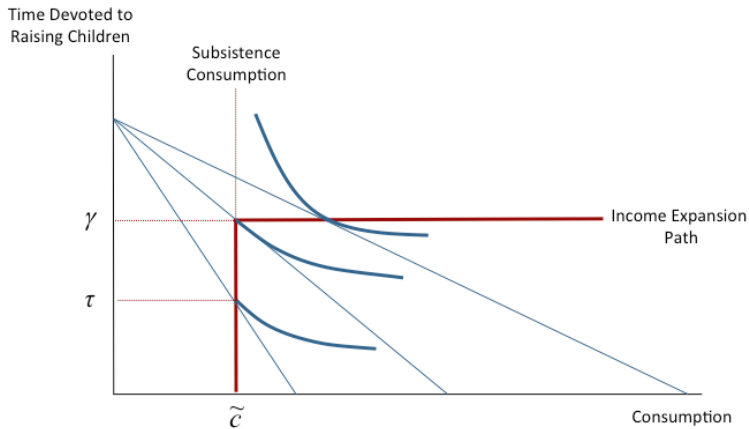
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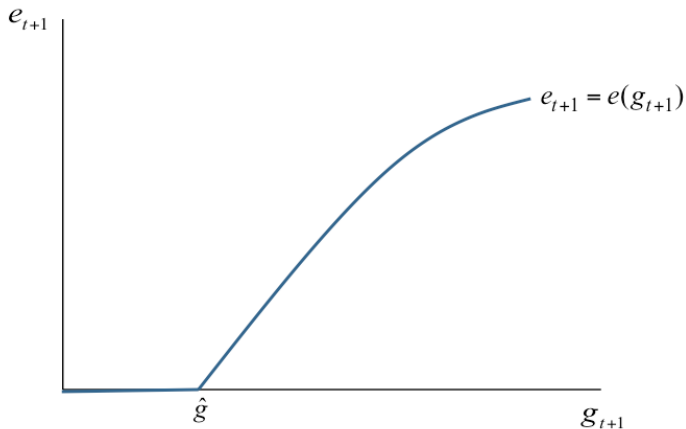
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Constraint and Optimization



Optimal Investment in Child Quality



Optimization: Quantity and Quality of Children

- Time devoted to children:

$$n_t(\tau + e_{t+1}) = \begin{cases} \gamma & \text{if } z_t \geq \tilde{z} \\ 1 - \frac{\tilde{c}}{z_t} & \text{if } z_t \leq \tilde{z} \end{cases}$$

- $z_t = \tilde{z}$ is the highest level of potential income such that the subsistence constraint is still binding

$$e_{t+1} = e(g_{t+1}) \implies$$

$$n_t = \begin{cases} \frac{\gamma}{\tau + e(g_{t+1})} \equiv n^b(g_{t+1}) & \text{if } z_t \geq \tilde{z} \\ \frac{1 - [\tilde{c}/z_t]}{\tau + e(g_{t+1})} \equiv n^a(g_{t+1}, z(e_t, g_t, x_t)) & \text{if } z_t \leq \tilde{z} \end{cases}$$

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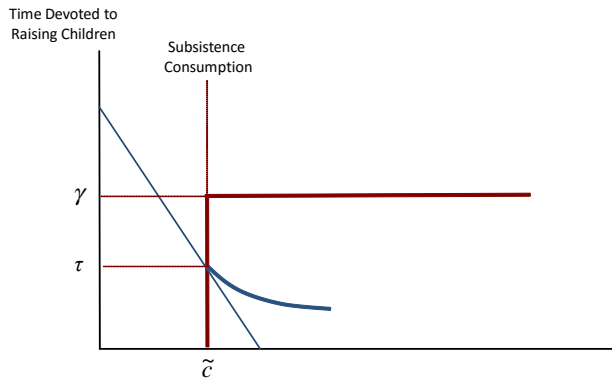
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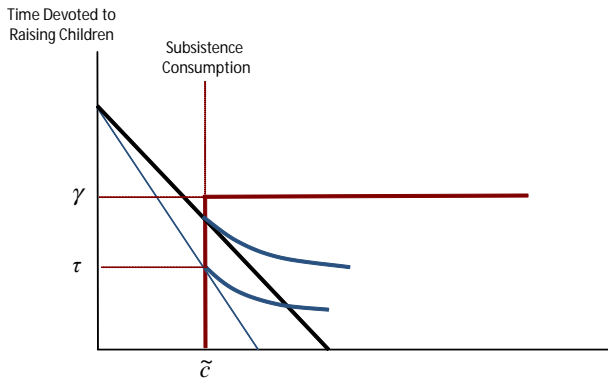
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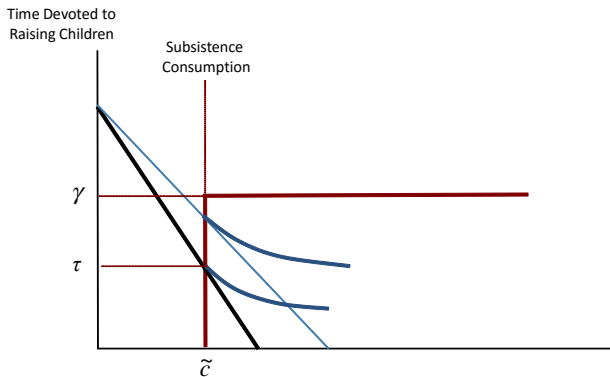
Optimization – Malthusian Epoch



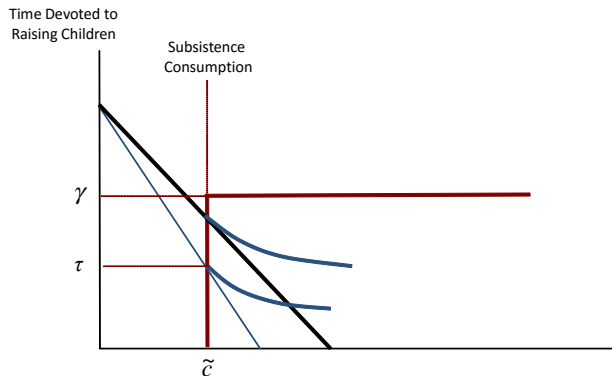
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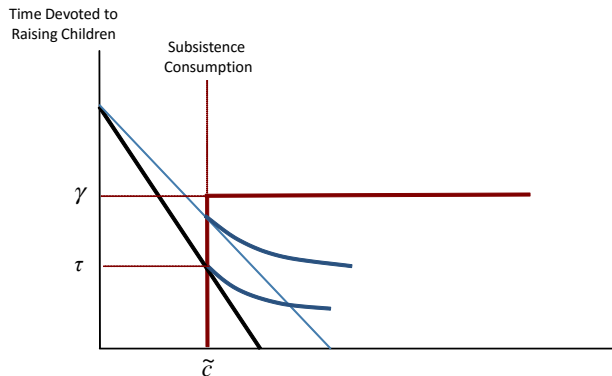
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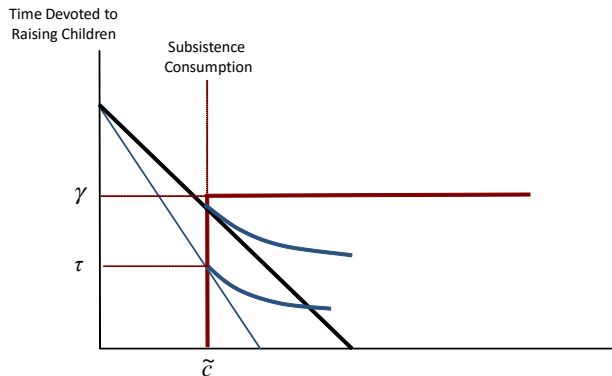
Income Expansion Path – Malthusian Epoch



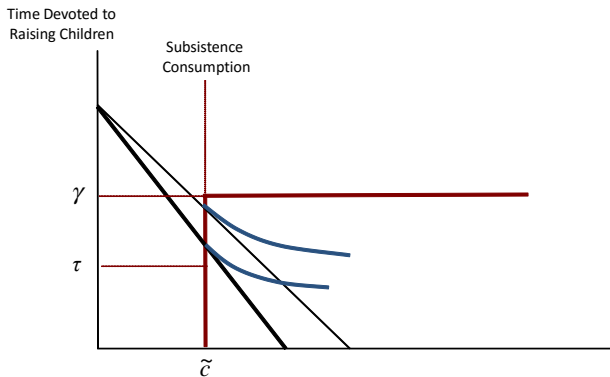
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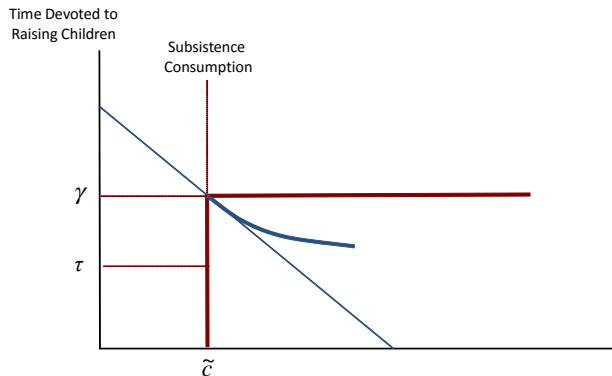
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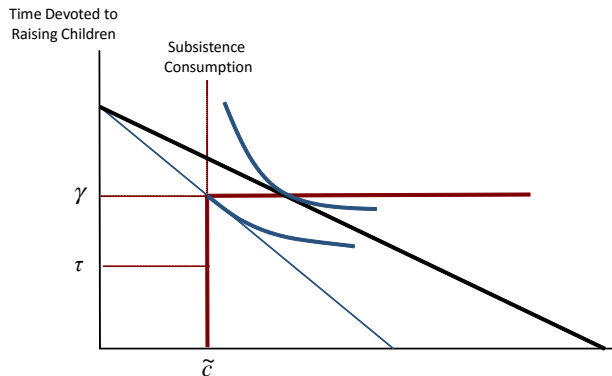
Income Expansion Path – Malthusian Epoch



Income Expansion Path – Post-Demographic Transition



Income Expansion Path – Post-Demographic Transition



Population Dynamics

$$L_{t+1} = n_t L_t$$

$$L_{t+1} = \begin{cases} n^b(g_{t+1})L_t & \text{if } z_t \geq \tilde{z} \\ n^a(g_{t+1}, z(e_t, g_t, x_t))L_t & \text{if } z_t \leq \tilde{z} \end{cases}$$

Dynamics of the Level of Resources per Worker

$$x_{t+1} = \frac{A_{t+1}X}{L_{t+1}} = \frac{(1 + g_{t+1})A_t X}{n_t L_t} = \frac{1 + g_{t+1}}{n_t} x_t$$

$$x_{t+1} = \begin{cases} \frac{[1+g(e_t, L_t)][\tau^q + \tau^e e(g(e_t, L_t))]}{\gamma} x_t \equiv \phi^b(e_t; L) x_t & z_t \geq \tilde{z} \\ \frac{[1+g(e_t, L_t)][\tau + e(g(e_t, L_t))]}{1 - [\tilde{c}/z(e_t, g_t, x_t)]} x_t \equiv \phi^a(e_t, g_t, x_t, L_t) x_t & z_t \leq \tilde{z}, \end{cases}$$

The Dynamical System

A sequence $\{x_t, e_t, g_t, L_t\}_{t=0}^{\infty}$ such that:

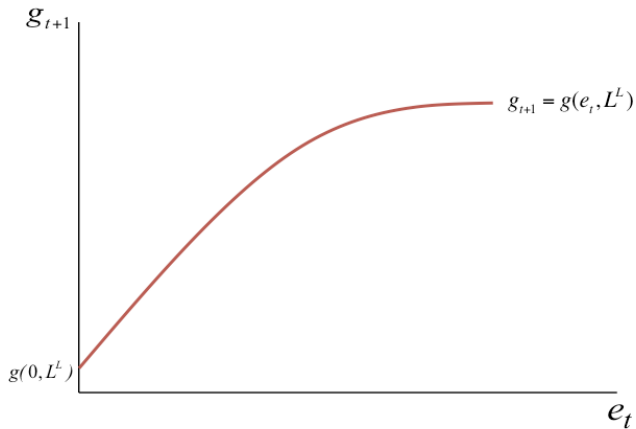
$$\begin{cases} x_{t+1} = \phi(e_t, g_t, x_t, L_t)x_t \\ e_{t+1} = e(g(e_t, L_t)) \\ g_{t+1} = g(e_t, L_t) \\ L_{t+1} = n(e_t, g_t, x_t, L_t)L_t \end{cases}$$

The Conditional Evolution of Technology and Education

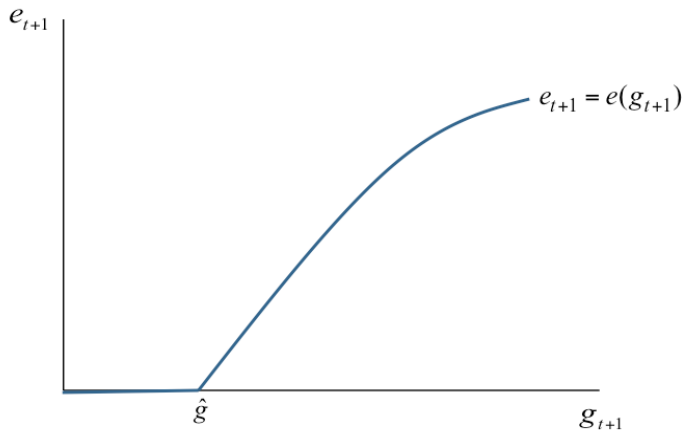
A sequence $\{g_t, e_t; L\}_{t=0}^{\infty}$ such that:

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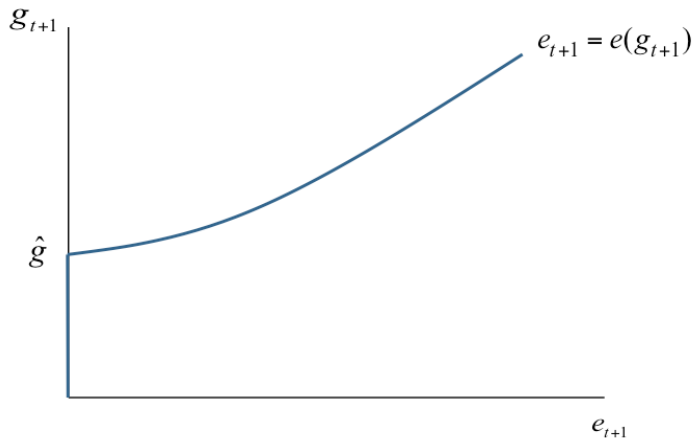
The Effect of Education on Technology



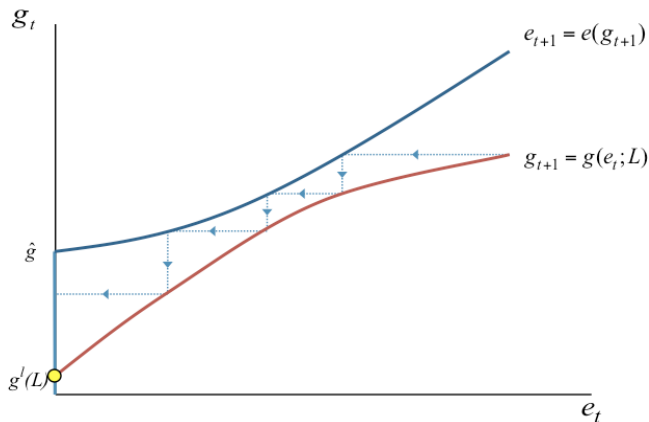
The Effect of Technology on Education



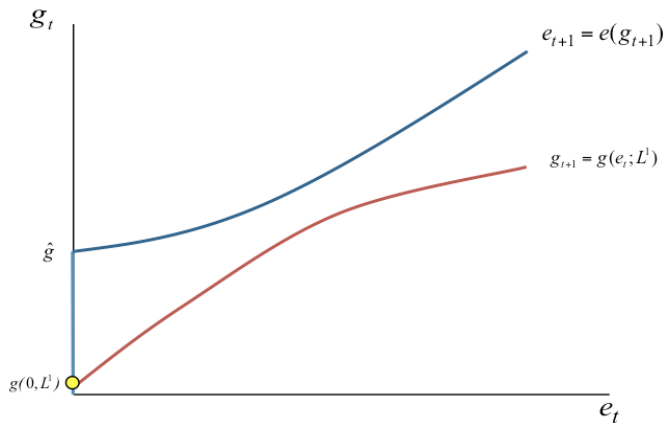
The Effect of Technology on Education: Flipped Axis



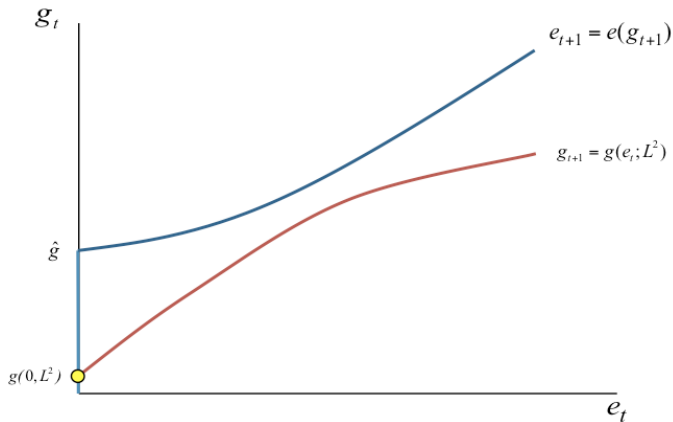
The Evolution of Education and Technology: For a Given Population Size



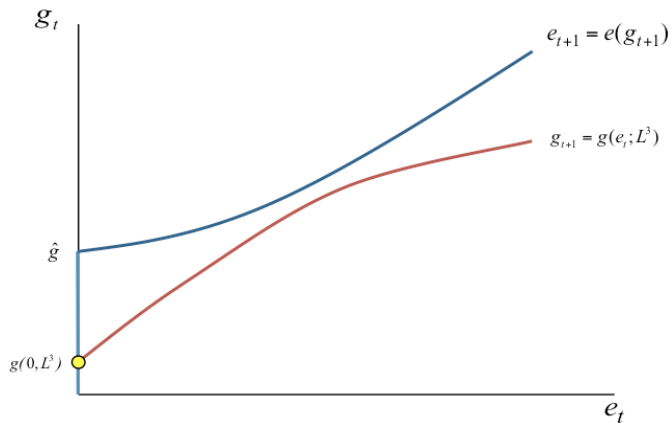
The Evolution of Education and Technology



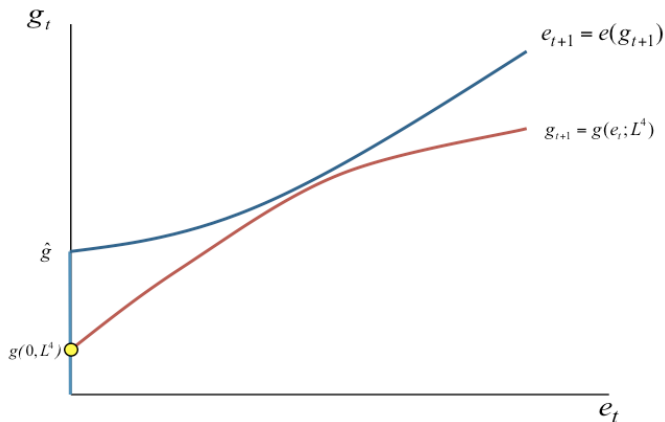
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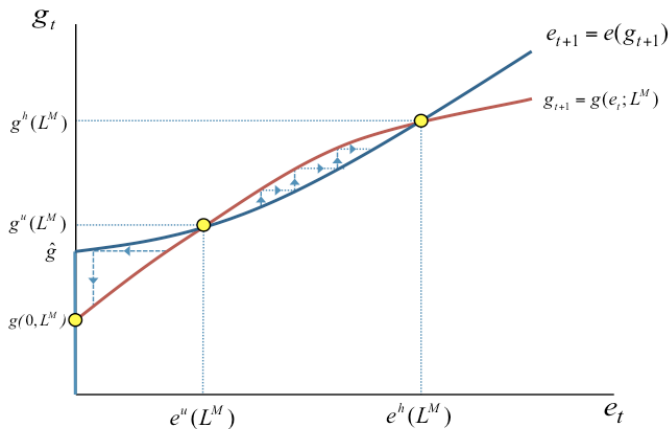
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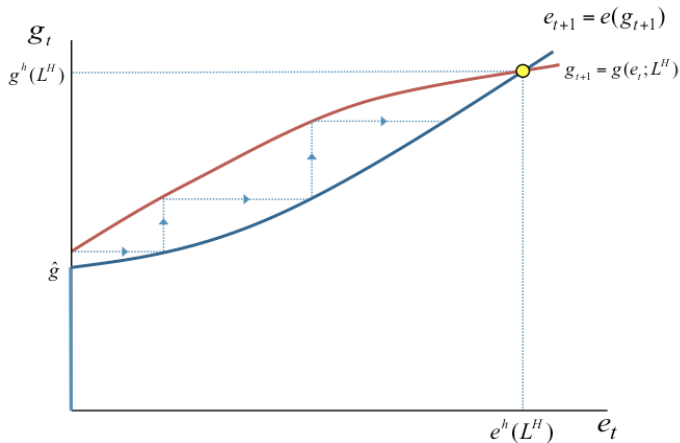
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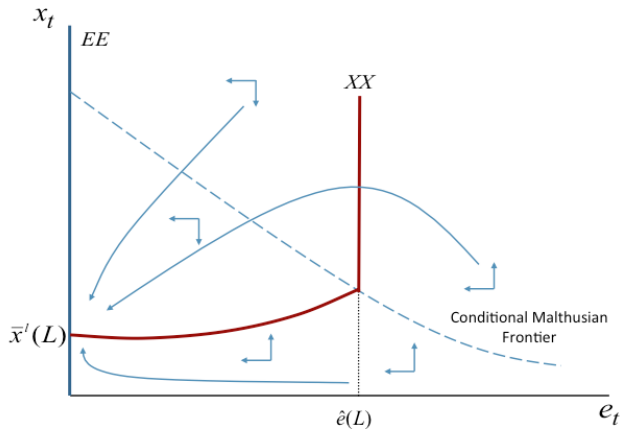
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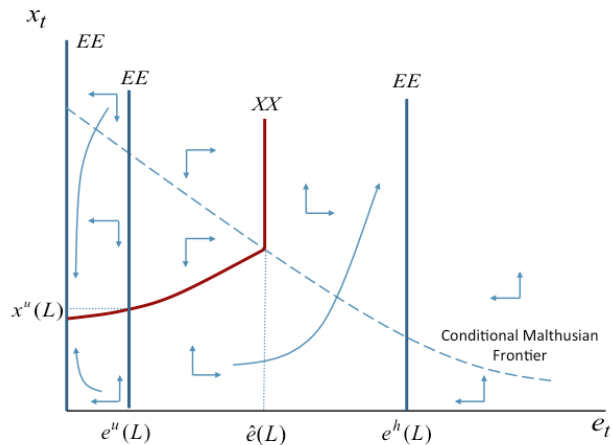
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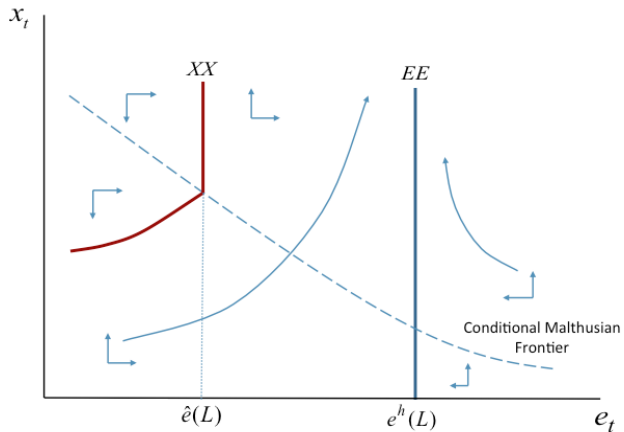
The Evolution of Education and Resources Per Worker: Small Population



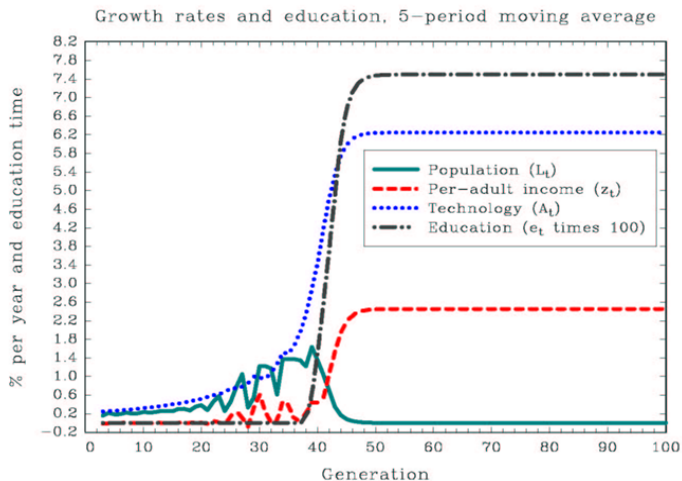
The Evolution of Education and Resources Per Worker: Intermediate Population



The Evolution of Education and Resources Per Worker: Large Population



Simulation



Source: Lagerlof (RED 2006)

- The Malthusian interaction between technology & population
 - Acceleration in technological progress
 - \Rightarrow Industrial demand for human capital
 - Human capital formation
 - \Rightarrow Decline in fertility rates
 - \Rightarrow Further technological progress
 - Decline in population growth
 - \Rightarrow Economic growth is freed from counterbalancing effects of population
 - Technological progress, human capital & decline in population growth
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Variations in Country-Specific Characteristics Conducive for Technological Progress

$$g_{t+1}^i = g(e_t^i, L_t^i, \Omega_t^i)$$

$\Omega_t^i \equiv$ characteristics affecting tech progress in country i :

- Protection of intellectual property rights (policy)
- The stock of knowledge within a society
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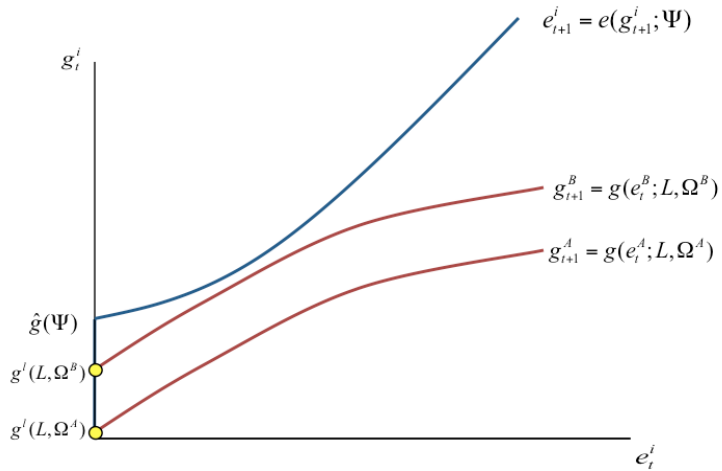
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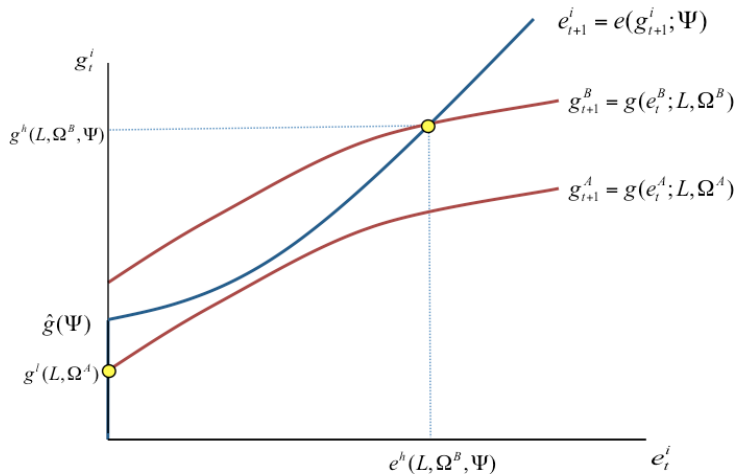
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Earlier Take-off in Country B



Variation in Characteristics Conducive for Human Capital Formation

- For country-specific characteristics Ψ_t^i

$$e_{t+1}^i = e(g_{t+1}^i; \Psi_t^i) \begin{cases} = 0 & \text{if } g_{t+1}^i \leq \hat{g}(\Psi_t^i), \\ > 0 & \text{if } g_{t+1}^i > \hat{g}(\Psi_t^i) \end{cases}$$

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 - Extent of human capital formation
- The availability, accessibility, and quality of public education (policy & interest groups)
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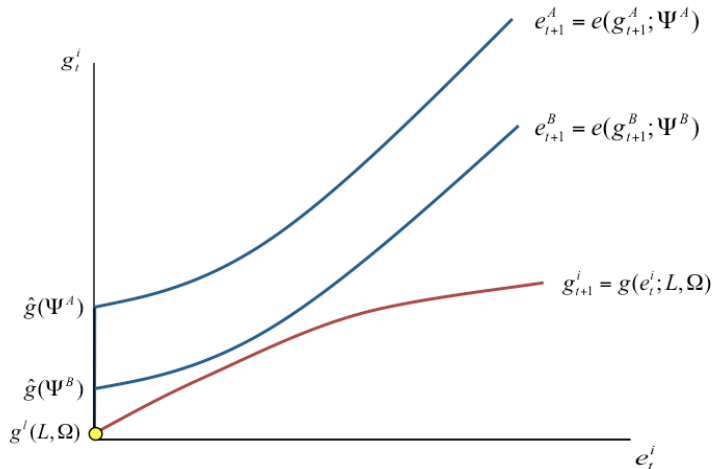
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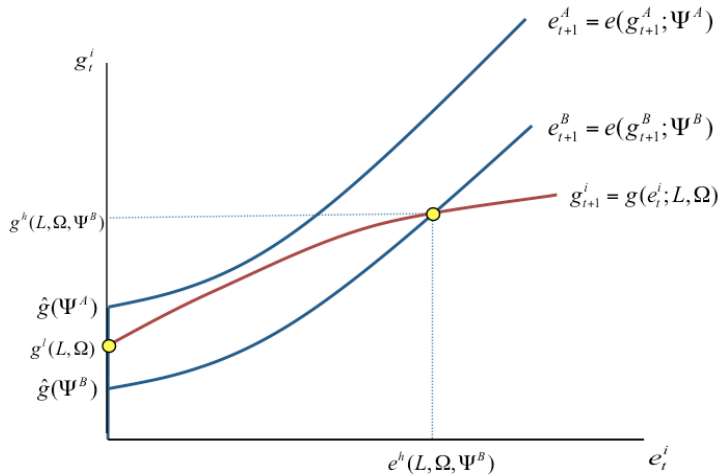
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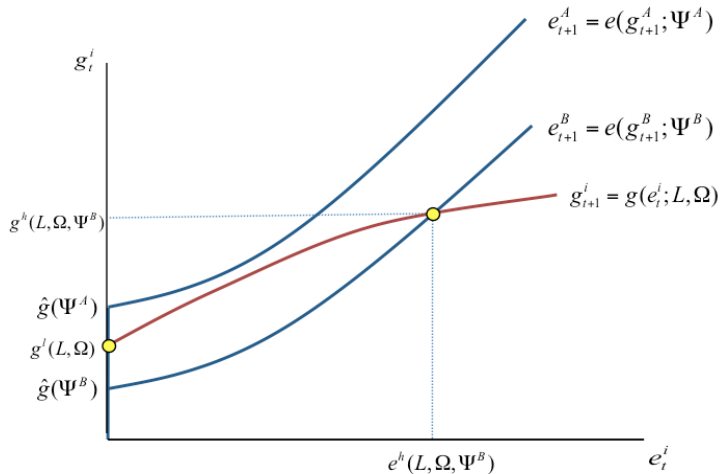
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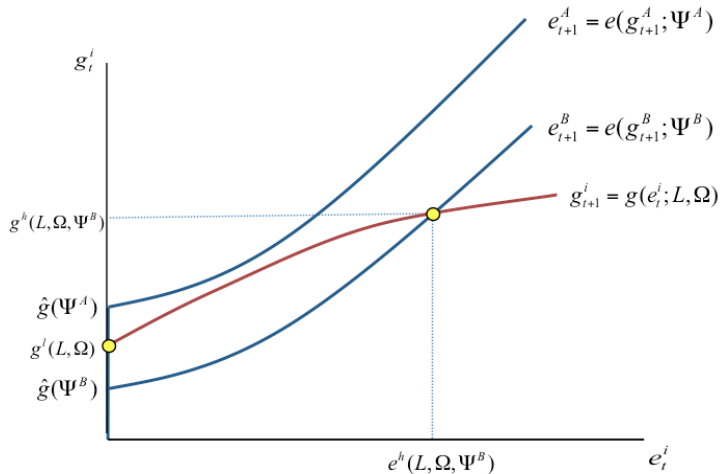
Concluding Remarks



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