

The Demographic Transition

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Economic Growth and Comparative Development

Phases of Development: Standard of Living

- The Malthusian Epoch

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- The Malthusian Epoch
- The Post-Malthusian Regime

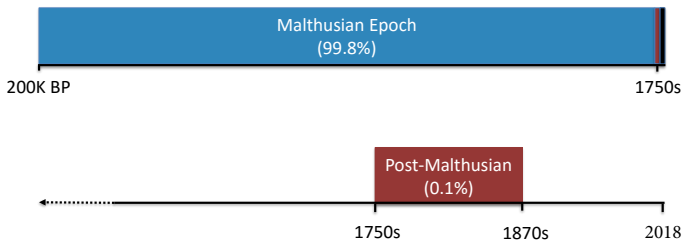
Phases of Development: Standard of Living

- The Malthusian Epoch
- The Post-Malthusian Regime
- The Modern Growth Regime

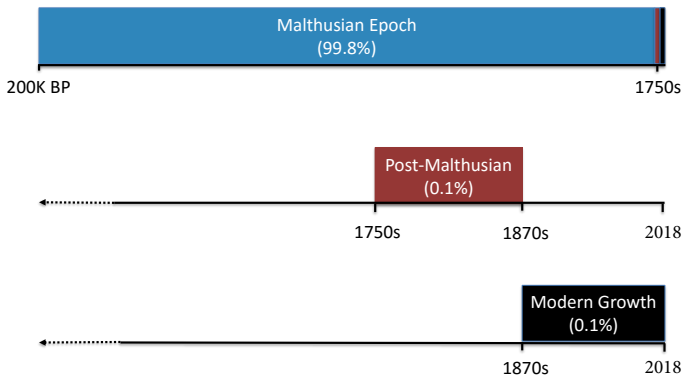
Phases of Development: Timeline of the Most Developed Economies



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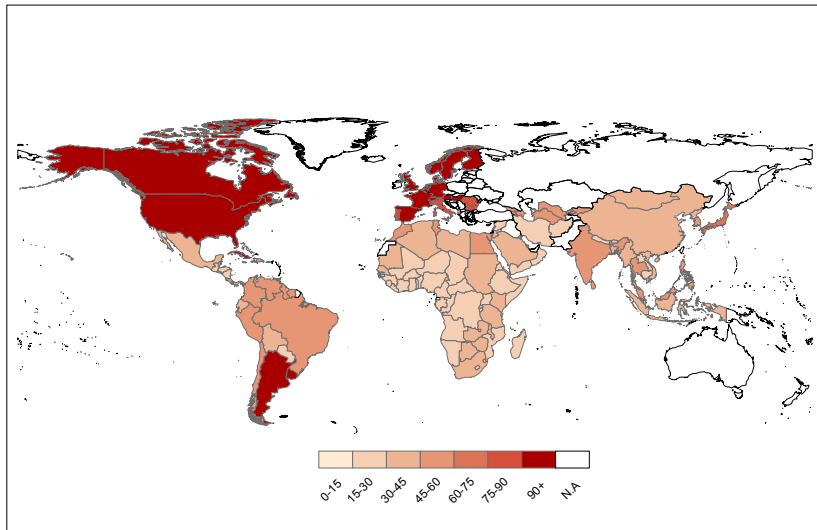
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- Transition to Modern Growth

Variation in years elapsed since the Onset of the Fertility Decline



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⇒ population growth excluding migration

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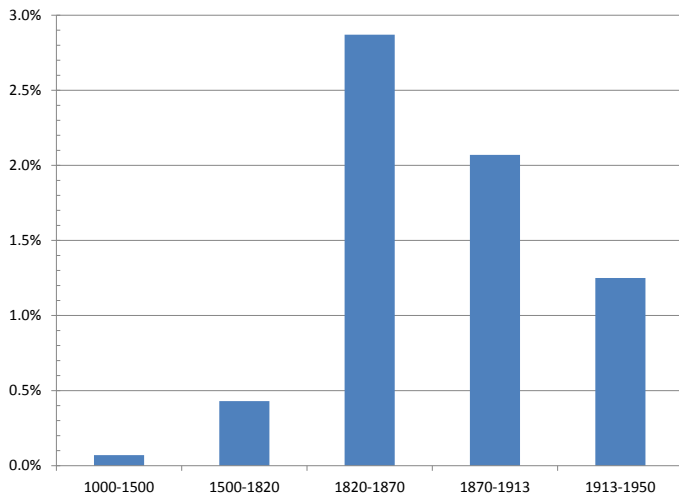
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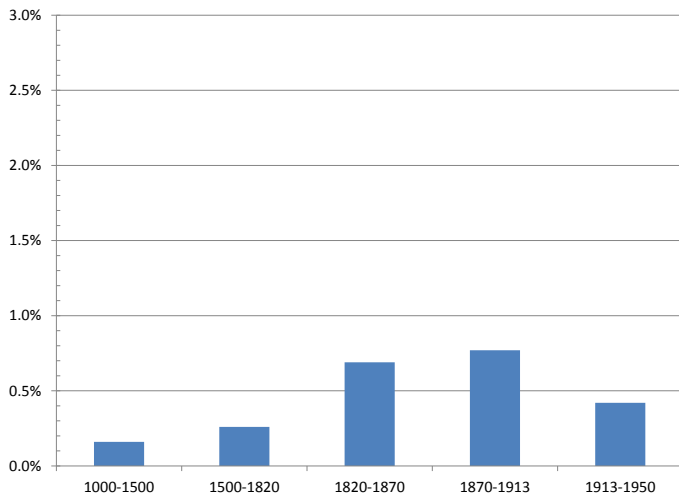
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- TFR & NRR are synthetic rates

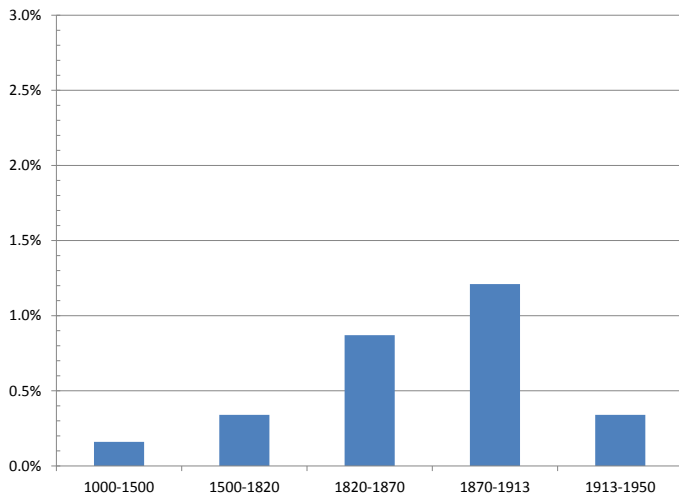
Early Fertility Decline – Western Offshoots



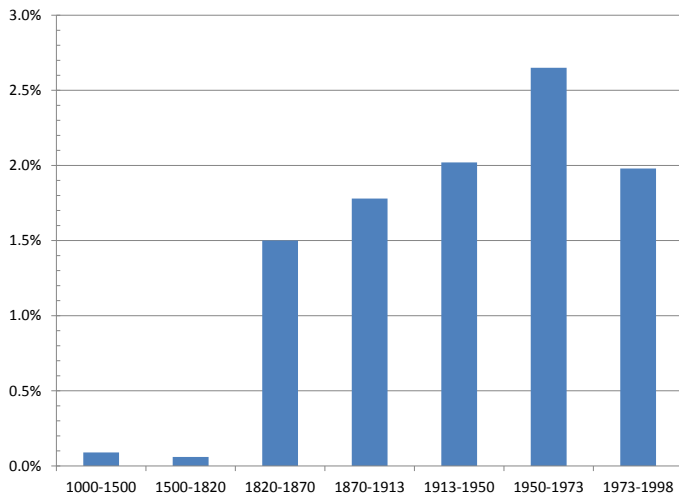
Early Fertility Decline – Western Europe



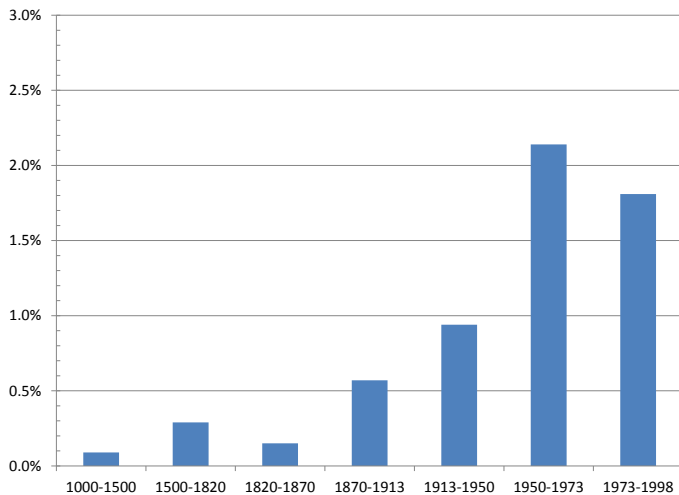
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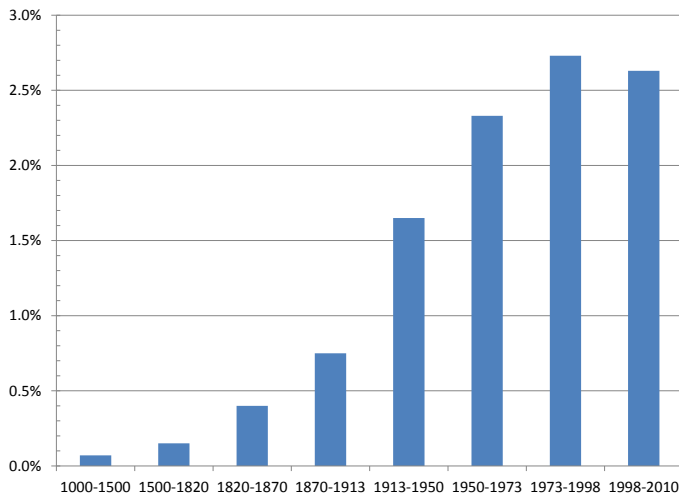
Late Fertility Decline – Latin America



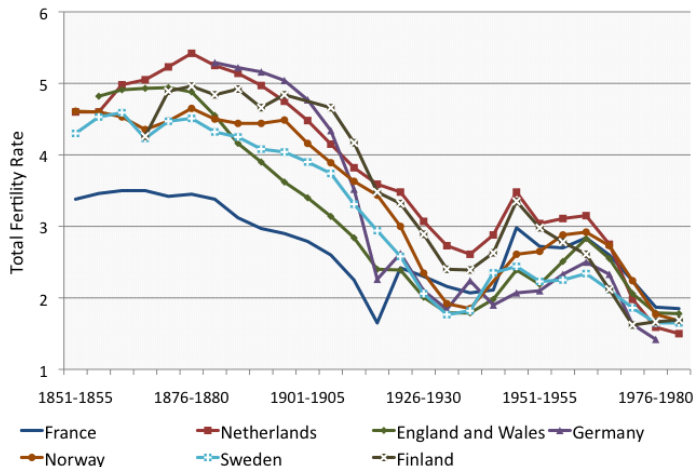
Late Fertility Decline – Asia



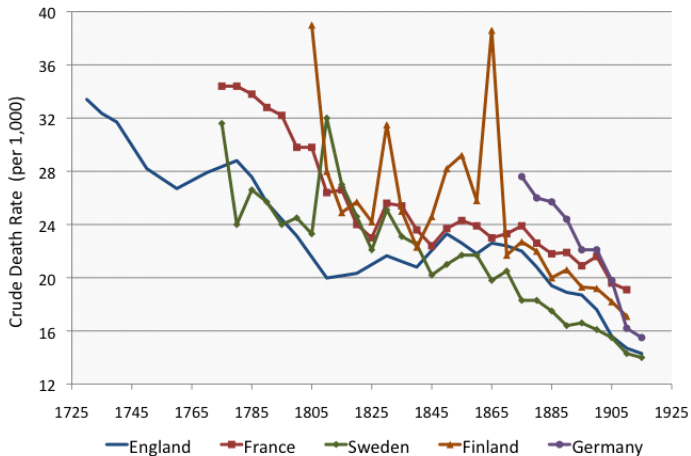
Late Fertility Decline – Africa



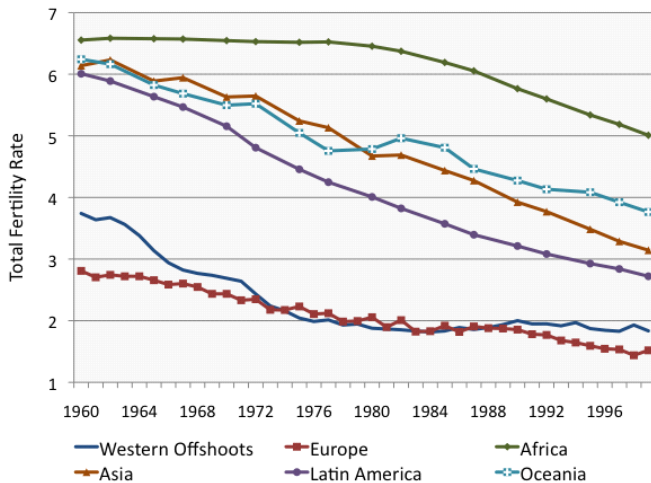
The Demographic Transition in Western Europe: Total Fertility Rates



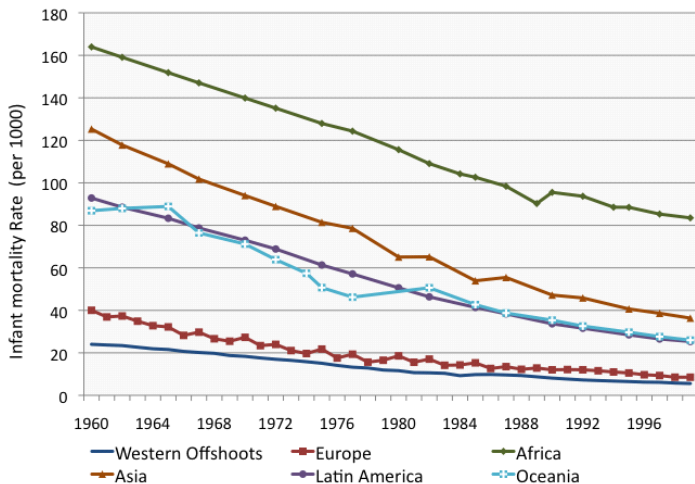
Mortality Decline Western Europe: 1730-1920



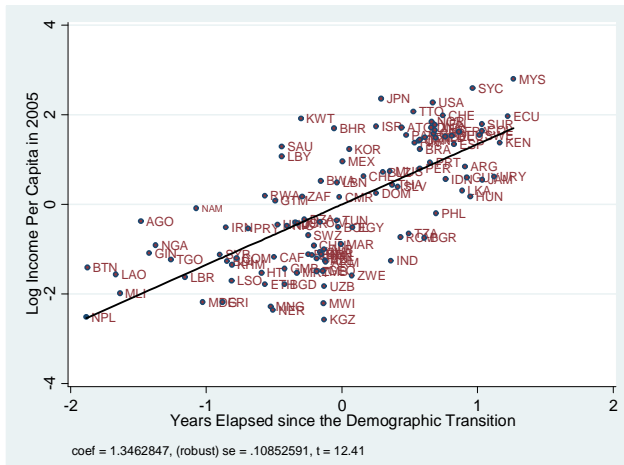
The Evolution of Total Fertility Rate across Regions, 1960-1999



Decline in infant mortality rates across regions, 1960-1999



Timing of the Demographic Transition and Current Income per Capita



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- The substitution effect dominates at a higher level of income
- As income increases fertility declines
- Fertility declines in the process of development

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 - Innate bias against child quantity beyond a certain level of income - non-refutable
 - Non-robust (e.g., the class of homothetic preferences will not trigger a fertility decline)

The Rise in Income - Homothetic Preferences

- Preferences:

$$u = n^{\gamma} c^{(1-\gamma)} \qquad 0 < \gamma < 1$$

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$$\begin{aligned} y\tau n &= \gamma y \\ c &= (1 - \gamma)y \end{aligned}$$

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- Fertility is unaffected by the process of development

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The Rise in Income: Testable predictions

- Across countries that are similar in sociocultural characteristics (and thus in noneconomic factors that may affect fertility decisions), the timing of the fertility decline is inversely related to the level of income per capita.

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- Within an economy, the number of (surviving) children across households is inversely related to their levels of income.

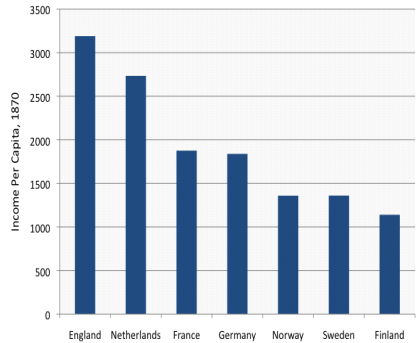
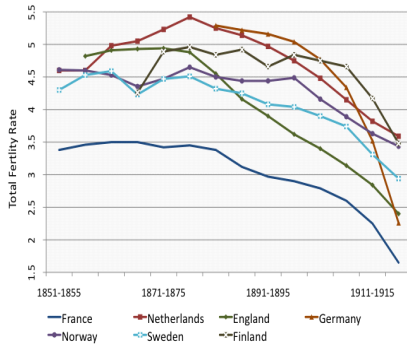
The Rise in Income: Refuting Cross Country Evidence

- Cross Section of Countries (1870-2000) - Income per worker was positively associated with fertility rates, accounting for mortality rates and education (Murtin 2013).

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- Western Europe (1870s) The DT occurred among countries that differed significantly in their income per capita.

Simultaneous DT across European Countries that Differ in Income per Capita



The Rise in Income: Refuting Evidence from Individual Countries

- France (1876–96) Income per capita had a positive effect on fertility rates during France's demographic transition, accounting for education, the gender literacy gap, and mortality rates (Murphy 2015)

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- England (1630s) Reproductive success increases with income (Clark and Hamilton JEH 2006)

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- A decline in child mortality permits parents to reach a given level of surviving children with lower fertility
- The decline in mortality triggered the subsequent decline in fertility

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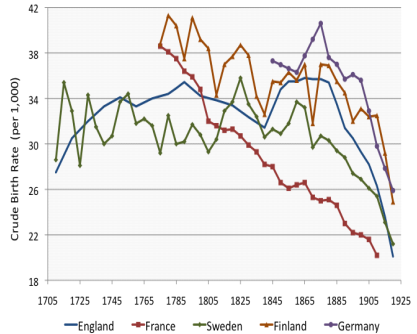
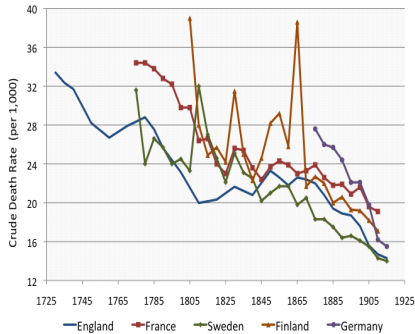
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- Prior to the demographic transition, richer individuals who presumably had better access to financial markets, had larger number of surviving children

The Decline in the Gender Wage Gap

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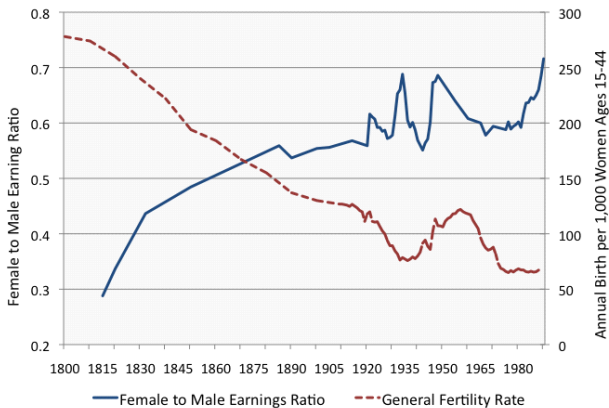
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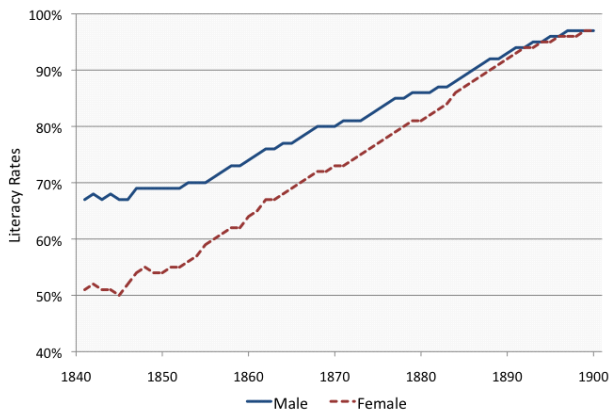
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Evolution of the Gender Earnings Ratio - US



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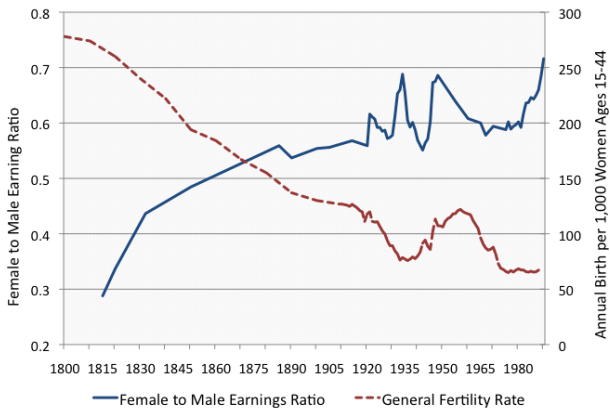
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Women's Relative Wages and Fertility - US



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- Sweden's demographic transition: $(w^F/w^M) \uparrow \implies n \downarrow$ (Schultz 1985)
- France (1876–1896): reduction in the gender literacy gap had an adverse effect on fertility, accounting for income per capita, educational attainment, and mortality rates (Murphy 2015)

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 - Education lessens the obsolescence of HC in a changing technological environment
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The Model - Human Capital Formation

$$h = h(e, g)$$

- $h_e(e, g) > 0$ & $h_{ee}(e, g) < 0$
 - HC is increasing (at decreasing rates) in the parental time investment in the education of the child
- $h_g(e, g) < 0$ & $h_{gg}(e, g) > 0$
 - HC is decreasing in the rate of technological progress (obsolescence of HC in a changing technological environment)
- $h_{eg}(e, g) > 0$
 - Education lessens the obsolescence of HC in a changing technological environment
- $h(0, g) > 0$ & $\lim_{e \rightarrow 0} h_e(e, g) = \infty$; $\lim_{e \rightarrow \infty} h_e(e, g) = 0$
 - Basic level of human capital & interior solution

The Model - Optimization

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$$\begin{aligned} \{n, e, c\} = \arg \max & \gamma [\ln n + \beta \ln h(e, g)] + (1 - \gamma) \ln c \\ \text{s.t.} & \quad yn(\tau^q + \tau^e e) + c \leq y \end{aligned}$$

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$$\frac{\gamma \beta h_e(e, g)}{h(e, g)} = n \tau^e \implies \frac{\beta h_e(e, g)}{h(e, g)} = \frac{\tau^e}{(\tau^q + \tau^e e)}$$

$$\beta h_e(e, g)(\tau^q + \tau^e e) = \tau^e h(e, g)$$

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$$\Rightarrow$$

$$e = e(g, \beta, \tau^e, \tau^q),$$

$$n = \gamma / [\tau^q + \tau^e e(g, \beta, \tau^e, \tau^q)]$$

Testable Predictions - Investment in Quality

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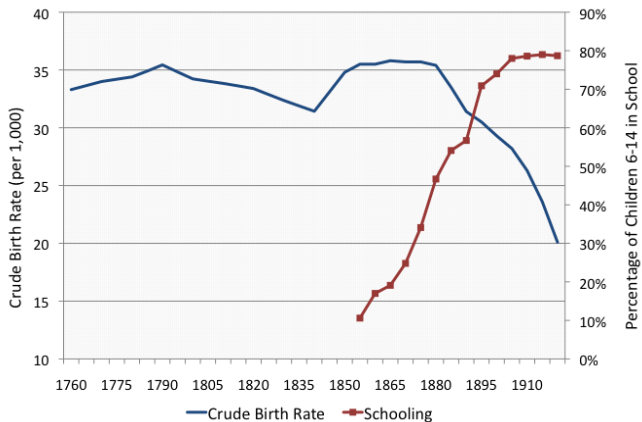
- The cost of raising a child (regardless of quality) increases

$$\partial n / \partial \tau^q < 0$$

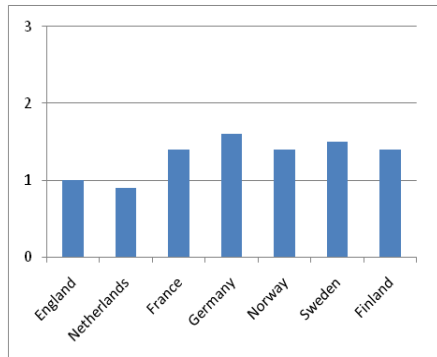
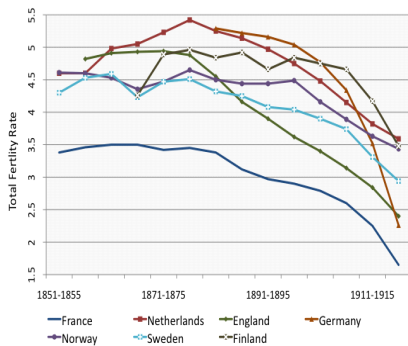
- The cost of educating a child increases and the elasticity of child quality with respect to the cost of child quality is smaller than one in absolute value

$$\partial n / \partial \tau^e < 0 \text{ if } [\partial e / \partial \tau^e][\tau^e / e] > -1$$

Human Capital Formation and the DT - England



Growth Rates 1870-1913 and DT



Supporting Evidence: Cross-Country Evidence

- Cross Section of Countries (1870-2000) - educational attainment has been negatively associated with fertility, accounting for income per worker and mortality rates (Murtin 2013).

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- Cross Section of Countries (1960-1999): adverse effect on net fertility of an increase in productivity in advanced stages of development, when education demand dominates (Lehr 2009)

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- England (1580-1871) Adverse effect of family size on children's literacy. (Klemp-Weisdorf 2016)