# The Malthusian Theory and Epoch

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Economic Growth and Comparative Development

## The Malthusian Epoch

- Technological progress and land expansion
    - $\Longrightarrow$ Temporary increase in the level of income per capita
    - $\Longrightarrow$ An increase in the size of the population
- $\Longrightarrow$ No effect on the level of income per capita in the long run
- Output per capita fluctuates around a subsistence level
- Technologically advanced or land rich economies
    - $\Longrightarrow$ Higher population density
    - $\Longrightarrow$ Similar level of income per-capita in the long-run

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## Central Elements

- Positive effect of income on population
  - $y \uparrow \implies L \uparrow$
- Fixed factor of production - Land
  - $L \uparrow \implies MPL \downarrow \implies y \downarrow$

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- Overlapping-generations economy
- $t = 0, 1, 2, 3...$
- One homogeneous good
- 2 factors of production:
  - Labor
  - Land

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- The output produced in period $t$

$$Y_t = (AX)^{\alpha} L_t^{1-\alpha} \qquad \alpha \in (0,1)$$

  - $L_t \equiv$ labor employed in period $t$
  - $X \equiv$ land
  - $A \equiv$ technological level
  - $AX \equiv$ effective resources

- Output per worker produced at time $t$

$$y_t = \frac{Y_t}{L_t} = \left[\frac{AX}{L_t}\right]^{\alpha}$$

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- Land is fixed over time
  - Surface of planet earth
- Labor evolves endogenously
  - Governed by the endogenous rate of population growth
  - Determined by households' optimal number of children

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## Individuals

- Live for 2 period
- Childhood: (1st Period):
  - Passive economic agents
  - Consume fixed amount of their parental resources
- Parenthood (2nd Period):
  - Work
  - Allocate income between consumption and children

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## Preferences and Budget Constraint

- Preferences of individual $t$(adult at time $t$)

$$u_t = (n_t)^\gamma (c_t)^{1-\gamma} \qquad \gamma \in (0,1)$$

  - $n_t \equiv$ number of children of individual $t$
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- Budget constraint:

$$\rho n_t + c_t \le y_t$$

  - $\rho \equiv$ cost of raising a child

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## Optimization

- Optimal expenditure on consumption and children

$$c_t = (1-\gamma)y_t$$

$$\rho n_t = \gamma y_t$$

- Optimal number of children (note $y_t = (AX/L_t)^\alpha$)

$$n_t = \frac{\gamma}{\rho} y_t = \frac{\gamma}{\rho}\left[\frac{AX}{L_t}\right]^\alpha$$

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## Population Dynamics

- The evolution of the size of the working population

$$L_{t+1} = n_t L_t$$

where

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- Population dynamics:

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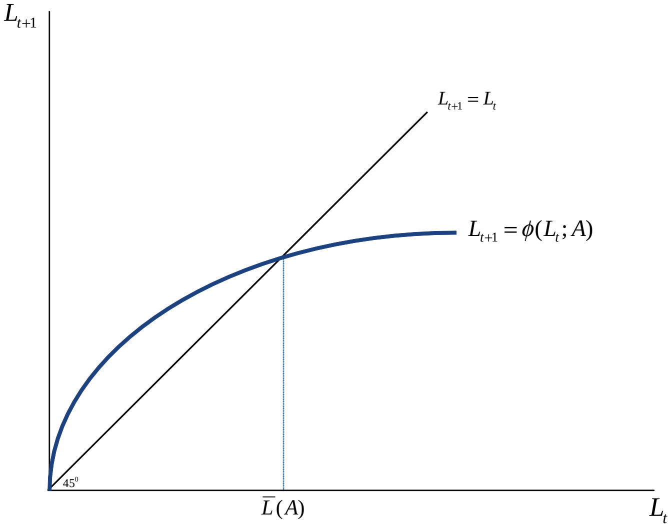

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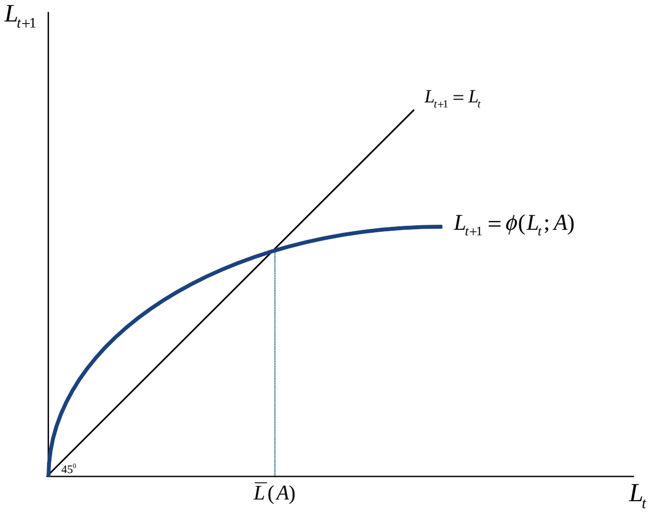
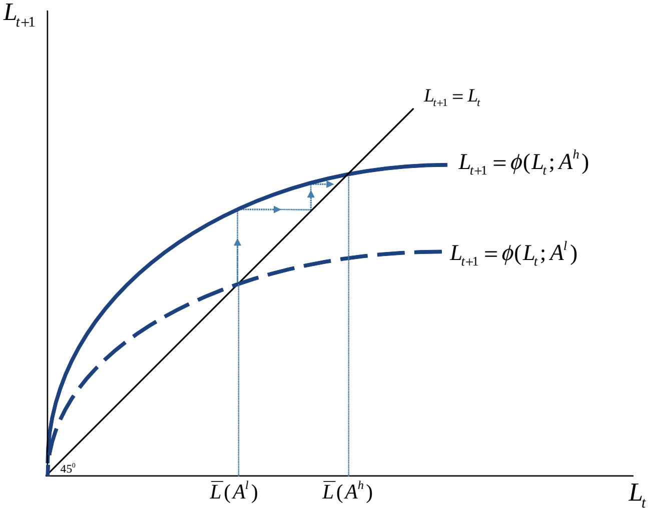


## Adjustment of Population to Advancements in Technology



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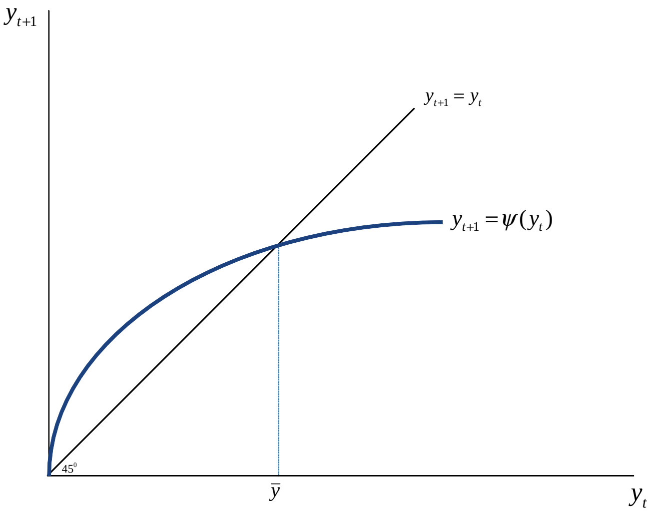

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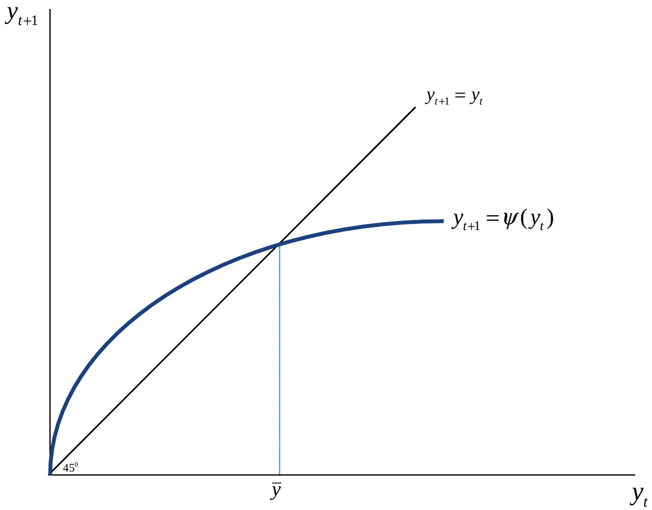
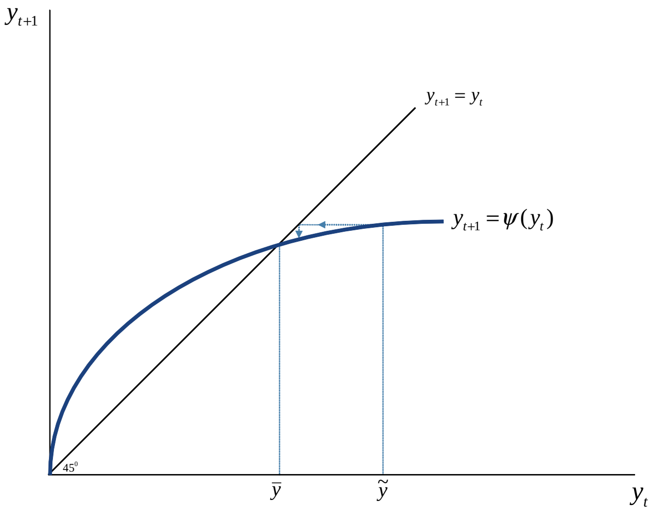
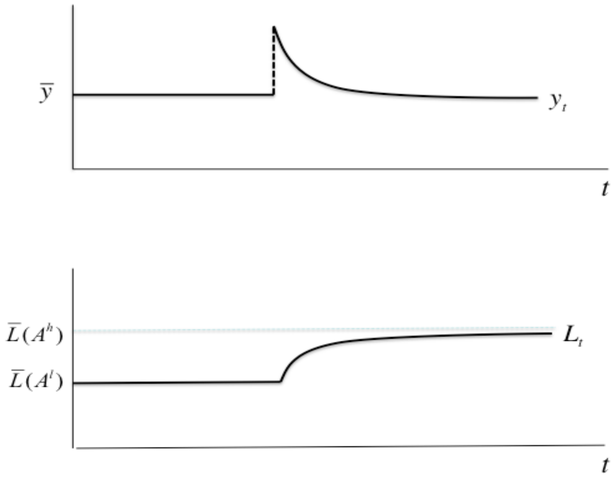


## The Effect of Technological Advancement on income per Worker



## The Effect of Technological Advancement on the Time Path of Population and Income per Worker



## The Effect of Advancement in Technology or Land Productivity

- Increases the short-run and the steady-state level of the working population

$$\frac{\partial L_t}{\partial A} > 0 \quad \text{and} \quad \frac{\partial \bar{L}}{\partial A} > 0$$

- Increases the level of income per capita in the short-run but does not affect the steady-state levels of income per worker

$$\frac{\partial y_t}{\partial A} > 0 \quad \text{and} \quad \frac{\partial \bar{y}}{\partial A} = 0$$

## Testable Implications

- Variations in technology and land quality across countries will be reflected primarily in variation in population density:
  - Technological superiority will result primarily in higher population density without any sizable effect on income per-capita in the long-run
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# Identification Strategy

Exploit exogenous sources of cross-country variation in technological level

- Resolving: reverse causality
  - Variation in the onset of the Neolithic Revolution (NR) across the globe - a proxy for variation in the technological level
- Resolving: omitted variable bias (i.e., 3rd factor (e.g., HC)) affected population & NR
  - Variation in prehistoric domesticable species of plants and animals – IV for the timing of the NR

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  - Variation in the onset of the Neolithic Revolution (NR) across the globe - a proxy for variation in the technological level
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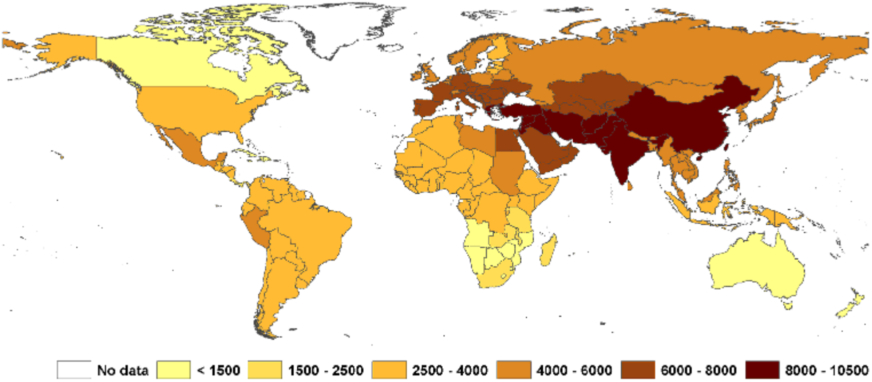
## Years Elapsed since the Onset of the Neolithic Revolution across the Globe



# The Neolithic Revolution as a proxy for Technological Advancement

| | OLS | OLS | OLS | OLS | OLS | OLS |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | Dependent Variable: | | | | | |
| | Log Communications Technology in: | | Log Industrial Technology in: | | Log Transportation Technology in: | |
| | 1000 CE | 1 CE | 1000 CE | 1 CE | 1000 CE | 1 CE |
| Log Years since Neolithic Transition | 0.368*** | 0.283*** | 0.074*** | 0.068*** | 0.380*** | 0.367*** |
| | (0.028) | (0.030) | (0.014) | (0.015) | (0.029) | (0.031) |
| Observations | 143 | 143 | 143 | 143 | 143 | 143 |
| $R^2$ | 0.48 | 0.26 | 0.17 | 0.12 | 0.52 | 0.51 |

Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

## Robustness of Identification Strategy

- Robustness to the inclusion of direct measures of technology
  - Exploit variation in a direct measure of the technology level
  - Variation in prehistoric biogeographic endowments – IV for this direct measure of technology
- Robustness to the exclusion of unobserved time-invariant country fixed effects
  - First-difference estimation strategy (with a lagged explanatory variable)
  - The effect of changes in the level of technology in 1000 BCE-1 CE on population density and income per capita in 1-1000CE

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## Empirical Model I

$$\ln P_{i,t} = \alpha_{0,t} + \alpha_{1,t} \ln T_{i,t} + \alpha_{2,t} \ln X_i + \alpha'_{3,t} \Gamma_i + \alpha'_{4,t} D_i + \delta_{i,t}$$

$$\ln y_{i,t} = \beta_{0,t} + \beta_{1,t} \ln T_{i,t} + \beta_{2,t} \ln X_i + \beta'_{3,t} \Gamma_i + \beta'_{4,t} D_i + \varepsilon_{i,t}$$

- $P_{i,t} \equiv$ population density of country $i$ in year $t$
- $y_{i,t} \equiv$ income per capita of country $i$ in year $t$
- $T_i \equiv$ years elapsed since the onset of agriculture in country $i$
- $X_i \equiv$ measure of land productivity for country $i$
- $\Gamma_i \equiv$ vector of geographical controls for country $i$
- $D_i \equiv$ vector of continental fixed effect in country $i$

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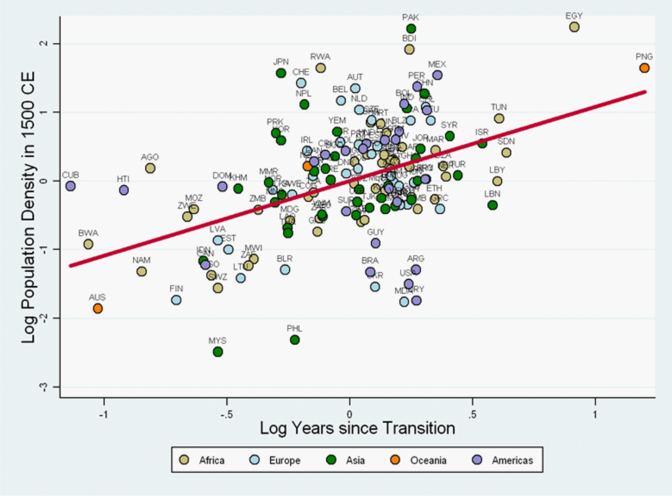
# Determinants of Population Density in 1500 CE

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | OLS | OLS | OLS | OLS | OLS | IV |
| | **Dependent Variable**: **Log population density in 1500 CE** | | | | | |
| Log years since Neolithic | **0.833*****<br>**(0.298)** | | **1.025)*****<br>**(0.223** | **1.087*****<br>**(0.184)** | **1.389*****<br>**(0.224)** | **2.077*****<br>**(0.391)** |
| Log land productivity | | **0.587*****<br>**(0.071)** | **0.641*****<br>**(0.059)** | **0.576*****<br>**(0.052)** | **0.573*****<br>**(0.095)** | **0.571*****<br>**(0.082)** |
| Log absolute latitude | | -0.425***<br>(0.124) | -0.353***<br>(0.104) | -0.314***<br>(0.103) | -0.278**<br>(0.131) | -0.248**<br>(0.117) |
| Distance to nearest coast or river | | | | -0.392***<br>(0.142) | 0.220<br>(0.346) | 0.250<br>(0.333) |
| % land within 100 km of coast or river | | | | 0.899***<br>(0.282) | 1.185***<br>(0.377) | 1.350***<br>(0.380) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 147 | 147 | 147 | 147 | 96 | 96 |
| $R^2$ | 0.40 | 0.60 | 0.66 | 0.73 | 0.73 | 0.70 |
| First-stage F-statistic | | | | | | 14.65 |
| Overident. p-value | | | | | | 0.44 |

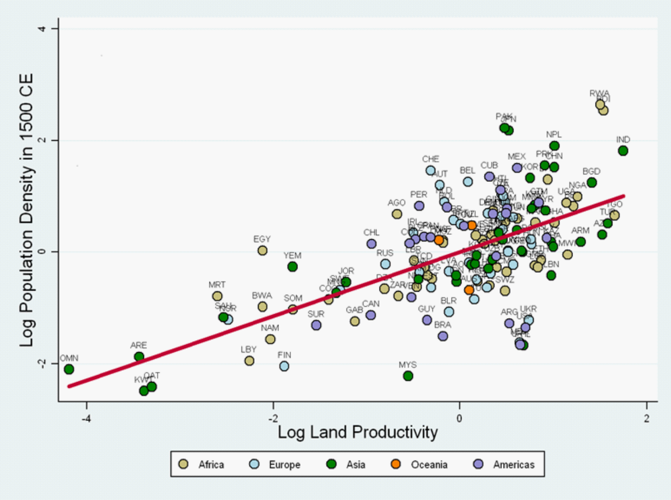
Notes: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

# Timing of Neolithic and Population Density in 1500 CE




Conditional on land productivity, geographical factors, and continental fixed effects

# Land Productivity and Population Density in 1500 CE




Conditional on transition timing, geographical factors, and continental fixed effects

# Determinants of Population Density in 1000 CE

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | OLS | OLS | OLS | OLS | OLS | IV |
| | **Dependent Variable**: **Log population density in 1000 CE** | | | | | |
| Log years since Neolithic | **1.232***** | | **1.435***** | **1.480***** | **1.803***** | **2.933***** |
| | **(0.293)** | | **(0.243)** | **(0.205)** | **(0.251)** | **(0.504)** |
| Log land productivity | | **0.470***** | **0.555***** | **0.497***** | **0.535***** | **0.549***** |
| | | **(0.081)** | **(0.065)** | **(0.056)** | **(0.098)** | **(0.092)** |
| Log absolute latitude | | -0.377** | -0.283** | -0.229** | -0.147 | -0.095 |
| | | (0.148) | (0.116) | (0.111) | (0.127) | (0.116) |
| Distance to nearest coast or river | | | | -0.528*** | 0.147 | 0.225 |
| | | | | (0.153) | (0.338) | (0.354) |
| % land within 100 km of coast or river | | | | 0.716** | 1.050** | 1.358*** |
| | | | | (0.323) | (0.421) | (0.465) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 142 | 142 | 142 | 142 | 94 | 94 |
| $R^2$ | 0.38 | 0.46 | 0.59 | 0.67 | 0.69 | 0.62 |
| First-stage F-statistic | | | | | | 15.10 |
| Overident. p-value | | | | | | 0.281 |

Notes: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

# Determinants of Population Density in 1 CE

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | OLS | OLS | OLS | OLS | OLS | IV |
| | Dependent Variable: Log population density in 1 CE | | | | | |
| Log years since Neolithic | **1.560*****<br>**(0.326)** | | **1.903*****<br>**(0.312)** | **1.930*****<br>**(0.272)** | **2.561*****<br>**(0.369)** | **3.459*****<br>**(0.437)** |
| Log land productivity | | **0.404*****<br>**(0.106)** | **0.556*****<br>**(0.081)** | **0.394*****<br>**(0.067)** | **0.421*****<br>**(0.094)** | **0.479*****<br>**(0.089)** |
| Log absolute latitude | | -0.080<br>(0.161) | -0.030<br>(0.120) | 0.057<br>(0.101) | 0.116<br>(0.121) | 0.113<br>(0.113) |
| Distance to nearest coast or river | | | | -0.685***<br>(0.155) | -0.418<br>(0.273) | -0.320<br>(0.306) |
| % land within 100 km of coast or river | | | | 0.857**<br>(0.351) | 1.108***<br>(0.412) | 1.360***<br>(0.488) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 128 | 128 | 128 | 128 | 83 | 83 |
| $R^2$ | 0.47 | 0.41 | 0.59 | 0.69 | 0.75 | 0.72 |
| First-stage F-statistic | | | | | | 10.85 |
| Overident. p-value | | | | | | 0.590 |

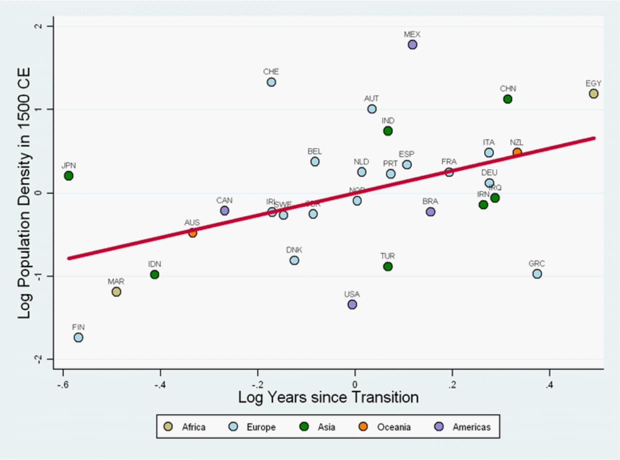
Notes: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

# Effects on Income per Capita versus Population Density

| | OLS | OLS | OLS | OLS | OLS | OLS |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | Log Income Per Capita in | | | Log Population Density in | | |
| | 1500 CE | 1000 CE | 1 CE | 1500 CE | 1000 CE | 1 CE |
| Log years since Neolithic | **0.159** | **0.073** | **0.109** | **1.337**$^{**}$ | **0.832**$^{**}$ | **1.006**$^{**}$ |
| | (0.136) | (0.045) | (0.072) | (0.594) | (0.363) | (0.483) |
| Log land productivity | **0.041** | **-0.021** | **-0.001** | **0.584**$^{***}$ | **0.364**$^{***}$ | **0.681**$^{**}$ |
| | (0.025) | (0.025) | (0.027) | (0.159) | (0.110) | (0.255) |
| Log absolute latitude | -0.041 | 0.060 | -0.175 | 0.050 | -2.140$^{**}$ | -2.163$^{**}$ |
| | (0.073) | (0.147) | (0.175) | (0.463) | (0.801) | (0.979) |
| Distance to nearest coast or river | 0.215 | -0.111 | 0.043 | -0.429 | -0.237 | 0.118 |
| | (0.198) | (0.138) | (0.159) | (1.237) | (0.751) | (0.883) |
| % land within 100 km of coast or river | 0.124 | -0.150 | 0.042 | 1.855$^{**}$ | 1.326$^{**}$ | 0.228 |
| | (0.145) | (0.121) | (0.127) | (0.820) | (0.615) | (0.919) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 31 | 26 | 29 | 31 | 26 | 29 |
| $R^2$ | 0.66 | 0.68 | 0.33 | 0.88 | 0.95 | 0.89 |

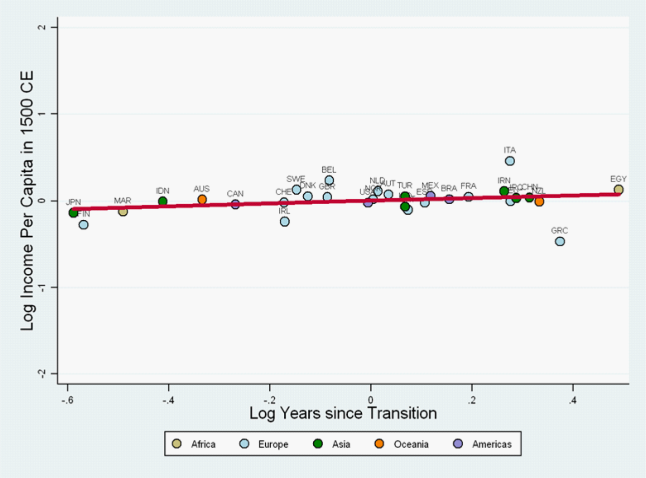
Notes: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

# Transition Timing and Population Density in 1500 CE



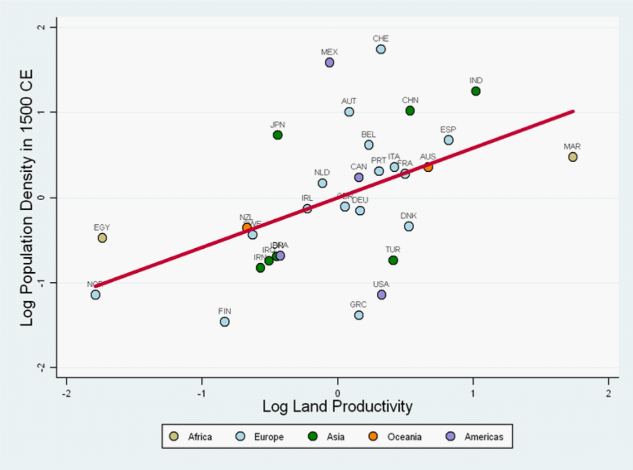

Conditional on land productivity, geographical factors, and continental fixed effects

# Transition Timing and Income Per Capita in 1500 CE



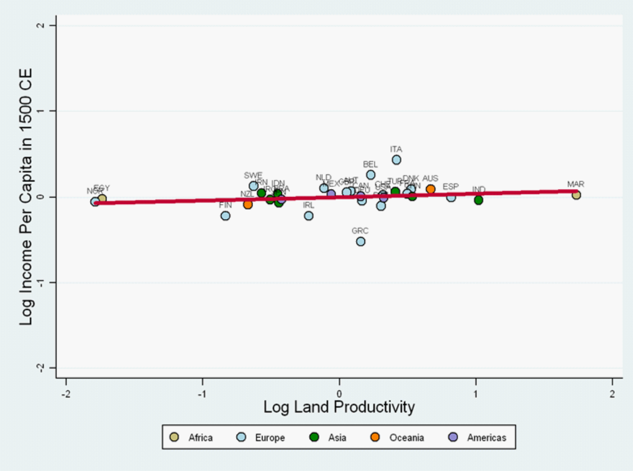

Conditional on land productivity, geographical factors, and continental fixed effects

# Land Productivity and Population Density in 1500 CE




Conditional on transition timing, geographical factors, and continental fixed effects

# Land Productivity and Income Per Capita in 1500 CE




Conditional on transition timing, geographical factors, and continental fixed effects

# Robustness to Income per Capita Data Quality Concerns

| | OLS | OLS | OLS | OLS | OLS | OLS |
|---|---|---|---|---|---|---|
| | *Weighted* | *Weigthed* | *Weighted* | *Weigthed* | *Weighted* | *Weigthed* |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | Weighted According to: | | | | | |
| | Income Data Frequency | | | Total Population Size | | |
| | Dependent Variable: Log Income Per Capita in: | | | | | |
| | 1500 CE | 1000 CE | 1 CE | 1500 CE | 1000 CE | 1 CE |
| Log years since Neolithic | **0.173** | **0.122*** | **0.189** | **0.278** | **0.143*** | **0.289** |
| | **(0.162)** | **(0.063)** | **(0.121)** | **(0.171)** | **(0.068)** | **(0.175)** |
| Log land productivity | **0.039** | **-0.045*** | **0.008** | **-0.005** | **-0.062*** | **-0.011** |
| | **(0.023)** | **(0.022)** | **(0.031)** | **(0.026)** | **(0.030)** | **(0.027)** |
| Log absolute latitude | -0.042 | 0.205* | -0.442 | -0.089 | 0.298*** | 0.080 |
| | (0.080) | (0.108) | (0.362) | (0.052) | (0.031) | (0.089) |
| Distance to nearest coast or river | 0.219 | -0.370** | 0.139 | 0.332** | -0.592*** | -0.180 |
| | (0.202) | (0.148) | (0.298) | (0.148) | (0.108) | (0.189) |
| % land within 100 km of coast or river | 0.153 | -0.228 | 0.159 | 0.329 | -0.477*** | 0.003 |
| | (0.169) | (0.137) | (0.257) | (0.227) | (0.122) | (0.277) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 31 | 26 | 29 | 31 | 26 | 29 |
| $R^2$ | 0.54 | 0.79 | 0.29 | 0.74 | 0.83 | 0.45 |

Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

# Robustness to Direct Measures of Technological Sophistication

| | OLS | OLS | OLS | OLS | OLS | OLS |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | Dependent Variable: | | | | | |
| | Log Population Density in: | | Log Income Per Capita in: | | Log Population Density in: | |
| | 1000 CE | 1 CE | 1000 CE | 1 CE | 1000 CE | 1 CE |
| Log Technology Index in Relevant Period | 4.315*** | 4.216*** | 0.064 | 0.678 | 12.762*** | 7.461** |
| | (0.850) | (0.745) | (0.230) | (0.432) | (0.918) | (3.181) |
| Log land productivity | 0.449*** | 0.379*** | -0.016 | 0.004 | 0.429** | 0.725** |
| | (0.056) | (0.082) | (0.030) | (0.033) | (0.182) | (0.303) |
| Log absolute latitude | -0.283** | -0.051 | 0.036 | -0.198 | -1.919*** | -2.350*** |
| | (0.120) | (0.127) | (0.161) | (0.176) | (0.576) | (0.784) |
| Distance to nearest coast or river | -0.638*** | -0.782*** | -0.092 | 0.114 | 0.609 | 0.886 |
| | (0.188) | (0.198) | (0.144) | (0.164) | (0.469) | (0.904) |
| % land within 100 km of coast or river | 0.385 | 0.237 | -0.156 | 0.092 | 1.265** | 0.788 |
| | (0.313) | (0.329) | (0.139) | (0.136) | (0.555) | (0.934) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 140 | 129 | 26 | 29 | 26 | 29 |
| $R^2$ | 0.61 | 0.62 | 0.64 | 0.30 | 0.97 | 0.88 |

Notes: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

# The Causal Effect of Technological Sophistication on Population Density

| | OLS | OLS | IV | OLS | OLS | IV |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | Dependent Variable: Population Density in: | | | | | |
| | 1000CE | | | 1CE | | |
| Log Technology Index in Relevant Period | 4.315*** | 4.198*** | 14.530*** | 4.216*** | 3.947*** | 10.798*** |
| | (0.850) | (1.164) | (4.437) | (0.745) | (0.983) | (2.857) |
| Log land productivity | 0.449*** | 0.498*** | 0.572*** | 0.379*** | 0.350** | 0.464** |
| | (0.056) | (0.139) | (0.148) | (0.082) | (0.172) | (0.182) |
| Log absolute latitude | -0.283** | -0.185 | -0.209 | -0.051 | 0.083 | -0.052 |
| | (0.120) | (0.151) | (0.209) | (0.127) | (0.170) | (0.214) |
| Distance to nearest coast or river | -0.638*** | -0.363 | -1.155* | -0.782*** | -0.625 | -0.616 |
| | (0.188) | (0.426) | (0.640) | (0.198) | (0.434) | (0.834) |
| % land within 100 km of coast or river | 0.385 | 0.442 | 0.153 | 0.237 | 0.146 | -0.172 |
| | (0.313) | (0.422) | (0.606) | (0.329) | (0.424) | (0.642) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 140 | 92 | 92 | 129 | 83 | 83 |
| $R^2$ | 0.61 | 0.55 | 0.13 | 0.62 | 0.58 | 0.32 |
| First-stage F-statistic | | | 12.52 | | | 12.00 |
| Overid. p-value | | | 0.941 | | | 0.160 |

Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

# Robustness to Technology Diffusion and Geographic Features

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | Log Population Density in 1500 | | Log Income Per Capita in 1500 | | Log Population Density in 1500 | |
| Log Technology Index in Relevant Period | 0.828*** | 0.877*** | 0.117 | 0.103 | 1.498** | 1.478** |
| | (0.208) | (0.214) | (0.221) | (0.214) | (0.546) | (0.556) |
| Log land productivity | 0.559*** | 0.545*** | 0.036 | 0.047 | 0.596*** | 0.691*** |
| | (0.048) | (0.063) | (0.032) | (0.037) | (0.123) | (0.122) |
| Log Distance to Frontier | -0.186*** | -0.191*** | -0.005 | -0.001 | -0.130* | -0.108* |
| | (0.035) | (0.036) | (0.011) | (0.013) | (0.066) | (0.055) |
| Small Island Dummy | 0.067 | 0.086 | -0.118 | -0.046 | 1.962** | 2.720*** |
| | (0.582) | (0.626) | (0.216) | (0.198) | (0.709) | (0.699) |
| Landlocked Dummy | 0.131 | 0.119 | 0.056 | 0.024 | 1.490*** | 1.269*** |
| | (0.209) | (0.203) | (0.084) | (0.101) | (0.293) | (0.282) |
| % Land in Temperate Climate Zones | | -0.196 | | -0.192 | | -1.624* |
| | | (0.513) | | (0.180) | | (0.917) |
| Continental dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 147 | 147 | 31 | 31 | 31 | 31 |
| $R^2$ | 0.76 | 0.76 | 0.67 | 0.67 | 0.94 | 0.96 |

## Robustness to Unobserved Heterogeneity across Countries

$$\ln P_{i,t} = \gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}$$

$$\ln y_{i,t} = \gamma_0 + \nu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + \gamma_2^{'} \Gamma_i + \gamma_3^{'} D_i + \xi_{i,t}^{y}$$

- $P_{i,t} \equiv$ population density in country $i$ in year $t$
- $y_{i,t} \equiv$ income per capita in country $i$ in year $t$
- $A_{i,t-1} \equiv$ technological level in country $i$ in year $t-1$
- $\xi_{i,t}^{p} \equiv$ disturbance term for population density in country $i$ in period $t$
- $\xi_{i,t}^{y} \equiv$ disturbance term for income per capita in country $i$ in period $t$

## Robustness to Unobserved Heterogeneity across Countries

$$\ln P_{i,t} = \gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}$$

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## Robustness to Unobserved Heterogeneity across Countries

$$\ln P_{i,t} = \gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}$$

$$\ln y_{i,t} = \gamma_0 + \nu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + \gamma_2^{'} \Gamma_i + \gamma_3^{'} D_i + \xi_{i,t}^{y}$$

- $P_{i,t} \equiv$ population density in country $i$ in year $t$
- $y_{i,t} \equiv$ income per capita in country $i$ in year $t$
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$$\ln P_{i,t} = \gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}$$

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$$\ln P_{i,t} = \gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}$$

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$$\ln P_{i,t} = \gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}$$

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- $\xi_{i,t}^{p} \equiv$ disturbance term for population density in country $i$ in period $t$
- $\xi_{i,t}^{y} \equiv$ disturbance term for income per capita in country $i$ in period $t$

## Unobserved Heterogeneity across Countries

$$\xi^p_{i,t} = \eta^p_i + \mu_0 t + \sigma^p_{i,t}$$

$$\xi^y_{i,t} = \eta^y_i + \nu_0 t + \sigma^y_{i,t}$$

- $\eta^p_i \equiv$ unobserved time-invariant country fixed effect on population density in country $i$
- $\sigma^p_{i,t} \equiv$ random disturbance in population density in country $i$ in period $t$
- $\mu_0$ $[\nu_0] \equiv$ global time fixed-effect on population density [income per capita]
- $\eta^y_i \equiv$ unobserved time-invariant country fixed effect on income per capita in country $i$
- $\sigma^y_{i,t} \equiv$ random disturbance in income per capita in country $i$ in period $t$

## Unobserved Heterogeneity across Countries

$$\xi^p_{i,t} = \eta^p_i + \mu_0 t + \sigma^p_{i,t}$$

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## Unobserved Heterogeneity across Countries

$$\xi^p_{i,t} = \eta^p_i + \mu_0 t + \sigma^p_{i,t}$$

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- $\eta^y_i \equiv$ unobserved time-invariant country fixed effect on income per capita in country $i$
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# Unobserved Heterogeneity across Countries

$$\xi^p_{i,t} = \eta^p_i + \mu_0 t + \sigma^p_{i,t}$$

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- $\eta^y_i \equiv$ unobserved time-invariant country fixed effect on income per capita in country $i$
- $\sigma^y_{i,t} \equiv$ random disturbance in income per capita in country $i$ in period $t$

## Unobserved Heterogeneity across Countries

$$
\begin{aligned}
\xi^{p}_{i,t} &= \eta^{p}_{i} + \mu_0 t + \sigma^{p}_{i,t} \\
\xi^{y}_{i,t} &= \eta^{y}_{i} + \nu_0 t + \sigma^{y}_{i,t}
\end{aligned}
$$

- $\eta^p_i \equiv$ unobserved time-invariant country fixed effect on population density in country $i$
- $\sigma^p_{i,t} \equiv$ random disturbance in population density in country $i$ in period $t$
- $\mu_0\ [\nu_0] \equiv$ global time fixed-effect on population density [income per capita]
- $\eta^y_i \equiv$ unobserved time-invariant country fixed effect on income per capita in country $i$
- $\sigma^y_{i,t} \equiv$ random disturbance in income per capita in country $i$ in period $t$

## Control for Unobserved Heterogeneity across Country: Population Density

Changes in Population density in country $i$ from time $t-1$ to time $t$:

$$\begin{aligned}\Delta \ln P_{i,t} &\equiv \ln P_{i,t} - \ln P_{i,t-1} \\ &= [\gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^p] \\ &\quad -[\gamma_0 + \mu_1 \ln A_{i,t-2} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t-1}^p]\end{aligned}$$

where,

$$\xi_{i,t}^p - \xi_{i,t-1}^p = [\eta_i^p + \mu_0 t + \sigma_{i,t}^p] - [\eta_i^p + \mu_0 (t-1) + \sigma_{i,t-1}^p] = \mu_0 + \phi_{i,t}$$

- $\phi_{i,t} \equiv \sigma_{i,t}^p - \sigma_{i,t-1}^p \equiv$ random disturbance in the change in population density in country $i$ from time $t$ to time $t+1$

$$\Delta \ln P_{i,t} = \mu_0 + \mu_1 \Delta \ln A_{i,t-1} + \phi_{i,t}$$

## Control for Unobserved Heterogeneity across Country: Population Density

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where,

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&\quad -[\gamma_0 + \mu_1 \ln A_{i,t-2} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t-1}^p]
\end{aligned}$$

where,

$$\xi_{i,t}^p - \xi_{i,t-1}^p = [\eta_i^p + \mu_0 t + \sigma_{i,t}^p] - [\eta_i^p + \mu_0 (t-1) + \sigma_{i,t-1}^p] = \mu_0 + \phi_{i,t}$$

- $\phi_{i,t} \equiv \sigma_{i,t}^p - \sigma_{i,t-1}^p \equiv$ random disturbance in the change in population density in country $i$ from time $t$ to time $t+1$

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## Control for Unobserved Heterogeneity across Country: Population Density

Changes in Population density in country $i$ from time $t-1$ to time $t$:

$$
\begin{aligned}
\Delta \ln P_{i,t} &\equiv \ln P_{i,t} - \ln P_{i,t-1} \\
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&\quad -[\gamma_0 + \mu_1 \ln A_{i,t-2} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t-1}^{p}]
\end{aligned}
$$

where,

$$\xi_{i,t}^{p} - \xi_{i,t-1}^{p} = [\eta_i^p + \mu_0 t + \sigma_{i,t}^{p}] - [\eta_i^p + \mu_0 (t-1) + \sigma_{i,t-1}^{p}] = \mu_0 + \phi_{i,t}$$

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$$\Delta \ln P_{i,t} = \mu_0 + \mu_1 \Delta \ln A_{i,t-1} + \phi_{i,t}$$

## Control for Unobserved Heterogeneity across Country: Population Density

Changes in Population density in country $i$ from time $t-1$ to time $t$:

$$\begin{aligned}\Delta \ln P_{i,t} &\equiv \ln P_{i,t} - \ln P_{i,t-1} \\ &= [\gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t}^{p}] \\ &\quad -[\gamma_0 + \mu_1 \ln A_{i,t-2} + \gamma_1 \ln X_i + + \gamma_2^{'} \Gamma_i + \gamma_3 D_i + \xi_{i,t-1}^{p}]\end{aligned}$$

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## Control for Unobserved Heterogeneity across Country: Population Density

Changes in Population density in country $i$ from time $t-1$ to time $t$:

$$\begin{aligned}\Delta \ln P_{i,t} &\equiv \ln P_{i,t} - \ln P_{i,t-1} \\ &= [\gamma_0 + \mu_1 \ln A_{i,t-1} + \gamma_1 \ln X_i + + \gamma_2' \Gamma_i + \gamma_3 D_i + \xi_{i,t}^p] \\ &\quad -[\gamma_0 + \mu_1 \ln A_{i,t-2} + \gamma_1 \ln X_i + + \gamma_2' \Gamma_i + \gamma_3 D_i + \xi_{i,t-1}^p]\end{aligned}$$

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$$\Delta \ln P_{i,t} = \mu_0 + \mu_1 \Delta \ln A_{i,t-1} + \phi_{i,t}$$

## Control for Unobserved Heterogeneity across Country: Population Density

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$$\Delta \ln P_{i,t} = \mu_0 + \mu_1 \Delta \ln A_{i,t-1} + \phi_{i,t}$$

## Control for Unobserved Heterogeneity across Country: Population Density

Changes in income per capita in country $i$ from time $t-1$ to time $t$:

$$\begin{aligned}\Delta \ln y_{i,t} &\equiv \ln y_{i,t} - \ln y_{i,t-1} \\ &= \upsilon_0 + \upsilon_1 \Delta \ln A_{i,t-1} + \psi_{i,t}\end{aligned}$$

- $\psi_{i,t} \equiv \sigma^y_{i,t} - \sigma^y_{i,t-1} \equiv$ random disturbance in the change in income per capita in country $i$

## Control for Unobserved Heterogeneity across Country: Population Density

Changes in income per capita in country $i$ from time $t-1$ to time $t$:

$$\begin{aligned}\Delta \ln y_{i,t} &\equiv \ln y_{i,t} - \ln y_{i,t-1} \\ &= \upsilon_0 + \upsilon_1 \Delta \ln A_{i,t-1} + \psi_{i,t}\end{aligned}$$

- $\psi_{i,t} \equiv \sigma^y_{i,t} - \sigma^y_{i,t-1} \equiv$ random disturbance in the change in income per capita in country $i$

## Control for Unobserved Heterogeneity across Country: Population Density

Changes in income per capita in country $i$ from time $t-1$ to time $t$:

$$\begin{aligned}\Delta \ln y_{i,t} &\equiv \ln y_{i,t} - \ln y_{i,t-1} \\ &= \upsilon_0 + \upsilon_1 \Delta \ln A_{i,t-1} + \psi_{i,t}\end{aligned}$$

- $\psi_{i,t} \equiv \sigma^y_{i,t} - \sigma^y_{i,t-1} \equiv$ random disturbance in the change in income per capita in country $i$

# Robustness to First Differences and Migration Theory

| | OLS | OLS | OLS |
|---|---|---|---|
| | (1) | (2) | (3) |
| | Dependent Variable Differences in: | | |
| | Log Population Density 1 CE - 1000 CE | | Log Income Per Capita 1 CE - 1000 CE |
| Diff. in Log Technology Index between 1000 BCE and 1 CE | 1.747*** | 3.133* | 0.073 |
| | (0.429) | (1.550) | (0.265) |
| Constant | 0.451*** | -0.026 | -0.040 |
| | (0.053) | (0.204) | (0.064) |
| Observations | 126 | 26 | 26 |
| $R^2$ | 0.17 | 0.34 | 0.00 |

Notes: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

## Robustness to Migration Theory

- Migration Theory: variations in technology and land quality across countries will induce migration from the low to the high productivity country:
  - Technological superiority will result primarily in higher population density
  - Technological superiority will not be reflected in higher income per capita
  - Advancements in technology will result in higher income per capita in all countries (i.e., $\upsilon_0 > 0$).
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