Unified Growth Theory and Comparative Economic Development

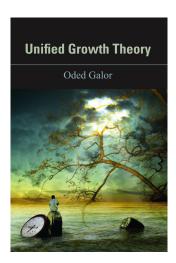
Ömer Özak

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Economic Growth and Comparative Development

Unified Growth Theory

Why?



- Inconsistent with the qualitative aspects of the growth process during the Malthusian epoch and the Post-Malthusian Regime
- Limited to the modern growth regime a miniscule fraction (0.1%) of the entire process of development
- Do not capture the forces that brought about the transition of developed countries from stagnation to growth and hence unable to shed light of the hurdles faced by LDCs in their attempt to take-off to a state of sustained economic growth
- Unable to capture the role of deep rooted factors in the contemporary disparity in income per-capita across the globe

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 - The forces that triggered the transition from stagnation to growth of the currently developed economies
 - hurdles faced by LDCs
 - The role played by deep rooted factors in the differential timing of the transition from stagnation to growth
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- Identifies the forces the permitted the currently developed economy to transit from an epoch of Malthusian stagnation to sustained economic growth
- Uncovers the hurdles faced by LDCs in their transitions from stagnation to growth
- Derives policies that may expedite the transition of LDCs to sustained economic growth

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- Identifies the persistent effect of initial biogeographical conditions on the growth process
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A dynamical system that permits an escape from a *stable* Malthusian Steady-State:

- A major shock in an environment characterized by multiple locally stable equilibria (inconsistent with evidence of a gradual transition)
- A gradual escape from an absorbing (stable) equilibrium (contradiction to the essence of a stable equilibrium)

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- The latent evolution of the demand for human capital ultimately changes the dynamical system qualitatively:
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 - Faster rates of technological progress
 - Faster rate of population growth
- Transition from the Post-Malthusian to Modern Growth Regime:
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 - Decline in population growth

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- t = 0, 1, 2, 3...
- One homogeneous good
- 2 factors of production:
 - Labor (measured in efficiency units)
 - Land

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 - e.g., surface of planet earth
- Efficiency units of labor evolves endogenously
 - determined by households' decisions about the number and level of human capital of their children

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 - $y \uparrow \Longrightarrow L \uparrow$
- Fixed factor of production Land
 - $L \uparrow \Longrightarrow AP_L \downarrow \Longrightarrow y \downarrow$
- Output per capita fluctuates without a significant trend around a constant level in the long-run

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$$Y_t = H_t^{\alpha}(A_t X)^{1-\alpha}$$

- $H_t \equiv$ efficiency units of labor
- $A_t \equiv$ technological level
- $\bullet X \equiv land$
- Output per worker produced at time t

$$y_t = \left[\frac{H_t}{L_t}\right]^{\alpha} \left[\frac{A_t X}{L_t}\right]^{(1-\alpha)} \equiv h_t^{\alpha} x_t^{1-\alpha}$$

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 - $A_t \uparrow \implies y_t \uparrow \text{ (above } \bar{y}\text{)}$
- Short-run:
 - $y_t \uparrow \implies L_t \uparrow$
- Long-run:
 - $L_t \uparrow \implies y \downarrow \text{ (back to } \bar{y}\text{)}$
- Output per capita is constant in the long-run

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Early stage of development

$$L_t \uparrow \implies A_t \uparrow$$

- Channels:
 - Supply of innovations
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Later Stages of Development

Human capital positively affects technological progress

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• Educated individuals have a comparative advantage in adopting and advancing new technologies (Nelson and Phelps, 1966)

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- ullet $g_{t+1} \equiv ext{ rate of tech progress}$
- $e_t \equiv \text{education}$
- $L_t \equiv$ population size
- $g(0, L_t) > 0$
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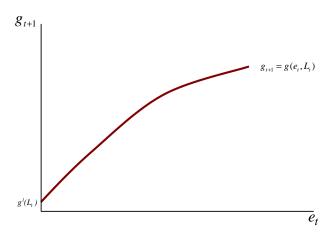
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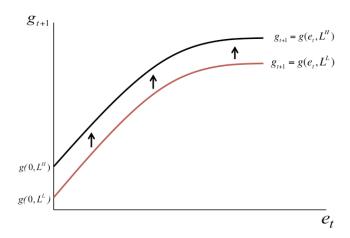
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The Effect of Population Size on Technological Progress



- The increase in the rate of technological progress increases the demand for human capital
 - Human capital permits individuals to better cope with the changes in the technological environment
 - The introduction of new technologies is skill-biased in the short-run, although the nature of the technology is skill-biased or skill-saving in the long run

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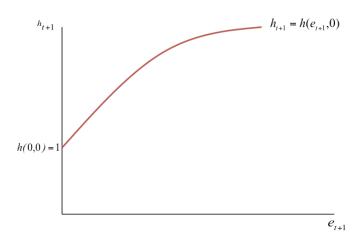
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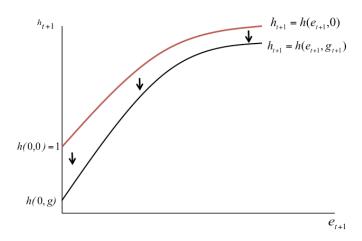
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 - Consume a fraction of their parental unit-time endowment
 - The required time increases with children's quality
 - $\tau \equiv$ time required to raise a child, regardless of quality
 - $\tau + e_{t+1} \equiv$ time to raise a child with education e_{t+1}
- Parenthood (2nd Period):
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 - The required time increases with children's quality
 - \bullet $\tau \equiv$ time required to raise a child, regardless of quality
 - $\tau + e_{t+1} \equiv \text{ time to raise a child with education } e_{t+1}$
- Parenthood (2nd Period):
 - Allocate time between childrearing and work
 - Choose the optimal mixture of quantity and quality of children
 - Consume

- Live for 2 period
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- Parenthood (2nd Period):
 - Allocate time between childrearing and work
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 - Consume

Preferences

The utility function of individual

$$u^t = (1 - \gamma) \ln(c_t) + \gamma \ln(n_t h_{t+1})$$

- $c_t \equiv$ consumption of individual t
- $n_t \equiv$ number of children of individual t
- $h_{t+1} \equiv$ level of human capital of each child

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$$z_t n_t (\tau + e_{t+1}) + c_t \leq z_t$$

- $z_t \equiv$ potential income of individual t
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$$z_t \equiv y_t = h_t^{\alpha} x_t^{1-\alpha} = z(e_t, g_t, x_t)$$

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Biological Constraint

$$c_t \geq \tilde{c}$$

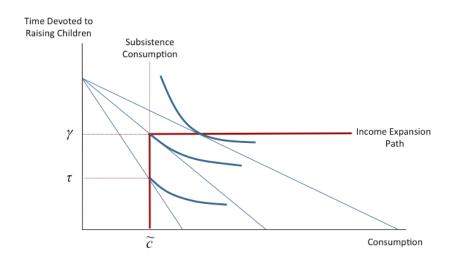
• $\tilde{c} \equiv$ subsistence consumption

Biological Constraint

$$c_t \geq \tilde{c}$$

 $\ \ \, ilde{c} \equiv {
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Constrainst and Optimization



Optimization: Quantity and Quality of Children

Budget constraint:

$$z_t n_t(\tau + e_{t+1}) + c_t \le z_t \iff c_t = z_t[1 - n_t(\tau + e_{t+1})]$$

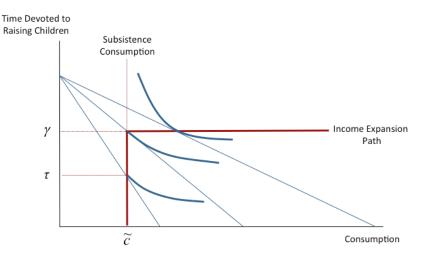
• For $c_t = \tilde{c}$,

$$[1-n_t(\tau+e_{t+1})]=\frac{\tilde{c}}{z_t}$$

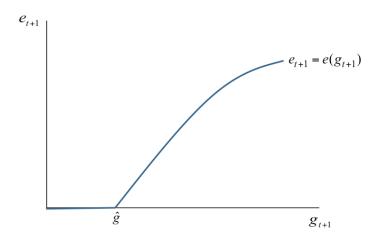
Time devoted to children:

$$n_t(au + e_{t+1}) = \left\{ egin{array}{ll} \gamma & ext{if} & z_t \geq ilde{z} \ & & & \ 1 - rac{ ilde{c}}{z_t} & ext{if} & z_t \leq ilde{z} \end{array}
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Constraints and Optimization



Optimal Investment in Child Quality

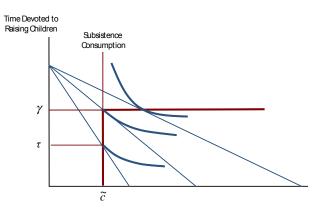


Optimization: Quantity and Quality of Children

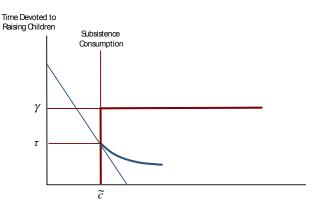
$$e_{t+1} = e(g_{t+1})$$

$$n_t = \left\{ egin{array}{ll} rac{\gamma}{ au + e(g_{t+1})} \equiv n^b(g_{t+1}) & ext{if} \quad z_t \geq ilde{z} \ \\ rac{1 - [ilde{c}/z_t]}{ au + e(g_{t+1})} \equiv n^a(g_{t+1}, z(e_t, g_t, x_t)) & ext{if} \quad z_t \leq ilde{z} \end{array}
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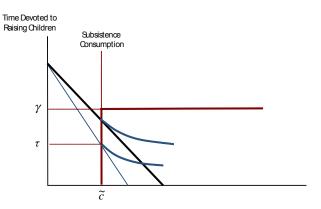
Optimization - Income Expansion Path



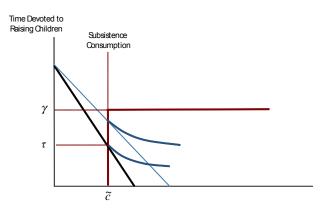
$Optimization - Malthusian \ Epoch$

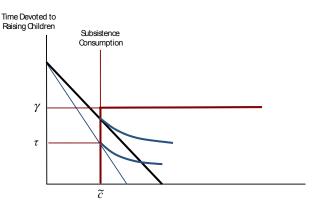


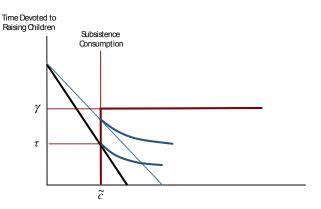
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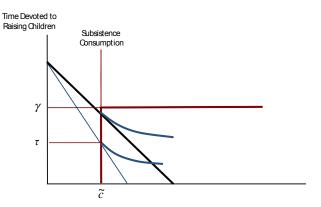


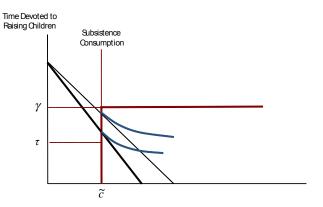
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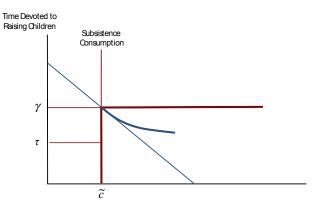




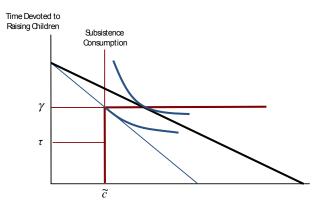




Income Expansion Path - Post-Demographic Transition



Income Expansion Path - Post-Demographic Transition



Technological Progress

Technological progress over time

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(e_t, L_t)$$

- $g(0, L_t) > 0$
- $g_i(e_t, L_t) > 0$ and $g_{ii}(e_t, L_t) < 0$, i = e, L

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Population Dynamics

$$L_{t+1} = n_t L_t$$

$$L_{t+1} = \left\{ egin{array}{ll} n^b(g_{t+1}) L_t & ext{if} & z_t \geq \hat{z} \ \\ n^a(g_{t+1}, z(e_t, g_t, x_t)) L_t & ext{if} & z_t \leq \hat{z} \end{array}
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Dynamics of the Level of Resources per Worker

$$x_{t+1} = \frac{A_{t+1}X}{L_{t+1}} = \frac{(1+g_{t+1})A_tX}{n_tL_t} = \frac{1+g_{t+1}}{n_t}x_t$$

$$x_{t+1} = \begin{cases} \frac{[1+g(e_t, L_t)][\tau^q + \tau^e e(g(e_t, L_t))]}{\gamma} x_t \equiv \phi^b(e_t; L) x_t & z_t \ge \tilde{z} \\ \frac{[1+g(e_t, L_t)][\tau + e(g(e_t, L_t))]}{1 - [\tilde{c}/z(e_t, g_t, x_t)]} x_t \equiv \phi^a(e_t, g_t, x_t, L_t) x_t & z_t \le \tilde{z} \end{cases}$$

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The Dynamical System

A sequence $\{x_t, e_t, g_t, L_t\}_{t=0}^{\infty}$ such that:

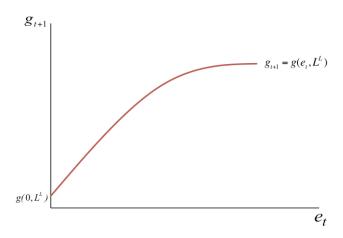
$$\begin{cases} x_{t+1} = \phi(e_t, g_t, x_t, L_t) x_t \\ e_{t+1} = e(g(e_t, L_t)) \\ g_{t+1} = g(e_t, L_t) \\ L_{t+1} = n(e_t, g_t, x_t, L_t) L_t \end{cases}$$

The Conditional Evolution of Technology and Education

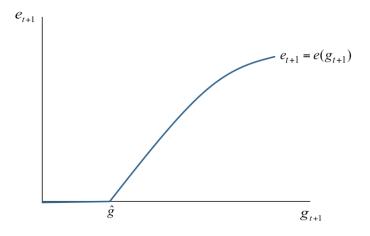
A sequence $\{g_t, e_t; L\}_{t=0}^{\infty}$ such that:

$$\left\{egin{array}{l} g_{t+1}=g(e_t;L) \ \ \ e_{t+1}=e(g_{t+1}) \end{array}
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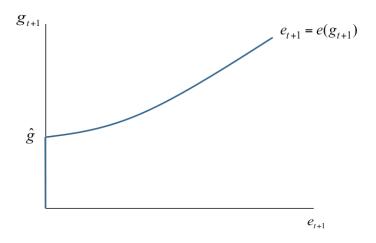
The Effect of Education on Technology



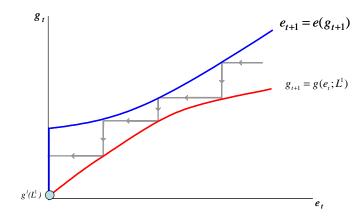
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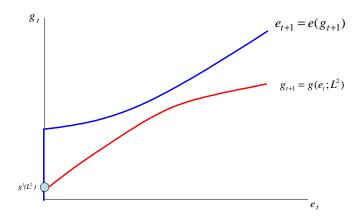


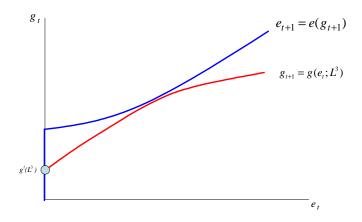
The Effect of Technology on Education: Flipped Axis

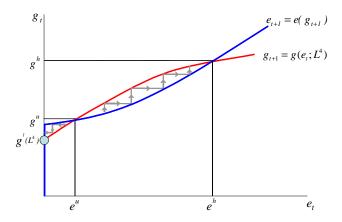


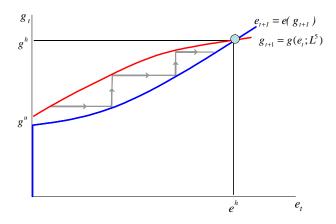
The Evolution of Education and Technology: For a Given Population Size



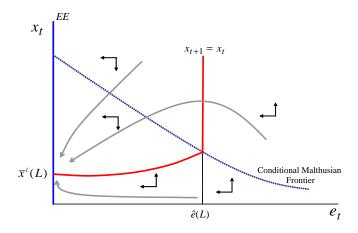




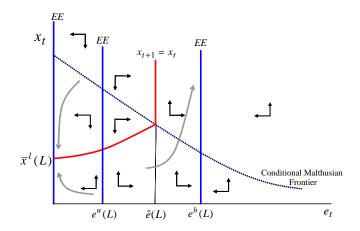




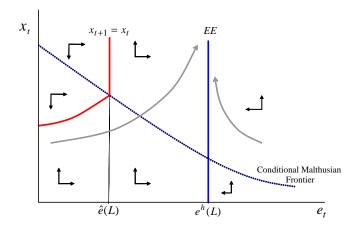
The Evolution of Education and Resources Per Worker: Small Population



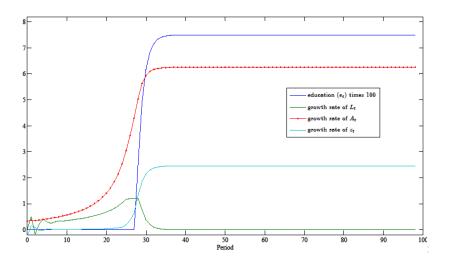
The Evolution of Education and Resources Per Worker: Intermediate Population



The Evolution of Education and Resources Per Worker: Large Population



Calibration



- The transition from stagnation to growth is an *inevitable* by-product of the process of development
- The inherent Malthusian interaction between technology and population, accelerated the pace of technological progress, and eventually brought an industrial demand for human capital
- Human capital formation, triggered a demographic transition, enabling economies to convert a larger share of the fruits of factor accumulation and technological progress into growth of income per capita
- Variations in the timing of the take-off contributed significantly to the divergence in income per capita in the past two centuries

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 Differential timing of takeoffs from stagnation to growth segmented economies into three fundamental regimes:

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 - Slow growing economies in the vicinity of a Malthusian regime
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- Variations in country-specific characteristics that affect:
 - The pace of technological progress
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$$g_{t+1}^i = g(e_t^i, L_t^i, \Omega_t^i)$$

- Protection of intellectual property rights (policy)
 - Positive effect on the incentive to innovate
 - Adverse effect on the proliferation of existing knowledge
- The stock of knowledge within a society
 - Rate of knowledge creation
 - Rate of knowledge diffusion
- The propensity of a country to trade (geography & policy)
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 - Rate of knowledge creation
 - Rate of knowledge diffusion
- The propensity of a country to trade (geography & policy)
 - Technological diffusion
 - Specialization and technological progress via learning by doing

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- The composition of interest groups in society
 - Incentives to block or promote technological innovation (e.g., Luddite, landowners)
- Cultural and genetic diversity
 - Diversity may allow for greater division of labor/economic specialization
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Production of Human Capital

$$h_{t+1}^{i} = h(e_{t+1}^{i}, g_{t+1}^{i}, \phi_{t}^{i})$$

- $\phi_t^i \equiv$ country-specific characteristics affecting HC formation:
- Preferences for Quality

$$u_t^i = (1 - \gamma) \ln(c_t^i) + \gamma \ln[(n_t^i)^{1 - \mu_t^i} (h_{t+1}^i)^{\mu_t^i}]$$

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Variations in Preferences for Human Capital Formation

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- Composition of religious groups within a society and their attitude towards literacy (e.g., Judaism, Protestantism)
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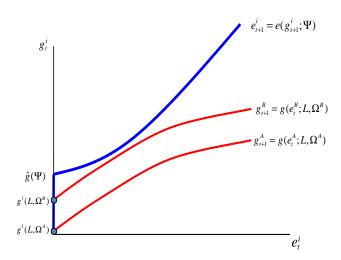
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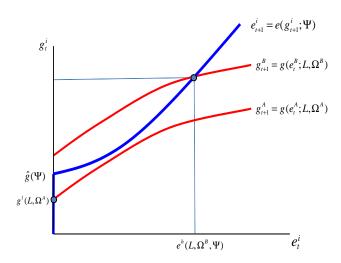
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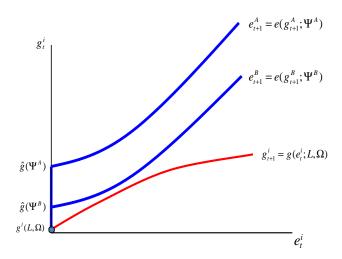
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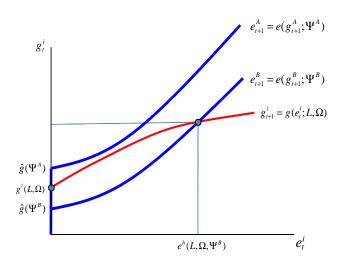
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Unified Growth Theory

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- Introduces role for deep factors
- Provides framework for comparative development
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