Bruce Campbell ST-617 Homework 2

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Chapter 4

Problem 7

Suppose that we wish to predict whether a given stock will issue a dividend this year ("Yes" or "No") based on X, last year's percent profit.We examine a large number of companies and discover that the mean value of X for companies that issued a dividend was $\overline{X} = 10$, while the mean for those that didn't was $\overline{x} = 0$. In addition, the variance of X for these two sets of companies was $\hat{\sigma^2} = 36$. Finally, 80% of companies issued dividends. Assuming that X follows a normal distribution, predict the probability that a company will issue a dividend this year given that its percentage profit was X = 4 last year.

Let's denote our 2 classes 0,1 where 0 indicates no dividend and 1 indicates a dividend. Then we will need to calculate the posterior probability $P(Y = 1|X = 4)$. We are given all of the information we need to do this. The prior probabilities are $\pi_1 = 0.8$ and $\pi_0 = 0.2$ and the likelihood of each class is given by $N(\mu_1, \hat{\sigma})(x)$ where $\mu_1 = 10$ and $N(\mu_0, \hat{\sigma})$ where $\mu_0 = 0$ and $\hat{\sigma} = 36$ in both cases. $N(\mu, \sigma)(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-1}{2\sigma^2}(x-\mu)^2}$ is the normal distribution with mean μ and variance $\hat{\sigma^2}$.

Putting this all together into Bayes theorem

$$P(Y = 1|X = 4) = \frac{N(\mu_1, \hat{\sigma})(4)\pi_1}{N(\mu_1, \hat{\sigma})(4)\pi_1 + N(\mu_0, \hat{\sigma})(4)\pi_0}$$

```r
mu_1 = 0.1
mu_0 = 0

pi_1 = 0.8
pi_0 = 0.2

sigma_sq = 36
stdev_est = 6

x = 4

posterior_probability_of_dividend_given_x <- function(x, mu_1, mu_0, stdev_est,
    pi_1, pi_0) {
    probability = (dnorm(x, mean = mu_1, sd = stdev_est) * pi_1)/(dnorm(x, mean = mu_1,
        sd = stdev_est) * pi_1 + dnorm(x, mean = mu_0, sd = stdev_est) * pi_0)
    return(probability)
}

posterior_probability_of_dividend <- posterior_probability_of_dividend_given_x(x,
    mu_1, mu_0, stdev_est, pi_1, pi_0)
```

We have that the probability of a dividend in the event the $X =$ is `0.8017498`.