SDS 383D Final exam

Spring 2018

For all questions, you do not need to repeat calculations that you have carried out in homework exercises; instead you may refer back to the appropriate exercise. Any derivations that do not refer back to homework exercises must be described in full.

1. Let
$$\begin{aligned}\lambda \sim& \text{Gamma}(\alpha, \beta)\\ X_i \sim& \text{Poisson}(\lambda)\end{aligned}$$
Derive the form of $P(X_{n+1}|X_{1:n})$

2. Now, let X be sampled from a mixture of two Poisson distributions, so that
$$\begin{aligned}\pi \sim& \text{Beta}(\tau, \tau)\\ \lambda_k \sim& \text{Gamma}(\alpha, \beta), \qquad k \in \{0, 1\}\\ Z_i|\pi \sim& \text{Bernoulli}(\pi)\\ X_i|Z_i = k \sim& \text{Poisson}(\lambda_k)\end{aligned}$$
Derive the form of $P(Z_i|Z_{-i}, X)$

3. Let
$$\begin{aligned}y \sim& \text{Normal}(X\beta, I)\\ \beta \sim& \text{Normal}(0, (X^T X)^{-1})\end{aligned}$$
Showing all work (i.e. *not* simply referring back to previous exercise questions), what is the posterior distribution over β given y?

4. You want to use a Gaussian process
$$f \sim \text{GP}(0, K)$$
to model annual rainfall at various geographic locations. Since you know that rainfall is always positive, a Gaussian likelihood is not appropriate here. Write down an appropriate likelihood model for this problem (there are a number of valid options!), and describe either:
   (a) A Gibbs sampler for posterior inference of the latent function f, providing all necessary conditional likelihoods.
   (b) Or, the steps required (i.e. an equation that must be solved to provide the mean, and an analytic form for the covariance) for a Laplace approximation to the posterior of f.