220311/205001 – Computational engineering

Assignment 3 : Numerical resolution of the Navier Stokes equations in laminar flows

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1 INTRODUCTION

To solve the Navier Stokes equation we need to use the fractional step method int hat case we will describe how to do it taking the differentially heated cavity example.

2 DIFFERENTIALLY HEATED CAVITY

I choose to describe the differentially heated cavity because it as also thermal calculations wich is a more global approach.

2.1 Fractional step mathod (FSM)

The fractional step method is a method to solve the incompressible Navier Stoke equation.

To solve Navier Stoke equation we need to solve 5 main step : (for each time step)

1. $R_T(T^n)$ and $\boldsymbol{R}(\boldsymbol{v}^n)$
2. $T^{n+1} = T^n + \frac{\Delta t}{\rho c_p}\left[\frac{3}{2}R_T(T^n) - \frac{1}{2}R_T(T^{n-1})\right]$ and $\boldsymbol{v}^p = \boldsymbol{v}^n + \frac{\Delta t}{\rho}\left[\frac{3}{2}\boldsymbol{R}(\boldsymbol{v}^n) - \frac{1}{2}R(\boldsymbol{v}^{n-1})\right]$
3. $\Delta p^{n+1} = \frac{\rho}{\Delta t}\nabla \cdot \boldsymbol{v}^p$
4. $\boldsymbol{v}^{n+1} = \boldsymbol{v}^p - \frac{\Delta t}{\rho}\nabla p^{n+1}$
5. New $\Delta t = min(\Delta t_c, \Delta t_d, \Delta t_t)$, where $\Delta t_t = 0.20\frac{\Delta x^2}{\lambda/\rho c_p}$

We can see that we need every step to do the next one.
So we will explain the steps in this order.

2.2 $\boldsymbol{R}(\boldsymbol{T^n})$ and $\boldsymbol{R}(\boldsymbol{v^n})$

$$R(u)\Omega_{xP} = -\left[\dot{m}_e u_e - \dot{m}_w u_w + \dot{m}_n u_n - \dot{m}_s u_s\right] + \left[\mu_e \frac{u_E - u_P}{d_{EP}} A_e - \mu_w \frac{u_P - u_W}{d_{WP}} A_w + \mu_n \frac{u_N - u_P}{d_{NP}} A_n - \mu_s \frac{u_P - u_S}{d_{SP}} A_s\right]$$

where $\dot{m}_e = (\rho u)_e A_e$, $\dot{m}_w = (\rho u)_w A_w$, $\dot{m}_n = (\rho v)_n A_n$, $\dot{m}_s = (\rho v)_s A_s$ (here $\dot{m}$ is positive in the positive coordinate direction).

$R_T(T) = -\rho c_p \boldsymbol{v} \cdot \nabla T + \nabla \cdot (\lambda \nabla T).$ then we would need to discretize this function in the same way to calculate RT.

In order to calculate $\boldsymbol{R}$(vn), we have to estimate the velocity for all facets of the control volume(ue,us,us uw).For that, we need to use the scheme of approximation for the convective terms (CDS,UDS,QUICK..). About the terms $(\rho u)e$,$(\rho u)w$,$(\rho u)n$,$(\rho u)s$, it depends on the type of mesh.
Once all facets velocity for each control volume are determined, we obtain the foolowing matrix system:
[R(vn)]= [A][u] .

2.3 T^{n+1} and v^p

$$v^P = v^n + \frac{\Delta t}{\rho}\left[\frac{3}{2}R(v^n)\ \frac{1}{2}R(v^{n-1})\right]$$

Computing every values ov v in the mesh with the Adams bendshforth scheme. If we can compute RT(Vn) we can do the calculation for R(Vn-1).

$$T^{n+1} = T^n + \frac{\Delta t}{\rho c_p}\left[\frac{3}{2}R_T(T^n) - \frac{1}{2}R_T(T^{n-1})\right]$$

Computing the temperature repartition for the new time step . If we can compute RT(Tn) we can do the calculation for RT(Tn-1).

2.4 ΔP^{n+1}

Then we need to take into account the pressure term.

To determine ΔP^{n+1} **:**

$$a_P p_P^{n+1} = a_E p_E^{n+1} + a_W p_W^{n+1} + a_N p_N^{n+1} + a_S p_S^{n+1} + b_P$$

$$a_P = a_E + a_W + a_N + a_S$$

$$a_E = \frac{A_e}{d_{EP}} \qquad a_N = \frac{A_n}{d_{NP}}$$

$$a_W = \frac{A_w}{d_{WP}} \qquad a_S = \frac{A_s}{d_{SP}}$$

$$b_P = -\frac{1}{\Delta t}[(\rho u^P)_e A_e - (\rho u^P)_w A_w + (\rho v^P)_n A_n - (\rho v^P)_s A_s]$$

The equation comes from the discretization of the pressure equation.
The matrix system will be : $A.P^{n+1} = \boldsymbol{B}$ solved by $P^{n+1} = \boldsymbol{A}^{-1}\boldsymbol{B}$.

2.5 $\boldsymbol{v^{n+1}}$

$$v^{n+1} = v^P - \frac{\Delta t}{\rho} \nabla p^{n+1}$$

if ${v^{n+1}}_P$ is known, we can set $v^P = {v^{n+1}}_P$. Thus,

$$\frac{\partial p}{\partial n} = 0$$

So we take into account the pressure term at the time step n+1 to have the correct velocity at the time step n+1. So we solve : (Where vn is equal to vp)

The matrix system is $v^{n+1} = v^n - \boldsymbol{C}.P^{n+1}$

2.6 $\boldsymbol{\Delta t}$

We need to calculate the time step of the convective terms, diffusive terms and the thermal terms according to the CFL conditions.

Then we need to choose the minimum time step to be conservative on the over terms.

Then we iterate on the time step to obtain the new values of the physical properties.

We need information to do those operations:

- the initial velocity (t=0)
- Fluid properties
- Boundary conditions
- Mesh properties
- The scheme used (CDS, UDS, PLDS…)

Once we have these data, we can use the FSM like described previously until the final time ($tsteady$).

3 CONCLUSION

The fractional step method can be divided into 5 steps and by doing so we can see the concept more clearly the key point is to calculate the pressure terms to finally calculate the velocity with those pressure terms.