Exercises
Computational Intelligence Lab
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Problem 1 (CNN computations):

1. The convolutional layer has two 2-channels filters K^1 and K^2, with each filter channel K^l_c having an odd number of pixels, so that we can index them as $(K^l_c)_{i,j}$ for $-k \leq i, j \leq k$. The convolutional layer can be seen as taking as input the image $(I_c)_{1 \leq c \leq 2}$ where $(I_c)_{i,j}$ is the pixel of position (i, j) and channel c, for $1 \leq i, j \leq 4$, and giving as output the following 4x4 2-channels image, whose pixel (i', j') of channel l is given by
$$(I \star K^l)_{i',j'} = \sum_{1 \leq c \leq 2} \sum_{-k \leq i,j \leq k} (I_c)_{i'+i,j'+j} (K^l_c)_{i,j},$$
where $(I_c)_{a,b} = 0$ for (a, b) outside of the range of pixels $\{1, ..., 4\} \times \{1, ..., 4\}$ of I, because of the zero-padding.

2. The ReLU activation function is defined by $ReLU(x) = \max(0, x)$. Applying such a real function to an image consists of applying it to each pixel. Hence after the convolutional layer and the non-linearity, we get the following two images, for $l \in \{1, 2\}$, whose pixels (i', j') are given by
$$(ReLU(I \star K^l))_{i',j'} = \max \left(0, \sum_{1 \leq c \leq 2} \sum_{-k \leq i,j \leq k} (I_c)_{i'+i,j'+j} (K^l_c)_{i,j} \right).$$
Then, applying a 3x3 max-pooling without stride to such a 1-channel image gives us the following image, whose pixel (i, j) is given by
$$\max_{-1 \leq i', j' \leq 1} (ReLU(I \star K^l))_{i+i',j+j'}.$$

3. We have
$$K^1_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
and
$$I_1 = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 5 & 7 & 9 \\ 4 & 7 & 10 & 13 \\ 5 & 9 & 13 & 17 \end{pmatrix}, \; I_2 = \begin{pmatrix} 3 & 4 & 5 & 6 \\ 4 & 6 & 8 & 10 \\ 5 & 8 & 11 & 14 \\ 6 & 10 & 14 & 18 \end{pmatrix},$$
hence
$$(I \star K^1)_{1,1} = (-2 + 3 + 3) + (-3 + 4 + 4) = 9.$$
Similarly,
$$(I \star K^1)_{1,2} = (-3 + 2 + 4 + 5) + (-4 + 3 + 5 + 6) = 18,$$
$$(I \star K^1)_{2,1} = (-3 + 2 + 4 + 5) + (-4 + 3 + 5 + 6) = 18,$$
$$(I \star K^1)_{2,2} = (-5 + 3 + 3 + 7 + 7) + (-6 + 4 + 4 + 8 + 8) = 33,$$
etc.

4. In this case, all pixel values happen to be non-negative, hence applying ReLU doesn't change the values.

5. For the first coefficient, we have

$$\max_{-1\leq i',j'\leq 1}(ReLU(I\star K^l))_{1+i',1+j'}=\max(0,9,18,33)=33.$$