Exercises
Computational Intelligence Lab
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Series 6, Solutions
(Word Embeddings and Text Classification)

Problem 1 (Theory of Word Embeddings):

Recall the GloVe objective that consists in weighted least squares fit of log-counts, written as

$$\mathcal{H}(\theta;\mathbf{N}) = \sum_{i,j} f(n_{ij}) \Bigg(\underbrace{\log n_{ij}}_{\text{target}} - \underbrace{\langle \mathbf{x}_i, \mathbf{y}_j \rangle}_{\substack{=\log \tilde{p}_\theta(w_i|w_j) \\ \text{model}}} \Bigg)^2,$$

where $\tilde{p}_\theta(w_i|w_j) = \exp\langle \mathbf{x}_i, \mathbf{y}_j \rangle$ (note that we here ignore the bias terms for simplicity).

1) Assume $f(.) := 1$ for all arguments, and write $m_{ij} := \log n_{ij}$.

   a) According to the chain rule:

$$\frac{\partial \mathcal{H}}{\partial \mathbf{x}_i} = \sum_j 2(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{y}_j \tag{1}$$

$$\frac{\partial \mathcal{H}}{\partial \mathbf{y}_j} = \sum_i 2(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{x}_i \tag{2}$$

   b) Let $\mathcal{H}_{ij} := f(n_{ij})(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})^2$, so the stochastic gradient is:

$$\frac{\partial \mathcal{H}_{ij}}{\partial \mathbf{x}_i} = 2(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{y}_j \tag{3}$$

$$\frac{\partial \mathcal{H}_{ij}}{\partial \mathbf{y}_j} = 2(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{x}_i \tag{4}$$

2) Plug in the specific weight function:

$$\mathcal{H} = \sum_{ij\,:\,n_{ij}>0} (m_{ij} - \langle \mathbf{x}_i, \mathbf{y}_j \rangle)^2 = \sum_{ij\,:\,n_{ij}>0} \big(m_{ij} - (\mathbf{X}^\top \mathbf{Y})_{ij}\big)^2 \tag{5}$$

   the minimization of which w.r.t. $\mathbf{X}, \mathbf{Y}$ is a matrix completion problem.

3)

$$\frac{\partial \mathcal{H}}{\partial \mathbf{x}_i} = \sum_j 2f(n_{ij})(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{y}_j \tag{6}$$

$$\frac{\partial \mathcal{H}}{\partial \mathbf{y}_j} = \sum_i 2f(n_{ij})(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{x}_i \tag{7}$$

$$\frac{\partial \mathcal{H}_{ij}}{\partial \mathbf{x}_i} = 2f(n_{ij})(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{y}_j \tag{8}$$

$$\frac{\partial \mathcal{H}_{ij}}{\partial \mathbf{y}_j} = 2f(n_{ij})(\langle \mathbf{x}_i, \mathbf{y}_j \rangle - m_{ij})\mathbf{x}_i \tag{9}$$

Problem 3 (GloVe Implementation):

Hint for Step 3): you can use the PCA provided by Scikit Learn, e.g. `http://scikit-learn.org/stable/auto_examples/decomposition/plot_pca_3d.html` *or any other library.*