

# Number Encoding of Problems in the Complexity Class P: A Number-Theoretic Perspective

## 1. Introduction: Setting the Stage for a Number-Theoretic View of Tractability

This report provides an expert-level analysis of the paper "Number Encoding of Problems in the Complexity Class P: A Number-Theoretic Exploration," which investigates the possibility of representing computational problems belonging to the complexity class P (Polynomial Time) as numerical values. The paper, attributed to Anand Kumar Keshavan and Google Deep Research AI, builds upon the foundational concepts of encoding in theoretical computer science, drawing a parallel with the profound technique of Gödel numbering. The central motivation of this exploration is to uncover a deeper mathematical structure underlying computational complexity, akin to how Gödel numbering arithmetized formal systems to explore their inherent limitations <sup>1</sup>.

Gödel numbering, a pivotal concept developed by Kurt Gödel for his groundbreaking incompleteness theorems, demonstrated the remarkable ability to map statements about a formal system onto statements within the system itself through the assignment of unique natural numbers to symbols and well-formed formulas <sup>1</sup>. This arithmetization allowed for the exploration of self-referential statements and ultimately revealed inherent limitations in the provability of certain mathematical truths within consistent axiomatic systems <sup>1</sup>. The current paper seeks to leverage this powerful idea by investigating whether a systematic encoding of computational problems, specifically those within the tractable class P, into numbers can reveal fundamental connections between the nature of efficient computation and the properties of numbers themselves.

As with Gödel numbering, where multiple encoding schemes are possible <sup>3</sup>, the paper acknowledges that various approaches can be explored for representing P problems numerically. The choice of encoding might significantly influence the number-theoretic properties observed in the resulting numerical representations. By examining different mappings from computational instances to numerical values, the research aims to provide a richer understanding of this interdisciplinary landscape.

The paper distinguishes its focus from the authors' prior work, which explored the encoding of NP-complete problems, often perceived as computationally intractable, as transcendental numbers <sup>1</sup>. This shift in focus to the complexity class P, which encompasses problems solvable in polynomial time, allows for an investigation into whether the inherent efficiency of these problems can be mirrored through specific number encodings and their algebraic properties.

## 2. Decoding the Encoding: Mechanisms for Representing P Problems as Numbers

The paper explores several representative problems within the complexity class P and proposes mechanisms for encoding their instances as numerical values. These examples illustrate different facets of P problems and how their structure can be translated into a numerical format.

- **Sorting:** The problem of sorting a sequence of  $n$  numbers into a specific order is a fundamental problem in P, with efficient algorithms like Merge Sort and Quick Sort achieving average-case time complexity of  $O(n \log n)$ <sup>1</sup>. The paper suggests a straightforward encoding where the input sequence is treated as a tuple of numbers. If these numbers are integers, the tuple can be directly represented. For a single numerical representation, the binary (or decimal) representations of the integers can be concatenated, possibly with delimiters to maintain separation<sup>1</sup>. For instance, the sequence (3, 1, 4) could be encoded as the binary string "11|1|100", which can then be interpreted as a unique natural number. This direct mapping from input data to a numerical representation results in an encoded number whose size grows polynomially with the input size, reflecting the tractability of the sorting problem. The order and comparability inherent in sortable data are preserved or can be efficiently reconstructed from this encoding.
- **Graph Connectivity:** Given a graph with  $n$  vertices and  $m$  edges, determining if there is a path between any two vertices is a classic problem solvable in linear time using algorithms like Breadth-First Search or Depth-First Search, thus belonging to P<sup>1</sup>. A common way to represent a graph is using an adjacency matrix, an  $n \times n$  matrix of 0s and 1s indicating the presence or absence of an edge between vertices. For numerical encoding, the adjacency matrix can be linearized by concatenating its rows (or columns) to form a binary string of length  $n^2$ . This binary string can then be interpreted as a natural number<sup>1</sup>. Alternatively, the adjacency list representation, where each vertex is listed along with its neighbors, can also be encoded numerically by representing vertices and their neighbors as numbers and then concatenating these representations with delimiters if necessary. The adjacency matrix encoding yields a number whose size is quadratic in the number of vertices, reflecting the complexity of representing all pairwise relationships in the graph. The connectivity properties of the graph are directly encoded in the patterns of 0s and 1s within the adjacency matrix, allowing for potential analysis using number-theoretic tools.
- **Linear Programming (with integer coefficients):** The problem of optimizing a linear objective function subject to a set of linear inequalities, where all coefficients are integers, is solvable in polynomial time<sup>1</sup>. An instance of this problem is defined by the integer coefficients of the inequalities and the objective function. The paper suggests encoding an instance by listing all these integer coefficients. Similar to the sorting problem, these integers can be concatenated in binary or decimal form to yield a single natural number encoding the entire instance<sup>1</sup>. The size of this

encoded number depends on the number of inequalities, the number of variables, and the magnitude of the integer coefficients. Assuming these parameters are polynomially bounded by the input size, the encoding remains within polynomial bounds. The algebraic nature of the integer coefficients and the polynomial-time solvability of linear programming hint at a potential connection between the number-theoretic properties of this encoding and the computational tractability of the problem.

- **Primality Testing (for numbers up to a certain bound):** Determining whether a given integer  $n$  is a prime number is a fundamental problem in number theory that has been shown to be in P [1, 65, 23, 22, 67, 23, 126, 117, 127, 128, 129, 130, 131, 132, 133, 134, 135, 23]. The paper notes that the integer  $n$  itself serves as a natural numerical encoding of the problem instance<sup>1</sup>. The size of this encoding, measured by the number of bits required to represent  $n$ , is logarithmic in the value of  $n$ . For problems that are inherently rooted in number theory, such as primality testing, the encoding naturally aligns with the mathematical domain, and the number-theoretic properties of the integer directly encode the complexity of the problem.

### 3. The Algebraic Nature of Tractability: Number-Theoretic Properties of P Encodings

The encoding mechanisms discussed in the paper primarily result in natural numbers, which are integers. When a problem instance is encoded as a finite string of symbols, such as binary digits, this string can be directly interpreted as the binary representation of a unique natural number<sup>1</sup>.

- **Integers:** The encodings of instances of P problems, using the methods discussed, invariably lead to integers. Integers are a fundamental class of numbers within mathematics and possess well-defined algebraic properties. Notably, integers are algebraic numbers, as any integer  $k$  is a root of the simple polynomial equation  $x - k = 0$ , which has integer coefficients<sup>1</sup>. This property suggests a potential link between the computational simplicity of P problems and the fundamental nature of algebraic numbers.
- **Rational Numbers:** While the direct encodings discussed so far yield integers, it is conceivable that alternative encoding schemes could involve rational numbers. For instance, if problem instances involved rational parameters, a direct encoding might retain these rational values. However, even rational numbers  $p/q$  (where  $p, q$  are integers and  $q \neq 0$ ) are algebraic, being roots of the polynomial equation  $qx - p = 0$ <sup>1</sup>. Thus, even with rational parameters, the encodings remain within the realm of algebraic numbers.
- **Algebraic Numbers:** Based on the encoding schemes considered in the paper, the resulting numerical representations of P problems appear to fall within the set of algebraic numbers, specifically integers and potentially rationals. This aligns with

the intuition that problems solvable in polynomial time exhibit a certain structural regularity that might be mirrored by the well-defined nature of algebraic numbers as roots of polynomials with integer coefficients <sup>1</sup>. This contrasts with transcendental numbers, which are not roots of such polynomials [

1, 70, 136, 137, 138, 76, 137, 139, 70, 70, 140, 141, 142, 143, 76, 144, 145, 146, 70, 147, 148, 107, 134, 149, 76, 150, 121, 136, 151, 152, 117, 153, 104, 134, 154, 155, 156, 154, 157, 158, 159, 160, 129, 161, 162, 163, 164, 163, 165, 166, 111, 167, 168, 74, 130, 109, 111, 169, 170, 171, 122, 172, 173, 174, 175, 167, 176, 177, 178, 179, 169, 170, 76, 73, 86, 171, 180, 122, 115, 172, 181, 182, 183, 184, 123, 185, 186, 183, 187, 188, 189, 190, 191, 192, 193, 194, 76, 195, 196, 197.

## 4. Alternative Encoding Schemes

Beyond direct concatenation of standard representations, the paper briefly touches upon other encoding schemes that could be explored for problems in P:

- **Polynomial Coefficients:** For P problems that naturally involve polynomials, such as polynomial evaluation, the coefficients of the polynomials themselves could serve as a natural encoding. These coefficients are typically integers or rational numbers, both of which are algebraic numbers <sup>1</sup>.
- **Matrix Representations:** Many P problems, particularly those solved using linear algebra (e.g., solving systems of linear equations, matrix multiplication), can be naturally represented using matrices. If the entries of these matrices are rational or integer, the resulting encoding would again consist of algebraic numbers <sup>1</sup>.
- **Ackermann Encoding Variation:** Inspired by Ackermann's encoding of hereditarily finite sets as natural numbers <sup>23</sup>, a variation could potentially map problem instances to real numbers. While the standard Ackermann encoding yields natural numbers for well-founded sets, a modified version <sup>23</sup> could result in real numbers for a broader class of sets, potentially including structures related to P problems. In certain cases, these real numbers might also be algebraic. The precise conditions under which this occurs require further investigation into the relationship between the underlying structures of P problems and hereditarily finite sets.
- **Church Encoding:** Church encoding provides a way to represent natural numbers and other data types using the lambda calculus <sup>1</sup>. While primarily a theoretical tool in the foundation of computation, it demonstrates that the discrete structures associated with P problems can be represented within a formal system. The numerical interpretation of Church numerals can be related to natural numbers, which are algebraic. This encoding highlights the fundamental representability of computational constructs within mathematical formalisms, although its direct contribution to finding algebraic number encodings for all P problems remains an open question.

## 5. Speculative Research Avenues

The paper concludes by proposing several intriguing avenues for future research based on the exploration of encoding P problems as algebraic numbers:

- **Degree of Algebraic Numbers and Complexity within P:** A key question raised is whether the degree of the minimal polynomial whose root is the encoding of a P problem instance could be related to the time complexity of solving that instance. For example, problems solvable in linear time might correspond to algebraic numbers of degree 1 (rationals), while problems with higher polynomial time complexity might map to algebraic numbers of higher degrees <sup>1</sup>. Investigating this potential hierarchy within algebraic numbers could provide a more refined number-theoretic classification of problems within P, potentially revealing a deeper connection between the algebraic complexity of the encoding and the computational complexity of the problem.
- **Computational Properties of Algebraic Number Fields:** If instances of P problems can be systematically encoded as elements of specific algebraic number fields, the paper speculates whether the properties of these fields, such as their ring of integers or Galois groups, could offer insights into the algorithmic techniques used to solve the corresponding problems <sup>1</sup>. Exploring connections between algebraic structures and algorithmic design might lead to new algorithmic approaches for P problems, where the algebraic properties of the encoding directly inform the development of efficient solutions.
- **Philosophical Implications of Tractability and Algebraicity:** The apparent correspondence between the computational tractability of P problems and the algebraic nature of their encodings prompts philosophical questions about the fundamental nature of efficient computation <sup>1</sup>. Is there an inherent mathematical harmony between problems that can be solved with limited resources and numbers that are defined by finite algebraic equations? Further investigation into this connection could contribute to a deeper understanding of the boundaries of feasible computation and the underlying mathematical principles that govern it.

## 6. Conclusion

This exploration into the number encoding of problems within the complexity class P suggests that using standard encoding techniques, instances of these computationally tractable problems can be naturally represented as integers, which are a subset of algebraic numbers. While alternative encoding schemes might yield other types of algebraic numbers, the overall trend indicates a potential correspondence between the inherent solvability of P problems and the algebraic nature of their numerical representations. This number-theoretic perspective offers a novel lens through which to examine computational complexity, potentially paving the way for future research into the intricate relationships between the algebraic properties of numbers and the inherent difficulty of computational problems. The speculative research avenues proposed by the

paper highlight the potential for this interdisciplinary approach to yield new insights and connections between the fields of computer science and pure mathematics.

Key Tables:

- **Table 1: Encoding Mechanisms for Problems in P**

Problem	Encoding Mechanism	Resulting Number Type	Algebraic Nature
Sorting	Concatenation of binary representations with delimiters	Integer	Algebraic
Graph Connectivity	Linearization of adjacency matrix into a binary string	Integer	Algebraic
Linear Programming	Concatenation of integer coefficients in binary or decimal form	Integer	Algebraic
Primality Testing	The integer itself	Integer	Algebraic

- **Table 2: Alternative Encoding Schemes and Potential Number Types**

Encoding Scheme	Potential Number Type	Algebraic Nature
Polynomial Coefficients	Integer/Rational	Algebraic
Matrix Representations	Integer/Rational	Algebraic

Ackermann Variation	Real	Potentially Algebraic
Church Encoding	Related to Integer	Algebraic

• **Table 3: Speculative Research Avenues**

Research Avenue	Potential Connection
Degree of Algebraic Numbers vs. Complexity within P	Could the degree of the polynomial relate to the time complexity (e.g., linear time $\rightarrow$ degree 1)?
Computational Properties of Algebraic Number Fields	Could properties of these fields (ring of integers, Galois groups) offer insights into algorithmic techniques for P problems?
Philosophical Implications of Tractability and Algebraicity	Is there an inherent mathematical harmony between efficiently solvable problems and numbers defined by finite algebraic equations? Could this connection reveal fundamental boundaries of feasible computation?

**Works cited**

1. How Gödel's Proof Works | Quanta Magazine, accessed March 30, 2025, <https://www.quantamagazine.org/how-godels-proof-works-20200714/>
2. dmi.unibas.ch, accessed March 30, 2025, [https://dmi.unibas.ch/fileadmin/user\\_upload/dmi/Studium/Computer\\_Science/Vorlesung\\_HS20/Seminar\\_Turing\\_Award\\_Winners\\_and\\_Their\\_Contributions/report-example.pdf](https://dmi.unibas.ch/fileadmin/user_upload/dmi/Studium/Computer_Science/Vorlesung_HS20/Seminar_Turing_Award_Winners_and_Their_Contributions/report-example.pdf)
3. Gödel numbering - Wikipedia, accessed March 30, 2025, [https://en.wikipedia.org/wiki/G%C3%B6del\\_numbering](https://en.wikipedia.org/wiki/G%C3%B6del_numbering)
4. Gödel's Incompleteness Theorems (Stanford Encyclopedia of ..., accessed March 30, 2025, <https://plato.stanford.edu/entries/goedel-incompleteness/>
5. History Of Gödel Numbering Part 1 - Durable Scope, accessed March 30, 2025, <https://blog.durablescope.com/post/HistoryOfGodelNumberingPart1/>
6. Gödel's Incompleteness Theorems and their Implications for Computing - TOM ROCKS MATHS, accessed March 30, 2025,



- <https://tomrocksmaths.com/wp-content/uploads/2023/06/godels-incompleteness-theorems-and-their-implications-for-computing.pdf>
7. On Formally Undecidable Propositions of Principia Mathematica and Related Systems, accessed March 30, 2025, [https://en.wikipedia.org/wiki/On\\_Formally\\_Undecidable\\_Propositions\\_of\\_Principia\\_Mathematica\\_and\\_Related\\_Systems](https://en.wikipedia.org/wiki/On_Formally_Undecidable_Propositions_of_Principia_Mathematica_and_Related_Systems)
  8. Could someone explain Gödel numbers? : r/learnmath - Reddit, accessed March 30, 2025, [https://www.reddit.com/r/learnmath/comments/s7wkkp/could\\_someone\\_explain\\_g%C3%B6del\\_numbers/](https://www.reddit.com/r/learnmath/comments/s7wkkp/could_someone_explain_g%C3%B6del_numbers/)
  9. Gödel Numbering in Automata Theory - TutorialsPoint, accessed March 30, 2025, [https://www.tutorialspoint.com/automata\\_theory/automata\\_theory\\_godel\\_numbering.htm](https://www.tutorialspoint.com/automata_theory/automata_theory_godel_numbering.htm)
  10. Gödel's incompleteness theorems - Wikipedia, accessed March 30, 2025, [https://en.wikipedia.org/wiki/G%C3%B6del%27s\\_incompleteness\\_theorems](https://en.wikipedia.org/wiki/G%C3%B6del%27s_incompleteness_theorems)
  11. Gödel and the limits of logic - plus.maths.org - Millennium Mathematics Project, accessed March 30, 2025, <https://plus.maths.org/content/godel-and-limits-logic>
  12. Richard Zach, Kurt Gödel, paper on the incompleteness theorems (1931) - PhilArchive, accessed March 30, 2025, <https://philarchive.org/rec/ZACKGP>
  13. Gödel's Substitution Function in his incompleteness proof - Logic, accessed March 30, 2025, [https://jamesrmeyer.com/ffgit/godel\\_sb](https://jamesrmeyer.com/ffgit/godel_sb)
  14. How Gödel Proved Math's Inherent Limitations - Infinity Plus One - WordPress.com, accessed March 30, 2025, <https://infinitypluseonemath.wordpress.com/2017/09/04/how-godel-proved-maths-inherent-limitations/>
  15. A simplified explanation of Gödel's incompleteness proof: 3 - Logic, accessed March 30, 2025, <https://jamesrmeyer.com/ffgit/GodelSimplified3>
  16. Meta-mathematics and Gödel Numbering, accessed March 30, 2025, <https://www.lucy.cam.ac.uk/sites/default/files/inline-files/Meta-mathematics%20and%20Godel%20Numbering.pdf>
  17. Gödel Numbering - Stanford Encyclopedia of Philosophy, accessed March 30, 2025, <https://plato.stanford.edu/entries/goedel-incompleteness/sup1.html>
  18. What is a Gödel Number? (Arithmatization) - YouTube, accessed March 30, 2025, <https://www.youtube.com/watch?v=F8KGidsuxgY>
  19. Computation | Instructions for Authors - MDPI, accessed March 30, 2025, <https://www.mdpi.com/journal/computation/instructions>
  20. Fifty Years of P vs. NP and the Possibility of the Impossible - Communications of the ACM, accessed March 30, 2025, <https://cacm.acm.org/research/fifty-years-of-p-vs-np-and-the-possibility-of-the-impossible/>
  21. A Quantum Perspective on P vs NP. When Numbers Start Acting Weird | by Sebastian Schepis | Medium, accessed March 30, 2025, <https://medium.com/@sschepis/a-quantum-perspective-on-p-vs-np-d2102f8cdfdd>
  22. Explained: P vs. NP | MIT News | Massachusetts Institute of Technology, accessed March 30, 2025, <https://news.mit.edu/2009/explainer-pnp>
  23. P versus NP problem - Wikipedia, accessed March 30, 2025,



- [https://en.wikipedia.org/wiki/P\\_versus\\_NP\\_problem](https://en.wikipedia.org/wiki/P_versus_NP_problem)
24. The History and Status of the P versus NP Question - Columbia University Computer Science, accessed March 30, 2025,  
<https://www.cs.columbia.edu/~tal/3261/sipserpnp.pdf>
  25. The Turing Machine Halting Problem | by Brent Morgan - Medium, accessed March 30, 2025,  
<https://medium.com/@BrentMorgan/the-turing-machine-halting-problem-cd5bfb7fbc9>
  26. Discrete mathematics - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Discrete\\_mathematics](https://en.wikipedia.org/wiki/Discrete_mathematics)
  27. www.brics.dk, accessed March 30, 2025,  
<https://www.brics.dk/RS/03/50/BRICS-RS-03-50.pdf>
  28. Complexity Classes in Wolfram|Alpha, accessed March 30, 2025,  
<https://blog.wolframalpha.com/2011/02/21/complexity-classes-in-wolframalpha/>
  29. Encoding Sets as Real Numbers - CEUR-WS, accessed March 30, 2025,  
<https://ceur-ws.org/Vol-2199/paper1.pdf>
  30. NP-completeness - Wikipedia, accessed March 30, 2025,  
<https://en.wikipedia.org/wiki/NP-completeness>
  31. What are the differences between NP, NP-Complete and NP-Hard? - Stack Overflow, accessed March 30, 2025,  
<https://stackoverflow.com/questions/1857244/what-are-the-differences-between-np-np-complete-and-np-hard>
  32. List of NP-complete problems - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/List\\_of\\_NP-complete\\_problems](https://en.wikipedia.org/wiki/List_of_NP-complete_problems)
  33. Computational complexity theory - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Computational\\_complexity\\_theory](https://en.wikipedia.org/wiki/Computational_complexity_theory)
  34. The P versus NP problem - Clay Mathematics Institute, accessed March 30, 2025,  
<https://www.claymath.org/wp-content/uploads/2022/06/pvsnp.pdf>
  35. Decoding Complexity: A Deep Dive into NP-complete Problems - Medium, accessed March 30, 2025,  
<https://medium.com/nerd-for-tech/decoding-complexity-a-deep-dive-into-np-complete-problems-fe72d8efc677>
  36. ICS 311 #24: NP-Completeness - University of Hawaii System, accessed March 30, 2025,  
<https://www2.hawaii.edu/~nodari/teaching/s18/Notes/Topic-24.html>
  37. Automata And Complexity Theory Insights | Restackio, accessed March 30, 2025,  
<https://www.restack.io/p/automata-answer-complexity-theory-cat-ai>
  38. Eli5: What is P vs NP? : r/explainlikeimfive - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/explainlikeimfive/comments/15fcign/eli5\\_what\\_is\\_p\\_vs\\_np/](https://www.reddit.com/r/explainlikeimfive/comments/15fcign/eli5_what_is_p_vs_np/)
  39. Nondeterministic Turing machine - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Nondeterministic\\_Turing\\_machine](https://en.wikipedia.org/wiki/Nondeterministic_Turing_machine)
  40. ELI5 the P vs NP problem in mathematics : r/explainlikeimfive - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/explainlikeimfive/comments/ywmr3y/eli5\\_the\\_p\\_vs\\_np\\_problem\\_in\\_mathematics/](https://www.reddit.com/r/explainlikeimfive/comments/ywmr3y/eli5_the_p_vs_np_problem_in_mathematics/)
  41. en.wikipedia.org, accessed March 30, 2025,

- [https://en.wikipedia.org/wiki/P\\_versus\\_NP\\_problem#:~:text=In%20this%20theory%2C%20the%20class,given%20the%20right%20information%2C%20or](https://en.wikipedia.org/wiki/P_versus_NP_problem#:~:text=In%20this%20theory%2C%20the%20class,given%20the%20right%20information%2C%20or)
42. Complexity class - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Complexity\\_class](https://en.wikipedia.org/wiki/Complexity_class)
  43. The relationship between P vs NP problem and "Kolmogorov complexity with time", accessed March 30, 2025,  
<https://mathoverflow.net/questions/142800/the-relationship-between-p-vs-np-problem-and-kolmogorov-complexity-with-time>
  44. Constructive vs computable real numbers - Mathematics Stack Exchange, accessed March 30, 2025,  
<https://math.stackexchange.com/questions/3884598/constructive-vs-computable-real-numbers>
  45. P vs NP Problems - GeeksforGeeks, accessed March 30, 2025,  
<https://www.geeksforgeeks.org/p-vs-np-problems/>
  46. Complexity Theory - Rutgers Computer Science, accessed March 30, 2025,  
<https://www.cs.rutgers.edu/~allender/papers/ALR12.pdf>
  47. P, NP, CoNP, NP hard and NP complete | Complexity Classes | GeeksforGeeks, accessed March 30, 2025,  
<https://www.geeksforgeeks.org/types-of-complexity-classes-p-np-conp-np-hard-and-np-complete/>
  48. Computational Complexity and Approximation - 625.690 | Hopkins EP Online, accessed March 30, 2025,  
<https://ep.jhu.edu/courses/625690-computational-complexity-and-approximation/>
  49. Number of NP-Complete Problems : r/math - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/math/comments/8w96fr/number\\_of\\_npcomplete\\_problems/](https://www.reddit.com/r/math/comments/8w96fr/number_of_npcomplete_problems/)
  50. Possible NP-Complete problem? : r/algorithms - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/algorithms/comments/17qsvnm/possible\\_npcomplete\\_problem/](https://www.reddit.com/r/algorithms/comments/17qsvnm/possible_npcomplete_problem/)
  51. Computational Complexity Theory - Stanford Encyclopedia of Philosophy, accessed March 30, 2025,  
<https://plato.stanford.edu/entries/computational-complexity/>
  52. On the computational complexity of the Dirichlet Problem for Poisson's Equation | Mathematical Structures in Computer Science - Cambridge University Press & Assessment, accessed March 30, 2025,  
<https://www.cambridge.org/core/journals/mathematical-structures-in-computer-science/article/on-the-computational-complexity-of-the-dirichlet-problem-for-poissons-equation/739D07DDDEA1E68E789B0ECF15707AE7>
  53. Why is "P vs. NP" necessarily relevant? - MathOverflow, accessed March 30, 2025,  
<https://mathoverflow.net/questions/54239/why-is-p-vs-np-necessarily-relevant>
  54. NP-hardness of an optimization problem with real value, accessed March 30, 2025,  
<https://cs.stackexchange.com/questions/29227/np-hardness-of-an-optimization-problem-with-real-value>

55. mathoverflow.net, accessed March 30, 2025,  
<https://mathoverflow.net/questions/252728/hard-to-compute-real-numbers#:~:text=Here%20is%20Turing's%20definition%3A%20A,number's%20decimal%20expansion%20as%20output>.
56. Sorting algorithm reference, for coding interviews and computer science classes, accessed March 30, 2025,  
<https://www.interviewcake.com/sorting-algorithm-cheat-sheet>
57. Sorting Algorithms - LAMFO, accessed March 30, 2025,  
<https://lamfo-unb.github.io/2019/04/21/Sorting-algorithms/>
58. Algebraic Positional Encodings - arXiv, accessed March 30, 2025,  
<https://arxiv.org/html/2312.16045v3>
59. The property of complexity class P - algorithm - Stack Overflow, accessed March 30, 2025,  
<https://stackoverflow.com/questions/15562025/the-property-of-complexity-class-p>
60. The Mathematics Behind Algorithm Design: Unlocking the Power of Computational Thinking, accessed March 30, 2025,  
<https://algotcademy.com/blog/the-mathematics-behind-algorithm-design-unlocking-the-power-of-computational-thinking/>
61. Computational complexity of mathematical operations - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Computational\\_complexity\\_of\\_mathematical\\_operations](https://en.wikipedia.org/wiki/Computational_complexity_of_mathematical_operations)
62. CS 535: Complexity Theory, Fall 2023 Homework 1, accessed March 30, 2025,  
[https://cs-people.bu.edu/mbun/courses/535\\_F23/handouts/hw1.pdf](https://cs-people.bu.edu/mbun/courses/535_F23/handouts/hw1.pdf)
63. Quantitative Coding and Complexity Theory of Compact Metric Spaces - PMC, accessed March 30, 2025, <https://pmc.ncbi.nlm.nih.gov/articles/PMC7309482/>
64. Talk like a graph: Encoding graphs for large language models - Google Research, accessed March 30, 2025,  
<https://research.google/blog/talk-like-a-graph-encoding-graphs-for-large-language-models/>
65. The Class P, accessed March 30, 2025,  
<https://homepages.gac.edu/~sskulrat/Courses/2019S-265/lectures/P.html>
66. Linear programming restricted to rational coefficients - Computer Science Stack Exchange, accessed March 30, 2025,  
<https://cs.stackexchange.com/questions/82086/linear-programming-restricted-to-rational-coefficients>
67. Algebraic numbers proof - Mathematics Stack Exchange, accessed March 30, 2025,  
<https://math.stackexchange.com/questions/1980911/algebraic-numbers-proof>
68. Implicit computation complexity in higher-order programming languages | Mathematical Structures in Computer Science | Cambridge Core, accessed March 30, 2025,  
<https://www.cambridge.org/core/journals/mathematical-structures-in-computer-science/article/implicit-computation-complexity-in-higherorder-programming-languages/0E9A155F520EA7294C6A64F039479D33>
69. Algebraic Complexity Theory: Where the Abstract and the Practical Meet,

- accessed March 30, 2025,  
<https://www.simonsfoundation.org/2021/02/24/algebraic-complexity-theory-where-t-he-abstract-and-the-practical-meet/>
70. Transcendental number theory - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Transcendental\\_number\\_theory](https://en.wikipedia.org/wiki/Transcendental_number_theory)
  71. Algebraic Theory of Automata Networks - SIAM Publications Library, accessed March 30, 2025, <https://epubs.siam.org/doi/book/10.1137/1.9780898718492>
  72. Arithmetic complexity computations | Algorithmics, complexity, computer algebra and computational geometry - Cambridge University Press & Assessment, accessed March 30, 2025,  
<https://www.cambridge.org/fk/universitypress/subjects/computer-science/algorithmics-complexity-computer-algebra-and-computational-g/arithmetic-complexity-computations>
  73. Transcendental Numbers: An Extended Example, accessed March 30, 2025,  
<https://www.math.wustl.edu/~freiwald/310transcendentals.pdf>
  74. Hard-to-compute real numbers - MathOverflow, accessed March 30, 2025,  
<https://mathoverflow.net/questions/252728/hard-to-compute-real-numbers>
  75. Geometry and Complexity Theory (Cambridge Studies in Advanced Mathematics, Series Number 169): 9781107199231: Landsberg, J. M. - Amazon.com, accessed March 30, 2025,  
<https://www.amazon.com/Geometry-Complexity-Cambridge-Advanced-Mathematics/dp/1107199239>
  76. Transcendental number - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Transcendental\\_number](https://en.wikipedia.org/wiki/Transcendental_number)
  77. Computational Complexity - Cambridge University Press & Assessment, accessed March 30, 2025,  
<https://www.cambridge.org/core/books/computational-complexity/3453CAFDEB0B4820B186FE69A64E1086>
  78. Independence in Algebraic Complexity Theory, accessed March 30, 2025,  
<https://d-nb.info/1045872121/34>
  79. ALGEBRAIC AND TRANSCENDENTAL NUMBERS FROM AN INVITATION TO MODERN NUMBER THEORY 1. Introduction 1 2. Russell's Paradox and the, accessed March 30, 2025,  
[https://web.williams.edu/Mathematics/sjmillier/public\\_html/238/currentnotes/AlgTrans\\_Chap5.pdf](https://web.williams.edu/Mathematics/sjmillier/public_html/238/currentnotes/AlgTrans_Chap5.pdf)
  80. Complexity Theory for Operators in Analysis - Department of Computer Science, accessed March 30, 2025,  
<https://www.cs.toronto.edu/~sacook/homepage/stoc.2010.pdf>
  81. ACM Transactions on Computation Theory (TOCT) Template ..., accessed March 30, 2025,  
<https://scispace.com/formats/association-for-computing-machinery/acm-transactions-on-computation-theory-toct/5f66dd32971e422fa6c9b28e3cec95e6>
  82. Computing with real numbers - Fredrik Johansson, accessed March 30, 2025,  
[https://fredrikj.net/math/ejcm2020\\_joh\\_en.pdf](https://fredrikj.net/math/ejcm2020_joh_en.pdf)
  83. Algebraic computation models (Chapter 16) - Cambridge University Press & Assessment, accessed March 30, 2025,

<https://www.cambridge.org/core/books/computational-complexity/algebraic-computation-models/4A62FD4F8AAEB1B42F29593D3E948DF5>

84. Algebraic complexity theory / Peter Bürgisser, Michael Clausen, M. Amin Shokrollahi ; with the collaboration of Thomas Lickteig. - University of California Berkeley - UC Library Search, accessed March 30, 2025,  
[https://search.library.berkeley.edu/discovery/fulldisplay?vid=01UCS\\_BER%3AUCB&search\\_scope=DN\\_and\\_CI&tab=Default\\_UCLibrarySearch&docid=alma991065176489706532&lang=en&context=L&adaptor=Local%20Search%20Engine&query=sub%2Cexact%2C%20Stochastic%20processes%2CAND&mode=advanced&offset=0](https://search.library.berkeley.edu/discovery/fulldisplay?vid=01UCS_BER%3AUCB&search_scope=DN_and_CI&tab=Default_UCLibrarySearch&docid=alma991065176489706532&lang=en&context=L&adaptor=Local%20Search%20Engine&query=sub%2Cexact%2C%20Stochastic%20processes%2CAND&mode=advanced&offset=0)
85. Logical characterizations of algebraic circuit classes over integral domains | Mathematical Structures in Computer Science - Cambridge University Press & Assessment, accessed March 30, 2025,  
<https://www.cambridge.org/core/journals/mathematical-structures-in-computer-science/article/logical-characterizations-of-algebraic-circuit-classes-over-integral-domains/94541B2F327A87136EE80DAAE763B706>
86. Recounting the History of Math's Transcendental Numbers - Quanta Magazine, accessed March 30, 2025,  
<https://www.quantamagazine.org/recounting-the-history-of-maths-transcendental-numbers-20230627/>
87. Computation over algebraic structures and a classification of undecidable problems, accessed March 30, 2025,  
<https://www.cambridge.org/core/journals/mathematical-structures-in-computer-science/article/computation-over-algebraic-structures-and-a-classification-of-undecidable-problems/64E8E1323A5E81D8BA5EE5769DC48C35>
88. The field of algebraic numbers as a recursive structure - Mathematics Stack Exchange, accessed March 30, 2025,  
<https://math.stackexchange.com/questions/3577474/the-field-of-algebraic-numbers-as-a-recursive-structure>
89. computational complexity - Algebraic P vs. NP - MathOverflow, accessed March 30, 2025, <https://mathoverflow.net/questions/57741/algebraic-p-vs-np>
90. Algebraic Positional Encodings - Chalmers Research, accessed March 30, 2025,  
[https://research.chalmers.se/publication/544856/file/544856\\_Fulltext.pdf](https://research.chalmers.se/publication/544856/file/544856_Fulltext.pdf)
91. Math 784: algebraic NUMBER THEORY, accessed March 30, 2025,  
<https://people.math.sc.edu/filaseta/gradcourses/TheMath784Notes.pdf>
92. Algebraic number field - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Algebraic\\_number\\_field](https://en.wikipedia.org/wiki/Algebraic_number_field)
93. Showing that  $7+3\sqrt{2}$  is an algebraic number - Mathematics Stack Exchange, accessed March 30, 2025,  
<https://math.stackexchange.com/questions/1300546/showing-that-7-sqrt32-is-an-algebraic-number>
94. 1 Basic facts - Berkeley Math Circle, accessed March 30, 2025,  
<https://mathcircle.berkeley.edu/sites/default/files/BMC3/bamc.pdf>
95. How to prove an algebraic integer. - Mathematics Stack Exchange, accessed March 30, 2025,  
<https://math.stackexchange.com/questions/387437/how-to-prove-an-algebraic-integer>



[ger](#)

96. Algebraic Number Theory, accessed March 30, 2025,  
<https://personalpages.manchester.ac.uk/staff/Martin.Orr/2018-9/ant/lectures.pdf>
97. Algebraic number theory - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Algebraic\\_number\\_theory](https://en.wikipedia.org/wiki/Algebraic_number_theory)
98. Algebraic number - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Algebraic\\_number](https://en.wikipedia.org/wiki/Algebraic_number)
99. WHAT IS ANDERSON'S LOG-ALGEBRAICITY? - OSU Math Department, accessed March 30, 2025,  
<https://math.osu.edu/sites/math.osu.edu/files/LogAlgebraicity.pdf>
100. How to Prove a Number is an Algebraic Number: Example with  $\sqrt[3]{2} + \sqrt{3}$ , accessed March 30, 2025,  
<https://www.youtube.com/watch?v=A2mK1Y6s0Ro>
101. Some NP Complete Problems Based on Algebra and Algebraic Geometry - arXiv, accessed March 30, 2025, <https://arxiv.org/html/2503.14715v1>
102. Is combinatorics the field of math with the most NP-complete problems? - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/math/comments/1i00tcp/is\\_combinatorics\\_the\\_field\\_of\\_math\\_with\\_the\\_most/](https://www.reddit.com/r/math/comments/1i00tcp/is_combinatorics_the_field_of_math_with_the_most/)
103. Satisfiability modulo theories - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Satisfiability\\_modulo\\_theories](https://en.wikipedia.org/wiki/Satisfiability_modulo_theories)
104. Semi-galois Categories II: An arithmetic analogue of Christol's theorem - ResearchGate, accessed March 30, 2025,  
[https://www.researchgate.net/publication/320195419\\_Semi-galois\\_Categories\\_II\\_An\\_arithmetic\\_analogue\\_of\\_Christol's\\_theorem](https://www.researchgate.net/publication/320195419_Semi-galois_Categories_II_An_arithmetic_analogue_of_Christol's_theorem)
105. arXiv:1512.07824v2 [math.NT] 11 Sep 2016, accessed March 30, 2025,  
<https://arxiv.org/pdf/1512.07824>
106. Complexity and Real Computation - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Complexity\\_and\\_Real\\_Computation](https://en.wikipedia.org/wiki/Complexity_and_Real_Computation)
107. Relation between transcendental numbers and computational complexity?, accessed March 30, 2025,  
<https://cstheory.stackexchange.com/questions/39922/relation-between-transcendental-numbers-and-computational-complexity>
108. On Transcendence of Numbers Related to Sturmian and Arnoux-Rauzy Words - People at MPI-SWS, accessed March 30, 2025,  
<https://people.mpi-sws.org/~joel/publications/sturmian-numbers24.pdf>
109. The Computational Complexity of Algebraic Numbers | SIAM Journal on Numerical Analysis, accessed March 30, 2025,  
<https://epubs.siam.org/doi/10.1137/0712008>
110. Computational complexity - CS@Cornell, accessed March 30, 2025,  
[https://www.cs.cornell.edu/gries/40brochure/pg38\\_39.pdf](https://www.cs.cornell.edu/gries/40brochure/pg38_39.pdf)
111. On Polynomial Time Computable Numbers - arXiv, accessed March 30, 2025,  
<https://arxiv.org/pdf/cs/0608067>
112. Math 249A Fall 2010: Transcendental Number Theory - Stanford Mathematics, accessed March 30, 2025, <http://math.stanford.edu/~ksound/TransNotes.pdf>
113. Computable and Automatic Numbers 1 Introduction 2 Algebraic and

- Transcendental Numbers, accessed March 30, 2025,  
<http://www.cs.ox.ac.uk/james.worrell/computable.pdf>
114. Is it possible that  $P=NP$  but any algorithm that is capable of solving problems like the TSP/SAT in polynomial time have a time complexity that is unprovable in ZFC : r/computerscience - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/computerscience/comments/1bz0yte/is\\_it\\_possible\\_that\\_pnp\\_but\\_any\\_algorithm\\_that\\_is/](https://www.reddit.com/r/computerscience/comments/1bz0yte/is_it_possible_that_pnp_but_any_algorithm_that_is/)
  115. Real-Time Computability of Real Numbers by Chemical Reaction Networks ?, accessed March 30, 2025,  
<https://cs.carleton.edu/faculty/tklinge/assets/papers/rtrcn2.pdf>
  116. Lecture Boolean Satisfiability (SAT) Solvers - cs.Princeton, accessed March 30, 2025,  
<https://www.cs.princeton.edu/courses/archive/fall21/cos326/lec/23-01-sat.pdf>
  117. Function Fields in Positivity characteristics: expansions and Cobham's theorem, accessed March 30, 2025,  
<https://www.math.uwaterloo.ca/~jpbell/ABK.pdf>
  118. Encoding Sets as Real Numbers - LIRMM, accessed March 30, 2025,  
[https://www.lirmm.fr/sets2018/papers/paper\\_4.pdf](https://www.lirmm.fr/sets2018/papers/paper_4.pdf)
  119. Groundbreaking Approach to Solving the P vs NP Problem - ResearchGate, accessed March 30, 2025,  
[https://www.researchgate.net/publication/374545591\\_Groundbreaking\\_Approach\\_to\\_Solving\\_the\\_P\\_vs\\_NP\\_Problem](https://www.researchgate.net/publication/374545591_Groundbreaking_Approach_to_Solving_the_P_vs_NP_Problem)
  120. Computational hardness assumption - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Computational\\_hardness\\_assumption](https://en.wikipedia.org/wiki/Computational_hardness_assumption)
  121.  $\{\text{transcendental numbers}\} \setminus \{\text{computable transcendental numbers}\}$  - MathOverflow, accessed March 30, 2025,  
<https://mathoverflow.net/questions/26402/transcendental-numbers-computable-transcendental-numbers>
  122. 1 Introduction 2 Extremely Sparse Sets - UMD Computer Science, accessed March 30, 2025, <https://www.cs.umd.edu/~gasarch/open/juris.pdf>
  123. Some NP Complete Problems Based on Algebra and Algebraic Geometry - ResearchGate, accessed March 30, 2025,  
[https://www.researchgate.net/publication/390018899\\_Some\\_NP\\_Complete\\_Problems\\_Based\\_on\\_Algebra\\_and\\_Algebraic\\_Geometry](https://www.researchgate.net/publication/390018899_Some_NP_Complete_Problems_Based_on_Algebra_and_Algebraic_Geometry)
  124. Church encoding - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Church\\_encoding](https://en.wikipedia.org/wiki/Church_encoding)
  125. Subset sum vs. Subset product (strong vs. weak NP hardness), accessed March 30, 2025,  
<https://cstheory.stackexchange.com/questions/16902/subset-sum-vs-subset-product-strong-vs-weak-np-hardness>
  126. Automata Methods in Transcendence - Dinesh S. Thakur - CiteSeerX, accessed March 30, 2025,  
<https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=5acd4cd13623b523a7ee49ce4c48b5377df280d3>
  127. Christol's theorem and its analogue for generalized power series, part 1, accessed March 30, 2025,



- <https://www.fields.utoronto.ca/programs/scientific/12-13/words/slides/Kedlaya1.pdf>
128. AN ANALOGUE OF COBHAM'S THEOREM FOR FRACTALS by Boris Adamczewski & Jason Bell, accessed March 30, 2025, [https://adamczewski.perso.math.cnrs.fr/Fractals\\_New.pdf](https://adamczewski.perso.math.cnrs.fr/Fractals_New.pdf)
  129. [math/0601604] Diophantine properties of real numbers generated by finite automata - arXiv, accessed March 30, 2025, <https://arxiv.org/abs/math/0601604>
  130. On the Complexity of Algebraic Numbers, and the Bit-Complexity of Straight-Line Programs - Rutgers Computer Science, accessed March 30, 2025, <https://www.cs.rutgers.edu/~allender/papers/algebraic.pdf>
  131. THE COMPUTATIONAL COMPLEXITY OF ALGEBRAIC NUMBERS\* Carnegie-Mellon University Pittsburgh, Pa. ABSTRACT Let  $\{x_i\}$  be a sequence ap - CiteSeerX, accessed March 30, 2025, <https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=14d4279d1022131ae766faf8c93fefeb8519baa3>
  132. A Short History of Computational Complexity - Uni Ulm, accessed March 30, 2025, [https://www.uni-ulm.de/fileadmin/website\\_uni\\_ulm/iui.inst.190/Mitarbeiter/toran/beatcs/column80.pdf](https://www.uni-ulm.de/fileadmin/website_uni_ulm/iui.inst.190/Mitarbeiter/toran/beatcs/column80.pdf)
  133. UCT Polynomial Time - CMU School of Computer Science, accessed March 30, 2025, <https://www.cs.cmu.edu/~15455/pdf/060-polynomial.pdf>
  134. On the computational complexity of algebraic numbers: the Hartmanis–Stearns problem revisited - Boris Adamczewski, accessed March 30, 2025, <https://adamczewski.perso.math.cnrs.fr/HS.pdf>
  135. Computability vs. Nondeterministic and P vs. NP - arXiv, accessed March 30, 2025, <https://arxiv.org/pdf/1305.4029>
  136. people.math.rochester.edu, accessed March 30, 2025, <https://people.math.rochester.edu/faculty/dthakur2/autbanffFinal.pdf>
  137. fse.studenttheses.ub.rug.nl, accessed March 30, 2025, [https://fse.studenttheses.ub.rug.nl/18517/1/bMATH\\_2018\\_deJagerRJ.pdf](https://fse.studenttheses.ub.rug.nl/18517/1/bMATH_2018_deJagerRJ.pdf)
  138. TRANSCENDENTAL NUMBERS 1. Introduction The Greeks tried unsuccessfully to square the circle with a compass and straightedge. In, accessed March 30, 2025, <https://people.clas.ufl.edu/jeremybooher/files/transcendence.pdf>
  139. How do you prove a number is transcendental? : r/askscience - Reddit, accessed March 30, 2025, [https://www.reddit.com/r/askscience/comments/21xyob/how\\_do\\_you\\_prove\\_a\\_number\\_is\\_transcendental/](https://www.reddit.com/r/askscience/comments/21xyob/how_do_you_prove_a_number_is_transcendental/)
  140. AWS Lecture 4 Auxiliary functions in transcendence proofs, accessed March 30, 2025, <https://swc-math.github.io/aws/2008/08WaldschmidtSlides4.pdf>
  141. Algebraic number theory and applications to properties of the natural numbers., accessed March 30, 2025, <https://mathoverflow.net/questions/16271/algebraic-number-theory-and-applications-to-properties-of-the-natural-numbers>
  142. en.wikipedia.org, accessed March 30, 2025, [https://en.wikipedia.org/wiki/Transcendental\\_number#:~:text=In%201882%20Ferdinand%20von%20Lindemann.%CF%80%20must%20therefore%20be%20transcendental.](https://en.wikipedia.org/wiki/Transcendental_number#:~:text=In%201882%20Ferdinand%20von%20Lindemann.%CF%80%20must%20therefore%20be%20transcendental.)

143. Transcendence of  $e$  and  $\pi$  - Jonathan Love, accessed March 30, 2025, <https://jonathanlove.info/teaching/23-NT/rhaouti.pdf>
144. Lindemann–Weierstrass theorem - Wikipedia, accessed March 30, 2025, [https://en.wikipedia.org/wiki/Lindemann%E2%80%93Weierstrass\\_theorem](https://en.wikipedia.org/wiki/Lindemann%E2%80%93Weierstrass_theorem)
145. How hard is the proof of  $\pi$  or  $e$  being transcendental? - Mathematics Stack Exchange, accessed March 30, 2025, <https://math.stackexchange.com/questions/12872/how-hard-is-the-proof-of-pi-or-e-being-transcendental>
146. how do you prove a (Pi) number is transcendental? : r/math - Reddit, accessed March 30, 2025, [https://www.reddit.com/r/math/comments/bjorgu/how\\_do\\_you\\_prove\\_a\\_pi\\_number\\_is\\_transcendental/](https://www.reddit.com/r/math/comments/bjorgu/how_do_you_prove_a_pi_number_is_transcendental/)
147. Transcendence of Periods: The State of the Art - Michel Waldschmidt, accessed March 30, 2025, [https://archive.intlpress.com/site/pub/files/\\_fulltext/journals/pamq/2006/0002/0002/PAMQ-2006-0002-0002-a003.pdf](https://archive.intlpress.com/site/pub/files/_fulltext/journals/pamq/2006/0002/0002/PAMQ-2006-0002-0002-a003.pdf)
148. Transcendentals - Complexity - follow the idea - Obsidian Publish, accessed March 30, 2025, <https://publish.obsidian.md/followtheidea/Content/Math/Transcendentals+-+Complexity>
149. Boolean Satisfiability, accessed March 30, 2025, <https://www.cs.cmu.edu/afs/cs/project/jair/pub/volume21/dixon04a-html/node2.html>
150. Cryptography based on transcendental numbers | Request PDF - ResearchGate, accessed March 30, 2025, [https://www.researchgate.net/publication/225720932\\_Cryptography\\_based\\_on\\_transcendental\\_numbers](https://www.researchgate.net/publication/225720932_Cryptography_based_on_transcendental_numbers)
151. Transcendence in positive characteristic and special values of hypergeometric functions, accessed March 30, 2025, [https://www.researchgate.net/publication/228929061\\_Transcendence\\_in\\_positive\\_characteristic\\_and\\_special\\_values\\_of\\_hypergeometric\\_functions](https://www.researchgate.net/publication/228929061_Transcendence_in_positive_characteristic_and_special_values_of_hypergeometric_functions)
152. A note on Christol's theorem - Boris Adamczewski, accessed March 30, 2025, [https://adamczewski.perso.math.cnrs.fr/Note\\_On\\_Christol.pdf](https://adamczewski.perso.math.cnrs.fr/Note_On_Christol.pdf)
153. Christol's theorem and its analogue for generalized power series, part 2, accessed March 30, 2025, <https://www.fields.utoronto.ca/programs/scientific/12-13/words/slides/Kedlaya2.pdf>
154. Computational Classification of Numbers and Algebraic Properties Robert M. Beals and Dinesh S. Thakur, accessed March 30, 2025, <https://people.math.rochester.edu/faculty/dthakur2/bealsthakur.pdf>
155. Finite automata and arithmetic, accessed March 30, 2025, <https://www.mat.univie.ac.at/~slc/s/s30allouche.pdf>
156. The Mathematics of Finite State Automata, accessed March 30, 2025, <https://math.stackexchange.com/questions/1635594/the-mathematics-of-finite-state-automata>
157. Cobham's theorem and its extensions - Numdam, accessed March 30, 2025, <https://www.numdam.org/item/10.5802/acirm.3.pdf>

158. arXiv:2304.09223v1 [cs.FL] 18 Apr 2023, accessed March 30, 2025, <https://arxiv.org/pdf/2304.09223>
159. Cobham's theorem and its extensions - Numdam, accessed March 30, 2025, <http://www.numdam.org/item/10.5802/acirm.3.pdf>
160. Diophantine properties of real numbers generated by finite automata - NASA/ADS, accessed March 30, 2025, <https://ui.adsabs.harvard.edu/abs/arXiv:math%2F0601604>
161. Diophantine properties of real numbers generated by finite automata - Cambridge University Press & Assessment, accessed March 30, 2025, <https://www.cambridge.org/core/services/aop-cambridge-core/content/view/447D3E82F7AAFBF389812E9189879E8E/S0010437X06002247a.pdf/diophantine-properties-of-real-numbers-generated-by-finite-automata.pdf>
162. Diophantine approximation and expansions of real numbers (Numeration and Substitution 2012), accessed March 30, 2025, [https://repository.kulib.kyoto-u.ac.jp/dspace/bitstream/2433/226198/1/B46\\_010.pdf](https://repository.kulib.kyoto-u.ac.jp/dspace/bitstream/2433/226198/1/B46_010.pdf)
163. Diophantine properties of real numbers generated by finite automata - Boris.Adamczewski, accessed March 30, 2025, [https://adamczewski.perso.math.cnrs.fr/article\\_Compositio.pdf](https://adamczewski.perso.math.cnrs.fr/article_Compositio.pdf)
164. NEW BOUNDS FOR IRRATIONALITY MEASURES OF SOME FAST CONVERGING SERIES, accessed March 30, 2025, <https://mathsapplication.com/wp-content/uploads/2023/07/10.13164-ma-2014-12.pdf>
165. THE IRRATIONALITY MEASURE OF  $\pi$  IS AT MOST 7.103205334137... - rutgers math, accessed March 30, 2025, <https://sites.math.rutgers.edu/~zeilberg/mamarim/mamarimPDF/pimeasNew.pdf>
166. arxiv.org, accessed March 30, 2025, <https://arxiv.org/pdf/cs/0608067#:~:text=Computable%20number%20means%20that%20it,%2C%20the%20Dedekind%20cut%2C%20etc.>
167. Characterizing Polynomial Time Computability of Rational and Real Functions - arXiv, accessed March 30, 2025, <https://arxiv.org/pdf/0911.2325>
168. Polynomial-time hierarchy of computable reals - Griffith Research Online, accessed March 30, 2025, <https://research-repository.griffith.edu.au/bitstreams/27e52ef3-b712-56dd-a055-521117aaaa41/download>
169. Polynomial-Time Computability in Analysis: A Survey Ker-I Ko, accessed March 30, 2025, <https://www3.cs.stonybrook.edu/~keriko/cca10.pdf>
170. Computable and Automatic Numbers 1 Introduction 2 Algebraic and Transcendental Numbers, accessed March 30, 2025, <https://www.cs.ox.ac.uk/people/james.worrell/computable.pdf>
171. Any computable transcendental number that is computable in P time but not  $\$O(n)$ , accessed March 30, 2025, <https://cstheory.stackexchange.com/questions/38541/any-computable-transcendental-number-that-is-computable-in-p-time-but-not-on>
172. Hartmanis-Stearns conjecture and the computable transcendental numbers, accessed March 30, 2025, <https://cstheory.stackexchange.com/questions/27992/hartmanis-stearns-conjectur>

[e-and-the-computable-transcendental-numbers](#)

173. Is Hartmanis-Stearns conjecture settled by this article?, accessed March 30, 2025,  
<https://cstheory.stackexchange.com/questions/39113/is-hartmanis-stearns-conjecture-settled-by-this-article>
174. Polynomial Time Computation in the Context of Recursive Analysis - ResearchGate, accessed March 30, 2025,  
[http://www.researchgate.net/profile/Walid\\_Gomaa/publication/227193071\\_Polynomial\\_Time\\_Computation\\_in\\_the\\_Context\\_of\\_Recursive\\_Analysis/links/0a85e539e969cb5958000000.pdf](http://www.researchgate.net/profile/Walid_Gomaa/publication/227193071_Polynomial_Time_Computation_in_the_Context_of_Recursive_Analysis/links/0a85e539e969cb5958000000.pdf)
175. Differentiability of polynomial time computable functions - DROPS, accessed March 30, 2025,  
<https://drops.dagstuhl.de/storage/00lipics/lipics-vol025-stacs2014/LIPIcs.STACS.2014.602/LIPIcs.STACS.2014.602.pdf>
176. Computable number - Wikipedia, accessed March 30, 2025,  
[https://en.wikipedia.org/wiki/Computable\\_number](https://en.wikipedia.org/wiki/Computable_number)
177. Online Encyclopedia of continuous and/or computable real valued functions?, accessed March 30, 2025,  
<https://math.stackexchange.com/questions/1841841/online-encyclopedia-of-continuous-and-or-computable-real-valued-functions>
178. Constructive analysis - Encyclopedia of Mathematics, accessed March 30, 2025, [https://encyclopediaofmath.org/wiki/Constructive\\_analysis](https://encyclopediaofmath.org/wiki/Constructive_analysis)
179. Computational complexity of zeros of an analytic function - MathOverflow, accessed March 30, 2025,  
<https://mathoverflow.net/questions/460326/computational-complexity-of-zeros-of-an-analytic-function>
180.  $e$  is transcendental, accessed March 30, 2025,  
<https://math.colorado.edu/~rohi1040/expository/eistranscendental.pdf>
181. arXiv:1805.02572v1 [cs.CC] 7 May 2018, accessed March 30, 2025,  
<https://arxiv.org/pdf/1805.02572>
182. A Short History of Computational Complexity - Lance Fortnow, accessed March 30, 2025, <https://lance.fortnow.com/papers/files/history.pdf>
183. SAT Encoding: An Introduction - Venkatesh-Prasad Ranganath - Medium, accessed March 30, 2025,  
<https://rvprasad.medium.com/sat-encoding-an-introduction-44d23049ab2a>
184. Encoding Problems in Boolean Satisfiability | Constraint Satisfaction and Optimization, accessed March 30, 2025,  
<https://ozanerdem.github.io/jekyll/update/2019/11/17/representation-in-sat.html>
185. computational complexity - Number theory and NP-complete - MathOverflow, accessed March 30, 2025,  
<https://mathoverflow.net/questions/72628/number-theory-and-np-complete>
186. NP-complete Problems and Physical Reality - Scott Aaronson, accessed March 30, 2025, <https://www.scottaaronson.com/papers/npcomplete.pdf>
187. An Approximated QUBO Formulation for Solving Practical SAT Problems - Qeios, accessed March 30, 2025, <https://www.qeios.com/read/JD6K4G>
188. Encoding Basic Arithmetic Operations for SAT-Solvers, accessed March 30,

- 2025,  
[https://yurichev.com/mirrors/SAT\\_factor/Encoding%20Basic%20Arithmetic%20Operations%20for%20SAT-Solvers.pdf](https://yurichev.com/mirrors/SAT_factor/Encoding%20Basic%20Arithmetic%20Operations%20for%20SAT-Solvers.pdf)
189. cc.complexity theory - Reducing P vs. NP to SAT, accessed March 30, 2025,  
<https://cstheory.stackexchange.com/questions/2247/reducing-p-vs-np-to-sat>
190. testbook.com, accessed March 30, 2025,  
<https://testbook.com/en-us/mathematics/complex-numbers#:~:text=Complex%20numbers%20are%20represented%20by,is%20the%20value%20'b'.>
191. Proof Complexity and SAT Solving - UCSD Math, accessed March 30, 2025,  
<https://www.math.ucsd.edu/~sbuss//ResearchWeb/ProofComplexitySAT/ProofComplexityChapter.pdf>
192. A Tricky Transcendental Equation - YouTube, accessed March 30, 2025,  
<https://m.youtube.com/watch?v=ubilaDckjDI&pp=ygUJI2VxdWI0aW9u>
193. Hey guys! Please help me understand what is a "transcendental" number? : r/learnmath - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/learnmath/comments/9rs0tj/hey\\_guys\\_please\\_help\\_me\\_understand\\_what\\_is\\_a/](https://www.reddit.com/r/learnmath/comments/9rs0tj/hey_guys_please_help_me_understand_what_is_a/)
194. Transcendental Numbers - Varsity Tutors, accessed March 30, 2025,  
[https://www.varsitytutors.com/hotmath/hotmath\\_help/topics/transcendental-numbers](https://www.varsitytutors.com/hotmath/hotmath_help/topics/transcendental-numbers)
195. why are transcendental numbers not the solution to any algebraic equation? - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/learnmath/comments/y5p7yl/why\\_are\\_transcendental\\_numbers\\_not\\_the\\_solution/](https://www.reddit.com/r/learnmath/comments/y5p7yl/why_are_transcendental_numbers_not_the_solution/)
196. Why Believe That  $P=NP$  Is Impossible?, accessed March 30, 2025,  
<https://rjlipton.com/2009/09/18/why-believe-that-pnp-is-impossible/>
197. Everything about P vs. NP : r/math - Reddit, accessed March 30, 2025,  
[https://www.reddit.com/r/math/comments/2x4rba/everything\\_about\\_p\\_vs\\_np/](https://www.reddit.com/r/math/comments/2x4rba/everything_about_p_vs_np/)