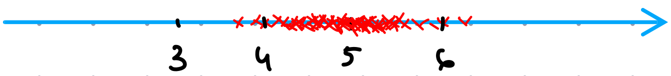
1 a)



$\Rightarrow \sim 50$

b) Ergebnisse:

$$D = \begin{pmatrix} 3{,}8, & 5{,}4, & 6{,}0, & 5{,}3, & 6{,}0, \\ 6{,}7, & 6{,}3, & 5{,}7, & 4{,}6, & 5{,}9, \\ 5{,}4, & 5{,}7, & 5{,}7, & 5{,}4, & 6{,}6, \\ 5{,}9, & 4{,}8, & 4{,}8, & 5{,}4, & 5{,}4, \\ 5{,}2, & 5{,}3, & 5{,}3, & 6{,}3, & 5{,}3, \\ 5{,}7, & 5{,}5, & 5{,}2, & 5{,}4, & 4{,}0, \\ 4{,}8, & 5{,}2, & 4{,}5, & 4{,}9, & 5{,}0, \\ 4{,}9, & 4{,}7, & 4{,}9, & 5{,}1, & 5{,}1, \\ 5{,}0, & 5{,}6, & 5{,}5, & 5{,}2, & 5{,}4, \\ 5{,}2, & 5{,}5, & 5{,}1, & 5{,}7, & 5{,}2 \end{pmatrix}$$

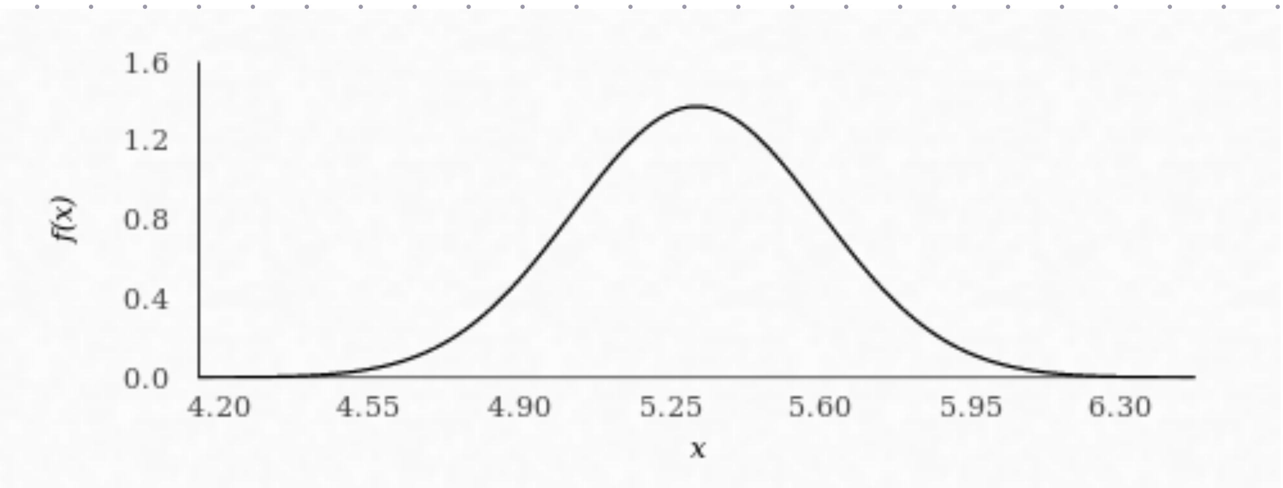
$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$\text{MLE } \hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} x_n = \frac{265{,}6}{50} = 5{,}312$$

$$\text{MLE } \hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - \hat{\mu})^2 = 0{,}289856$$

$$\Rightarrow X \sim \mathcal{N}(5{,}312,\ 0{,}289856)$$



c) ω_1: $\mu_1 = 5{,}312$ $\sigma_1^2 = 0{,}289856 \Rightarrow \sigma_1 = 0{,}538382763$

ω_2: $\mu_2 = 5{,}5$ $\sigma_2^2 = 1 \Rightarrow \sigma_2 = 1$

Ergebnis: $x = 5{,}3$

$P(\omega_1) = 0{,}3$

$P(\omega_2) = 0{,}7$

$$p(x|\omega_1) = \frac{1}{\sqrt{2\pi}\, 0{,}538382763} \exp\left[-\frac{1}{2}\left(\frac{x - 5{,}312}{0{,}538382763}\right)^2\right]$$

$$p(x|\omega_2) = \frac{1}{\sqrt{2\pi} \cdot 1} \exp\left[-\frac{1}{2}\left(\frac{x - 5{,}5}{1}\right)^2\right]$$

$p(5{,}3|\omega_1) = 0{,}740817$

$p(5{,}3|\omega_2) = 0{,}391043$

$$P(\omega_1|5{,}3) = \frac{p(5{,}3|\omega_1)\, 0{,}3}{p(5{,}3|\omega_1)\, 0{,}3 + p(5{,}3|\omega_2)\, 0{,}7}$$

$$= \frac{0{,}740817 \cdot 0{,}3}{0{,}740817 \cdot 0{,}3 + 0{,}391043 \cdot 0{,}7} = 0{,}448097$$

$p(\omega_2|5{,}3) =$ analog $= 0{,}551903$

$\Rightarrow$ Ergebnis passt zu ω_2.

(2) a) $P(\omega_1) = \frac{3}{5}$

$P(\omega_2) = \frac{2}{5}$

$p(x|\omega_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x+1)^2}{2}}$

$p(x|\omega_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-3)^2}{2}}$

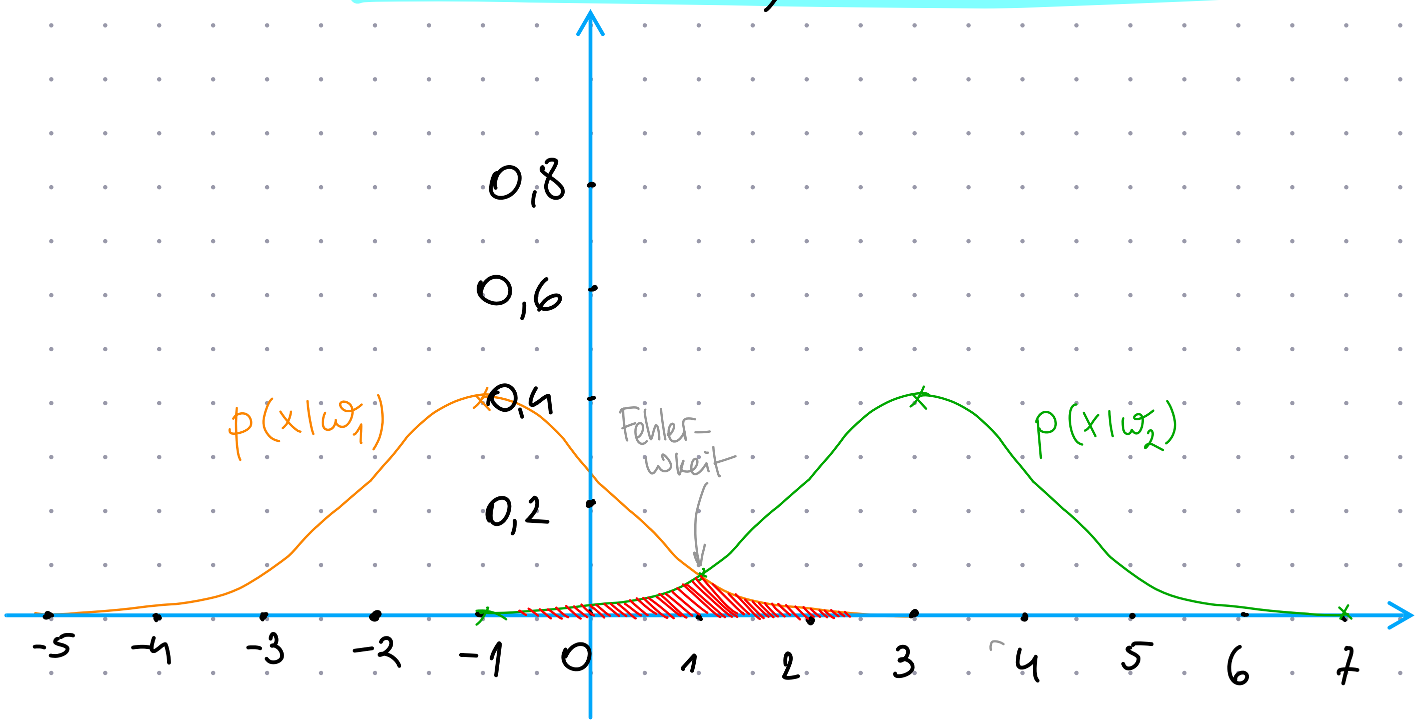
$P(\text{error}|x) = \begin{cases} P(\omega_1|x), & \text{falls entschieden für } \omega_2 \\ P(\omega_2|x), & \text{falls entschieden für } \omega_1 \end{cases}$

$$\rightarrow p(\text{error}) = p(\omega_1 \cap x \in R_2) + p(\omega_2 \cap x \in R_1)$$

$$= \int_{R_2} p(x|\omega_1) \cdot p(\omega_1)\,dx + \int_{R_1} p(x|\omega_2) \cdot p(\omega_2)\,dx$$

$$= \int_{\theta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x+1)^2}{2}\right) \cdot \frac{3}{5}\,dx + \int_{-\infty}^{\theta} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-3)^2}{2}\right) \cdot \frac{2}{5}\,dx$$

$$= \frac{3}{5} \int_{\theta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x+1)^2}{2}\right) dx + \frac{2}{5} \int_{-\infty}^{\theta} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-3)^2}{2}\right) dx$$



b) $p(x|\omega_1) \cdot p(\omega_1) = p(x|\omega_2) \cdot p(\omega_2)$

$$\frac{1}{\cancel{\sqrt{2\pi}}} \exp\left(-\frac{(x+1)^2}{2}\right) \cdot \frac{3}{\cancel{5}} = \frac{1}{\cancel{\sqrt{2\pi}}} \exp\left(-\frac{(x-3)^2}{2}\right) \cdot \frac{2}{\cancel{5}} \quad \Big| \cdot \frac{5 \cdot \sqrt{2\pi}}{2}$$

$$\frac{3}{2} \exp\left(-\frac{(x+1)^2}{2}\right) = \exp\left(-\frac{(x-3)^2}{2}\right) \quad \Big| / \exp\left(-\frac{(x+1)^2}{2}\right)$$

$$\frac{3}{2} = \frac{\exp\left(-\frac{(x-3)^2}{2}\right)}{\exp\left(-\frac{(x+1)^2}{2}\right)}$$

$$\frac{3}{2} = \exp\left(\frac{-(x-3)^2 + (x+1)^2}{2}\right)$$

$$\frac{3}{2} = \exp\left(\frac{-(x^2-6x+9) + x^2 + 2x + 1}{2}\right)$$

$$\frac{3}{2} = \exp\left(\frac{\cancel{-x^2} + 6x - 9 \cancel{+x^2} + 2x + 1}{2}\right)$$

$$\frac{3}{2} = \exp\left(\frac{8x-8}{2}\right) \quad \Big| \ln$$

$$\ln\left(\frac{3}{2}\right) = \frac{8x-8}{2}$$

$$x = \frac{2 \cdot \ln\left(\frac{3}{2}\right) + 8}{8}$$

$$x = 1{,}101366277 \Rightarrow \text{(ii)}\ 1{,}101$$

$$c)\ \underbrace{\frac{3}{5}\int_{1,101}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x+1)^2}{2}\right)dx}_{m(x)} + \underbrace{\frac{2}{5}\int_{-\infty}^{1,101}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x-3)^2}{2}\right)dx}_{h(x)}$$

(!) μ muss 0 werden

$$m(x) = \int_{1,101}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x+1)^2}{2}\right)dx\ ;\ u = -(x+1) \Rightarrow du = -dx$$

$$= -\frac{1}{\sqrt{2\pi}}\int_{-2,101}^{-\infty}\exp\left(-\frac{u^2}{2}\right)du$$

$$= +\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-2,101}\exp\left(-\frac{u^2}{2}\right)du = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{2,101}\exp\left(-\frac{u^2}{2}\right)du$$

$$= 1 - \Phi(2,101) = 1 - 0,98214 = 0,01786$$

$$h(x) = \int_{-\infty}^{1,101}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x-3)^2}{2}\right)dx\ ;\ u = x-3 \Rightarrow du = dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-1,899}\exp\left(-\frac{u^2}{2}\right)du$$

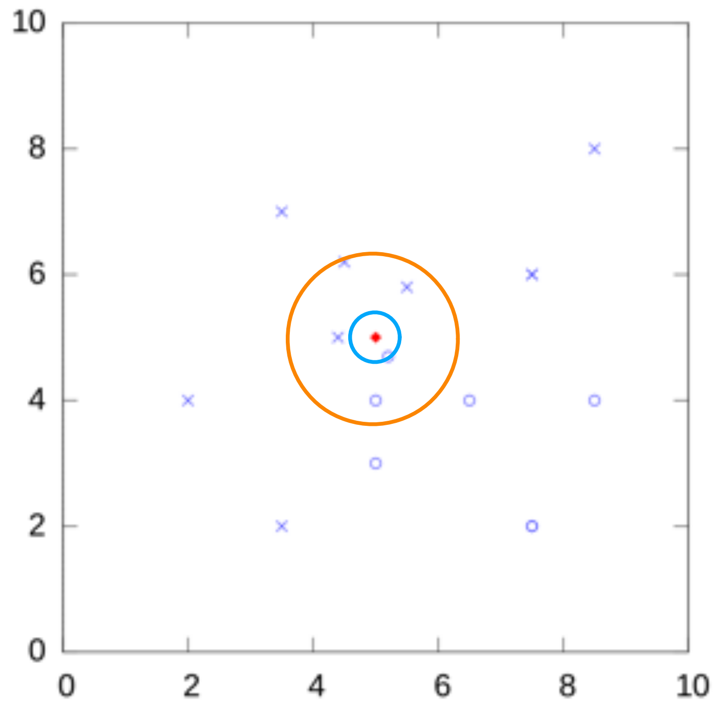
$$= 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{1,895}\exp\left(-\frac{u^2}{2}\right)du$$

$$= 1 - \Phi(1,899) = 1 - 0,97062 = 0,02938$$

$$P_{\text{Min-Fehler}} = \frac{3}{5}\cdot 0,01786 + \frac{2}{5}\cdot 0,02938 = \underline{0,022468}$$

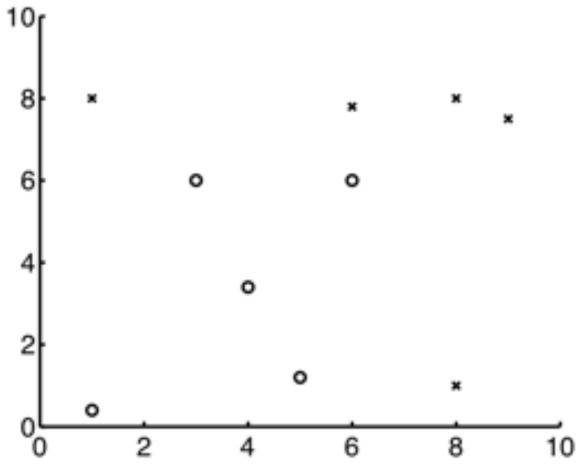
d) • Verteilung keine Gauß-Verteilung?
• Parameter der Gauß-Verteilung falsch geschätzt?
• Daten reichen nicht aus?
• Fehlerwahrscheinlichkeit ist nur eine Schätzung, die der realen Verteilung nicht genau entsprechen muss.

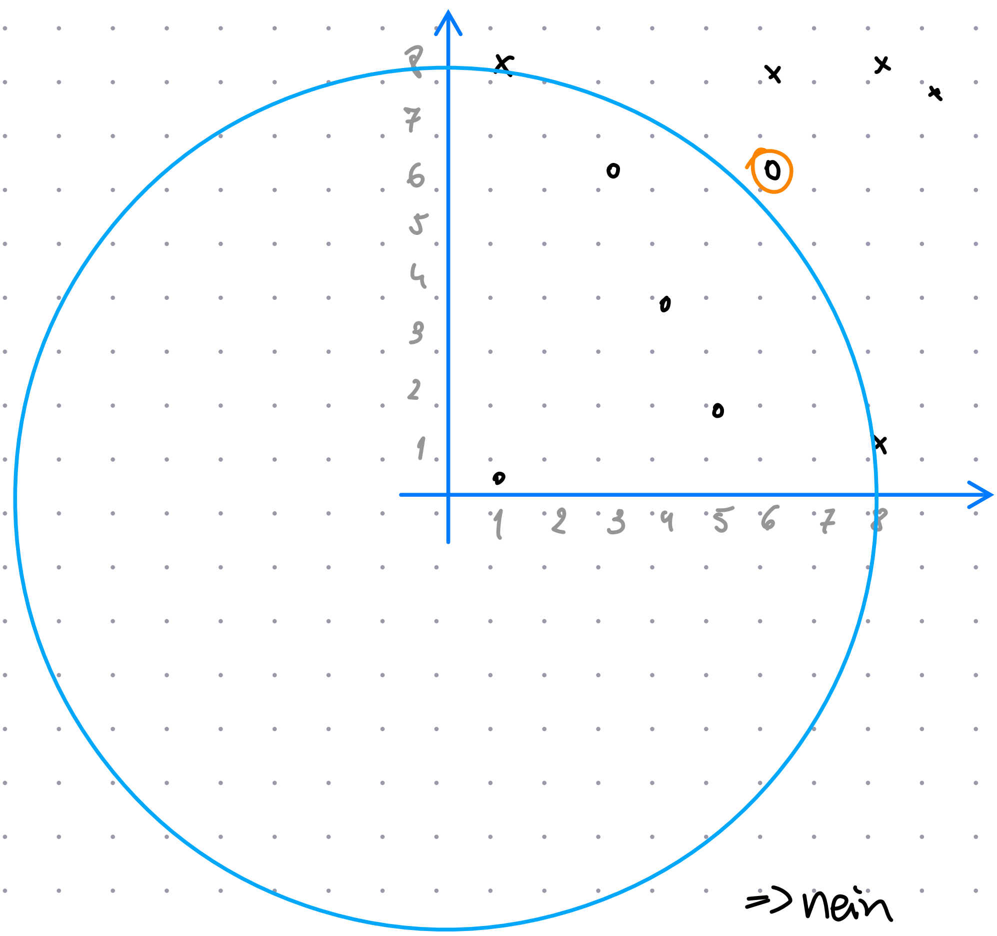
3) kNN, $k=1$: 1o => klassifiziere „o"
, $k=5$: 2o, 3x => klassifiziere „x"



Online-Frage Nr. 2: (iii) Kreuz

④ Online-Frage Nr. 3: (iii), (vi)

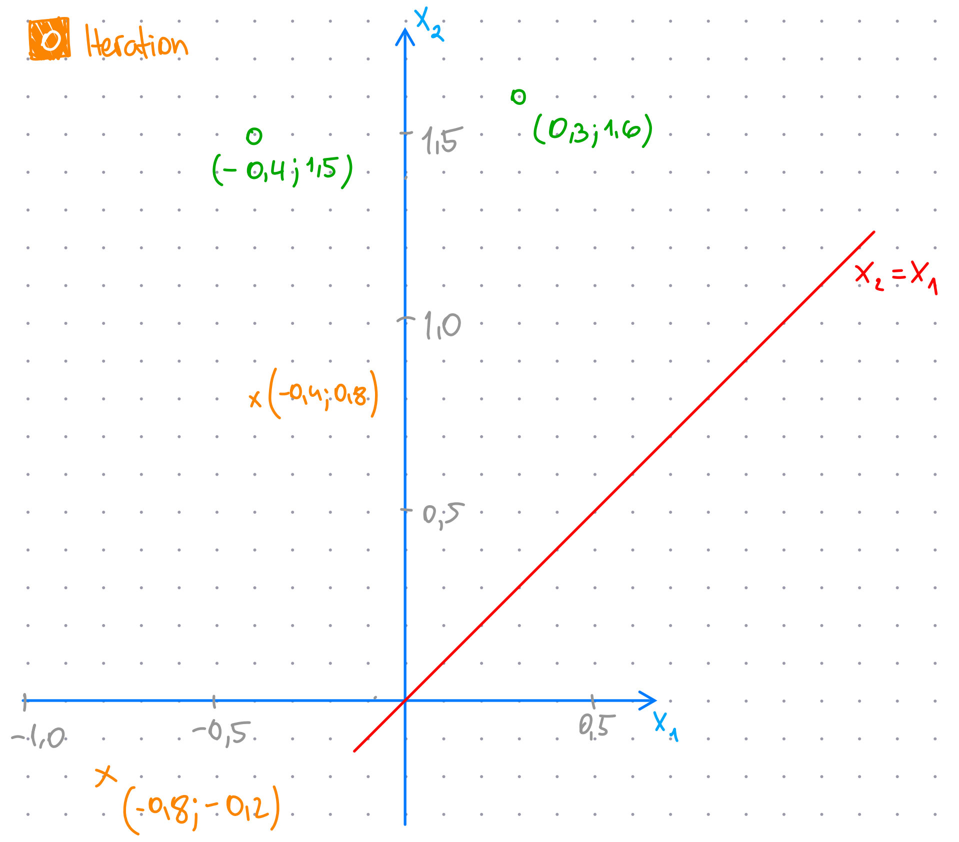




=> nein

5

0 Iteration



$\forall x \in A;\quad \forall o \in B$

$g(x) = \sum_{i=1}^{n} \omega_i x_i + b_0 = 0$

$\Rightarrow \quad x_1 - x_2 = 0$

$x_1 = x_2$

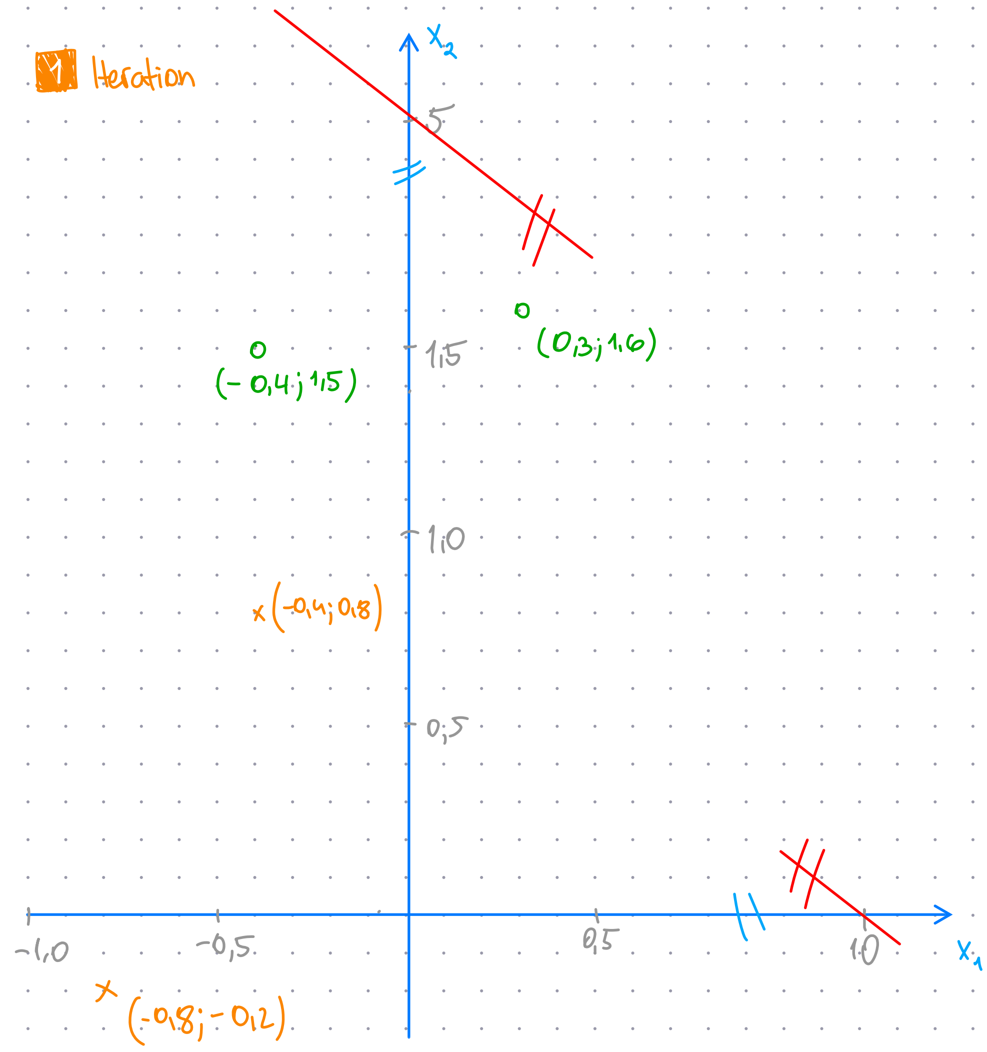
Korrekt für: $g(x) > 0 \Rightarrow A$; $g(x) < 0 \Rightarrow B$

$g((-0{,}4; 0{,}8)) = -1{,}2 < 0$ ↯ $\qquad g((-0{,}8; -0{,}2)) = -1 < 0$ ↯

$g((-0{,}4; 1{,}5)) = -1{,}9 < 0$ ✓ $\qquad g((0{,}3; 1{,}6)) = -1{,}3 < 0$ ✓

$E_0 = \left\{ \begin{pmatrix} -0{,}4 \\ 0{,}8 \end{pmatrix}; \begin{pmatrix} -0{,}8 \\ -0{,}2 \end{pmatrix} \right\}$

1 Iteration



$$\begin{pmatrix} \vec{\omega}_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} \vec{\omega}_0 \\ b_0 \end{pmatrix} - \sum_{\vec{e} \in E} \left[\delta_{\vec{e}} \cdot \begin{pmatrix} \vec{e} \\ 1 \end{pmatrix} \right] =$$

$$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} - \underbrace{\left[\begin{pmatrix} 0{,}4 + 0{,}8 \\ -0{,}8 + 0{,}2 \\ -1 - 1 \end{pmatrix} \right]}_{\begin{pmatrix} 1{,}2 \\ -0{,}6 \\ 2 \end{pmatrix}} = \begin{pmatrix} -0{,}2 \\ -0{,}4 \\ +2 \end{pmatrix}$$

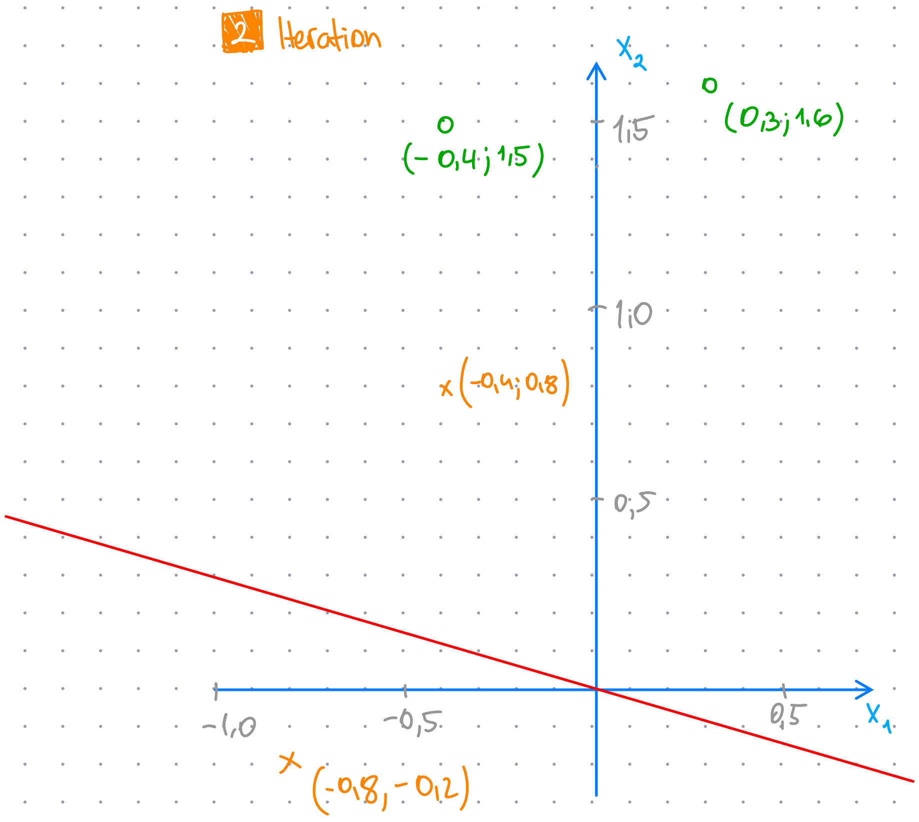
$g_1(x) = -0{,}2x_1 - 0{,}4x_2 + 2$; $x_2 = \dfrac{0{,}2x_1 - 2}{-0{,}4}$

$g((-0{,}4; 0{,}8)) = 1{,}76 > 0$ ✓ $\qquad g((-0{,}8; -0{,}2)) = 2{,}24 > 0$ ✓

$g((-0{,}4; 1{,}5)) = 1{,}32 > 0$ ↯ $\qquad g((0{,}3; 1{,}6)) = 1{,}3 > 0$ ↯

$E_1 = \left\{ \begin{pmatrix} -0{,}4 \\ 1{,}5 \end{pmatrix}; \begin{pmatrix} 0{,}3 \\ 1{,}6 \end{pmatrix} \right\}$

2 Iteration



$$\begin{pmatrix} \vec{\omega}_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} -0{,}2 \\ -0{,}4 \\ +2 \end{pmatrix} - \begin{pmatrix} -0{,}1 \\ 3{,}1 \\ 2 \end{pmatrix} = \begin{pmatrix} -0{,}1 \\ -3{,}5 \\ 0 \end{pmatrix}$$

$g_2(x) = -0{,}1x_1 - 3{,}5x_2$

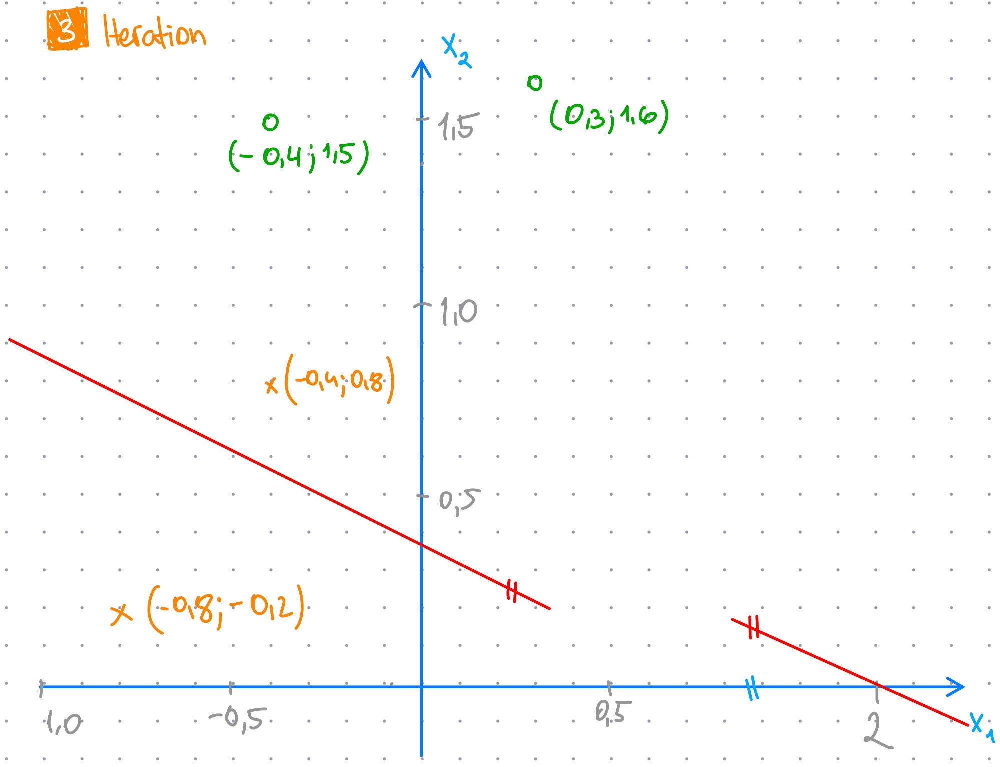
$x_2 = \dfrac{0{,}1x_1}{-3{,}5}$

$g((-0{,}4; 0{,}8)) = -2{,}76 < 0$ ↯ $\qquad g((-0{,}8; -0{,}2)) = 0{,}62 > 0$ ✓

$g((-0{,}4; 1{,}5)) = -5{,}21 < 0$ ✓ $\qquad g((0{,}3; 1{,}6)) = -5{,}63 < 0$ ✓

$E_2 = \left\{ \begin{pmatrix} -0{,}4 \\ 0{,}8 \end{pmatrix} \right\}$

3 Iteration



$$\begin{pmatrix} \vec{w}_3 \\ b_3 \end{pmatrix} = \begin{pmatrix} -0{,}1 \\ -3{,}5 \\ 0 \end{pmatrix} - \begin{pmatrix} 0{,}4 \\ -0{,}8 \\ -1 \end{pmatrix} = \begin{pmatrix} -0{,}5 \\ -2{,}7 \\ 1 \end{pmatrix}$$

$$g_3(x) = -0{,}5x_1 - 2{,}7x_2 + 1$$

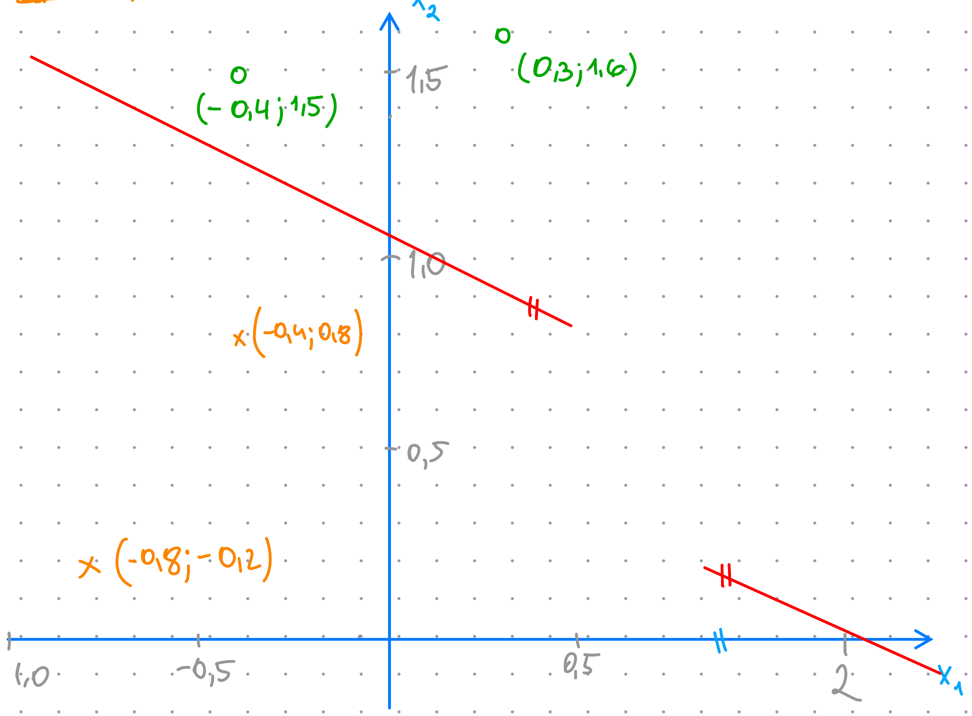
$$x_2 = \frac{0{,}5x_1 - 1}{-2{,}7}$$

$g((-0{,}4;0{,}8)) = -0{,}96 < 0$ ↯ $\qquad g((-0{,}8;-0{,}2)) = 1{,}94 > 0$ ✓

$g((-0{,}4;1{,}5)) = -2{,}85 < 0$ ✓ $\qquad g((0{,}3;1{,}6)) = -3{,}47 < 0$ ✓

$$E_3 = \left\{ \begin{pmatrix} -0{,}4 \\ 0{,}8 \end{pmatrix} \right\}$$

4 Iteration



$$\begin{pmatrix} \vec{w}_4 \\ b_4 \end{pmatrix} = \begin{pmatrix} -0{,}5 \\ -2{,}7 \\ 1 \end{pmatrix} - \begin{pmatrix} 0{,}4 \\ -0{,}8 \\ -1 \end{pmatrix} = \begin{pmatrix} -0{,}9 \\ -1{,}9 \\ 2 \end{pmatrix}$$

$$g_4(x) = -0{,}9x_1 - 1{,}9x_2 + 2$$

$$x_2 = \frac{0{,}9x_1 - 2}{-1{,}9}$$

$g((-0{,}4;0{,}8)) = 0{,}84 > 0$ ✓ $\qquad g((-0{,}8;-0{,}2)) = 2{,}34 > 0$ ✓

$g((-0{,}4;1{,}5)) = -0{,}49 < 0$ ✓ $\qquad g((0{,}3;1{,}6)) = -1{,}31 < 0$ ✓

$E_4 = \{\}$ ✓

Online-Frage Nr.4: (iii)