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Determining informative priors for cognitive models

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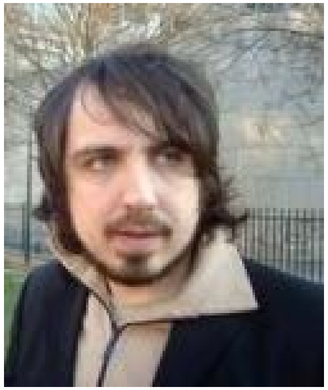

Wolf Vanpaemel

The Prevailing View

- The Bayesian approach to implementing cognitive models is unique in its requirement that a prior distribution over parameters be specified
  - Often this is seen as an unfortunate price to pay for all of the advantages of Bayesian methods (representing uncertainty, controlling complexity …)
  - The reaction of modelers is then often to try and minimize the impact of priors by making them vague, or flat, or weak, or non-informative
  - Critics of Bayesian approaches see priors as additional “researcher degrees of freedom” that influence the results, and highlight an undesirable subjectivity

Our View

- The Bayesian approach to implementing cognitive models is unique in its requirement that a prior distribution over parameters be specified
  - This is (yet another) major advantage of Bayesian methods, since prior distributions are a natural way to formalize modeling assumptions that can otherwise be hard to capture
  - Priors should be informative, which means that they should capture the relevant theoretical, logical, and empirical information about the psychological variables they represent
  - Critics of Bayesian approaches who see priors as additional “researcher degrees of freedom” do not understand what a model is, or how empirical science works

Feynman View of Science

- First you guess at how something works (build a model)
- Second, you compute the consequences (use the model to make predictions)
- Third you do an experiment to test the predictions, and if the predictions are wrong then the model is wrong

The Scientific Method-Richard Feynman

Making Predictions

- In a Bayesian setting, the model predictions are the prior predictive distribution
  - The expectations you have about the outcomes of the experiment, based on your prior and likelihood
- The prior and likelihood **together** make the model, and both have equal status as ways to formalize theory
  - Usually, the likelihood will formalize assumptions about the psychological processes that generate data (e.g., memory, learning, decision making processes…)
  - Usually, the priors will formalize assumptions about the psychological variables that control these processes (memory capacity, learning rate, decision strategy, …)

Predictions Come Before You See Data

- Our view is that both the original Nosofsky (1991) paper and the Farrell and Lewandowsky (2018) recount are wrong, in the same way "predicting" Germany to win the 2016 World Cup is not a prediction

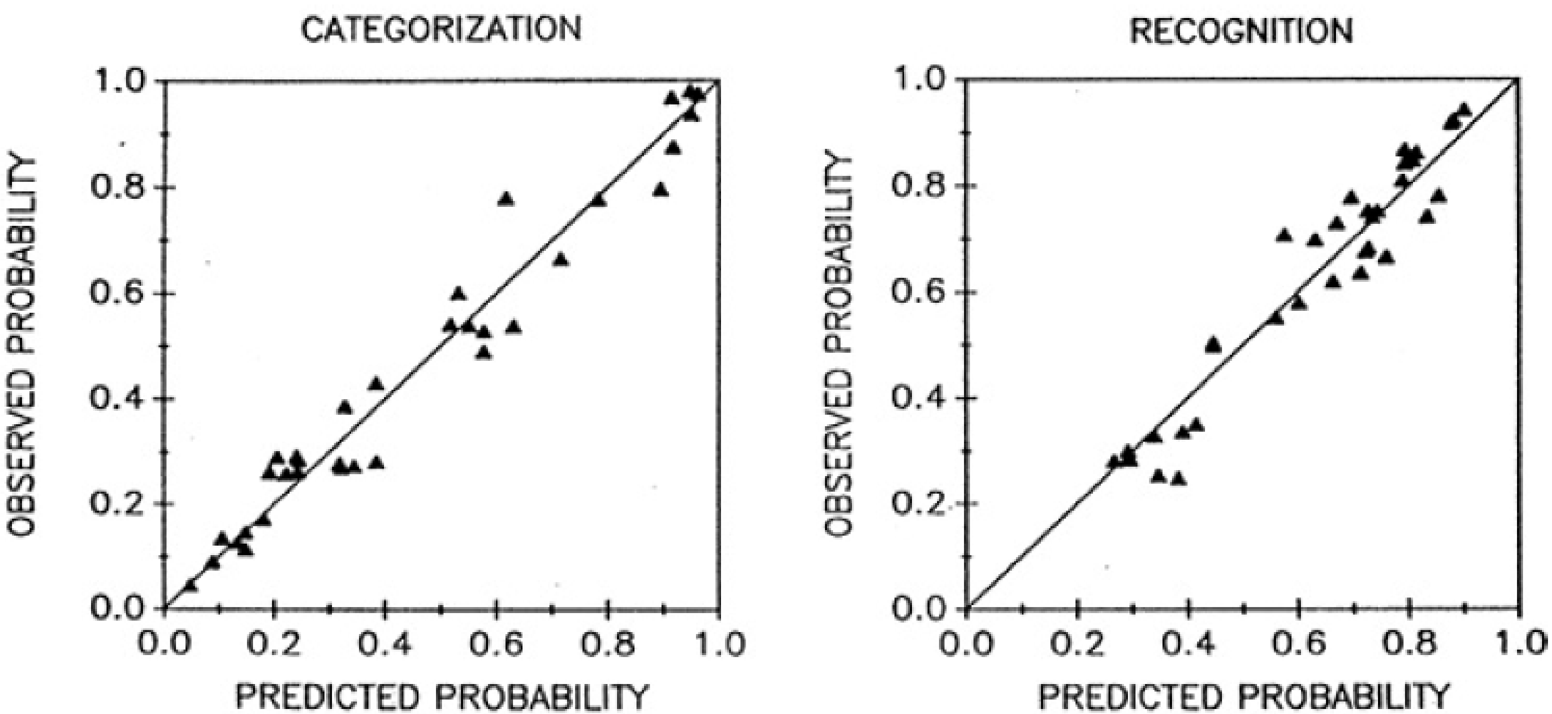

Figure 1.4 Observed and predicted classification (left panel) and recognition (right panel). Predictions are provided by the GCM; see text for details. Perfect prediction is represented by the diagonal lines. Figure reprinted from Nosofsky, R. M., Tests of an exemplar mode

Farrell and Lewandowsky (2018)

- The precision of predictions in each panel is remarkable: If the model's predictions were 100% perfect, then all points would fall on the diagonal. They do not, but they come close (accounting for 96% and 92% of the variance in classification and recognition, respectively). The fact that these accurate predictions were provided by the same model tells us that classification and recognition can be understood and related to each other within a common psychological theory. Thus, notwithstanding the low correlation between the two measures, there is an underlying model that explains how both tasks are related and permits accurate prediction of one response from knowledge of the other."

Farrell and Lewandowsky (2018)

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Building Models is a Creative Process

- Determining a prior is subjective in the same way as determining a likelihood as subjective (i.e., unavoidably and non-problematically)
  - "The difference between a fact and an opinion for purposes of decision making and inference is that when I use opinions, I get uncomfortable. I am not too uncomfortable with the opinion that error terms are normally distributed because most econometricians make use of that assumption. This observation has deluded me into thinking that the opinion that error terms are normal may be a fact, when I know deep inside that normal distributions are actually used only for convenience. In contrast, I am *quite* uncomfortable using a prior distribution, mostly I suspect because hardly anyone uses them. If convenient prior distributions were used as often as convenient sampling distributions, I suspect that I could be as easily deluded into thinking that prior distributions are facts as I have been into thinking that sampling distributions are facts." (Leamer, 1983)

Sources for Determining Priors

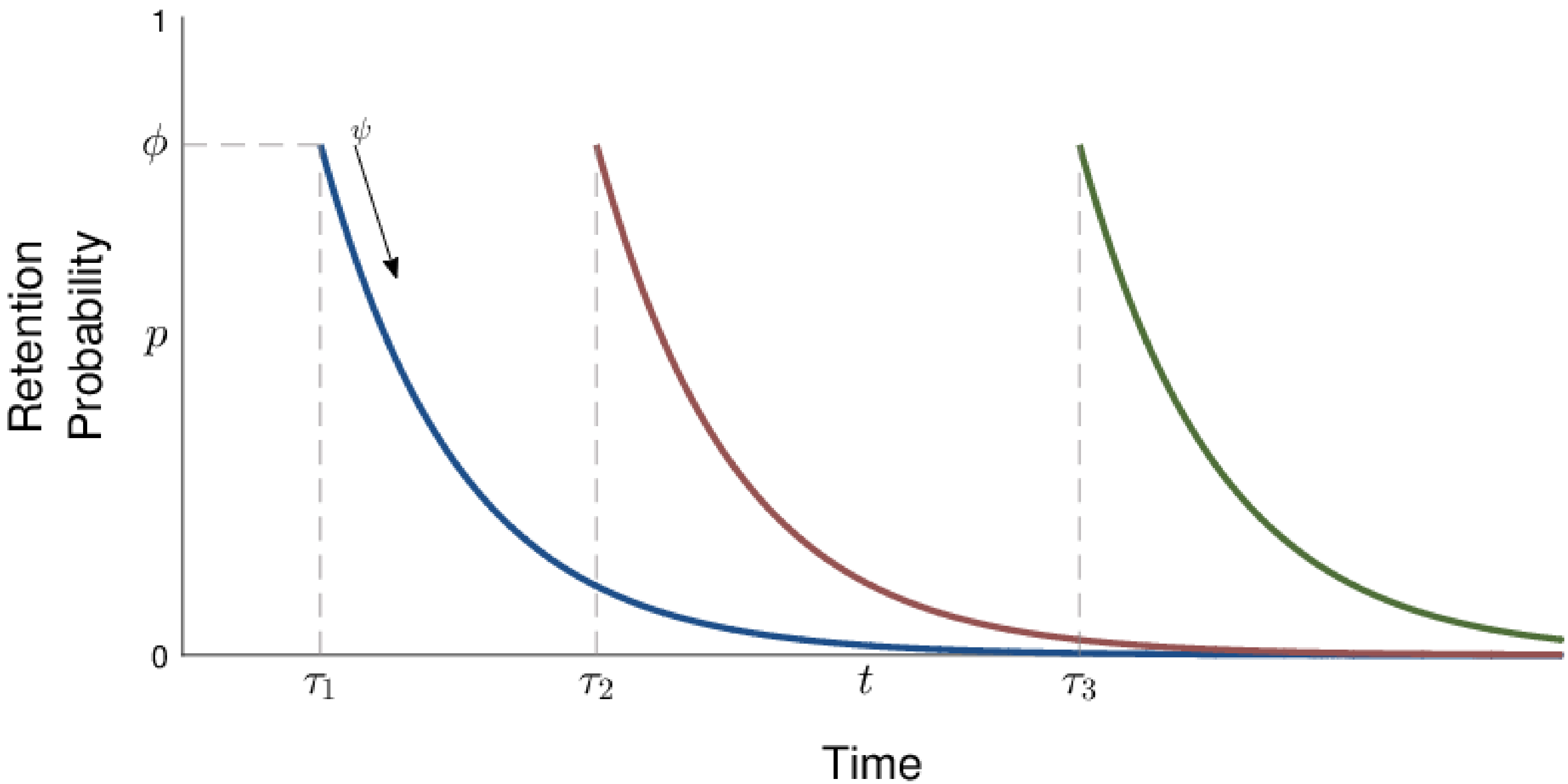
- Psychological and other scientific theory
  - Theories like working memory capacity, population IQ distributions, but also from biology, physics, …
- Logic and invariance
  - Probabilities lie between 0 and 1, scale parameters are positive, inferences should not depend upon the scale of measurement
- Previous data and modeling
  - Use parameters inferred from previous relevant data to determine a plausible range for priors on the same or similar parameters
- Elicitation
  - There are various experimental procedures for inferring prior from experts

Methods for Determining Priors

- Constraint satisfaction
  - Use the logic inherent in the modeling problem and data to determine constraints that the prior must satisfy
    - Maximum entropy principles are one interested and under-explored way of doing this
- Prior prediction
  - Using the Feynman philosophy, priors can be justified by making predictions that are the ones the modeler wants their model to make
- Hierarchical extension
  - Build a model of how the parameters themselves are psychologically generated, and this additional theory will help define a prior on the parameters

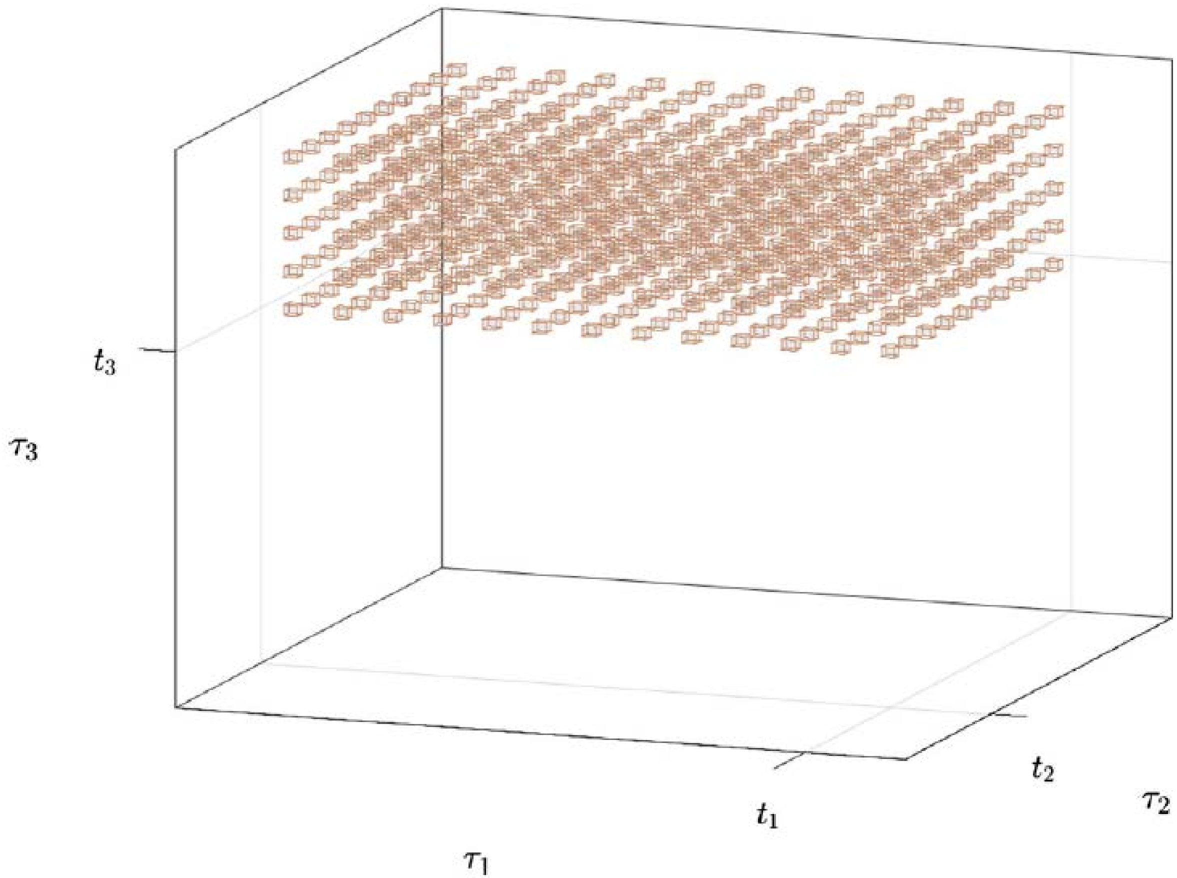
Constraint Satisfaction

- A simple memory decay model that assumes there are latent rehearsals of items



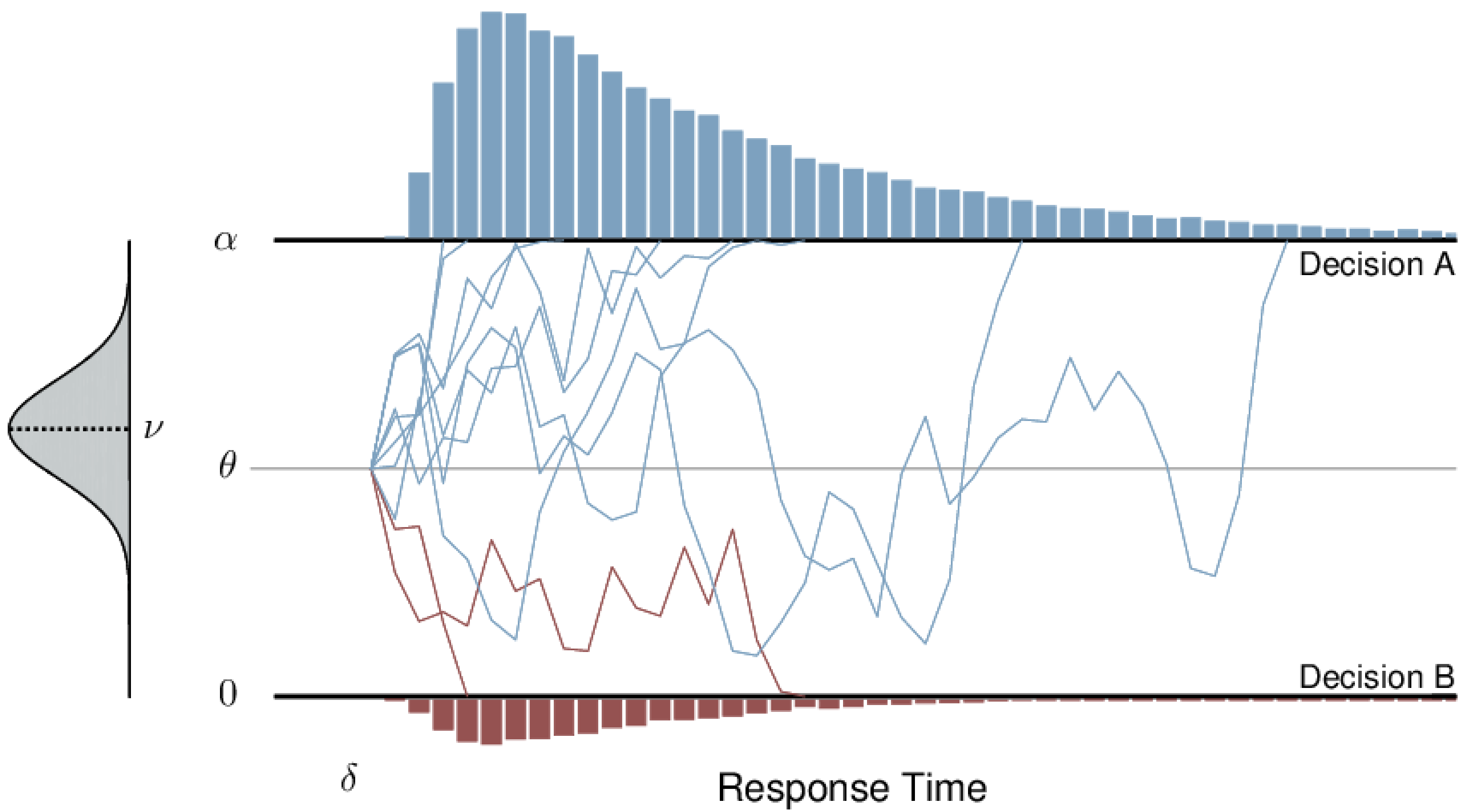
Constraint Satisfaction

- The obvious (physical) constraint that an item cannot be rehearsed before it is presented simplifies the model by constraining the prior distribution



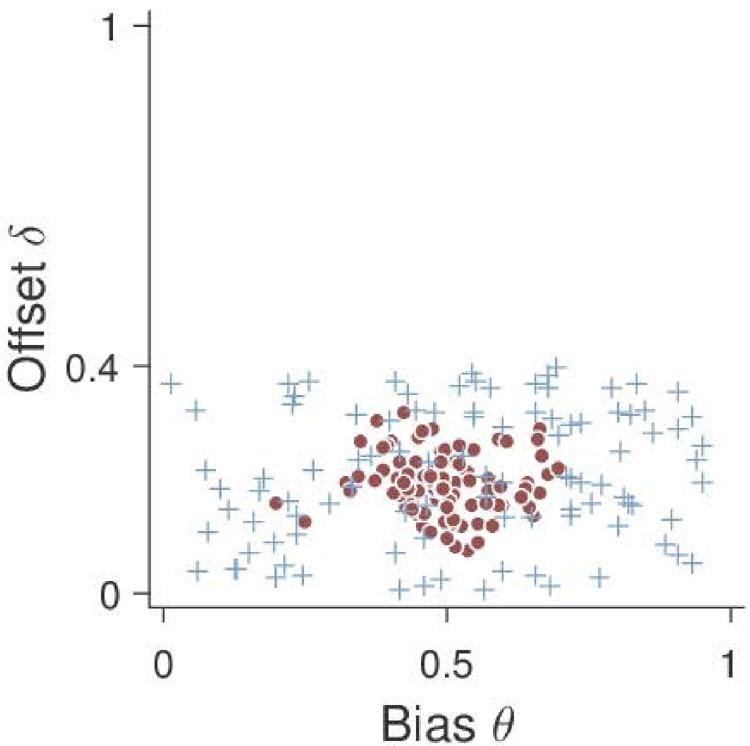
Prior Prediction

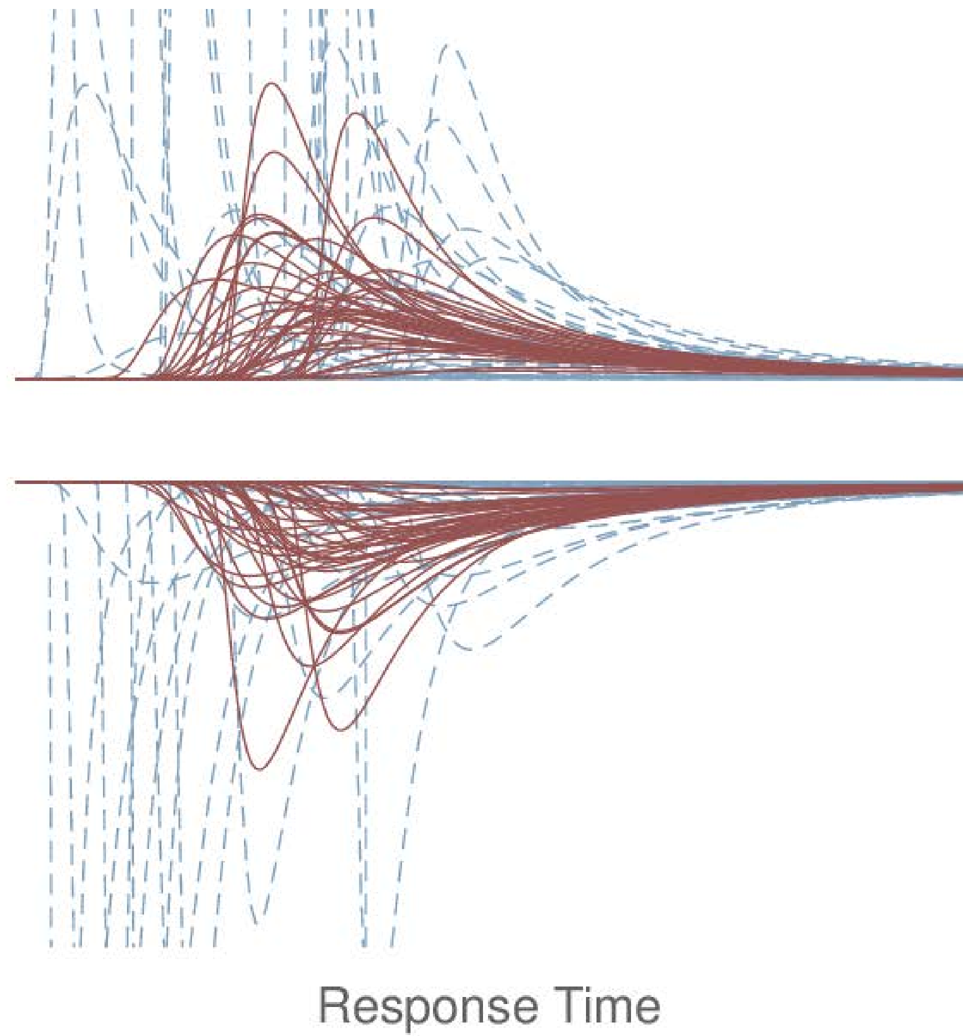
- A basic diffusion model with bias, offset, drift rate and boundary separation parameters



Prior Prediction

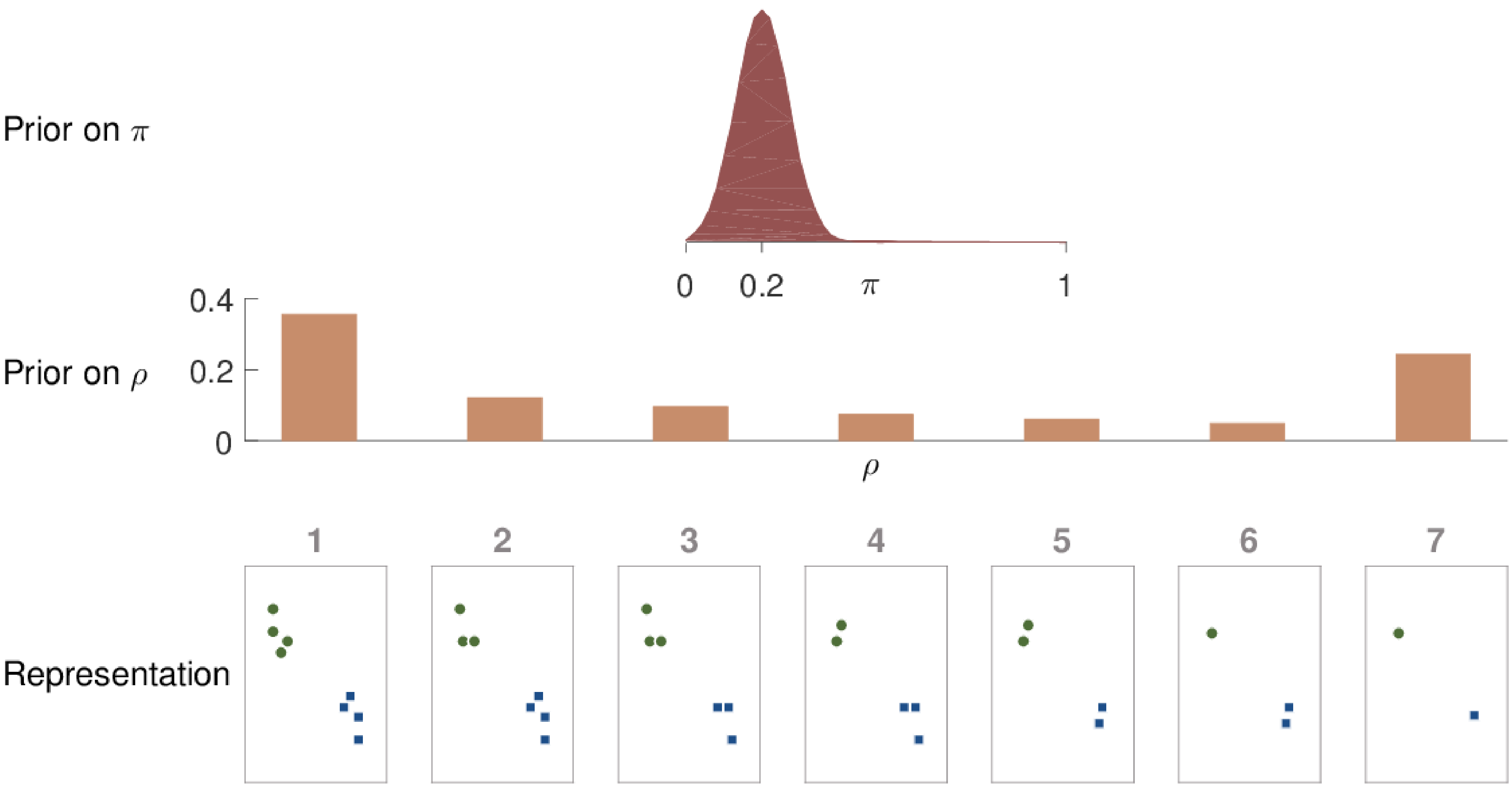
- Much more sensible prior prediction arises from considering more plausible bias and offset parameter priors





Hierarchical Extension

- A model for how people summarize stimuli by merging unifies exemplar and prototype representations, and automatically puts a prior on different category representations



Benefits of Informative Priors

- Help solve modeling problems
  - Make a model statistically identified (e.g., by placing order constraints on mixture components)
  - Make a model theoretically clear (e.g., when a parameter has multiple possible interpretations, and the prior makes clear which one is intended)
- Make a model simpler
  - Adding more theoretical content makes a model more precise, testable, and falsifiable because it makes stronger claims and narrower predictions
    - Response determinism example $p_{iA} = \beta_A s_{iA}^{\gamma} / \sum_C \beta_C s_{iC}^{\gamma}$
- Make a model more complete
  - Priors allow more theoretical assumptions to be captured formally within the model