Consider the following model of a bioreactor in continuous operation mode

$$\dot{b} = -\frac{q_{in}}{V} b + \mu(s) b \tag{1a}$$

$$\dot{s} = \frac{q_{in}}{V}(s_{in} - s) + \mu(s) b \tag{1b}$$

$$\dot{V} = q_{in} - q_{out} \tag{1c}$$

with biomass concentration b, substrate concentration s, volume V, volumetric feed flow rate q_{in}, and extraction flow rate q_{out}, feed concentration s_{in}, specific growth rate

$$\mu(s) = \mu_{\max} \frac{s}{s^2/k_i + k_s + s} \tag{1d}$$

and the following set of parameters

$$s_{in} = 2, \quad \mu_{\max} = 10, \quad k_s = 0.5, \quad k_i = 0.05. \tag{1e}$$

1. Perform numerical simulations for the following scenarios:
   - Initial conditions $b(0) = 0.01, s(0) = 0.5, V(0) = 0.5$ and two arbitrary feed and extraction flow rates q_{in}, q_{out}. Discuss the observed behavior shortly.
   - Initial conditions $b(0) = 0.5, s(0) = 1.4, V(0) = 1$, and $q_{in} = q_{out} = 0.4$.
   - Initial conditions $b(0) = 5, s(0) = 1.4, V(0) = 1$, and $q_{in} = q_{out} = 0.4$.
   - Initial conditions $b(0) = 5, s(0) = 1.4, V(0) = 1$, and $q_{in} = q_{out} = 1.4$.

   Discuss the different observations from these simulations.

2. Analyze the different types of bifurcations taking place for the reactor model regarding the feed flow rate for the case that $q_{in} = q_{out}$ and provide a solution diagram for s and for b in dependency of q_{in}. Discuss the simulation results from task 1 in the context of the obtained bifurcation analysis.

3. Assuming again that $q_{in} = q_{out}$, design an input-output linearizing state feedback control for the dilution rate in order to achieve a constant biomass production rate $P = q_{out} b$. Visualize the performance of the closed-loop system using numerical simulations, for the above mentioned initial conditions.