Homework 4

The Lane-Emden Equation of gravitational potential can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2 \frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right) = -\,\xi^2\theta^n.$$

Our goal today is to explore numerical solutions to this famous equation, and compare them to the "Standard Solar Model" we saw in class (BP2000).

You're welcome to solve the Lane-Emden Equation yourself (there are many tutorials and info online if you're interested in how to numerically solve such differential equations). However, Earl Bellinger at the Stellar Astrophysics Centre in Denmark made a handy Jupyter Notebook that solves the equation using a couple approaches:
https://github.com/earlbellinger/Lane-Emden/blob/master/Lane-Emden.ipynb

Part 1
(10pts)

Read the BP2000 model file in, and recreate the same sorts of plots we saw in the Feb 6 lecture as a function of **radius** (i.e. put radius on the x-axis). Create a plot for each of these as a function of radius:

1. Mass
2. Temperature
3. Pressure
4. Density
5. Mass Fractions (X and Y)

Be sure your plots are clearly labeled!

Part 2
(5pts)

Recreate the plot from the Feb 8 lecture showing a range of solutions for the Lane-Emden Equation. Solve it for values of `n=0,1,2,3,4,5.`

Your plot should be ξ on the x-axis, and θ on the y-axis. Clearly indicate which curve is which.

Hint 1: if using the Jupyter Notebook solution listed above, turn the $\log \Delta\xi$ resolution down to -2 (the default is -4).

Hint 2: if using the Jupyter Notebook solution listed above, you can just show `ycs` and ignore the `ys` output.

State in words: What are the variables ξ and θ? What is happening as we increase `n`?

Part 3

(15pts)

Now we want to combine these two models, the Lane-Emden Equation and the BP2000 solar structure model, to find what polytropic index best describes the Sun. (Note: we are only going to use integer values of *n* here!)

Create two plots as a function of solar radius: density relative to the core density (ρ/ρ_c), and pressure relative to the core pressure (P/P_c).

Find the Polytrope index (n) that best describes the Sun (either look around online - though please state your source if you take this approach - or find it experimentally), and overplot the Lane-Emden solution for both of these plots.

Hint 1: Recall the definition of dimensionless density (or temperature): $\rho = \rho_c\ \theta^n$, and the resulting polytropic pressure: $P = P_c\ \theta^{n+1}$

Hint 2: Recall the definition of the dimensionless radius: $r = \alpha\xi$. Rather than trying to solve for α (which you *CAN* do, but don't need to), recall the boundary conditions at the core ($\xi = 0$, $\theta = 1$) and the surface ($r = R_\odot$, $\theta = 0$). You can assume the maximum value of ξ produced in your Lane-Emden solver (i.e. the first time $\theta = 0$) is the "surface", and scale by that value.

Turn in your write-up, including the labeled plots, as a PDF. Remember to include an attribution for any group work! Also turn in your code or Jupyter Notebook used to solve the assignment. Note: we'd like to be able to run your code to check that it actually works, so be sure (if using Jupyter notebooks) to check that it runs "top down"!